Error Analysis for the LHC Beam Optics

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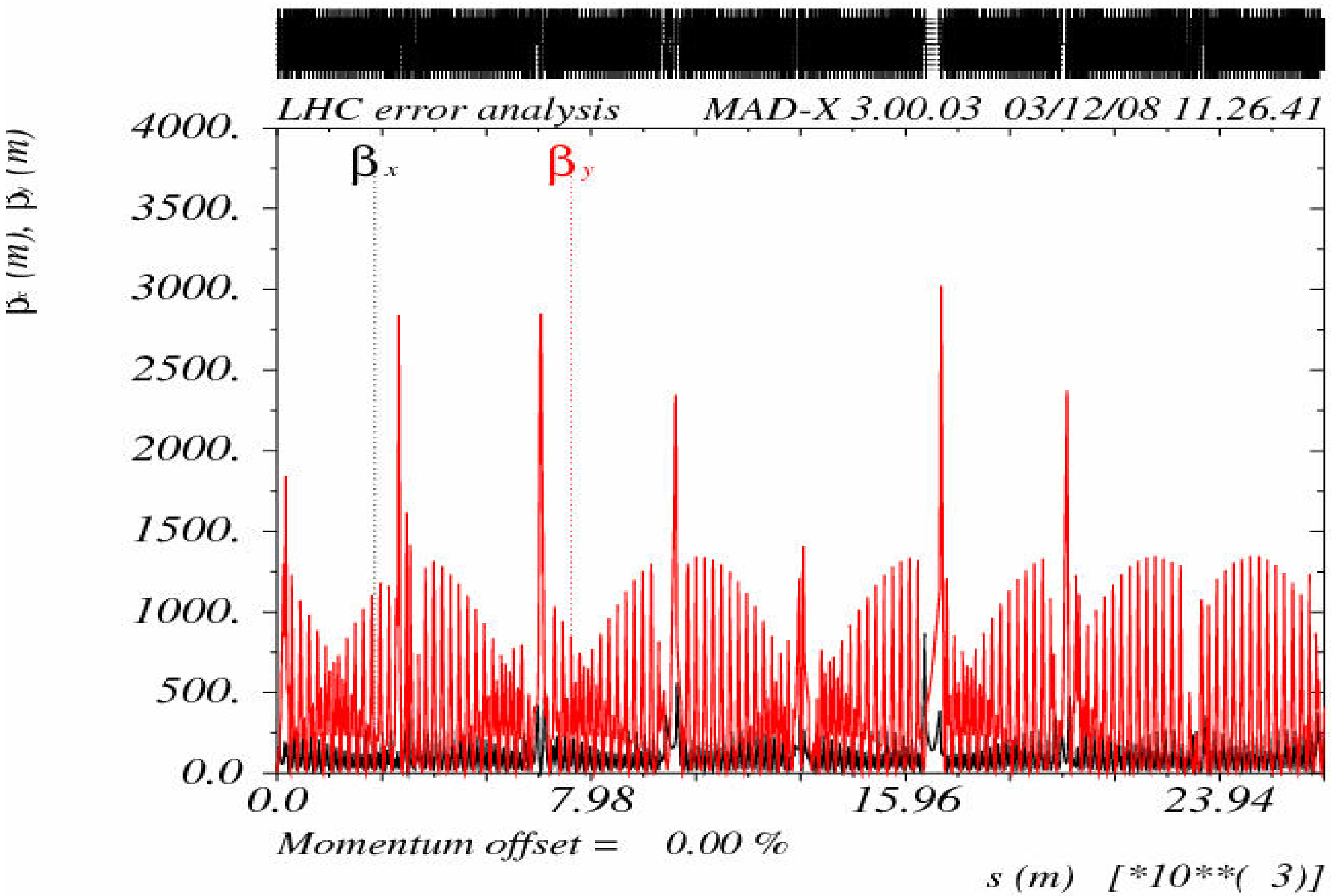
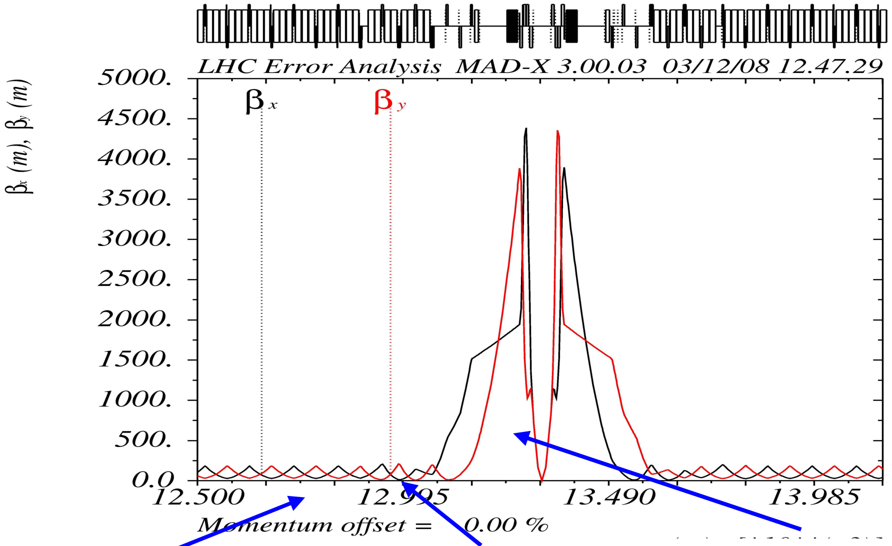
Quadrupole Error

... about things that should not happen and still occur from time to time.

