Acausality in Superfluid Dark Matter and MOND-like Theories

Neil Shah

Tufts University

neil.shah@tufts.edu

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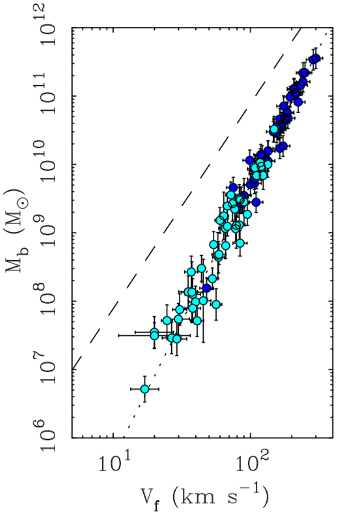
Successes/Issues of ΛCDM

- ΛCDM is very successful on cosmological scales
  - Large Scale Structure
  - CMB fluctuations
- However on galactic scales, some potential issues:
  - Dwarf satellite observations
  - Core-cusp
- Primary focus: Baryonic Tully-Fisher Relation (BTFR)
- Mismatch: Simple collapse model predicts $M_b \propto v_r^3 \neq v_r^4$



MOND

- MOdified Newtonian Dynamics (MOND) is a phenomenological theory:

$$\boldsymbol{f}_N = m\mu\left(\frac{a}{a_0}\right)\boldsymbol{a} = \frac{GmM_{\text{enc}}}{R^2} \quad \text{where} \quad \mu\left(\frac{a}{a_0}\right) \to \begin{cases} 1 & a \gg a_0 \\ a/a_0 & a \sim a_0 \end{cases} \tag{1}$$

  where $a_0 = 1.2 * 10^{-10}$
- This leads to $a_{\text{MOND}} = \sqrt{a_0 GM_{\text{enc}}}/R$ vs. $a_N = GM_{\text{enc}}/R^2$
- For circular motion, $a \propto v_r^2 \Rightarrow M_{b,\text{MOND}} \propto v_r^4$. Matches BTFR!
- Disadvantages: Empirical, no microscopic construction, unsuccessful on cosmological scales
- MOND and ΛCDM perform well on different scales. What if we combine them?

Unified Theory

- New unified theory developed by Berezhiani and Khoury[1]
- Introduce a massive scalar Φ with $U(1)$ symmetry.
  - On large scales, acts as CDM.
  - On galactic scales, undergoes a phase transition to a new superfluid phase of goldstone phonons. Mediate MONDian force between baryons.
- One such theory:

$$F_{\text{SFDM}} = \frac{1}{2}(X + m^2|\Phi|^2) + \frac{\Lambda^4}{6(\Lambda_c^2 + |\Phi|^2)^6}(X + m^2|\Phi|^2)^3 \tag{2}$$

where $X = g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^*$

- On cosmological scales, $|\Phi|$ and X are small:

$$F_{\text{CDM}} \approx \frac{1}{2}(X + m^2|\Phi|^2) \tag{3}$$

[1] **L. Berezhiani**, **J. Khoury**, **Phys. Rev. D**, **(2015)**.

Unified Theory

- Decompose field into $\Phi = \rho e^{i(\theta + mt)}$
- For the low energy effective action, assume ρ, θ are slowly varying.
- With some additional work, we obtain the low-energy effective theory:

$$F_{\text{MOND}} = -\frac{2\Lambda(2m)^{3/2}}{3} Y \sqrt{|Y|} \tag{4}$$

where $Y = \dot{\theta} - m\phi_N - \frac{1}{2m}(\nabla\theta)^2$

- With the inclusion of a coupling to baryons $\beta\theta\sigma_B$, in the static limit:

$$\vec{a} = \beta\nabla\theta = -\sqrt{\frac{|\beta|^3 M_{\text{enc}}}{8\pi\Lambda}}\frac{\hat{r}}{R} = -\frac{\sqrt{a_0 G M_{\text{enc}}}}{R}\hat{r} \tag{5}$$

