

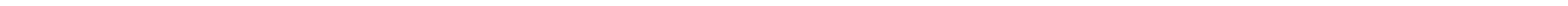


# Longitudinal Beam Diagnostics

6 – 18 November 2022

Neoclub, Sévrier, France

T. Lefevre, CERN



# Outline

- Longitudinal beam profile in accelerators
- Invasive and Non-invasive techniques
  - Explain concepts
  - Review performances and limitations

# Accelerating charged particles

# Acceleration techniques

DC Accelerator



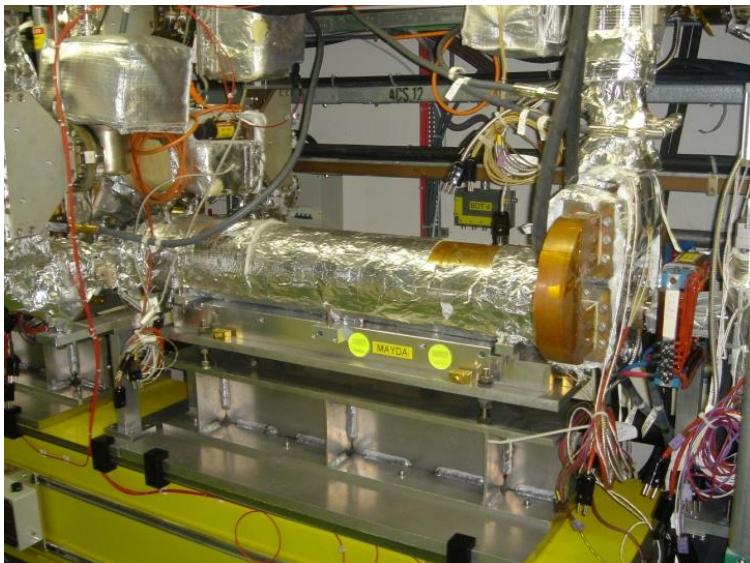
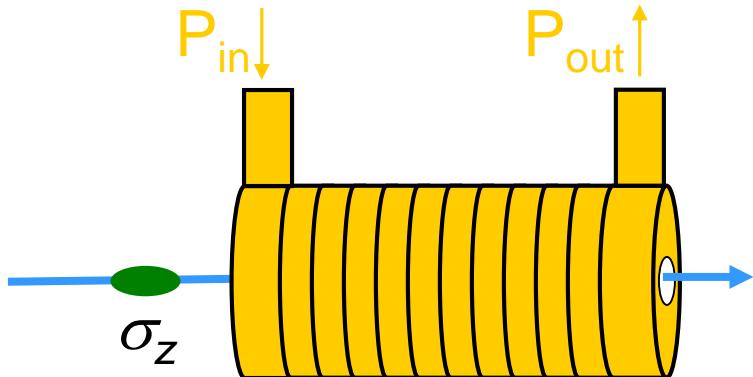
RF Accelerator



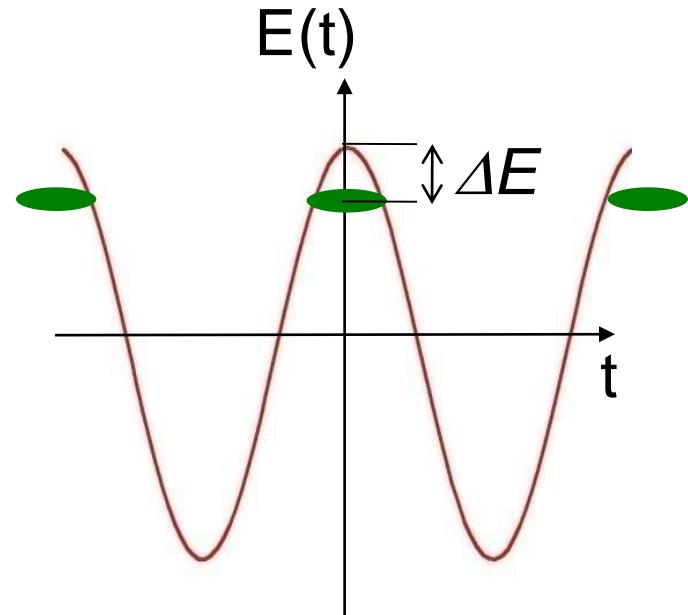
synchronizing particle  
with an  
electromagnetic wave!

# Acceleration techniques

## RF Accelerating structures



## RF Accelerating Field



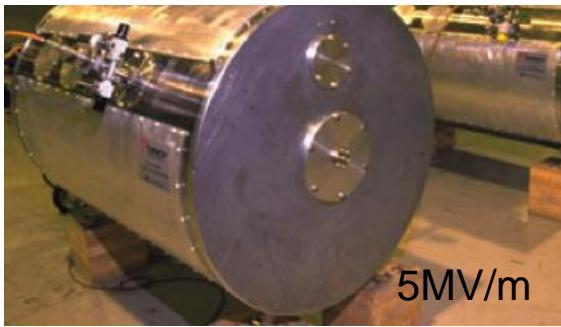
At 3GHz accelerating frequency

1 period = 333ps : Bunch spacing  
 Typical bunch length : few deg ~ few ps

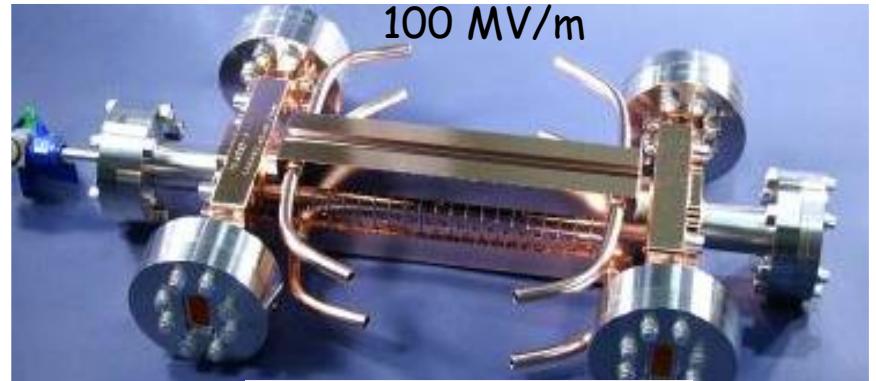
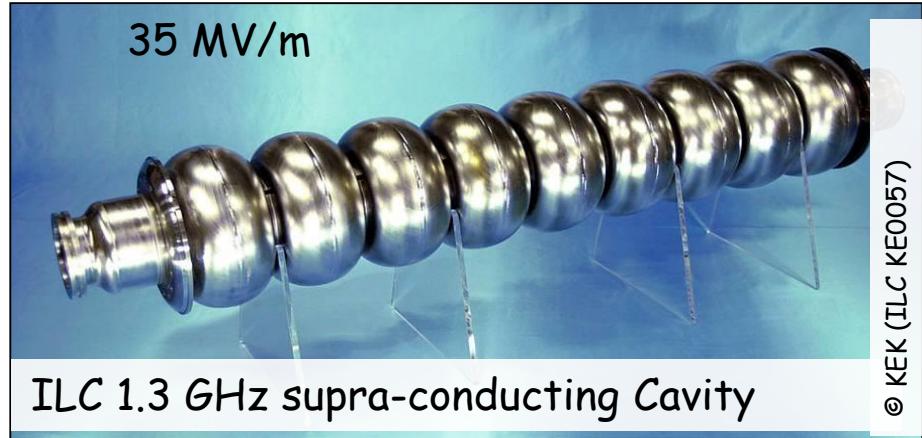
# Accelerating cavities



CERN PS 19 MHz Cavity (prototype 1966)



400MHz LHC Cavity in its cryo-module



CLIC 12 GHz Cavity

# Dielectric Wakefield Acceleration



## ARTICLE

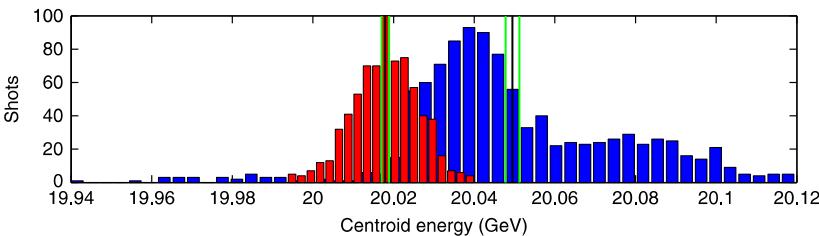
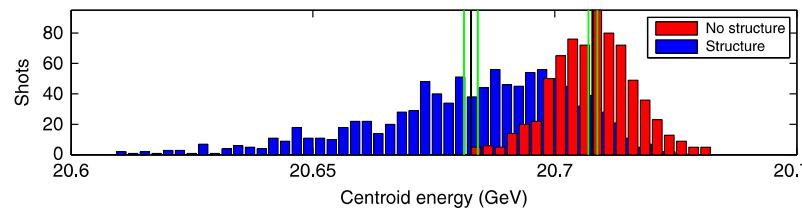
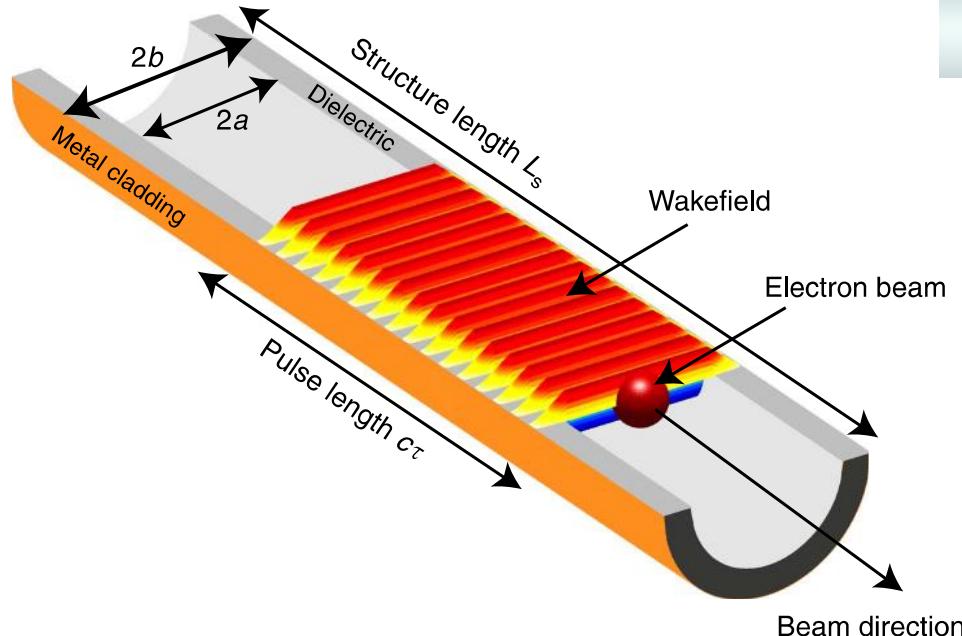
Received 10 Mar 2016 | Accepted 29 Jul 2016 | Published 14 Sep 2016

DOI: 10.1038/ncomms12763

OPEN

## Observation of acceleration and deceleration in gigaelectron-volt-per-metre gradient dielectric wakefield accelerators

B.D. O'Shea<sup>1,2</sup>, G. Andonian<sup>1</sup>, S.K. Barber<sup>1</sup>, K.L. Fitzmorris<sup>1</sup>, S. Hakimi<sup>1</sup>, J. Harrison<sup>1</sup>, P.D. Hoang<sup>1</sup>, M.J. Hogan<sup>2</sup>, B. Naranjo<sup>1</sup>, O.B. Williams<sup>1</sup>, V. Yakimenko<sup>2</sup> & J.B. Rosenzweig<sup>1</sup>



### SiO<sub>2</sub> - 15cm long dielectric

Outer diameter : 2b-400um

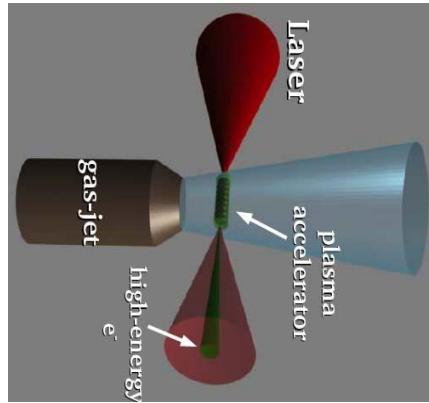
Inner diameter : 2a-300um

Beam size 30um

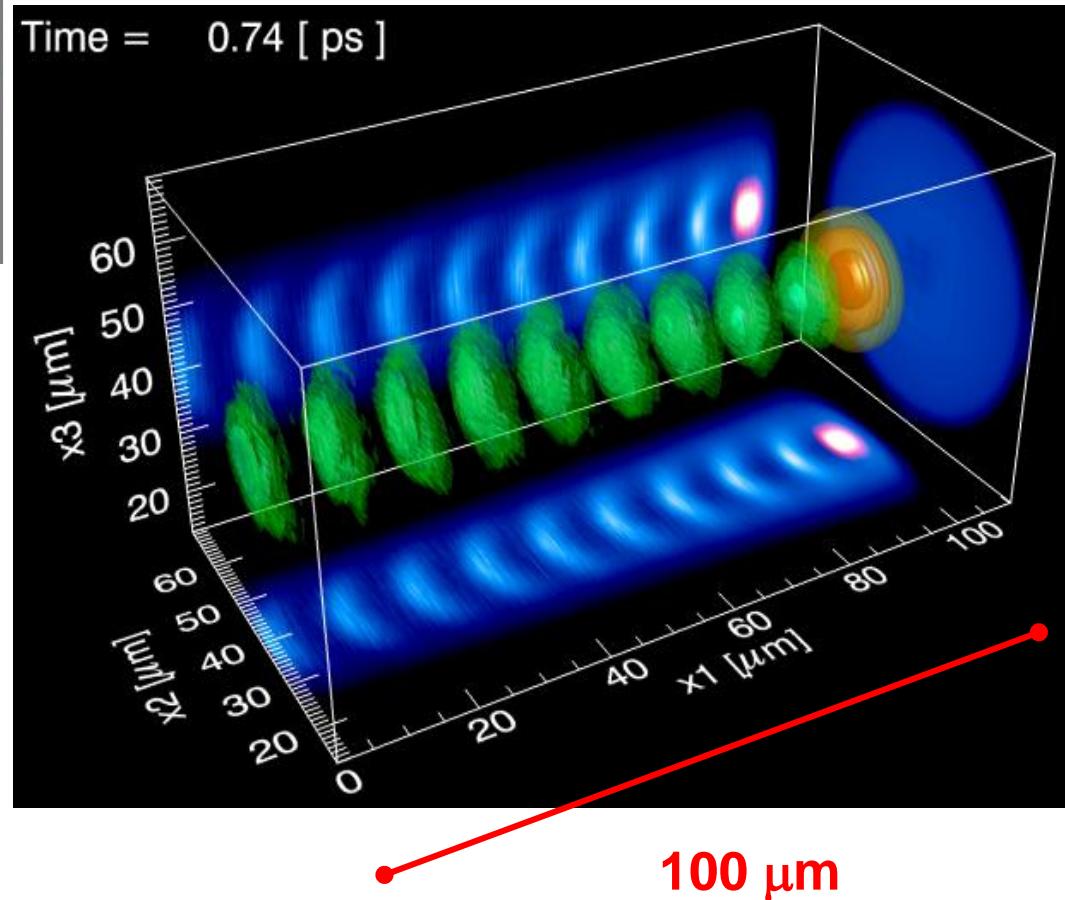
Bunch length 25um (W) and 55um (D)

