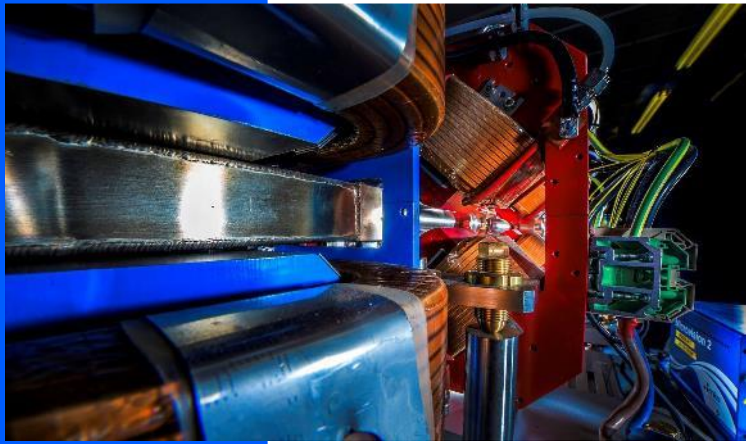
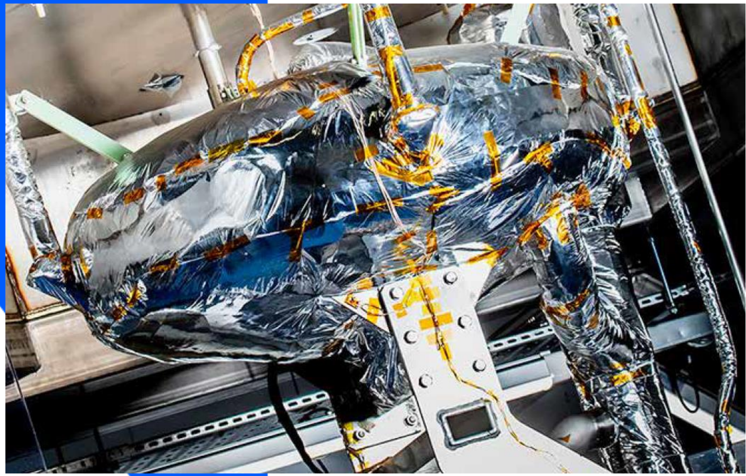
ASTeC

Making a brighter future through advanced accelerators

Insertion Devices
Lecture 1
Radiation





Jim Clarke
STFC Daresbury Laboratory

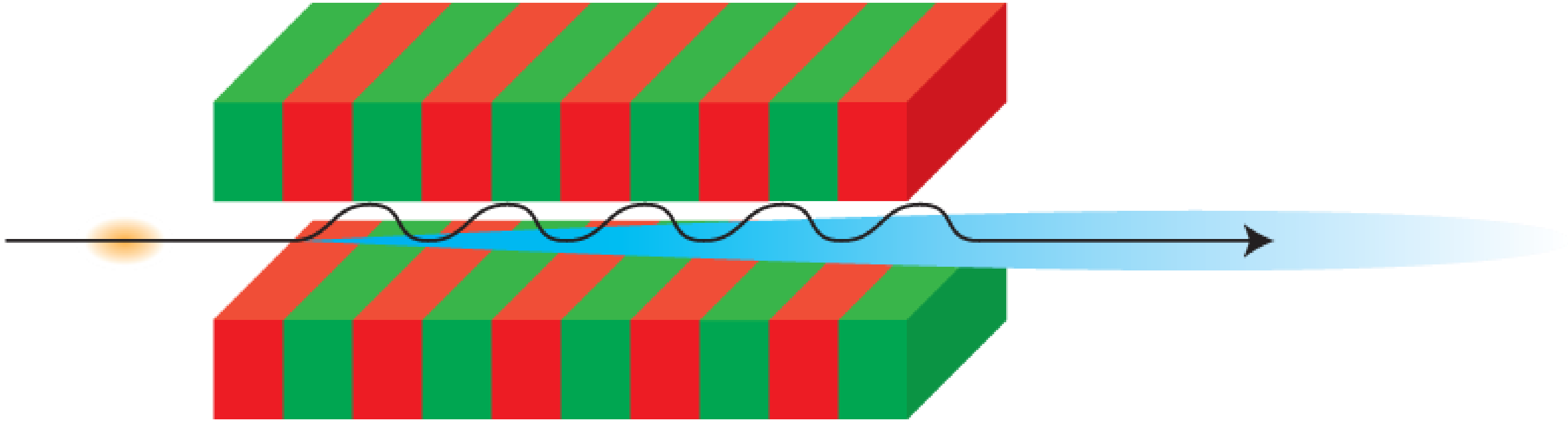
12th November 2022

Introduction

- Insertion Devices are special magnets that are specifically designed to generate synchrotron radiation
- In this first lecture we'll discuss the synchrotron radiation properties that are generated by insertion devices
- In the second lecture we'll look at how the required magnetic fields can be generated and the engineering that enables these to be built

What are Insertion Devices?

- Insertion Devices are special magnets that are specifically designed to generate synchrotron radiation
- They are quite literally **Devices** that are **Inserted** into an accelerator straight section (drift space)
- There are two basic types; Undulators and Wigglers
- They both have a periodic, sinusoidal magnetic field



Science and
Technology
Facilities Council
ASTeC

Why generate synchrotron radiation?

- Synchrotron radiation can be generated at any wavelength and with any polarization
- It is the brightest source of X-rays on the planet (by far!) – both in terms of peak and average brightness
- The pulses of synchrotron radiation are typically 10s of picoseconds (10^{-12}) in a synchrotron light source and 10s of femtoseconds (10^{-15}) down to 100s of attoseconds (10^{-18}) in a free electron laser
- The purpose of many advanced accelerators is to generate **synchrotron radiation from undulators and wigglers** so it can be used for **scientific research in all areas of science – this is the main application of IDs**
- **Damping rings use wigglers** to reduce the beam emittance for use in particle colliders
- **Undulators** are used in the proposed International Linear Collider to generate **gamma photons** which then create the **positrons**

X-rays are Vital for Science Research

Paul D. Boyer

John E. Walker

1997

Peter Agre

Roderick MacKinnon

2003

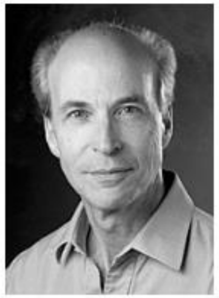


Photo: Stanford University

Roger D. Kornberg

2006

Photo: U. Montan

Venkatraman Ramakrishnan

Photo: U. Montan

Thomas A. Steitz

Photo: U. Montan

Ada E. Yonath

2009

Photo: U. Montan

Robert J. Lefkowitz

Photo: U. Montan

Brian K. Kobilka

2012

Five Nobel prizes have been awarded for research that would not have been possible without synchrotron X-ray sources

Science and Technology Facilities Council

ASTeC

Is Synchrotron Radiation Important to YOU?

All accelerator scientists and engineers need to understand SR as it impacts directly on many areas of accelerator design and performance

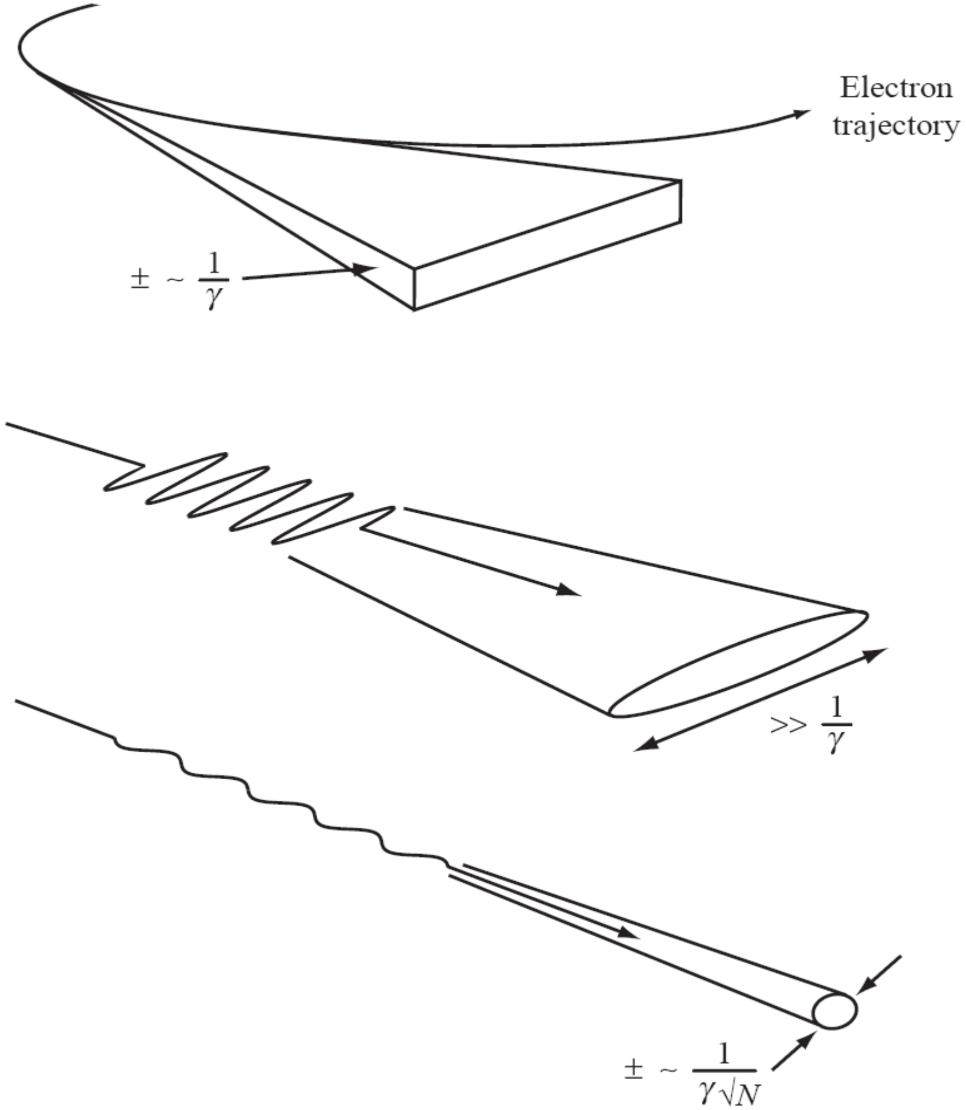
- RF
- Diagnostics
- Vacuum design
- Magnets
- Beam Dynamics

It is generated by all relativistic charged particles

Storage Ring Light sources and Free Electron Lasers are a major “customer” of accelerator scientists and engineers!

