The Muon Collider
Lecture 1

International
MUON Collider
Collaboration

C. T. Rogers
Rutherford Appleton Laboratory

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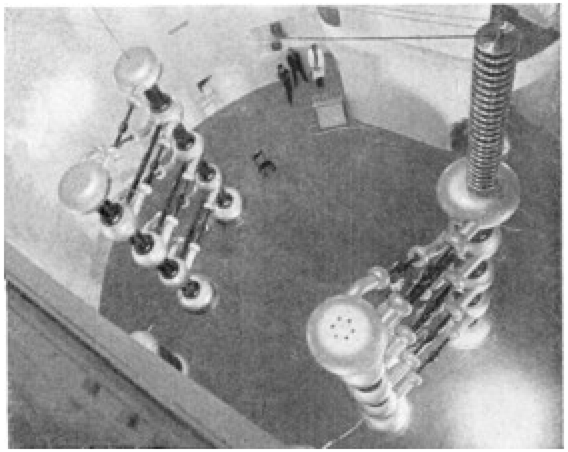
Accelerators in Physics

- First accelerators built in 1920s/30s
  - Accelerating protons, ions and electrons
  - Positrons in 1960s
  - Antiprotons in 1980s
- Tools for fundamental physics
- Hadron colliders
  - E.g. LHC
  - “Discovery machines”
- Electron positron colliders
  - E.g. Large Electron Positron Collider (LEP)
  - “Precision machines”
- Growing interest in building muon collider
  - Muons first accelerated in 2017 – new tech
  - Why muons? How?



Muon Collider

- Lecture 1
  - Why are muon collisions interesting?
  - What are the ingredients required to make muons?
  - Production of low emittances
- Lecture 2
  - Rapid acceleration to fight muon lifetime
  - Extremely low β^* at the focus
  - Experimental demonstration and staging
    - how to make it happen

Why Muons?

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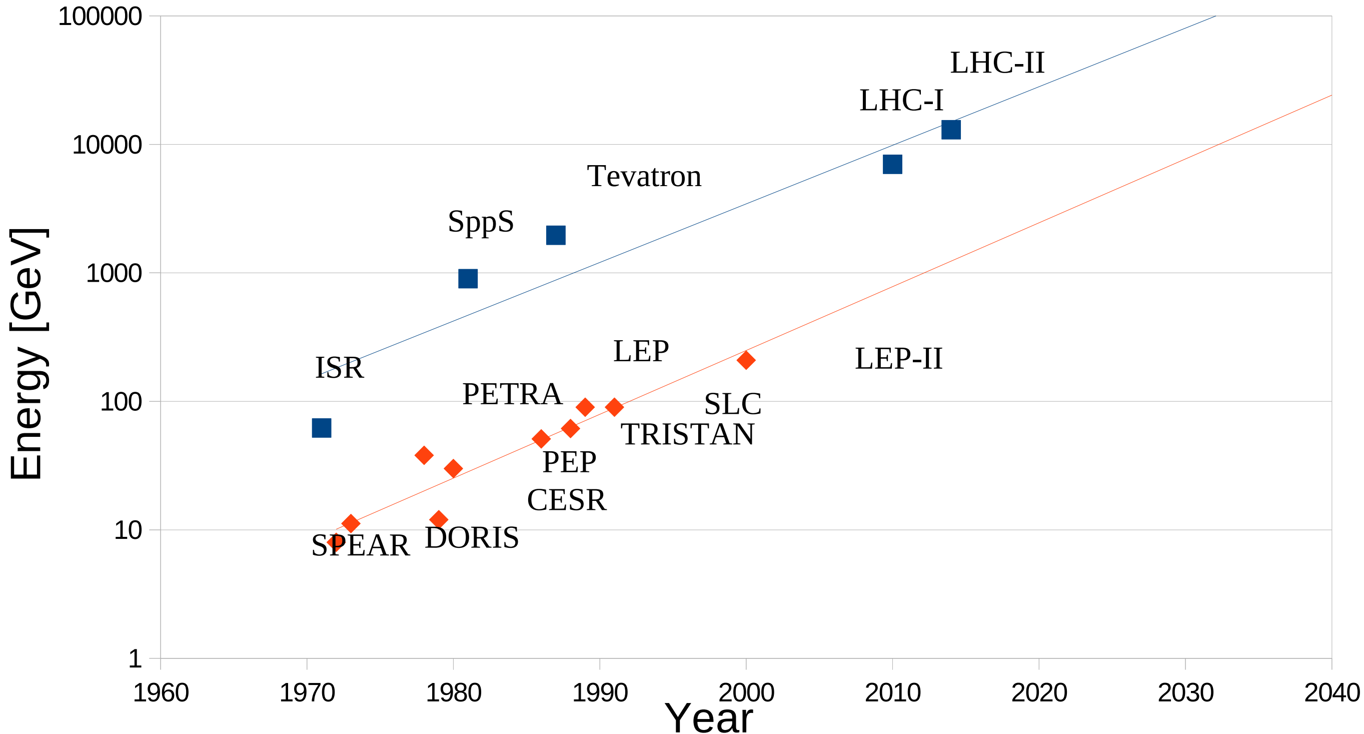
Back to the Future...

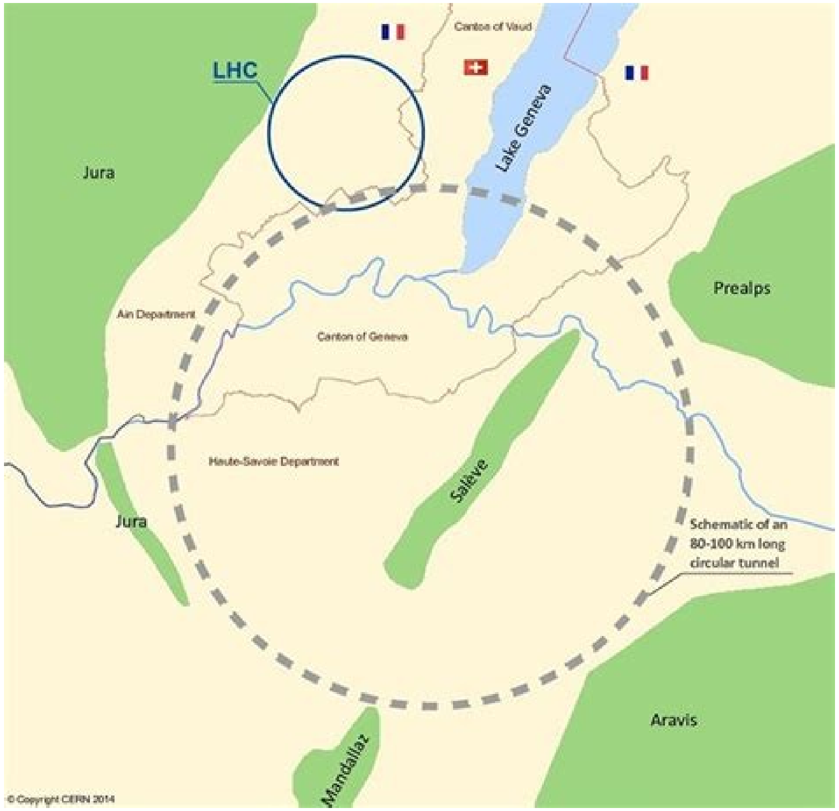


- Effort to explore phenomena at higher and higher energies
- Corresponds to smaller scales
- Higher energy → bigger, more expensive, more power hungry

E.g. circular colliders

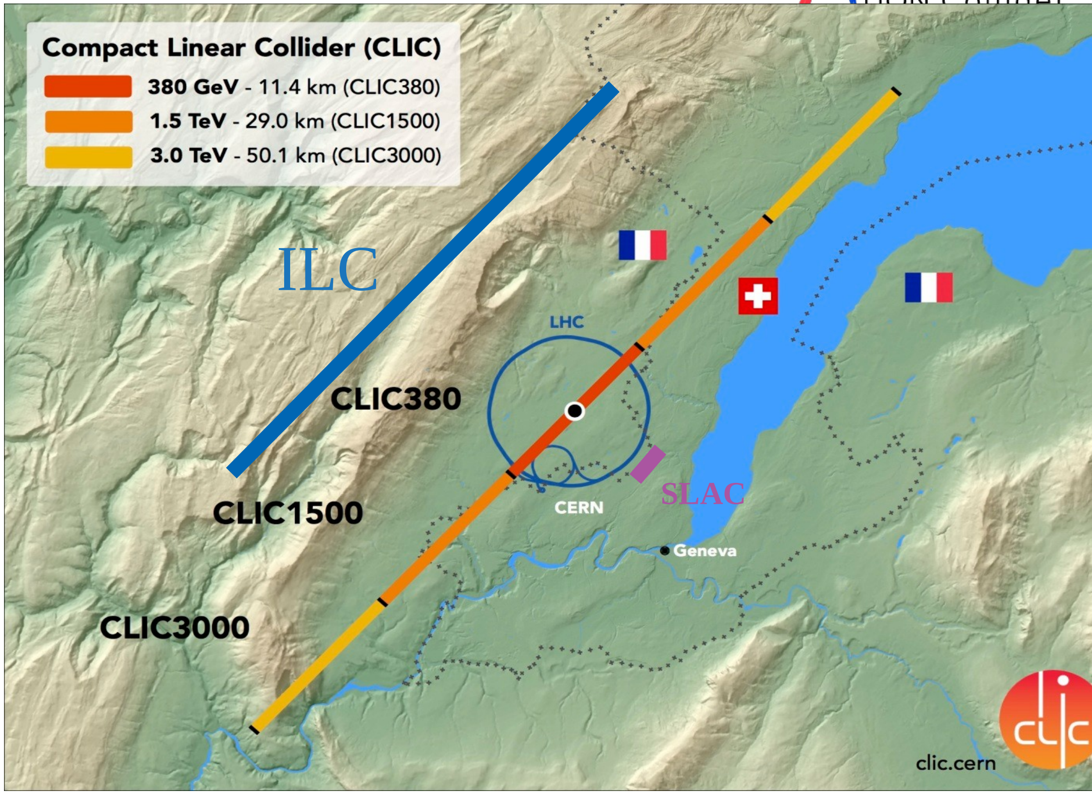
- Tevatron
  - 1.96 TeV proton - antiproton
  - 6.2 km circumference
- LEP/LHC
  - 14 TeV proton proton (LHC)
  - 209 GeV e$^+$e$^-$ (LEP)
  - 27 km circumference
- FCC (proposed)
  - 90 – 350 GeV e$^+$e$^-$
  - 100 TeV proton-proton
  - 90-100 km circumference

