Beam Emittance
by
3 Screen Method

Task: Measure the Emittance of the Laser Beam

Your tasks in green frames

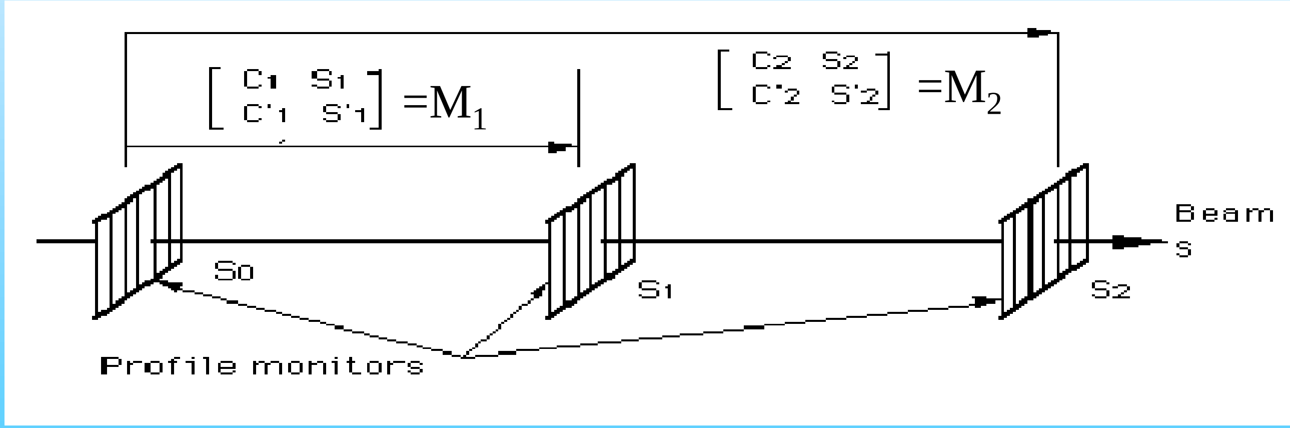
Introduction

Three screen method:

If β is known unambiguously as in a circular machine, then a single profile measurement determines ε by

$$\sigma_y^2 = \varepsilon\beta_y.$$

But it is not easy to be sure in a transfer line which β to use, or rather, whether the beam that has been measured is matched to the β-values used for the line. This problem can be resolved by using **three monitors (see Fig. 1), i.e. the three width measurement determines the three unknown α, β and ε of the incoming beam.**



Introduction

Introduction of σ-Matrix (see for example: K. Wille; Physik der Teilchenbeschleuniger, Teubner)

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma\ matrix$$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

$$(\beta\gamma - \alpha^2 = 1)$$

Beam width$_{rms}$ of measured profile = σ_y $= \sqrt{\sigma_{11}} = \sqrt{\beta(s) \cdot \varepsilon}$

Transformation of σ-Matrix through the elements of an accelerator:

$$\sigma_{s1} = M \cdot \sigma_{s0} \cdot M^t$$

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}; M^t = \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix}$$

L_1, L_2 = distances between screens or from Quadrupole to screen and Quadrupole field strength are given, therefore the transport matrix M is known.
Applying the transport matrix gives:

Introduction

$$\sigma_{s_1} = M \cdot \sigma_{s_0} \cdot M^t$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}_{s_0} \cdot \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix} = \sigma^{measured} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{y'y} & \sigma_{y'}^2 \end{pmatrix}_{s_1}^{measured} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11}M_{11} + \sigma_{12}M_{12} & \sigma_{11}M_{21} + \sigma_{12}M_{22} \\ \sigma_{21}M_{11} + \sigma_{22}M_{12} & \sigma_{12}M_{21} + \sigma_{22}M_{22} \end{pmatrix}$$

$$= \begin{pmatrix} M_{11}(\sigma_{11}M_{11} + \sigma_{12}M_{12}) + M_{12}(\sigma_{21}M_{11} + \sigma_{22}M_{12}) & \cdots \\ \cdots & \cdots \end{pmatrix}$$

