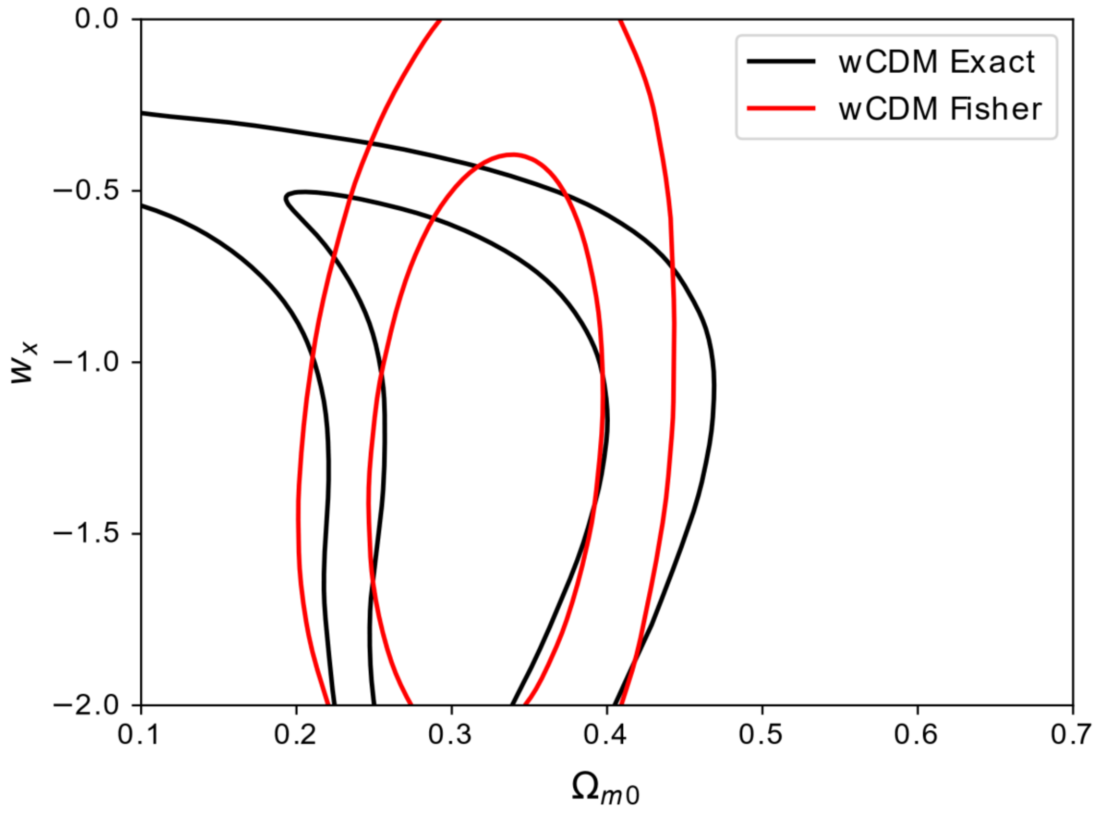
Beyond Fisher Forecasting for Cosmology



Joseph Ryan

Based on work in progress being conducted with Joel Meyers, Brandon Stevenson, and Cynthia Trendafilova (paper currently in preparation).

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Why forecasting?

» Upcoming cosmological surveys are expected to produce vast quantities of data.

» It is important to be able to accurately forecast the constraints those data can place on cosmological models, so that instrumental and computational time and resources can be used most effectively.

Fisher forecasting

» Forecasts of the parameter constraints on a given model often assume:

$$P = N \exp\left[-\frac{1}{2} F_{ab} \Delta p^a \Delta p^b\right]$$

» Where P is the posterior probability of the model, F_{ab} is known as the "Fisher matrix", and $\Delta p := p - p_{fid}$. p_{fid} is the fiducial value of the parameter p.

E. Sellentin, M. Quartin, and L. Amendola, "Breaking the spell of Gaussianity: forecasting with higher order Fisher matrices," Mon. Not. Roy. Astron. Soc. 441 no. 2, (2014) 1831–1840, arXiv:1401.6892 [astro-ph.CO].

Fisher forecasting

» Given a data covariance matrix $\overleftrightarrow{C}$ and a vector of model predictions $\vec{\mu}$, the Fisher matrix can be written in the form

$$F_{ab} := \vec{\mu}_{,a}\overleftrightarrow{M}\vec{\mu}_{,b}$$

» Where $\overleftrightarrow{M} = \overleftrightarrow{C}^{-1}$, the subscript "$,\mathrm{a}$" refers to a partial derivative taken with respect to the parameter p^a, and the model vectors are contracted with $\overleftrightarrow{M}$ in the data space (in Einstein notation, $\vec{\mu}_{,a}\overleftrightarrow{M}\vec{\mu}_{,b} = \mu^i_{,a}M_{ij}\mu^j_{,b}$ where i and j are indices in the data space).

E. Sellentin, M. Quartin, and L. Amendola, "Breaking the spell of Gaussianity: forecasting with higher order Fisher matrices," Mon. Not. Roy. Astron. Soc. 441 no. 2, (2014) 1831–1840, arXiv:1401.6892 [astro-ph.CO].

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Fisher forecasting

» Advantages of Fisher forecasting: speed and computational simplicity.

» Disadvantage: assumes that the components of $\vec{\mu}$ are linear in the model parameters.

» Bottom line: Fisher forecasting can, in some situations, produce oversimplified constraints.

E. Sellentin, M. Quartin, and L. Amendola, “Breaking the spell of Gaussianity: forecasting with higher order Fisher matrices,” Mon. Not. Roy. Astron. Soc. 441 no. 2, (2014) 1831–1840, arXiv:1401.6892 [astro-ph.CO].

Beyond Fisher forecasting

» For an arbitrary posterior P, the Derivative Approximation for Likelihoods (DALI), to second order in the model vector derivatives, is

$$P = N \exp\left[-\frac{1}{2}F_{ab}\Delta p^a \Delta p^b - \frac{1}{2}G_{abc}\Delta p^a \Delta p^b \Delta p^c - \frac{1}{8}H_{abcd}\Delta p^a \Delta p^b \Delta p^c \Delta p^d\right],$$

» where $G_{abc} := \vec{\mu}_{,ab}\overleftrightarrow{M}\vec{\mu}_{,c}$, $H_{abcd} := \vec{\mu}_{,ab}\overleftrightarrow{M}\vec{\mu}_{,cd}$, and N is a normalization constant.

» Note: this order of approximation is known as “Doublet-DALI”, or “Doublet”, for short.