○ Tevatron



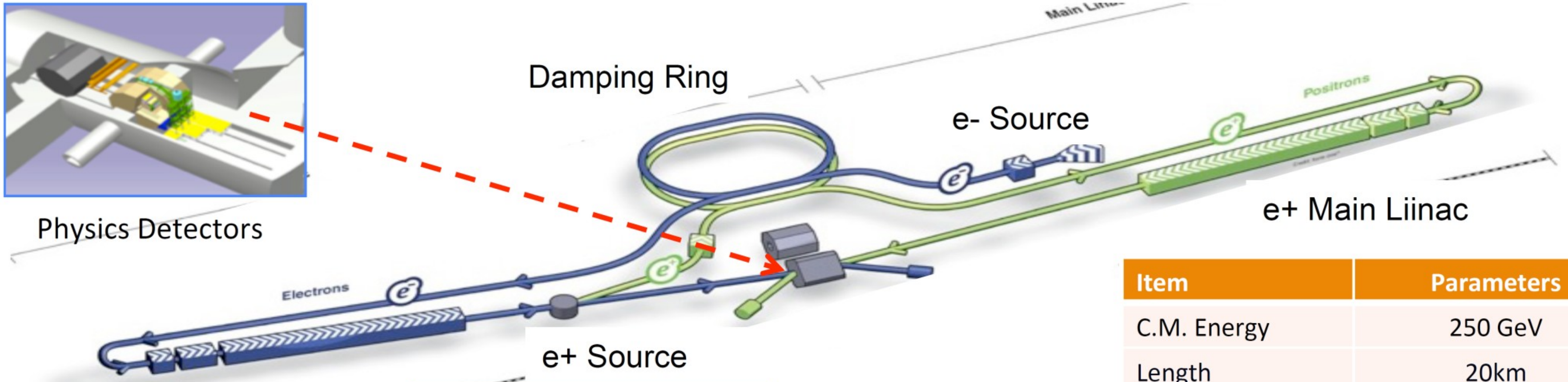
E.g. linear colliders

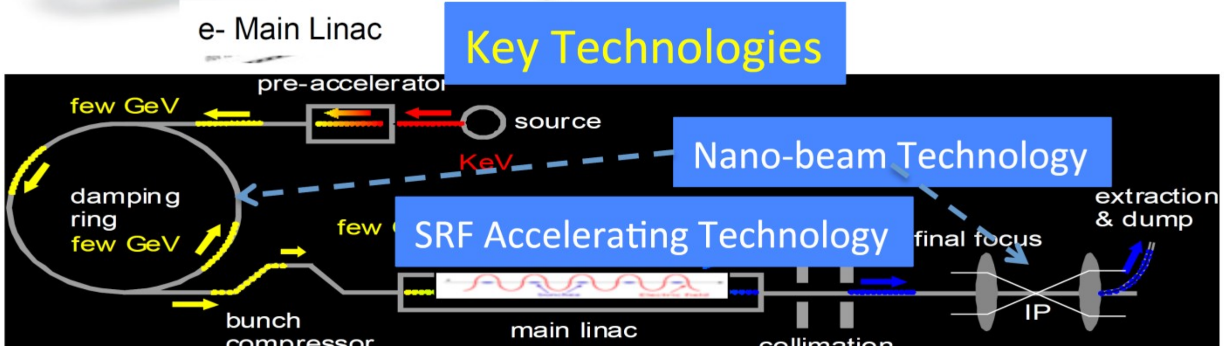
- SLAC (California)
  - 3 km length
  - 90 GeV e^+e^-
- ILC (proposed)
  - 31 km
  - 500 GeV e^+e^-
- CLIC (proposed)
  - 380 GeV e^+e^-
  - 11 km



Electron-positron colliders

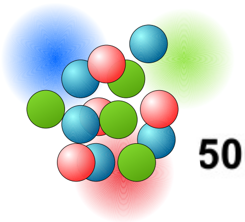
- Circular machines limited by synchrotron radiation
  - Power emitted ~ E^4/m^4
  - Practically limits centre-of-mass energy to ~ low 100s GeV
- Linear machines limited by available RF acceleration
  - Practically limits centre-of-mass energy to ~ 100s GeV (TeV)

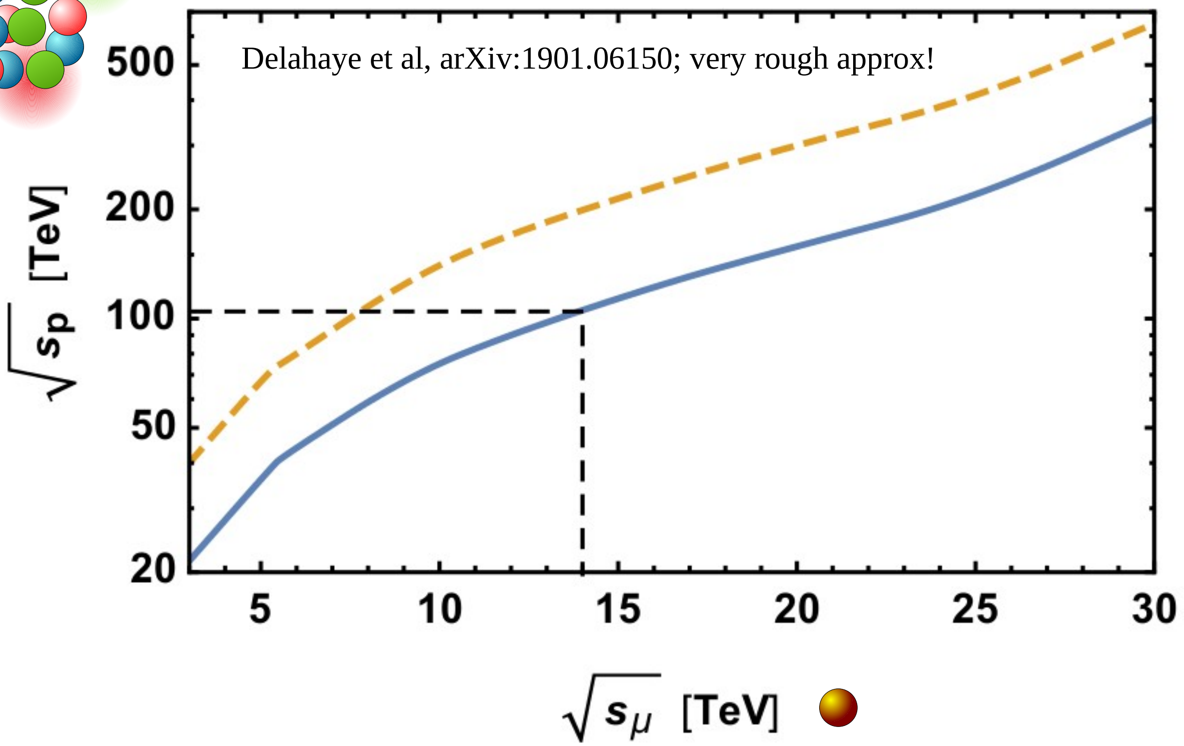




Item	Parameters
C.M. Energy	250 GeV
Length	20km
Luminosity	1.35 x10^{34} $cm^{-2}s^{-1}$
Repetition	5 Hz
Beam Pulse	0.73 ms
Beam Current	5.8 mA (in pulse)
Beam size (y) at FF	7.7 nm@250GeV
SRF Cavity G.	31.5 MV/m

Muons Physics Reach





Delahaye et al, arXiv:1901.06150; very rough approx!

Energy at which cross-section is equal

- - - Assuming equal Feynman amplitude (EW)

▬ Assuming factor 10 enhancement in pp (EW+QCD)

- Seek a particle which
  - Is not so low mass as an electron
  - Is a fundamental particle
- **Muons!**

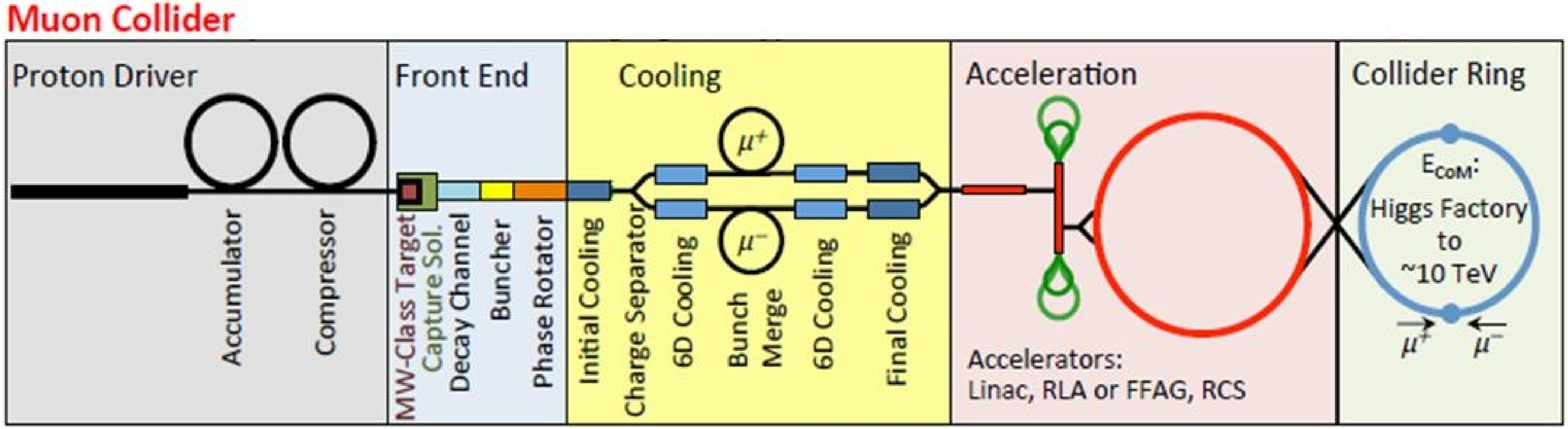
Muons

Le bestiaire

European Organization for Nuclear Research | Organisation européenne pour la recherche nucléaire

- Muon
  - Half-life 2.2 μs
  - Mass 105.658 MeV/c
  - 207 times electron mass
- What would a muon collider look like?