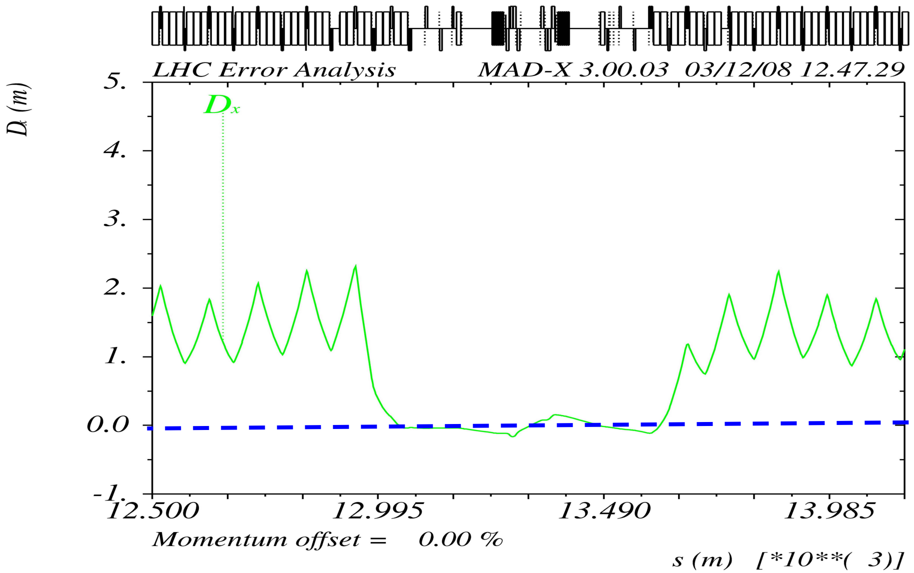


Basic Layout of the Machine



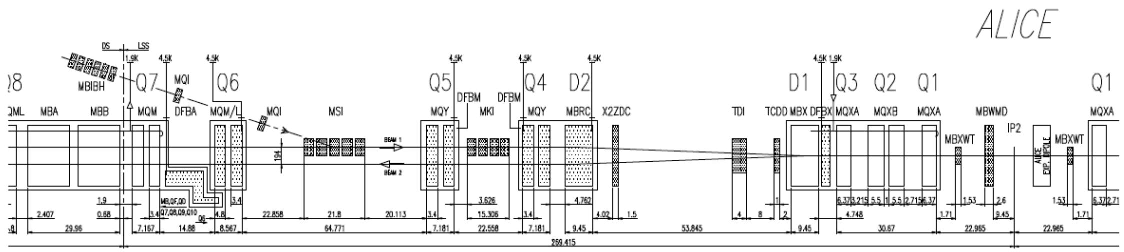
LHC Luminosity Optics in IR_1 / 5

Arc ... Dispersion Suppressor ... Matching Section ... Triplett

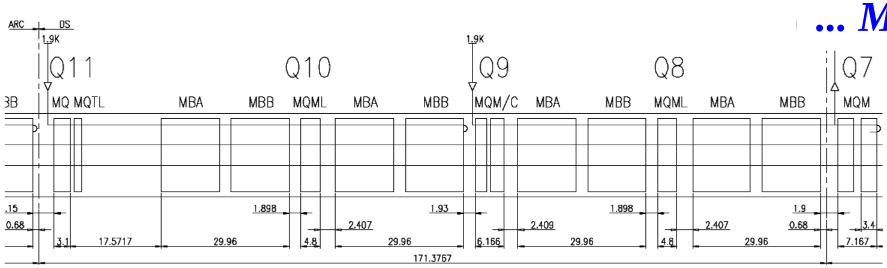


LHC Lattice Layout

IR layout



... Matching Section ***Triplett***



Dispersion Suppressor ...

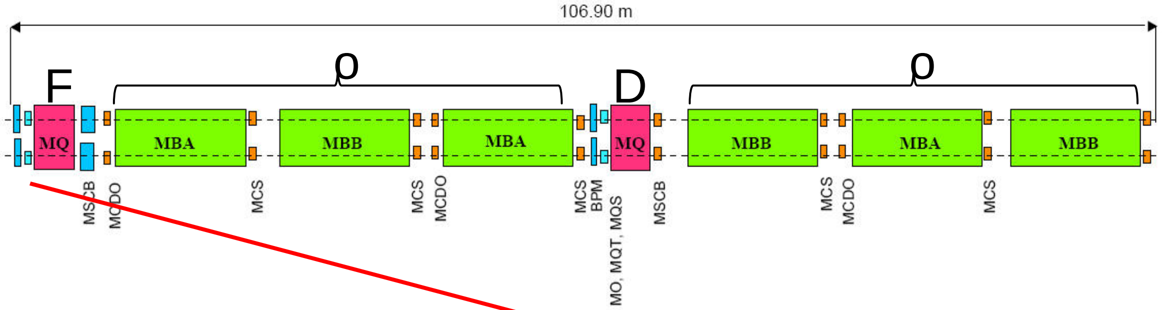


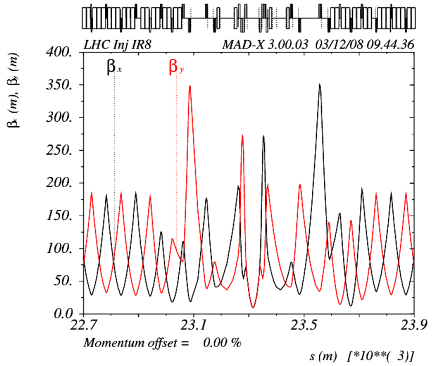
Table 3.3: *Magn*

	LSS													
	low-β triplet			MS					DS				arc-cell	
Magnet	Q1	Q2	Q3	Q4	Q5L	Q5R	Q6	Q7	Q8	Q9	Q10	QT11	QT12	QT13
#	1	2	1	2					1	2	1		1	
Type: MQ-	XL	X	XL	Y	Y	M	M		ML	MC	ML	TL	T	
L [m]	6.3	5.5	6.3	3.4					4.8	3.4 2.4	4.8	1.15	0.32	
T [K]	1.9			4.5			1.9		1.9				1.9	
B [T/m]	215 → 220			160			200		200			110	110	
r [mm]	22.2 17.3	28.95 24.05		27.2 22.3	27.2 22.3	20.6 15.75		22.2 17.3	22.2 17.3				22.2 17.3	

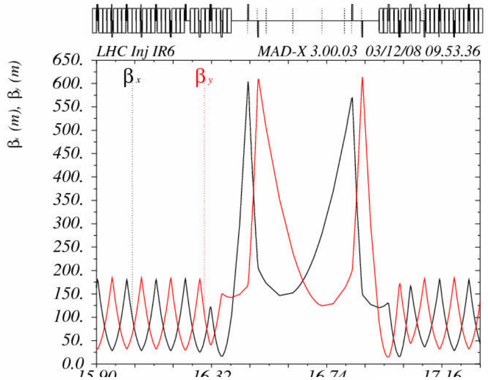
individuallly powered quadrupoles and trims at the beginning of the arc structure

LHC: ... it is a messsss: eight IR's

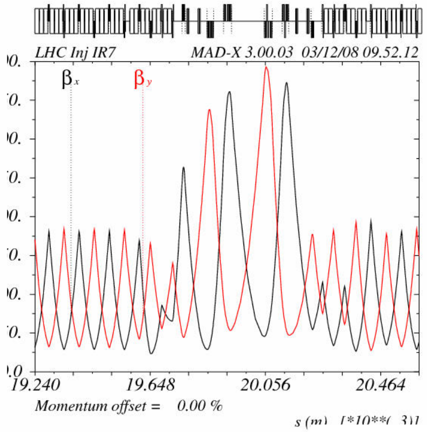
with individual lattices (i.e. quad positions),
optics (i.e. quad values)
with individual beat functions