E. Sellentin, M. Quartin, and L. Amendola, “Breaking the spell of Gaussianity: forecasting with higher order Fisher matrices,” Mon. Not. Roy. Astron. Soc. 441 no. 2, (2014) 1831–1840, `arXiv:1401.6892 [astro-ph.CO]`.

Beyond Fisher forecasting at low redshift

» Flat ΛCDM, characterized by the Hubble parameter

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}$$

» Flat wCDM, characterized by the Hubble parameter

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})(1+z)^{3(1+w_X)}}$$

Low redshift data

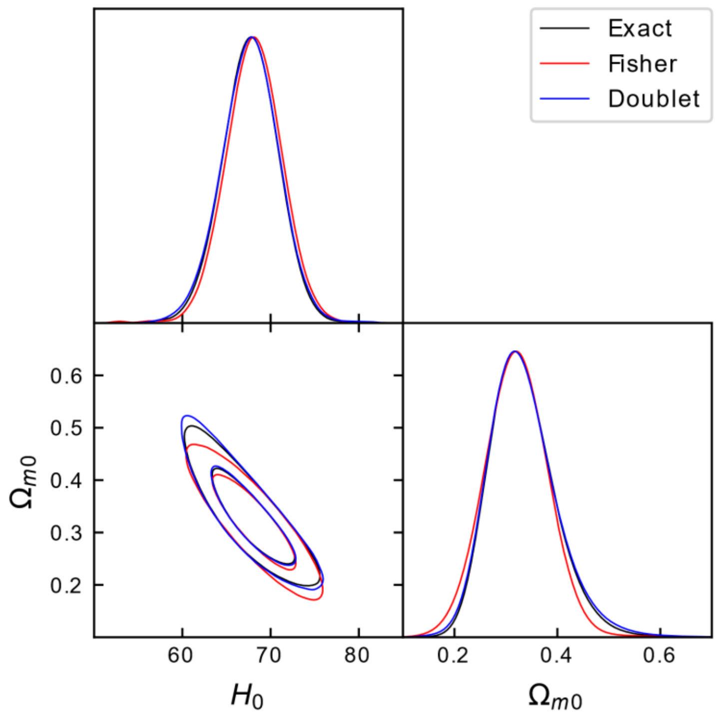
» 31 $H(z)$ measurements from cosmic chronometers.

» 6 comoving distance and $H(z)$ measurements from BAO.

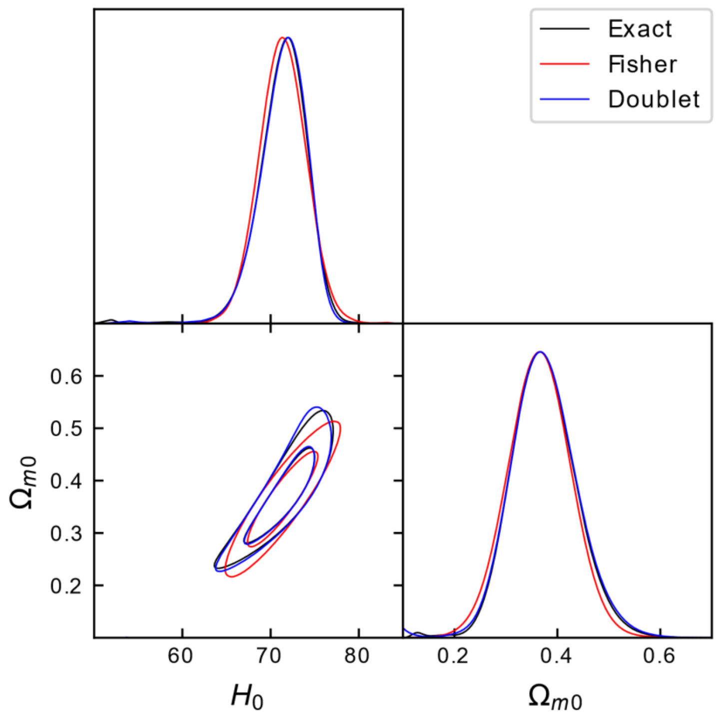
» 120 standard ruler measurements from QSOs.

See J. Ryan, Y. Chen, and B. Ratra, "Baryon acoustic oscillation, Hubble parameter, and angular size measurement constraints on the Hubble constant, dark energy dynamics, and spatial curvature," MNRAS 488 no. 3, (Sept., 2019) 3844–3856, `arXiv:1902.03196[astro-ph.CO]`, and J. Ryan, S. Doshi, and B. Ratra, "Constraints on dark energy dynamics and spatial curvature from Hubble parameter and baryon acoustic oscillation data," MNRAS 480 (Oct., 2018) 759–767, `astro-ph/1805.06408` for details.

Constraints on Flat ΛCDM



H(z) data only



6BAO data

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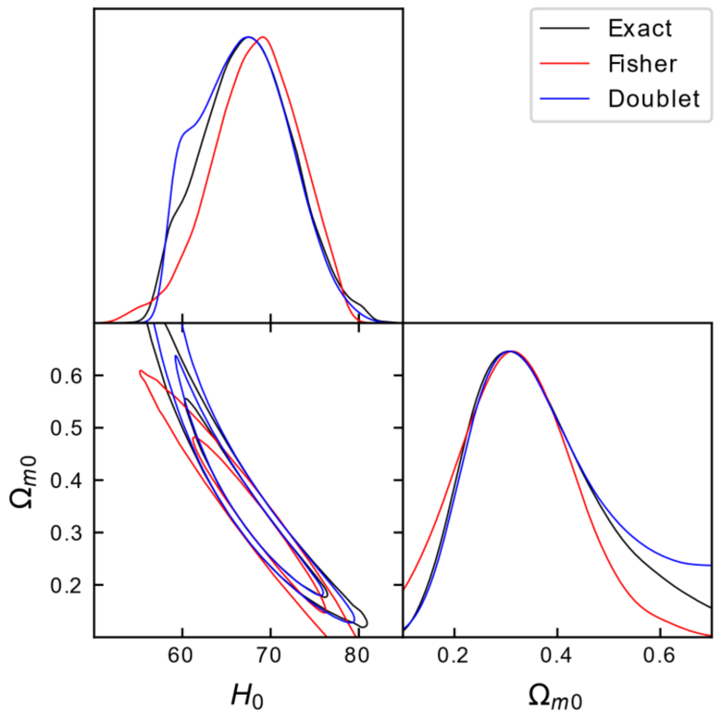
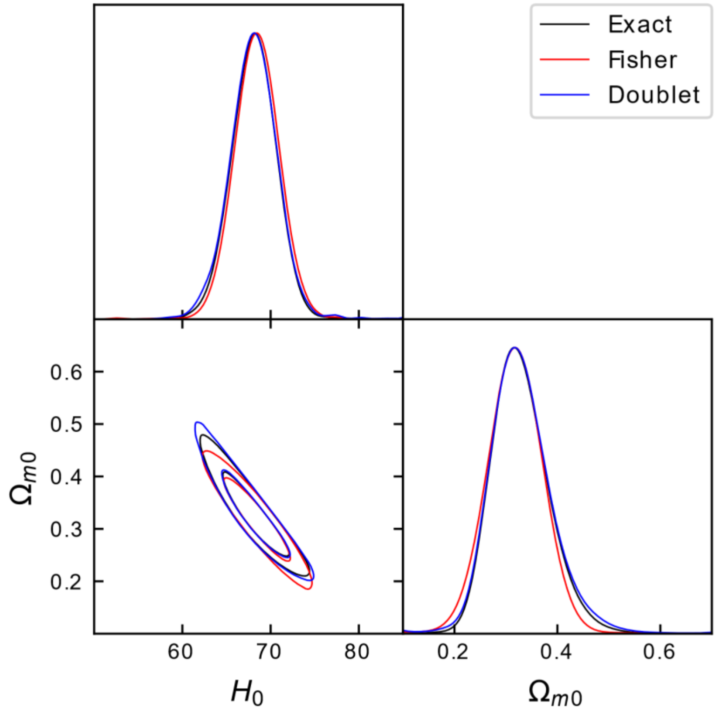
Constraints on Flat ΛCDM