Causality Constraints

- In this work, we wished to put these theories to some theoretical tests
- Consider a k-essence theory:

$$\mathcal{L}_k = \sqrt{-g}F(X,\phi) \quad \text{where} \quad X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \tag{6}$$

- Want to study high-energy perturbations $\phi = \phi_b + \epsilon$. Eqs. of motion are:

$$G_\phi^{\mu\nu}\partial_\mu\partial_\nu\epsilon = \left(F'(X_b,\phi_b)g^{\mu\nu} + F''(X_b,\phi_b)\partial^\mu\phi_b\partial^\nu\phi_b\right)\partial_\mu\partial_\nu\epsilon = 0 \tag{7}$$

  where $' = \partial/\partial X$, and we work to order $\mathcal{O}(\partial^2\epsilon)$
- High energy perturbations propagate on effective background metric $G_\phi^{\mu\nu}$

Global Hyperbolicity

- To prevent CTCs, demand global hyperbolicity of the background metric (well-defined Cauchy problem[2])
- Leads to:[3] $\text{sig}(G_\phi^{\mu\nu}) = \text{sig}(g^{\mu\nu}) = \{-,+,+,+\}$
  - Two eigenvalues of $G_\phi^{\mu\nu}$ are $F' \Rightarrow F' > 0$
  - From the determinant: $\text{Det}[G_\phi^{\mu\nu}] < 0 \Rightarrow F' + 2XF'' > 0$
  - No superluminal signal propagation $\Rightarrow F'' \leq 0$
- Together, we have 3 "causal" conditions:

$$\begin{aligned} (A) \quad & A \equiv F' > 0 \\ (B) \quad & B \equiv F' + 2XF'' > 0 \\ (C) \quad & C \equiv -F'' \geq 0 \end{aligned} \tag{8}$$

[2] **Y. Aharonov**, **A. Komar**, **L. Susskind**, **Phys. Rev. (1969)**.
[3] **J. P. Bruneton**, **Phys. Rev. D (2007)**.

Complex Scalar Causality Constraints

- Generalize our causality constraints to scalar with $U(1)$ symmetry: $\Phi = \phi_1 + i\phi_2$.
- Examine high energy perturbations: $\epsilon_j = \tilde{\epsilon}_j e^{ik_\mu x^\mu} + \text{c.c.}$, $j \in \{1, 2\}$
- Find that high-energy perturbations have two normal modes:
- First mode: $k^2 = 0 \Rightarrow$ corresponds to massless goldstone
- Second mode: More interesting, obeys $\mathcal{G}_\phi^{\mu\nu} \partial_\mu \partial_\nu \psi = 0$, where:

$$\mathcal{G}_\phi^{\mu\nu} = F' g^{\mu\nu} + F'' \sum_j \partial^\mu \phi_j^{(b)} \partial^\nu \phi_j^{(b)} \quad \text{and} \quad \psi = \sum_j \partial^\mu \phi_j^{(b)} \partial_\mu \epsilon_j \qquad (9)$$

  where $' = \partial/\partial X$, $X = \sum_j (\partial \phi_j)^2/2$
- Recover the same constraints A, B, C

Acauslity in Superfluid DM

- Causality tests of $F_{\rm SFDM}$:

$$A = \frac{1}{2} + \frac{\Lambda^4}{2\zeta(\Phi)}(X + m^2|\Phi|^2)^2 > 0$$
$$B = A - 2XC \overset{?}{<} 0 \qquad (10)$$
$$C = -\frac{\Lambda^4}{2\zeta(\Phi)}(X + m^2|\Phi|^2) \overset{?}{<} 0$$

where $\zeta(\Phi) = (\Lambda_c^2 + |\Phi|^2)^6$

- While A is always positive, what about B and C?
- In the MONDian regime:

$$B = 4m^3\Lambda^4\rho^4 Y < 0$$
$$C = 2m\Lambda^4\rho^2 Y < 0 \qquad (11)$$

- In the MONDian regime, $Y = \dot{\theta} - m\phi_N - \frac{1}{2m}(\nabla\theta)^2 \approx -\frac{1}{2m}(\nabla\theta)^2$
- Manifestly break causality constraints!