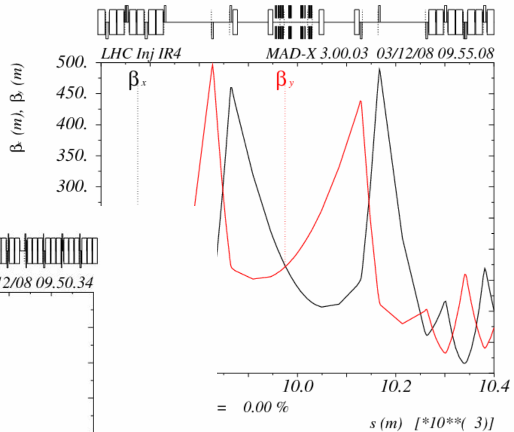
IR_8



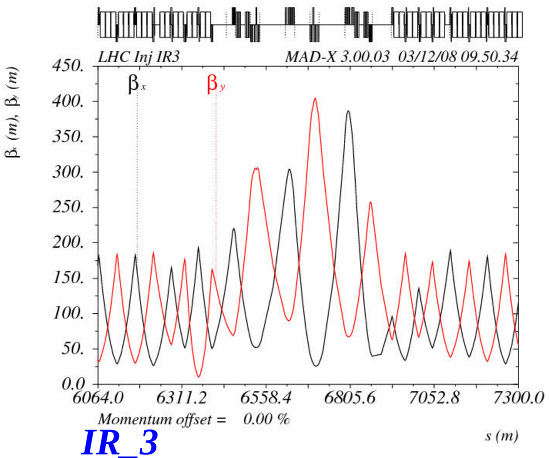
IR_6



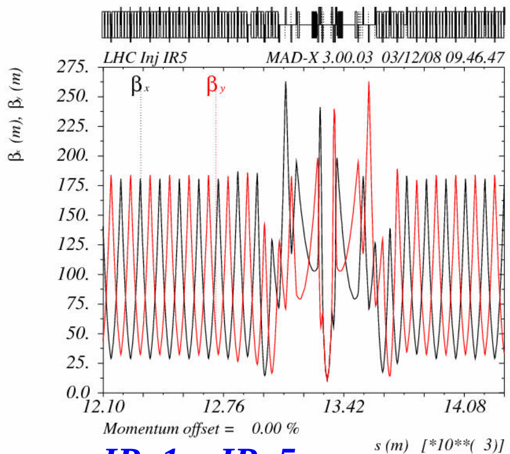
IR_7



IR_4



IR_3



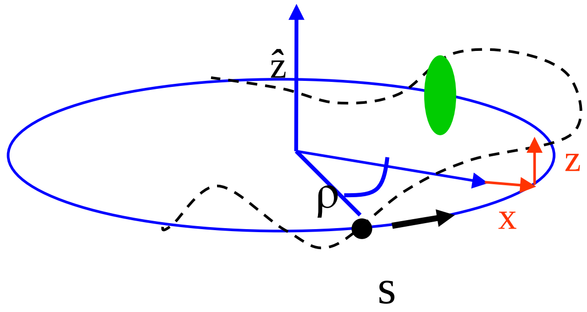
IR_1 = IR_5

Quadrupole Errors

optic perturbation described by thin lens quadrupole

$$M_{dist} = M_{\Delta k} \cdot M_0 = \begin{pmatrix} 1 & 0 \\ \Delta k ds & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\psi_{turn} + \alpha\sin\psi_{turn} & \beta\sin\psi_{turn} \\ -\gamma\sin\psi_{turn} & \cos\psi_{turn} - \alpha\sin\psi_{turn} \end{pmatrix}$$

quad error — *ideal storage ring*



$$M_{dist} = \begin{pmatrix} \cos\psi_0 + \alpha\sin\psi_0 & \beta\sin\psi_0 \\ \Delta k ds(\cos\psi_0 + \alpha\sin\psi_0) - \gamma\sin\psi_0 & \Delta k ds\,\beta\sin\psi_0 + \cos\psi_0 - \alpha\sin\psi_0 \end{pmatrix}$$

rule for getting the tune

$$Trace(M) = 2\cos\psi = 2\cos\psi_0 + \Delta k ds\,\beta\sin\psi_0$$

Quadrupole error → Tune Shift

$$\psi = \psi_0 + \Delta\psi \quad \longrightarrow \quad \cos(\psi_0 + \Delta\psi) = \cos\psi_0 + \frac{\Delta k ds\, \beta \sin\psi_0}{2}$$

remember the old fashioned trigonometric stuff and assume that the error is small !!!

$$\cos\psi_0 \underbrace{\cos\Delta\psi}_{\approx 1} - \sin\psi_0 \underbrace{\sin\Delta\psi}_{\approx \Delta\psi} = \cos\psi_0 + \frac{kds\,\beta \sin\psi_0}{2}$$

$$\Delta\psi = \frac{kds\,\beta}{2}$$

and referring to Q instead of ψ:

$$\psi = 2\pi Q$$

$$\Delta Q = \int_{s0}^{s0+l} \frac{\Delta k(s)\beta(s)ds}{4\pi}$$

... for small errors only !!!

the tune shift is proportional

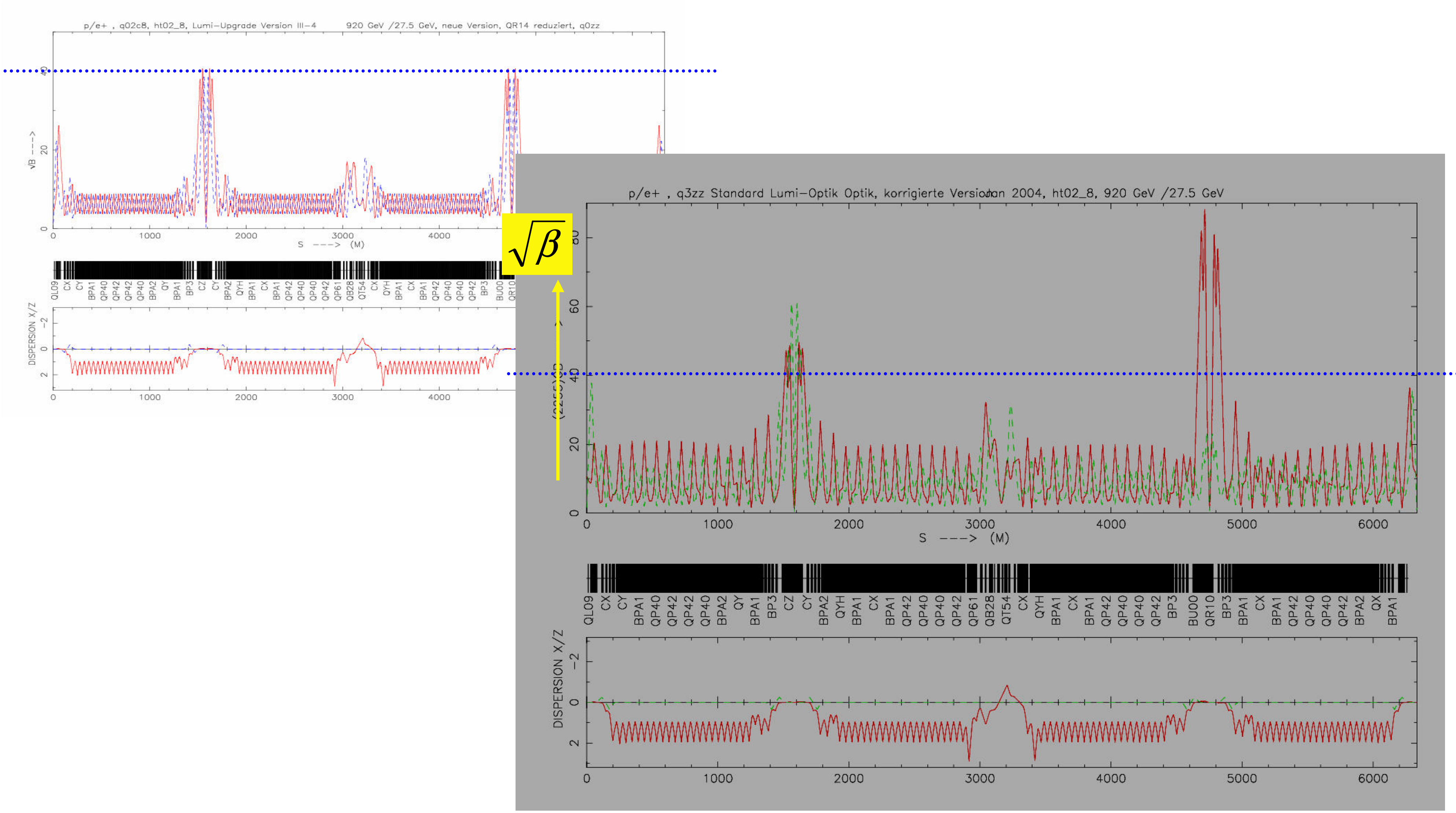
! *... to the β-function at the quadrupole : $\beta \approx 50$ m in the arc, $\beta \approx 500$ m in the matching section*