Muon Collider



- MW-class proton driver → target
- Pions produced; decay to muons
- Muon capture and cooling
- Acceleration to TeV & Collisions
- Designed for high energy while **maximising luminosity**
  - Luminosity is key

It’s All About Luminosity

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Luminosity

- **Luminosity** is key challenge
  - Number of events per cross-section per time
  - Diffuse beam → low chance of particles colliding → low luminosity
- Change in integrated luminosity per beam jth crossing

$$\Delta\mathfrak{L} = \frac{N_{+,j}N_{-,j}}{4\pi\sigma_{\perp}^{2}}$$

$N_{+/-}$ = Number of μ^{+} or μ^{-} on jth crossing

$\sigma_{\perp}$ = size of the beam in x/y … assume cylindrical symmetry

- What drives luminosity? Can we relate luminosity to
  - Repetition rate of accelerator
  - Efficiency of muon creation
  - Proton beam parameters
  - Etc

Luminosity – stored muons (1)

$$\Delta\mathfrak{L} = \frac{N_{+,j}N_{-,j}}{4\pi\sigma_\perp^2}$$

- The number of particles is always falling due to muon decay

Number of particles entering collider

Distance between collisions i.e. collider circumference

$$N_{\pm,j} = N_\pm \exp\left(\frac{-2\pi R j}{c\gamma\tau_\mu}\right)$$

Muon speed

Muon lifetime in the lab

Luminosity – stored muons (2)

- Luminosity (change in integrated luminosity per time)

$$\mathfrak{L} = f_r n_b \sum_{j=0}^{\infty} \Delta\mathfrak{L}$$

Number of bunches per acceleration cycle

Repetition rate (number of acceleration cycles per second)

$$\Delta\mathfrak{L} = \frac{N_{+,j}N_{-,j}}{4\pi\sigma_{\perp}^{2}}$$

$$N_{\pm,j} = N_{\pm} \exp\left(\frac{-2\pi R j}{c\gamma\tau_{\mu}}\right)$$

$$\mathfrak{L} = f_r n_b \frac{N_+ N_-}{4\pi\sigma_{\perp}^{2}} \sum_{j=0}^{\infty} \exp\left(-\frac{4\pi R}{\gamma c \tau_{\mu}} j\right)$$

Luminosity – stored muons (3)

$$\mathfrak{L} = f_r n_b \frac{N_+ N_-}{4\pi\sigma_\perp^2} \sum_{j=0}^{\infty} \exp\left(-\frac{4\pi R}{\gamma c \tau_\mu} j\right)$$

- Assuming muon lifetime is long compared to ring time-of-flight

$$\boxed{R = p/(e\bar{B}) \approx \gamma m_\mu c/(e\bar{B})}$$

$$\mathfrak{L} \approx f_r n_b \frac{N_+ N_-}{(4\pi)^2 \sigma_\perp^2} \frac{\gamma c \tau_\mu}{R}$$

- So

$$\mathfrak{L} \approx f_r n_b \frac{N_+ N_-}{(4\pi)^2 \sigma_\perp^2} \frac{\tau_\mu e\bar{B}}{m_\mu}.$$

Luminosity – Facility efficiency

$$\mathfrak{L} \approx f_r n_b \frac{N_+ N_-}{(4\pi)^2 \sigma_\perp^2} \frac{\tau_\mu e \bar{B}}{m_\mu}.$$

$$N_\pm = \frac{\eta_\tau \eta_\pm P_p}{n_b f_r}$$

Efficiency of muon acceleration

Number of muons per proton beam power

Luminosity – $\sigma_\perp$ (1)

$$\mathfrak{L} \approx f_r n_b \frac{N_+ N_-}{(4\pi)^2 \sigma_\perp^2} \frac{\tau_\mu e \bar{B}}{m_\mu}.$$

Transverse beam size

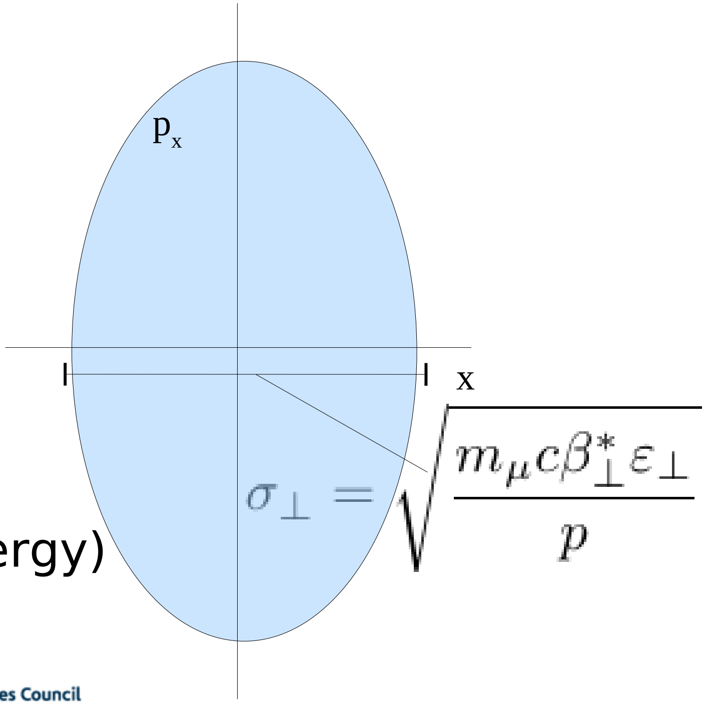
- Definition of emittance

$$\sigma_\perp = \sqrt{\frac{m_\mu c \beta_\perp^* \varepsilon_\perp}{p}}$$

Twiss function

Emittance

- Need very tight focusing!
- Limits:
  - Focusing strength of magnets
  - Chromaticity (focusing depends on energy)
  - Hour glass effect (next slide)



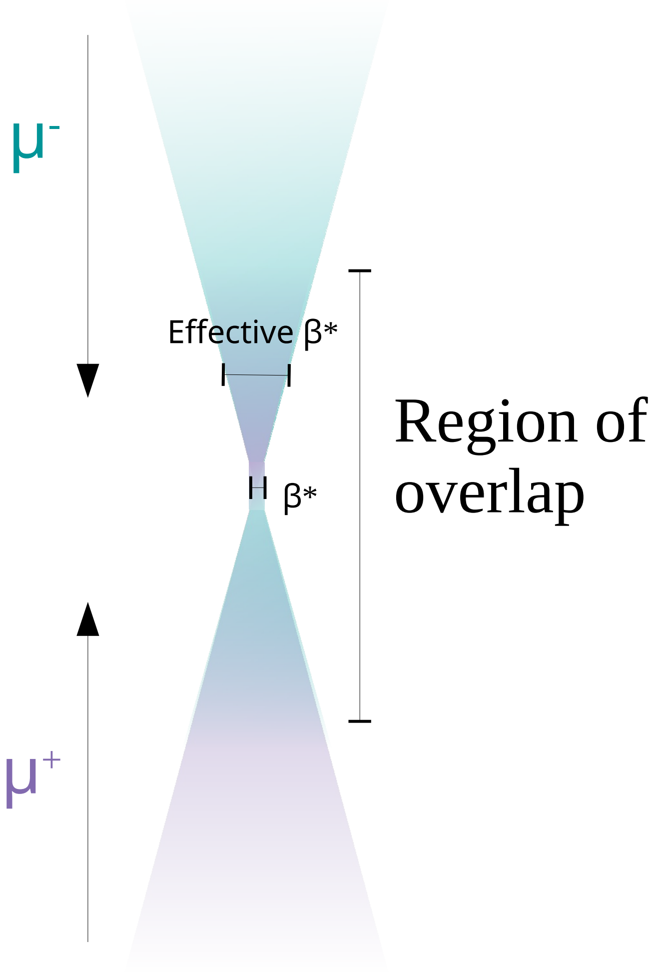
Luminosity – $\sigma_\perp$ (2)

- Hour glass effect
  - Even for collider ring with super small β*
  - Small β* means short focal length
  - Region of overlap is very short
  - Bunch needs to be short as well!
- Introduce hour-glass factor f_{hg}
  - Relates the effective to lattice β^*

$$\beta^*_{eff} = \beta^* f_{hg}$$

- If $\sigma_z = \beta^*$
  - Hour-glass factor is 0.76

$$\sigma_\perp = \sqrt{\frac{m_\mu c \sigma_z \varepsilon_\perp}{p f_{hg}}}$$



Luminosity – $\sigma_\perp$ (3)

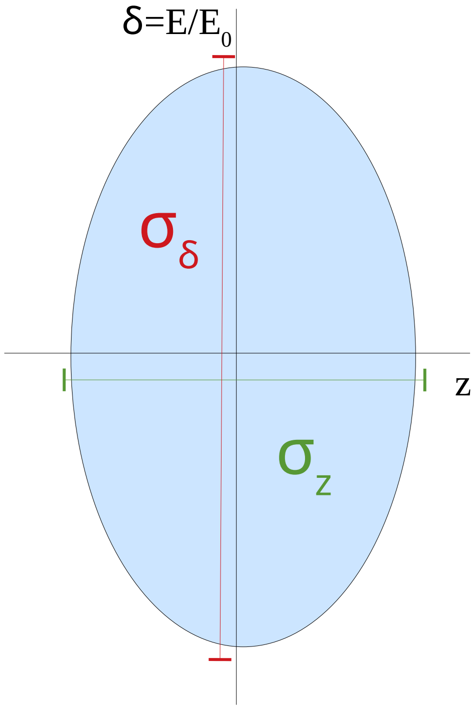
- Definition of longitudinal emittance

$$\varepsilon_l = \gamma m_\mu c^2 \sigma_\delta \sigma_z$$

- So

$$\sigma_\perp = \sqrt{\frac{m_\mu c \sigma_z \varepsilon_\perp}{p f_{hg}}}$$

$$\sigma_\perp^2 = \frac{\varepsilon_l \varepsilon_\perp}{p f_{hg} \gamma c \sigma_\delta}$$



- Recalling the expression for luminosity and N

$$\boxed{\mathfrak{L} \approx f_r n_b \frac{N_+ N_-}{(4\pi)^2 \sigma_\perp^2} \frac{\tau_\mu e \bar{B}}{m_\mu}.}$$

$$\boxed{N_\pm = \frac{\eta_\tau \eta_\pm P_p}{n_b f_r}}$$

- Bringing everything together

$$\mathfrak{L} \approx \underbrace{\frac{e\tau_\mu}{(4\pi m_\mu c)^2}}_{K_L} \frac{f_{hg} \sigma_\delta \bar{B}}{\varepsilon_\perp \varepsilon_L n_b f_r} \underbrace{\eta_+ \eta_- (\eta_\tau P_p \gamma m_\mu c^2)^2}_{P_+ P_-}$$

Recap

$$\mathfrak{L} \approx \underbrace{\frac{e\tau_\mu}{(4\pi m_\mu c)^2}}_{K_L} \frac{f_{hg}\sigma_\delta \overset{\boxed{2}}{\bar{B}}}{\underset{\boxed{4}}{\varepsilon_\perp \varepsilon_L} \underset{\boxed{3}}{n_b f_r}} \underbrace{\overset{\boxed{4}}{\eta_+\eta_-}(\overset{\boxed{5}}{\eta_\tau} P_p \gamma \overset{\boxed{1}}{m_\mu c^2})^2}_{P_+P_-}$$

1) Luminosity increases with the square of muon energy/power
   - Number of collisions per bunch increases as muon lifetime increases
   - Beam size decreases as energy increases (geometric emittance)
2) High field, low circumference collider ring → more luminosity
   - Shorter path length, more collisions before muon decay
3) Low repetition rate, few bunches is best
   - Assume that the bottleneck is in the number of protons
   - Fewer collisions, but each collision is more intense
4) High quality muon source is essential
   - Low emittance, good capture efficiency
5) Good efficiency acceleration is essential
   - High voltage systems

- The whole muon collider is designed to maximise luminosity!