$\Delta t$  (D-W) = 250um – 833fs

# Laser Plasma Wakefield Acceleration



Courtesy of W. Mori & L. da Silva



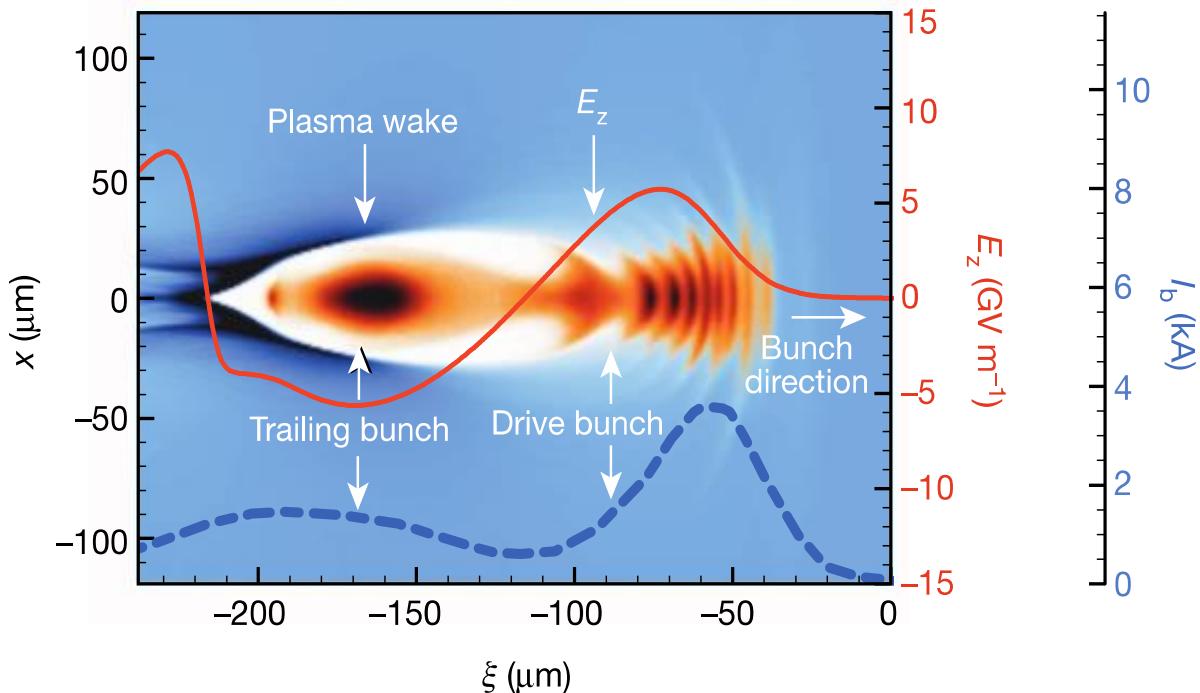
# Beam Plasma Wakefield Acceleration

## LETTER

doi:10.1038/nature13882

### High-efficiency acceleration of an electron beam in a plasma wakefield accelerator

M. Litos<sup>1</sup>, E. Adli<sup>1,2</sup>, W. An<sup>3</sup>, C. I. Clarke<sup>1</sup>, C. E. Clayton<sup>4</sup>, S. Corde<sup>1</sup>, J. P. Delahaye<sup>1</sup>, R. J. England<sup>1</sup>, A. S. Fisher<sup>1</sup>, J. Frederico<sup>1</sup>, S. Gessner<sup>1</sup>, S. Z. Green<sup>1</sup>, M. J. Hogan<sup>1</sup>, C. Joshi<sup>4</sup>, W. Lu<sup>5</sup>, K. A. Marsh<sup>4</sup>, W. B. Mori<sup>3</sup>, P. Muggli<sup>6</sup>, N. Vafaei-Najafabadi<sup>4</sup>, D. Walz<sup>1</sup>, G. White<sup>1</sup>, Z. Wu<sup>1</sup>, V. Yakimenko<sup>1</sup> & G. Yocky<sup>1</sup>



# Typical bunch length

H <sup>-</sup> @ SNS	100ps
H <sup>+</sup> @ LHC	230ps
e <sup>-</sup> @ CLIC	130fs
e <sup>-</sup> @ XFEL	10fs
e <sup>-</sup> @ DWFA	<60fs
e <sup>-</sup> @ PWFA	<30fs
e <sup>-</sup> @ LWFA	<10fs

# Bunch length measurement techniques

# Bunch length measurement techniques

## Radiative techniques

### Optical Method

1. Produce visible light
2. Analyse the light pulse using dedicated instruments

### Bunch Frequency Spectrum

The shorter the bunches, the broader the bunch frequency spectrum

### RF manipulation

Use RF techniques to convert time information into transverse information

### Laser-based beam diagnostic

Using short laser pulses and sampling techniques

# Bunch length measurement techniques

## 1- Longitudinal Profile

 $\leftarrow$  $\rightarrow$  $\sigma$ 

RMS or FWHM values

- *More precise information on the beam characteristic*

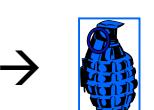
## 2- Single shot measurements

 $\leftarrow$  $n!$ 

Sampling measurements

- *Do not care about the beam reproducibility*
- *No additional problem due to timing jitter*

## 3- Non interceptive

 $\leftarrow$ 

Destructive Devices

- *Can be used for beam study and beam control for on-line monitoring*
- *Beam Power density : No risk of damage by the beam itself*

# Bunch length measurement techniques

**Do not forget about Simplicity and Reliability**

‘Beam diagnostics should help you to understand the beam properties, **it should not be the opposite**’

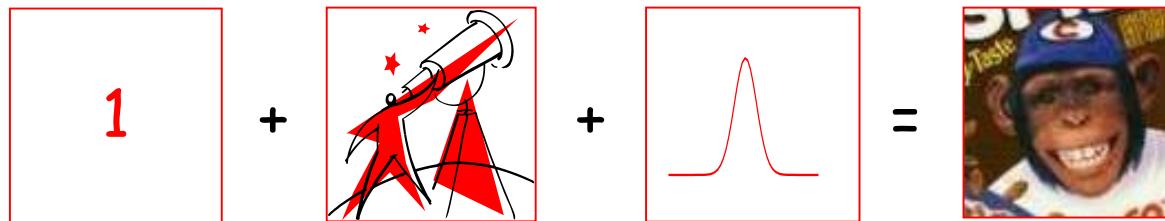


A detector, what for ?

- Online Beam stability → Non-intercepting and **reliable**  
*Only have access to a partial information (RMS values)*
- Beam characterization and beam physics study → **Full information**  
*Complexity and time consuming*

# Bunch length measurement techniques

**Can we perform non intercepting, single shot, beam profile measurement in an easy way ?**

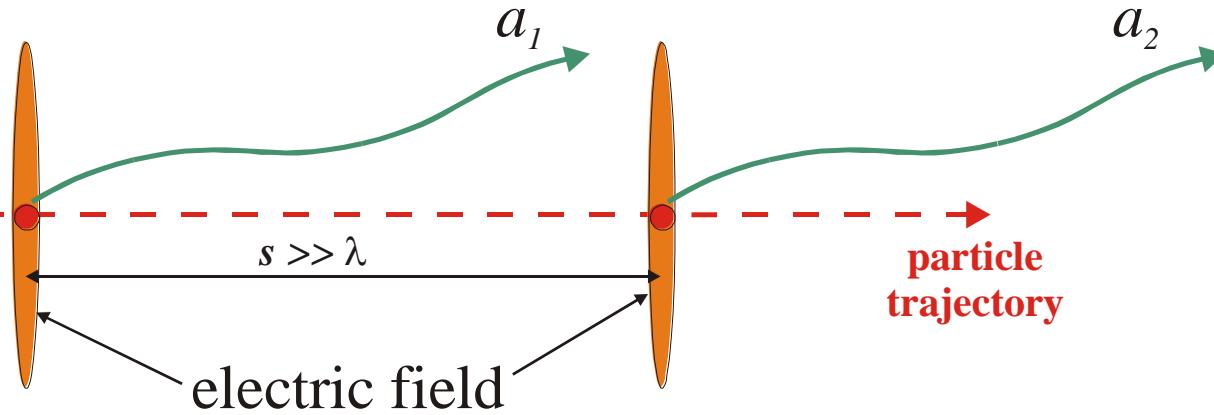


# Radiative techniques

**'Converting particles into photons'**

# Incoherent versus Coherent Radiation

*At wavelength much shorter than the bunch length, the radiation is emitted incoherently because each particle emits photon independently from the others without a defined phase relation*

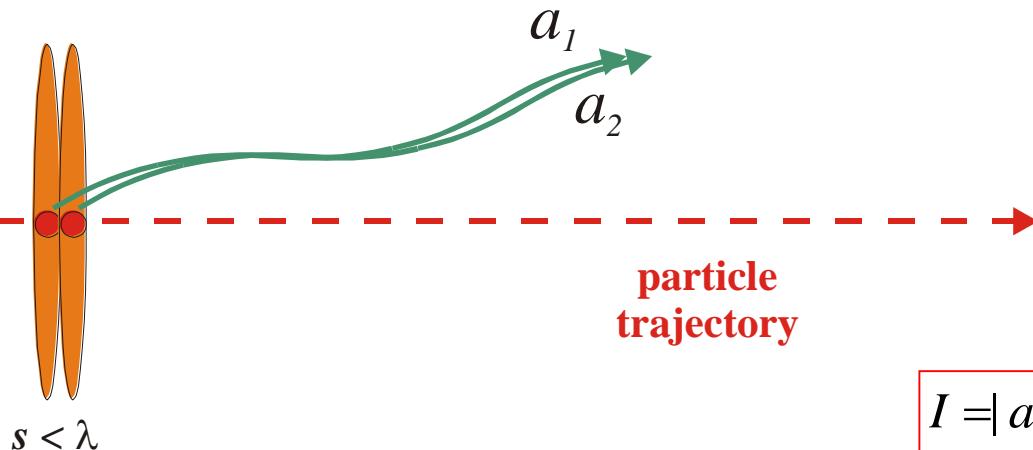


$$I = |a_1|^2 + |a_2|^2 = 2|a|^2 \rightarrow N|a|^2$$

***Incoherent radiation***

# Incoherent versus Coherent Radiation

*A coherent enhancement occurs at wavelengths which are equal to or longer than the bunch length, where fixed phase relations are existing, resulting in the temporal coherence of the radiation*



particle  
trajectory

$$I = |a_1 + a_2|^2 = |2a|^2 = 4|a|^2 \rightarrow N^2 |a|^2$$

**Coherent radiation**

# Total radiation spectrum

$$S(\omega) = S_p(\omega) \bar{N} + N(N-1)F(\omega)$$

Incoherent term                                  Coherent term

$S(\omega)$  – radiation spectrum

$S_p(\omega)$  – single particle spectrum

$N$  – number of electrons in a bunch

$F(\omega)$  – longitudinal bunch form factor

$$F(\omega) = \left| \int_{-\infty}^{\infty} r(s) e^{-i\frac{\omega}{c}s} ds \right|^2$$

$\rho(s)$  – Longitudinal particle distribution in a bunch

# Radiative processes



- Transition radiation                      Better for  $\gamma > 100$
- Cherenkov radiation                       $\beta > 1/n$



- Diffraction radiation
- Cherenkov Diffraction radiation
- Synchrotron radiation

For incoherent radiation  
 $\gamma > 3000$

For coherent radiation  
$$\gamma > \frac{0.06}{\sigma_z(m)}$$

# Optical method using Incoherent light

# Time correlated single photon counting



Geiger-mode Avalanche photodiode converts photon to electrical pulse

Visible photon



Precise trigger synchronized with the beam

Time to Digital converter records pulse arrival time

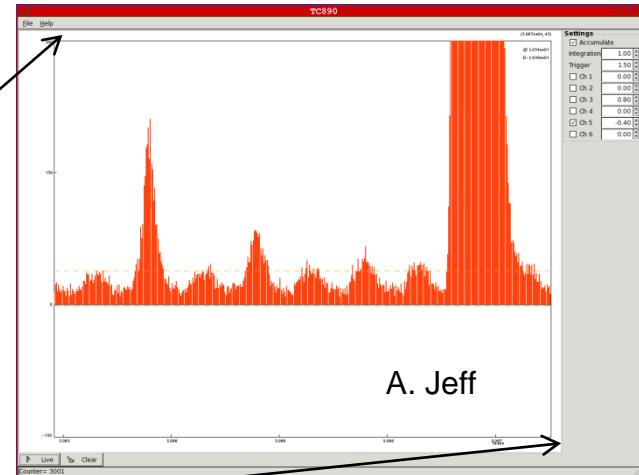
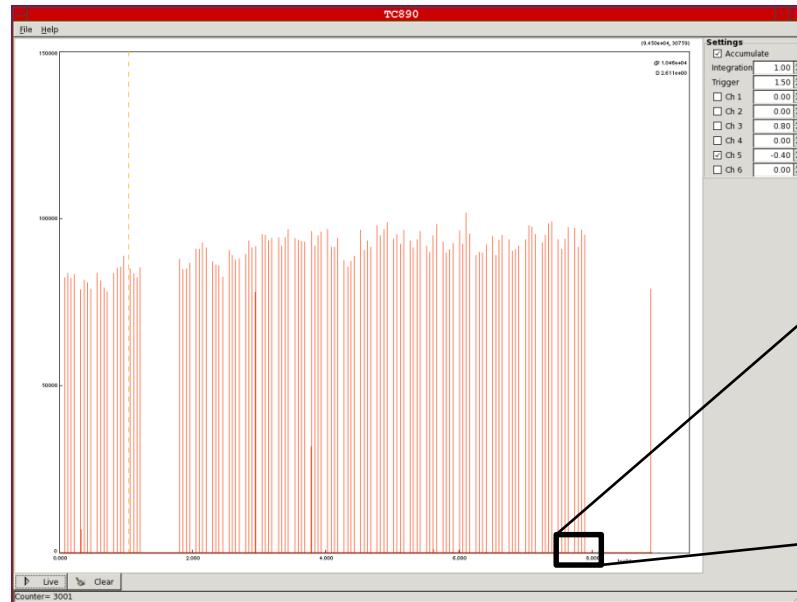
- Sampling Method allowing very high dynamic range if you measure long enough
- Avalanche photodiode have deadtime and are subject to after-pulsing
- State of the art TDC typically limited to 10ps sampling

D.V. O'Connor, D. Phillips, *Time-correlated Single Photon Counting*, Academic Press, London, 1984  
C.A. Thomas et al., Nucl. Instr. and Meth. A566 (2006) p.762

# Time correlated single photon counting



Longitudinal profile of the entire LHC ring  
(89us) with 50ps resolution using SR light

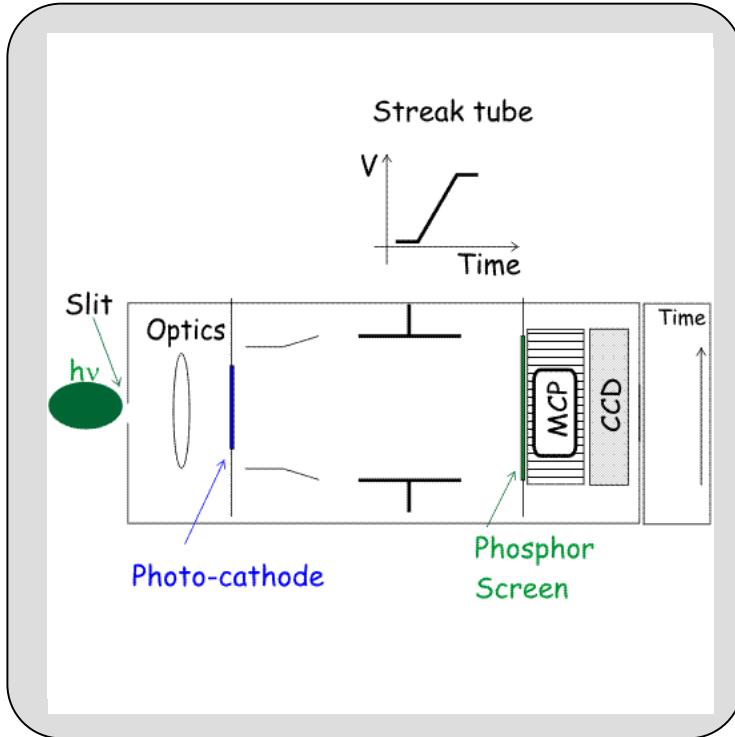


A very large dynamic range should make it possible to see ghost bunches as small as 5e5 protons / 50ps with long integration

# Streak Camera



1



*'Streak cameras uses a time dependent deflecting electric field to convert time information in spatial information on a CCD'*

M. Uesaka et al, *NIMA 406 (1998) 371*

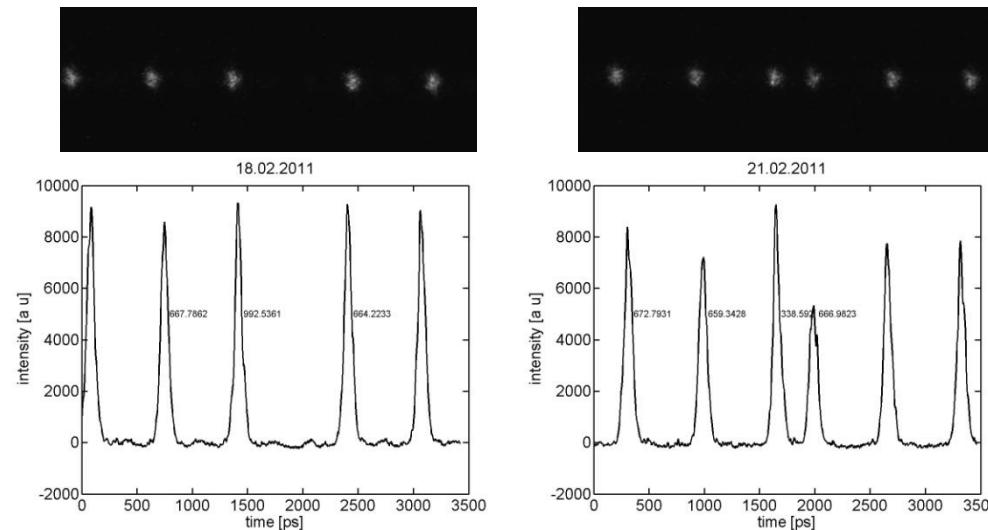
200fs time resolution obtained using reflective optics and 12.5nm bandwidth optical filter (800nm) and the Hamamatsu FESCA 200