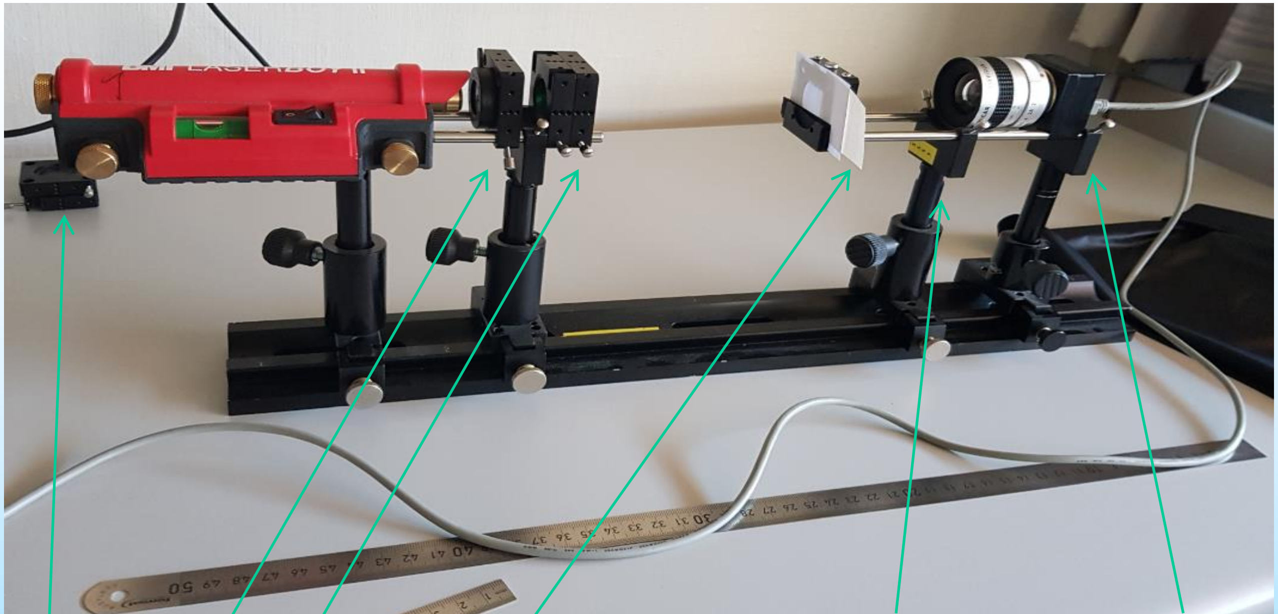
$$\sigma(s_n)_{11}^{new} = \sigma^2(s_n)_y^{new} = M_{11}^2 \sigma(s_0)_{11} + 2M_{11} M_{12} \sigma(s_0)_{12} + M_{12}^2 \sigma(s_0)_{22} \quad (\sigma_{12} = \sigma_{21}) \qquad (1)$$

transferred beam width at s_n measured beam width at s_0 unknown

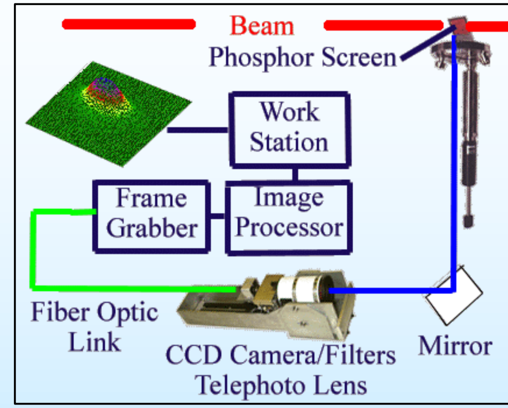
Solving $\sigma(s_n)_{11}$ $\sigma(s_0)_{12}$ and $\sigma(s_0)_{22}$ while Matrix elements are known: Needs minimum of three (n=0,1,2) different measurements, either three screens or three different Quadrupole settings with different field strength.