The Facility – From protons to muons

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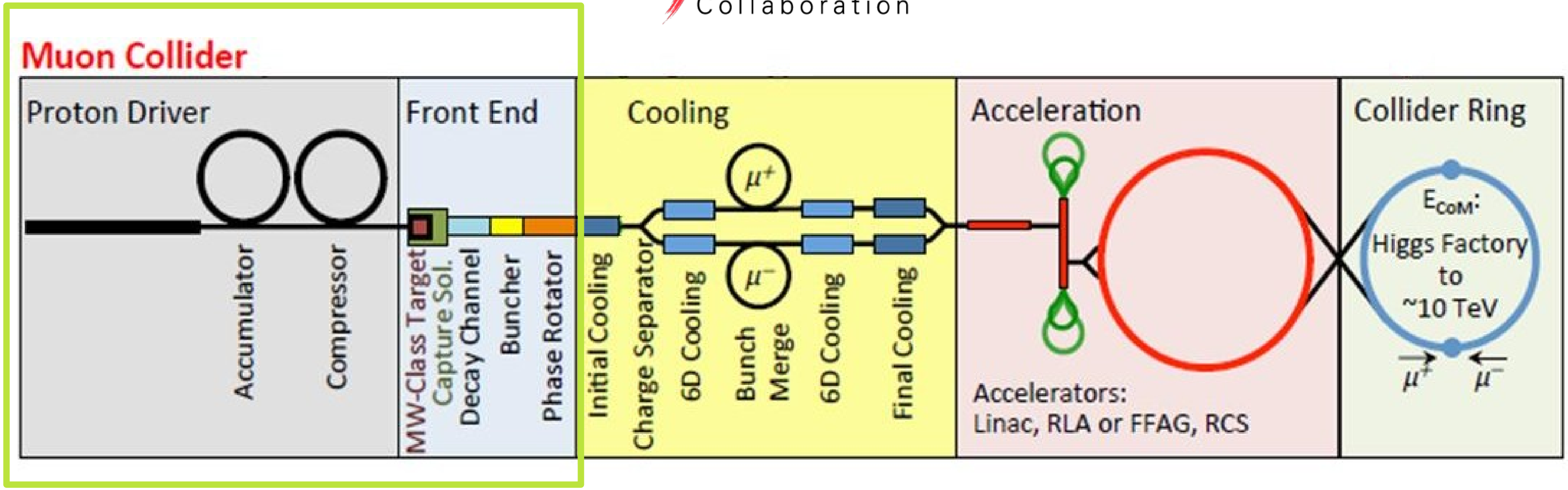
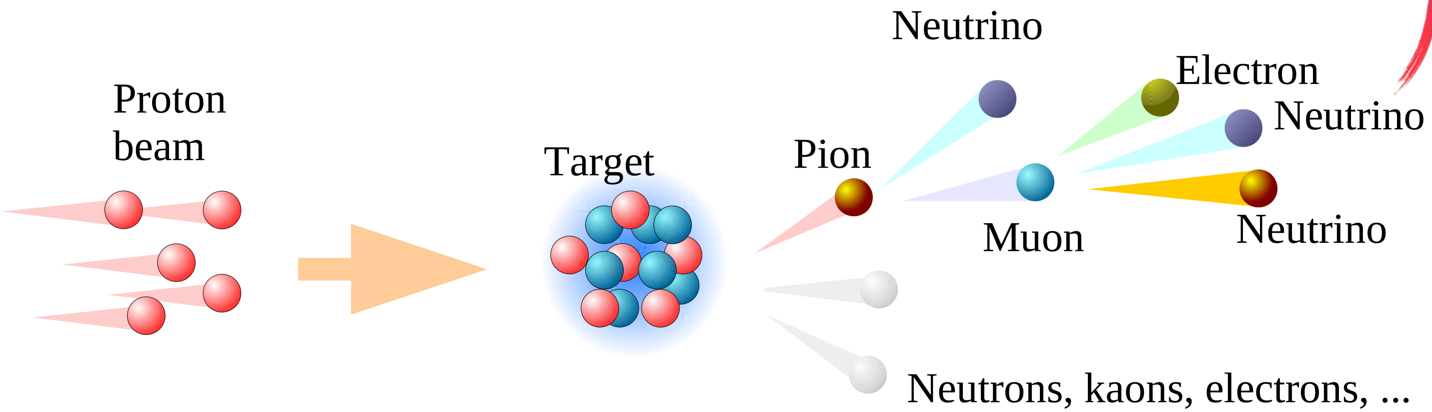
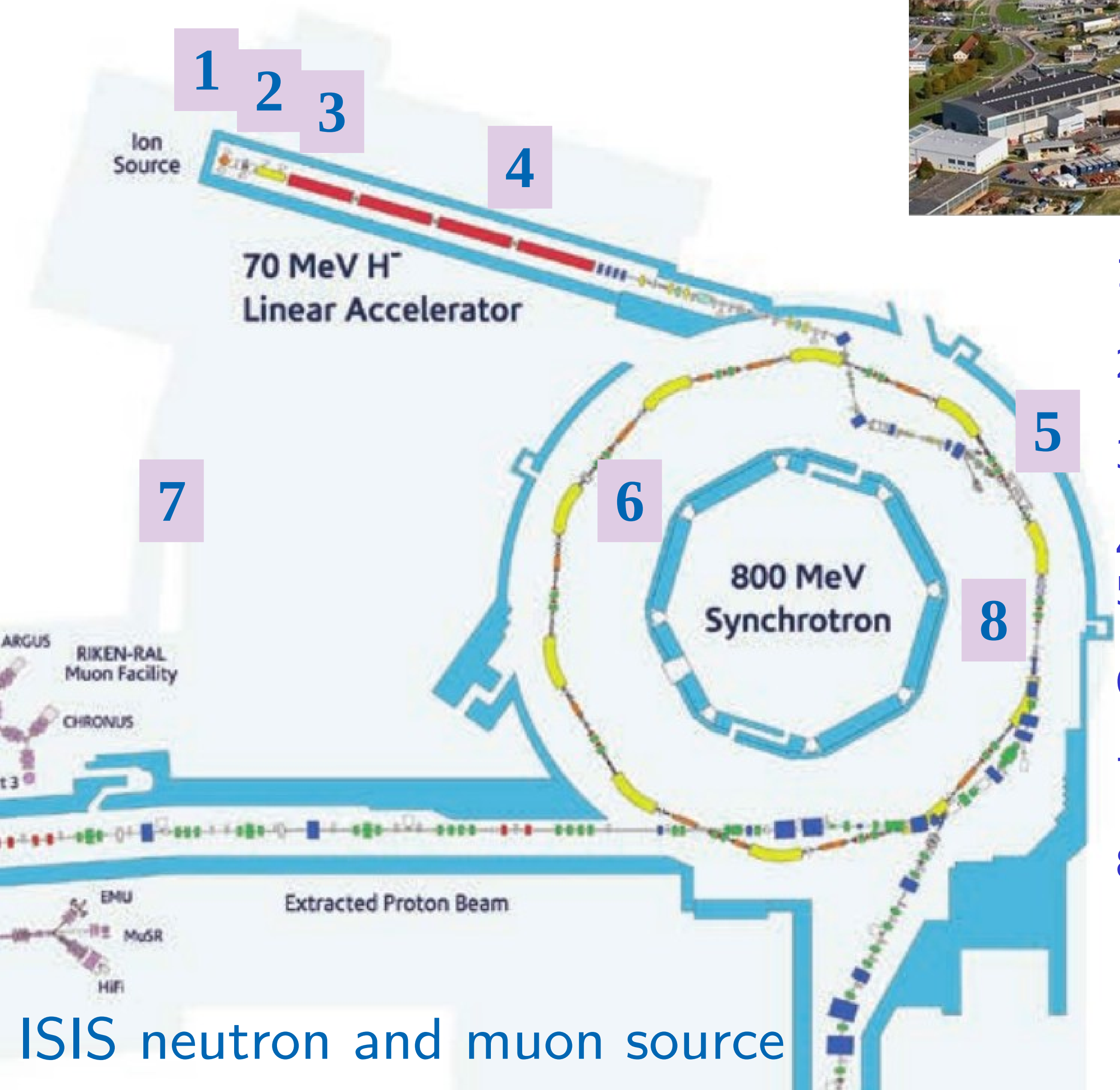


Artificial Muons



- Muons produced by putting protons onto target
- Pions come out
- Pions decay radioactively to muons
- Enables an intense muon source

