# Stories from inside a magnet: solenoidal spectrometers



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### Solar system abundances of heavy elements

Beyond Fe, Ni  $\rightarrow$  neutron-capture reactions

- **s-process** (slow neutron-capture process)
- r-process (rapid neutron-capture process)







# Studies on fission of neutron-rich nuclei



### Direct kinematics reactions



- limited choice of targets material (only stable or close-to-stable nuclei);
- relatively low energies of FFs (Fission Fragments) result in difficulties; mostly for their Z identification.

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### Inverse kinematics using radioactive ion beams (RIBs)

The main focus is on the beam!



Projectile-like products  $\sim$  beam energy

Advantages

- Large kinematic **boost** in forward direction for fission fragments
- Study of fission barriers for very exotic nuclei.
- By measuring energy of the proton one can determine the excitation energy of the fissioning nucleus.

But...

# Inverse kinematics challenges

Typical experimental problems

- Strong angular dependence of proton energy on the LAB angle.
- Kinematic compression → much worse resolution in backward angles.
- Low intensity beams (detection efficiency).



beams. NIM A, J.S. Winfield. Neutron transfer reactions with radioactive 396(1-2):147 . 164, 1997

### An important difference



- Target inside the solenoid
- Fission fragments
- Proton follows helical trajectory and then is detected in a position-sensitive silicon array



# What do we get?



# An ideal spectrometer with a stationary source

Formula for magnetic rigidity:

$$B\rho = \frac{p_{xy}}{Q} \rightarrow \rho = \frac{p_{xy}}{QB} = \frac{p\sin\theta}{QB}$$

The radius can be also expressed as:

$$\rho = \frac{mv_{xy}}{QB}.$$

The cyclotron period:

$$T_{cyc} = \frac{2 \pi \rho}{v_{xy}} = \frac{2\pi}{B} \frac{m}{Q}.$$

Distance:  $z = v_z T_{cvc}$ 



https://physexams.com/blog/Motion-of-a-charged-particle-in-a uniform-magnetic-field\_13



### Finite size detector



- the projection of the particle trajectory onto the xy plane
- one of the detector planes
- $\phi_p$  the angle between the normal of a detector plane and the x-axis
- **a** the shortest distance between a detector plane and the center of the detector
- **ρ** the particle bending radius

The normal of the detector plane:

$$\hat{n} = \left(\cos\phi_p, \sin\phi_p, 0\right)$$

The equation of the locus of the + charged particle when the B-field is directed along the z-axis:

$$\binom{x}{y} = \binom{x_0 + \rho \cos \Delta \phi}{y_0 + \rho \sin \Delta \phi}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 + \rho \cos \Delta \phi \\ y_0 + \rho \sin \Delta \phi \end{pmatrix} \longrightarrow \begin{cases} v_z = v \cos \theta = \frac{z}{t} \\ v_{xy} = v \sin \theta = \omega \rho = \frac{\Delta \phi}{t} \rho \end{cases} \quad \Delta \phi = \tan \theta \cdot \frac{z}{\rho}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \rho \begin{pmatrix} -\sin \phi + \sin \left( \tan \theta \cdot \frac{z}{\rho} + \phi \right) \\ \cos \phi - \cos \left( \tan \theta \cdot \frac{z}{\rho} + \phi \right) \end{pmatrix}$$





$$\binom{x}{y} = \rho \begin{pmatrix} -\sin\phi + \sin\left(\tan\theta \cdot \frac{z}{\rho} + \phi\right) \\ \cos\phi - \cos\left(\tan\theta \cdot \frac{z}{\rho} + \phi\right) \end{pmatrix}$$

$$y = bx + c$$
  

$$b = \tan \alpha = \tan \left( \pi - \left(\frac{\pi}{2} - \phi_p\right) \right) = -\frac{1}{\tan \phi_p}$$
  

$$y = -\frac{\cos \phi_p}{\sin \phi_p} x + c$$
  

$$a = c \sin \phi_p$$
  

$$x \cos \phi_p + y \sin \phi_p = a$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \rho \begin{pmatrix} -\sin\phi + \sin\left(\tan\theta \cdot \frac{z}{\rho} + \phi\right) \\ \cos\phi - \cos\left(\tan\theta \cdot \frac{z}{\rho} + \phi\right) \end{pmatrix}$$
• The particle can cross the detector plane n times
• The hit-point is from outside -> the dot product of the direction vector with the detector plane normal is less than 0
$$\hat{x} \cos\phi_p + y \sin\phi_p = a$$

$$\hat{n} \cdot \frac{d}{dz} \begin{pmatrix} x \\ y \end{pmatrix} < 0$$

$$\tan \theta \cdot \frac{z_{hit}}{\rho} = \phi_p - \phi + \arcsin\left(\frac{a}{\rho} + \sin(\phi - \phi_p)\right)$$
for  $\phi = 0, \phi_p = \pi, n = 1$ 

$$z_{hit} = \frac{2\pi \rho}{\tan \theta} \left(1 - \frac{1}{2\pi} \arcsin\left(\frac{a}{\rho}\right)\right)$$

### We know:

- $m_a$ ,  $\bullet$
- $m_b$ , ۲
- kinetic energy of the projectile. •

### We measure:

- $E_p$ , •
- Z<sub>hit</sub>, •
- $T_{cyc}$ .