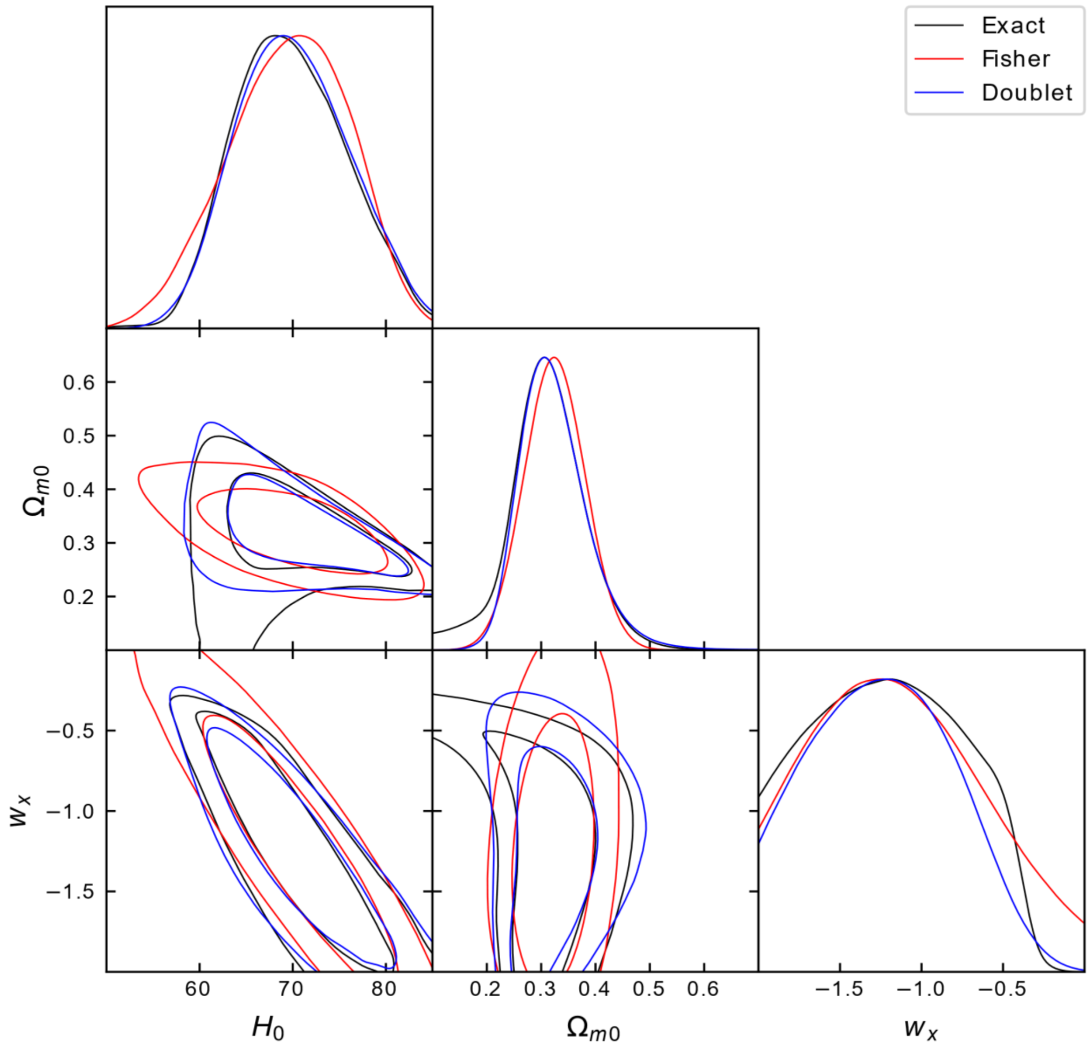


QSO data only

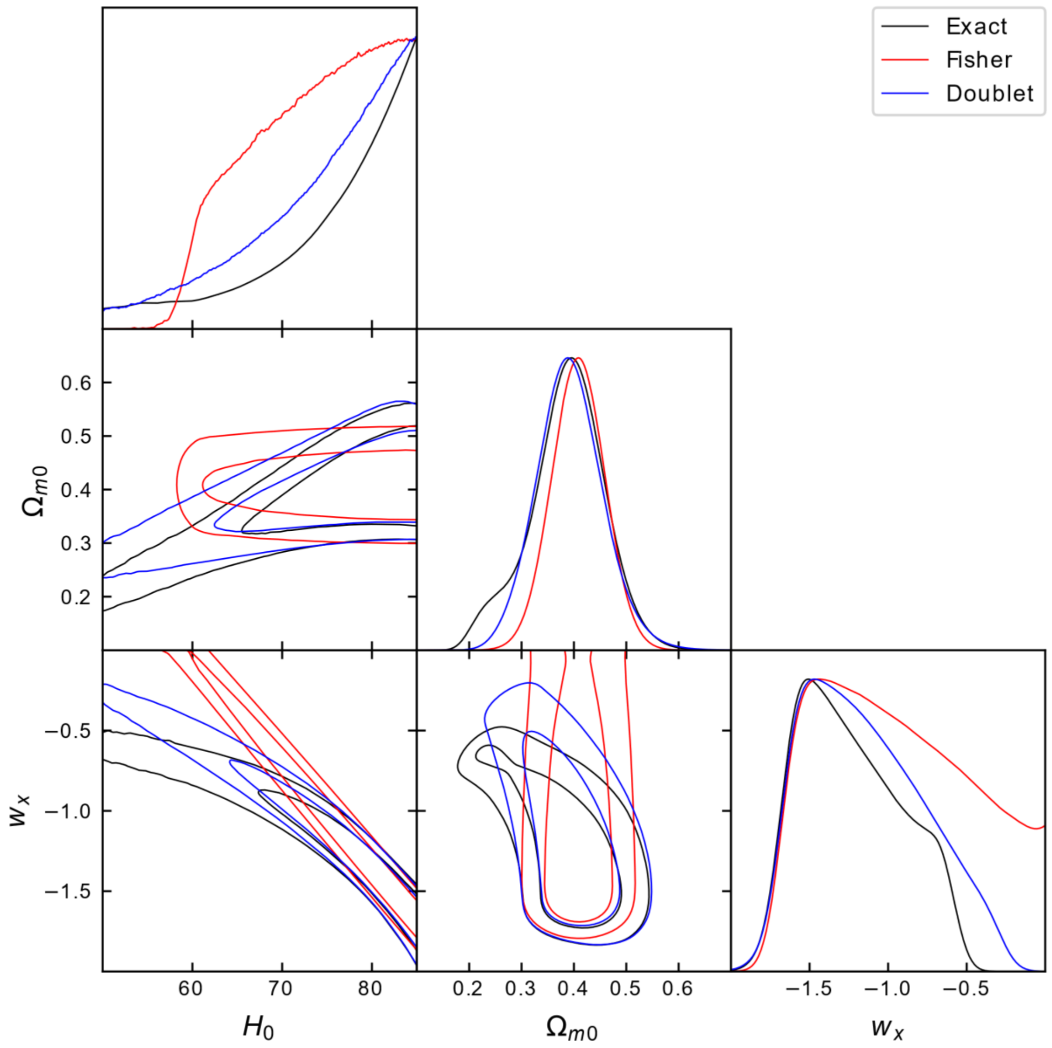


H(z) + QSO data

Constraints on flat wCDM

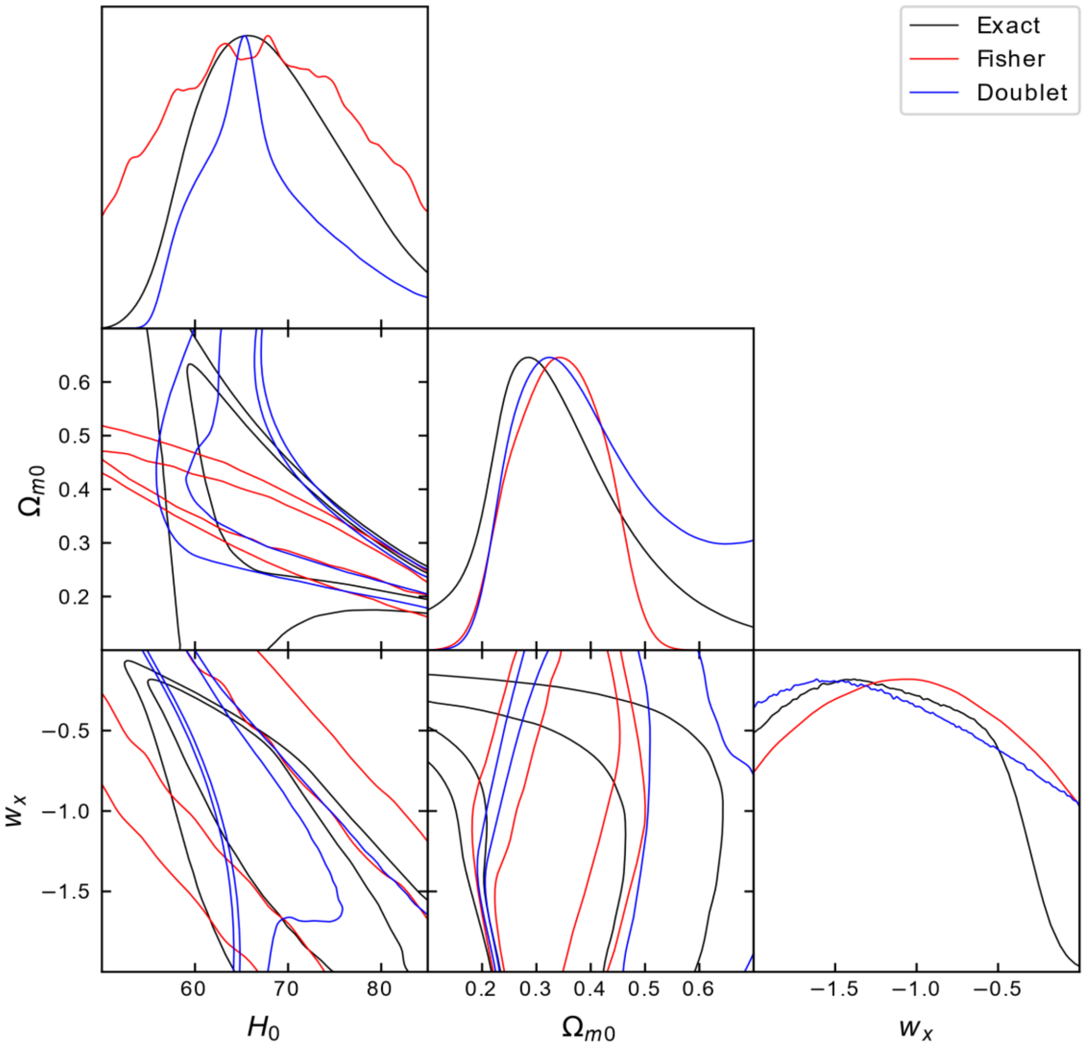


H(z) data

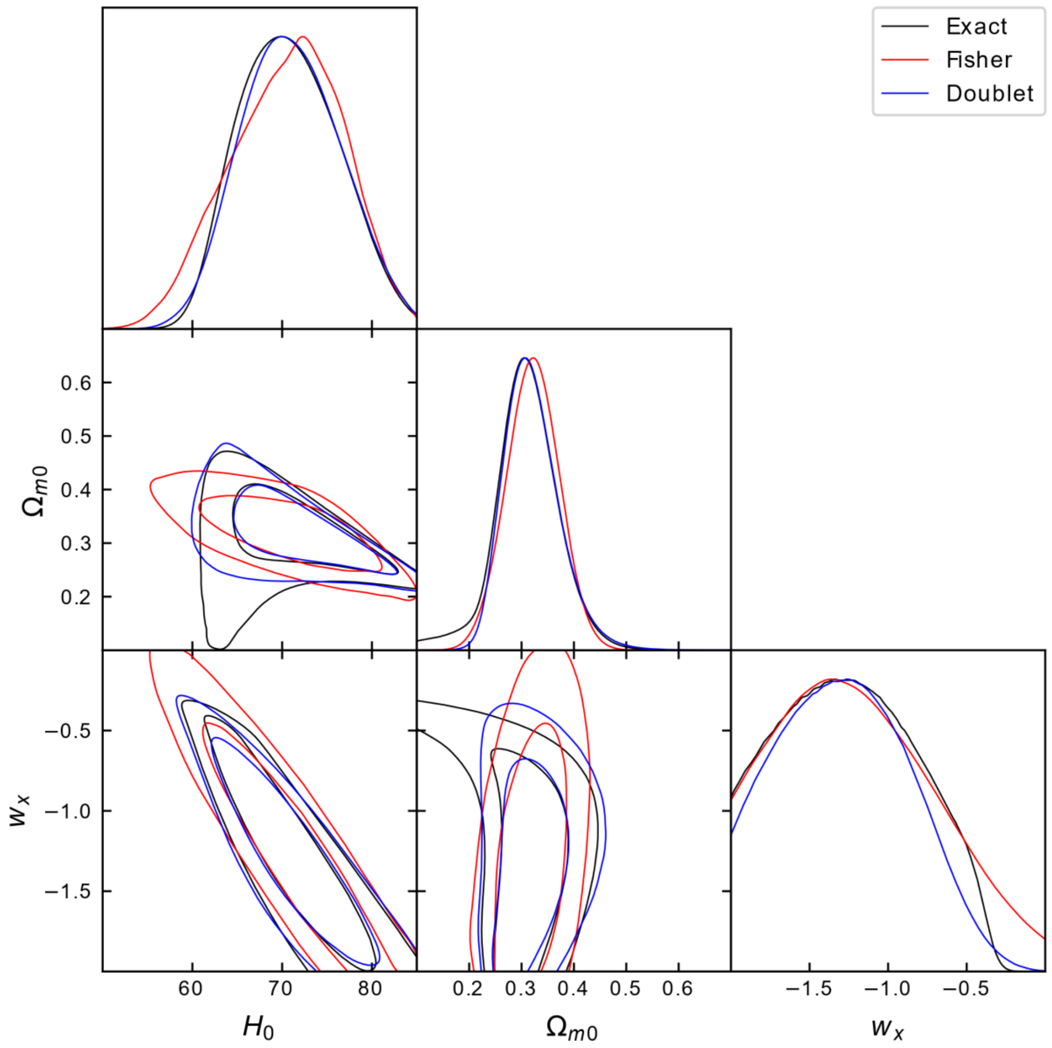


6BAO data

Constraints on flat wCDM



QSO data



H(z) + QSO data

When do we need to use DALI?

» If exact likelihood is concave in one or more directions, Fisher approximation will break down.

» Not easy to tell, a priori, when this will happen. Therefore, it’s not easy to tell, ahead of time, whether we need to use the DALI approximation.

» Can we quantify the difference between Fisher and DALI without sampling?