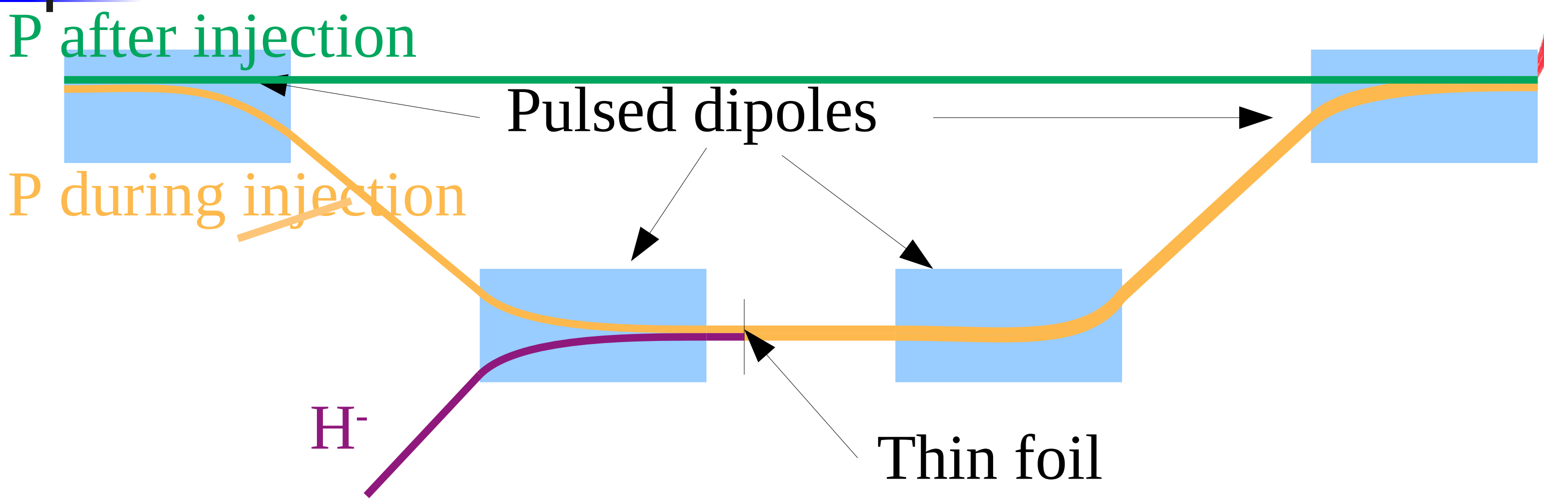
Proton Source



1) Ion source: spark across H gas to make H^- ions
2) Accelerate and focus in Radiofrequency Quadrupole
3) Chop into pulsed beam using fast/slow kicker
4) Accelerate in linac
5) Inject into a ring through a foil
6) Accelerate some more (maybe)
7) Compress the proton bunch to very short length
8) Extract and bring onto a target

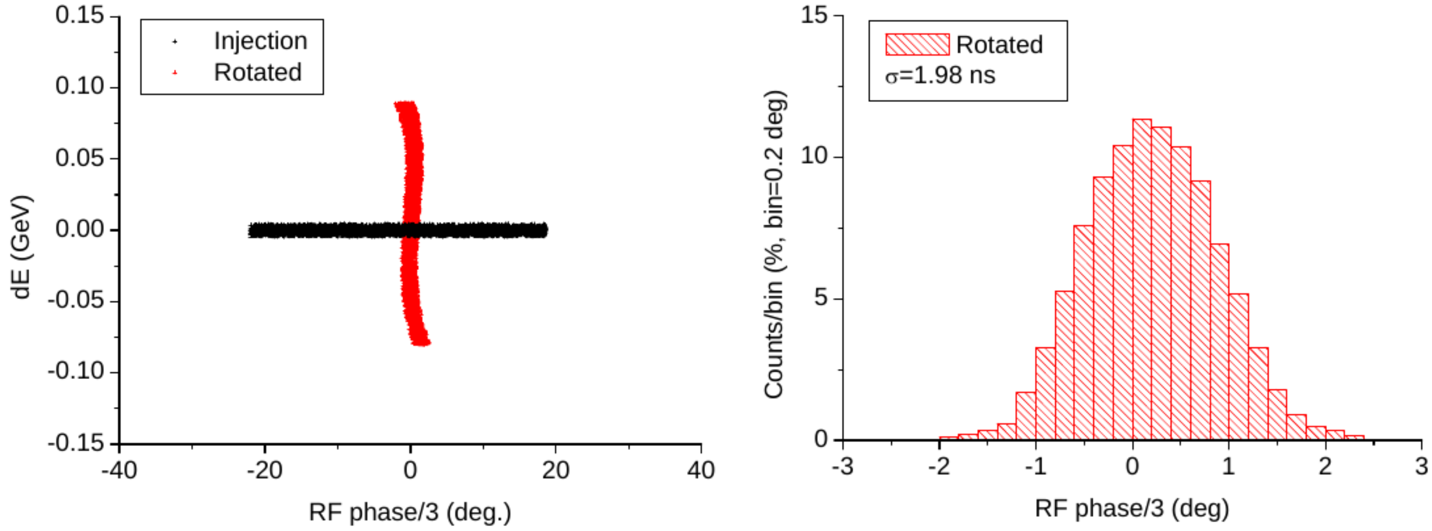
ISIS neutron and muon source

Charge Exchange Injection



- High current → accumulate beam over many turns
  - Charge exchange injection of H^- ions through a thin foil
  - Foil removes electrons
  - Issues: Scattering and energy loss of protons in foil
- Painting of beam into synchtron acceptance using fast “bumper” magnets
  - Move recirculating/injected beam phase space
- Foil lifetime is critical limit
- Space charge at injection is critical limit

Bunch Compression

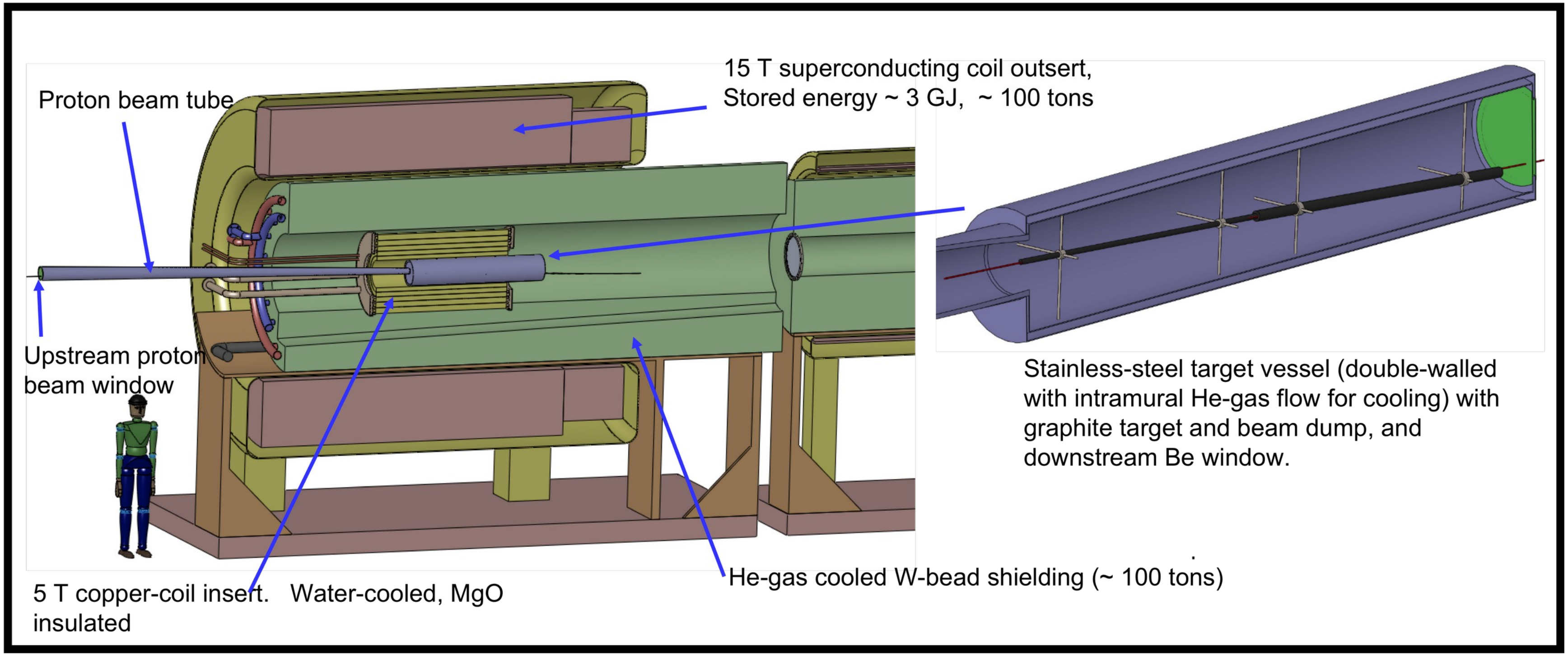


M. Aiba, CERN-AB-2008-060 BI (2008)

- Aim is to rotate the beam in longitudinal phase space
    - Short proton bunch → short muon bunch
    - Reduce longitudinal emittance of the muons
- Achieve bunch compression by rotation in the RF bucket
- Limitations:
    - Microwave instability → higher energy
    - Space charge → higher energy

MC Target

X. Ding et al, Carbon and Mercury target system for muon colliders and neutrino factories, IPAC16



Proton beam tube

15 T superconducting coil outsert, Stored energy ~ 3 GJ, ~ 100 tons

Upstream proton beam window

Stainless-steel target vessel (double-walled with intramural He-gas flow for cooling) with graphite target and beam dump, and downstream Be window.

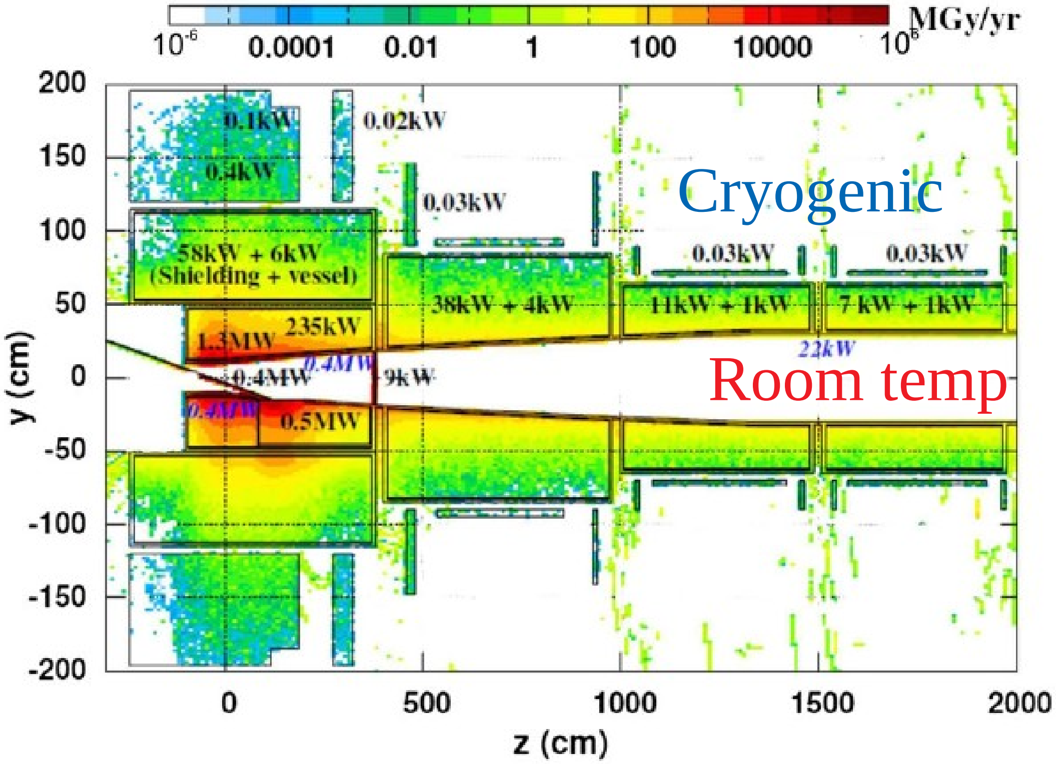
5 T copper-coil insert. Water-cooled, MgO insulated