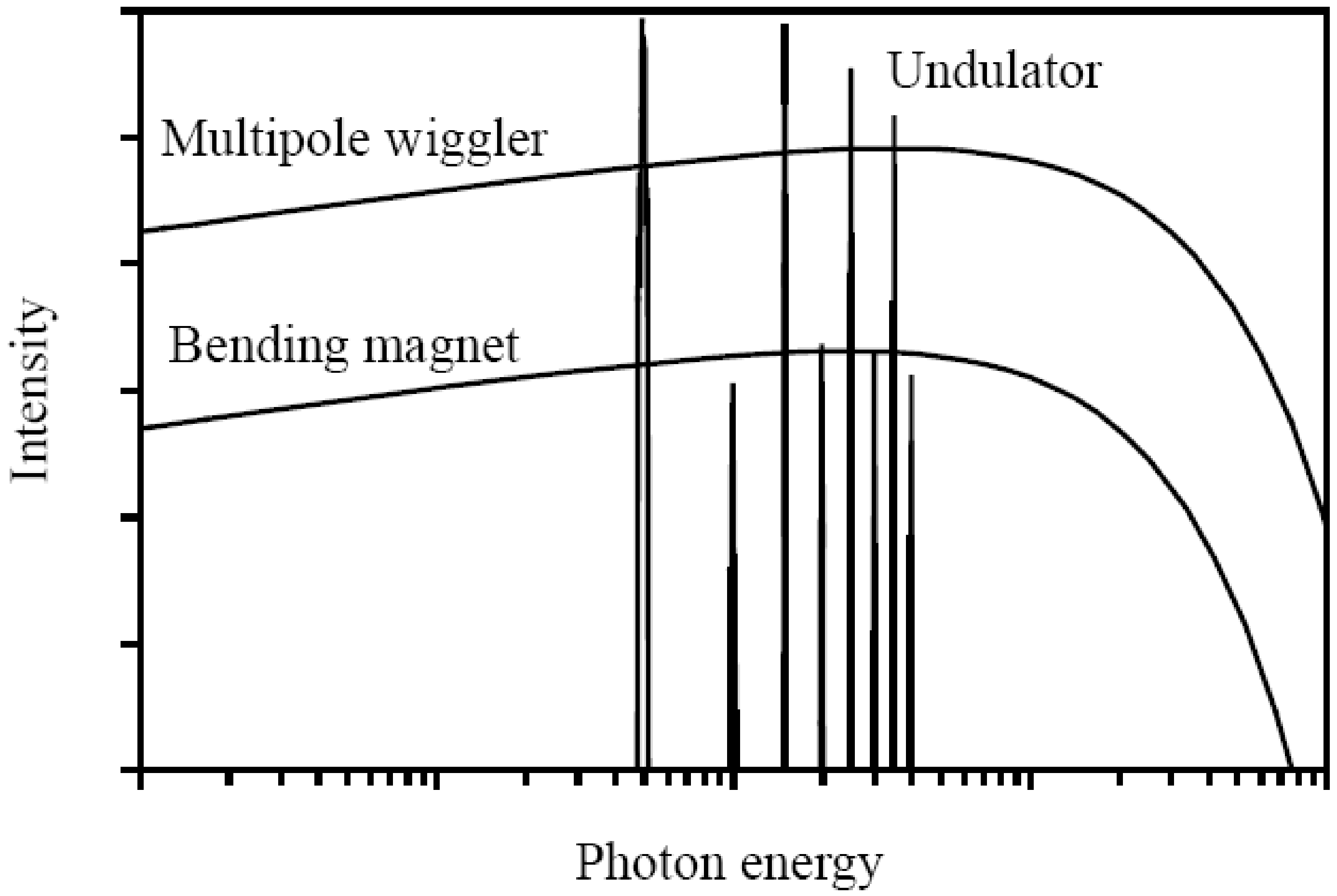
Three basic sources of SR

- Dipole
- (Multipole) Wiggler
- Undulator



Science and Technology Facilities Council
ASTeC

Typical Spectrum of the Three Sources



Researchers typically only use one photon energy or wavelength at a time (they use a monochromator to select this wavelength) for their experiments so undulators, which have the highest intensities, are the most popular sources of SR

Science and Technology Facilities Council
ASTeC

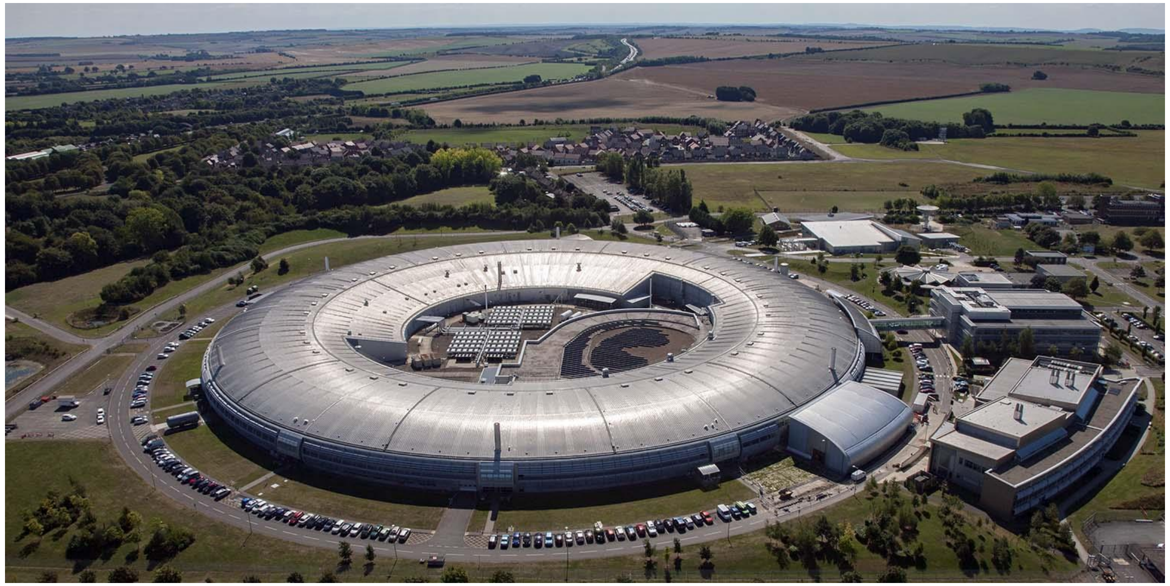
Synchrotron Light Sources

In all modern storage ring based light sources the majority of experiments rely upon undulator sources, these are typically 2 to 5m long magnets