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \qquad (2)$$

Introduction



Laser Aperture Lense Calibration Screen Moveable Part Camera



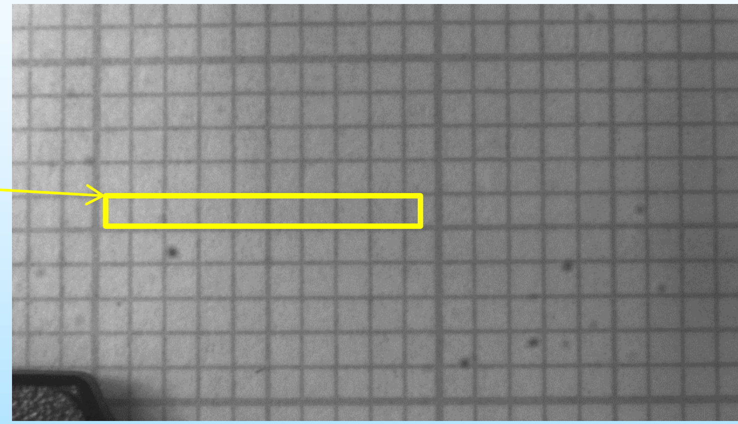
By moving screen (together with the camera) one can take pictures from three positions (avoid the focus due to limited resolution of the optic system). The distance of the lens to various screen positions can be measured by a simple ruler. The camera is connected to a Computer where the readout software is installed. The pictures (.jpg) can be saved and can be loaded into a free software called “ImageJ” where a profile of an area can be displayed and the curser position and the value is displayed (8 bit). The σ of the profile have to be found for each screen position and the emittance have to be calculated.

Your tasks in green frames

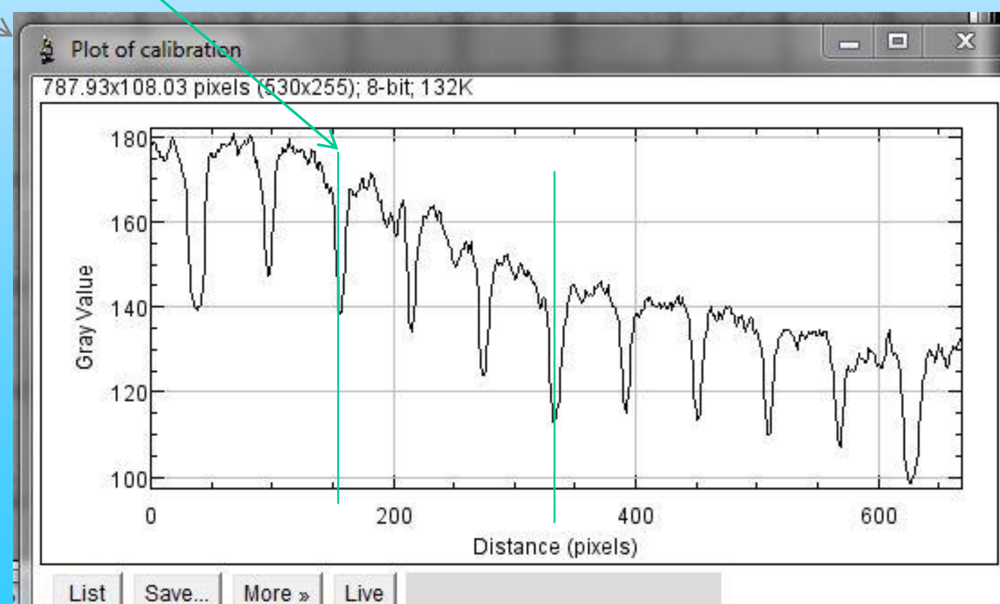
Check: Do not saturate (255)

Calibration

Use mm-grid to calibrate the readout setup.
Select ROI (where beam image will appear), plot profile, use cursor and enter measurement into pre-prepared Excel sheet “Laser emittance.xlsx”