He-gas cooled W-bead shielding (~ 100 tons)

- Protons on target → pions → muons
  - Heavily shielded, very high field solenoid captures π^+ and π^-
- Challenge: Energy deposition on solenoid
- Challenge: Solid target lifetime

Radiation issues (magnet)

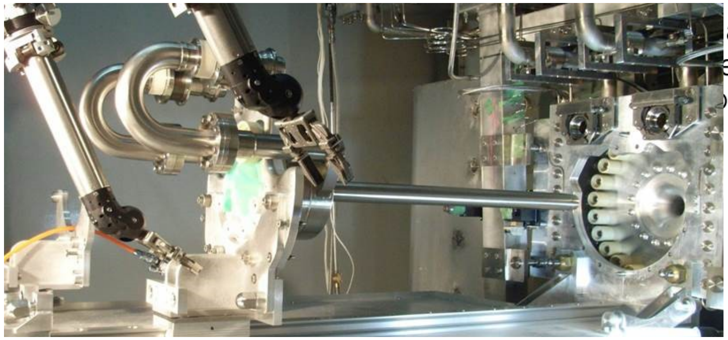
- Radiation load significant issue
  - Degrades insulation/glue
  - Requires more cooling
    - 1 kW heat → O(200) kW electricity
- Shield at room temperature
- Magnet at superconducting temperature
  - HTS → warmer, more efficient

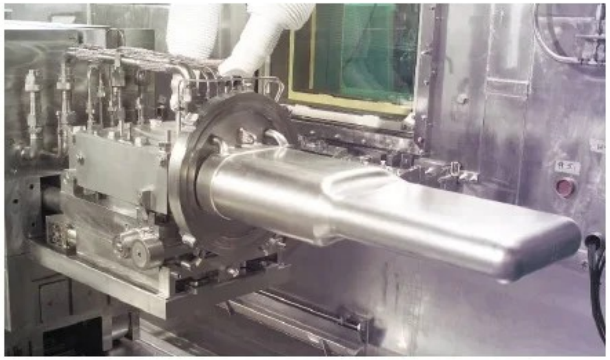


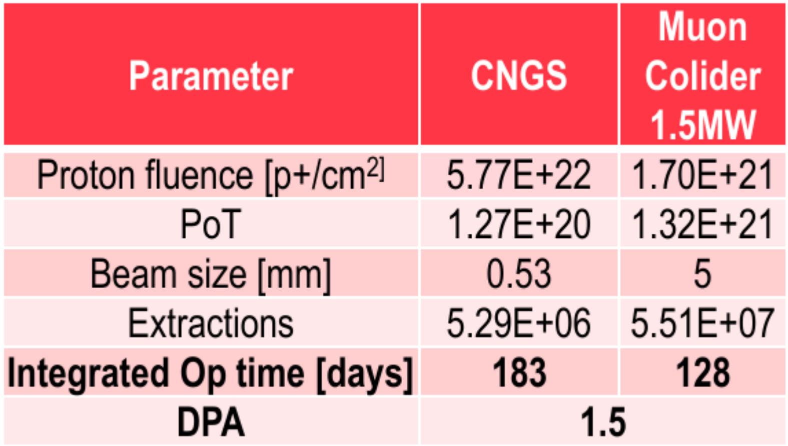
Neutrino factory, Bogomilov et al, PRSTAB 17 (2014)

Radiation issues (target)

- Radiation on target can make an issue
  - Instantaneous shock
  - Long term radiation damage
- Liquid metal targets (Pb)
  - Cavitation issues
  - Specific issues around Hg
- Flowing/moving solid targets
  - Geometry issues
  - Target wheels – e.g. PSI
  - Fluidised powder

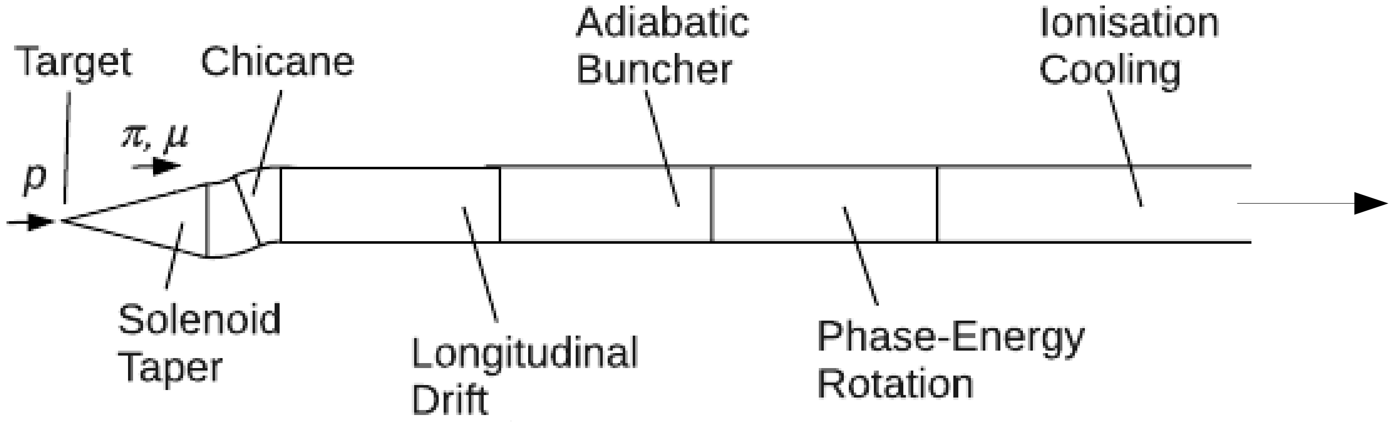






Parameter	CNGS	Muon Colider 1.5MW
Proton fluence [p+/cm^2]	5.77E+22	1.70E+21
PoT	1.27E+20	1.32E+21
Beam size [mm]	0.53	5
Extractions	5.29E+06	5.51E+07
Integrated Op time [days]	**183**	**128**
DPA	**1.5**	

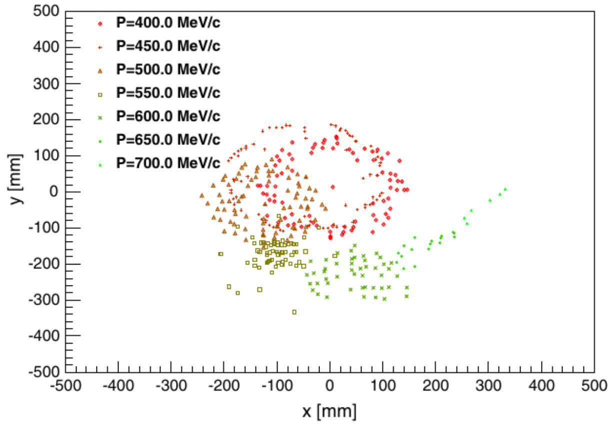
Muon front end

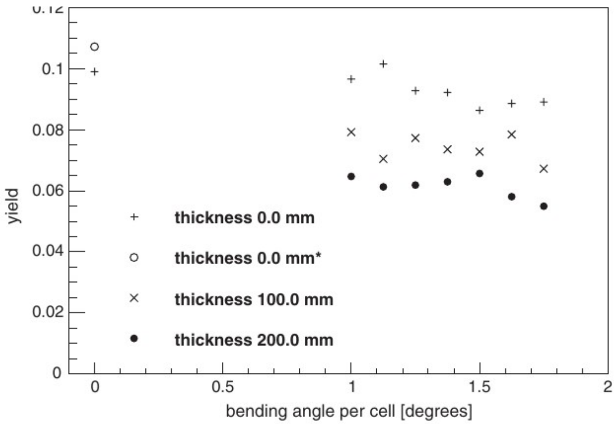


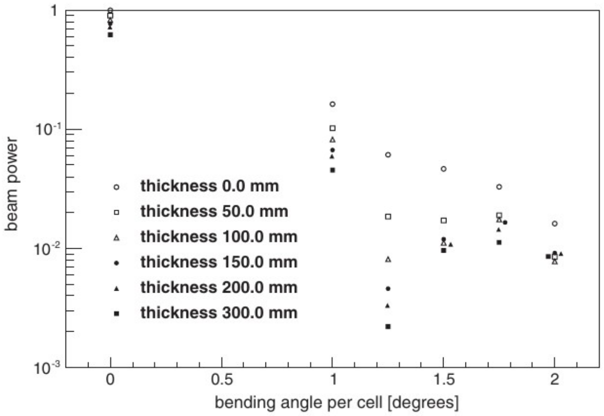
- Muon front-end to capture muon beam
- Solenoid taper
- Solenoid chicane removes high momentum particles
- Beryllium plug removes low momentum impurities
- Longitudinal capture system
    - Adiabatically bunch beam
    - Phase rotate

Chicane/proton absorber

- Solenoid chicane
  - No dipoles!
  - Vertical dispersion → low pass filter
  - Excellent transport properties within acceptance
- Beryllium plug
  - Protons stop more quickly than muons/pions
  - Removes low momentum protons