### We want:

- $\theta_{cm}$ , •
- $E_{\chi}$ . •

total energy $E_t$ in the CM frame or the mass of the system $M_c$ ( $E = mc^2$ in the CM frame appl	ics), q		
and Q as the total energy of particles 1 and 2 in the CM frame, respectively, one gets: $E = T + m \rightarrow T - E = m - \sqrt{m^2 + M^2}$		$e + m_1 = \gamma q - \gamma \beta^2 q + \alpha \beta z =$	(94)
$E_a = 1 + m_a \Rightarrow 1 = E_a - m_a = \sqrt{m_a^2 + k_a^2 - m_a}$ , $M_e^2 = E_t^2 = E_e^2 - k_a^2 = m_a^2 + k_a^2 + m_b^2 + 2m_b\sqrt{m_a^2 + k_a^2} - k_a^2 =$		$= \gamma q(1 - \beta^{*}) + \alpha \beta z = -\gamma q + \alpha \beta z,$	
$m_a^2 + m_b^2 + 2m_b\sqrt{m_a^2 + k_a^2} = m_a^2 + m_b^2 + 2m_b(T + m_a) =$		$e = \frac{1}{2}q - m_1 + \alpha\beta z =$	
$(m_a + m_b) + 2m_b I$ , $q = \sqrt{m_s^2 + k^2} = \frac{1}{m_s} (E_s^2 - m_s^2 + m_s^2)$ .		$\gamma = \frac{1}{-\pi^{10}} (M_e^2 + m_1^2 - m_2^2) - m_1 + \alpha\beta z.$	(95)
$Q = \sqrt{m^2 + k^2} = \frac{1}{(E^2 + m^2 - m^2)}$	(82)	$\gamma 2E_t^*$ Please note that it depends only on the excitation energy.	
$k^2 - \frac{1}{2E_t} \left( (E_t^2 - (m_0 + m_1)^2) (E_t^2 - (m_0 - m_1)^2) \right)$		The intercept with the kinetic energy axis is:	
$s = \frac{1}{4E_t^2} \left( (m_t - (m_2 + m_1)) (m_t - (m_2 - m_1)) \right),$ $k_n = \sqrt{(T + m_0)^2 - m_1^2}$		$c_0 = c _{t=0} = \frac{M_c^2 + m_1^2 - m_2^2}{2\gamma E_t} - m_1.$	(96)
$\beta = \frac{1}{E_e} = \frac{\sqrt{1 + m_b + T}}{m_a + m_b + T}, \qquad \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$		The only non-constant is $m_2$ , which can be excited. For small excitation energy $E_x \ll m_2$	2:
Let's calculate $p$ now. We start with:		$\frac{m_2^2}{2\gamma E_t} \rightarrow \frac{(m_2 + E_x)^2}{2\gamma E_t} \approx \frac{m_2^2}{2\gamma E_t} \left(1 + \frac{2E_x}{m_2}\right) = \frac{m_2^2}{2\gamma E_t} + \frac{m_2}{\gamma E_t} E_x.$	(97)
$\sin^2 \theta_{em} + \cos^2 \theta_{em} = 1.$	(83)	At small incident energy, $M_c = m_1 + m_2 + T_{cm} \approx m_2, \gamma \approx 1$ :	
From Eq. 81 we know that:		$c_0 \approx \frac{M_c^2 + m_1^2 - m_2^2}{2\gamma E_t} - m_1 - E_x.$	(98)
$p \sin \theta$		Now we eliminate $e_i$ so that:	
$k \sin \theta_{em} = p \sin \theta \Rightarrow \sin \theta_{em} = \frac{1}{k}$ , $\beta q = p \cos \theta$	(84)	$\cos \theta_{em} = \frac{\beta q}{k} - \frac{\alpha}{\gamma k} z.$	(99)
$\gamma p q - \gamma \kappa \cos \theta_{em} = p \cos \theta \implies \cos \theta_{em} = \frac{1}{k} - \frac{1}{\gamma k}$		This is the relationship between the center-of-mass angle and the z-position. The depen the excitation energy is inside the term $q$ .	dency on
$\left(\frac{p \sin \theta}{k}\right)^2 + \left(\frac{\beta q}{k} - \frac{p \cos \theta}{\gamma k}\right)^2 - 1 = 0,$		3.3.2 The constant $\theta_{rm}$ line	
$p^2\left(\sin^2\theta + \frac{\cos^2\theta}{\gamma^2}\right) - 2\beta qp \frac{\cos\theta}{\gamma} + \beta^2 q^2 - k^2 = 0,$		Next, we eliminate $m_2$ in Eq. 93	
$\tan \theta = \frac{\sin \theta}{\sin \theta} \Rightarrow \sin^2 \theta = \tan^2 \theta \cos^2 \theta,$	(85)		
$\frac{p^2 \cos^2 \theta}{r^2} (\gamma^2 \tan^2 \theta + 1) - 2\beta q p \frac{\cos \theta}{r^2} + (\beta^2 q^2 - k^2) = 0.$		$(\alpha z)^{-} = (\gamma \beta q - \gamma \kappa \cos \theta_{em})^{-},$ $(e + m_1)^2 = (\gamma q - \gamma \beta k \cos \theta_{em})^2,$	(100)
$\gamma^{*} = \gamma$ After solving this 2nd order equation in p:		$(-\frac{1}{4})$	(101)
$p = \frac{\frac{\gamma}{\cos\theta}}{\frac{1}{2}} \left(\beta q + \sqrt{k^2 + (k^2 - q^2\beta^2)\gamma^2 \tan^2\theta}\right),$	(86)	$A - C = = q^{-}(1 - \cos^{-}\theta_{cm}) + m_{1}^{-}\cos^{-}\theta_{cm},$	(101)
$1 + \gamma^2 \tan^2 \theta$ ( ) and [54] gives $\theta_{cm}$ :		$q = \gamma(e + m_1 - \alpha\beta z) = \gamma(\sqrt{A} - \beta\sqrt{C}),$ $\Downarrow$	(102)
$\tan \theta_{em} = \frac{\sin \theta_{em}}{a} = \frac{p \sin \theta}{a}$	(87)	$q^2 = \gamma^2 \left(A + \beta^2 C - 2\beta \sqrt{C} \sqrt{A}\right).$	()
$\rho q - \frac{1}{2} \cos \theta$ The basic formula for the curvature radius due to the presence of a magnetic field (Eq. [2]):		For more detailed calculations see App. A After inserting [157] into [101] and some rearran we get:	ngements
$\rho = \frac{p_{TB}}{2p}$ .	(88)		
ZB		$A(1 - \sin^2 \theta_{em} \gamma^2) + \sqrt{A}  2\gamma^2 \beta \sqrt{C} \sin^2 \theta_{em} - m_1^2 \cos^2 \theta_{em} - \gamma^2 \beta^2 C \sin^2 \theta_{em} - C = 0.$	(103)
15		17	
Under the kinematics of transfer reaction:		The solution for $\sqrt{3}$ is:	
Under the kinematics of transfer reaction: $a_{m} \frac{p_{T}}{p_{m}} \frac{k \sin \theta_{m}}{m}$	(89)	The solution for $\sqrt{A}$ is: $\sin^2 \theta_{} \approx \theta_{-}^2 + \cos \theta_{} \sqrt{\alpha_{+}^2 + 1 + \alpha_{-}^2 (1 - \sin^2 \theta_{} - 2)}$	
Under the kinematics of transfer reaction: $\rho = \frac{p_{T}}{ZB} = \frac{k \sin \theta_{em}}{ZB}.$ The time for the cycle is given by Eq. (3)	(59)	The solution for $\sqrt{A}$ is: $\sqrt{A} = \frac{\sin^2 \theta_{em} \alpha \beta \gamma^2 z + \cos \theta_{em} \sqrt{\alpha^2 z^2} + m_1^2 (1 - \sin^2 \theta_{em} \gamma^2)}{1 - 4 \sin^2 \theta_{em} \gamma^2}.$	(104)
Under the kinematics of transfer reaction: $\rho = \frac{p_{\rm H}}{ZB} = \frac{k \sin \theta_{\rm em}}{ZB}.$ The time for the cycle is given by Eq. (3) $T_{\rm eq} = \frac{2\pi p}{ZB} = \frac{2\pi k \sin \theta_{\rm em}}{ZB}.$	(89)	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{\sin^2 \theta_{em} \alpha \beta \gamma^2 z + \cos \theta_{em} \sqrt{\alpha^2 z^2 + m_t^2 (1 - \sin^2 \theta_{em} \gamma^2)}}{1 - \sin^2 \theta_{em} \gamma^2}.$ Using that, we can obtain $c$ :	(104)
Under the kinematics of transfer reaction: $\begin{split} \rho &= \frac{p_{T}}{ZB} = \frac{k\sin\theta_{\rm res}}{ZB}. \end{split}$ The time for the cycle is given by Eq. B $\frac{r_{Tge} - \frac{2\pi r_{T}}{ZB}}{T_{Tge} - \frac{2\pi r_{T}}{ZB}} = \frac{2\pi k\sin\theta_{\rm res}}{ZB}. \end{split}$ The time for the cycle is kine. These this distance overwell alogs the beam axis over a cycle	(89) (90) sc	The solution for $\sqrt{A}$ is: $\sqrt{A} = \frac{-\sin^2 \theta_{em} n\beta\gamma^2 z + \cos \theta_{em} \sqrt{\alpha^2 z^2} + m_1^2 (1 - \sin^2 \theta_{em} \gamma^2)}{1 - \sin^2 \theta_{em} \gamma^2}.$ Using that, we can obtain $e$ : $\sqrt{A} = -\frac{-\sin^2 \theta_{em} \alpha\beta\gamma^2 + \cos \theta_{em} \sqrt{\alpha^2 z^2} + m_1^2 (1 - \sin^2 \theta_{em} \gamma^2)}{1 - \sin^2 \theta_{em} \gamma^2}.$	(104)
Under the kinematics of transfer reaction: $\begin{split} & \mu = \frac{p_T p}{ZB} = \frac{k \sin \theta_{\rm em}}{ZB}. \end{split}$ The time for the cycle is given by Eq. (3) $T_{\rm opt} = \frac{2\pi r}{v_{\rm opt}} = \frac{2\pi k \sin \theta_{\rm em}}{ZB}.$ The time for the cycle is fixed. Then the distance covered along the beam axis over a cycle $z_0 = 2\pi r \frac{p_{\rm em}}{r_{\rm opt}} = \frac{2\pi r}{ZB} \frac{p_{\rm em}}{r_{\rm opt}}.$	(89) (90) sc	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{\sin^2 \theta_{em} \alpha \beta \gamma^2 z + \cos \theta_{em} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{em} \gamma^2)}}{1 - \sin^2 \theta_{em} \gamma^2}$ . Using that, we can obtain $c$ : $c = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{em} \alpha \beta \gamma^2 z + \cos \theta_{em} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{em} \gamma^2)}}{1 - \sin^2 \theta_{em} \gamma^2}$ .	(104)
Under the kinematics of transfer reaction: $\begin{split} & \mu = \frac{p_T y}{ZB} = \frac{k \sin \theta_{mn}}{ZB}. \end{split}$ The time for the cycle is given by Eq. (3) $T_{epc} = \frac{2\pi x}{r_{epc}} = \frac{2\pi k \sin \theta_{em}}{ZB}. \end{split}$ The time for the cycle is fixed. The distance cover along the beam axis over a cycle $z_{e} = v_{eT} \frac{r_{epc}}{r_{epc}} = \frac{2\pi}{ZB} \frac{v_{epc}}{v_{epc}} \frac{\theta_{epc}}{\theta_{epc}} \frac{1}{r_{epc}} = \frac{1}{2B_{epc}}, \\ z_{e} = z_{eT} \frac{1}{r_{epc}} = \frac{2\pi}{R_{epc}} \frac{v_{epc}}{R_{epc}} \frac{1}{R_{epc}} $	(89) (90) æ	The solution for $\sqrt{A}$ is: $\sqrt{A} = \frac{-\sin^2 \theta_{em} \alpha \beta \gamma^2 z + \cos \theta_{em} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{em} \gamma^2)}}{1 - \sin^2 \theta_{em} \gamma^2}.$ Using that, we can obtain $c$ : $c = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{em} \alpha \beta \gamma^2 z + \cos \theta_{em} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{em} \gamma^2)}}{1 - \sin^2 \theta_{em} \gamma^2}.$ When $\theta_{em} = 0$ , it reduces to:	(104)
Under the kinematics of transfer reaction: $\begin{split} & \mu = \frac{p_{TT}}{ZB} - \frac{k \sin \theta_{rm}}{ZB}. \end{split}$ The time for the cycle is given by Eq. 6 $T_{eq} = \frac{2\pi c_{p}}{ZB} - \frac{2\pi k \sin \theta_{rm}}{k m_{p}}. \end{split}$ The time for the cycle is fixed. Thus the distance covered along the beam axis over a cycle $z_{z} = v_{z}T_{qq} = 2\pi \rho_{eq}^{-2} - \frac{2\pi k}{ZB} \frac{k \sin \theta_{rm}}{k m_{q}}. \qquad $	(89) (90) &: (91)	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{\sin^2 \theta_{\mu m}}{1 - \sin^2 \theta_{\mu m}} \frac{\alpha \beta \gamma^2 z}{1 - \sin^2 \theta_{\mu m}} \frac{\alpha (1 - \sin^2 \theta_{\mu m} \gamma^2)}{1 - \sin^2 \theta_{\mu m}}$ . Using that, we can obtain $c$ : $e = -m_1 + \sqrt{A} = -m_1 + -\frac{\sin^2 \theta_{\mu m}}{1 - \sin^2 \theta_{\mu m}} \frac{\alpha \beta \gamma^2 z}{2 + \cos^2 (1 - \sin^2 \theta_{\mu m} \gamma^2)}$ . When $\theta_{\mu m} = 0$ , it reduces to: $e = -m_1 + \sqrt{a^2 z^2 + m_1^2}$ . which is computed of the dotted curve in the <b>R</b> . When $\theta_{\mu m} = -\mathbf{s}$ :	(104) (105) (106)