Generalized SFDM Model

- Can we generalize the Berezhiani, Khoury model to include the missing quadratic term, as well as higher order terms? We can!
- Our generalized SFDM model[4]:

$$F_{\text{gen}}(X,\Phi) = (X + m^2|\Phi|^2)\mathcal{F}(\mathcal{Z}) \quad \text{where} \quad \mathcal{Z} \equiv \frac{\Lambda^2(X + m^2|\Phi|^2)}{(\Lambda_c^2 + |\Phi|^2)^3} \tag{12}$$

  The model studied above is: $\mathcal{F}(\mathcal{Z}) = \frac{1}{2} + \frac{1}{6}\mathcal{Z}^2$

- The causality constraints are:

$$\begin{aligned} A &= \mathcal{F} + \mathcal{Z}\mathcal{F}_{\mathcal{Z}} \\ B &= A - 2XC \\ C &= -\frac{\Lambda^2}{(\Lambda_c^2 + |\Phi|^2)^3}(2\mathcal{F}_{\mathcal{Z}} + \mathcal{Z}\mathcal{F}_{\mathcal{Z}\mathcal{Z}}) \end{aligned} \tag{13}$$

[4] **M. P. Hertzberg**, **J. A. Litterer**, **N. Shah**, **JCAP (2021)**.

Generalized SFDM Model

- In the MONDian regime:

$$F_{\text{gen,MOND}} = -2m\rho^2 Y \mathcal{F}\left(-\frac{2\Lambda^2 m}{\rho^4} Y\right) \tag{14}$$

- Impose the following criteria in the MONDian regime:
  - The resulting force law must be attractive: $\Rightarrow \mathcal{F} > 0$
  - After the phase transition, ρ is at the minimum of its effective potential F: $\Rightarrow \mathcal{F}_{\mathcal{Z}} > 0$
  - To avoid tachyonic instabilities, $F_{,\rho\rho} > 0$: $\Rightarrow \mathcal{F}_{\mathcal{Z}\mathcal{Z}} < \frac{\rho^4}{4m\Lambda^2 Y}\mathcal{F}_{\mathcal{Z}}$
- This results in: $A > 0, B < 0, C < 0$
- Conclusion: Precisely in the MONDian regime of interest, the most general SFDM theory manifestly breaks causality constraints

Conclusions

- ΛCDM has successes on cosmological scales, while MOND has successes on galactic scales.
- Superfluid DM is a novel theory that retains dark matter on cosmological scales, and on galactic scales undergoes a phase transition into a superfluid of phonons which mediate a MONDian force between baryons.
- For relativistic fields which propagate on an effective background metric, the field's evolution is causal if the metric is globally hyperbolic. This introduces constraints on the eqs. of motion.
- We show that for the SFDM model, these causality constraints are broken.

Thank you!

References

[1] Y Aharonov, A Komar, and Leonard Susskind. "Superluminal behavior, causality, and instability". In: *Physical Review* 182.5 (1969), p. 1400.

[2] Lasha Berezhiani and Justin Khoury. "Theory of dark matter superfluidity". In: *Physical Review D* 92.10 (2015), p. 103510.

[3] Jean-Philippe Bruneton. "Causality and superluminal behavior in classical field theories: Applications to k-essence theories and modified-Newtonian-dynamics-like theories of gravity". In: *Physical Review D* 75.8 (2007), p. 085013.

[4] Mark P Hertzberg, Jacob A Litterer, and Neil Shah. "Acausality in superfluid dark matter and MOND-like theories". In: *Journal of Cosmology and Astroparticle Physics* 2021.11 (2021), p. 015.