Beam Emittance
by
QP Scan Method

Task: Measure the Emittance of the Laser Beam

Your tasks in green frames

Introduction

Quadrupole scan method:

If β is known unambiguously as in a circular machine, then a single profile measurement determines ε by

$$\sigma_y^2 = \varepsilon\beta_y.$$

But it is not easy to be sure in a transfer line which β to use, or rather, whether the beam that has been measured is matched to the β-values used for the line. This problem can be resolved by using **a single quadrupole scan system consists of a quadrupole magnet and a drift space s (Fig. 1). The transformation matrix M of this system for the Y direction can be obtained Using a thin-lens approximation for the quadrupole with K=±1/f, where f is the focusing strength of the quadrupole (- for focusing and + for defocusing)**

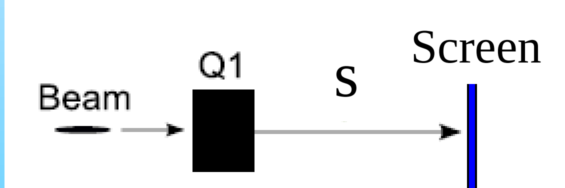


Fig. 1

$$Q = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \quad S = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

$$M = S\,Q = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1 - s/f & s \\ -s/f & 1 \end{pmatrix}$$

Introduction

Introduction of σ-Matrix (see for example: K. Wille; Physik der Teilchenbeschleuniger, Teubner)

beam size²

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma\, matrix$$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

$$(\beta\gamma - \alpha^2 = 1)$$

Beam width$_{rms}$ of measured profile = σ_y $= \sqrt{\sigma_{11}} = \sqrt{\beta(s)\cdot\varepsilon}$

Transformation of s-Matrix through the elements of an accelerator: The evolution of a beam matrix between two points s_1 and s_0 of an uncoupled transfer line is described by the following matrix equation:

$$\sigma_{s1} = M \cdot \sigma_{s0} \cdot M^t$$

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix};\; M^t = \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix}$$

The distances between screens or from Quadrupole to screen s and Quadrupole field strength 1/f are given, therefore the transport matrix M is known.

Applying the transport matrix gives:

Introduction

$$\sigma_{s_1} = M \cdot \sigma_{s_0} \cdot M^t$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}_{s_0} \cdot \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix} = \sigma^{measured} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{y'y} & \sigma_{y'}^2 \end{pmatrix}_{s_1}^{measured} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11}M_{11} + \sigma_{12}M_{12} & \sigma_{11}M_{21} + \sigma_{12}M_{22} \\ \sigma_{21}M_{11} + \sigma_{22}M_{12} & \sigma_{12}M_{21} + \sigma_{22}M_{22} \end{pmatrix}$$

$$= \begin{pmatrix} M_{11}(\sigma_{11}M_{11} + \sigma_{12}M_{12}) + M_{12}(\sigma_{21}M_{11} + \sigma_{22}M_{12}) & \cdots \\ \cdots & \cdots \end{pmatrix}$$

$$\sigma_{11}^{new} = \sigma_y^{2\,new} = M_{11}^2 \sigma(s_0)_{11} + 2M_{11}\, M_{12} \sigma(s_0)_{12} + M_{12}^2 \sigma(s_0)_{22} \quad (\sigma_{12} = \sigma_{21}) \qquad (1)$$

Transferred/measured beam width^2 from s_n

Unknown at QP (at s_0)

Solving $\sigma(s_n)_{11}$ $\sigma(s_0)_{12}$ and $\sigma(s_0)_{22}$ while Matrix elements are known: Needs minimum of three different measurements, either three screens or **three different Quadrupole settings with different field strength K = 1/f. We will use in the following some more focal length values and use a fit.**

$$\sigma_{11}^{new} = \sigma^2{}_y^{new} = M_{11}{}^2\sigma(s_0)_{11} + 2M_{11}\, M_{12}\sigma(s_0)_{12} + M_{12}{}^2\, \sigma(s_0)_{22}$$

with $\quad M = S\,Q = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} = \begin{pmatrix} 1 - s/f & s \\ -s/f & 1 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$

$$\sigma^2{}_y = (1\text{-}s/f)^2\, \sigma(s_0)_{11} + 2s(1\text{-}s/f)\sigma(s_0)_{12} + s^2\, \sigma(s_0)_{22}$$

$$\sigma^2{}_y = (s/f)^2\, \sigma_{11} + (s/f)(2\sigma_{11} + 2s\sigma_{12}) + (\sigma_{11} + 2s\sigma_{12} + s^2\sigma_{22})$$