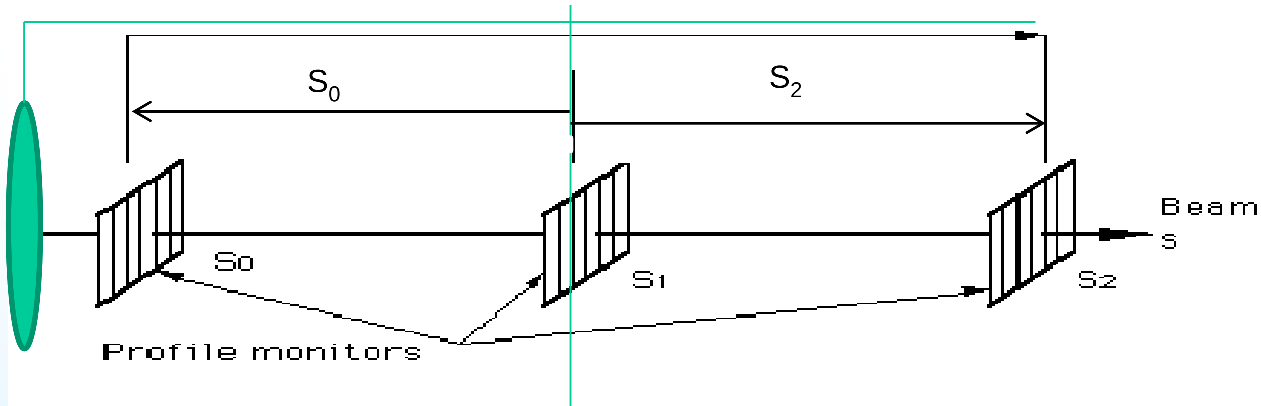


Calibration			
Line-distance [mm]	pixel 1	pixel 2	
3	66	401	
[meter]		Cal. Result	
3.00E-03	=>	111.6667	pixel/mm



All yellow cells will be calculated automatically

Take profiles at 3 different distances lens-screen. Move lense to simulate 3 positions.



Hint1: Make the distances equal, set $s_1 = 0$, $s_2 = s$, $s_0 = -s$
Hint 2: Avoid position at waist (why?)

$s_1 = 0$

Check: Do not saturate (255)

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma\ matrix$$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

$$\sigma_{y\ measured}^2 = M_1^2\sigma_{11} + 2M_{11} M_{12}\sigma_{12} + M_{12}^2 \sigma_{22}$$

No optical elements => $M = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$

$$\sigma^2(s_1)_{measured} = \sigma_{11} + 2s_1 \sigma_{12} + s_1^2 \sigma_{22}$$
$$\sigma^2(s_0)_{measured} = \sigma_{11} + 2s_0 \sigma_{12} + s_0^2 \sigma_{22}$$
$$\sigma^2(s_2)_{measured} = \sigma_{11} + 2s_2 \sigma_{12} + s_2^2 \sigma_{22}$$

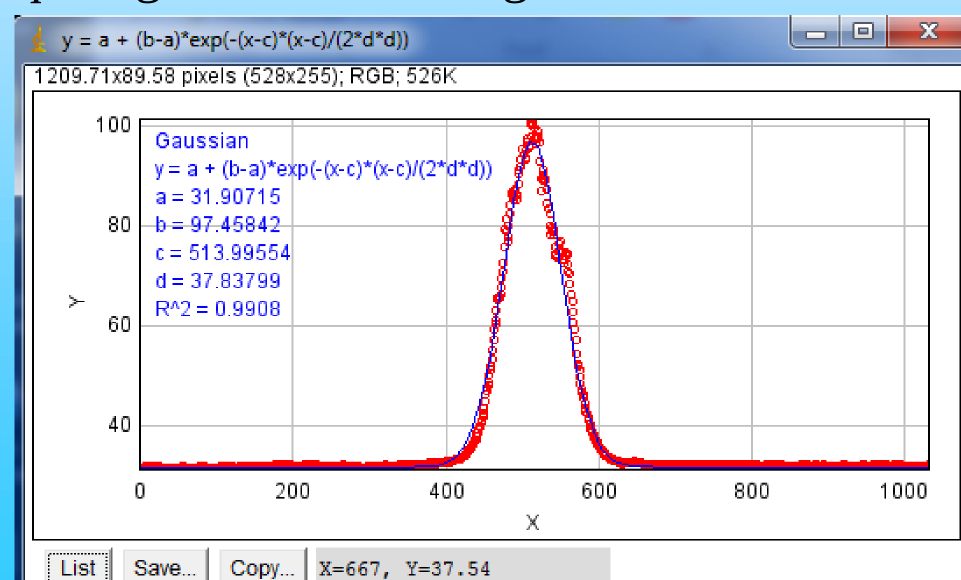
With $s_1=0$, $s_0=-s$, $s_2=+s$, measured = y