!! *to the number of quadrupoles in series*

!!! *to the strength of the quadrupole*

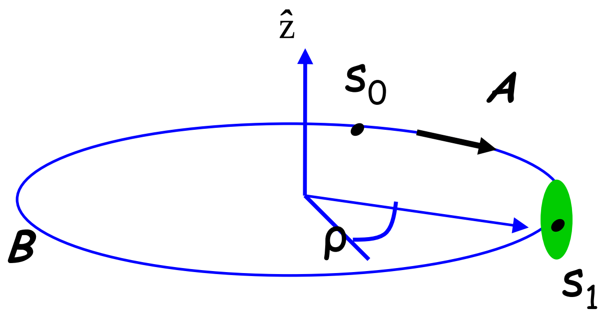
Quadrupole error: Beta Beat

$$\Delta\beta(s_0) = \frac{\beta_0}{2\sin 2\pi Q} \int_{s1}^{s1+l} \beta(s_1)\Delta K \cos\left(2\left|\psi_{s1} - \psi_{s0}\right| - 2\pi Q\right) ds$$



Quadrupole Errors and Beta Function

a quadrupole error will not only influence the oscillation frequency … „tune"
… but also the amplitude … „beta function"



$$M_{turn} = B * A \qquad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

distorted matrix $M_{dist} = \begin{pmatrix} m^*_{11} & m^*_{12} \\ m^*_{21} & m^*_{22} \end{pmatrix} = B \begin{pmatrix} 1 & 0 \\ -\Delta kds & 1 \end{pmatrix} A$

$$M_{dist} = B \begin{pmatrix} a_{11} & a_{12} \\ -\Delta kdsa_{11} + a_{12} & -\Delta kdsa_{12} + a_{22} \end{pmatrix}$$

$$M_{dist} = \begin{pmatrix} \sim & b_{11}a_{12} + b_{12}(-\Delta kdsa_{12} + a_{22}) \\ \sim & \sim \end{pmatrix}$$

the beta function is usually obtained via the matrix element „m12", which is in Twiss form for the undistorted case

$$m_{12} = \beta_0 \sin 2\pi Q$$

and including the error:

$$m_{12}^{*} = \underbrace{b_{11}a_{12} + b_{12}a_{22}}_{m_{12} = \beta_0 \sin 2\pi Q} - b_{12}a_{12}\Delta k ds$$

$$(1) \quad m_{12}^{*} = \beta_0 \sin 2\pi Q - a_{12}b_{12}\Delta k ds$$

As M is still a matrix for one complete turn we still can express the nt m_{12} in twiss form:*

$$(2) \quad m_{12}^{*} = (\beta_0 + d\beta) * \sin 2\pi (Q + dQ)$$

Equalising (1) and (2) and assuming a small error

$$\beta_0 \sin 2\pi Q - a_{12}b_{12}\Delta k ds = (\beta_0 + d\beta) * \sin 2\pi (Q + dQ)$$

$$\beta_0 \sin 2\pi Q - a_{12}b_{12}\Delta k ds = (\beta_0 + d\beta) * \sin 2\pi Q \underbrace{\cos 2\pi dQ}_{\approx 1} + \cos 2\pi Q \underbrace{\sin 2\pi dQ}_{\approx 2\pi dQ}$$

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds = \beta_0 \sin 2\pi Q + \beta_0 2\pi dQ \cos 2\pi Q + d\beta_0 \sin 2\pi Q + d\beta_0 2\pi dQ \cos 2\pi Q$$

ignoring second order terms

$$-a_{12} b_{12} \Delta k ds = \beta_0 2\pi dQ \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

remember: tune shift dQ due to quadrupole error:
(index „1" refers to location of the error)

$$dQ = \frac{\Delta k \beta_1 ds}{4\pi}$$

$$-a_{12} b_{12} \Delta k ds = \frac{\beta_0 \Delta k \beta_1 ds}{2} \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

solve for dβ

$$d\beta_0 = \frac{-1}{2 \sin 2\pi Q} \left\{ 2 a_{12} b_{12} + \beta_0 \beta_1 \cos 2\pi Q \right\} \Delta k ds$$

express the matrix elements a_{12}, b_{12} in Twiss form

$$M = \begin{pmatrix} \sqrt{\frac{\beta_s}{\beta_0}} \left(\cos\psi_s + \alpha_0 \sin\psi_s \right) & \sqrt{\beta_s \beta_0} \sin\psi_s \\ \frac{(\alpha_0 - \alpha_s)\cos\psi_s - (1 + \alpha_0 \alpha_s)\sin\psi_s}{\sqrt{\beta_s \beta_0}} & \sqrt{\frac{\beta_0}{\beta s}} \left(\cos\psi_s - \alpha_s \sin\psi_s \right) \end{pmatrix}$$

$$d\beta_0 = \frac{-1}{2\sin 2\pi Q}\left\{2a_{12}b_{12} + \beta_0\beta_1 \cos 2\pi Q\right\}\Delta k ds$$

$$\boldsymbol{a}_{12} = \sqrt{\boldsymbol{\beta}_0\boldsymbol{\beta}_1}\sin\Delta\boldsymbol{\psi}_{0\to 1}$$

$$\boldsymbol{b}_{12} = \sqrt{\boldsymbol{\beta}_1\boldsymbol{\beta}_0}\sin(2\pi\boldsymbol{Q} - \Delta\boldsymbol{\psi}_{0\to 1})$$

$$d\beta_0 = \frac{-\beta_0\beta_1}{2\sin 2\pi Q}\left\{2\sin\Delta\psi_{12}\sin(2\pi Q - \Delta\psi_{12}) + \cos 2\pi Q\right\}\Delta k ds$$

... after some TLC transformations ... $= \cos(2\Delta\psi_{01} - 2\pi Q)$

$$\Delta\beta(s_0) = \frac{-\beta_0}{2\sin 2\pi Q}\int_{s1}^{s1+l}\beta(s_1)\Delta k\cos\left(2(\psi_{s1} - \psi_{s0}) - 2\pi Q\right)ds$$