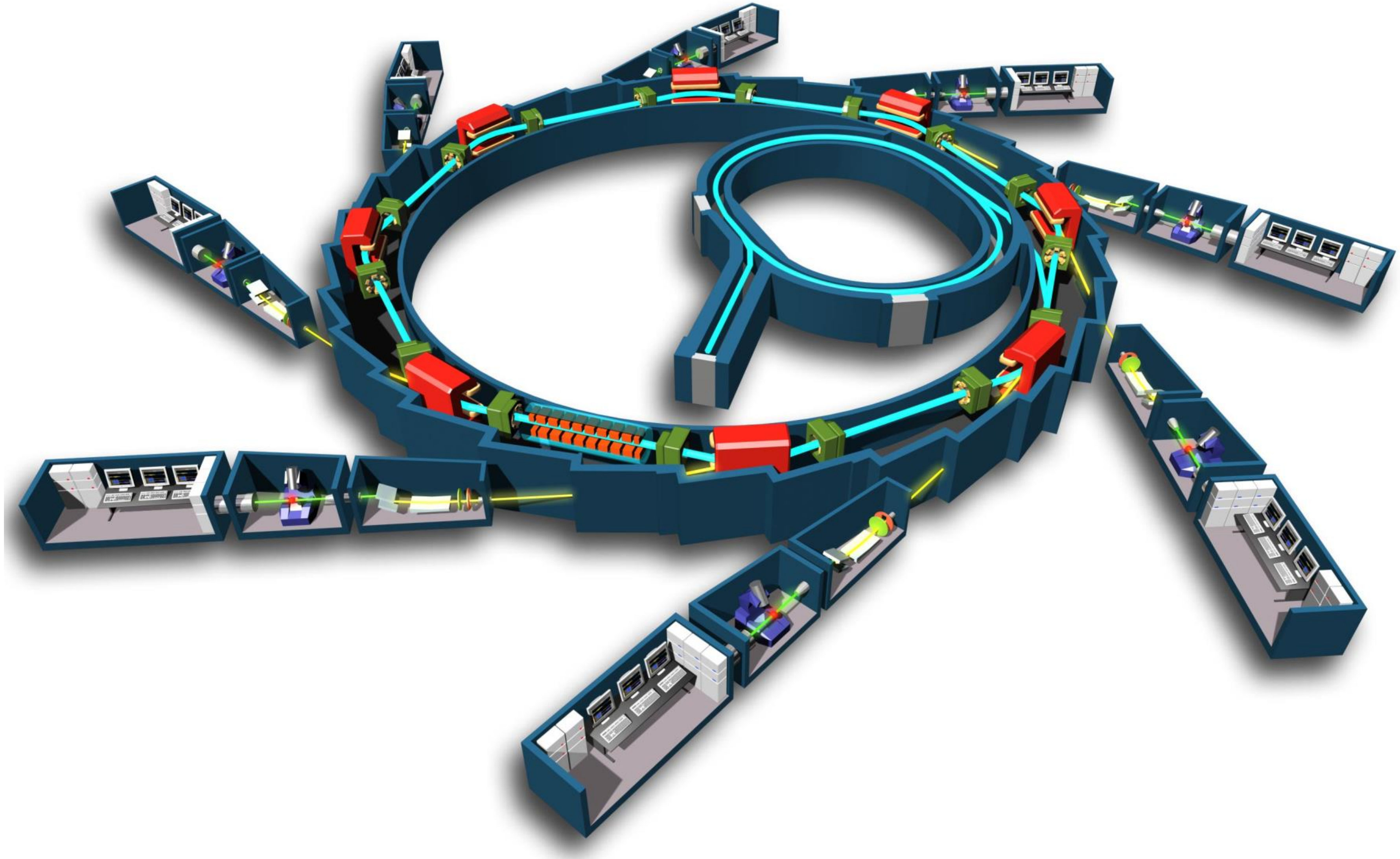
~50 user facilities worldwide

ESRF, Grenoble

Diamond Light Source, UK



Synchrotron Light Sources



By EPSIM 3D/JF Santarelli, Synchrotron Soleil - Synchrotron Soleil, Attribution, https://commons.wikimedia.org/w/index.php?curid=376907

Science and Technology Facilities Council

ASTeC

Free Electron Lasers

The most advanced light source is the **X-ray Free Electron Laser**

Requires an undulator ~50 to 100 m long

Brighter than a synchrotron by ~1,000,000,000 times

LCLS in USA, the first hard X-ray FEL opened in 2009 and is now being upgraded

Science and Technology Facilities Council

ASTeC

Introduction to Synchrotron Radiation

Synchrotron Radiation (SR) is a relativistic effect

Many features can be understood in terms of a combination of two relativistic processes:

Lorentz contraction and

Doppler shift

I will talk about electrons but the effect is present for all charged particles

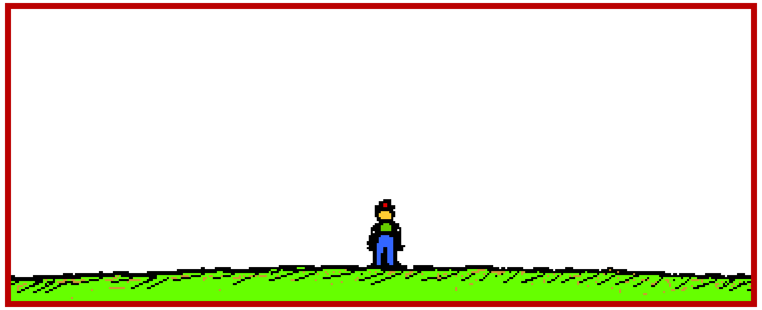
Lorentz Contraction

Special Relativity tells us that moving objects shorten in length along their direction of travel.

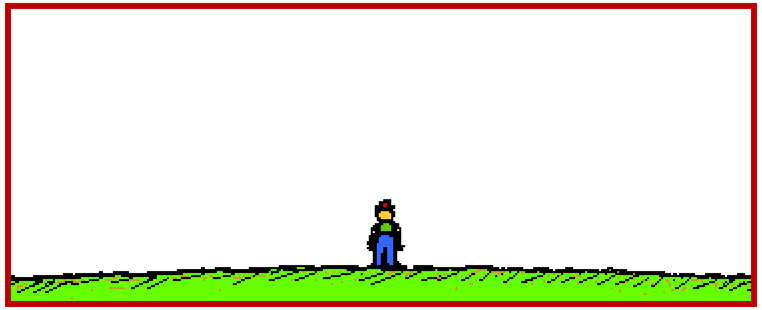
$$\gamma = \frac{E}{E_0}$$

E_0 = 0.511 MeV for an electron

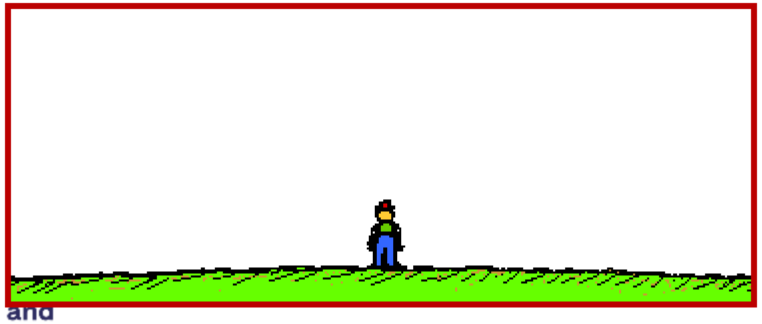
Spaceship Moving at the 10 % the Speed of Light



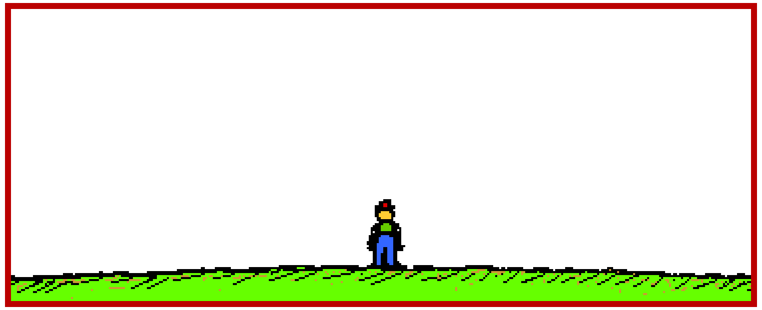
Spaceship Moving at the 86.5 % the Speed of Light



Spaceship Moving at the 99 % the Speed of Light



Spaceship Moving at the 99.99 % the Speed of Light



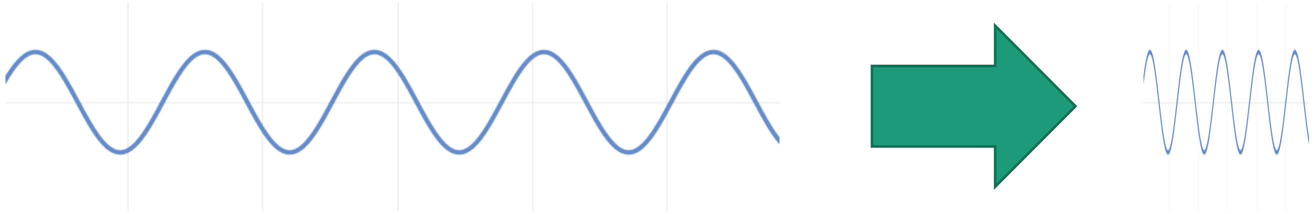
http://www.physicsclassroom.com

Lorentz Contraction

Imagine that a relativistic electron is travelling through an undulator with a periodic magnetic field (i.e. the field has a sinusoidal variation)

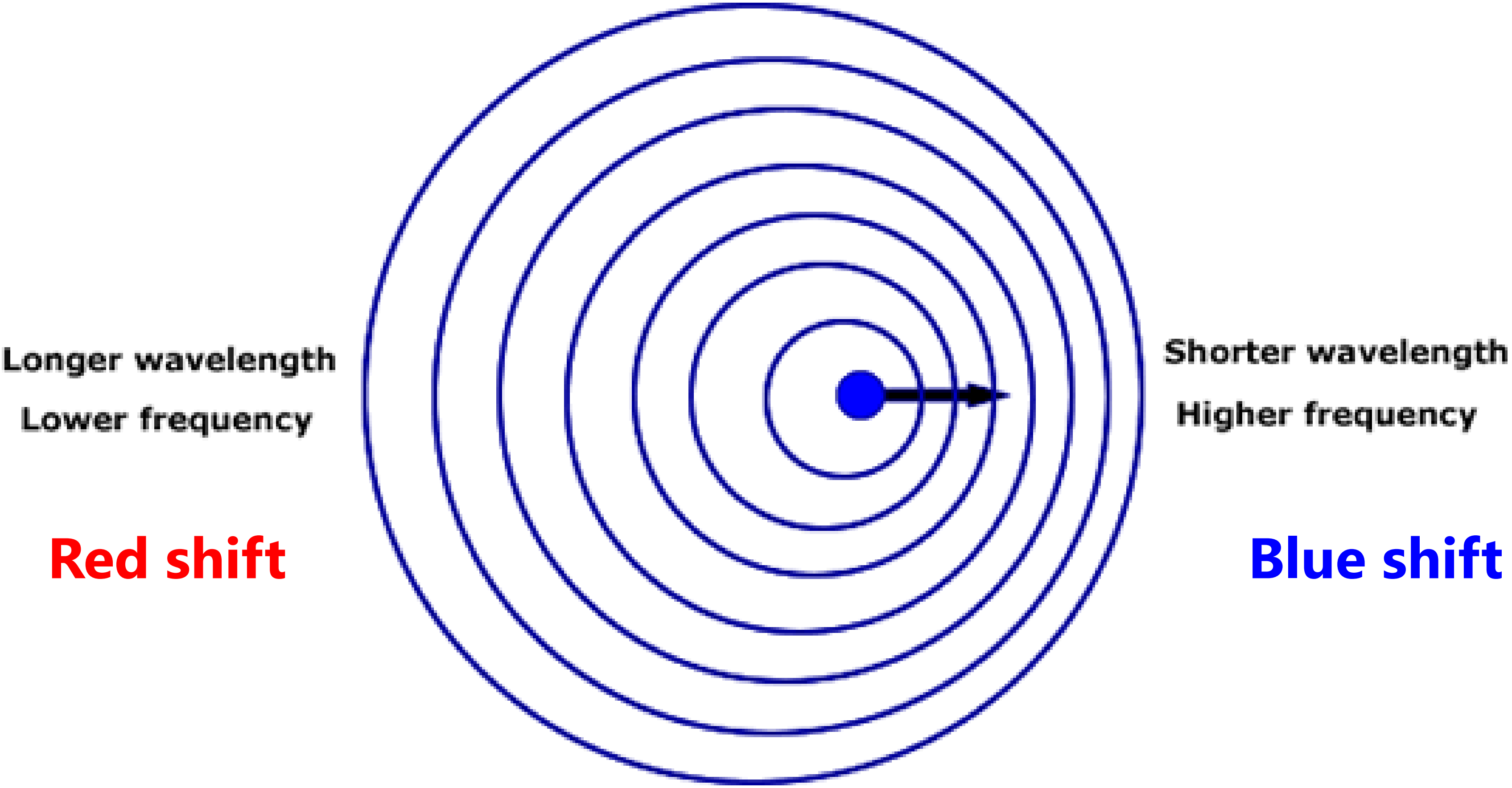
To the electron it seems like a magnetic field is rushing towards it