When do we need to use DALI?

- Think of likelihood contours as (D-1)-dimensional hypersurfaces embedded D-dimensional parameter space:

$$\mathcal{L}(p^a) \coloneqq -\ln[P(p^a)] = C$$

- C is a constant, and $P(p^a)$ is the posterior probability as a function of the model parameters $\{p^a\}$. This is a constraint equation of the form

$$\Phi(p^a) = 0$$

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When do we need to use DALI?

» We can define a unit vector that is normal to the likelihood hypersurfaces:

$$n_a = \frac{\Phi_{,a}}{\sqrt{g^{ab}\Phi_{,a}\Phi_{,b}}}$$

» Where $g^{ab} = \delta^{ab}$. The divergence of the unit normal vector field is

$$K = n^a_{,a}$$

E. Poisson, *A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics*. Cambridge University Press, Cambridge, UK, 2004

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When do we need to use DALI?

» The divergence $K = h^{AB}K_{AB}$ is the trace of the extrinsic curvature K_{AB}, where "A" and "B" refer to coordinates on the hypersurface, and h_{AB} is the metric on the hypersurface.

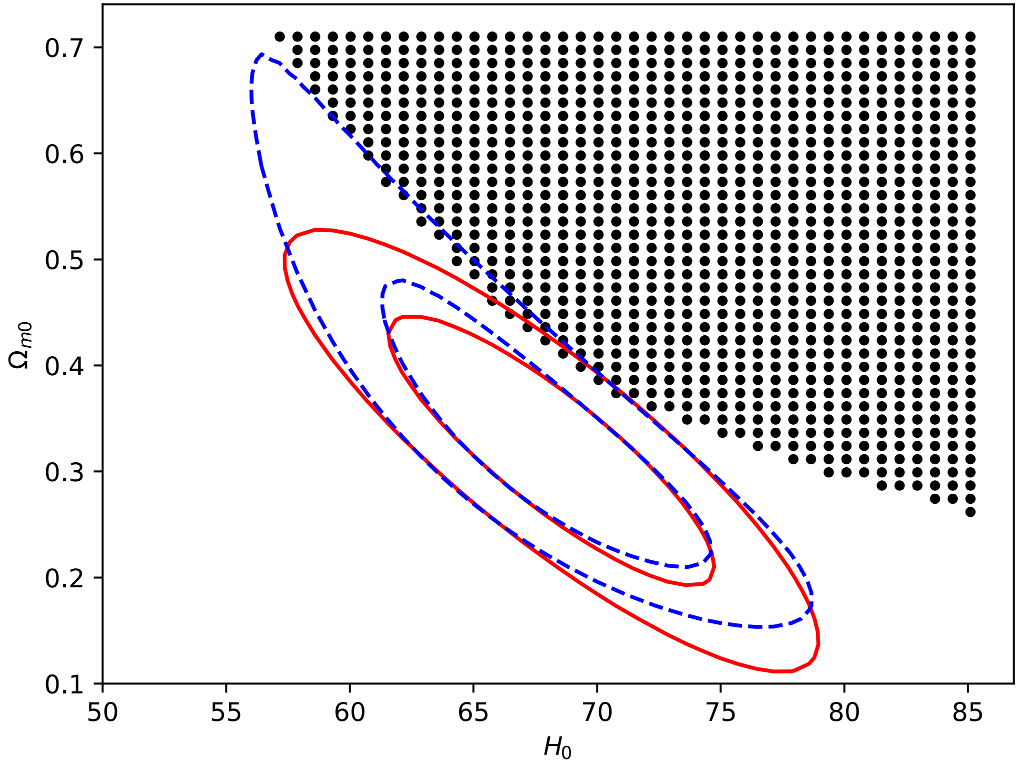
» If $K > 0$ at a given point, then the hypersurface is convex there. On the other hand, if $K < 0$ at a given point, then the hypersurface is concave there.