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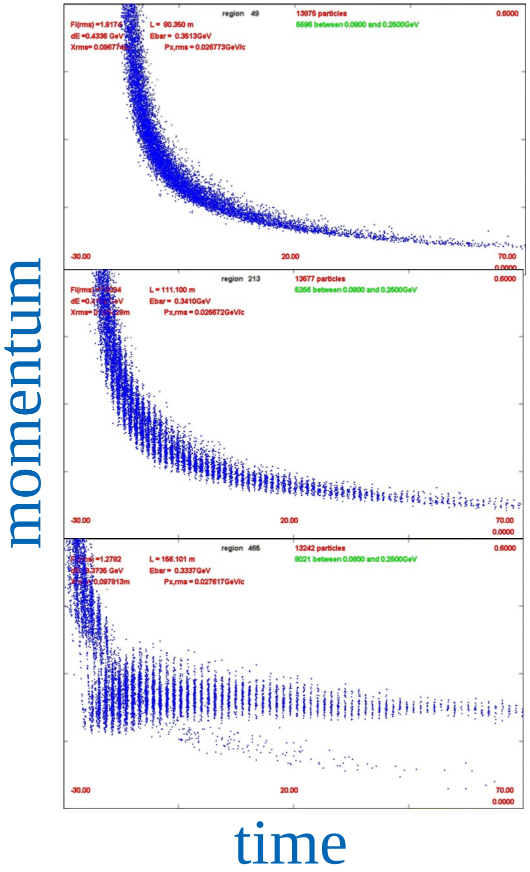
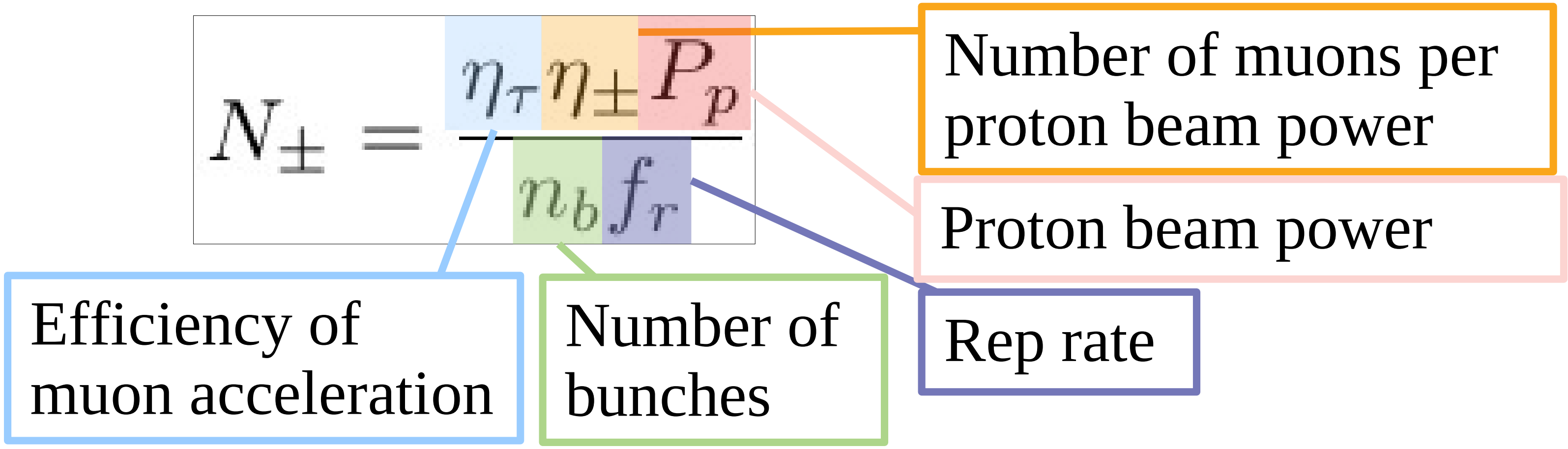
Buncher/Phase Rotator

- Drift to develop energy-time relation
- Buncher adiabatically ramp RF voltages
- Phase rotator → misphase RF
    - High energy bunches decelerated
    - Low energy bunches accelerated



Luminosity consideration

$$\mathfrak{L} \approx \underbrace{\frac{e\tau_\mu}{(4\pi m_\mu c)^2}}_{K_L\ =\ 4.4\text{e}36\ \text{MeV MW}^{-2}\ \text{T}^{-1}\ \text{s}^{-2}} \frac{f_{hg}\sigma_\delta \bar{B}}{\varepsilon_\perp \varepsilon_L n_b f_r} \underbrace{\eta_+\eta_-(\eta_\tau P_p \gamma m_\mu c^2)^2}_{P_+P_-}$$



$$N_\pm = \frac{\eta_\tau \eta_\pm P_p}{n_b f_r}$$

Number of muons per proton beam power

Proton beam power

Efficiency of muon acceleration

Number of bunches

Rep rate

- Proton beam power ~ 1-2 MW → (FNAL, JPARC, SNS)
- Approx 0.1 $\mu^{+/-}$ per 8 GeV proton → O(1e14) muons per MW
- BUT: muon front end produces multiple bunches (about 20)
- Rep rate is between 60 Hz (SNS) and 0.1 Hz (JPARC)
- Emittance is huge

The Facility – Ionisation Cooling

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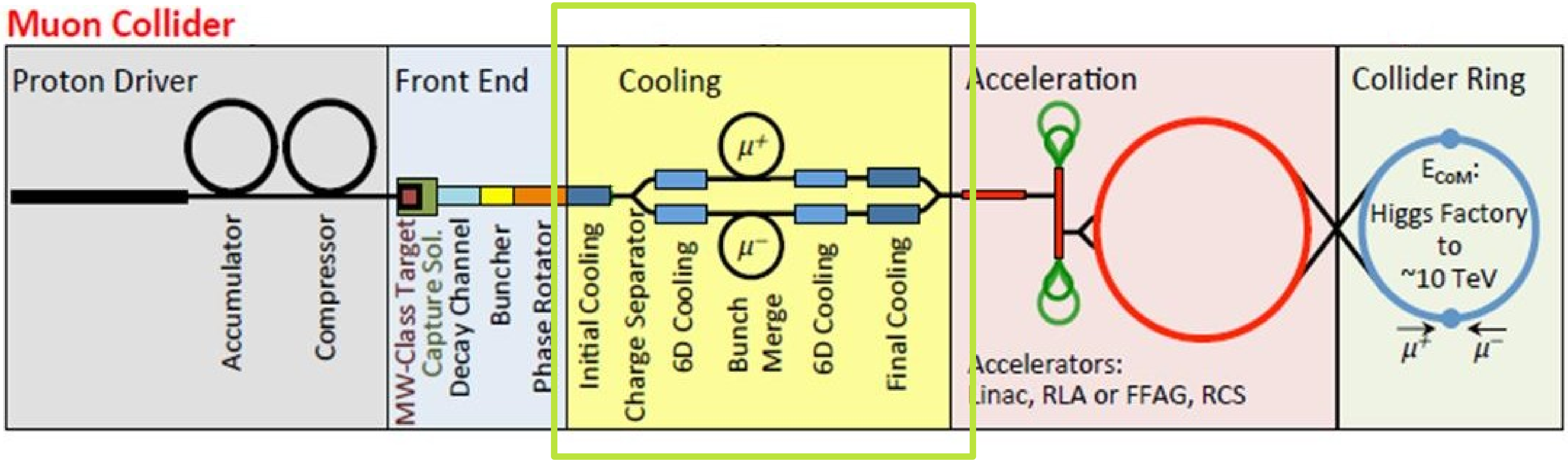
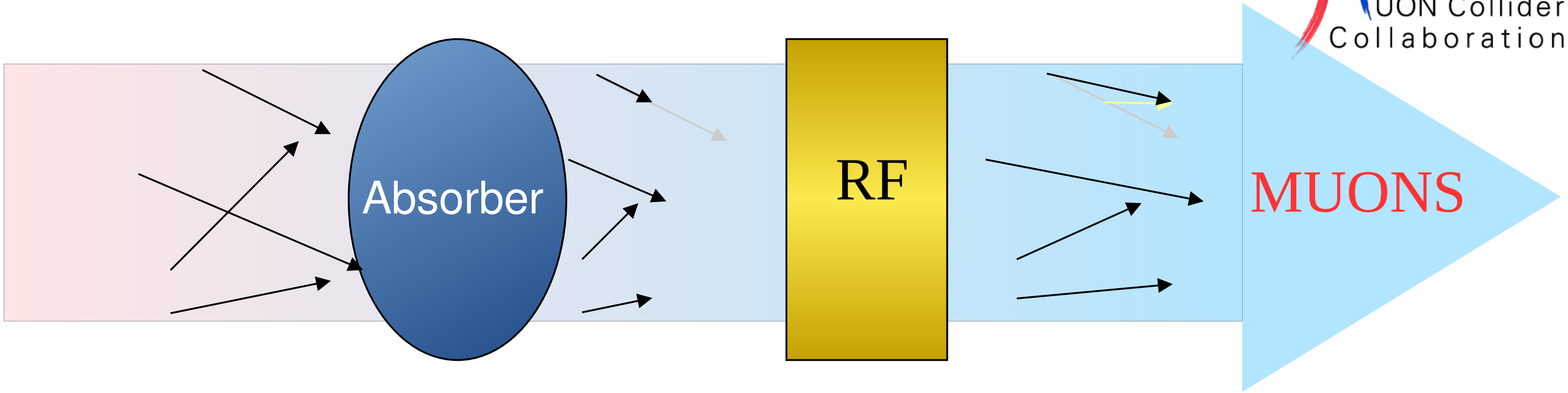


Ionisation Cooling - intro

- Muon front end produces huge flux of muons
- Muons have too large emittance at the source
- How can we reduce beam emittance? COOLING!
  - Laser cooling
  - Stochastic cooling
  - Electron cooling
  - **Too slow**
- Ionisation cooling (and Frictional cooling)

Ionisation Cooling



- Beam loses energy in absorbing material
  - Absorber removes momentum in all directions
  - RF cavity replaces momentum only in longitudinal direction
  - End up with beam that is more straight
- Multiple Coulomb scattering from nucleus ruins the effect
  - Mitigate with tight focussing
  - Mitigate with low-Z materials
  - Equilibrium emittance where MCS completely cancels the cooling

Beam emittance in 4D

- Normalised RMS beam emittance in 2D
  - area of ellipse aligned with beam

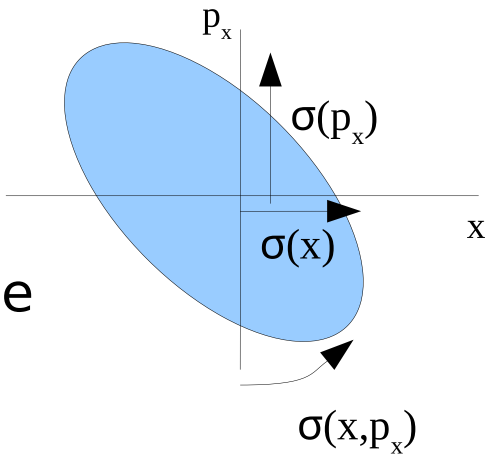
$$\varepsilon_{2d} = \frac{1}{m}\sqrt{\sigma^2(x)\sigma^2(p_x) - \sigma^2(x, p_x)}$$