## Limitations : Time resolution of the streak camera :

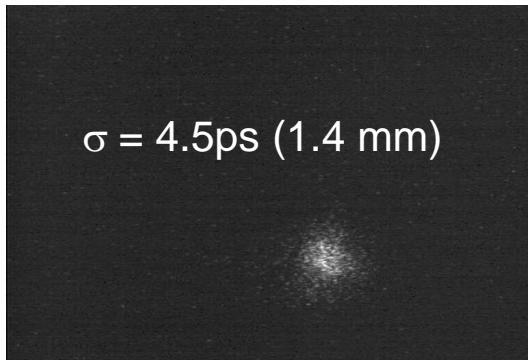
- (i) Initial velocity distribution of photoelectrons : *narrow bandwidth optical filter*
- (ii) Spatial spread of the slit image: *small slit width*
- (iii) Dispersion in the optics

# Streak Camera

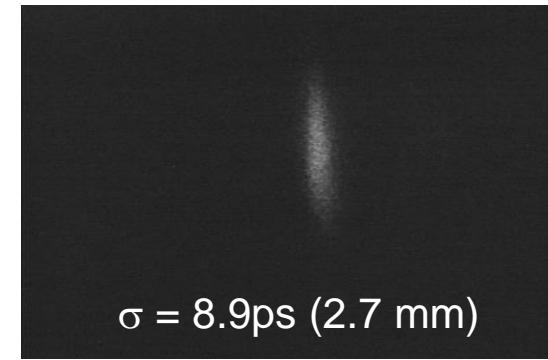
Observation of 5MeV electron bunch train using Cherenkov radiation  
 Sweep speed of 250ps/mm



Measure of bunch length using OTR and OSR



*Sweep  
speed of  
10ps/mm*



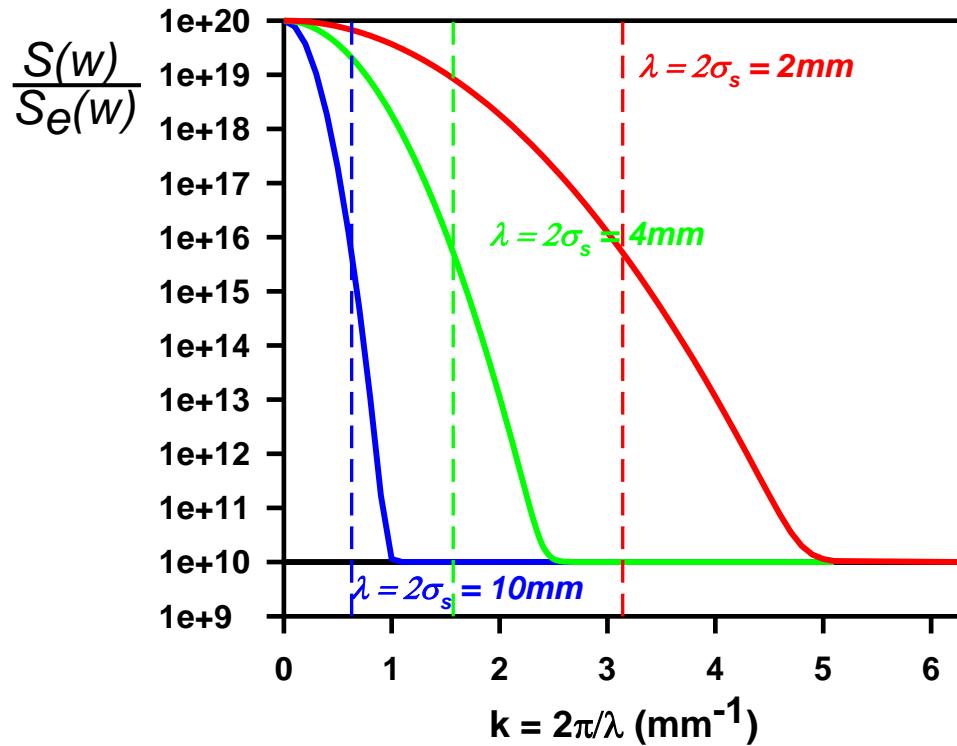
# Bunch length measurement using using Coherent light

**‘The shorter in time the broader in  
frequency’**

# Bunch form factor

$$F(\omega) = \left| \int_{-\infty}^{\infty} dz \rho(z) e^{i(\omega/c)z} \right|^2$$

$$\rho(z) = \frac{1}{\pi c} \int_0^{\infty} d\omega \sqrt{F(\omega)} \cos\left(\frac{\omega z}{c}\right)$$



Coherent radiation appears when the bunch length is comparable to or shorter than the emitted radiation wavelength

# Measuring the Radiation Spectrum

$$S(\omega) \gg N^2 S_p(\omega) F(\omega)$$

- ✓  $S(\omega)$  – radiation spectrum (known in the experiment)
- ✓  $N$  – number of particles s / bunch (known from the experiment)
- ✓  $F(\omega)$  – bunch form factor (what you want to find out)
- ✓  $S_p(\omega)$  – single particle spectrum (should be known)



## Coherent Transition Radiation (CTR)

P. Kung et al, **Physical review Letters** **73** (1994) 96



## Coherent Diffraction (CDR) or Coherent Synchrotron (CSR)

B. Feng et al, **NIM A** **475** (2001) 492–497 ; A.H. Lumpkin et al, **NIM A** **475** (2001) 470–475  
C. Castellano et al, **Physical Review E** **63** (2001) 056501  
T. Watanabe et al, **NIM A** **437** (1999) 1-11 & **NIM A** **480** (2002) 315–327

# Measuring the Radiation Spectrum

## Frequency Domain

Spectral Intensity  
 $S(\omega)$

Extrapolation  
(high and low frequencies)

Correction  
(transfer function of detection system)

Form Factor  
 $|F(\omega)|$

Inverse Fourier Transform for  
symmetric bunch distribution

Kramers-Kronig relation  
for non symmetric bunches

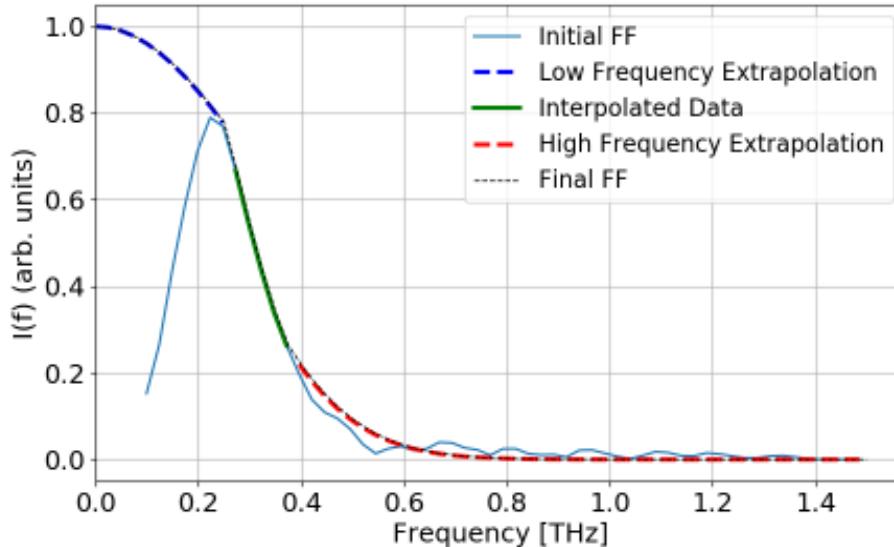
Long. Bunch profile  $\sigma(z)$

R. Lai and A.J. Sievers, NIM-A 397 (1997) 221 -231

## Time Domain

# Measuring the Radiation Spectrum

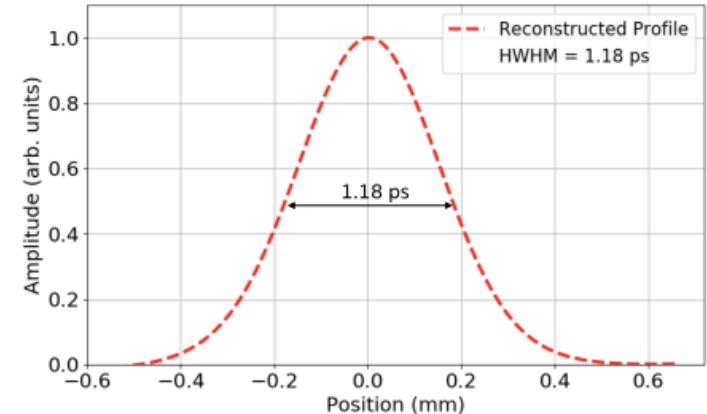
## Frequency Domain



- **Extrapolation**  
(high and low frequencies)
- **Correction**  
(transfer function of detection system)



Inverse Fourier Transform using  
Kramer kronig relation



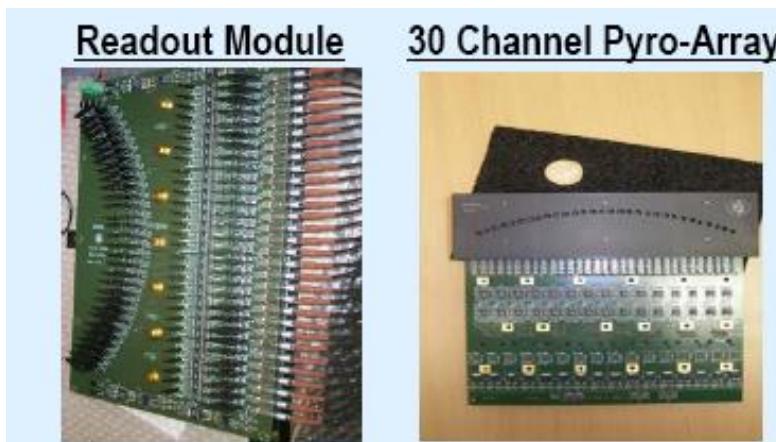
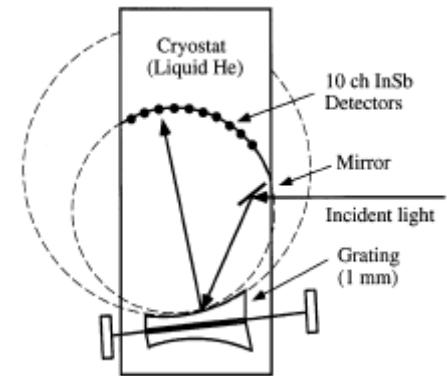
## Time Domain

# Measuring the Radiation Spectrum

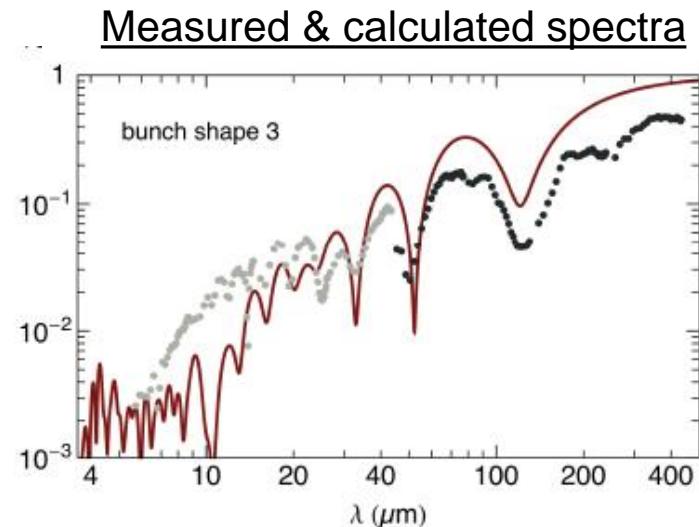
1

*'The **polychromator** enables to get the spectrum directly by a single shot. The radiation is deflected by a grating and resolved by a multi-channels detector array'*

T. Wanatabe et al., NIM-A 480 (2002) 315-327  
 H. Delsim-Hashemiet al., Proc. FLS, Hamburg 2006, WG512



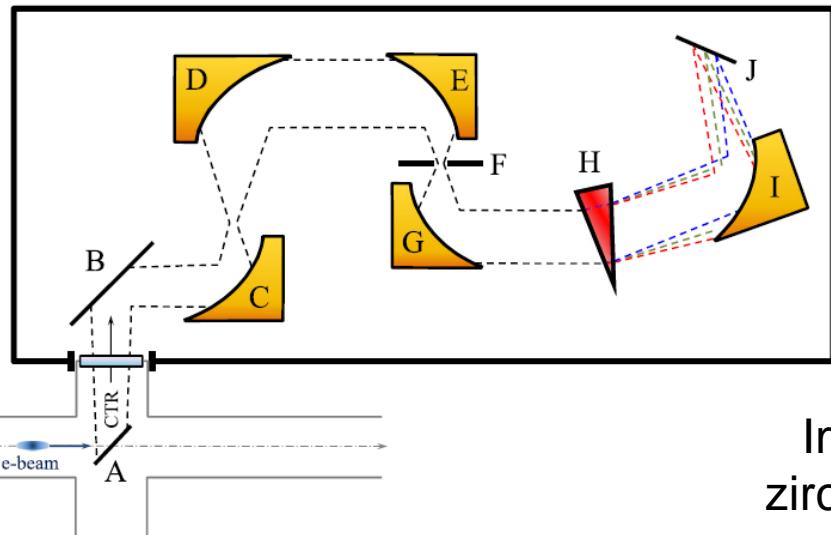
B. Schmidt, DESY



(E. Hass et al., Proc. SPIE 8778, May 2013)

# Single shot CTR measurements

- T. J. Maxwell et al. "Coherent-radiation spectroscopy of few-femtosecond electron bunches using a middle-infrared prism spectrometer." *Physical review letters* 111.18 (2013)



KRS-5 (thallium bromoiodide) prism based spectrometer

Images CTR from foil onto 128 lead zirconate titanate pyroelectric elements with  $100 \mu\text{m}$  spacing line array

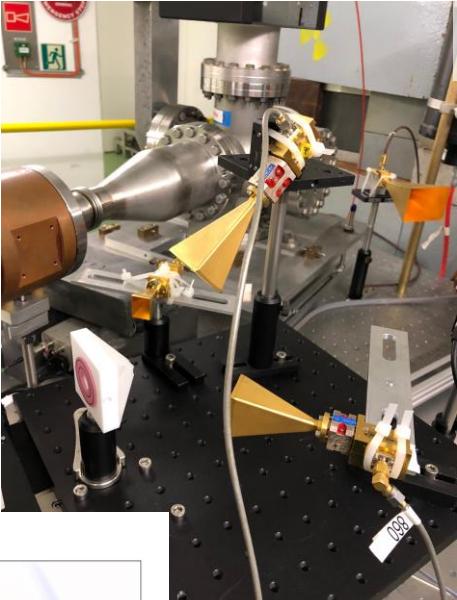


# Single-shot Cherenkov diffraction measurement

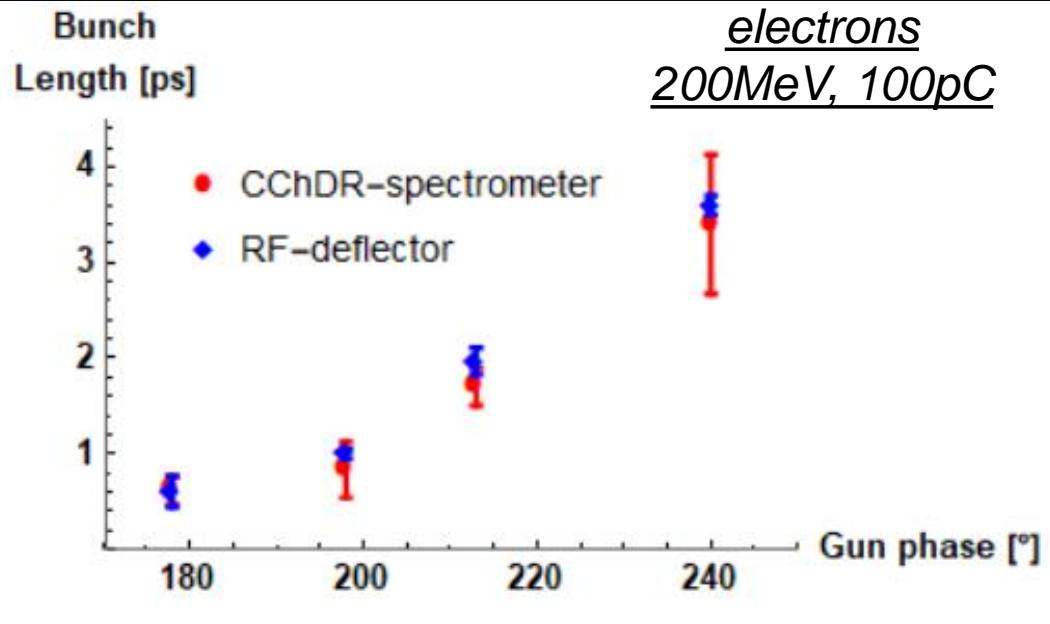
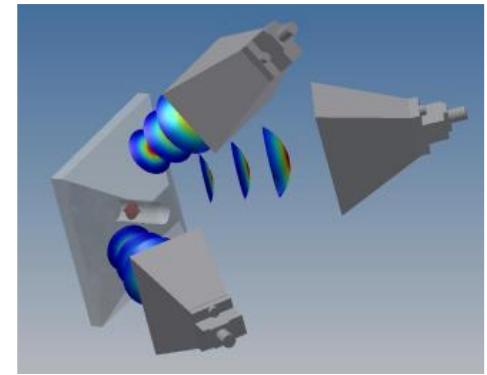
1