$$\sigma_{11} = \sigma_y^2(0) = \sigma^2(s_1)$$

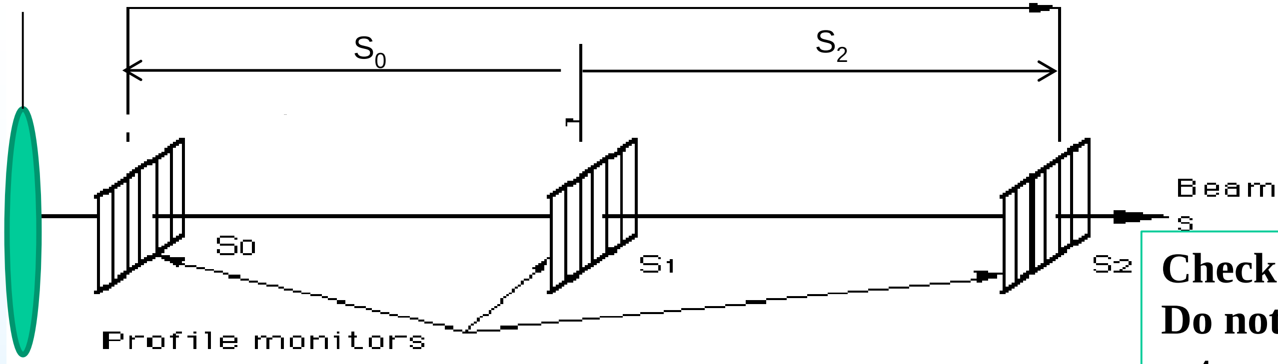
$$\sigma_{12} = \frac{\sigma_y^2(+s) - \sigma_y^2(-s)}{4s}$$

$$\sigma_{22} = \frac{\sigma_y^2(+s) - 2\cdot\sigma^2(0) + \sigma_y^2(-s)}{2\cdot s^2}$$

Adjust the aperture so that the image looks quite gaussian at its larges size!



Take profiles at 3 different distances lens-screen.

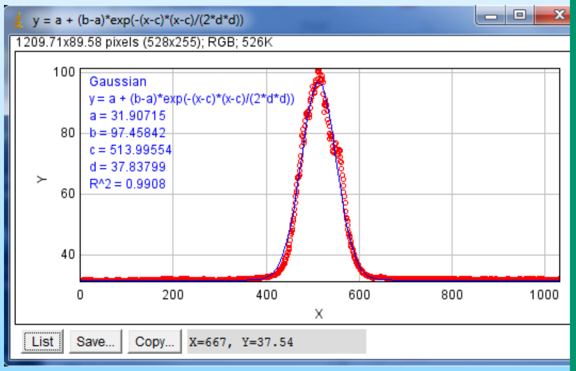


Check: Do not saturate (255)

Make the distances equal, set $s_1 = 0$, $s_2 = s$, $s_0 = -s$

Enter the screen positons +s, 0, -s and the measured σ from fits (in pixel) at +s, 0 and –s into the Excel sheet:

			sigma [p]	sigma(s) [m]	
			PIXEL		
S_0	-1,00E-02		37.8	3.38E-04	Take FWHM/2.36 if not Gaussian!
		1,00E-02			
S_1=0	0,00E+00		33.5	2.99E-04	Take FWHM/2.36 if not Gaussian!
		1,00E-02			
S_2	1,00E-02		30.2	2.70E-04	Take FWHM/2.36 if not Gaussian!