Nota bene:

! the beta beat is proportional to the strength of the error Δk

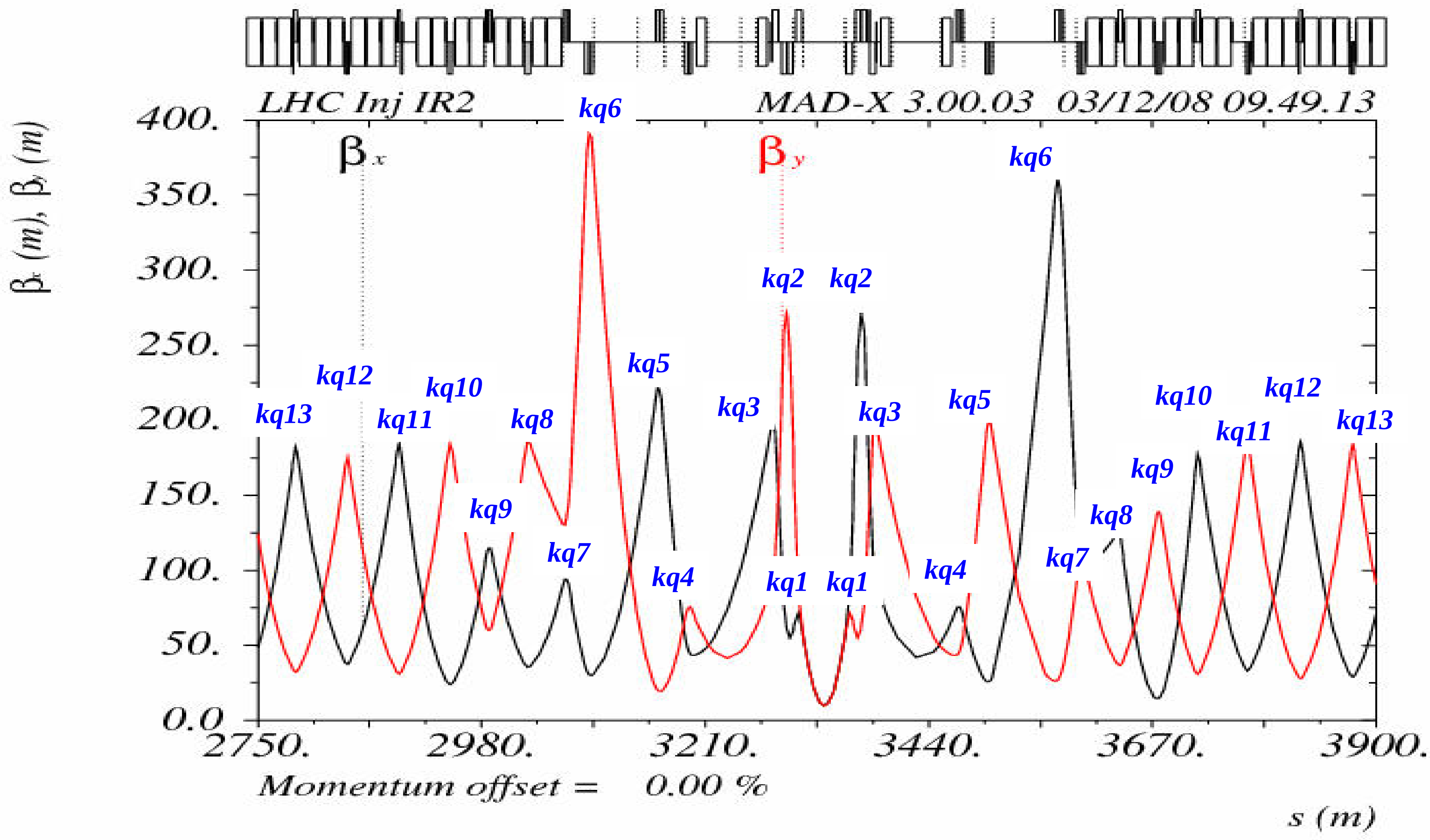
!! and to the β function at the place of the error ,

!!! and to the β function at the observation point,

!!!! proceeds as double the phase advance

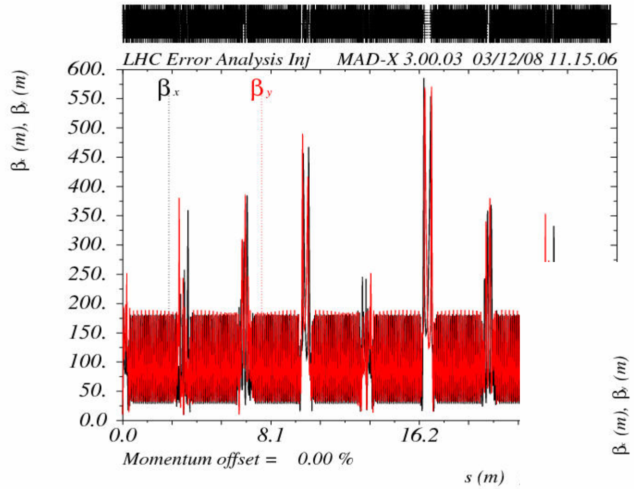
!!!!! there is a resonance denominator

en detail: LHC Injection Optics IR_2

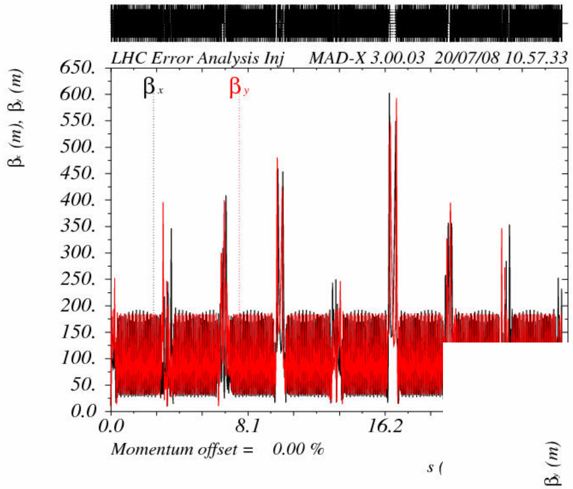


Absolute Beta function

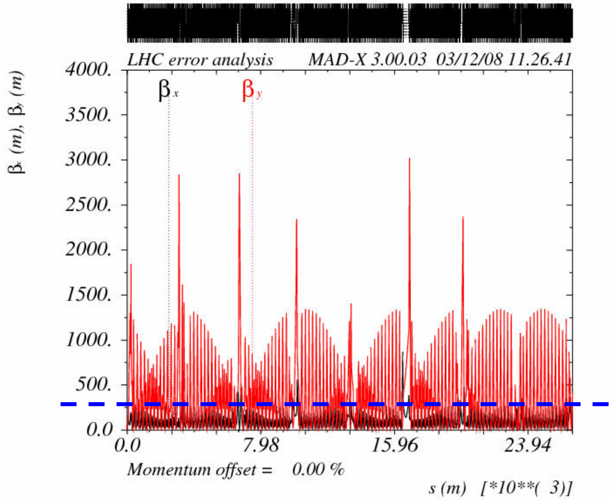
after trip of different quadrupoles in IR_2



kqt12 0.6 / 2.4 %



kqt12 12 / 2.4 %



kq 5 52 / 650 %

Cross Check with analytical formula:

Example: IR 2 ... Inj. Optics

kqtl11.r2.b1

$$\Delta\beta(s_0) = \frac{-\beta_0}{2\sin 2\pi Q}\int_{s1}^{s1+l}\beta(s_1)\Delta k\,\underbrace{\cos\left(2(\psi_{s1}-\psi_{s0})-2\pi Q\right)}_{\approx 1}ds$$

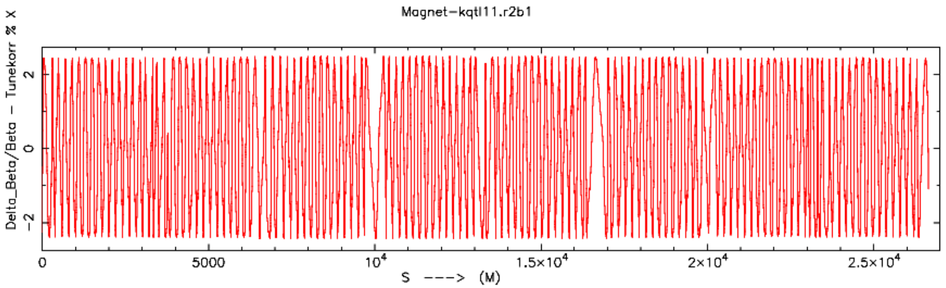
$Q_x \approx 64.31$

$Q_y \approx 59.32$

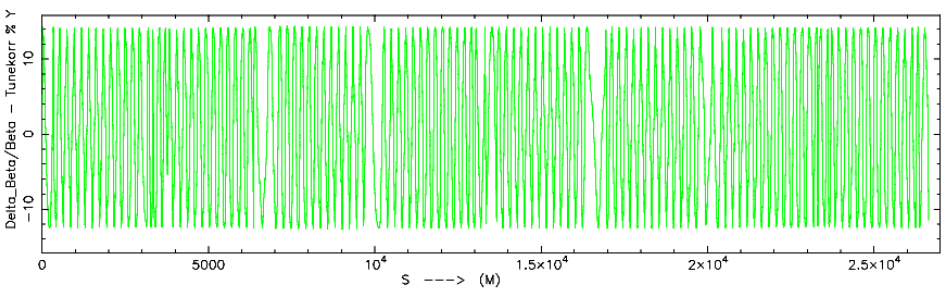
$$\frac{\Delta\beta}{\beta} \approx \frac{1}{2\sin 2\pi Q}\beta_Q\ \Delta k * l_Q$$

$$\frac{\Delta\beta}{\beta} \approx \frac{1}{2\sin\,(2\pi * 0.31)}180\,m * 1.06 * 10^{-3}\frac{1}{m^2},$$

$$\frac{\Delta\beta}{\beta} \approx 13\ \%$$



$$\frac{\Delta\beta}{\beta} \approx 12\ \%$$

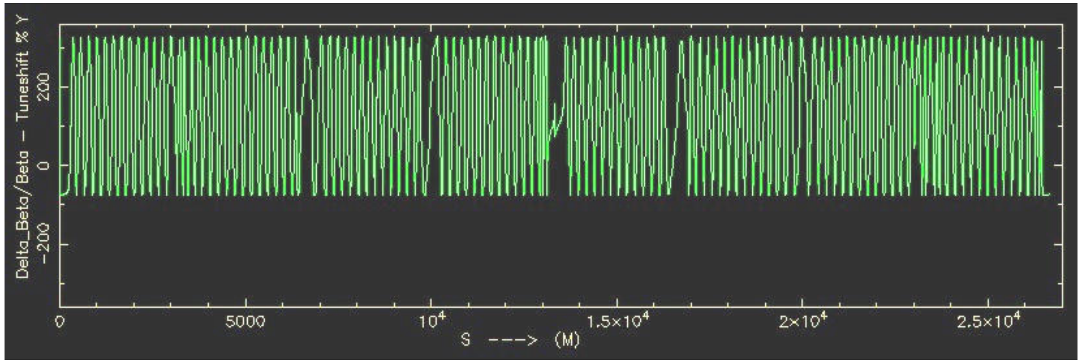


Tune Problem:

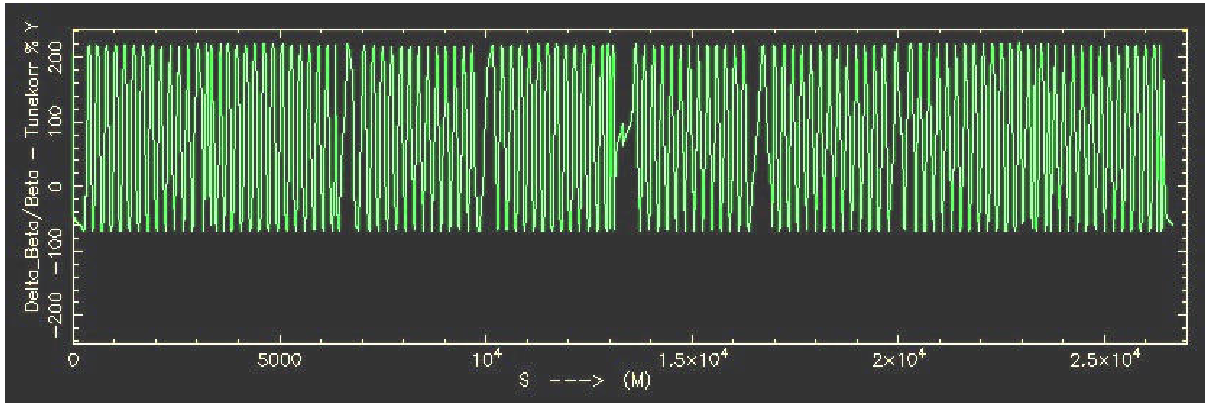
tune shift can increase the beta beat "ad infinitum"

$$\Delta\beta(s_0) = \frac{\beta_0}{2\sin 2\pi Q}\int_{s1}^{s1+l}\beta(s_1)\Delta K\cos\left(2|\psi_{s1}-\psi_{s0}|-2\pi Q\right)ds$$

$$\Delta Q = \int_{s0}^{s0+l}\frac{\Delta k(s)\beta(s)ds}{4\pi}$$



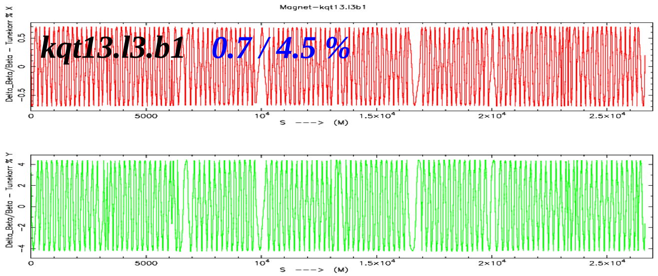
beta beat $\Delta\beta / \beta \approx$ 350 %
without tune correcťion