Cherenkov diffraction radiation  
Measured in 3 bands (60-90-110GHz)



Pyramidal cone  
 with 1cm aperture

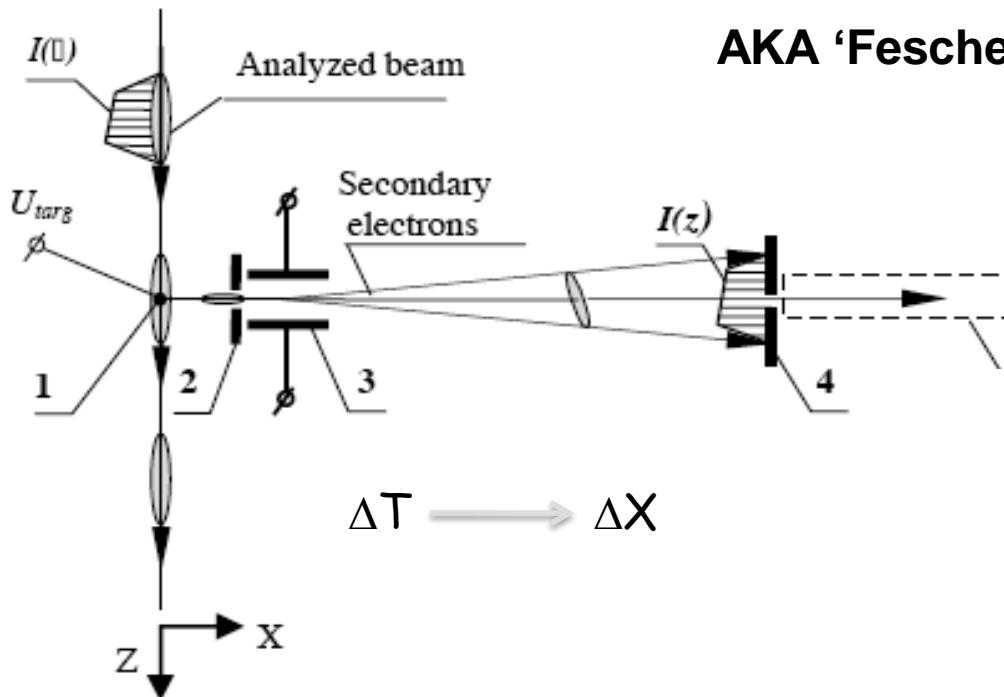
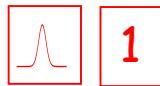


A. Curcio et al, PRAB 23 (2018) 022802

# Radiofrequency manipulation

**‘How to transform time information  
into spatial information’**

# Bunch shape monitor



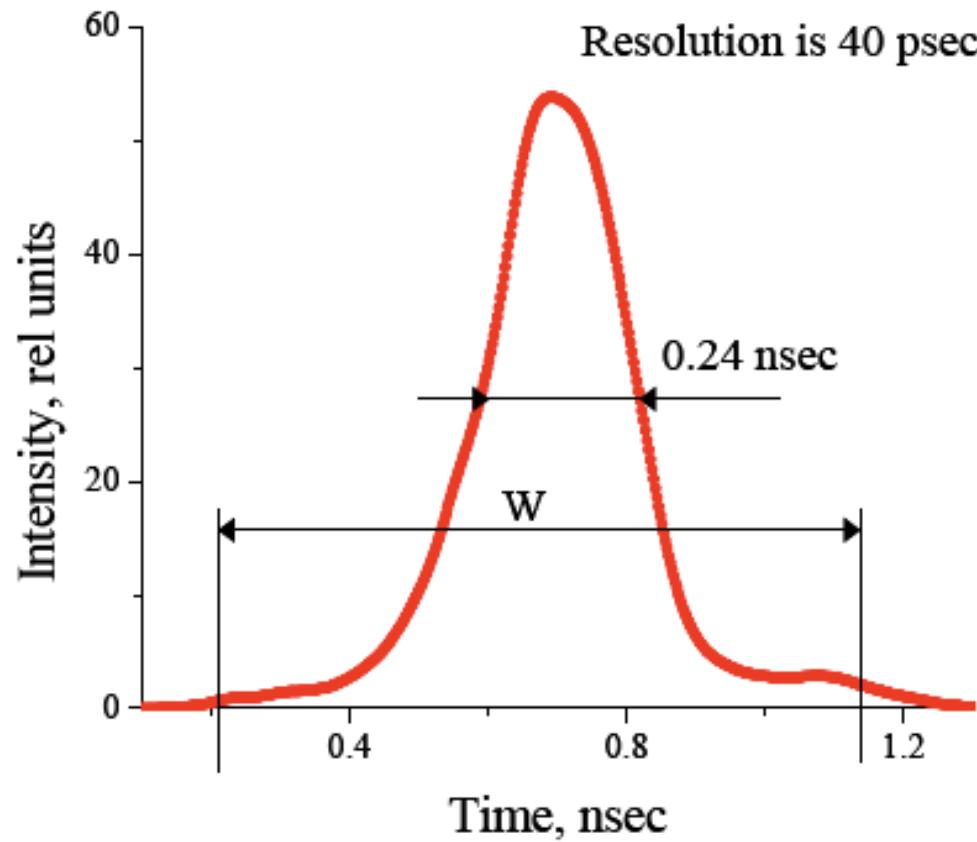
**AKA ‘Feschenko monitor’**

- 1 - Target (wire, screen, laser for  $H^-$ ) : Source of secondary electrons
- 2 - Input collimator
- 3 - RF deflector (100MHz, 10kV) combined with electrostatic lens
- 4 - Electron Beam detector (electron multiplier, ..)

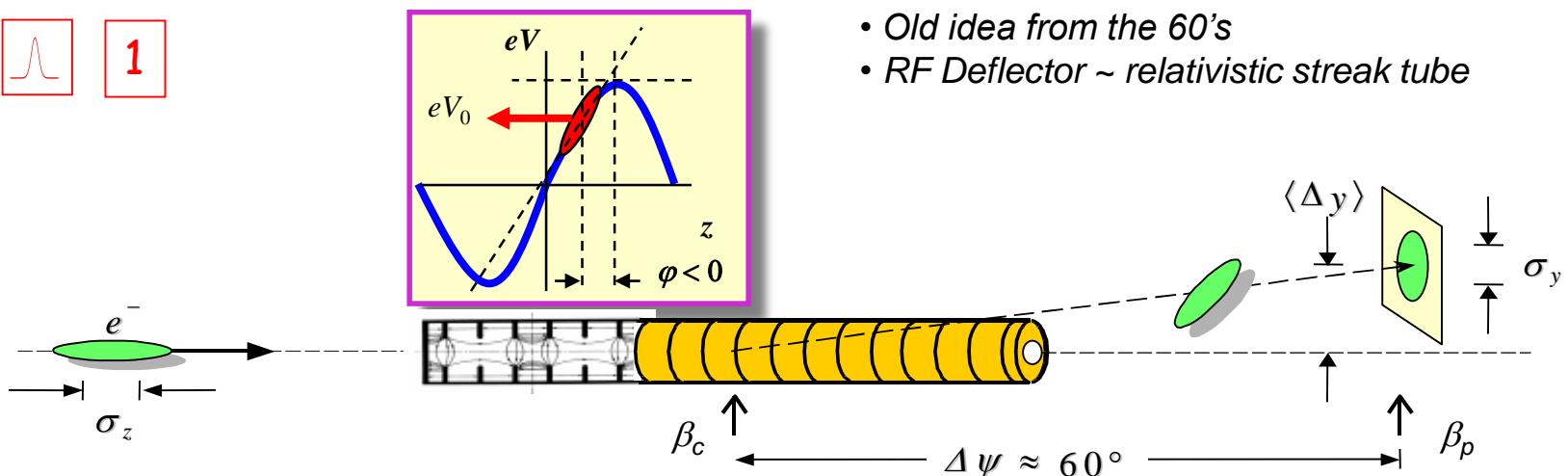
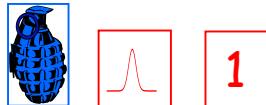
# Bunch shape monitor

## Longitudinal Bunch profile @ SNS

A. Feschenko *et al*, Proceedings of LINAC 2004, Lübeck, p408



# RF Deflecting cavities



Beam profile RF on

Beam profile RF off      Deflecting Voltage

$$\sigma_y = \sqrt{\sigma_{y0}^2 + \sigma_z^2 \beta_c \beta_p \left( \frac{2\pi}{\lambda} \frac{eV_0}{E_0} \sin(\Delta\Psi) \cos(\varphi) \right)^2}$$

Bunch length      Beta function at cavity and profile monitor      Beam energy

RF deflector wavelength      Betatron phase advance (cavity-profile monitor)

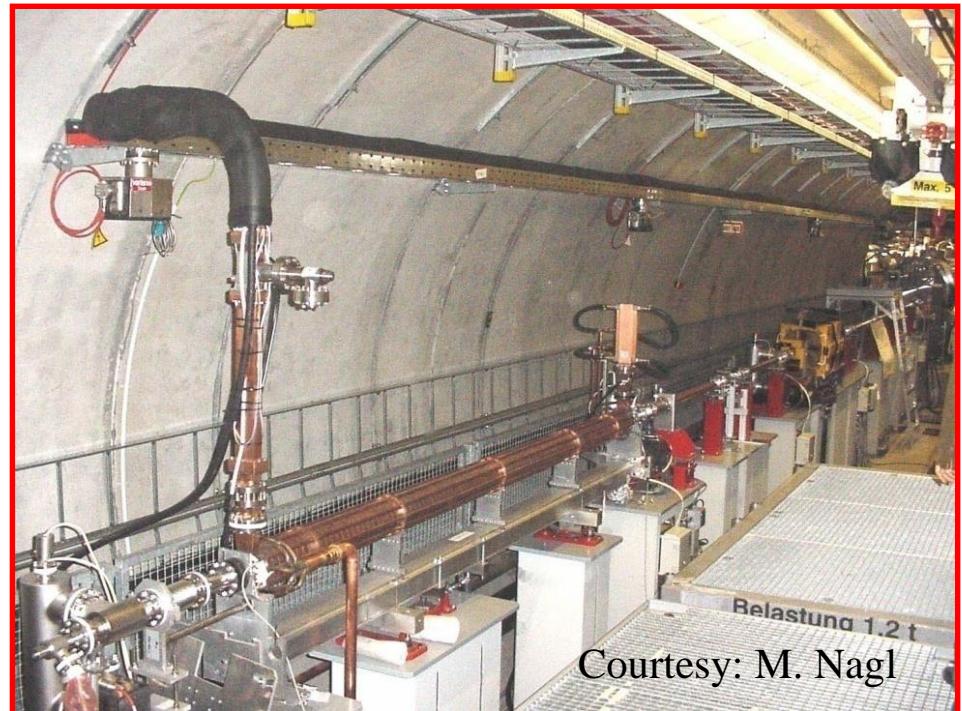
$\sin\Delta\Psi = 1$ ,  $\beta_p$  small  
 Make  $\beta_c$  large

# RF Deflecting cavities

CTF3



LOLA @ Flash



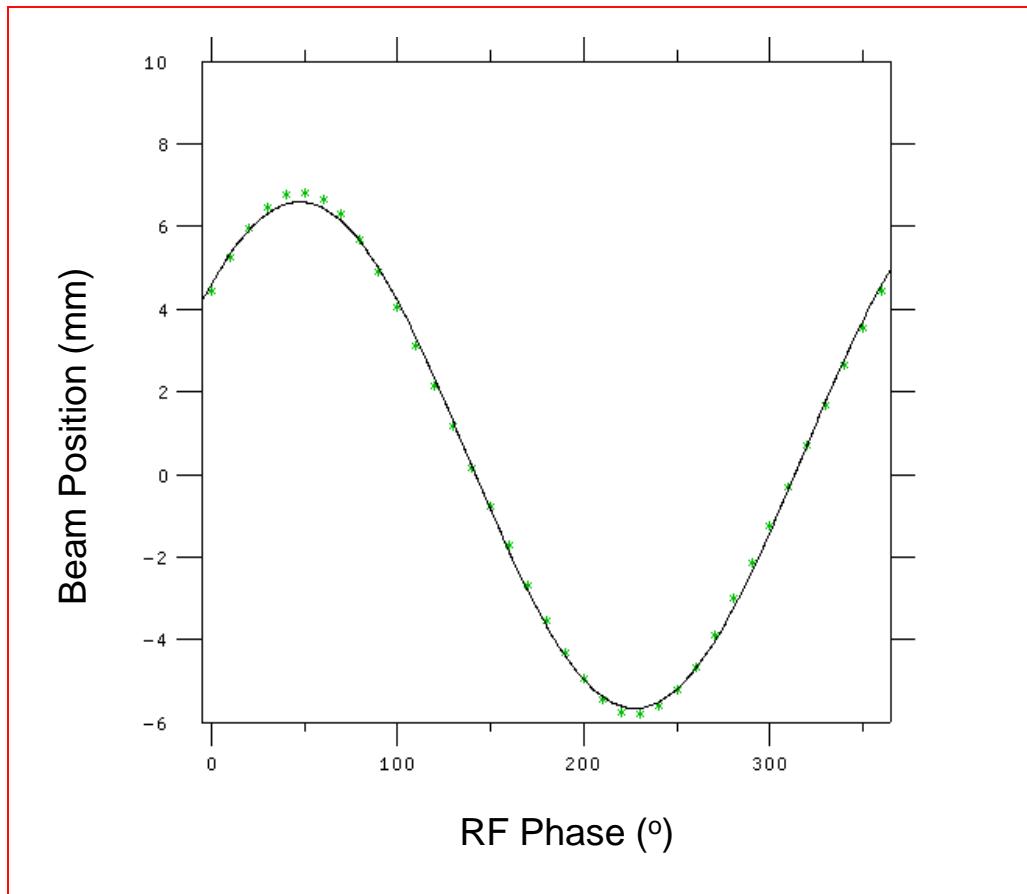
Courtesy: M. Nagl

# RF Deflecting cavities

## Calibration of RF Deflector

$$\Delta X(\text{mm}) \longrightarrow \Delta\varphi(^{\circ}) \\ \Delta T(\text{ps})$$

Monitor the Beam Position on (or close to) the Profile monitor to calibrate the deflection angle