distance s	[meter]			
s=	1.00E-02	(s equal between 3 positions)		
		Emitt^2	2.38E-12	should not be negativ!
σ_{11}^0 =	8.95E-08	Emittance:	1.5415E-06	
σ_{12}^0 =	1.03E-06			
σ_{22}^0 =	3.84E-05			

expected Emittance: around 1 * 10E-6 [m rad]

(often the unit is written in [π mm mrad] while π indicates that ε is the area of an ellipse)

Unfortunately it depends sometimes from one pixel in sigma only, in this experiment

↑

$$\sigma^2(s_1)_{measured} = \sigma_{11} + 2s_1\,\sigma_{12} + s_1^2\,\sigma_{22}$$
$$\sigma^2(s_0)_{measured} = \sigma_{11} + 2s_0\,\sigma_{12} + s_0^2\,\sigma_{22}$$
$$\sigma^2(s_2)_{measured} = \sigma_{11} + 2s_2\,\sigma_{12} + s_2^2\,\sigma_{22}$$

Ref. Criegee: Trick: **Three screens at –s, s_1=0, +s** with s_0 =-s and s_2 = +s and some little algbra

measured = y

$$\sigma_{11} = \sigma_y^2(0)$$

$$\sigma_{12} = \frac{\sigma_y^2(+s) - \sigma_y^2(-s)}{4s}$$

$$\sigma_{22} = \frac{\sigma_y^2(+s) - 2\cdot\sigma^2(0) + \sigma_y^2(-s)}{2\cdot s^2}$$

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

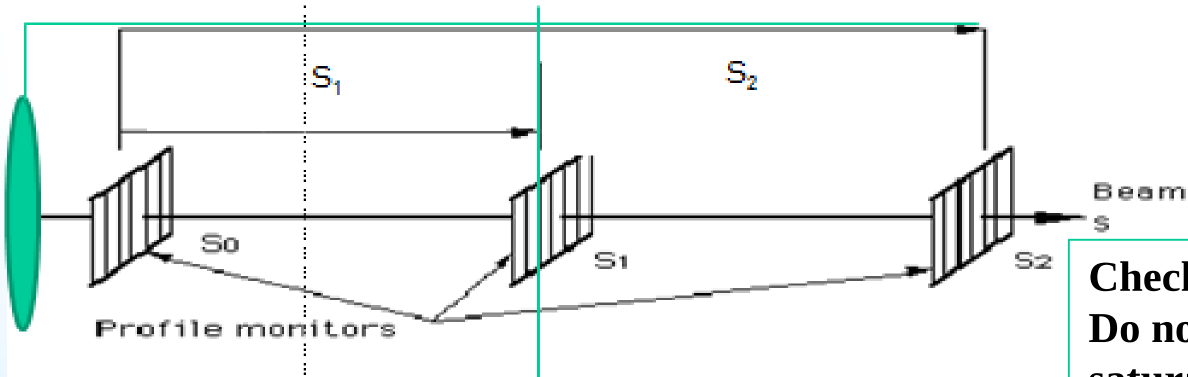
with s=L Ref. Criegee

$$\left.\begin{aligned} \hat{x}_1^2 &= \epsilon\beta_2 - 2L\cdot\epsilon\alpha_2 + L^2\cdot\epsilon\gamma_2 \\ \hat{x}_2^2 &= \epsilon\beta_2 \\ \hat{x}_3^2 &= \epsilon\beta_2 + 2L\cdot\epsilon\alpha_2 + L^2\cdot\epsilon\gamma_2 \end{aligned}\right\} \text{ with the solution } \left\{\begin{aligned} \epsilon\beta_2 &= \hat{x}_2^2 \\ \epsilon\alpha_2 &= (\hat{x}_3^2 - \hat{x}_1^2)/(4L) \\ \epsilon\gamma_2 &= (\hat{x}_1^2 - 2\hat{x}_2^2 + \hat{x}_3^2)/(2L^2) \\ \epsilon^2 &= \epsilon\beta_2\cdot\epsilon\gamma_2 - (\epsilon\alpha_2)^2 \end{aligned}\right.$$

The emittance is calculated by the formulas.
Since the Laser is not gaussian, vary a little(!) the fitted widths. Note how sensitive the emittance behave.
=> Need of **good resolution** and good fits.