Under the kinematics of transfer reaction: $\begin{split} & \rho = \frac{p_{TH}}{ZB} = \frac{k \sin \theta_{em}}{ZB}. \end{split}$ The time for the cycle is given by Eq. (B) $T_{epc} = \frac{2\pi e}{v_{epc}} - \frac{2\pi k \sin \theta_{em}}{ZB}. \end{split}$ The time for the cycle is fixed. Thus the distance covered along the beam axis over a cycle $z_{z} = v_{z}T_{epc} = 2\pi P_{epc}^{-2} = \frac{2\pi}{Z} \frac{p_{epc}}{p_{epc}} k \sin \theta_{em},  \frac{v_{epc}}{v_{epc}} = \frac{1}{\sin \theta}, \\ z_{z} = z_{z} - \frac{\theta_{epc}}{t_{end}} = \frac{2\pi}{Z} \frac{p_{epc}}{P_{epc}} \frac{2\pi}{Z} \frac{p_{epc}}{P_{epc}} p_{epc} \frac{p_{epc}}{P_{epc}},  \alpha = \frac{2\theta}{Z}, \\ \alpha = p_{z} - \gamma D = \gamma D (p_{epc} - \gamma E \alpha \Theta_{epc}). \end{cases}$ Together with the energy equation of (B), we have 2 coupled equation:	(89) (90) sc (91) (92)	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{i\hbar r^2}{\theta_{mn}} \frac{n\beta\gamma^2 z}{n\beta\gamma^2} + \cos \theta_{mn} \sqrt{\alpha^2 z^2} + m_1^2(1 - i\hbar n^2 \theta_{mn}\gamma^2)}{1 - i\hbar n^2 \theta_{mn}\gamma^2}$ . Using that, we can obtain $c$ : $c = -m_1 + \sqrt{A} = -m_1 + -\frac{-i\hbar n^2 \theta_{mn}}{2\theta_{mn}} \frac{\alpha\beta\gamma^2 z}{2} + \cos \theta_{mn} \sqrt{\alpha^2 z^2} + m_1^2(1 - i\hbar n^2 \theta_{mn}\gamma^2)}{1 - i\pi n^2 \theta_{mn}}$ . When $\theta_{mn} = 0$ , it reduces to: $e = -m_1 + \sqrt{\alpha^2 z^2 + m_1^2}$ , which is equation of the dotted curve in Fig. When $\theta_{mn} = \frac{\pi}{2}$ .	(104) (105) (106) (107)
Under the kinematics of transfer reaction: $\begin{split} \mu &= \frac{2\pi g}{ZB} = \frac{k \sin \theta_{\rm em}}{ZB}. \end{split}$ The time for the cycle is given by Eq. (3) $T_{\rm egc} = \frac{2\pi g}{z_{\rm egc}} - \frac{2\pi k \sin \theta_{\rm em}}{ZB}. \end{split}$ The time for the cycle is fixed. Thus the distance covered along the beam axis over a cycle: $z_{\rm e} = v_{\rm e}T_{\rm egc} = 2\pi \frac{g}{ZB} \frac{2\pi}{z_{\rm egc}} \frac{k \sin \theta_{\rm em}}{z_{\rm egc}}. \qquad \frac{v_{\rm egc}}{v_{\rm egc}} = \frac{1}{a_{\rm egc}}, \qquad \\ z_{\rm e} = v_{\rm e}T_{\rm egc} = \frac{2\pi}{z_{\rm egc}} - \frac{2\pi}{ZB} \frac{v_{\rm egc}}{v_{\rm egc}} \frac{1}{v_{\rm egc}} \frac{v_{\rm egc}}{z_{\rm egc}} = \frac{1}{a_{\rm egc}}, \qquad \\ z_{\rm e} = z_{\rm egc} - \frac{v_{\rm egc}}{(a_{\rm egc})^2} - \frac{v_{\rm egc}}{ZB} \frac{1}{v_{\rm egc}} - v_{\rm ecced}, \qquad \\ \omega = z_{\rm egc} - \frac{v_{\rm egc}}{v_{\rm egc}} - v_{\rm ecced}, \qquad \\ z_{\rm egc} = z_{\rm egc} - \frac{v_{\rm egc}}{v_{\rm egc}} - v_{\rm ecced}, \qquad \\ z_{\rm egc} = z_{\rm egc} - \frac{v_{\rm egc}}{v_{\rm egc}} - v_{\rm ecced}, \qquad \\ z_{\rm egc} = z_{\rm egc} - v_{\rm e$	(89) (90) ac (91) (92) (03)	The solution for $\sqrt{A}$ is: $\frac{\sqrt{A}}{\sqrt{A}} = -\frac{\sin^2}{6m} \frac{a_0 \gamma^2 z}{a_0 m} \sqrt{a^2 z^2} + \frac{m_1^2(1-\sin^2\theta_{mn}\gamma^2)}{1-\sin^2\theta_{mn}\gamma^2}.$ Using that, we can obtain $c$ : $c = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2\theta_{mn}\alpha\beta\gamma^2 z + \cos^2\theta_{mn}\sqrt{a^2 z^2} + \frac{m_1^2(1-\sin^2\theta_{mn}\gamma^2)}{1-\sin^2\theta_{mn}\gamma^2}.$ When $\theta_{mn} = 0$ , it reduces to: $c = -m_1 + \sqrt{a^2 z^2 + m_1^2}.$ which is equation of the dotted curve in Fig. [] When $\theta_{mn} = \frac{3}{2}$ : $c = -m_1 + \frac{\beta^2}{\beta^2}.$ which is equation of the dinted lung in Fig. [] Min $\theta_{mn} = \frac{3}{\beta}$ .	(104) (105) (106) (107)
Under the kinematics of transfer reaction: $\begin{split} \rho &= \frac{p_H}{2B} = \frac{k \sin \theta_{min}}{2B}. \end{split}$ The time for the cycle is given by Eq. § $T_{\rm eq} &= \frac{2 \pi p}{v_{\rm eq}} = \frac{2 \pi k \sin \theta_{min}}{2B}. \end{split}$ The time for the cycle is fixed. Thus the distance covered along the beam axis over a cycle: $z_{\rm c} = v_{\rm e} T_{\rm eq} = 2\pi \frac{p_{\rm eq}}{2B} = \frac{2\pi k}{2B} \frac{k \sin \theta_{min}}{v_{\rm eq}}. $ The time for the cycle is fixed. Thus the distance covered along the beam axis over a cycle: $z_{\rm c} = v_{\rm e} T_{\rm eq} = \frac{2\pi p}{2B} - \frac{2\pi k}{2B} \frac{k \sin \theta_{min}}{v_{\rm eq}} = \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm c} = \frac{p_{\rm eq}}{2B} - \frac{2\pi k}{2B} \frac{1}{v_{\rm eq}} = \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm c} = \frac{p_{\rm eq}}{2B} - \frac{2\pi k}{2B} \frac{1}{v_{\rm eq}} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} = -\frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} = -\frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} = -\frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{p_{\rm eq}}{2B} - \frac{1}{2\pi a} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{1}{2} \theta_{\rm e} - \frac{1}{2} \theta_{\rm e} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{1}{2} \theta_{\rm e} - \frac{1}{2} \theta_{\rm e} - \frac{1}{2} \theta_{\rm e} - \frac{1}{2} \theta_{\rm e}, \\ z_{\rm c} = z_{\rm e} - \frac{1}{2} \theta_{\rm $	(99) (90) 55 (91) (92) (93)	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{\sin^2 \theta_{mn} n\beta\gamma^2 z}{1 - \sin^2 \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2(1 - \sin^2 \theta_{mn} \gamma^2)}}$ . Using that, we can obtain $c$ : $e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha\beta\gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2(1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}$ . When $\theta_{mn} = 0$ , it reduces to: $e = -m_1 + \sqrt{\alpha^2 z^2 + m_1^2}$ , which is equation of the dotted curve in Fig. <b>B</b> When $\theta_{mn} = \frac{3}{2}$ ; $e = -m_1 + \frac{\beta^2}{\beta^2}$ , which is equation of the dotted line in Fig. <b>B</b>	(104) (105) (106) (107)
Under the kinematics of transfer reaction: $\begin{split} \rho &= \frac{p_H}{2B} = \frac{k \sin \theta_{\rm em}}{2B}. \end{split}$ The time for the cycle is given by Eq. (3) $T_{\rm eq.} = \frac{2\pi p}{\pi p} = \frac{2\pi k \sin \theta_{\rm em}}{2B}. \end{split}$ The time for the cycle is fixed. Thus the distance covered along the beam axis over a cycle: $z_0 = v_s T_{\rm eq.} = 2\pi \frac{p_{\rm eq.}}{2B} \frac{2\pi k}{2B} \frac{k \sin \theta_{\rm em.}}{k_{\rm eq.}} \frac{v_{\rm eq.}}{k_{\rm eq.}} = \frac{1}{2\pi q}, \\z_0 = 2\pi \frac{p_{\rm eq.}}{k_{\rm eq.}} = \frac{2\pi k}{2B} \frac{p_{\rm eq.}}{k_{\rm eq.}} \frac{2\pi k}{2B} \frac{1}{k_{\rm eq.}} \frac{p_{\rm eq.}}{k_{\rm eq.}} = \frac{1}{k_{\rm eq.}}, \\z_0 = 2\pi \frac{p_{\rm eq.}}{k_{\rm eq.}} = \frac{2\pi k}{2B} \frac{p_{\rm eq.}}{k_{\rm eq.}} \frac{2\pi k}{k_{\rm eq.}} \frac{p_{\rm eq.}}{k_{\rm eq.}} = \frac{1}{k_{\rm eq.}}, \\\alpha_{\rm eq.} = p_{\rm eq.} - \gamma \beta q - \gamma \log \theta_{\rm eq.}. \\\text{Together with the energy equation of (6), we have 2 coupled equations:} \\\left\{\alpha_{\rm eq.} = p_{\rm eq.} - \gamma \beta_{\rm eq.} - \gamma \delta \alpha_{\rm eq.}, \\e_{\rm eq.} = p_{\rm eq.} - \gamma \beta_{\rm eq.} - \gamma \delta \alpha_{\rm eq.}, \\e_{\rm eq.} = p_{\rm eq.} - \gamma \beta_{\rm eq.} - \gamma \delta \alpha_{\rm eq.}, \\\text{where } e \text{ is the kinet: energy of the lighter particle 2.} \\ \end{cases}$	(99) (90) ac (91) (92) (93)	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{\sin^2 \theta_{mn} \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}$ . Using that, we can obtain $c$ : $e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}$ . When $\theta_{mn} = 0$ , it reduces to: $e = -m_1 + \sqrt{\alpha^2 z^2 + m_1^2}$ , which is equation of the dotted curve in Fig. [5]. When $\theta_{mn} = \frac{\pi}{2}$ : $e = -m_1 + \frac{\pi}{\beta^2}$ , which is equation of the dotted line in Fig. [5]. <b>3.13</b> The Bores radius line The dotector may have maximum radius $R$ and $2p \leq R$ . Considering Eq. [5].	(104) (105) (106) (107)
Under the kinematics of transfer reaction: $\begin{split} & \rho = \frac{p_H}{2B} = \frac{k \sin \theta_{\rm em}}{2B}. \end{split}$ The time for the cycle is given by Eq. (3) $T_{\rm epc} = \frac{2\pi}{v_{\rm ep}} = \frac{2\pi}{2E} \frac{k \sin \theta_{\rm em}}{k_{\rm em}}. \end{split}$ The time for the cycle is fixed. Thus the distance covered along the beam axis over a cycle $z_0 = v_0 T_{\rm epc} = 2\pi \frac{v_{\rm ep}}{2E} - \frac{2\pi}{2E} \frac{k_{\rm em}}{k_{\rm em}} = \frac{m}{v_{\rm ep}} = \frac{1}{2\pi k_{\rm em}}, \newline z_0 = v_0 T_{\rm epc} = 2\pi \frac{v_{\rm ep}}{2E} - \frac{2\pi}{2E} \frac{k_{\rm em}}{k_{\rm em}} = \frac{m}{v_{\rm ep}} = \frac{1}{2\pi k_{\rm em}}, \newline z_0 = z_0 = v_0 T_{\rm epc} = 2\pi \frac{k_{\rm em}}{2E} = 2\pi \frac{k_{\rm em}}{k_{\rm em}} = \frac{2\pi}{2E}, \newline z_0 = z_0 = v_0 T_{\rm epc} = 2\pi \frac{k_{\rm em}}{2E} = 2\pi \frac{k_{\rm em}}{k_{\rm em}} = \frac{2\pi}{2E}, \newline z_0 = z_0 = v_0 = $	(99) 50 (91) (93)	The solution for $\sqrt{A}$ is: $\sqrt{A} = \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}.$ Using that, we can obtain $c$ : $e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}.$ When $\theta_{mn} = 0$ , it reduces to: $e = -m_1 + \sqrt{\alpha^2 z^2 + m_1^2}.$ which is equation of the dotted curve in Fig. [3] When $\theta_{mn} = \frac{\pi}{2}$ : $e = -m_1 + \frac{\pi}{\beta^2}.$ which is equation of the disted curve in Fig. [3] <b>3.13</b> The Bore radius line The denotor may have maximum radius $R$ and $2p \leq R$ . Considering Eq. [69] $\mu = \frac{k \sin \theta_{mn}}{R} \leq \frac{R}{2} \implies k \sin \theta_{mn} = R \frac{R^2}{2} = Rar.$	(104) (105) (106) (107) (108)