E. Poisson, *A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics*. Cambridge University Press, Cambridge, UK, 2004

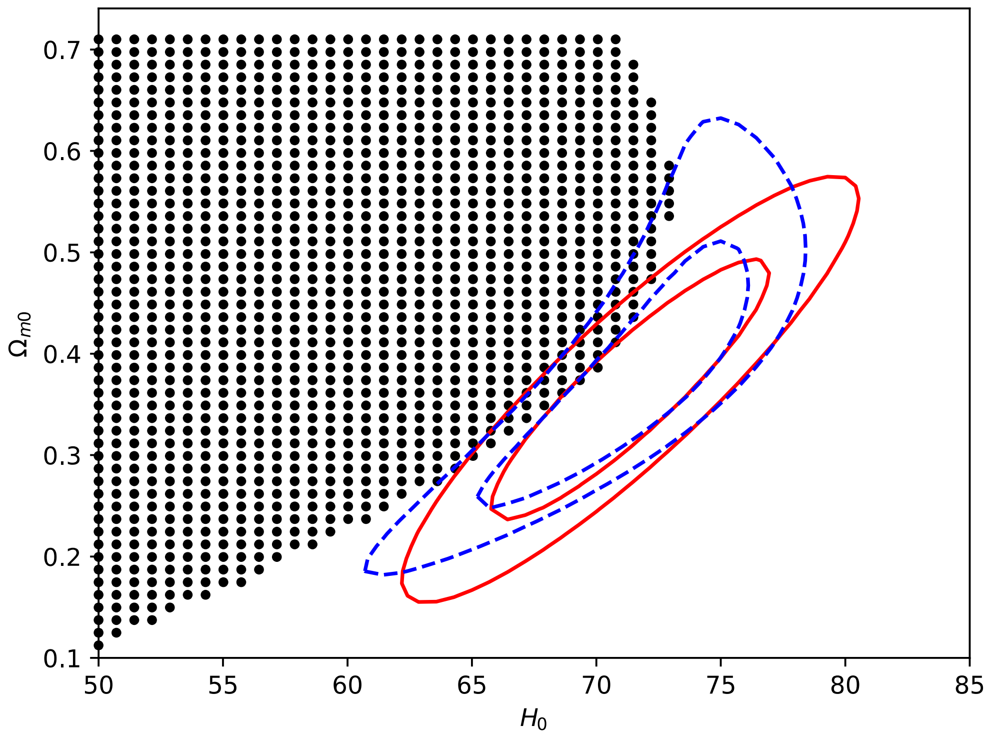
Extrinsic curvature test

- Step 1: compute K on grids covering every possible cross-section of the exact posterior.
  - Why cross-sections? K can be computed very quickly in two dimensions, with results that are easier to interpret.
- Step 2: Plot cross-sections on which negative (or zero) curvature has been detected, visually inspect plots to determine extent of deviation from Fisher.
  - Alternatively, compute K/K_F, where K_F is the curvature of the Fisher hypersurfaces.

Extrinsic curvature test applied to flat ΛCDM



H(z) data
Solid red contours: Fisher
Dashed blue contours: doublet-DALI
Black dots: points where $K \leq 0$



BAO data
Solid red contours: Fisher
Dashed blue contours: doublet-DALI
Black dots: points where $K \leq 0$

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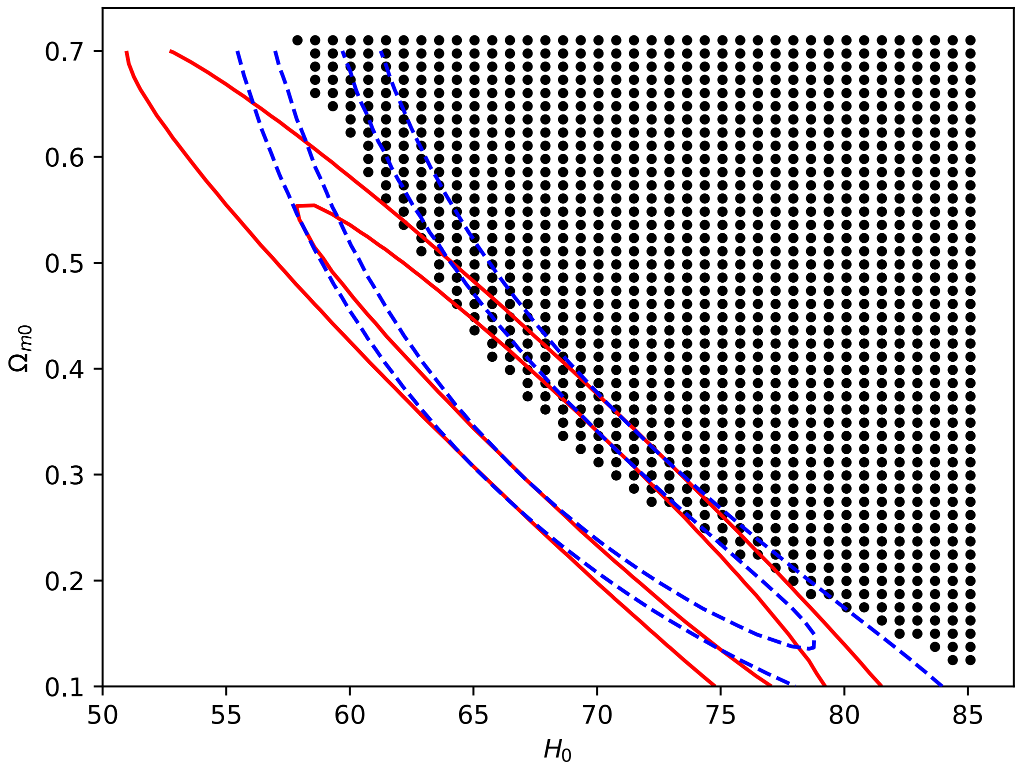
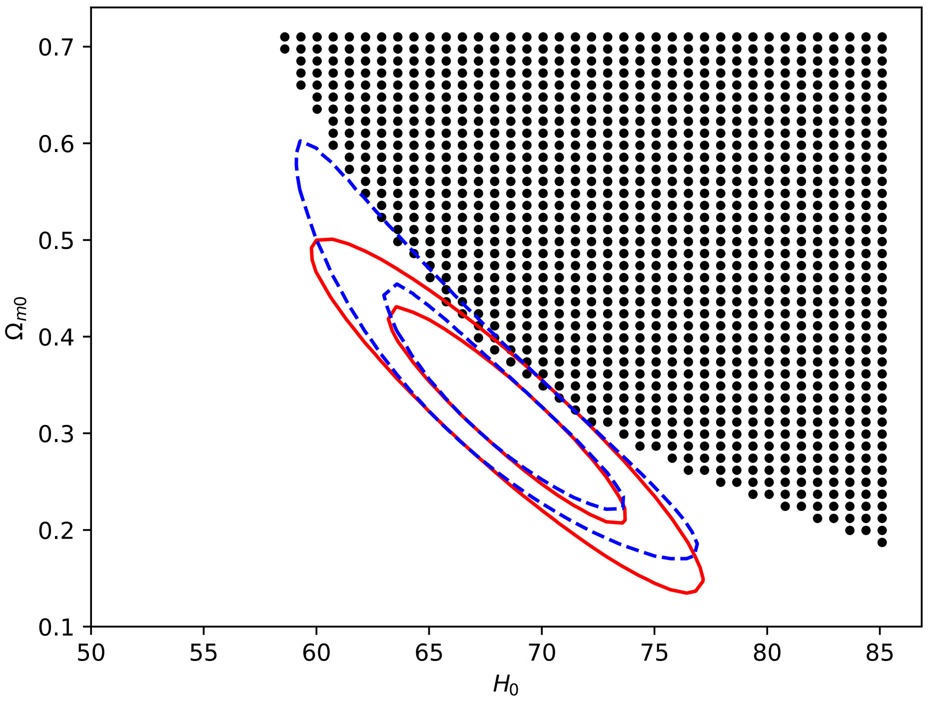
Extrinsic curvature test applied to flat ΛCDM