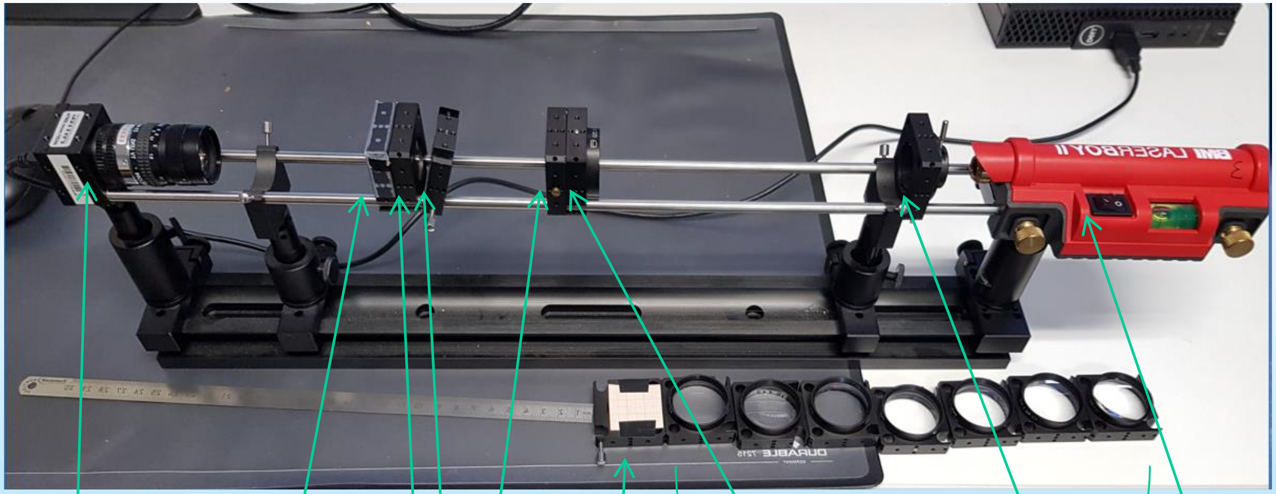
Parabola fit as a function of the quadrupole excitation (s/f) and the parameters (a,b,c) of $\sigma^2{}_y = ax^2+bx+c$ (**width²!!!**) contain the unknown beam properties σ_{11} ,σ_{12} and σ_{22}.

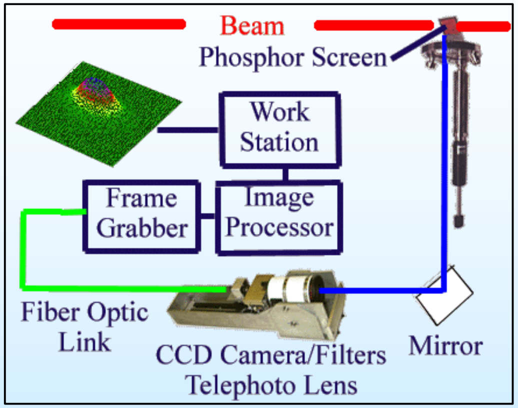
$a = \sigma_{11} \qquad b = 2\sigma_{11} + 2s\sigma_{12} \qquad c = \sigma_{11} + 2s\sigma_{12} + s^2\sigma_{22}$ or

$\sigma_{11} = a \qquad \sigma_{12} = (b\text{-}2\sigma_{11}) / 2s \qquad \sigma_{22} = (c - \sigma_{11} - 2s\sigma_{12}) / s^2$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

Setup





Camera Screen Filters Calibration various Lenses Aperture Laser

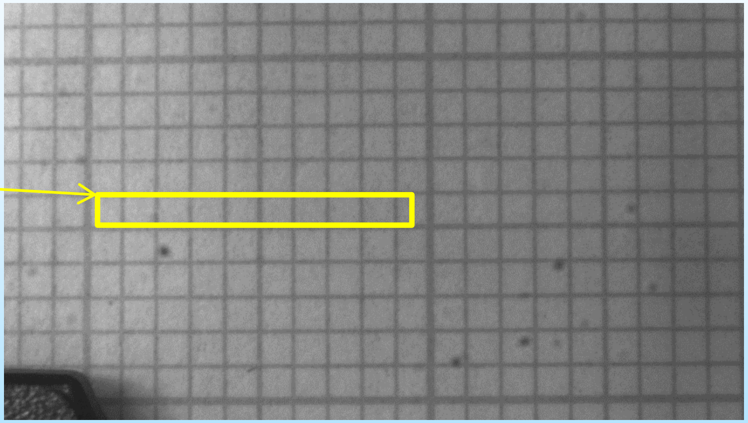
By changing the lenses with different focal length f one can take pictures from the camera. The distance of the lens to the screen can be measured by a simple ruler. The camera is connected to a Computer where the readout software is installed. The pictures (.jpg) can be saved and can be loaded into a free software called “ImageJ” where a profile of an area can be displayed and the curser position and the value is displayed (8 bit). The σ of the profiles have to be found for each focal length and the emittance have to be calculated on an prepared Excel sheet..