Under the kinematics of transfer reaction: $\begin{aligned} \mu &= \frac{2\pi g}{ZB} - \frac{k \sin \theta_{\rm eff}}{ZB}. \end{aligned}$ The time for the cycle is given by Eq. (2) $T_{\rm eff} &= \frac{2\pi g}{ZB} - \frac{2\pi k \sin \theta_{\rm eff}}{k m_{\rm eff}}. \end{aligned}$ The time for the cycle is fixed. The the distance covered along the beam axis over a cycle $z_{\rm e} = v_{\rm e} T_{\rm eff} = \frac{2\pi}{ZB} \frac{2\pi}{k m_{\rm eff}} 2$	(89) (90) (91) (92) (93)	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{\sin^2 \theta_{mn} \alpha \beta \gamma^2 z}{1 - \sin^2 \theta_{mn} \gamma^2} - \frac{\sin^2 \theta_{mn} \alpha \beta \gamma^2 z}{1 - \sin^2 \theta_{mn} \gamma^2}.$ Using that, we can obtain $c$ : $e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma^2 z}{1 - \sin^2 \theta_{mn} \gamma^2} - \frac{\sin^2 \theta_{mn} \gamma^2}{1 - \sin^2 \theta_{mn} \gamma^2}.$ When $\theta_{mn} = 0$ , it reduces to: $e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma^2 z}{1 - \sin^2 \theta_{mn} \gamma^2}.$ which is equation of the dotted curve in Fig. [9]. When $\theta_{mn} = \frac{\pi}{2}$ . which is equation of the dotted line in Fig. [9]. <b>1.3.1 The Bore radius IIM</b> The detector may have maximum radius $R$ and $2p \leq R$ . Considering Eq. [9]. $\mu = \frac{\sin^2 \theta_{mn}}{2B} \leq \frac{R}{2} \Rightarrow kint \theta_{mn} = R\frac{ZB}{2} = Rax.$ Inserting into Eq. [9].	(104) (105) (106) (107) (108)
Under the kinematics of transfer reaction: $\begin{split} & \rho = \frac{p_{T}}{ZB} = \frac{k \sin \theta_{m}}{ZB}. \\ & The time for the cycle is given by Eq. (B) \\ & T_{ep} = \frac{2\pi}{v_{ep}} - \frac{2\pi}{ZB} \frac{k \sin \theta_{m}}{v_{ep}}. \\ & The time for the cycle is fixed. Then the distance covered along the beam axis over a cycle \\ & z_{e} = v_{e}T_{ep} = 2\pi \frac{p_{ep}}{v_{ep}} - \frac{2\pi}{ZB} \frac{k \sin \theta_{m}}{v_{ep}}. \\ & z_{e} = z_{e} - \frac{\theta_{ep}}{v_{ep}} - \frac{2\pi}{ZB} \frac{p_{ep}}{v_{ep}} p_{ep} = \frac{1}{v_{ep}} - \frac{1}{2\pi}, \\ & z_{e} = z_{e} - \frac{\theta_{ep}}{v_{ep}} - \frac{2\pi}{ZB} \frac{p_{ep}}{v_{ep}} p_{ep} = \frac{1}{v_{ep}}, \\ & \alpha = p_{e} - \gamma k \cos \theta_{m}, \\ & \text{Together with the energy equation of (B), we have 2 coupled equations:} \\ & \left\{x_{e} = \gamma \beta_{e} - \gamma k \cos \theta_{m}, \\ & w_{em} = \gamma \alpha - \gamma \beta k \cos \theta_{m}, \\ & \text{where } e \text{ is the kinetic energy of the lighter particle 2} \\ & u = \frac{1}{v_{ep}} - \frac{1}{v_{ep}} 1$	(89) (90) ac (91) (92) (93)	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{shr^2}{m} \frac{\theta_{mn} \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2(1 - shr^2 \theta_{mn} \gamma^2)}}{1 - shr^2 \theta_{mn} \gamma^2}.$ Using that, we can obtain $c$ : $c = -m_1 + \sqrt{A} = -m_1 + \frac{-shr^2 \theta_{mn} \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2(1 - shr^2 \theta_{mn} \gamma^2)}}{1 - shr^2 \theta_{mn} \gamma^2}.$ When $\theta_{mn} = 0$ , it reduces to: $e = -m_1 + \sqrt{A^2} = -m_1 + \frac{\sigma^2}{m^2}.$ Which is equation of the dotted curve in Fig. When $\theta_{mn} = \frac{\pi}{2}$ . $e = -m_1 + \frac{\sigma^2}{2}.$ which is equation of the dotted curve in Fig. [3] When $\theta_{mn} = \frac{\pi}{2}.$ which is equation of the dotted line in Fig. [3] <b>1.16 Hore realities Homestrian</b> The dotector may have maximum random R and $2p \leq R$ . Considering Eq. [6] $\mu = \frac{k \sin \theta_{mn}}{2R} \leq \frac{R}{2} \Rightarrow k \sin \theta_{mn} = R\frac{2R}{2} - Rox.$ Inserting into Eq. [6] $\begin{cases} ex = -\eta_2 + \sqrt{\gamma e^2 - (Rox)^2}. \\ ex = -\eta_2 + \sqrt{\gamma e^2 - (Rox)^2}. \end{cases}$	(104) (105) (106) (107) (108) (109)
Under the kinematics of transfer reaction: $\begin{split} & \mu = \frac{2\pi g}{2M} = \frac{k \sin \theta_{\rm eff}}{2B}. \end{split}$ The time for the cycle is given by Eq. (3) $T_{\rm eff} = \frac{2\pi g}{v_{\rm eff}} = \frac{2\pi k \sin \theta_{\rm eff}}{2B - v_{\rm eff}}. \end{split}$ The time for the cycle is fixed. Thus the distance covered along the beam axis over a cycle: $z_{\rm e} = v_{\rm e} T_{\rm eff} = \frac{2\pi}{16m} \frac{2}{\theta} \frac{2\pi}{2B} \frac{v_{\rm eff}}{2B - v_{\rm eff}}. \qquad $	(99) (90) ac (91) (92) (93)	The solution for $\sqrt{A}$ is: $\begin{array}{l} \sqrt{A} = \frac{-\sin^2 \theta_{mn} \alpha \beta^2 z^2 + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}.\\ \text{Using that, we can obtain c: e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}.\\ \text{When } \theta_{mn} = 0, \text{ it reduces to:} \\ e = -m_1 + \sqrt{\alpha^2 z^2 + m_1^2},\\ \text{which is equation of the dotted curve in Fig. [I]. When } \theta_{mn} = \frac{\pi}{2}:\\ e = -m_1 + \frac{\pi}{\beta^2},\\ \text{which is equation of the dotted curve in Fig. [I]. When } \theta_{mn} = \frac{\pi}{2}:\\ e = -m_1 + \frac{\pi}{\beta^2},\\ \text{which is equation of the dotted curve in Fig. [I]. When } \theta_{mn} = \frac{\pi}{2}:\\ e = -m_1 + \frac{\pi}{\beta^2},\\ \text{which is equation of the thin sold line in Fig. [I]. The dottect may have maximum radius R and 2p \leq R. Considering Eq. [I]. The dottect may have maximum radius R and 2p \leq R. Considering Eq. [I]. \mu = \frac{k \sin \theta_{mn}}{RB} \leq \frac{R}{2} \Rightarrow k \sin \theta_{mn} = R\frac{R}{2} = Rax. Inserting into Fig. [I]. \left\{ az = \beta \beta q - \gamma \sqrt{k^2 - (Ran)^2}, \\ e + m = \gamma q - \gamma \sqrt{k^2 - (Ran)^2}. \end{aligned} Let's replace \sqrt{k^2 - (Ran)^2}.$	(104) (105) (106) (107) (108) (109)
Under the kinematics of transfer reaction: $\begin{aligned} \mu &= \frac{2\pi}{2B} - \frac{k \sin \theta_m}{2B}. \end{aligned}$ The time for the cycle is given by Eq. (B) $T_{eq} &= \frac{2\pi}{2w_0} - \frac{2\pi}{2B} \frac{k \sin \theta_m}{w_0}. \end{aligned}$ The time for the cycle is fixed. Thus the distance covered along the beam ach over a cycle is fixed. Thus the distance covered along the beam ach over a cycle is $2\pi + \pi^2 - \pi^2 - \pi^2 + \frac{2\pi}{2B} - \frac{2\pi}{2W} + \frac{2\pi}{2B} - \frac{\pi}{2W} + \frac{\pi}{2W} - \frac{\pi}{2W} = \frac{\pi}{2W}, \\ z_0 &= 2\pi - \frac{\pi}{2W} - \frac{2\pi}{2B} - \frac{\pi}{2W} - \frac{\pi}{2W} - \frac{\pi}{2W} = \frac{\pi}{2W}, \\ z_0 &= 2\pi - \frac{\pi}{2W} - \frac{\pi}{2W} - \frac{\pi}{2W} - \frac{\pi}{2W} - \frac{\pi}{2W} - \frac{\pi}{2W}, \\ z_0 &= \pi - \frac{\pi}{2W} - \frac{\pi}{2W} - \frac{\pi}{2W} - \frac{\pi}{2W} - \frac{\pi}{2W}, \\ z_0 &= \pi - \frac{\pi}{2W} - \frac{\pi}{2$	(89) (90) ac (91) (92) (93)	The solution for $\sqrt{A}$ is: $\frac{\sqrt{A}}{\sqrt{A}} = \frac{-8\pi^2 \ \theta_{mn} \ \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \ \theta_{mn} \gamma^2)}}{1 - 4\pi^2 \ \theta_{mn} \gamma^2}.$ Using that, we can obtain $c$ : $e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \ \theta_{mn} \ \alpha \beta \gamma^2 z + \cos \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2 (1 - \sin^2 \ \theta_{mn} \gamma^2)}}{1 - \sin^2 \ \theta_{mn} \gamma^2}.$ When $\theta_{mn} = 0$ , it reduces to: $e = -m_1 + \sqrt{a^2 z^2 + m_1^2}.$ which is equation of the dotted curve in Fig. (Billshift $\theta_{mn} = \frac{2}{3}$ : $e = -m_1 + \sqrt{a^2 z^2 + m_1^2}.$ which is equation of the dotted curve in Fig. (Billshift $\theta_{mn} = \frac{2}{3}$ : which is equation of the dotted curve in Fig. (Billshift $\theta_{mn} = \frac{2}{3}$ . which is equation of the dotted curve in Fig. (Billshift $\theta_{mn} = \frac{2}{3}$ . which is equation of the dotted the in Fig. (Billshift $\theta_{mn} = \frac{2}{3}$ . The dotted restor may have maximum radius R and $2p \in SI$ . Considering Fig. (Billshift $\theta_{mn} = \frac{2}{3} \frac{2}{3} = 0 \ k \sin \theta_{mn} = \frac{R^2 B}{2} = R \alpha \pi.$ Inserting into Fig. (Billshift $\frac{2}{2} = \frac{2}{3} = \frac{1}{3} \ k \sin \theta_{mn} = \frac{R^2 B}{2} = R \alpha \pi.$ Let's replace $\sqrt{k^2 - (h \pi \pi)^2}$ into the axe of simplety: $\left\{ (\alpha z)^2 - (\gamma \beta \eta_2 - \gamma \gamma)^2 - (\omega \pi)^2 \right\}$	(104) (105) (106) (107) (108) (109) (110)
Under the kinematics of transfer reaction: $\begin{aligned} \mu &= \frac{p_H}{2R} = \frac{k \sin \theta_m}{2R}. \end{aligned}$ The time for the cycle is given by Eq. (3) $F_{eq} &= \frac{2\pi e_F}{2R} = \frac{2\pi k \sin \theta_m}{2R}. \end{aligned}$ The time for the cycle is fixed. Thus the distance overed along the beam axis over a cycle: $x_0 = v_0 T_{eq} = 2\pi \frac{p_{eq}}{2R} = \frac{2\pi}{2R} \frac{h_{eff}}{h_{eff}} \frac{h_{eff}} \frac{h_{eff}}{h_{e$	(89) (90) ж (91) (92) (93)	The solution for $\sqrt{A}$ is: $\sqrt{A} = \frac{-\sin^2 \theta_{mn} n\beta\gamma^2 z}{1 - \sin^2 \theta_{mn} \sqrt{\alpha^2 z^2 + m_1^2(1 - \sin^2 \theta_{mn} \gamma^2)}}$ Using that, we can obtain $c$ : $e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha\beta\gamma^2 z}{1 - \sin^2 \theta_{mn} \gamma^2}$ When $\theta_{m} = 0$ , it reduces to: $e = -m_1 + \sqrt{\alpha^2 z^2 + m_1^2},$ which is equation of the dotted curve in Fig. <b>B</b> . When $\theta_{m} = \frac{1}{2}$ : $e = -m_1 + \frac{\sqrt{\alpha^2 z^2 + m_1^2}}{1 - \sin^2 \theta_{mn} \gamma^2},$ which is equation of the dotted curve in Fig. <b>B</b> . When $\theta_{m} = \frac{1}{2}$ : $e = -m_1 + \frac{\sqrt{\alpha^2 z^2 + m_1^2}}{1 - \sin^2 \theta_{mn} \gamma^2},$ which is equation of the dotted curve in Fig. <b>B</b> . When $\theta_{m} = \frac{1}{2}$ : $e = -m_1 + \frac{1}{2},$ which is equation of the dotted line in Fig. <b>B</b> . <b>1.5 dotter radius line</b> The dotter or may have maximum radua R and $2\mu \leq R$ . Considering Eq. <b>B</b> . $\mu = \frac{k \sin \theta_{mn}}{2R} = \frac{R}{2} = k \sin \theta_{mn} = R \frac{Z}{2} = Rox.$ Inserting into Eq. <b>B</b> . $\begin{cases} a_1 z = \gamma \beta q - \gamma \sqrt{k^2 - (Rox)^2}, \\ (z + m_1)^2 = (\gamma q - \gamma \beta)^2, \\ (z + m_1)^2 = (\gamma q - \gamma \beta)^2, \end{cases}$ Let's replace $\sqrt{k^2 - (Rox)^2}$ with it for the solve of simplicity: $\begin{cases} (a_1^2 - (\gamma \beta q - \gamma)^2), \\ ((z + m_1)^2 = (\gamma q - \gamma \beta)^2, \end{cases}$	(104) (105) (106) (107) (108) (109) (110)