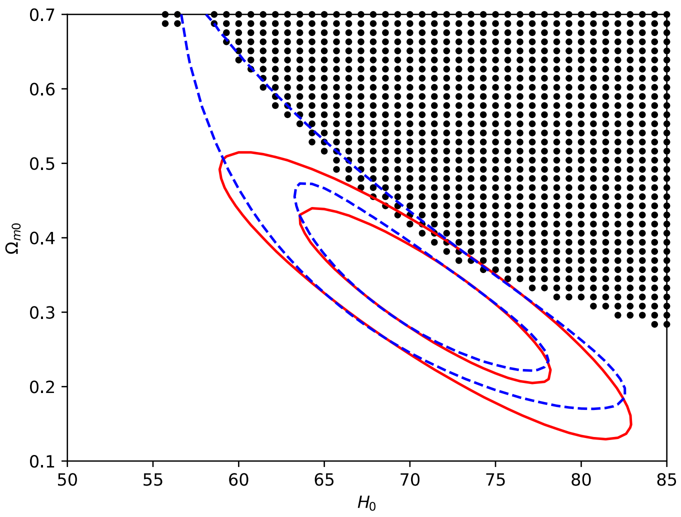


QSO data
Solid red contours: Fisher
Dashed blue contours: doublet-DALI
Black dots: points where $K \leq 0$

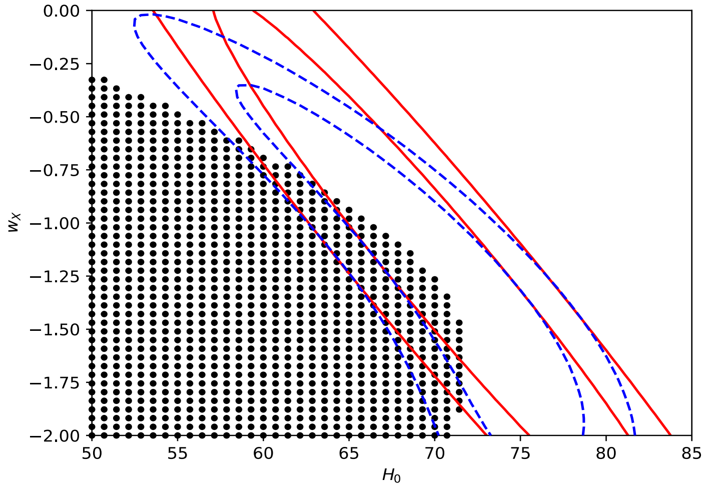


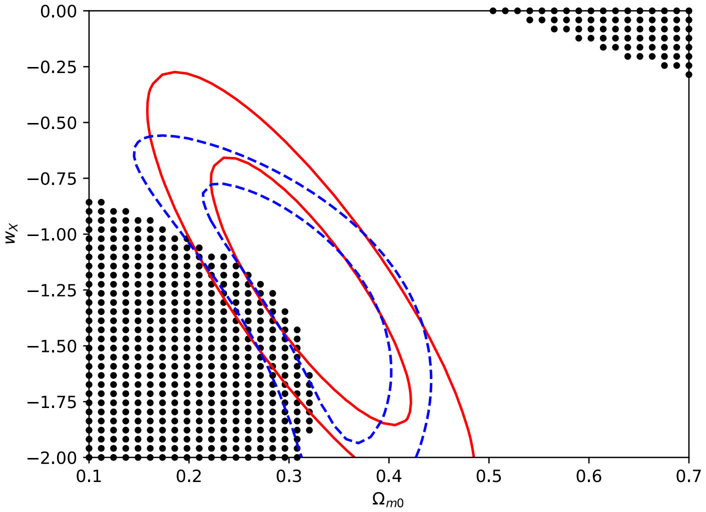
H(z) + QSO data
Solid red contours: Fisher
Dashed blue contours: doublet-DALI
Black dots: points where $K \leq 0$

Extrinsic curvature test applied to flat wCDM

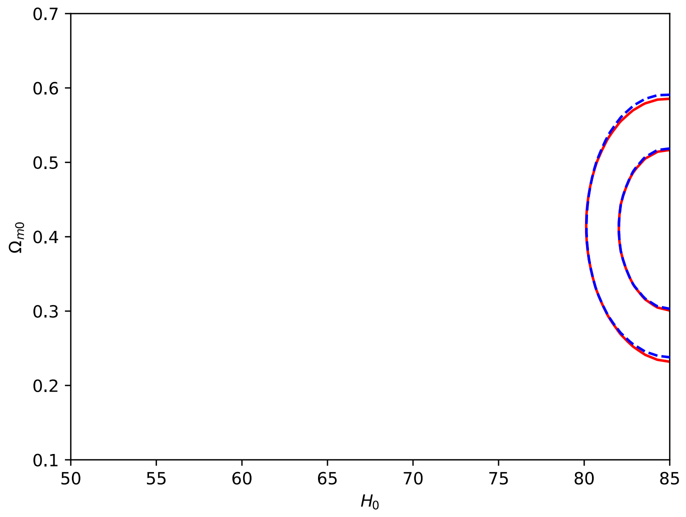


H(z) data
Solid red contours: Fisher
Dashed blue contours: doublet
Black dots: points where $K \leq 0$.