If in our rest frame the magnet period is λ_u then because of Lorentz contraction the electron sees it as λ_u/γ



UKRI Science and Technology Facilities Council
ASTeC

Doppler Shift



Relativistic Doppler Shift

In the **relativistic** version of the Doppler effect the ***frequency*** of light seen by an observer at rest is

$$f = \gamma f'(1 + \beta)$$

Source travelling towards the observer

where f’ is the frequency emitted by the moving source and β = v/c ~1 when the electron is relativistic.

So, in terms of wavelength $\lambda \sim \frac{\lambda'}{2\gamma}$

Combining Lorentz and Doppler

So the electron emits light of wavelength λ_u/γ

Lorentz contraction

And since it is travelling towards us this wavelength is further reduced by a factor 2γ

Doppler shift

So the wavelength observed will be ~ $\lambda_u/2\gamma^2$

For GeV electron energies with γ of 1000's, an undulator with a period of a **few cm** will provide radiation with wavelengths of **nm (X-rays)**

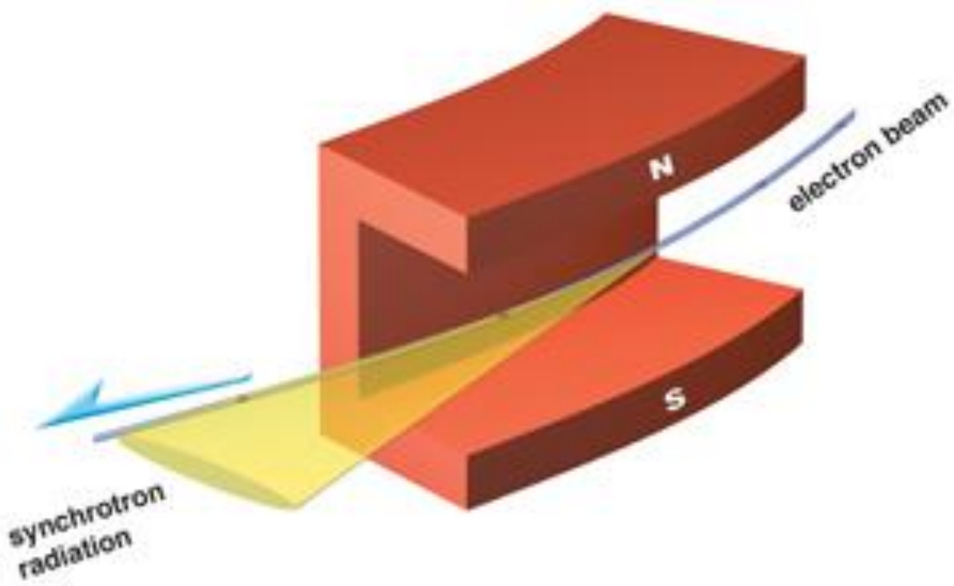
SR from Dipoles

A bending magnet or dipole has a uniform magnetic field

The electron travels on the arc of a circle of radius set by the magnetic field strength

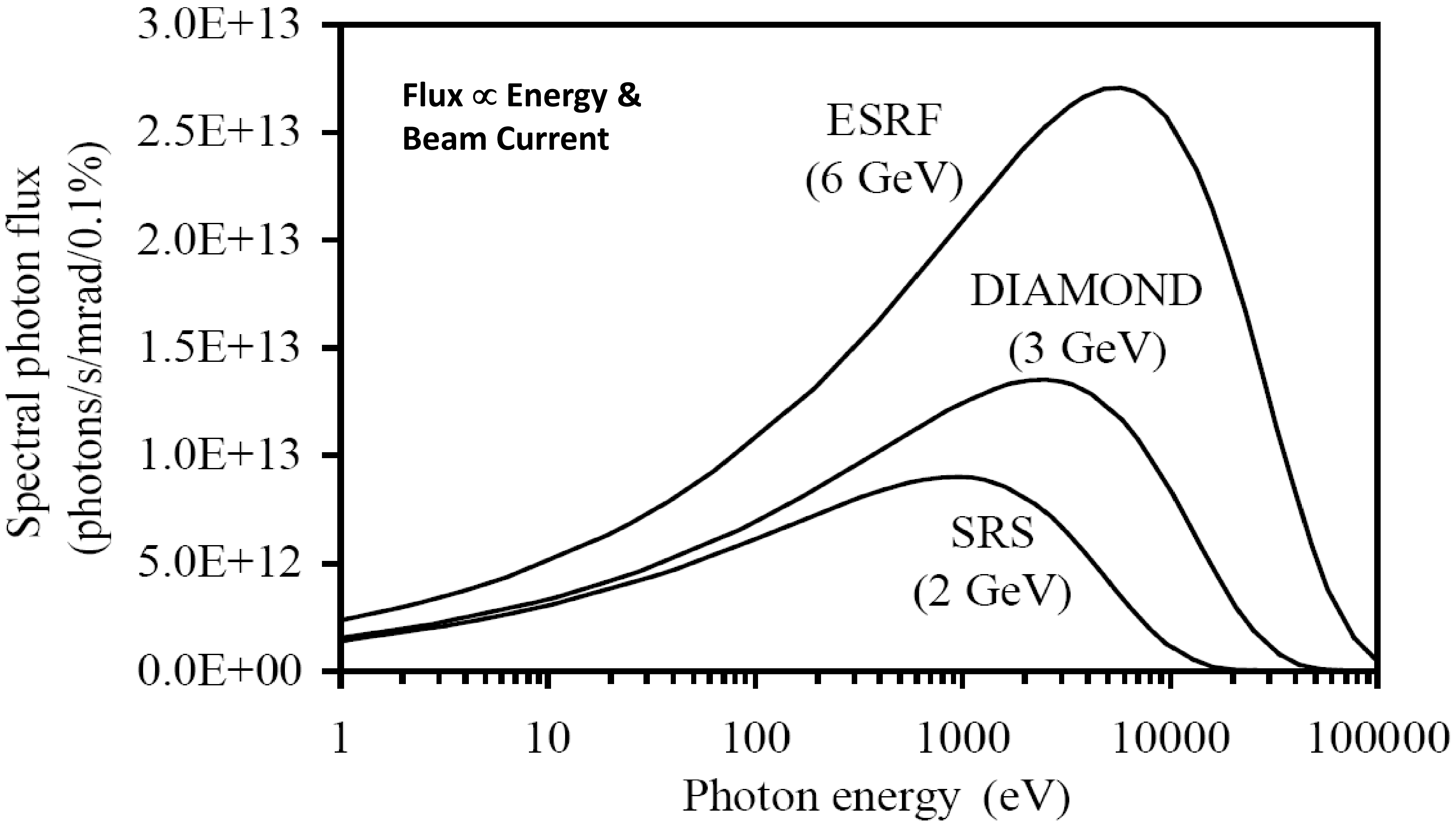
Horizontally the light beam sweeps out like a lighthouse - the intensity is flat with horizontal angle

Vertically it is in a narrow cone of angle typically $\pm 1/\gamma$



Examples for Photon Flux

log-linear scale, 200mA beam current assumed for all sources



SR Power

Virtually all SR facilities have melted vacuum chambers or other components due to the SR hitting an uncooled surface by accident

The average SR power is very high and the **power density is even higher**

The total power emitted by the electron beam in the dipoles is

$$P_{\text{total}} = 88.46 \, \frac{E^4 I_b}{\rho_0}$$

where the power is in **kW**, E is in GeV, I_b is in A, bend radius ρ_0 is in m.

and **power density on axis** (in W/mrad^2)

$$\left.\frac{dP}{d\Omega}\right|_{\psi=0} = 18.08 \, \frac{E^5 I_b}{\rho_0}$$

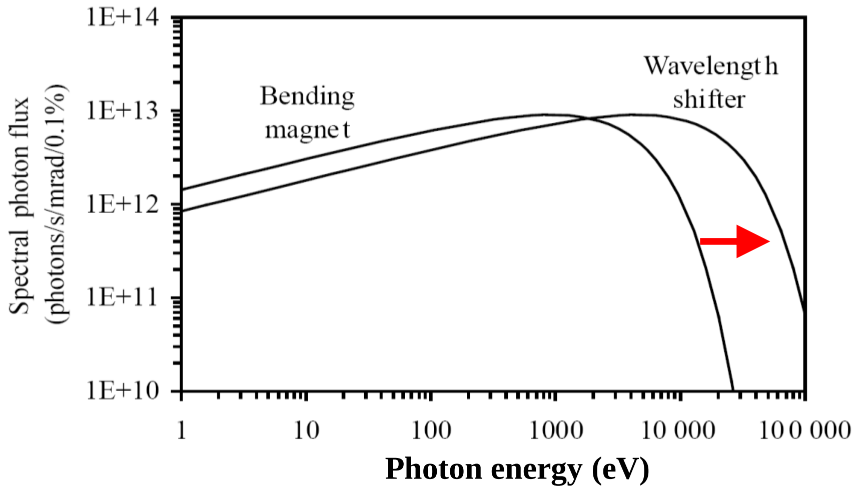
Examples

Ring	Energy (GeV)	ρ (m)	I_b (mA)	P_{total} (kW)	$dP/d\Omega$ (W/mrad^2)
SRS	2	5.56	200	50.9	20.8
DIAMOND	3	7.15	300	300.7	184.4
ESRF	6	25.0	200	916.5	1124.0