Under the kinematics of transfer reaction: $\begin{aligned} & \mu = \frac{2\pi}{2M} \frac{\mu}{R} + \frac{k \sin \theta_m}{2R}. \end{aligned}$ The time for the cycle is given by Eq. (B) $& T_{eq} = 2\pi \frac{e_{eq}}{e_{eq}} = \frac{2\pi}{2R} \frac{h \sin \theta_m}{R}. \end{aligned}$ The time for the cycle is fixed. Thus the distance covered lange the beam axis over a cycle. $& z = v_{e}T_{eq} = 2\pi \frac{e_{eq}}{R} \frac{2\pi}{2R} \frac{1}{R} \frac{e_{eq}}{e_{eq}} = \frac{1}{2\pi} \frac{1}{R} \frac{1}$	(89) (90) ** (91) (92) (93)	The solution for $\sqrt{A}$ is: $\sqrt{A} = -\frac{\sin^2 \theta_{mn} n\beta\gamma^2 z}{1 - \sin^2 \theta_{mn} \gamma^2}$ Using that, we can obtain $c$ : $e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} n\beta\gamma^2 z + \cos \theta_{mn} \sqrt{a^2 z^2 + m_1^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}$ When $\theta_{mn} = 0$ , it reduces to: $e = -m_1 + \sqrt{a^2 z^2 + m_1^2}$ which is equation of the dotted curve in Fig. [3] When $\theta_{mn} = \frac{\pi}{2}$ ; which is equation of the dotted curve in Fig. [3] When $\theta_{mn} = \frac{\pi}{2}$ ; which is equation of the dotted curve in Fig. [3] <b>All The Bore radius</b> Ine The dottector may have maximum radius $R$ and $2\mu \leq R$ . Considering Eq. [6] $\mu = \frac{k \sin \theta_m}{2R} \frac{R}{2} = k \sin \theta_m = \frac{R^2 B}{2} - Rox.$ Inserting into Eq. [6] $\begin{cases} a_1 = -\beta q - \sqrt{A^2 - (Rox)^2}, \\ c + m_1 = -\gamma q - \sqrt{A^2 - (Rox)^2}, \\ c + m_1 = -\gamma q - \sqrt{A^2 - (Rox)^2}, \end{cases}$ Let's replace $\sqrt{k^2 - (Rox)^2}$ with f for based simplicity: $\begin{cases} a_1 = (\gamma (\beta q - \gamma)q)^2, \\ (c + m_1)^2 = (\gamma (q - \gamma)\theta)^2, \\ (c + m_1)^2 = (\gamma (q - \gamma)\theta)^2, \end{cases}$ After some calculations we us: $(c + m_1)^2 - (n2)^2 = q^2 - r^2,$	(104) (105) (106) (107) (108) (109) (110) (111)
Under the kinematics of transfer reaction: $\begin{aligned} & \mu = \frac{2\pi g}{2M} = \frac{k \ln \theta}{2M} \\ & \mu = \frac{2\pi}{2M} = \frac{k \ln \theta}{2M} \\ & \mu = \frac{2\pi}{2M} = \frac{2\pi}{2M} \frac{k \ln \theta}{2M} \\ & \mu = \frac{2\pi}{2M} = \frac{2\pi}{2M} \frac{k \ln \theta}{2M} \\ & \mu = \frac{2\pi}{2M} = \frac{2\pi}{2M} \frac{k \ln \theta}{2M} \\ & \mu = \frac{2\pi}{2M} \frac{2\pi}{2M} \frac{2\pi}{2M} \frac{k \ln \theta}{2M} \\ & \mu = \frac{2\pi}{2M} \frac{2\pi}{2M} \frac{2\pi}{2M} \frac{k \ln \theta}{2M} \\ & \mu = \frac{2\pi}{2M} \frac{2\pi}{2M} \frac{2\pi}{2M} \frac{k \ln \theta}{2M} \\ & \mu = \frac{2\pi}{2M} \frac{2\pi}{2M} \frac{2\pi}{2M} \\ & \mu = \frac{2\pi}{2M} \frac{2\pi}{2M} \frac{2\pi}{2M} \\ & \mu = \frac{2\pi}{2M} \\ & \mu = \frac{2\pi}{2M} \frac{2\pi}{2M} \\ & \mu = \frac{2\pi}{2M} \\ & $	(39) (90) sc (91) (92) (93)	The solution for $\sqrt{A}$ is: $\begin{split} & \sqrt{A} = \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma_{\pi}^2 + \cos \theta_{mn} \sqrt{\alpha_{\pi}^2 z^2 + m_{\pi}^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}.\\ & \text{Using that, we can obtain c: & e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma^2 + \cos \theta_{mn} \sqrt{\alpha_{\pi}^2 z^2 + m_{\pi}^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}.\\ & \text{When } \theta_{mn} = 0, \text{ it reduces to:}\\ & e = -m_1 + \sqrt{A} = -m_1 + \frac{-\sin^2 \theta_{mn} \alpha \beta \gamma^2 + \cos \theta_{mn} \sqrt{\alpha_{\pi}^2 z^2 + m_{\pi}^2 (1 - \sin^2 \theta_{mn} \gamma^2)}}{1 - \sin^2 \theta_{mn} \gamma^2}.\\ & \text{Which is equation of the dotted curve is Fig. (5) When \theta_{mn} = \frac{\pi}{2};\\ & e = -m_1 + \frac{\alpha}{\beta^2},\\ & \text{which is equation of the dotted line in Fig. (5) \\ & \textbf{Lat B to extend in Mall III in Fig. (6) \\ & \mu = \frac{-\sin \theta_{mn}}{2D} \leq \frac{R}{2} \Rightarrow k \sin \theta_{mn} = R \frac{Z}{2} = R \tan.\\ & \text{Inserting into Eq. (6) } \\ & e^{1} + \frac{\sin \theta_{mn}}{2D} \leq \frac{R}{2} \Rightarrow k \sin \theta_{mn} = R \frac{Z}{2} = R \tan.\\ & \text{Inserting into Eq. (6) } \\ & e^{1} + \frac{\cos \theta_{mn}}{2D} \leq \frac{R}{2} \Rightarrow k \sin \theta_{mn} = R \frac{Z}{2} = R \tan.\\ & \text{Inserting into Eq. (6) } \\ & e^{1} + \frac{\cos \theta_{mn}}{2} + \frac{2}{2} + \frac{\cos \theta_{mn}}{2} + \frac{2}{2},\\ & \text{Let's replace } \sqrt{k^2 - (R \tan^2)^2} & \text{with for the asked simplicity:} \\ & e^{1} + (m_1)^2 - (m_1)^2 + (m_1)^2 $	(104) (105) (106) (107) (108) (109) (110) (111)
Under the kinematics of transfer reaction: $\begin{aligned} & \mu = \frac{2\pi}{2B} = \frac{k \sin \theta_m}{2B}. \end{aligned}$ The time for the cycle is given by Re B $T_{eq} = \frac{2\pi}{2e_{e}} = \frac{2\pi}{2B} \frac{k \sin \theta_m}{e_{eq}}. \end{aligned}$ The time for the cycle is find. The the distance covered along the beam acts over a cycle is $(2\pi) = \sqrt{2g_e} = 2\pi \frac{p_{e}}{2B} - \frac{2\pi}{2B} \frac{p_{eq}}{p_{eq}} = \frac{2\pi}{e_{e}}, \frac{p_{e}}{2B} - \frac{1}{e_{e}}, \frac{p_{e}}{e_{e}} = \frac{1}{e_{e}}, \frac{p_{e}}{e_{e}}, \frac{p_{e}}{e_{e}} = \frac{1}{e_{e}}, \frac{p_{e}}{e_{e}}, \frac{p_{e}}{e$	(89) (90) s: (91) (92) (93)	The solution for $\sqrt{A}$ is: $\begin{split} & \sqrt{A} = \frac{-i\hbar^2}{\theta_m} \frac{\theta_m \alpha\beta^2 r^2 + \cos\theta_m \sqrt{\alpha^2 z^2 + m_1^2(1 - i\hbar n^2 \theta_m \gamma^2)}}{1 - i\hbar n^2 \theta_m \gamma^2}. \end{split}$ Using that, we can obtain $c$ : $& c = -m_1 + \sqrt{A} = -m_1 + \frac{-i\hbar^2}{\theta_m} \frac{\theta_m \alpha\beta\gamma^2 + \cos\theta_m \sqrt{\alpha^2 z^2 + m_1^2(1 - i\hbar n^2 \theta_m \gamma^2)}}{1 - i\hbar n^2 \theta_m \gamma^2}. \end{split}$ When $\theta_m = 0$ , it reduces to: $& c = -m_1 + \sqrt{A} = -m_1 + \frac{-i\hbar^2}{\theta_m} \frac{\theta_m \alpha\beta\gamma^2 + \cos\theta_m \sqrt{\alpha^2 z^2 + m_1^2(1 - i\hbar n^2 \theta_m \gamma^2)}}{1 - i\hbar n^2 \theta_m \gamma^2}. \end{split}$ When $\theta_m = 0$ , it reduces to: $& c = -m_1 + \sqrt{A} = -m_1 + \frac{-i\hbar^2}{\theta_m} \frac{\theta_m \alpha\beta\gamma^2 + \cos\theta_m \sqrt{\alpha^2 z^2 + m_1^2(1 - i\hbar n^2 \theta_m \gamma^2)}}{1 - i\hbar n^2 \theta_m \gamma^2}. \end{split}$ Which is equation of the dotted curve in Fig. When $\theta_{m} = \frac{1}{\beta^2}$ . Which is equation of the dotted line in Fig. $B$ The dottector may have maximum radius $R$ and $2p \leq R$ . Considering Eq. $B$ Let us replace the maximum radius $R$ and $2p \leq R$ . Considering Eq. $B$ Let us replace $\sqrt{k^2 - (Ram)^2}$ with the orthe solution simplicity: $& \left\{ \frac{e_1}{e_1} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} \right\}$ . Let us replace $\sqrt{k^2 - (Ram)^2}$ with the orthe solution simplicity: $& \left\{ \frac{(e_1)^2}{(e_1)^2 - (e_1)^2} - \frac{1}{2} - \frac{1}{2} \right\}$ . There some calculations we get: $& (e_1 + m_1)^2 - (\alpha_2)^2 - \alpha^2 - t^2.$ This formulas for $q$ and $P$ if $Eq. Que obtain: & 2m_1 + e^2 - \frac{1}{4} \left\{ (2\pi - 1)^2 + z^2 \right\},$	(104) (105) (106) (107) (108) (109) (110) (111)
Under the kinematics of transfer reaction: $\begin{aligned} & \qquad $	(99) (00) ac (01) (02) (03)	The solution for $\sqrt{A}$ is: $\begin{aligned} & \sqrt{A} = \frac{-atr^2}{a_{mn}} \frac{a_{mn} a_p^2 z}{1 - atm^2 a_{mn} \sqrt{a^2 z^2 + m_1^2 (1 - atm^2 a_{mn} \gamma^2)}}{1 - atm^2 a_{mn} \gamma^2}. \end{aligned} The solution (z_1 + z_1) = \frac{-atr^2}{a_{mn}} \frac{a_{mn} a_p^2 z^2 + con a_{mn} \sqrt{a^2 z^2 + m_1^2 (1 - atm^2 a_{mn} \gamma^2)}}{1 - atm^2 a_{mn} \gamma^2}. \end{aligned} The solution (z_1 + z_2) = \frac{-atr^2}{a_{mn}} \frac{a_{mn} a_p^2 z^2 + con a_{mn} \sqrt{a^2 z^2 + m_1^2 (1 - atm^2 a_{mn} \gamma^2)}}{1 - atm^2 a_{mn} \gamma^2}. \end{aligned}  The solution (z_1 + z_2) = \frac{-atr^2}{a_{mn}} \frac{a_{mn} a_p^2 z^2 + con a_{mn} \sqrt{a^2 z^2 + m_1^2 (1 - atm^2 a_{mn} \gamma^2)}}{1 - atm^2 a_{mn} \gamma^2}. \end{aligned}  The solution (z_1 + z_2) = \frac{-atr^2}{a_{mn}} \frac{a_{mn} (z_1 + z_2)}{1 - atm^2 a_{mn} \gamma^2}. \end{aligned}  The solution (z_1 + z_1) = \frac{-atr^2}{a_{mn}} \frac{a_{mn} (z_1 + z_2)}{a_{mn}} \frac{a_{mn} ($	(104) (105) (106) (107) (108) (109) (110) (111) (112)
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Under the kinematics of transfer reaction: $\begin{aligned} & \qquad $	(89) (00) ac (91) (92) (03) dotted acregation LIOS	<text><text><text><equation-block><text><text><text><text><text><text><text><text><text><text><text><text><text><equation-block></equation-block></text></text></text></text></text></text></text></text></text></text></text></text></text></equation-block></text></text></text>	<ul> <li>(104)</li> <li>(105)</li> <li>(106)</li> <li>(107)</li> <li>(108)</li> <li>(109)</li> <li>(110)</li> <li>(111)</li> <li>(112)</li> </ul>
	(199) (100) sc (101) (102) (103) dotted stergy.	<text><text><text><text><text><text><text><text><text><text><text><text><text><text><text><text><equation-block></equation-block></text></text></text></text></text></text></text></text></text></text></text></text></text></text></text></text>	<ul> <li>(104)</li> <li>(105)</li> <li>(106)</li> <li>(107)</li> <li>(108)</li> <li>(109)</li> <li>(110)</li> <li>(111)</li> <li>(112)</li> </ul>
<text><text><text><equation-block><equation-block><equation-block><text><equation-block><text><equation-block><equation-block><equation-block></equation-block></equation-block></equation-block></text></equation-block></text></equation-block></equation-block></equation-block></text></text></text>	(89) (90) sc (91) (92) (93) dotted dotted LIOS equa- ments	<text><text><text><text><text><text><text><text><text><text><text><text><text><text><text><text><text><text></text></text></text></text></text></text></text></text></text></text></text></text></text></text></text></text></text></text>	<ul> <li>(104)</li> <li>(105)</li> <li>(106)</li> <li>(107)</li> <li>(108)</li> <li>(109)</li> <li>(110)</li> <li>(111)</li> <li>(112)</li> </ul>
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Jun 15, 2022 | Fysikdagarna 2022 | A. Kawecka