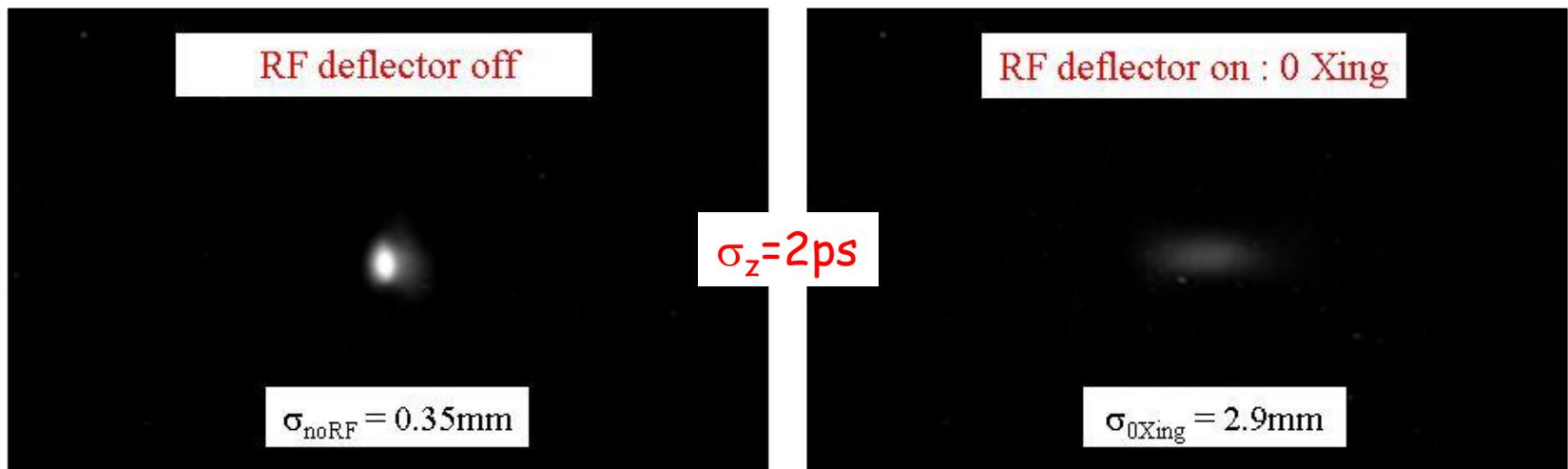
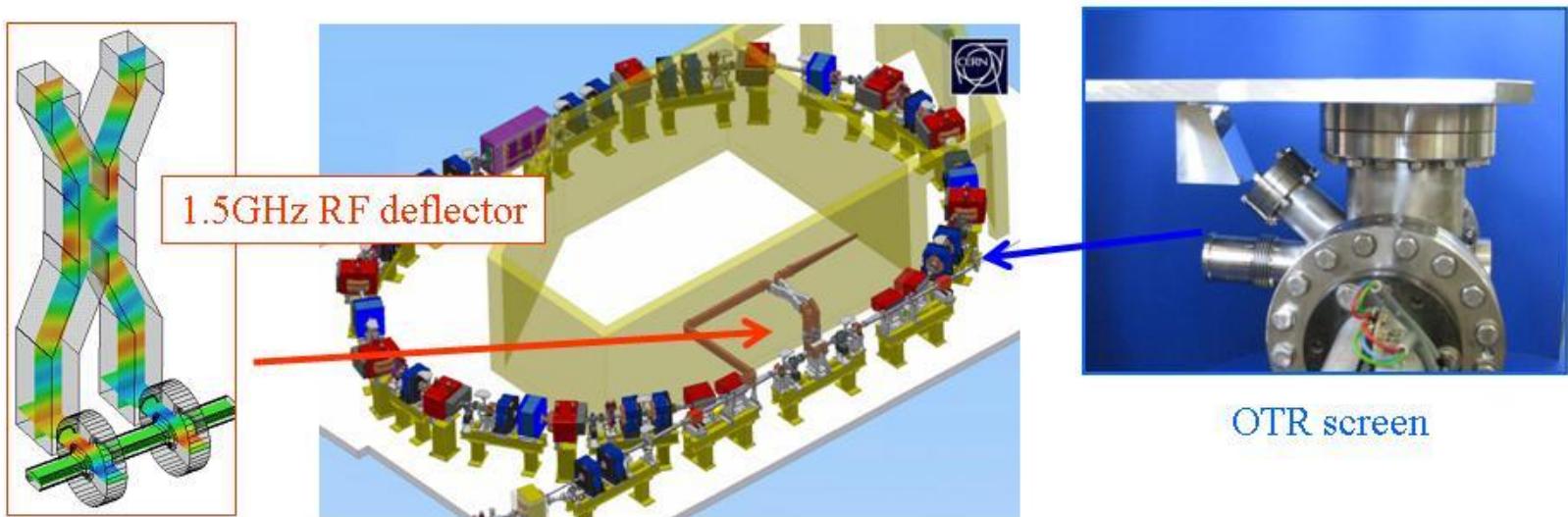


## Beam offset on the screen

$$\Delta y(z) \approx \frac{eV_0}{E_0} \cdot \sqrt{\beta_c \beta_p} \sin(\Delta\Psi) \left( \frac{2\pi}{\lambda} - z \cos(\varphi) + \sin(\varphi) \right)$$

RF deflector phase

# RF Deflecting cavities

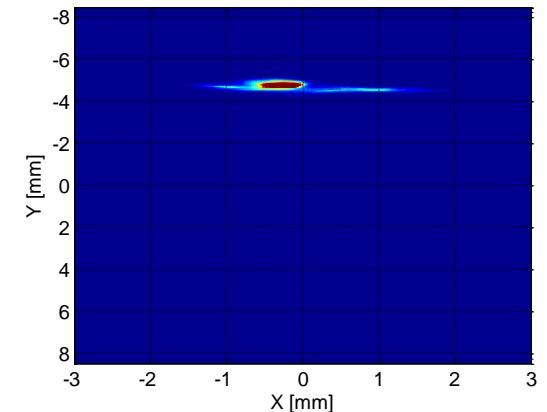
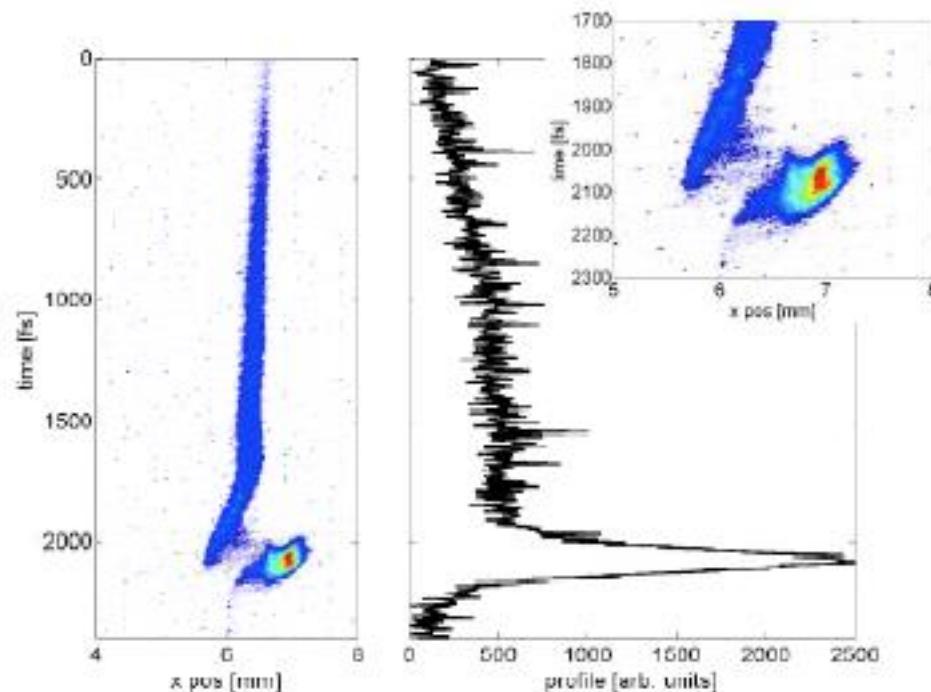


# RF Deflecting cavities

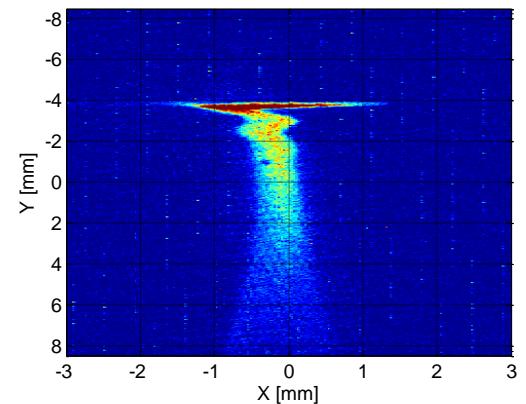
## Bunch length measurement @ Flash

M. Hüning *et al*, Proceeding of the 27<sup>th</sup> FEL conference, Stanford, 2005, pp538

**LOLA off:**



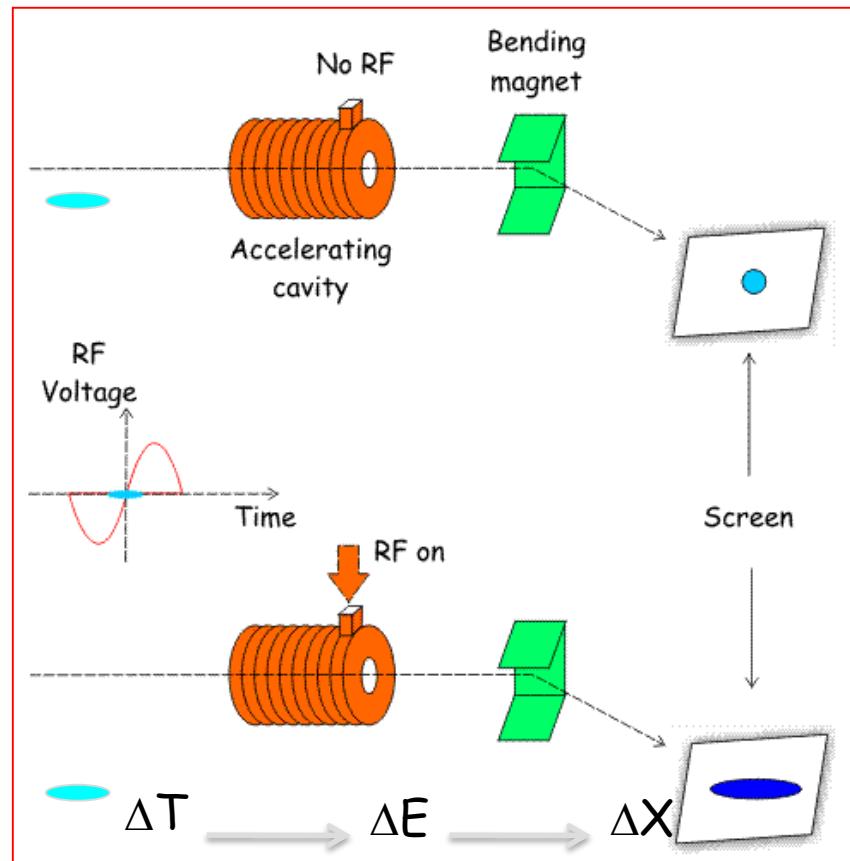
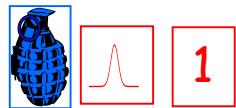
**LOLA on:**



→ Resolution of 4fs/pixels

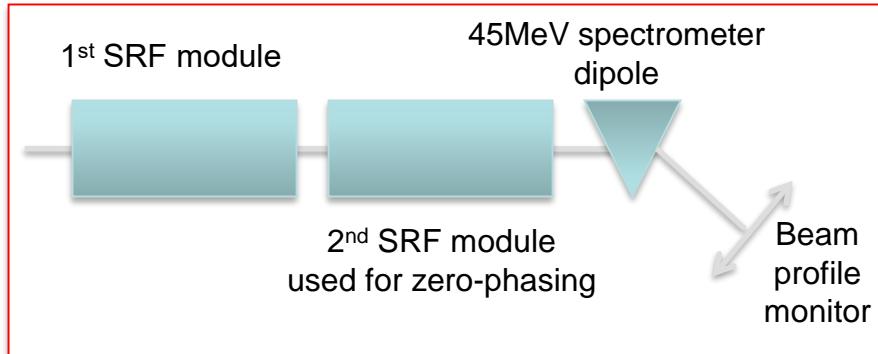
# RF Accelerating cavities

'The electron energy is modulated by the **zero-phasing** RF accelerating field and the bunch distribution is deduced from the **energy dispersion** measured downstream using a spectrometer line'



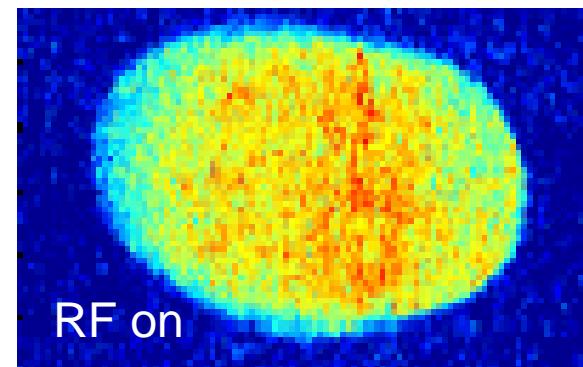
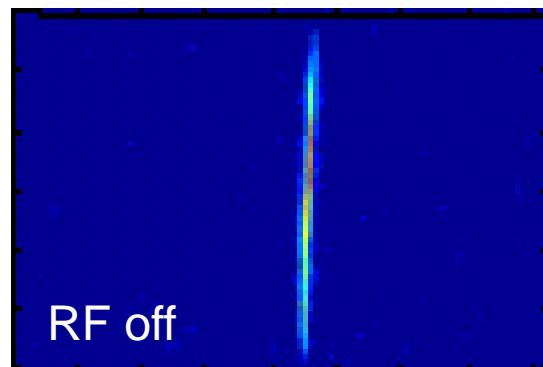
# RF Accelerating cavities

CEBAF injector, Newport News



D. X. Wang *et al*, Physical Review E57 (1998) 2283

**84fs, 45MeV beam but low charge beam**



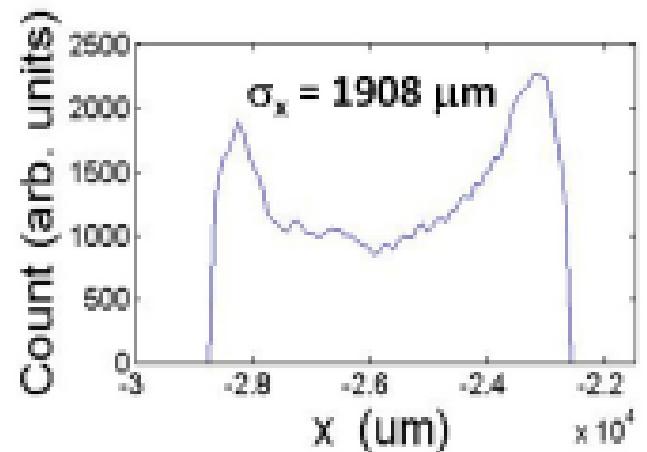
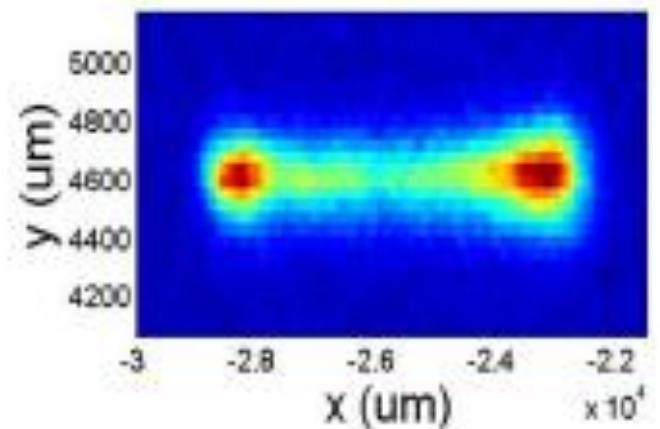
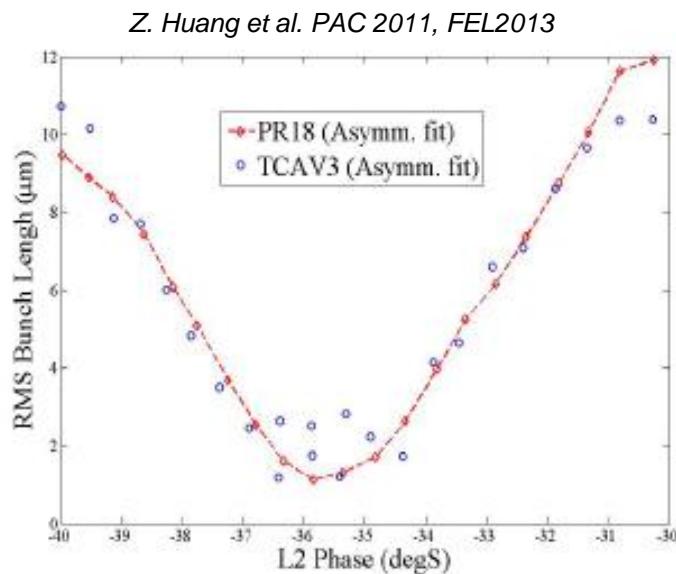
## Limitations