The RF system must replace this lost power continuously to keep the electrons in an equilibrium state

Adjusting the Dipole Spectrum

- In a storage ring of fixed energy, the spectrum can be shifted sideways along the photon energy axis if a different bending radius can be generated.
- Requires a different B Field – higher fields give higher energy photons
- This will then shift the rapidly falling edge (higher energy photons, shorter wavelengths)
- Special magnets that do this are called **wavelength shifters**
- Wavelength Shifters are also Insertion Devices but they are rarely used these days as wigglers and undulators have superior properties except in some extreme examples



Science and Technology Facilities Council

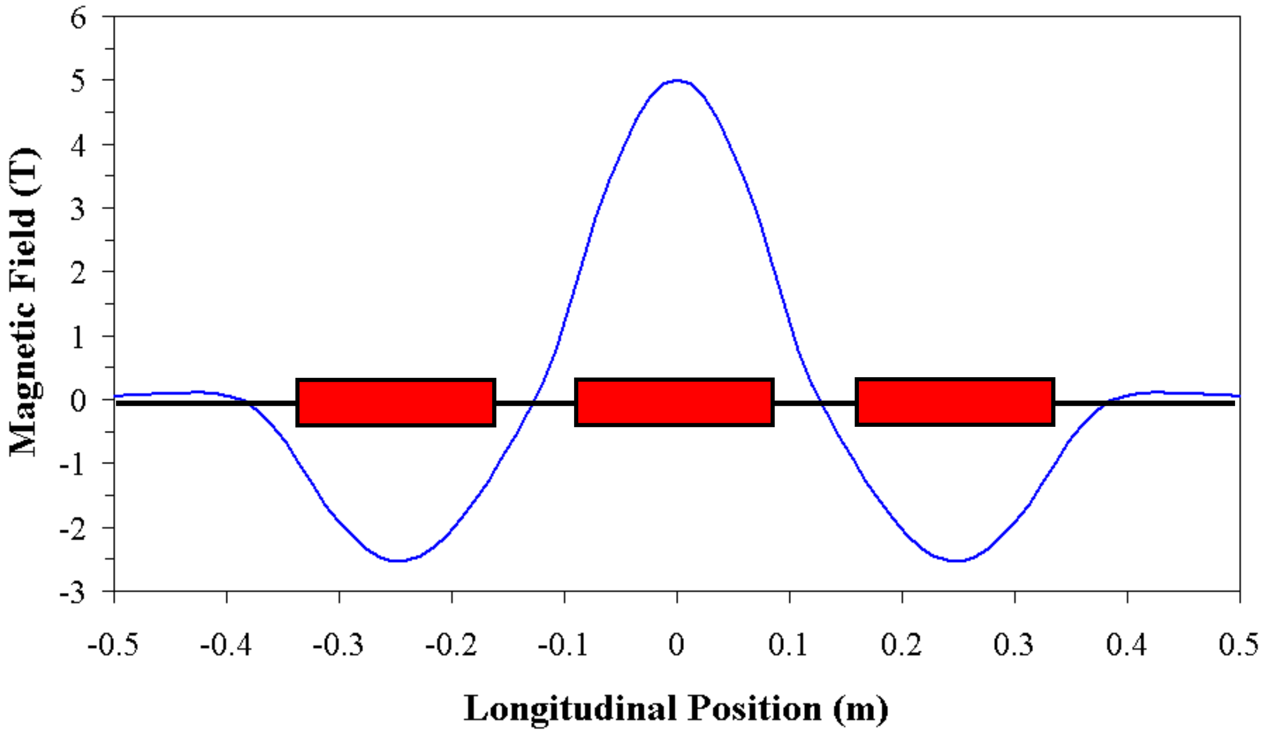
ASTeC

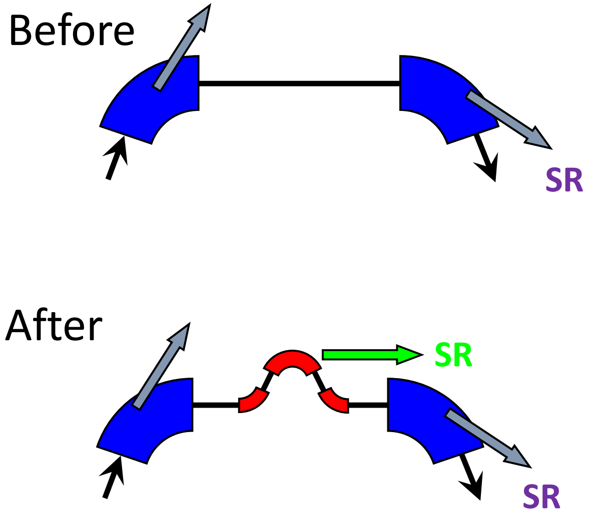
Wavelength Shifters

How can you introduce a high magnetic field into a storage ring?

A simple solution is to use 3 magnets to create a chicane-like trajectory on the electron beam in a straight section

The central magnet is the **high field** bending magnet source



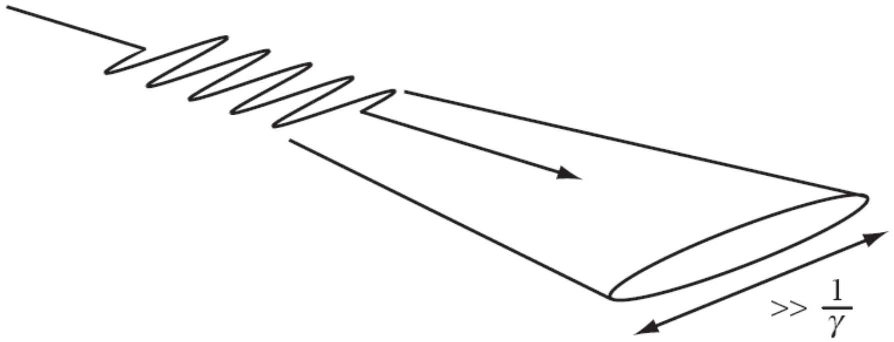


Multiple Wavelength Shifters?

- If we can fit many wavelength shifters one after the other in a straight section then we will get more photons!
- Each Wavelength Shifter would be an independent source of SR – all emitting in the same forward direction.
- The observer will see SR from all the Source points – the flux would just add up linearly

Multipole Wigglers

- This idea is the basic concept for a **Multipole Wiggler**
- A Multipole Wiggler has lots of high field magnets giving both shorter wavelengths and even more photons
- Separate Wavelength Shifters is not the most efficient use of the space! A better way of packing more high field emitters into a straight is a sinusoidal field shape



Wigglers – Electron Trajectory

Electrons are travelling in the **s** direction

The equations of motion for the electron are

$$\ddot{x} = \frac{d^2x}{ds^2} = \frac{e}{\gamma m_0 c}(B_y - \dot{y}B_s)$$

$$\ddot{y} = \frac{d^2y}{ds^2} = \frac{e}{\gamma m_0 c}(\dot{x}B_s - B_x)$$

If we have a wiggler which only makes the electron oscillate in the horizontal plane (x) (i.e. $B_x = B_s = 0$) then:

$$\ddot{x} = \frac{eB_y}{\gamma m_0 c}$$

$$\ddot{y} = 0 \quad .$$

Deflection Parameter, K

B_y is **sinusoidal** with period λ_u

$$B_y(s) = -B_0 \sin\left(\frac{2\pi s}{\lambda_u}\right)$$

Integrate once to find $\dot{x}$ which is the ***horizontal angular deflection*** from the s axis

$$\dot{x}(s) = \frac{B_0 e}{\gamma m_0 c} \frac{\lambda_u}{2\pi} \cos\left(\frac{2\pi s}{\lambda_u}\right)$$

Therefore, the maximum angular deflection is $\frac{B_0 e}{\gamma m_0 c} \frac{\lambda_u}{2\pi}$

We define K as the **"deflection parameter"**

$$K = \frac{B_0 e}{m_0 c} \frac{\lambda_u}{2\pi} = 93.36\, B_0 \lambda_u$$

Trajectory of the Electron

One more integration gives the electron path, which is also a sine wave

$$x(s) = \frac{K}{\gamma}\frac{\lambda_u}{2\pi}\sin\left(\frac{2\pi s}{\lambda_u}\right)$$

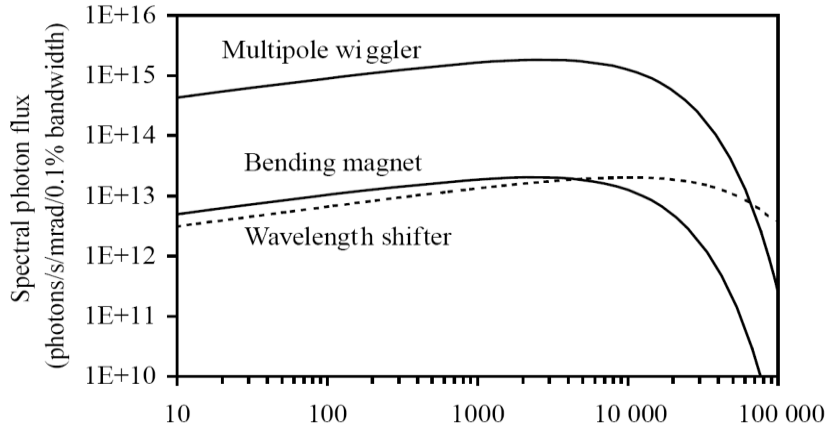
The maximum angular deflection is $$\frac{K}{\gamma}$$