16

$ \begin{pmatrix} \frac{da}{dz} \end{pmatrix}_{SOLID} = \begin{pmatrix} \frac{da}{dz} \end{pmatrix}_{DOTTED}, \\ 4 \\ \alpha\beta = \frac{2h^2 \frac{\pi^2}{2\sqrt{a^2 \frac{d}{dx}} + m_1^2}}, \\ \beta^2 = \frac{2h^2 \frac{\pi^2}{2\sqrt{a^2 \frac{d}{dx}} + m_1^2}}, \\ z_M = \frac{h^2}{a^2 m_1}, \\ z_M = \frac{h^2}{a^2 m_1}. \end{cases} $ Let's now find ear: $ \frac{e_M = -m_1 + \sqrt{a^2 \frac{d}{dx} + m_1^2}}{2\gamma M_1} = m_1 + \alpha\beta^2 m_1, \\ \gamma m_1 - m_1 = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_1} = m_1 + \alpha\beta^2 m_1, \\ \gamma m_1 - m_1 = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_1} = m_1 + \alpha\beta^2 m_1, \\ \gamma m_1 - m_1 = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_1} = m_1 + \alpha\beta^2 m_1, \\ \vdots \\ m_2 = M_e - m_1 = \sqrt{(m_e + m_0)^2 + 2m_e T} = m_1. \end{cases} $ At the non-relativistic limit where $m_e + m_\theta \gg m_e T$ : $ \frac{m_1 - m_1 - m_2}{2\gamma M_1} = m_1 - m_1 = m_1 + m_1 - m_1 = m_1 + m_2 - m_2 = m_1 + m_2 - m_2 - m_1 + m_2 + m_2 + m_2 - m_2 = m_1 + m_1 + m_2 - m_2 = m_1 + m_2 - m_2 - m_1 + m_2 + m_2 - m_2 = m_1 + m_2 - m_2 - m_1 = m_1 + m_2 + m_2 - m_2 = m_1 + m_2 + m_2 - m_2 = m_1 + m_2 + m_2 - m_2 = m_1 + m_2 + m_2 + m_2 + m_2 + m_2 = m_2 + m_$	(113) (114) (115) (116)
$\begin{split} & \alpha\beta = \frac{2n^2\tilde{\pi}_1^2}{2\sqrt{n^2}\tilde{x}_1^2 + m_1^2} \\ & \alpha\beta = \frac{2n^2\tilde{\pi}_1^2}{a^2\tilde{x}_2^2 + m_1^2} \\ & \beta^2 = \frac{2n^2\tilde{\pi}_1^2}{a^2\tilde{x}_2^2 + m_1^2} \\ & zM = \frac{P_1}{a^2\tilde{x}_2^2 + m_1^2} \\ & zM = \frac{P_2}{a^2\tilde{x}_2^2 + m_1^2} \\ & zM = \frac{P_1}{a^2\tilde{x}_2^2 + m_1^2} \\ & = -m_1 + \sqrt{n^2\tilde{x}_2^2 + m_1^2} \\ & m_1 - m_1 = (\gamma - 1)m_1. \end{split}$ Let us find the maximum $m_{II}$ : Let us find the maximum $m_{II}$ : $\begin{split} & e_{II} &= \frac{M_2^2 + m_1^2 - m_1^2}{2\gamma M_1} \\ & \gamma m_1 - m_1 = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_1} \\ & m_1 + \alpha\beta z_{II}, \\ & \gamma m_1 - m_1 = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_1} \\ & m_2 = M_4 - m_1 = \sqrt{(m_4 + m_3)^2 + 2m_4 T} \\ & m_1 + \alpha\beta z_{II}, \\ \end{split}$ At the non-relativistic limit where $m_4 + m_8 \gg m_4 T$ : $\begin{split} & m_{2m_m} = \sqrt{(m_m + m_3)^2 + 2m_m T} \\ & m_{2m_m} = \sqrt{(m_m + m_3)^2 + 2m_m T} \\ & m_{2m_m} = \sqrt{(m_m + m_3)^2 + 2m_m T} \\ & m_1 + m_2 \\ & m_2 + m_2 - m_1 + m_2 + m_2 + \frac{m_1 T}{m_2 + m_3} \\ & m_1 + m_2 - m_2 + m_2 + \frac{m_1 T}{m_2 + m_2} \\ & m_2 + m_2 - m_2 + m_2 + \frac{m_1 T}{m_2 + m_2} \\ & m_1 + m_2 + m_2 + m_2 \\ & m_2 + m_2 - m_2 + m_2 + m_2 + \frac{m_2 T}{m_2 + m_2} \\ & m_1 + m_2 + m_2 + m_2 \\ & m_2 + m_2 - m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 - m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 - m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 - m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 - m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 - m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 - m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 + m_2 + m_2 \\ & m_2 + m_2 + m_2 $	(113) (114) (115)
$\begin{split} &\alpha\beta = \frac{\alpha^{2}}{\alpha^{2}\tau_{M}^{2}} + \frac{1}{m^{2}}, \\ &\beta^{2} = \frac{\alpha^{2}\tau_{M}^{2}}{\alpha^{2}\tau_{M}^{2} + m_{1}^{2}}, \\ &z_{M} = \frac{b\gamma}{\alpha^{2}m_{1}}, \\ &z_{M} = \frac{b\gamma}{\alpha^{2}m_{1}}, \\ \text{Let's now find } e_{M}: \\ &e_{M} = -m_{1} + \sqrt{\alpha^{2}\tau_{M}^{2} + m_{1}^{2}} \\ &= -m_{1} + \sqrt{(\beta^{2}\gamma^{2} + 1)m_{1}^{2}} \\ &= \gamma m_{1} - m_{1} = (\gamma - 1)m_{1}. \end{split}$ Let us find the maximum $m_{H}: \\ &e_{M} = \frac{M_{2}^{2} + m_{1}^{2} - m_{1}^{2}}{2\gamma M_{M}} - m_{1} + \alpha\beta\beta z_{M}, \\ &\gamma m_{1} - m_{1} - \frac{M_{2}^{2} + m_{1}^{2} - m_{1}^{2}}{2\gamma M_{M}} - m_{1} + \alpha\beta\beta z_{M}, \\ &\vdots \\ &m_{2} = M_{e} - m_{1} = \sqrt{(m_{e} + m_{0})^{2} + 2m_{u}T} - m_{1}. \end{split}$ At the non-relativistic limit where $m_{u} + m_{0} \gg m_{u}T: \\ &m_{2m_{m}} = \sqrt{(m_{m} + m_{0})^{2} + 2m_{u}T} - m_{1} - m_{m} + m_{0} + \frac{m_{u}T}{m_{u} + m_{0}} - m_{1} = \frac{m_{u} - m_{u} - m_{1} - m_{1} + m_{u} - \frac{m_{u}T}{m_{u} + m_{0}}} - m_{1} = \frac{m_{u} - m_{u} - m_{u} - m_{u} + m_{u} - \frac{m_{u}T}{m_{u} + m_{u}}} - m_{u} + m_{u} + \frac{m_{u}T}{m_{u} + m_{u}}} - m_{u} = \frac{m_{u} - m_{u} - m_{u} - m_{u} + m_{u} - \frac{m_{u}T}{m_{u} + m_{u}}} - m_{u} = \frac{m_{u} - m_{u} - m_{u} - m_{u} + m_{u} - m_{u} - m_{u}}{m_{u} + m_{u}} - m_{u} - \frac{m_{u}}{m_{u} + m_{u}}} - m_{u} + m_{u} + m_{u} + m_{u}} - m_{u} + m_{u} + m_{u} + m_{u} + m_{u}} - m_{u} + m_{u} + m_{u} + m_{u}} - m_{u} + m_{u} + m_{u}} - m_{u} + m_{u} + m_{u} + m_{u}} - m_{u} + m_{u} + m_{u} + m_{u}} - m_{u} + m_{u}} - m_{u} + m_{u} + m_{u}} - m_{u} + m_{u}$	(114) (115) (116)
$\begin{split} & z_M = \frac{\delta \gamma}{a} m_1. \end{split}$ Let's now find eur: $\begin{split} & \varepsilon_M = -m_1 + \sqrt{a^2 z_M^2 + m_1^2} \\ & = -m_1 + \sqrt{(\beta^2 \gamma^2 + 1)m_1^2} \\ & = -m_1 + \sqrt{(\beta^2 \gamma^2 + 1)m_1^2} \\ & = -m_1 + \sqrt{(\beta^2 \gamma^2 + 1)m_1^2} \\ & = -m_1 - (\gamma - 1)m_1. \end{split}$ Let us inf the maximum mult: $\begin{split} & \varepsilon_M = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_2} - m_1 + \alpha \beta z_M, \\ & \gamma_{H-1} = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_2} - m_1 + \alpha \beta z_M, \\ & \gamma_{H-1} = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_2} - m_1 + \alpha \beta z_M, \\ & \vdots \\ & m_2 = A_c - m_1 = \sqrt{(m_a + m_b)^2 + 2m_a T} - m_1. \end{split}$ At the non-relativistic limit where $m_a + m_b \gg m_s T$ : $\begin{split} & m_{m_a} = \sqrt{(m_a + m_b)^2 + 2m_a T} - m_1 - m_b + m_b + \frac{m_a T}{m_a + m_b} - m_1 = \\ & m_1 + m_2 - m_2 - m_1 + m_b + \frac{m_b T}{m_a + m_b} - m_b = m_a + m_b - m_b - m_b + m_b - m_b T \end{split}$	(114) (115) (116)
Let's now find eur: $\begin{split} & \epsilon_M = -m_1 + \sqrt{a^2 x_M^2 + m_1^2} \\ & = -m_1 + \sqrt{(\beta^2 \gamma^2 + 1)m_1^2} \\ & = -m_1 + \sqrt{(\beta^2 \gamma^2 + 1)m_1^2} \\ & = m_1 - m_1 - (\gamma - 1)m_1. \end{split}$ Let us find the maximum mu: $& \epsilon_M = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_0} - m_1 + \alpha \beta z_M, \\ & \gamma m_1 - m_2 = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_0} - m_1 + \alpha \beta z_M, \\ & \gamma m_1 - m_1 = \frac{M_2^2 + m_1^2 - m_2^2}{2\gamma M_0} - m_1 + \alpha \beta z_M, \\ & \vdots \\ & m_2 = M_e - m_1 = \sqrt{(m_e + m_0)^2 + 2m_u^2} - m_1. \end{split}$ At the non-relativistic limit where $m_e + m_b \gg m_e 2$ : $& m_{a_{m_m}} = \sqrt{(m_e + m_0)^2 + 2m_u^2} - m_1 - m_m + m_0 + \frac{m_u T}{m_e + m_0} - m_1 = \frac{m_e + m_e - m_1 - m_1}{2\gamma M_0} - m_1 + \frac{m_u T}{m_e + m_0} - m_1 = \frac{m_e + m_e - m_1 - m_1}{2\gamma M_0} - m_1 + \frac{m_u T}{m_e + m_0} - m_1 = \frac{m_1 + m_2 - m_1}{2\gamma M_0} - m_1 + \frac{m_u T}{m_e + m_0} - m_1 = \frac{m_1 + m_2 - m_1}{2\gamma M_0} - m_1 + \frac{m_u T}{m_e + m_0} - m_1 = \frac{m_1 + m_2 - m_1}{2\gamma M_0} - m_1 + \frac{m_u T}{2\gamma M_0} - m_1 + \frac{m_u T}{2\gamma M_0} - m_1 + \frac{m_u T}{2\gamma M_0} - m_1 = \frac{m_u + m_1 - m_1}{2\gamma M_0} - m_1 + \frac{m_u T}{2\gamma M_0} - m_1 + \frac{m_u T}{2\gamma$	(115)
$\begin{split} e_{M} &= -m_{1} + \sqrt{a^{2} \frac{2}{M} + m_{1}^{2}} \\ &= -m_{1} + \sqrt{(\beta^{2} \gamma^{2} + 1)m_{1}^{2}} \\ &= \gamma m_{1} - m_{1} = (\gamma - 1)m_{1}. \end{split}$ Let us find the maximum $m_{H}$ : $e_{M} &= \frac{M_{2}^{2} + m_{1}^{2} - m_{2}^{2}}{2\gamma M_{1}} - m_{1} + \alpha\beta z_{M}, \\ \gamma m_{1} - m_{1} - \frac{M_{2}^{2} + m_{1}^{2} - m_{2}^{2}}{2\gamma M_{2}} - m_{1} + \alpha\beta z_{M}, \\ &\vdots \\ m_{2} = M_{x} - m_{1} = \sqrt{(m_{a} + m_{b})^{2} + 2m_{a}T} - m_{1}. \end{split}$ At the non-relativistic limit where $m_{a} + m_{b} \gg m_{a}T$ : $m_{2m_{a}} = \sqrt{(m_{a} + m_{b})^{2} + 2m_{a}T} - m_{1} - m_{1} + m_{b} + \frac{m_{a}T}{m_{b} + m_{b}} - m_{1} = \underbrace{m_{a} - m_{b} - m_{1} + m_{b} + \frac{m_{a}T}{m_{a} + m_{b}} - m_{1} = \underbrace{m_{a} - m_{b} - m_{1} + m_{b} + \frac{m_{a}T}{m_{a} + m_{b}} - m_{1} = \underbrace{m_{a} - m_{b} - m_{1} - m_{1} + m_{b} + \frac{m_{a}T}{m_{a} + m_{b}} - m_{1} = \underbrace{m_{a} - m_{b} - m_{1} - m_{1} + m_{a} + \frac{m_{a}T}{m_{a} + m_{b}} - m_{1} = \underbrace{m_{a} - m_{b} - m_{1} - m_{1} + m_{a} + \frac{m_{a}T}{m_{a} + m_{b}} - m_{1} = \underbrace{m_{a} - m_{b} - m_{1} - m_{1} + m_{a} + \frac{m_{a}T}{m_{a} + m_{b}} - m_{1} = \underbrace{m_{a} - m_{b} - m_{1} - m_{1} + m_{a} + m_{b}} - \underbrace{m_{a} - m_{b} - m_{1} - m_{1} + m_{a} + m_{b} - m_{a} + m_{b}} - \underbrace{m_{a} - m_{b} - m_{b} - m_{b} + m_{b} + m_{b}} - \underbrace{m_{a} - m_{b} - m_{b} + m_{b} + m_{b} - m_{b} - m_{b} + m_{b} + m_{b}} - \underbrace{m_{a} - m_{b} - m_{b} - m_{b} + m_{b} - m_{b} - m_{b} + m_{b} - m_{b} - m_{b} + m_{b} - m_{b} - m_{b} - \underbrace{m_{a} - m_{b} $	(115) (116)
$\begin{split} &= \gamma m_1 - m_1 = (\gamma - 1)m_1.\\ \text{Let us find the maximum mu:}\\ & e_M = \frac{M_2^2 + m_1^2 - m_1^2}{2\gamma M_1} - m_1 + \alpha\beta z_M,\\ & \gamma m_1 - m_1 = \frac{M_2^2 + m_1^2 - m_1^2}{2\gamma M_2} - m_1 + \alpha\beta z_M,\\ & \vdots\\ & m_2 = M_s - m_1 = \sqrt{(m_a + m_b)^2 + 2m_b T} - m_1.\\ \text{At the non-relativistic limit where } m_a + m_b \gg m_a T:\\ & m_{2m_a} = \sqrt{(m_a + m_b)^2 + 2m_a T} - m_1 - m_a + m_b + \frac{m_b T}{m_a + m_b} - m_1 - m_1 - m_1 - m_1 - m_1 + m_2 + \frac{m_b T}{m_a + m_b} - m_1 - m_1 - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_1 - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_1 - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_2 - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_2 - m_1 - m_1 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_2 - m_1 - m_1 - m_1 - m_2 - m_2 + m_2 + \frac{m_b T}{m_a + m_b} - m_2 - m_1 - m_1 - m_1 - m_2 - m_2 - m_2 - m_1 - m_2 $	(116)
$\begin{split} e_M &= \frac{M_1^2 + m_1^2 - m_1^2}{2\gamma M_1} - m_1 + \alpha \beta z_M, \\ \gamma m_1 - m_1 &= \frac{M_2^2 + m_1^2 - m_1^2}{2\gamma M_1} - m_1 + \alpha \beta z_M, \\ \vdots \\ m_2 &= M_t - m_1 = \sqrt{(m_a + m_b)^2 + 2m_b T} - m_1. \end{split}$ At the non-relativistic limit where $m_a + m_b \gg m_a T$ : $\begin{split} m_{2aaa} &= \sqrt{(m_a + m_b)^2 + 2m_b T} - m_1 - m_1 + m_b - m_b T \\ m_{2aaa} &= \sqrt{(m_a - m_b)^2 + 2m_b T} - m_1 - m_b + m_b + \frac{m_b T}{m_b + m_b} - m_1 - m_b + m_b - \frac{m_b T}{m_b + m_b} - m_b - m_b - m_b - m_b + m_b + \frac{m_b T}{m_b + m_b} - m_b - m_b + m_b + \frac{m_b T}{m_b + m_b} - m_b - m_b + m_b + m_b + m_b + m_b + m_b - m_b - m$	(116)
$\begin{split} e_{M} &= \frac{m_{e}}{2\pi M} - \frac{m_{e}}{2\pi M} - m_{e} + \alpha \beta 2M_{e}, \\ \gamma m_{1} - m_{1} &= \frac{M_{e}^{2} + m_{1}^{2} - m_{2}^{2}}{2\gamma M_{e}} - m_{1} + \alpha \beta 2M_{e}, \\ \vdots \\ m_{2} &= M_{e} - m_{1} = \sqrt{(m_{e} + m_{0})^{2} + 2m_{0}T} - m_{1}. \end{split}$ At the non-relativistic limit where $m_{e} + m_{b} \gg m_{e}T$ : $m_{2m_{e}} = \sqrt{(m_{e} + m_{0})^{2} + 2m_{0}T} - m_{1} - m_{1} + m_{e} + m_{0}} - m_{1} - \frac{m_{e}}{m_{e}} - m_{1} - m_{1} - m_{1} - m_{1}}{m_{e} - m_{0} - m_{1}} - m_{1} - m_{1} - m_{1} - m_{1} - m_{1}} - m_{1} - m_$	(116)
$\begin{split} \gamma m_1 - m_1 &= \frac{-\pi}{2\gamma M_1} - m_1 + \alpha \beta z_M, \\ &\vdots \\ m_2 &= M_4 - m_1 = \sqrt{(m_4 + m_b)^2 + 2m_b T} - m_1. \end{split}$ At the non-relativistic limit where $m_a + m_b \gg m_a T$ : $m_{2aaa} = \sqrt{(m_a + m_b)^2 + 2m_a T} - m_1 - m_a + m_b + \frac{m_b T}{m_a + m_b} - m_1 - \frac{m_a + m_b - m_1 - m_1}{m_a + m_b - m_b + m_b + \frac{m_b T}{m_a + m_b}} = Q_{mal} + m_2 + \frac{m_b T}{m_a + m_b} - \frac{m_b + m_b - m_b - m_b}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} - \frac{m_b + m_b - m_b - m_b}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + m_b + \frac{m_b T}{m_a + m_b} = Q_{mal} + \frac{m_b T}{m_a + $	(116)
$\begin{split} m_2 &= M_{\pi} - m_1 = \sqrt{(m_a + m_b)^2 + 2m_bT} - m_1. \end{split}$ At the non-relativistic limit where $m_a + m_b \gg m_sT$ : $m_{2m_a} = \sqrt{(m_a + m_b)^2 + 2m_sT} - m_1 - m_a + m_b + \frac{m_bT}{m_a + m_b} - m_1 = \frac{m_a + m_b - m_1 - m_a}{m_a + m_b - m_a + m_b} + \frac{m_bT}{m_a + m_b} = Q_{mad} + m_2 + \frac{m_bT}{m_a + m_b}. \end{split}$	
At the non-relativistic limit where $m_a + m_b \gg m_a T$ : $m_{2max} = \sqrt{(m_a + m_b)^2 + 2m_b T} - m_1 \rightarrow m_a + m_b + \frac{m_a T}{m_a + m_b} - m_1 = \frac{m_a + m_b - m_1 - m_2 + m_b + m_b m_a T}{m_a + m_b} = Q_{rad} + m_2 + \frac{m_a T}{m_a + m_b}.$	
$\begin{split} m_{2mu} &= \sqrt{(m_a + m_b)^2 + 2m_bT} - m_1 \rightarrow m_a + m_b + \frac{m_aT}{m_a + m_b} - m_1 = \\ &\underbrace{m_a + m_b - m_1 - m_2}_{Q_{av}} + m_2 + \frac{m_aT}{m_a + m_b} = Q_{m1} + m_2 + \frac{m_aT}{m_a + m_b}. \end{split}$	
$\underbrace{m_a + m_b - m_1 - m_2}_{Q_{area}} + m_2 + \frac{m_a T}{m_a + m_b} = Q_{val} + m_2 + \frac{m_a T}{m_a + m_b}.$	
1-00	(117)
$m_{2_{\rm max}} = E_{\pi_{\rm max}} + m_2, \label{eq:m2max}$	(118)
$E_{x_{\rm max}} = Q_{\rm val} + \frac{m_a T}{m_a + m_b} = Q_{\rm val} + T_{cm}, \label{eq:eq:expansion}$	(119)
where $Q_{val}$ is the $Q$ -value of the reaction and $T_{cm}$ is the kinetic energy in the center-of-mass.	
3.3.5 Minimum incident energy	
The minimum inclusive energy requires show. $M_e \ge m_1 + m_2 \implies (m_a + m_b)^2 + 2m_a T_{min} = (m_1 + m_2)^2,$	
$T_{min} = \frac{(m_1 + m_2)^2 - (m_a + m_b)^2}{2m_a} \cong -Q \left(1 + \frac{m_b}{m_a}\right) \neq Q.$	(120)
3.3.6 Tilted reaction	
When the incident particle is shifted by the initial angle $\theta_A$ , the four-momentum vector of part will be tilted by the $\theta_A$ angle:	ticle 1
$\mathbb{P}_{1} = \begin{pmatrix} E \\ p_{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{A} & -\sin \theta_{A} \\ 0 & \cos -\theta_{A} & -\sin -\theta_{A} \end{pmatrix} \begin{pmatrix} \gamma q - \gamma \beta k \cos \theta_{cm} \\ \gamma \beta q - \gamma k \cos \theta_{cm} \end{pmatrix}.$	
$\langle p_{xy} \rangle = \langle 0 \sin \theta_A \cos \theta_A \rangle \langle \kappa \sin \theta_{cm} \rangle$	(121)
$\langle p_{TF} \rangle = 0$ $\sin \theta_A = \cos \theta_A / (k \sin \theta_{em} - j)$ Since the z-position of the detector hit is $\alpha z = p_2 = (\gamma \beta q - \gamma k \cos \theta_{em}) \cos \theta_A - k \sin \theta_{em} \sin \theta_A,$	(121)
Dray ( $\partial \sin \beta_A = \cos \theta_A / \sqrt{\sin \theta_{ab}} \cos \theta_A / \sqrt{\sin \theta_{ab}}$ ) Since the is-position of the detected ht is $\alpha z = p_x = (\gamma \beta q - \gamma k \cos \theta_{ab}) \cos \theta_A - k \sin \theta_{ab} \sin \theta_A,$ and the energy:	(121)
Dray ( $\partial \sin \beta_A \cos \theta_A / \langle e \sin \theta_{em} \rangle$ ) Since the z-position of the detected ht is $\alpha z = p_a = (\gamma \beta q - \gamma k \cos \theta_{em}) \cos \theta_A - k \sin \theta_{em} \sin \theta_A$ , and the energy: $e + m_1 = \gamma q - \gamma \beta k \cos \theta_{em}$ , by eliminating $\theta_{em}$ we get:	(121) (122) (123)
Since the z-position of the detected h f is $\alpha z = p_{4} = (\gamma \beta q - \gamma k \cos \theta_{0m}) \cos \theta_{4} - k \sin \theta_{0m} \sin \theta_{4},$ and the energy: $e + m_{1} = \gamma q - \gamma \beta k \cos \theta_{0m},$ by eliminating $\theta_{0m}$ we get: $\alpha \beta z = \left(e + m_{1} - \frac{q}{2}\right) \cos \theta_{4} - \frac{1}{2} \sqrt{(\gamma \beta k)^{2} - (\gamma q - e - m_{1})^{2}} \sin \theta_{4}.$	(121) (122) (123) (124)