Your tasks in green frames

Check: Do not saturate (255)

Calibration

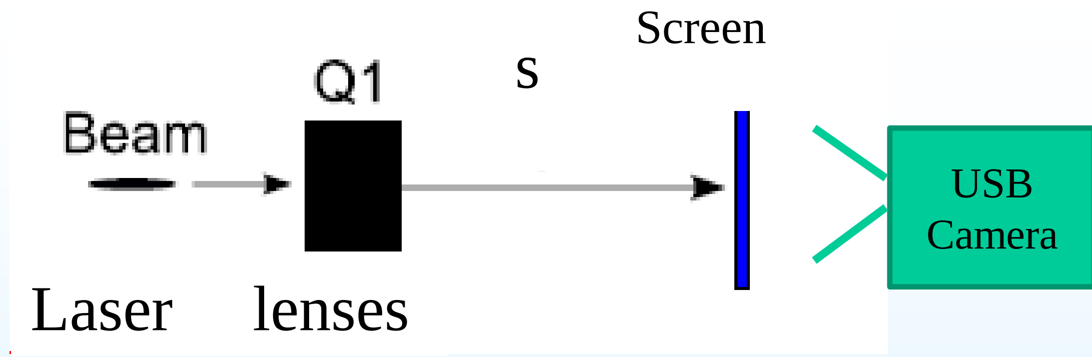
Use mm-grid to calibrate the readout setup.
Select ROI (where beam image will appear), plot profile, use cursor and enter measurement into pre-prepared Excel sheet “QP emittance.xlsx”



Calibration			
Line-distance [mm]	pixel 1	pixel 2	
3	66	401	
[meter]		Cal. Result	
3.00E-03	=>	111.6667	pixel/mm

All yellow cells will be calculated automatically

Take some profiles at same distance lens-screen s with different focal length f.



$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma\ matrix$$

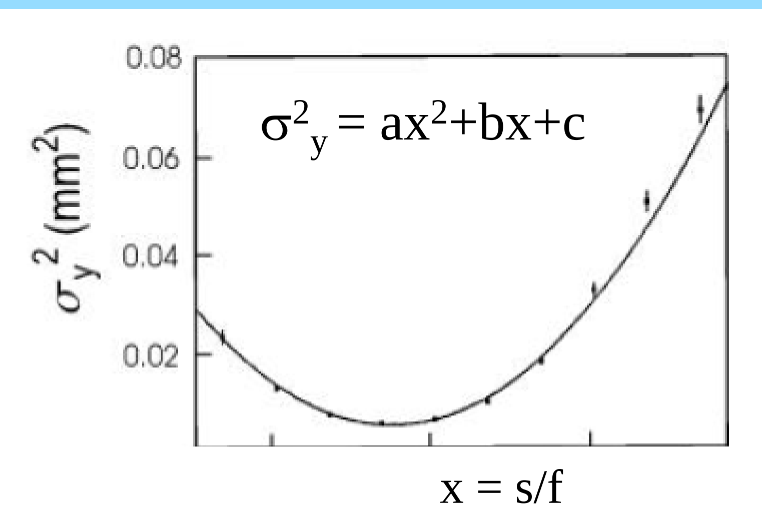
$$\sigma_{y\ measured}^2 = M_1^2\sigma_{11} + 2M_{11}M_{12}\sigma_{12} + M_{12}^2\sigma_{22}$$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

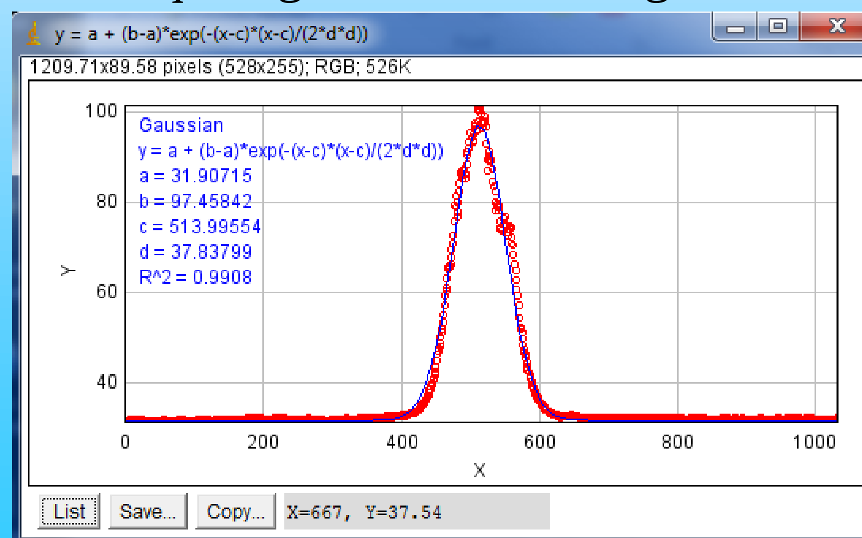
Check: Do not saturate (255)

$$M = S\ Q = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} = \begin{pmatrix} 1 - s/f & s \\ -s/f & s \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