- RF non linearities
- Beam loading and wakefield for high charge beam

# RF Accelerating cavities

## SLAC LCLS: at 4.7 GeV

- 550m of linac at RF zero crossing!
- 6m dispersion in spectrometer line



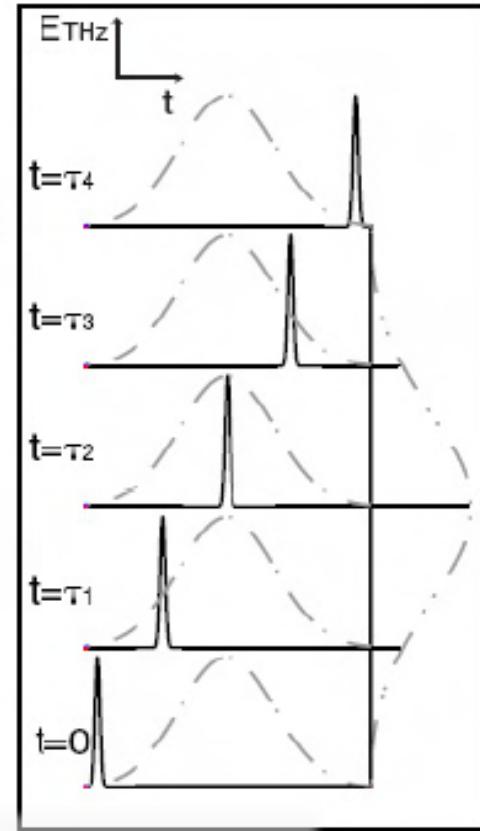
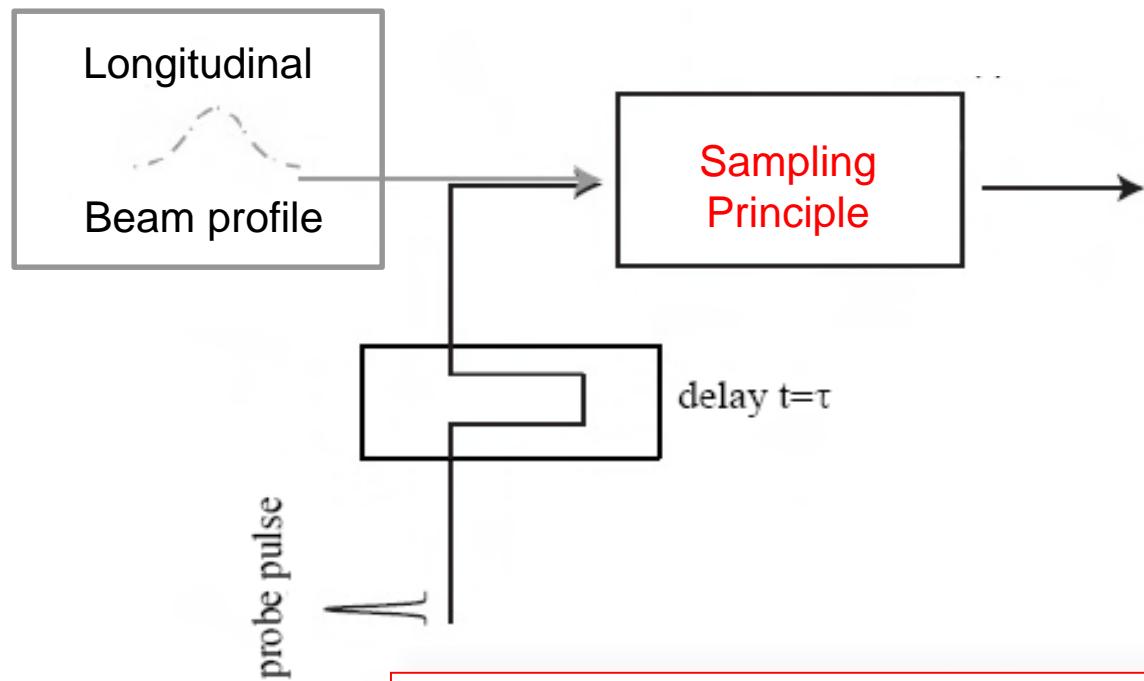
**TCAV3**

**~ 1 fs rms bunch length at 4.7 GeV**

# Laser-based diagnostics

# Sampling techniques

Using a short laser pulse to scan through the beam profile

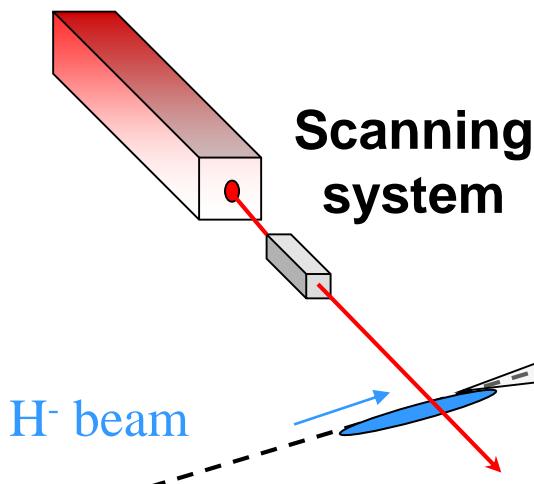


## Limitation

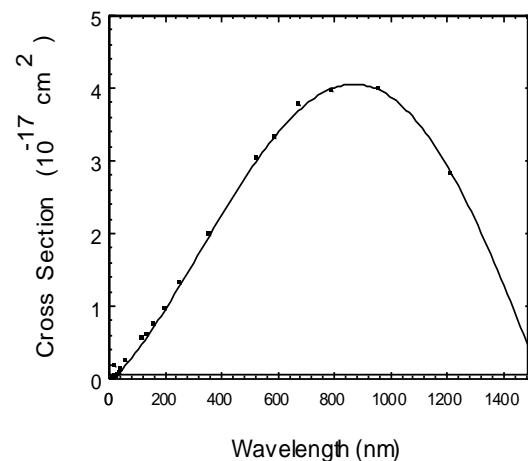
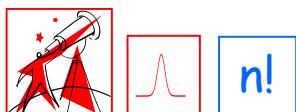
Laser-beam synchronization jitter should be smaller than the bunch length to measure

# Laser Wire Scanner

High power laser

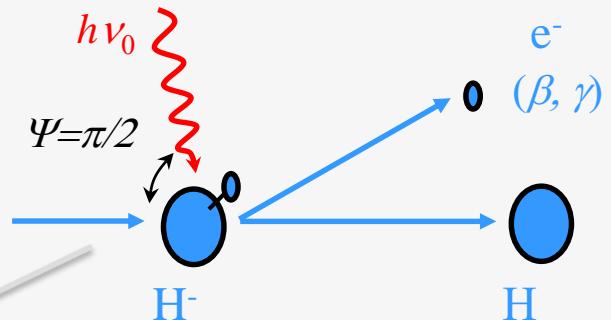


- First ionization potential for H<sup>-</sup> ions is 0.75eV
- Photo-neutralization cross section :  $\sigma \sim 4.10^{-17} \text{ cm}^2$



T. Lefevre - CAS Advanced Accelerator Physics, 6-  
18 November 2022, Sevrier, France

## Photo-neutralization



## Detection system based on

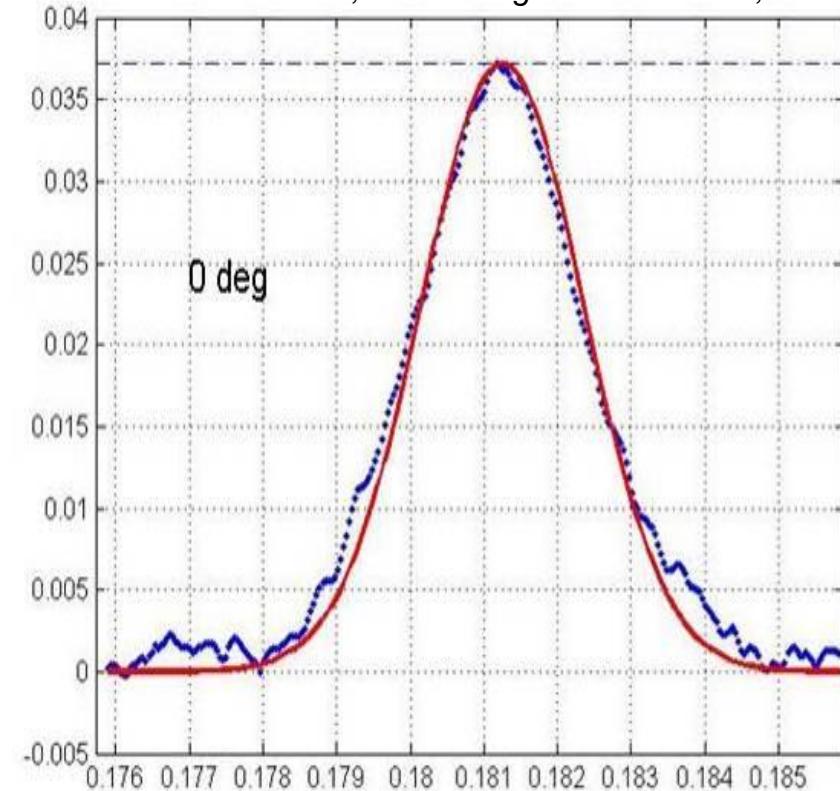
- The measurement of released electrons using a magnet and a collector (faraday cup, MCP,...)
- Measured the conversion of H<sup>-</sup> into H with a current monitor

# Laser Wire Scanner

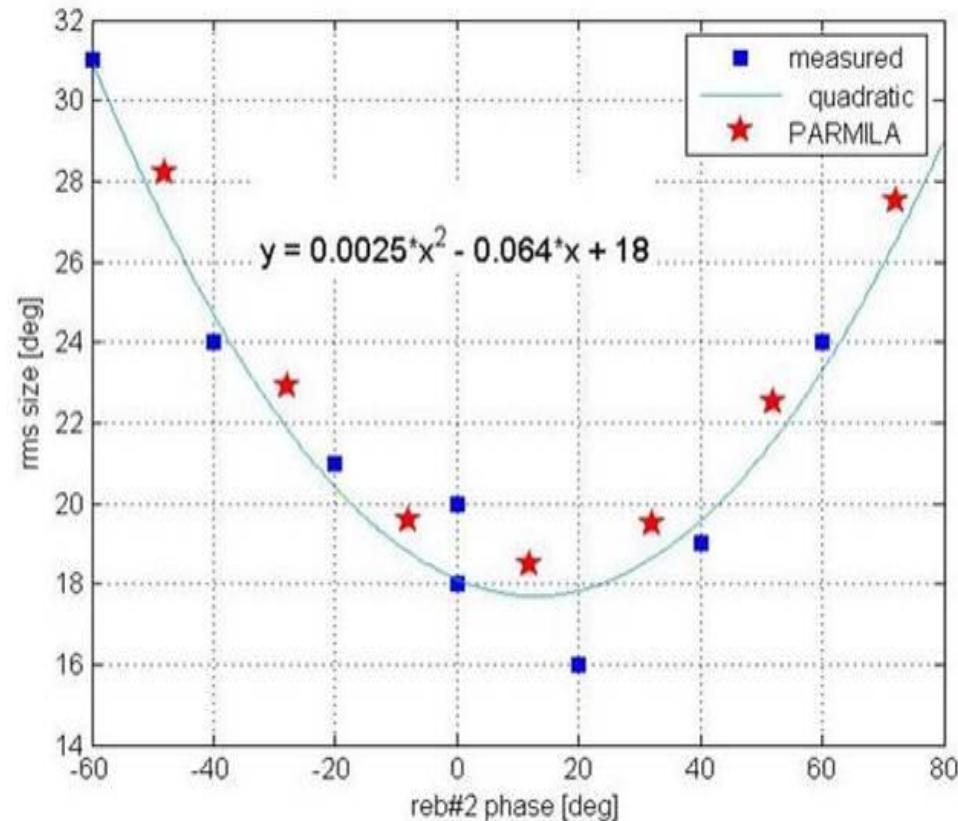
## Longitudinal Measurements @ SNS

2.5 MeV H<sup>+</sup>, 402.5 MHz bunching freq, Ti-Sapphire laser phase-locked @ 1/5<sup>th</sup> bunching frequency

S. Assadi et al, Proceedings of EPAC 2006, Edinburgh, pp 3161



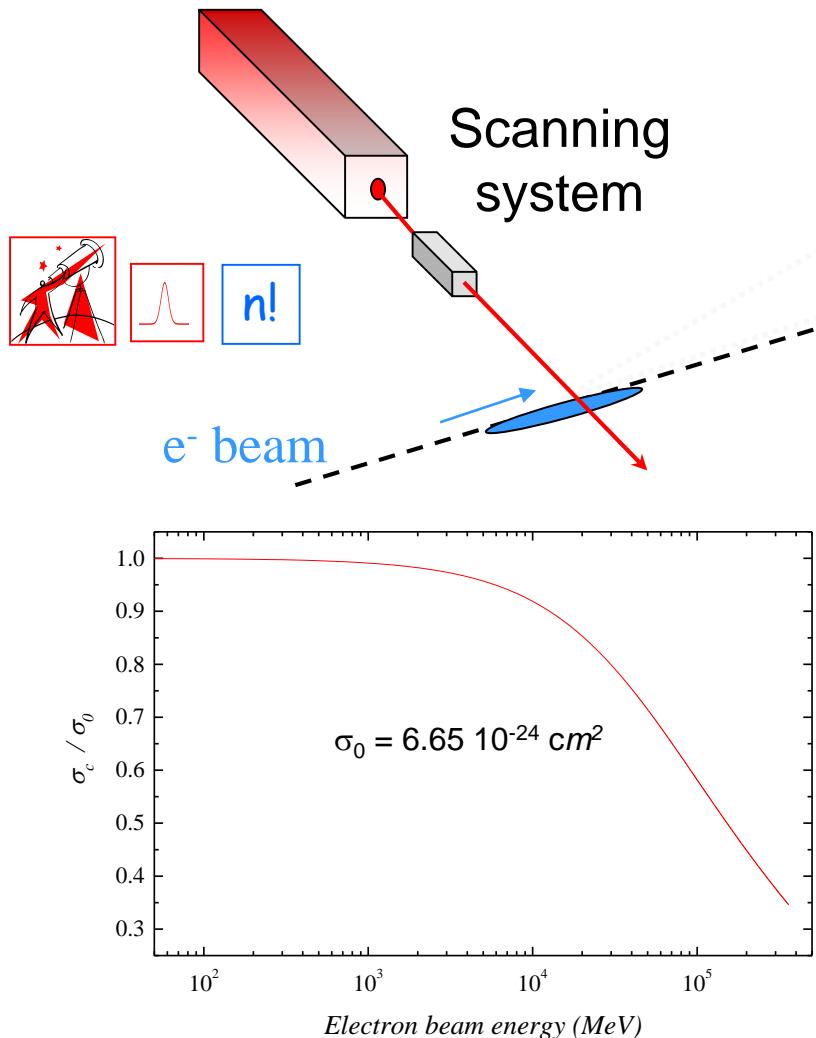
Collected electron signal plotted vs. phase



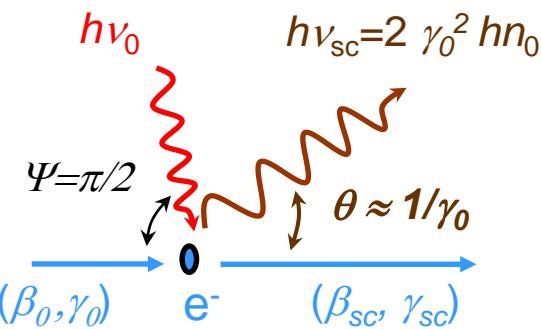
Measured and predicted bunch length  
vs. cavity phase setting