4 Inverse problem	
As already discussed in Sec. 1, the HELIOS set-up provides information about the the energy $E$ deposited by the detected particle. The main goal of HELIOS anal translate this information into the excitation energy $E_x$ of the compound nucleus angle $\theta_{cm}$ of the produced proton. In other words, a mapping of the kind	e z-coordinate and ysis is therefore to s and the emission
$\begin{pmatrix} E \\ z \end{pmatrix} \longrightarrow \begin{pmatrix} E_x \\ \theta_{cm} \end{pmatrix}$	(125)
must be found.	
4.1 From $E$ , $z$ to $E_{cm}$ $\theta_{cm}$	
One can express $(z, E)$ in terms of $(E_x, \theta_{cm})$ . Starting from the coupled Eq. 93 expressions for q and k found in Eq. 82, one obtains	and replacing the
$E = \frac{\gamma}{2E_t}(M_e^2 + m_1^2 - (m_2 + E_x)^2)$	(126)
$-\beta \cos \theta_{cm} \sqrt{(M_c^2 - (m_1 + m_2 + E_x)^2)(M_c^2 - (m_1 - m_2 - E_x)^2)}$	)
$z = \frac{1}{\alpha} [\beta (M_c^2 + m_1^2 - (m_2 + E_x)^2)]$	(1. m)
$-\cos \theta_{cm} \sqrt{\left(M_e^2 - (m_1 + m_2 + E_x)^2\right)\left(M_e^2 - (m_1 - m_2 - E_x)^2\right)}$	(127)
with $E_t^2 = M_c^2$ and $E = e + m_1$ . The inverse transformation can be derived by including in Eq. 126 the expression from Eq. 127, obtaining	of $\cos\theta_{cm}$ derived
$E_x^2 + 2m_2E_x + m_2^2 - m_1^2 - M_c^2 + 2\gamma E_t(E - \alpha\beta z) = 0$	(128)
which, after solving for $E_x$ , gives:	
$E_x=-m_2+\sqrt{M_c^2+m_1^2-2\gamma M_c(E-\alpha\beta z)}.$	(129)
By rearranging Eq. 126 one can also write:	
$M_{c}^{2} + m_{1}^{2} - (m_{1} + E_{x})^{2} = \frac{2E_{t}}{\gamma} E + \beta \cos \theta_{cm} \sqrt{(M_{c}^{2} - (m_{1} + m_{2} + E_{x})^{2})(M_{c}^{2} - (m_{1} - m_{2})^{2})} + \frac{2E_{t}}{\gamma} E + \beta \cos \theta_{cm} \sqrt{(M_{c}^{2} - (m_{1} + m_{2} + E_{x})^{2})}$	$m_2 - E_x)^2$ (130)
which if replaced in Eq. 127 leads to:	
$\cos \theta_{em} = \frac{\gamma(E\beta - \alpha z)}{\sqrt{\gamma^2(E - \alpha\beta z)^2 - m_1^2}}$	(131)
From $E$ and $z$ the reaction constant reads as:	
$k^2 = \gamma^2 (y - \alpha \beta z)^2 - m^2.$	(132)
where $y = e + m$ . Furthermore, knowing that: $\beta q = \alpha$	(400)
$\cos \theta = -z$	(133)
$k \gamma k'$	()
21	()
21	()
$g_{em}$ can be expressed as a function of y and the position 2:	()
$\theta_{em}$ can be expressed as a function of y and the position :: $\cos \theta_{em} = \frac{\gamma(y \beta - x)}{k} = \frac{\sqrt{\sqrt{m^2 + k^2} - y}}{\gamma \beta k}$	(134)
$k = \gamma k^{-1}$ 21 $\theta_{em}$ can be expressed as a function of y and the position z: $\cos \theta_{em} = \frac{\gamma(ye^2 - k^2 - y)}{k} - \frac{\gamma\sqrt{m^2 + k^2} - y}{\gamma 3k}$ From $k^2$ , the total mass of the particle 2 can be written as:	(134)
$\theta_{em}$ can be expressed as a function of y and the position z: $\cos \theta_{em} = \frac{\gamma(y\sigma^2 - xz)}{k} = \frac{\sqrt{m^2 + k^2} - y}{\gamma \beta k}$ From $k^3$ , the total mass of the particle 2 can be written as: $m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2}$	(134) (135)
$\theta_{\rm cm}$ can be expressed as a function of y and the position z: $\cos \theta_{\rm cm} = \frac{\gamma(y - z)}{k} = \frac{\gamma \sqrt{m^2 + k^2} - y}{\gamma \beta k}$ From $k^2$ , the total mass of the particle 2 can be written as: $m_2^2 = m_1^2 + M_c^2 - 2M_c \sqrt{k^2 + m_1^2}$ where $M_c$ is the total mass of the system.	(134) (135)
$\theta_{cm}$ can be expressed as a function of y and the position z: $\cos \theta_{cm} = \frac{\gamma(g\beta - \alpha_2)}{\gamma \beta k} = \frac{\sqrt{m^2 + k^2} - y}{\gamma \beta k}$ From $k^2$ , the total mass of the particle 2 can be written as: $m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2}$ where $M_e$ is the total mass of the system. <b>4.2 Finite detector</b>	(134) (135)
$\theta_{cm}$ can be expressed as a function of y and the position z: $\cos \theta_{cm} = \frac{\gamma(g\beta - \alpha_2)}{\gamma \alpha_1} = \frac{\gamma(m^2 + k^2 - y)}{\gamma \beta_1}$ From $k^2$ , the total mass of the particle 2 can be written as: $m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2}$ where $M_e$ is the total mass of the system. <b>4.2 Finite detector</b> In case the detector has a finite size (see Sec. 2.1) the hit position $z_{hB}$ is given by $\gamma_{-m} = -\gamma (z_1 - \frac{1}{2}, 0)$	(134) (135) :
$\theta_{cm}$ can be expressed as a function of y and the position z: $\cos \theta_{cm} = \frac{\gamma(g\beta - \alpha_2)}{2\pi} = \frac{\sqrt{m^2 + 1^2} - y}{\gamma\beta k}$ From $k^2$ , the total mass of the particle 2 can be written as: $m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2}$ where $M_e$ is the total mass of the system. <b>4.2 Finite detector</b> In case the detector has a finite size (see Sec. 2.1) the hit position $z_{ha}$ is given by $z_{hal} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{\rho}\right)$ . Recalling Eq. 93	(134) (135) : (136)
$\theta_{cm} \text{ can be expressed as a function of y and the position z:}$ $\cos \theta_{cm} = \frac{\gamma(g\beta - \alpha_z)}{2\pi} = \frac{\sqrt{m^2 + 1^2} - y}{\gamma\beta k}$ From k <sup>2</sup> , the total mass of the particle 2 can be written as: $m_z^2 = m_z^2 + M_z^2 - 2M_z \sqrt{k^2 + m_z^2}$ where M <sub>z</sub> is the total mass of the system. <b>4.2 Finite detector</b> In case the detector has a finite size (see Sec. 2.1) the hit position $z_{h,u}$ is given by $z_{h,l} \approx z(1 - \frac{1}{2\pi} \frac{a}{p}).$ Recalling Eq. 93 $y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma\beta \cos c_{hm},$	(134) (135) : (136) (137)
$\begin{split} \theta_{cm} & can be expressed as a function of y and the position z:: \\ & \cos \theta_{cm} = \frac{\gamma(y\beta - \alpha z)}{k} = \frac{\sqrt{m^2 + k^2} - y}{\gamma k} \\ & From k^2, the total mass of the particle 2 can be written as: \\ & m_2^2 = m_1^2 + M_2^2 - 2M_2 \sqrt{k^2 + m_1^2} \\ & where M_i is the total mass of the system. \\ \\ \hline A.2  Finite detector \\ In case the detector has a finite size (see Sec. 2.1) the hit position z_{hui} is given by z_{hui} \approx z \left(1 - \frac{1}{2\pi} \frac{p}{p}\right). \\ & Recalling Eq. 93 \\ & y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta A \cos \theta_{cm}, \\ & \sqrt{k^2 + m^2} = \gamma y - \gamma \beta \alpha z. \\ & \text{After combining Eq. 136 and 137 one has} \end{split}$	(134) (135) : (136) (137)
$\theta_{em} \text{ can be expressed as a function of y and the position z::}$ $\cos \theta_{em} = \frac{\gamma(y\beta - \alpha z)}{k} = \frac{\sqrt{m^2 + k^2} - y}{\gamma k}$ From $k^3$ , the total mass of the particle 2 can be written as: $m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2}$ where $M_e$ is the total mass of the system. <b>4.2 Finite dotector</b> In case the detector has a finite size (see Sec. 2.1) the hit position $z_{hai}$ is given by $z_{hai} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{p}\right)$ . Recalling Eq. 93 $y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \\ \sqrt{k^2 + m^2} = \gamma y - \gamma \beta \alpha z.$ After combining Eq. 136 and 137 one has $\alpha \beta \gamma z = (y\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{p}\right)$	(134) (135) : (136) (137) (138)
$\begin{split} \theta_{em} & \text{can be expressed as a function of y and the position z:} \\ & \cos \theta_{em} = \frac{\gamma(y \beta - \alpha_s)}{k} = \frac{\sqrt{m^2 + k^2} - y}{\gamma \beta k} \\ & \text{From } k^2, \text{ the total mass of the particle 2 can be written as:} \\ & m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2} \\ & \text{where } M_e \text{ is the total mass of the system.} \\ \hline & \textbf{A2 - Finite dotectore } \\ & \text{In case the detector has a finite size (see Sec. 2.1) the hit position z_{hit} is given by z_{hal} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{p}\right). \\ & \text{Recalling Eq. 93} \\ & y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{mi}, \\ & \sqrt{k^2 + m^2} = \gamma y - \gamma \beta \alpha z. \\ & \text{After combining Eq. 136 and 137 one has} \\ & = \alpha \beta \gamma z = (y\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{p}\right). \\ & \text{So the coupled solution is:} \end{split}$	(134) (135) : (136) (137) (138)
$\begin{split} \theta_{em} & can be expressed as a function of y and the position z: \\ & \cos \theta_{em} = \frac{\gamma(g - ax)}{k} = \frac{\gamma\sqrt{m^2 + k^2} - y}{\gamma 3k} \\ & Form k^3, the total mass of the particle 2 can be written as: \\ & m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2} \\ & where M_e$ is the total mass of the system. <b>4.1 Entite detector</b> <b>In</b> case the detector has a finite size (see Sec. 21.1) the hit position $z_{hal}$ is given by $z_{hal} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{g}\right). \\ & \text{Recalling Eq. 33} \\ & y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \\ & \sqrt{k^2 + m^2} = \gamma y - \gamma \delta az. \\ & \text{After combining Eq. 136 and 137 one has} \\ & = \left(y - \sqrt{m^2 + k^2} - \gamma H_e \cos \theta_{em}, \\ & \alpha \beta \gamma z = (y \gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{g}\right) \\ & \text{So the coupled solution is:} \\ & \begin{cases} y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma H_e \cos (\theta_{em}) \\ & \alpha \beta \gamma z = (\gamma \gamma - \sqrt{m^2 + k^2}) - \gamma H_e \cos (\theta_{em}) \\ & \alpha \beta \gamma z = (\gamma \gamma - \sqrt{m^2 + k^2}) - \gamma H_e \cos (\theta_{em}) \\ & \alpha \beta \gamma z = (\gamma \gamma - \sqrt{m^2 + k^2}) - \gamma H_e \cos (\theta_{em}) \end{cases}$	(134) (133) : : (136) (137) (138) (139)
$k - \gamma k^{-1}$ $21$ $\theta_{em} \text{ can be expressed as a function of y and the position z: \cos \theta_{em} = \frac{\gamma(g^2 - ax)}{k} = \frac{\gamma\sqrt{m^2 + k^2} - y}{\gamma k} From k^3, the total mass of the particle 2 can be written as:m_2^2 = m_1^2 + M_c^2 - 2M_c \sqrt{k^2 + m_1^2} where M_c is the total mass of the system.4.1 Entite detectorIn case the detector has a finite in (see Sec. 2.1) the hit position z_{hal} is given byz_{hall} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{g}\right). Recalling Eq. 33y = c + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \frac{\sqrt{k^2 + m_1^2}}{\sqrt{k^2 + m^2}} - y - \gamma \delta az. After combining Eq. 136 and 137 one has= \frac{\alpha \beta \gamma z - (y - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{g}\right) So the coupled solution is:\begin{cases} p = k - m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos(\theta_{em}) \\ (\alpha \gamma z = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{g}\right) \end{cases} Sincep = \frac{k \sin \theta_{em}}{\pi},$	(134) (133) : : : : : : : : : : : : : : : : : :
$\begin{split} \theta_{em} & can be expressed as a function of y and the position z: \\ & cos \theta_{em} = \frac{\gamma(g-a_{e})}{k} = \frac{\sqrt{m^2 + k^2} - y}{\gamma 3k} \\ & from k^3, the total mass of the particle 2 can be written as: \\ & m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2} \\ & where M_e$ is the total mass of the system. <b>4.1 Entite detector</b> <b>Base :</b> $(a - \frac{1}{2\pi} \frac{a}{g})$ . Recalling Eq. 93 $y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \\ & \omega \beta \gamma z = (yr) - \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \\ & \omega \beta \gamma z = (yr) - \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \\ & \omega \beta \gamma z = (yr) - \sqrt{m^2 + k^2} - \gamma k \cos \theta_{em}, \\ & \beta \gamma z = (yr) - \sqrt{m^2 + k^2} - (1 - \frac{1}{2\pi} \frac{a}{g}). \\ & \text{So the coupled solution is:} \\ & \qquad \qquad$	(134) (133) : (136) (137) (138) (139) (140)
$\theta_{cm} \text{ can be expressed as a function of y and the position z:} \\ \cos \theta_{cm} = \frac{\gamma(y\beta - \alpha x)}{k} = \frac{\sqrt{m^2 + k^2} - y}{\gamma k}$ From $k^2$ , the total mass of the particle 2 can be written as: $m_2^2 = m_1^2 + 4k^2 - 2M_c \sqrt{k^2 + m_1^2}$ where $M_c$ is the total mass of the system. <b>4.2 Finite dotector</b> In case the detector has a finite size (see Sec. 2.1) the hit position $z_{hui}$ is given by $z_{hui} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{\rho}\right)$ . Recalling Eq. 93 $y = c + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{cm}, \frac{\sqrt{k^2 + m^2}}{\sqrt{k^2 + m^2}} + \gamma - \gamma \beta \alpha z$ . After combining Eq. 136 and 137 one has $\alpha\beta\gamma z = (y\gamma - \sqrt{m^2 + k^2} - \gamma\beta k \cos \theta_{cm}), \frac{1}{\alpha\beta\gamma z} = (\gamma - \sqrt{m^2 + k^2} - \gamma\beta k \cos \theta_{cm}), \frac{1}{\alpha\beta\gamma z} = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{\rho}\right)$ . So the coupled solution is: $\left\{ \begin{array}{c} y = c + m = \gamma \sqrt{m^2 + k^2} - \gamma\beta k \cos \theta_{cm}, \\ \alpha\beta\gamma z = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{\rho}\right) \end{array} \right\}$ . Since $p = \frac{k \sin \theta_{cm}}{\frac{cB}{cB}}, \frac{1}{2B}$ where the expression for $\cos \theta_{cm}$ can be found in Eq. 134, after some algebraic p that $(y = k + m = \gamma - \beta m)$ .	(134) (135) : (136) (137) (138) (139) (140) assages it turns out
$\beta_{em} \text{ can be expressed as a function of y and the position z:} \\ \cos \theta_{em} = \frac{\gamma(y\beta - \alpha_z)}{k} = \frac{\sqrt{m^2 + k^2} - y}{\gamma k}$ From $k^2$ , the total mass of the particle 2 can be written as: $m_2^2 = m_1^2 + 3k^2 - 2M_c \sqrt{k^2 + m_1^2}$ where $M_c$ is the total mass of the spatial can be written as: $m_2^2 = m_1^2 + 3k^2 - 2M_c \sqrt{k^2 + m_1^2}$ where $M_c$ is the total mass of the system: <b>4.2 Finite dotector</b> In case the detector has a finite size (see Sec. 2.1) the hit position $z_{hai}$ is given by $z_{hai} \approx z \left(1 - \frac{1}{2\pi} \frac{p}{p}\right).$ Recalling Eq. 33 $y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{m_1}, \frac{\sqrt{k^2 + m^2}}{\sqrt{k^2 + m^2}} - \gamma \beta - \beta \alpha z.$ After combining Eq. 136 and 137 use has $\alpha\beta\gamma z = (\gamma p - \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_m), \frac{1}{\alpha\beta\gamma z = (\gamma p - \sqrt{m^2 + k^2})} \left(1 - \frac{1}{2\pi} \frac{p}{p}\right)$ Since $p = \frac{k \sin \theta_{m_1}}{cZB}, \frac{1}{cZB}, \frac{p}{m_1} = 1 - \frac{p \alpha}{\sqrt{2\gamma_2 \sqrt{m^2 + k^2} - p^2 - m^2 + k^2}}$	(134) (135) : (135) (136) (138) (139) (140) assages it turns out (141)
$\begin{split} & \mu = \frac{1}{24} - \frac{1}{24} \sum_{\alpha} \frac{1}{24} \\ \theta_{em} \ \text{can be expressed as a function of y and the position ::} \\ & \cos \theta_{em} = \frac{1}{2(\frac{1}{2}-\alpha)} = \frac{1}{\sqrt{n^2 + k^2} - y}}{\sqrt{n^2}} \\ & \text{From } k^3, \text{ the total mass of the particle 2 can be written as:} \\ & m_2^2 = m_1^2 + M_e^2 - 2M_e\sqrt{k^2 + m_1^2} \\ & \text{where } M_e \text{ is the total mass of the system.} \\ \hline \textbf{A2 Finite detector} \\ & \text{In case the detector bas a finite size (see Sec. 2.1) the hit position z_{hal} is given by z_{hal} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{\alpha}\right). \\ & \text{Recalling Eq. 93} \\ & y = e + m = \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \\ & \sqrt{k^2 + m^2} - \eta - \gamma \beta az. \\ & \text{After combining Eq. 136 and 137 one has} \\ & \alpha_\beta z_2 = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{\beta}\right). \\ & \text{So the coupled solution is } \\ & \qquad \qquad$	(134) (135) : (135) (136) (137) (138) (139) (140) assages it turns out (141) 2) (142)
$\begin{split} & (m-k-\gamma k^{-}) \\ & 2i \end{split}$ $\theta_{em} \text{ can be expressed as a function of y and the position :: \\ & \cos \theta_{em} = \frac{\gamma(g^2-g^2)}{k} = \frac{\gamma(m^2+k^2-y)}{\gamma k} \\ & \cos \theta_{em} = \frac{\gamma(g^2-g^2)}{k} = 2\sqrt{m^2+k^2-y} \\ & \text{Trom } k^3, \text{ the total mass of the particle 2 can be written as: } \\ & m_2^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2} \\ & \text{where } M_e \text{ is the total mass of the system.} \end{split}$ <b>4.2 Finite detector In</b> case the detector has a finite size (see Sec. 2.1) the hit position $z_{hal}$ is given by $z_{hal} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{g}\right). \\ & \text{Recalling Eq. 93} \\ & y = e + m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \\ & \sqrt{k^2 + m^2} - m^2 - m^2 - \gamma \delta x co \theta_{em}, \\ & \sqrt{k^2 + m^2} - m^2 - m^2 - m^2 \delta x co \theta_{em}, \\ & \sqrt{k^2 + m^2} - m^2 - m^2 - m^2 \delta x co \theta_{em}, \\ & \beta_0 \gamma_2 = (\gamma \gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{g}\right) \\ & \text{So the coupled solution is } \\ & \beta_0 \gamma_2 = (\gamma \gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{g}\right) \\ & \text{Since} \\ & \qquad \qquad$	(134) (135) : (135) (136) (137) (138) (139) (140) assages it turns out (141) (141)
$\begin{split} & (m-k-\gamma k)^{-} \\ & 2i \end{split}$ $\theta_{em} \text{ can be expressed as a function of y and the position :: \\ & \cos \theta_{em} = \frac{\gamma(g^2-a_e)}{k} = \frac{\gamma(m^2+k^2-y)}{\gamma k} \\ & \cos \theta_{em} = \frac{\gamma(g^2-a_e)}{k} = 2\sqrt{m^2+k^2-y} \\ & \text{From } k^3, \text{ the total mass of the particle 2 can be written as: } \\ & m_{\pi}^2 = m_1^2 + M_e^2 - 2M_e \sqrt{k^2 + m_1^2} \\ & \text{where } M_e \text{ is the total mass of the system.} \end{split}$ Breach det detector In case the detector has a finite size (see Sec. 2.1) the hit position $z_{kal}$ is given by $z_{kal} \approx z \left(1 - \frac{1}{2\pi} \frac{a}{g}\right).$ Recalling Eq. 93 $y = e^+ m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos \theta_{em}, \\ \sqrt{k^2 + m^2} - \eta - \beta k \cos \theta_{em}, \\ \sqrt{k^2 + m^2} - \eta - \beta k \cos \theta_{em}, \\ \cos \theta_{em} = (\gamma y - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{g}\right).$ So the coupled solution is: $\begin{cases} y = e^+ m = \gamma \sqrt{m^2 + k^2} - \gamma \beta k \cos(\theta_{em}) \\ \cos \theta_{em} = (\gamma y - \sqrt{m^2 + k^2}) \left(1 - \frac{1}{2\pi} \frac{a}{g}\right). \end{cases}$ Since $y = \frac{k \sin \theta_{em}}{(2B_{em})}, \\ \sin \theta_{em} = \sqrt{1 - \cos^2 \theta_{em}}, \\ \sin \theta_{em} = \sqrt{1 - \cos^2 \theta_{em}}, \\ (1 - \frac{1}{2\pi} \frac{a}{g}) = 1 - \frac{\beta \cos m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \end{cases}$ where the expression for core $\theta_{em}$ on the fourth IE F_4.13, $4\pi^2 - y^2 - m^2 \gamma^2 - k^2$ which leads tr: $\left[ \frac{a}{\alpha \beta \gamma z} - (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \beta \gamma z} - (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \beta \gamma z} = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \beta \gamma z} = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \gamma z} = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \gamma z} = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \gamma z} = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \gamma z} = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \gamma z} = (\gamma - \sqrt{m^2 + k^2}) \left(1 - \frac{\beta \gamma m}{\sqrt{2\gamma y \sqrt{m^2 + k^2} - y^2 - m^2 \gamma^2 - k^2}}, \\ \frac{a}{\alpha \gamma z}$	(134) (135) : (135) (135) (137) (138) (139) (139) (140) assages it turns out (141) $\overline{2}$ (142) (143)