- $\sigma^2(u_i)$ and $\sigma^2(u_i, u_j)$ are variance and covariance
  - Also written as $<u_i u_j>$
- Can be written as

$$\varepsilon_{2d} = \frac{1}{m}\sqrt{|\mathbf{V_{2d}}|}$$

- In higher dimensions the definition generalises

$$\varepsilon_{2nd} = \frac{1}{m_\mu}\sqrt[n]{|\mathbf{V}|}$$



Transverse cooling (1)

- Say we pass through some material at a focus
  - P decreases due to ionisation
  - Multiple Coulomb Scattering increases angular spread
- For a cylindrically symmetric beam with angular divergence Θ_x

$$\begin{aligned}\sqrt{|\mathbf{V}_\perp|} &= (<x^2><p_x^2> - <xp_x>^2 - <xp_y>^2)\\ &\approx p_z^2(<x^2><\Theta_x^2> - <x\Theta_x>^2 - <x\Theta_y>^2)\end{aligned}$$

The change in emittance is given by

$$\frac{d\epsilon_n}{dz} = \frac{1}{2m^2\epsilon_n}\frac{d\sqrt{|\mathbf{V}_\perp|}}{dz}$$

$$\boxed{\varepsilon_{2nd} = \frac{1}{m_\mu}\sqrt[n]{|\mathbf{V}|}}$$

Transverse cooling (2)

$$\frac{d\epsilon_n}{dz} = \frac{1}{2m^2\epsilon_n}\frac{d\sqrt{|\mathbf{V}_\perp|}}{dz}$$

- Only p$_z$ and $<\Theta_i^2>$ change; applying product rule

$$\frac{d\epsilon_n}{dz} \approx \frac{1}{2m^2\epsilon_n}\left(2\frac{dp_z}{dz}\frac{\sqrt{|\mathbf{V}_\perp|}}{p_z} + <x^2> p_z^2\frac{d<\Theta_x^2>}{dz}\right)$$

- Use (from *E*^{2} + *p*^{2} = *m*^{2}) $E\,dE/dz \approx p_z\,dp_z/dz$
- Use standard formula $\beta_\perp = \frac{<x^2>p}{m\epsilon_n}$
- Use scattering (from atomic physics) $\frac{d<\Theta_x^2>}{dz} \approx \frac{13.6^2}{(p\beta_{rel})^2 L_R}.$
- Gives $$\frac{d\epsilon_n}{dz} \approx \frac{1}{\beta_{rel}^2 E}\left\langle\frac{dE}{dz}\right\rangle\epsilon_n + \frac{1}{2m^2\epsilon_n} <x^2> \frac{13.6^2}{(\beta_{rel})^2 L_R}.$$

Transverse cooling (3)

- Rearranging $\frac{d\epsilon_n}{dz} \approx \frac{1}{E}\left\langle \frac{dE}{dz} \right\rangle \epsilon_n + \frac{1}{2m}\frac{13.6^2}{L_R}\frac{\beta_\perp}{\beta_{rel}^3 E}.$

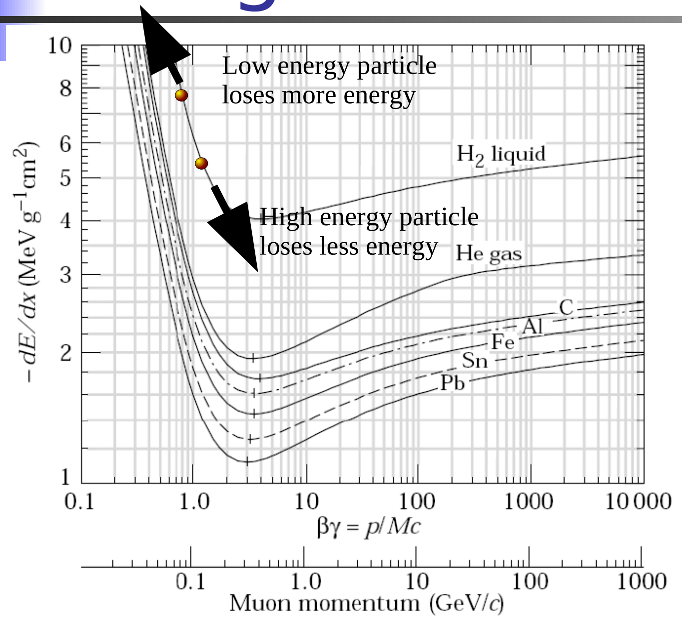
dE/dz is negative!
Cooling

Heating

- There exists an equilibrium emittance where the two terms balance (no emittance change)

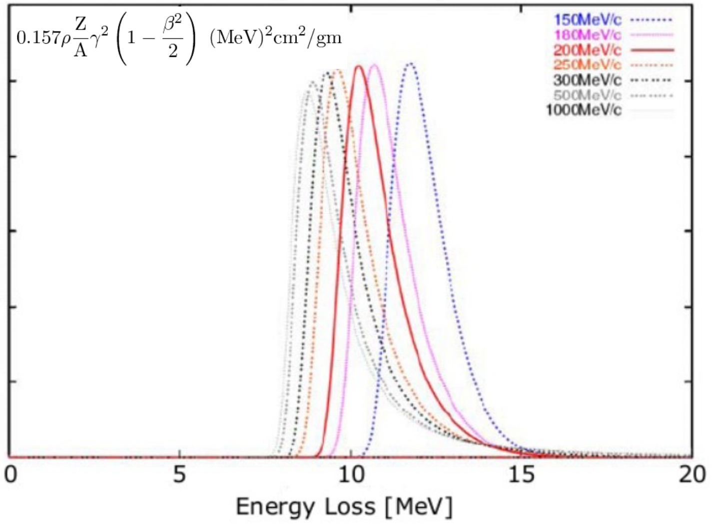
$$\epsilon_n(equilibrium) = \frac{1}{2m}\frac{13.6^2}{L_R}\frac{\beta_\perp}{\beta_{rel} < \frac{dE}{dz} >}$$

Longitudinal Heating



Low energy particle loses more energy

High energy particle loses less energy

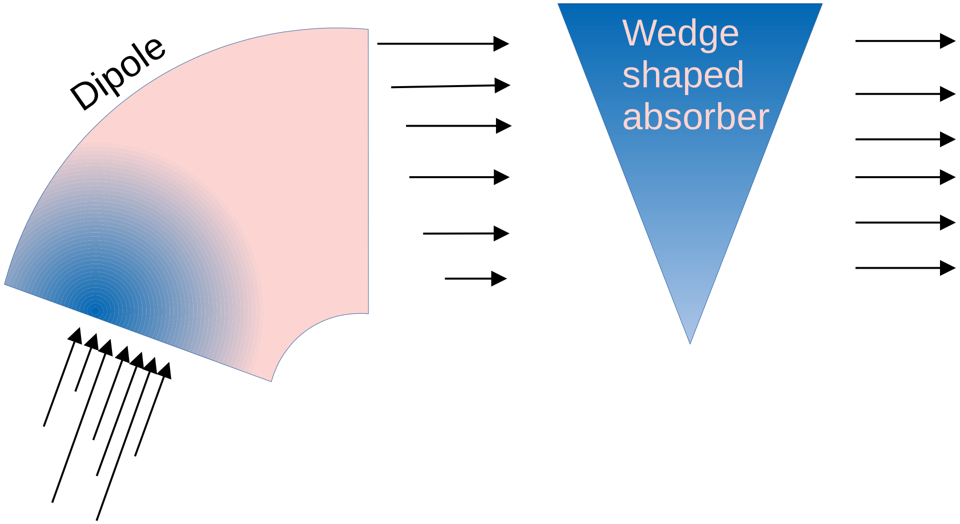


- In longitudinal phase space, the beam is usually heated
  - Heating due to random noise in the energy loss I.e. “straggling”
  - Heating due to curvature in energy loss (heating or weak cooling)

$$\frac{d<E^2>}{dz} = \left(2\frac{d}{dE}\frac{dE}{dz}\right) < E^2 > + \left(\frac{d<E^2>}{dz}\right)_{Vlasov}$$

- Mitigate using emittance exchange
  - Move emittance from longitudinal to transverse phase space

Emittance exchange



- Initial beam is narrow with some momentum spread
  - Low transverse emittance and high longitudinal emittance
- Beam follows curved trajectory in dipole
  - Higher momentum particles have higher radius trajectory
  - Beam leaves dipole wider with energy-position correlation
- Beam goes through wedge shaped absorber
  - Beam leaves wider without energy-position correlation
  - High transverse emittance and low longitudinal emittance

Emittance exchange

- Longitudinal emittance change becomes

$$\frac{d<E^2>}{dz} = \left(2\frac{d}{dE}\frac{dE}{dz}\right)<E^2> + \left(\frac{d<E^2>}{dz}\right)_{Vlasov}$$

$$\frac{\partial\frac{\mathrm{dE}}{\mathrm{ds}}}{\partial\mathrm{E}} \Rightarrow \left.\frac{\partial\frac{\mathrm{dE}}{\mathrm{ds}}}{\partial\mathrm{E}}\right|_0 + \frac{\mathrm{dE}}{\mathrm{ds}}\frac{\eta\rho'}{\beta\mathrm{cp}\rho_0}$$

dispersion

Effective density
Variation with position

- Transverse emittance change becomes

$$\frac{d\epsilon_n}{dz} \approx \frac{1}{\beta_{rel}^2 E}\left\langle\frac{dE}{dz}\right\rangle\epsilon_n + \frac{1}{2m^2\epsilon_n}<x^2>\frac{13.6^2}{(\beta_{rel})^2 L_R}.$$

$$\frac{1}{\beta^2\mathrm{E}}\frac{\mathrm{dE}}{\mathrm{ds}}\left(1-\frac{\eta\rho'}{\rho_0}\right)\epsilon_{\mathrm{N}}$$

Summary – Lecture 1

International
MUON Collider
Collaboration

C. T. Rogers
Rutherford Appleton Laboratory

Science & Technology Facilities Council
ISIS

Final Word – Part 1

International
UON Collider
Collaboration

- The muon collider is an exciting prospect
- An opportunity to build a sustainable path in HEP
  - Wall plug power
  - Energy reach
- Many challenges remain on the road
  - Luminosity is the key
  - Capture and cooling of the muon beam
    - Ionisation cooling
- Next lecture, explore
  - How ionisation cooling may be realised in practice
  - Acceleration
  - Collision
  - The path to a muon collider