# Laser Wire Scanner

High power laser



Thomson/Compton scattering

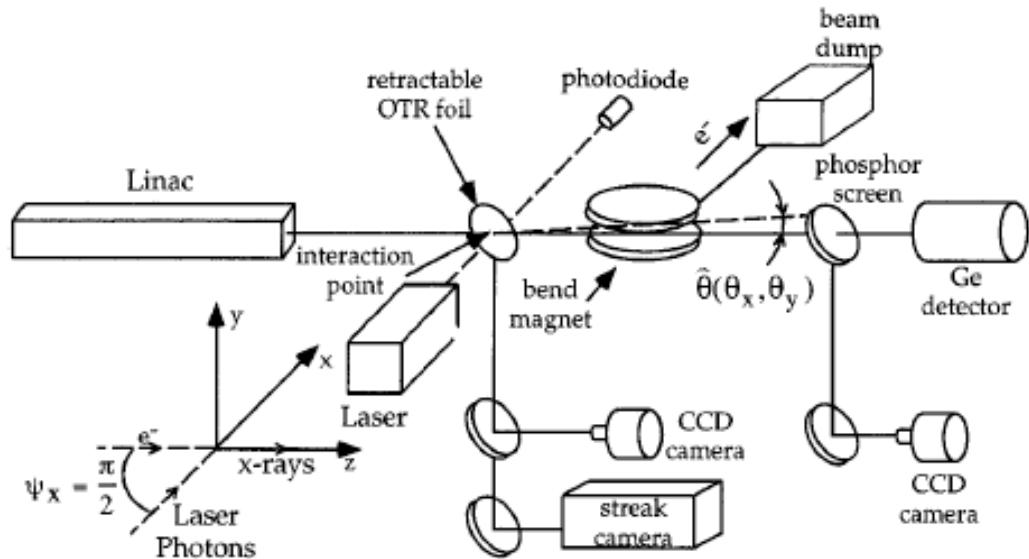


## Detection system based on

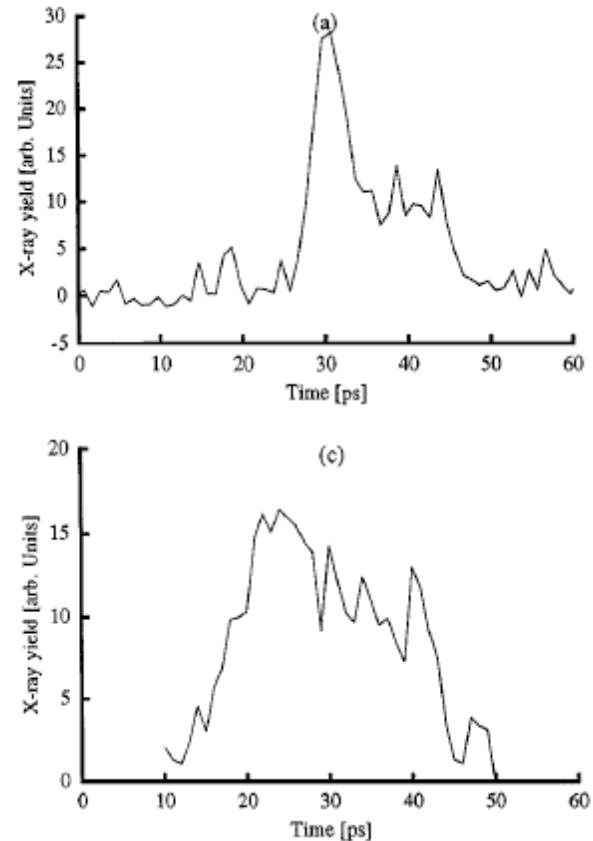
- The measurement of the scattered photons
- The measurement of degraded electrons

# Laser Wire Scanner

ALS @ LBNL



W.P. Leemans et al, PRL 77 (1996) 4182

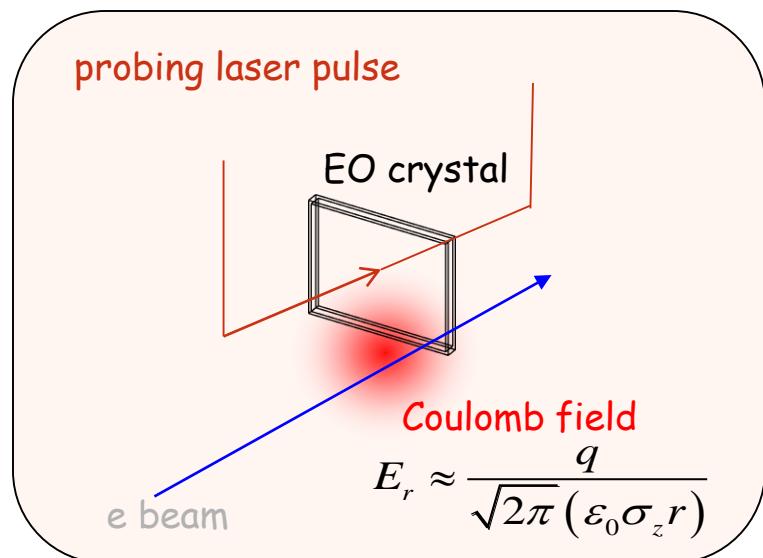


Using a 10TW Ti:Al<sub>2</sub>O<sub>3</sub> laser system. Detecting 5.10<sup>4</sup> 10-40 keV X-rays using either an X-ray CCD and Ge detector.

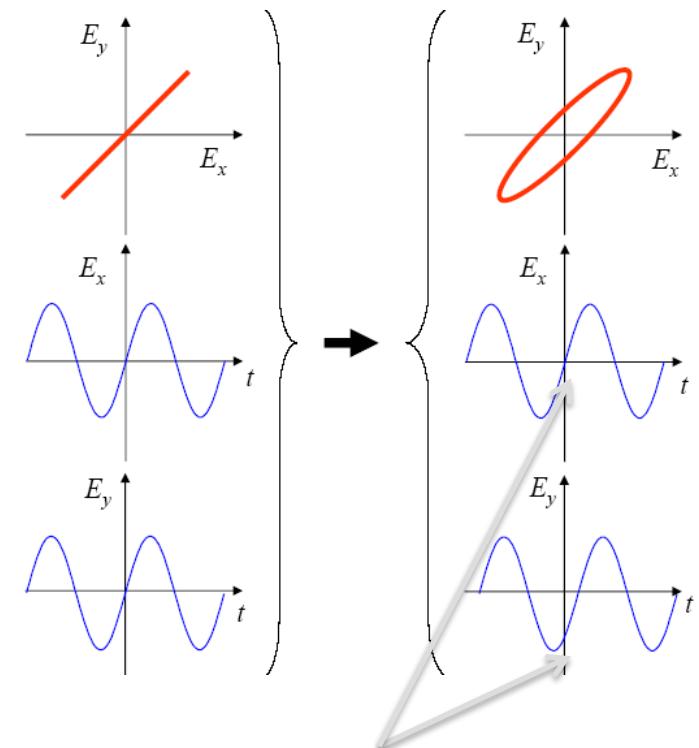
# Electro-optical techniques

*'This method is based on the polarization change of a laser beam which passes through a crystal itself polarized by the electrons electric field'*

## E-field induced birefringence in EO-crystal : Pockel/Kerr effect



- Polarization diagram
- electric field of the horizontal polarization
- electric field of the vertical polarization

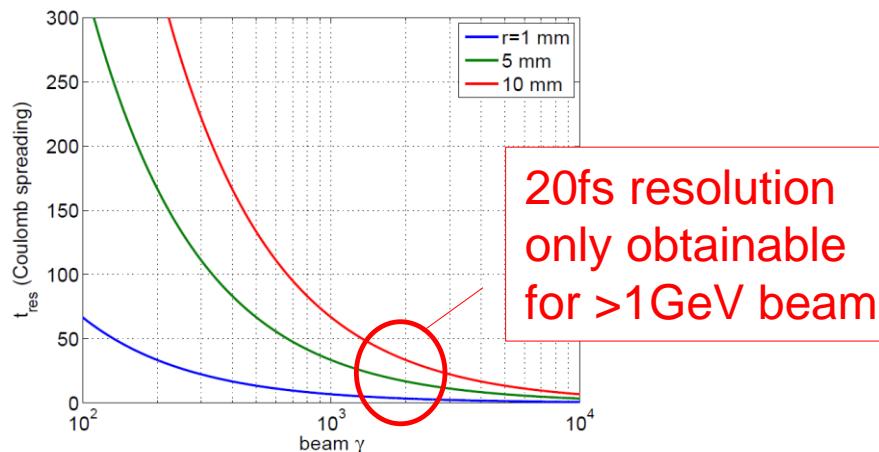
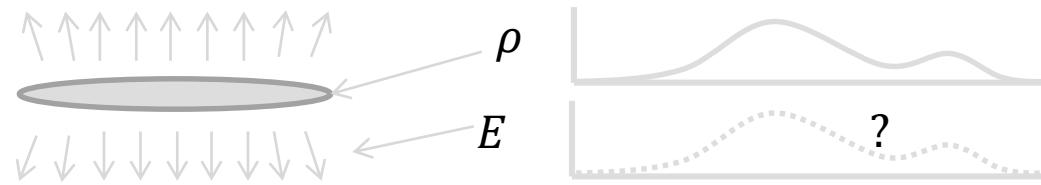


$$G = \frac{2pd}{I_0} (n_x - n_y) = \frac{2pd}{I_0} n_0^3 r_{41} E_r$$

**Relative phase shift between polarizations increases with the beam electric field**

# Bunch length and bunch field

Field radiated or probed is related to Coulomb field near the electron bunch



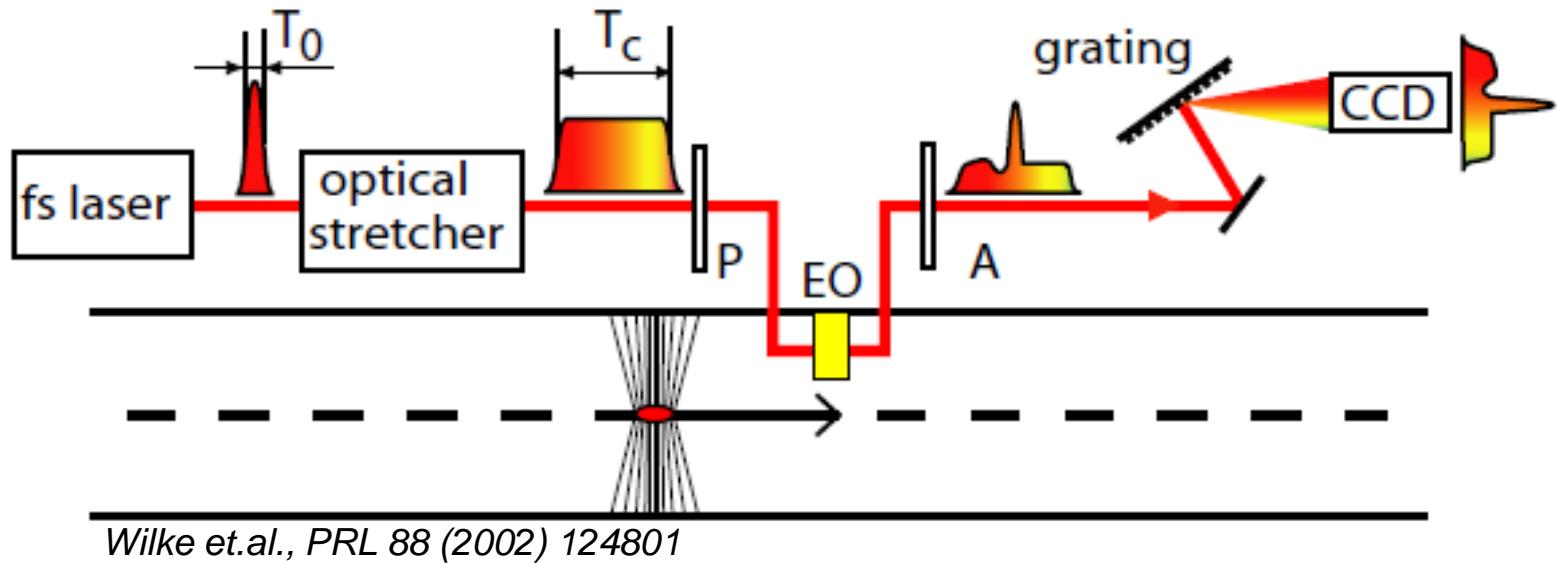
High  $\gamma$  is an advantage!

Time response & spectrum of field  
is dependent on spatial position,  $r$  :

$$\delta t \sim 2r / c\gamma$$

→ ultrafast time resolution requires close proximity to bunch

# Spectral decoding



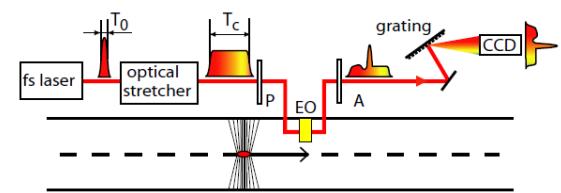
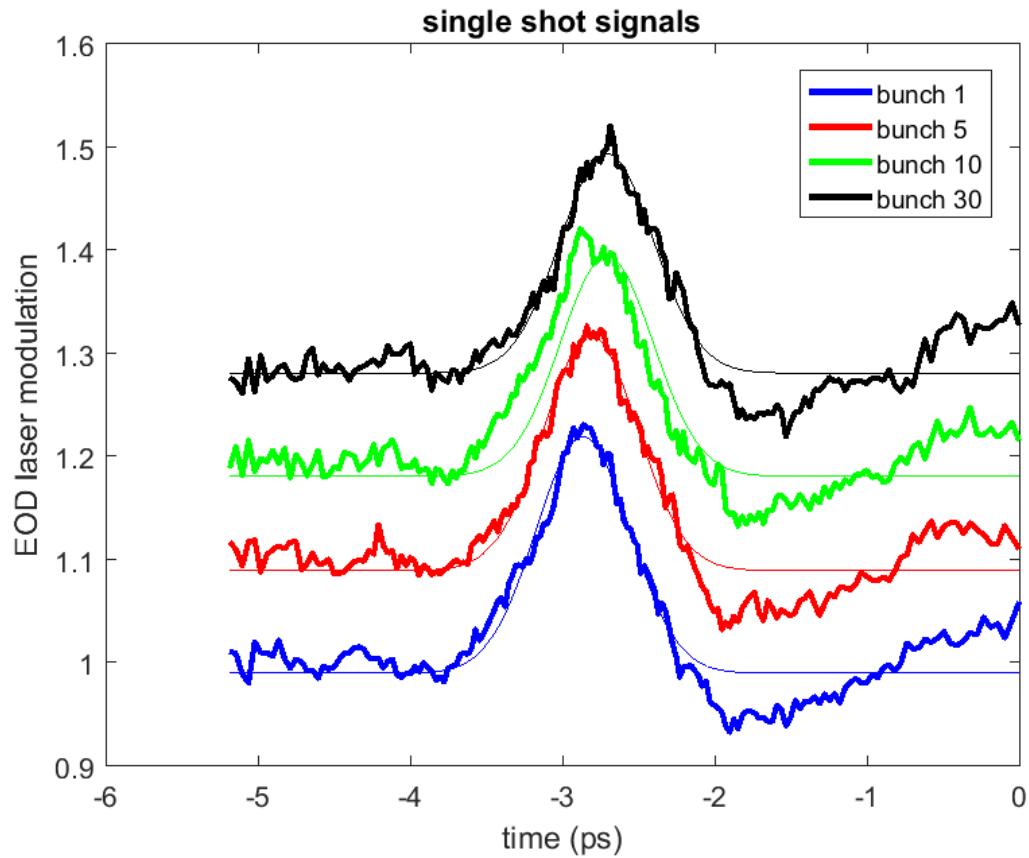
Single bunch measurements by detection the wavelength spectrum in spectrometer (position vs wavelength) of a chirped laser pulse (time vs wavelength)