The maximum transverse displacement is $$\frac{K}{\gamma}\frac{\lambda_u}{2\pi}$$

Wiggler Flux

- A Wiggler can be considered a series of dipoles one after the other
- There are two source points per period (two poles per period)
- The flux is simply the number of source points multiplied by the dipole flux for one pole
- The Wiggler has two clear advantages:
  - **The spectrum can be set to suit the science need**
  - **The Flux is enhanced by the number of poles**

Example
300mA, 3 GeV beam
1.4T dipole
6T WS
1.6T MPW with 45 periods



Science and Technology Facilities Council
ASTeC

Wiggler Power

The total power emitted by a beam of electrons passing through **any magnet system** is

$$P_{\text{total}} = 1265.5\, E^2 I_b \int_0^L B(s)^2 ds$$

For a sinusoidal magnetic field the total power emitted is (in W)

$$P_{\text{total}} = 632.8\, E^2 B_0^2 L I_b$$

Power Examples

Dipoles

Ring	Energy (GeV)	ρ (m)	I_b (mA)	P_{total} (kW)	$dP/d\Omega$ (W/mrad^2)
SRS	2	5.56	200	50.9	20.8
DIAMOND	3	7.15	300	300.7	184.4
ESRF	6	25.0	200	916.5	1124.0

Wigglers

Diamond Wiggler, 45 poles at 4.2T, period of 48mm

Power emitted = 33kW

This is equivalent to ~10% all the dipole SR from just one magnet

Undulators

- Undulators are very similar magnetically to wigglers
- They are also periodic magnets that make the electrons take a sinusoidal path
- The key difference compared to a wiggler is that the undulator output is due to ***interference of the light*** emitted by a single electron at the poles of the magnet
    - ***A Wiggler is basically just raw power***
    - ***An Undulator is more subtle and precise and ultimately far more popular!***

Trajectory of the Electron

SR is emitted with a typical angle of $\sim 1/\gamma$

So if $K < 1$ the electron trajectory will always overlap with the emitted SR and constructive interference will enhance the output at certain wavelengths (**an undulator**)

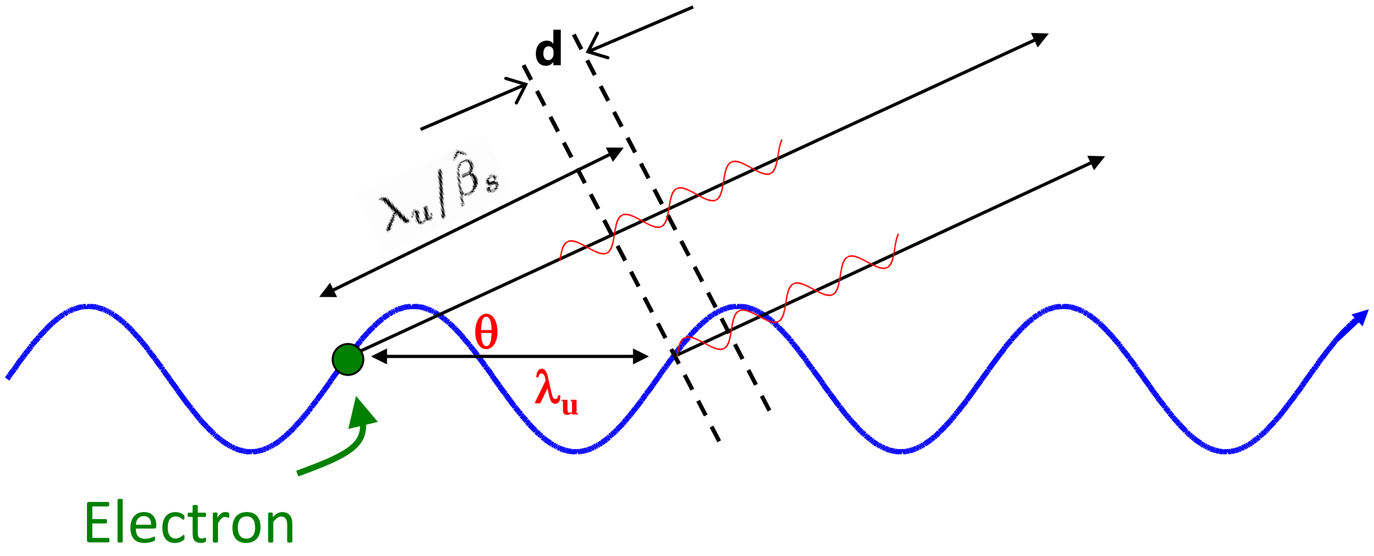
If $K \gg 1$ there will be very little overlap and the source points are effectively independent – this is the case for a **Wiggler**

Warning, be careful with the language:

Some groups, especially FEL people in the USA, refer to undulators as wigglers!

The Condition for Interference

For constructive interference between wavefronts emitted by the same electron
the electron must slip back by a whole number of wavelengths over one period



The Undulator Equation

Solving for λ we get **the undulator equation** for constructive interference

$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \theta^2\gamma^2\right)$$

Example, 3GeV electron passing through a 50mm period undulator with K = 3. First harmonic (n = 1), on-axis is ~4 nm.

So **cm** periods translate to **nm** wavelengths because of the γ^2 in the denominator

Science and Technology Facilities Council

ASTeC

Undulator equation implications

$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \theta^2\gamma^2\right)$$

- The wavelength primarily depends on the **period** and the **electron energy** but also on K and the observation angle θ.

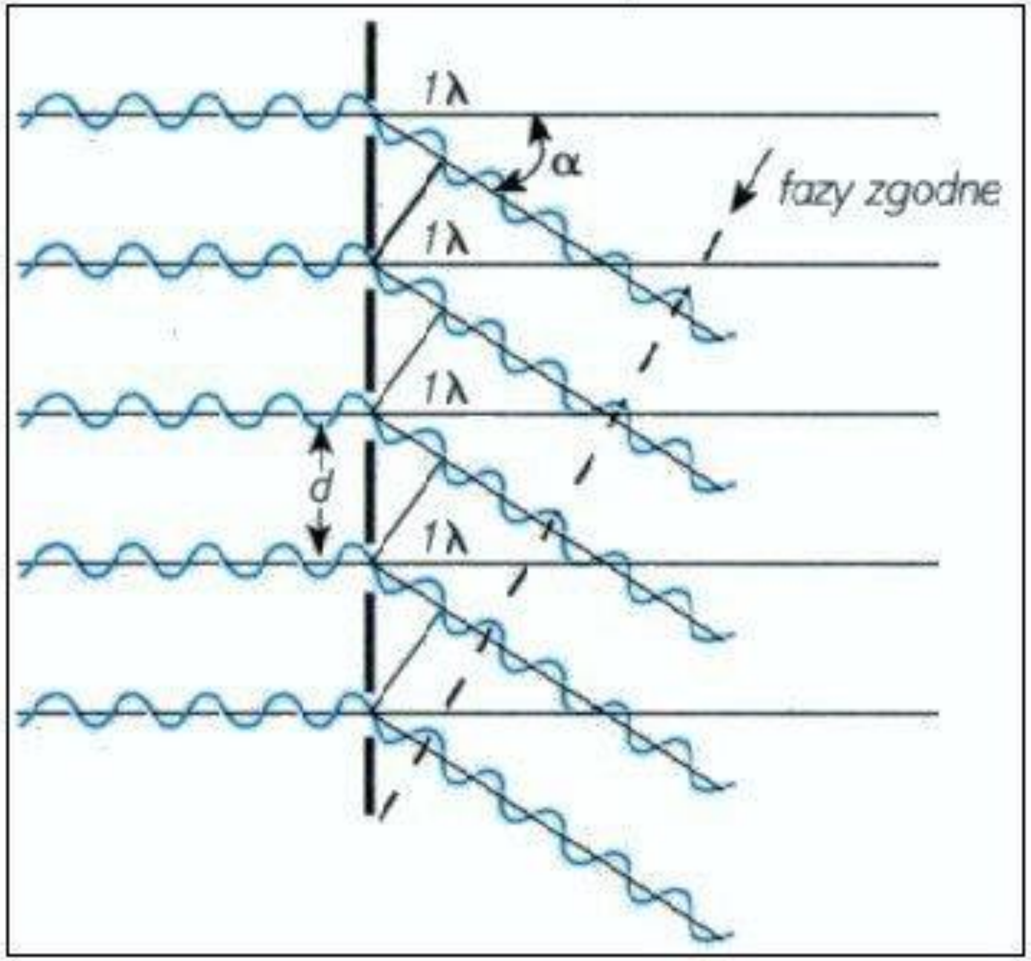
$$K = \frac{B_0 e}{m_0 c}\frac{\lambda_u}{2\pi} = 93.36\, B_0 \lambda_u$$

- **If we can change B_0 we can change λ.** For this reason, undulators are built with an adjustable magnetic field. The amount of adjustability determines the wavelength tuning range of the undulator.