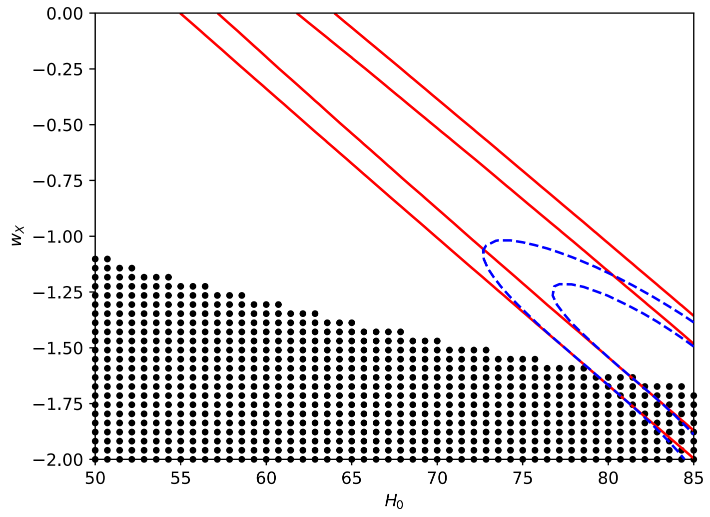


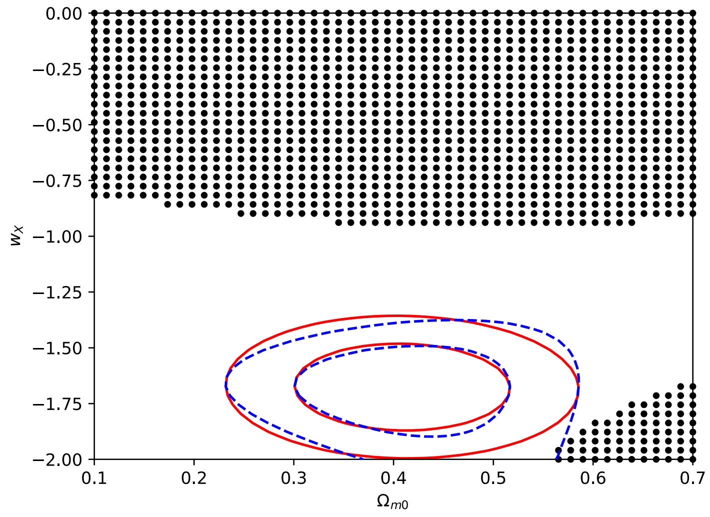


Extrinsic curvature test applied to flat wCDM

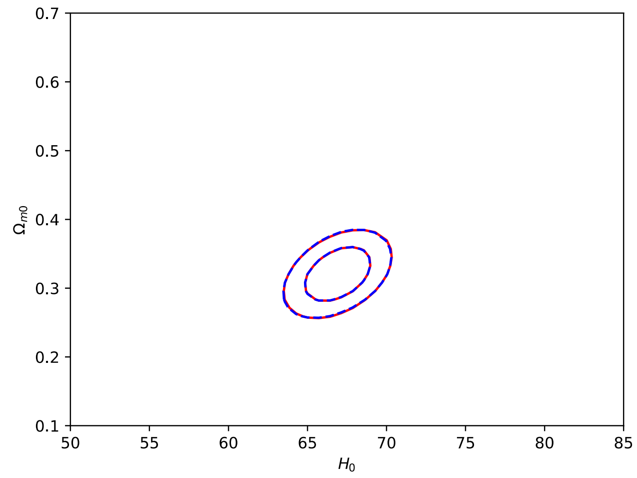


BAO data
Solid red contours: Fisher
Dashed blue contours: doublet
Black dots: points where $K \leq 0$.

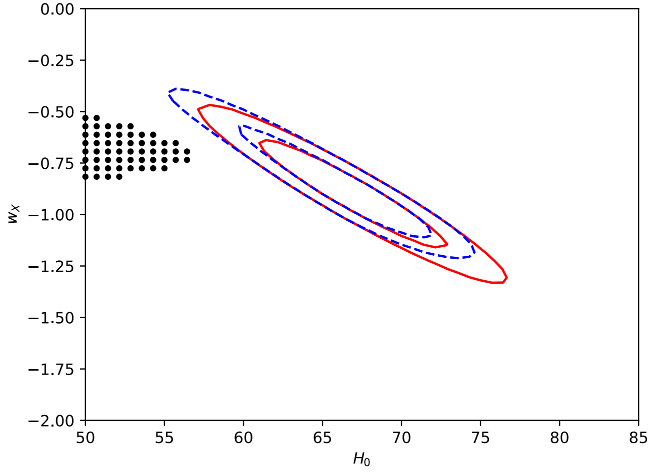


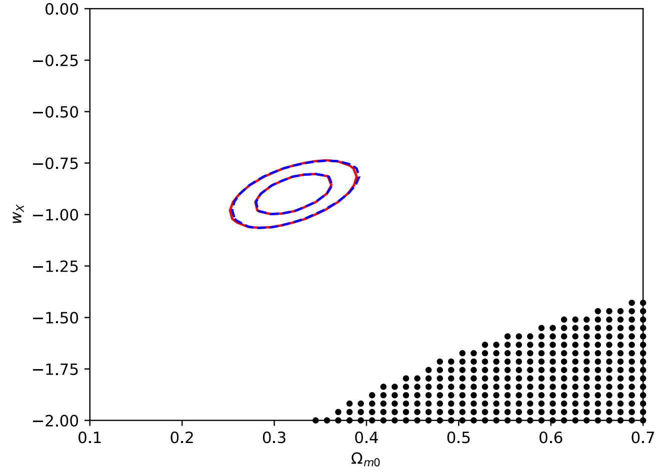


Extrinsic curvature test applied to flat wCDM



H(z) + BAO data
Solid red contours: Fisher
Dashed blue contours: doublet
Black dots: points where $K \leq 0$.





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Conclusion

- Testing the DALI approach on real data, we found it to be particularly useful in cases for which the data have little constraining power, and there are strong degeneracies between model parameters.

- We have also demonstrated a simple test that can be used to assess whether it is necessary to go beyond the Fisher approximation when forecasting parameter constraints.

Acknowledgements

» Computational resources for this research were provided by SMU's Center for Research Computing.

» Funding for this research was provided by DOE grant DE-SC0010129.