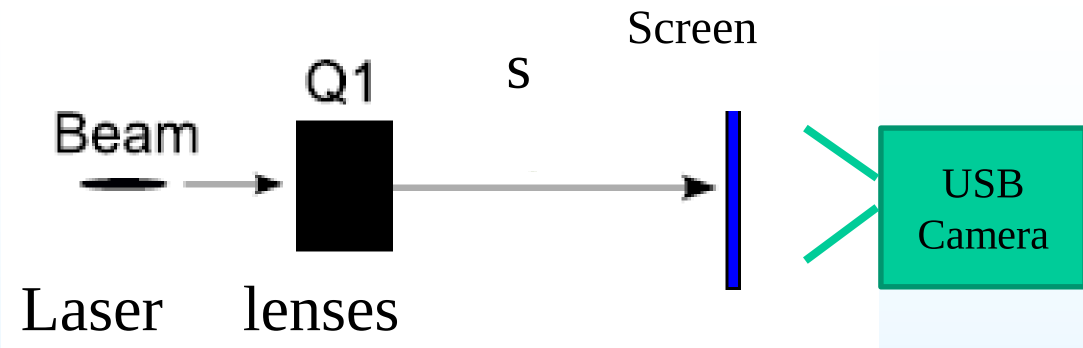
$$\sigma_y^2 = (s/f)^2\ \sigma_{11} + (s/f)(2\sigma_{11} + 2s\sigma_{12}) + (\sigma_{11} + 2s\sigma_{12} + s^2\sigma_{22})$$



Adjust the aperture so that the image looks quite gaussian at its larges size!



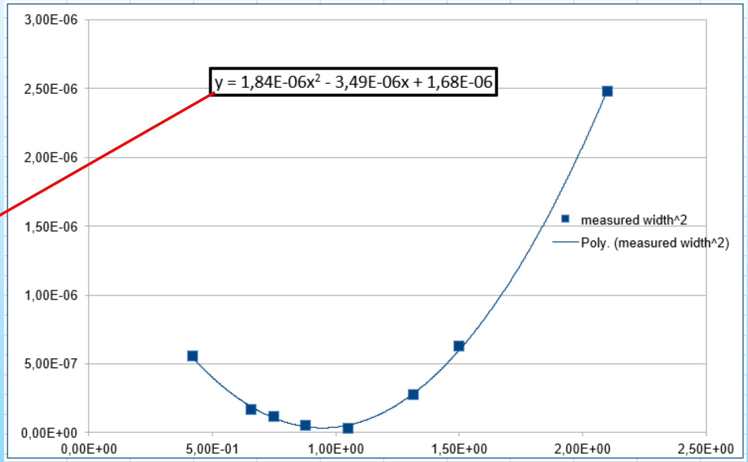
Take some profiles at same distance lens-screen s with different focal length f.



Enter s, f and the measured σ from fits (in pixel) into the Excel sheet:

Check: Do not saturate (255)

	Distance s METER!!!	Focus f [m]	s/f	sigma [p] PIXEL	sigma [m] measured	sigma^2
s =	1,05E-01	0,050	2,10E+00	207,7	1,58E-03	2,48E-06
		0,070	1,50E+00	104,37	7,92E-04	6,27E-07
		0,080	1,31E+00	69,23	5,25E-04	2,76E-07
		0,100	1,05E+00	22,71	1,72E-04	2,97E-08
		0,120	8,75E-01	29,69	2,25E-04	5,07E-08
		0,140	7,50E-01	44,57	3,38E-04	1,14E-07
		0,160	6,56E-01	54,37	4,13E-04	1,70E-07
		0,250	4,20E-01	98,13	7,45E-04	5,54E-07



From Fit:	y=ax^2 + bx + c with x=s/f and y=σ^2
a=	1,84E-06
b=	-3,49E-06
c=	1,68E-06

Emittance:					
σ_{11} =	1,84E-06				
σ_{12} =	-3,41E-05		Emitt^2	4,19E-12	should not be negativ!
σ_{22} =	6,36E-04		Emittance:	2,0465E-06	m rad

Enter the fit parameters a, b, c from the fit displayed in the picture. The emittance is calculated by the formulas:

$\sigma_{11} = a \quad \sigma_{12} = (b - 2\sigma_{11}) / 2s \quad \sigma_{22} = (c - \sigma_{11} + 2s\sigma_{12}) / s^2$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$