Diffraction Gratings

Undulators and diffraction gratings have much in common

This is because the diffraction grating acts as a large number of periodically spaced sources –
very similar concept to an undulator



Harmonic bandwidth

If we calculate at what wavelength destructive interference will occur we can estimate the harmonic bandwidth (width of the harmonic line) to be

$$\frac{\Delta\lambda}{\lambda} \sim \frac{1}{Nn}$$

So, if the undulator has 100 periods then the first harmonic (n = 1) line will have a bandwidth of ~1%

With a harmonic wavelength of 4 nm, the undulator will provide output between approx 3.98 and 4.02 nm

Example Angular Flux Density

- An Undulator with 50mm period and 100 periods with a 3GeV, 300mA electron beam will generate:
- Angular flux density of **8 x 10^{17}** photons/sec/mrad^2/0.1% bw

- For a dipole with the same electron beam we get a value of **~ 5 x 10^{13}** photons/sec/mrad^2/0.1% bw

- **The undulator has a flux density ~10,000 times greater than a dipole because it scales with N^2**
- N is the number of undulator periods

Example Flux

- Undulator with 50mm period, 100 periods
- 3GeV, 300mA electron beam
- Our example undulator has a flux of **4 x 10^{15}** photons/s/0.1% bandwidth compared with the dipole of **~ 10^{13}** photons/s/0.1%
- **The flux is proportional to N**

Undulator Tuning Curves

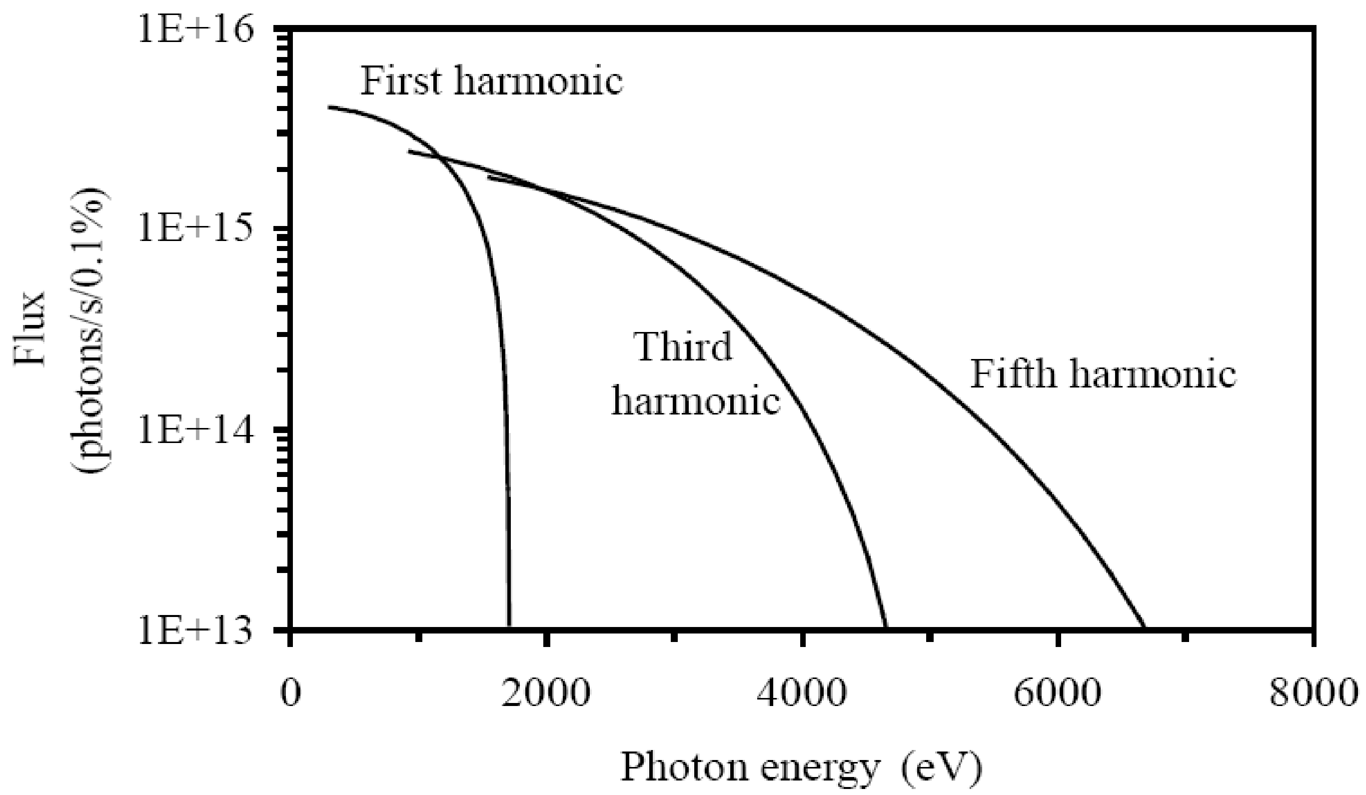
- Undulator outputs are often described by **"tuning curve"** plots
- These show the flux (or brightness) ***envelope*** for an undulator.
- The tuning of the undulator is achieved by **varying the K parameter**
- Obviously K can only be one value at a time !
- These curves represent what the undulator is capable of as the K parameter is varied – but not all of this radiation is available at the same time!

- The undulator user selects whatever wavelength they need by choosing the appropriate K value, to change wavelength they choose a different K value

Example Undulator Tuning Curve

Undulator with 50mm period, 100 periods

3GeV, 300mA electron beam



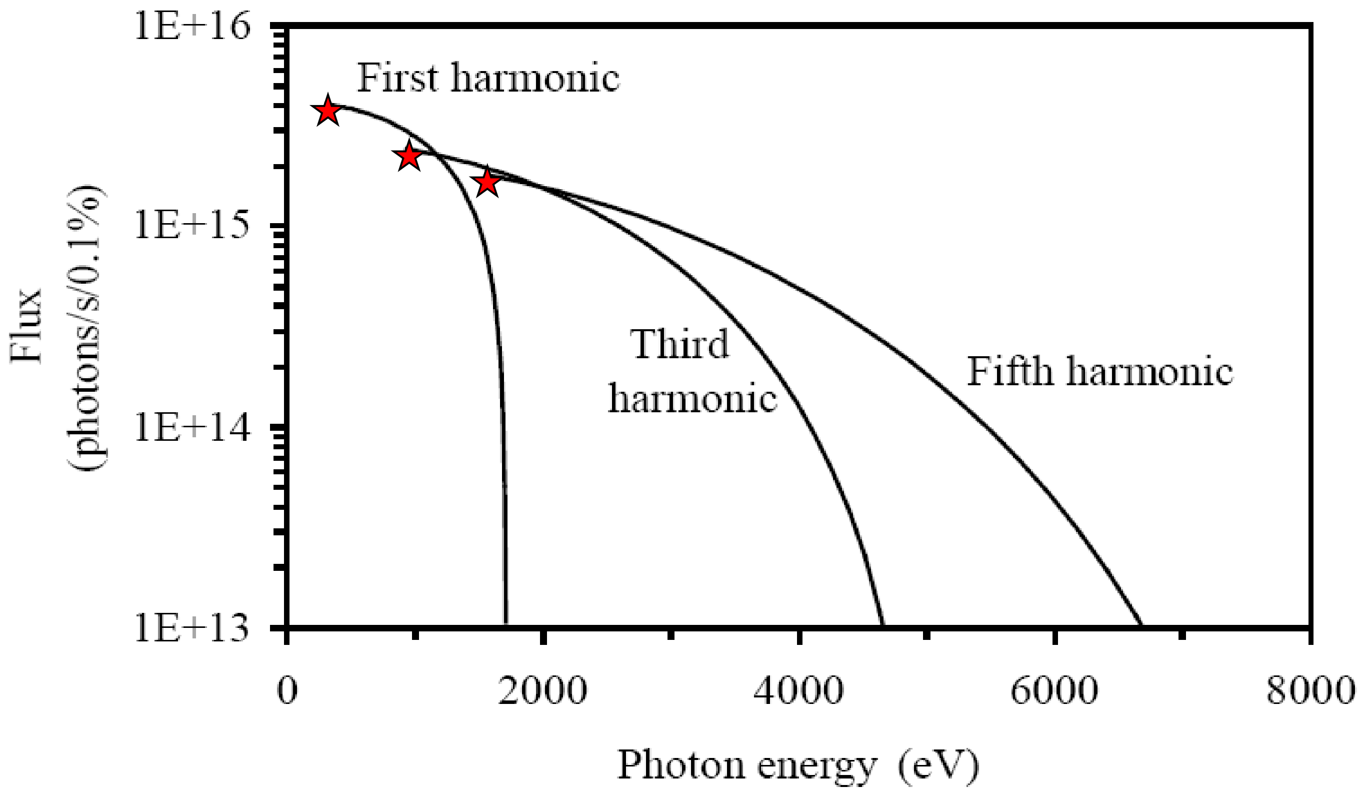
Science and Technology Facilities Council