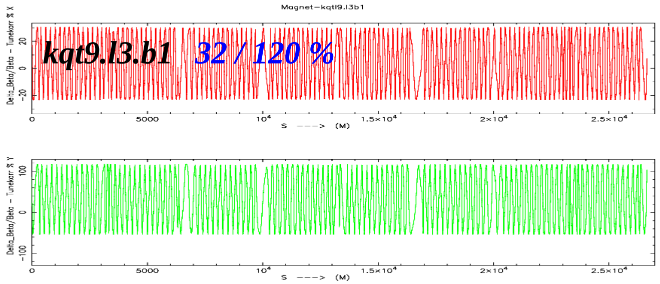


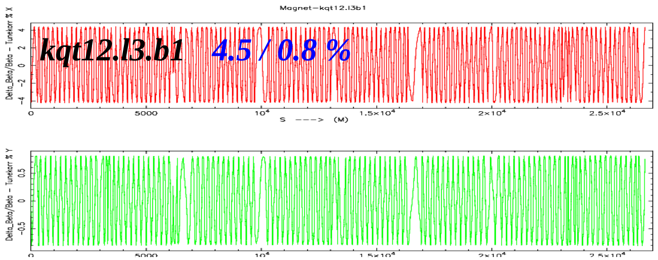
beta beat $\Delta\beta / \beta \approx$ 220 %
with tune correcťion

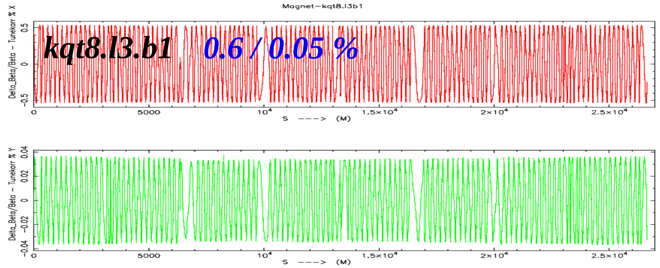
Example: IR_3

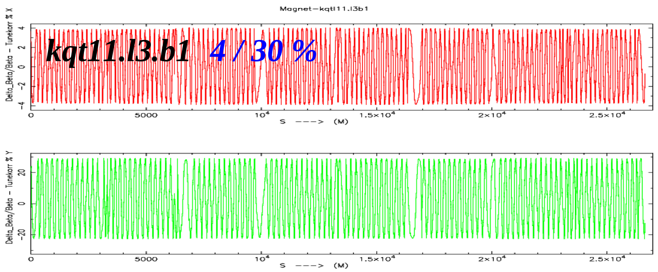
Δβ / β ... in %

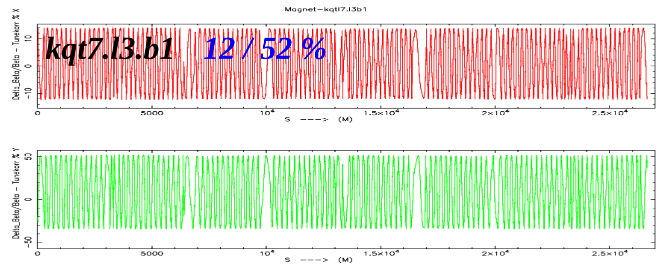


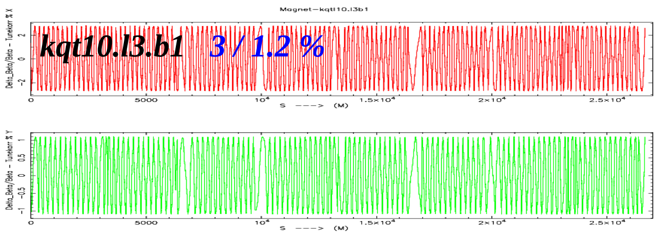


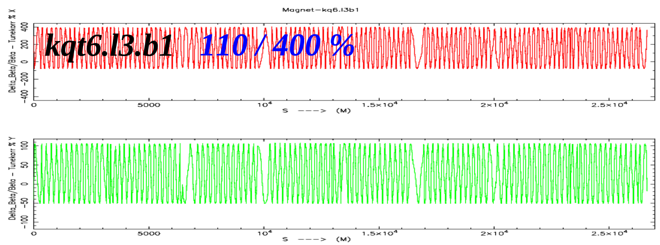






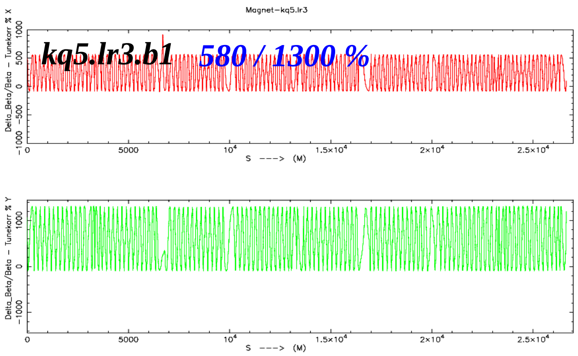


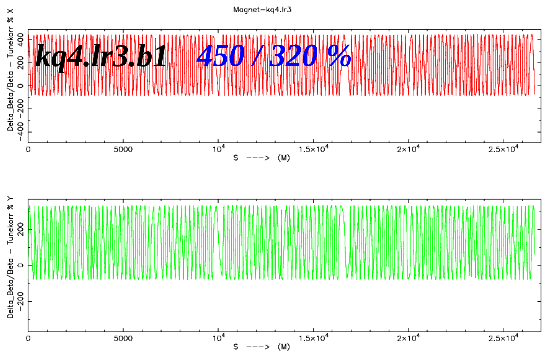


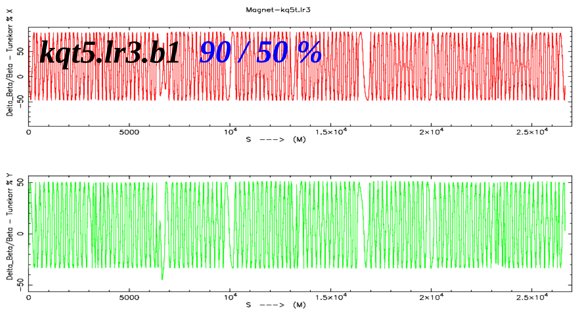


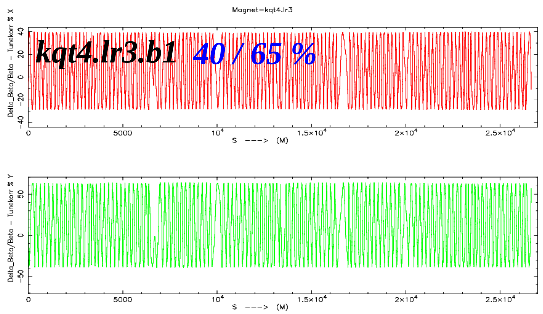
Example: IR_3

Δβ / β … in %









Résumé:

beta beat Δβ/β for the horizontal / vertical plane

IR_8

kqt 13.r8.b1	1 / 6 %	kqt 13.l8.b1	1.4 / 0.3 %
kqt 12.r8.b1	1.4 / 10 %	kqt 12.l8.b1	1.1 / 7.2 %
kqt l11.r8.b1	6 / 33 %	kqt l11.l8.b1	7.5 / 1.3 %
kq 10.r8.b1	600 / 50 %	kq 10.l8.b1	52 / 520 %
kq 9.r8.b1	28 / 600 %	kq 9.l8.b1	420/ 85 %
kq 8.r8.b1	420 / 27 %	kq 8.l8.b1	22 / 210 %
kq 7.r8.b1	290 / 550 %	kq 7.l8.b1	410 / 85 %
kq 6.r8.b1	1000 / 70 %	kq 6.l8.b1	50 / 1400 %
kq 5.r8.b1	60 / 600 %	kq 5.l8.b1	600 / 100 %
kq 4.r8.b1	210/ 105 %	kq 4.l8.b1	85/ 190 %

IR_7

kqt 13.r7.b1	9 / 1.9 %	kqt 13.l7.b1	0.3 / 1.4 %
kqt 12.r7.b1	1.3 / 6.4 %	kqt 12.l7.b1	0.6 / 0.1 %
kqt l11.r7.b1	6.2 / 1 %	kqt l11.l7.b1	2.0 / 12 %
kqtl 10.r7.b1	0.05 / 0.3 %	kqtl 10.l7.b1	50 / 11 %
kqtl 9.r7.b1	25 / 14 %	kqtl 9.l7.b1	0.3/ 2.0 %
kqtl 8.r7.b1	8 / 48 %	kqtl 8.l7.b1	8 / 7 %
kqtl 7.r7.b1	6.5 / 5 %	kqtl 7.l7.b1	4 / 8 %
kq 6.r7.b1	75 / 330 %	kq 6.l7.b1	620 / 80 %
kq 5.lr7.b1	1000 / 1250 %	kq 4.lr7.b1	1100 / 1200 %
kqt 5.lr7.b1	2/ 1.6 %	kqt 4.lr7.b1	19/ 21 %

Résumé

IR_6

kqt 13.r6.b1	1 / 1 %	kqt 13.l6.b1	1 / 1 %
kqt 12.r6.b1	16 / 3 %	kqt 12.l6.b1	0.8 / 6 %
kqt l11.r6.b1	3 / 16 %	kqt l11.l6.b1	10 / 1.8 %
kq 10.r6.b1	500 / 110 %	kq 10.l6.b1	42 / 500 %
kq 9.r6.b1	32 / 600 %	kq 9.l6.b1	420/ 120 %
kq 8.r6.b1	230 / 25 %	kq 8.l6.b1	14 / 200 %
kq 5.r6.b1	280 / 1400 %	kq 5.l6.b1	850 / 220 %
kq 4.r6.b1	900/ 300 %	kq 4.l6.b1	----- / ---- %

IR_5 & IR_1

kqt 13.r5.b1	6.5 / 1.7 %	kqt 13.l5.b1	1.4 / 11 %
kqt 12.r5.b1	1.4 / 6.0 %	kqt 12.l5.b1	3 / 0.5 %
kqt l11.r5.b1	2.2 / 0.4 %	kqt l11.l5.b1	3.1 / 0.5 %
kq 10.r5.b1	55 / 650 %	kq 10.l5.b1	600 / 56 %
kq 9.r5.b1	500 / 34 %	kq 9.l5.b1	33/ 520 %
kq 8.r5.b1	32/ 380 %	kq 8.l5.b1	360 / 33 %
kq 7.r5.b1	320 / 210 %	kq 7.l5.b1	170 / 350 %
kq 6.r5.b1	30 / 650 %	kq 6.l5.b1	620 / 140 %
kq 5.r5.b1	420/ 240 %	kq 5.l5.b1	260 / 450 %
kq 4.r5.b1	80/ 220 %	kqt 4.l5.b1	220/ 85 %

Résumé

IR_4

kqt 13.r4.b1	1 / 1 %	kqt 13.l4.b1	1 / 1 %
kqt 12.r4.b1	12 / 2 %	kqt 12.l4.b1	0.2 / 1.5 %
kqt l11.r4.b1	2 / 10 %	kqt l11.l4.b1	4 / 0.8 %
kq 10.r4.b1	510 / 100 %	kq 10.l4.b1	55 / 520 %
kq 9.r4.b1	30 / 480 %	kq 9.l4.b1	430/ 75 %
kq 8.r4.b1	310 / 70 %	kq 8.l4.b1	28 / 280 %
kq 7.r4.b1	90 / 140 %	kq 7.l4.b1	280 / 80 %
kq 6.r4.b1	900 / 180 %	kq 6.l4.b1	210 / 990 %
kq 5.r4.b1	200 / 700 %	kq 5.l4.b1	700 / 240 %

IR_3

kqt 13.r3.b1	13 / 3 %	kqt 13.l3.b1	0.7 / 4.5 %
kqt 12.r3.b1	3 / 12 %	kqt 12.l3.b1	4.5 / 0.8 %
kqt l11.r3.b1	60 / 13 %	kqt l11.l3.b1	4 / 30 %
kqtl 10.r3.b1	11 / 36 %	kqtl 10.l3.b1	3 / 1.2 %
kqtl 9.r3.b1	4.5 / 2.7 %	kqtl 9.l3.b1	32/ 120 %
kqtl 8.r3.b1	6/ 35 %	kqtl 8.l3.b1	0.6 / 0.05 %
kqtl 7.r3.b1	2.4 / 2 %	kqtl 7.l3.b1	12 / 52 %
kq 6.r3.b1	52 / 480 %	kq 6.l3.b1	110 / 400 %
kq 5.lr3.b1	580/ 1300 %	kq 4.lr3.b1	450 / 320 %
kqt 5.lr3.b1	90/ 50 %	kqt 4.lr3.b1	40/ 65 %

Résumé

IR_2

kqt 13.r2.b1	1.2 / 9	%	kqt 13.l2.b1	12 / 2.4	%
kqt 12.r2.b1	9 / 1.4	%	kqt 12.l2.b1	0.6 / 2.4	%
kqt l11.r2.b1	2.5 / 12	%	kqt l11.l2.b1	0.6 / 0.2	%
kq 10.r2.b1	580 / 72	%	kq 10.l2.b1	38 / 450	%
kq 9.r2.b1	25 / 400	%	kq 9.l2.b1	350/ 150	%
kq 8.r2.b1	270 / 70	%	kq 8.l2.b1	38 / 290	%
kq 7.r2.b1	380 / 420	%	kq 7.l2.b1	310 / 480	%
kq 6.r2.b1	1250 / 70	%	kq 6.l2.b1	63 / 1400	%
kq 5.r2.b1	52 / 650	%	kq 5.l2.b1	650 / 40	%
kq 4.r2.b1	200 / 110	%	kq 4.l2.b1	120 / 230	%