Without equal distances:

Take profiles at 3 different distances lens-screen.

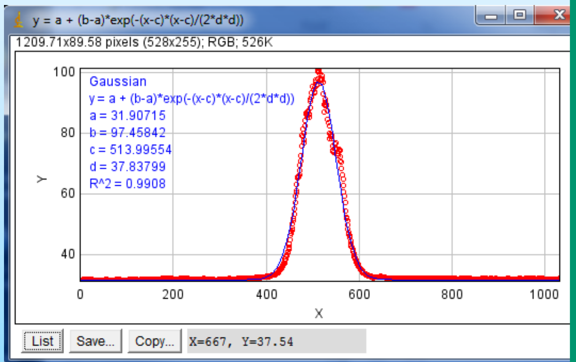


Check: Do not saturate (255)

Set $s_1 = L_1$, $s_2 = L_2$, $s_0 = 0$

Enter the screen positons 0, s_1, s_2 and the measured σ from fits (in pixel) at 0, s_1, s_2 into the Excel sheet:

	Width Measurement					$S_0 = 0$
	Position[m]	L [m]	sigma [p]	sigma(s) [m]		
	METER!!!		PIXEL	measured		
s_0	0,00E+00		37.8	3.38E-04	Take FWHM/2.36 if not Gaussian!	
s_1	1.00E-02		33.5	2.99E-04	Take FWHM/2.36 if not Gaussian!	
s_2	2.00E-02		30.2	2.70E-04	Take FWHM/2.36 if not Gaussian!	



		Emitt^2	2.38E-12
σ_{11}^0 =	1.14E-07	Emittance:	1.5415E-06
σ_{22}^0 =	3.84E-05		
σ_{12}^0 =	-1.41E-06		

should not be negativ!

expected Emittance: around 1 * 10E-6 [m rad]

(often the unit is written in [π mm mrad] while π indicates that ε is the area of an ellipse)

Unfortunately it depends sometimes from one pixel in sigma only, in this experiment

Three screens at s0, s1 and s2 and L1 = s1-s0 and L2 = s2-s0

$\sigma_{11}^0 =$ $=\sigma_{measured}^2$ at s_0

$\sigma_{11}^1 = \sigma_{11}^0 + 2L_1\sigma_{12}^0 + L_1^2\sigma_{22}^0$, and $=\sigma_{measured}^2$ at s_1

$\sigma_{11}^2 = \sigma_{11}^0 + 2L_2\sigma_{12}^0 + L_2^2\sigma_{22}^0$ $=\sigma_{measured}^2$ at s_2

We can solve for other elements of beam matrix at s_0 from the above two equations and with little algebra we find,

s=L

$$\sigma_{22}^0 = \frac{L_2(\sigma_{11}^1 - \sigma_{11}^0) - L_1(\sigma_{11}^2 - \sigma_{11}^0)}{L_1 L_2 (L_1 - L_2)}, \text{ and}$$

$$\sigma_{12}^0 = \frac{(\sigma_{11}^1 - \sigma_{11}^0 - L_1^2\sigma_{22}^0)}{2L_1}.$$

Knowing all the matrix elements of beam matrix at s_0, $\sigma^0 = \begin{pmatrix} \sigma^0_{11} & \sigma^0_{12} \\ \sigma^0_{12} & \sigma^0_{22} \end{pmatrix}$ the beam optics parameters at s_0 is then easily calculated as follows,

$$\varepsilon = \sqrt{\sigma_{11}^0\sigma_{22}^0 - (\sigma_{12}^0)^2}$$

The emittance is calculated by the formulas.
Since the Laser is not gaussian, vary a little(!) the fitted widths. Note how sensitive the emittance behave.
=> Need of **good resolution** and good fits.