ASTeC

Example Undulator Tuning Curve

Undulator with 50mm period, 100 periods

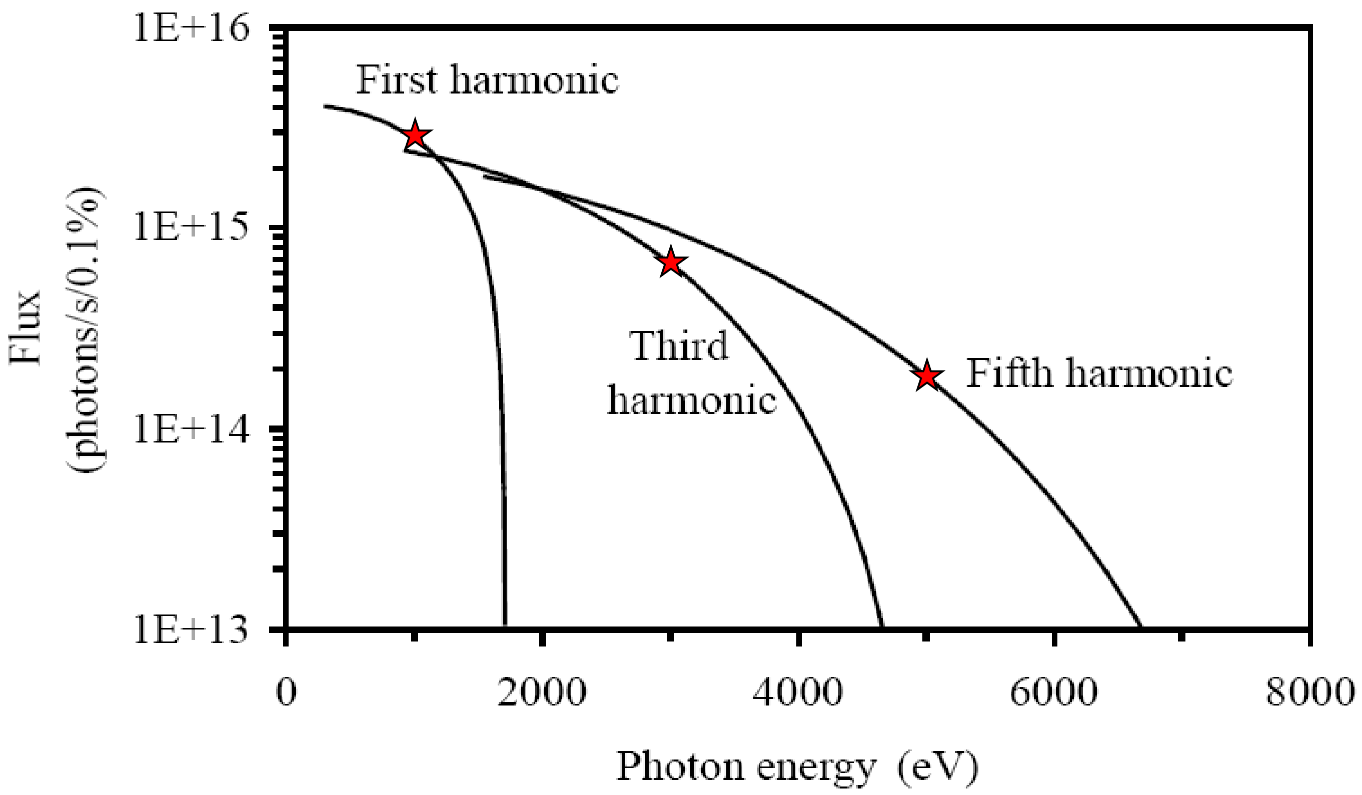
3GeV, 300mA electron beam



Example Undulator Tuning Curve

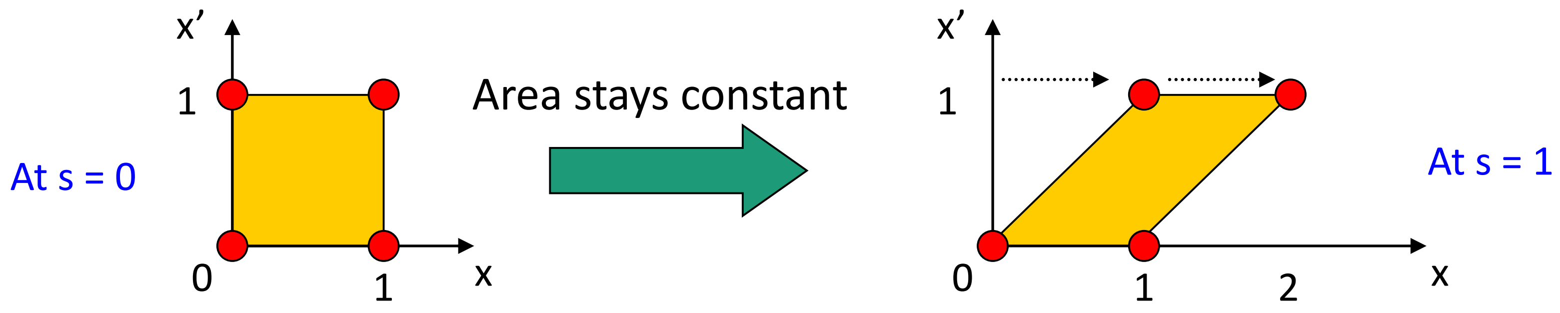
Undulator with 50mm period, 100 periods

3GeV, 300mA electron beam



Undulator Brightness

- All emitted photons have a position and an angle in phase space (x, x')
- Phase space evolves as photons travel but the area stays constant (Liouville's theorem)
- The emittance of an electron beam is governed by the same theorem
- **Brightness is the phase space density of the flux** – takes account of the number of photons and their concentration
- **Brightness (like flux) is conserved** by an ideal optical transport system, unlike angular flux density for instance
- Since it is conserved it is a **good figure of merit** for comparing sources (like electron beam emittance)



Undulator Brightness

To calculate the brightness therefore we need the "phase space areas"

We need to include the photon and electron beam contributions

We add contributions in quadrature as both are ***assumed*** to be Gaussian distributions

The photon beam size is found by assuming the source is the fundamental mode of an optical resonator (L is the undulator length)

$$2\pi\sigma_r\sigma_{r'} = \frac{\lambda}{2}$$

Where:

$$\sigma_r = \frac{1}{4\pi}\sqrt{\lambda L}$$

Radiation width (in x and y)

$$\sigma_{r'} = \sqrt{\frac{\lambda}{L}} \ .$$

Radiation divergence (in x and y)

Science and Technology Facilities Council

ASTeC

Undulator Brightness

Our example undulator has a source size and divergence of **11μm and 28μrad** respectively

The electron beam can also be described by Gaussian shape (genuinely so in a storage ring!) and so the **effective** source size and divergence is given by

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_r^2} \qquad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{r'}^2}$$

$$\Sigma_y = \sqrt{\sigma_y^2 + \sigma_r^2} \qquad \Sigma_{y'} = \sqrt{\sigma_{y'}^2 + \sigma_{r'}^2}$$

The units of brightness are photons/s/solid area/solid angle/spectral bandwidth

Example Brightness

Undulator brightness is the flux divided by the phase space volume given by these **effective** values

$$B = \frac{\dot{N}}{4\pi^2 \Sigma_x \Sigma_y \Sigma_{x'} \Sigma_{y'}}$$

For our example undulator, using electron beam parameters:

$$\sigma_x = 100\ \mu\text{m},\ \sigma_y = 10\ \mu\text{m},\ \sigma_{x'} = 20\ \mu\text{rad},\ \text{and}\ \sigma_{y'} = 2\ \mu\text{rad}$$

The brightness is

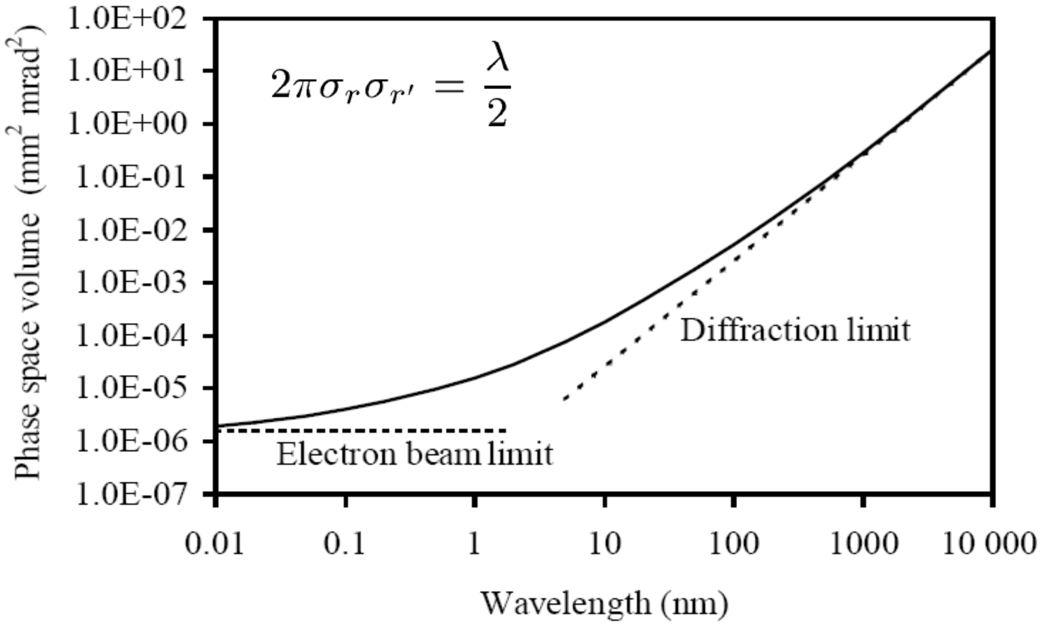
$$7 \times 10^{19}\ \text{photons/s/mrad}^2/\text{mm}^2/0.1\%\ \text{bandwidth}$$

Diffraction Limited Light Sources

Light source designers strive to reduce the electron beam size and divergences to maximise the brightness

But when $\sigma_r \gg \sigma_{x,y}$ and $\sigma_{r'} \gg \sigma_{x',y'}$ then there is nothing more to be gained

In this case, the source is said to be **diffraction limited**



Example **phase space volume** of a light source

$$4\pi^2 \Sigma_x \Sigma_y \Sigma_{x'} \Sigma_{y'}$$

In this example, for wavelengths of >100nm, the electron beam has virtually no impact on the undulator brightness

Summary

- Synchrotron Radiation is emitted by accelerated charged particles - the brightest source of X-rays available
- The combination of Lorentz contraction and the Doppler shift turns the **cm length scale into nm wavelengths**
- Wigglers are periodic, high field devices, used to generate broadband SR at short wavelengths
- Undulators are periodic, lower K, devices which generate radiation at specific harmonics – similar conceptually to diffraction gratings