# Inverse problem



$$E_{X} = -m_{2} + \sqrt{M_{C}^{2} + m_{1}^{2} - 2\gamma M_{C}(E - \alpha\beta z_{hit})}$$

$$E_{X} = -m_{2} + \sqrt{M_{C}^{2} + m_{1}^{2} - 2\gamma M_{C}(E - \alpha\beta z_{hit})}$$

$$\frac{6}{4}$$

J.C. Lighthall et al., NIMA 622 (2010) 97-106



Jun 15, 2022 | Fysikdagarna 2022 | A. Kawecka

# Summary

- The r-process is responsible for the creation of heavy elements in the universe
- Fission plays a crucial role in limiting the r-process
- Thus, fission cross-sections of neutron-rich nuclei are an essential input to theoretical modeling of the r-process
- Inverse kinematics studies using RIBs are promising tools for fission studies of neutron-rich nuclei
- Solenoidal spectrometers allow for precision studies of fission cross-sections

# Thank you for your attention!





#### **Position-sensitive Si Array**







- 24 double-sided silicon strip detectors (DSSD), four per side.
- 128x0.95 mm pitch strips on the front (p-side)
- 11x2 mm on the back (n-side).
- Solid angle coverage ~ 94% ( $\theta$ ), ~70% ( $\varphi$ )
- Length of active area (z axis) 501.5 mm
- Minimum distance to the target 14.5 mm
- Q-value resolutions approaching 20 keV



### Solenoidal spectrometers

Active area Si 1000 mm<sup>2</sup> - 20 mm x 50 mm



Square-shaped Si array





ISOLDE Solenoidal Spectrometer

Hexagonalshaped Si array



 $\mathbb{C}\mathsf{FR}$ 

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# Back up slides



# Finite size detector

Here (in reality) things get definitely worse:

the beam pipe

magnets to focus the beam

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# The origin of elements in the universe



### What happens inside a solenoidal field?





**Energy** of a proton is related to its *z* position along the beam axis

#### What do we measure?

- Position along the magnetic axis *z*,
- cyclotron period *T<sub>cyc</sub>*,
- energy of the proton in the laboratory frame *E<sub>LAB</sub>*

- Target inside the solenoid
- Fission fragments
- Proton follows helical trajectory and then is detected in a position-sensitive silicon array

### An important difference

- Particles are NOT detected at a fixed laboratory angle (conventional approach), but rather at a fixed distance from the target.
- The effective resolution with the solenoid can be <u>considerably better</u> than with a conventional detector array.

$$E_{\rm cm} = E_{\rm lab} + \frac{mV_{\rm cm}^2}{2} - \frac{mzV_{\rm cm}}{T_{\rm cyc}}$$

• Large background reduction



### Off-axis effect



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# Moving source

3D problem - Kinematics of 2-body scattering



### **Transfer reaction kinematics**

The 4-momentum of particle 1 using CM coordinates

a(b, 1)2

$$P_{1} = \begin{pmatrix} E \\ p_{z} \\ p_{xy} \end{pmatrix} = \begin{pmatrix} \gamma q - \gamma \beta k \cos \theta_{cm} \\ \gamma \beta q - \gamma k \cos \theta_{cm} \\ k \sin \theta_{cm} \end{pmatrix} = \begin{pmatrix} E \\ p \cos \theta \\ p \sin \theta \end{pmatrix}$$

$$P_{2} = \begin{pmatrix} E \\ p'_{z} \\ p'_{xy} \end{pmatrix} = \begin{pmatrix} \gamma Q + \gamma \beta k \cos \theta_{cm} \\ \gamma \beta Q + \gamma k \cos \theta_{cm} \\ -k \sin \theta_{cm} \end{pmatrix} = \begin{pmatrix} E' \\ p' \cos \theta \\ p' \sin \theta \end{pmatrix}$$

(those equations are derived using Lorentz transformation and kinematics of 2-body scattering)

#### the total energy in the CM frame

### **Transfer reaction kinematics**

a(b, 1)2

$$q = \frac{1}{2E_{t}} (E_{t}^{2} - m_{2}^{2} + m_{1}^{2})$$
$$P_{1} = \begin{pmatrix} E \\ p_{z} \\ p_{xy} \end{pmatrix} = \begin{pmatrix} \gamma q - \gamma \beta k \cos \theta_{cm} \\ \gamma \beta q - \gamma k \cos \theta_{cm} \\ k \sin \theta_{cm} \end{pmatrix} = \begin{pmatrix} E \\ p \cos \theta \\ p \sin \theta \end{pmatrix}$$

$$P_{2} = \begin{pmatrix} E \\ p'_{z} \\ p'_{xy} \end{pmatrix} = \begin{pmatrix} \gamma Q + \gamma \beta k \cos \theta_{cm} \\ \gamma \beta Q + \gamma k \cos \theta_{cm} \\ -k \sin \theta_{cm} \end{pmatrix} = \begin{pmatrix} E' \\ p' \cos \theta \\ p' \sin \theta \end{pmatrix}$$
(these equations are derived using Lorentz transformation and kinemaktics of 2-body scattering)
$$Q = \frac{1}{2E_{t}} (E_{t}^{2} + m_{2}^{2} - m_{1}^{2})$$

scattering)

Transfer reaction kinematics  
the momentum of particle 1 or 2 in  
the center-of-mass frame (CM)  
using LAB coordinates  

$$P_{1} = \begin{pmatrix} E \\ p_{z} \\ p_{xy} \end{pmatrix} = \begin{pmatrix} \gamma q - \gamma \beta k \cos \theta_{cm} \\ \gamma \beta q - \gamma k \cos \theta_{cm} \\ k \sin \theta_{cm} \end{pmatrix} = \begin{pmatrix} E \\ p \cos \theta \\ p \sin \theta \end{pmatrix}$$

$$P_{2} = \begin{pmatrix} E \\ p'_{z} \\ p'_{xy} \end{pmatrix} = \begin{pmatrix} \gamma Q + \gamma \beta k \cos \theta_{cm} \\ \gamma \beta Q + \gamma k \cos \theta_{cm} \\ -k \sin \theta_{cm} \end{pmatrix} = \begin{pmatrix} E' \\ p' \cos \theta \\ p' \sin \theta \end{pmatrix}$$

$$R^{\bullet} = \frac{1}{4E_{t}^{2}} ((E_{t}^{2} - (m_{2} + m_{1})^{2})(E_{t}^{2} - (m_{2} + m_{1})^{2}))$$

# Some simulations..