- Artifacts due to frequency mixing
- Minimum resolution in the order

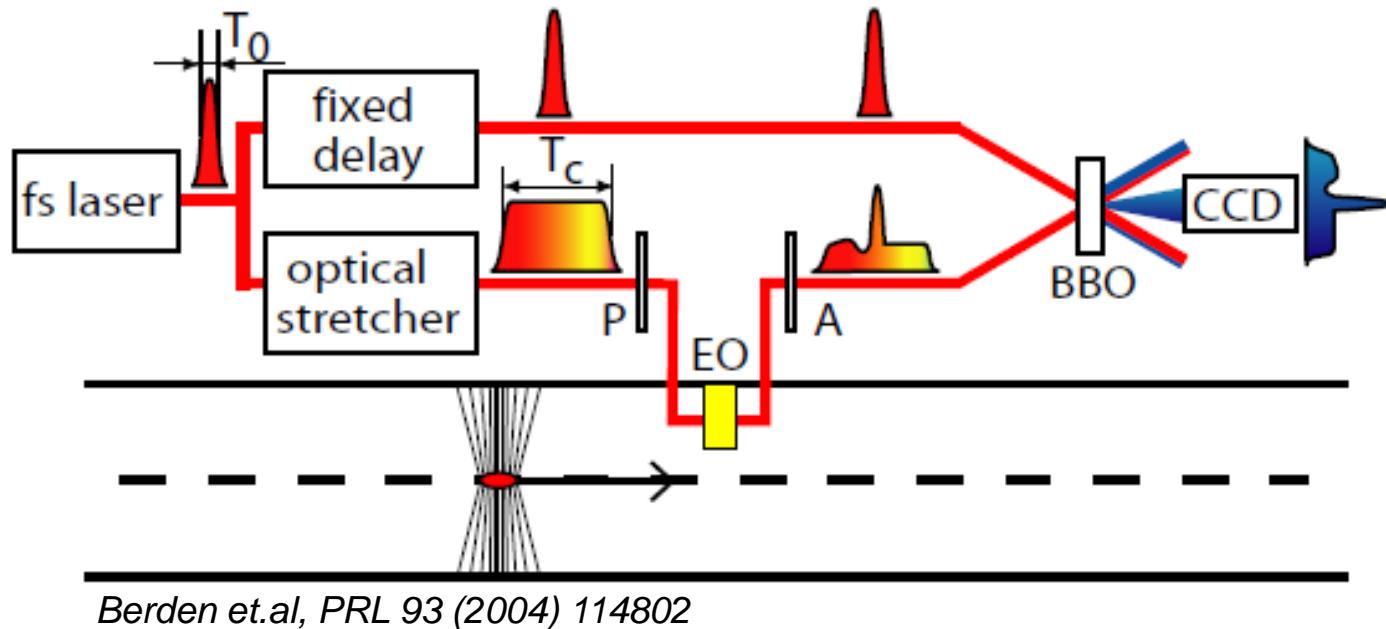
$$T_{\text{lim}} \approx 2.6 \sqrt{T_0 T_c}$$

# Spectral decoding

## Single shot measurements at the XFEL bunch compressor 1

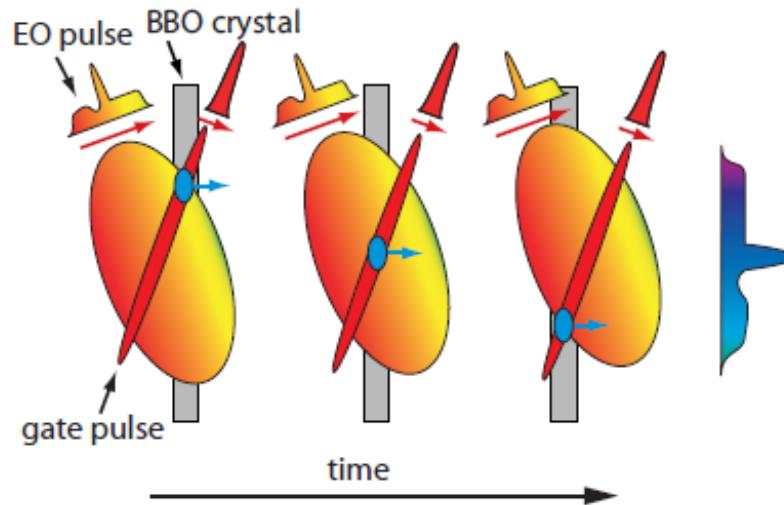


# Temporal decoding



- low efficiency of Second Harmonic Generation process, approx. 1mJ laser pulse energy necessary
- Resolution : duration of the gate beam, thickness of the SHG crystal – 50 fs or slightly better

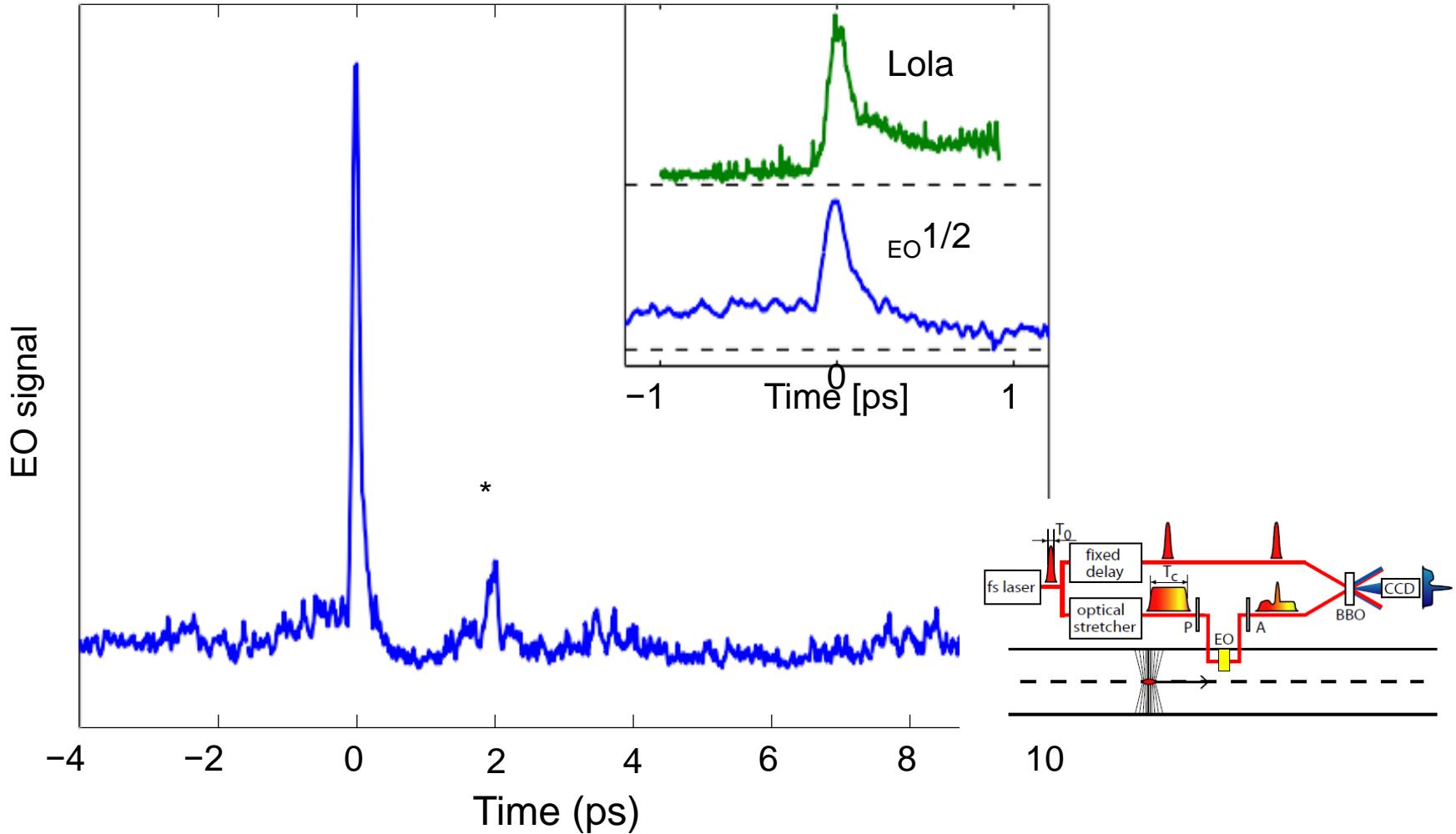
# Temporal decoding



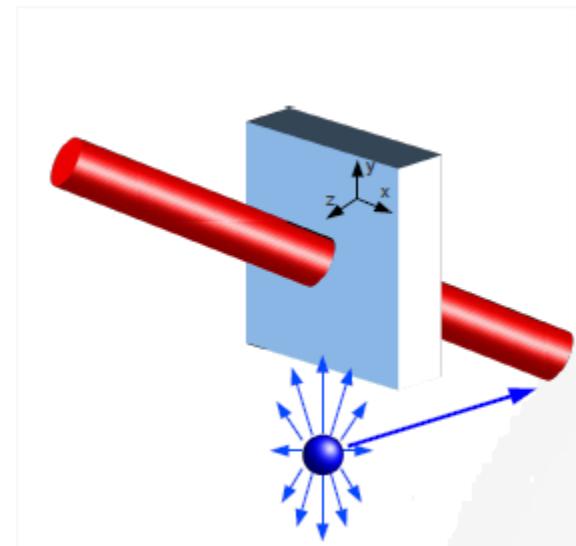
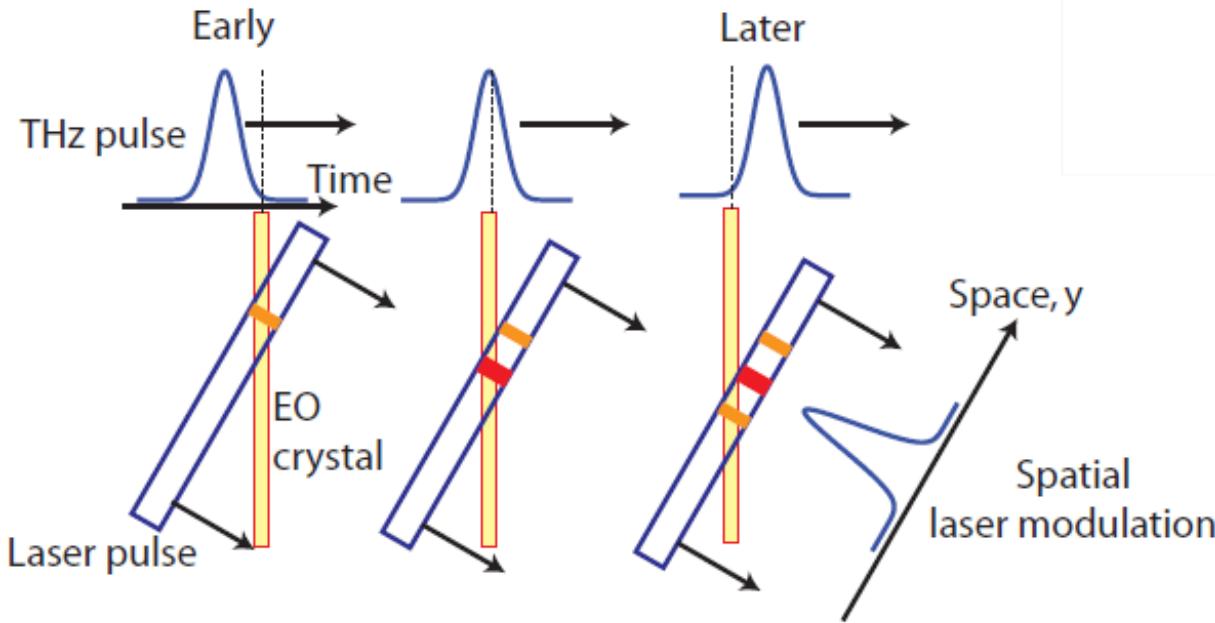
- The short gate pulse overlaps with different temporal slices of the EO pulse at different spatial positions of the BBO crystal.
- Thus the temporal modulation of the EO pulse is transferred to spatial distribution of the SHG laser beam.

# Temporal decoding

## Measurement performed at FLASH/DESY



# Spatial decoding



Cavalieri *et. al*, PRL 94 (2005) 114801  
Jamison *et. al*, Opt. Lett. 28 (2003) 1710  
Van Tilborg *et. al*, Opt. Lett. 32 (2007) 313

# Summary

- Optical radiation
  - Cherenkov / OTR radiation
  - ODR / OSR Radiation
  - Streak camera
- Coherent radiation : Bunch spectrum
  - Interferometry
  - Polychromator
- RF techniques
  - 'Feschenko' monitor
  - RF Deflector
  - Zero phasing techniques
- Laser based Method
  - Sampling
    - Non linear mixing
    - Thomson/Compton scattering
    - Photo-neutralization
    - Electro-Optic Sampling
  - E-O Spectral decoding
  - E-O Spatial decoding
  - E-O Temporal decoding

			$\sigma$	1	$n!$	Limitations
Optical radiation	X					200fs
Coherent radiation		X				
RF techniques	X	X	X	X	X	Hadron, 20ps
Laser based Method	X	X	X	X	X	1fs
Sampling		X	X	X	X	10fs
Non linear mixing						Jitter (10fs)
Thomson/Compton scattering		X	X			
Photo-neutralization	X	X	X			
Electro-Optic Sampling	X	X	X			
E-O Spectral decoding	X	X		X		~ 200fs
E-O Spatial decoding	X	X		X		~ 50fs
E-O Temporal decoding	X	X		X		~ 50fs

# Conclusions

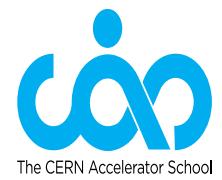
- Short bunch length measurements are challenging
- Resolution of few fs achieved operationally
- Field in constant move driven by the advances in FELs and novel accelerating technologies
- Another exciting field of R&D !

# Thank you for your attention



# Advanced Accelerator Physics

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## Extra slides

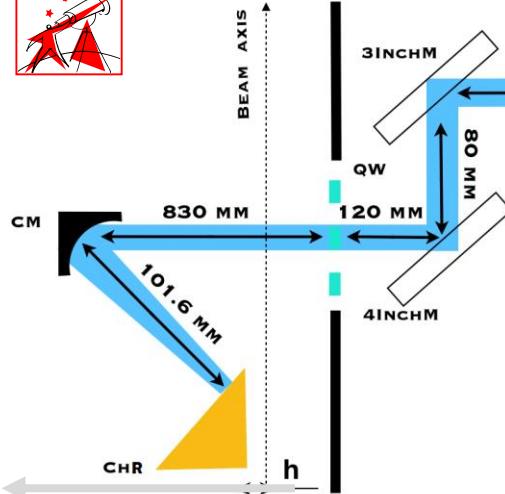
“When you are courting a nice girl an hour seems like a second. When you sit on a red-hot cinder a second seems like an hour. That's relativity. “

Albert Einstein

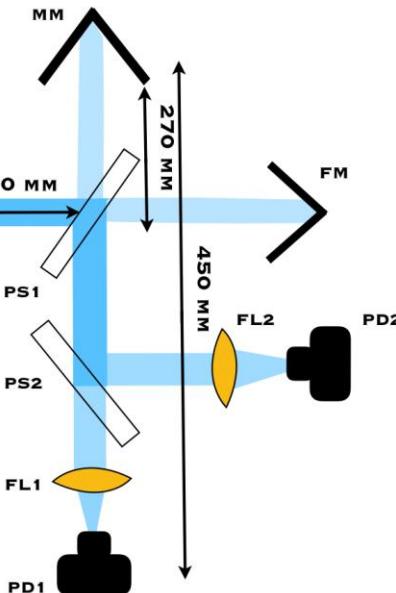


# Martin-Puplett Interferometer

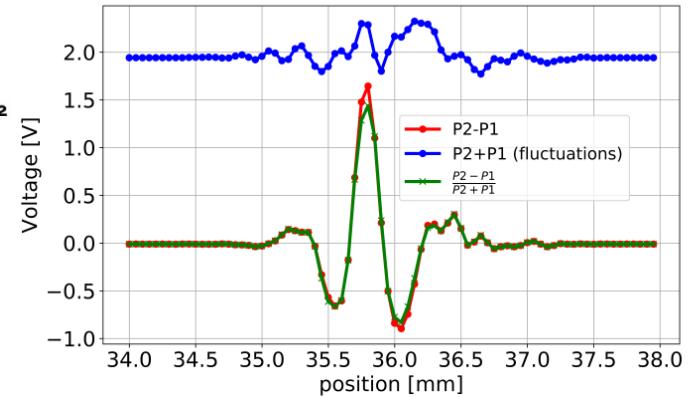
## *Cherenkov diffraction radiation*



electrons  
35MeV, 70pC

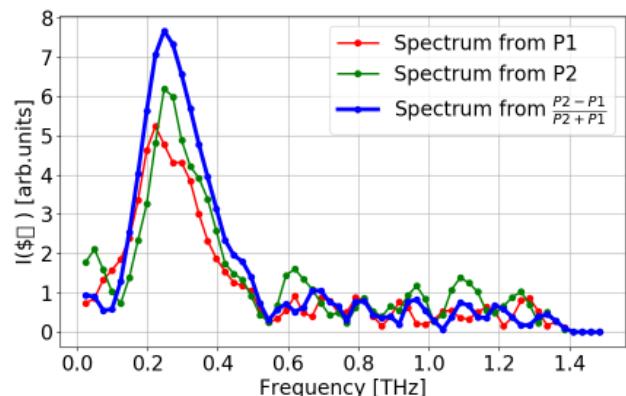


$$I(\delta) \propto \int_{-\infty}^{\infty} |E(t) + E(t + \delta/c)|^2 dt$$



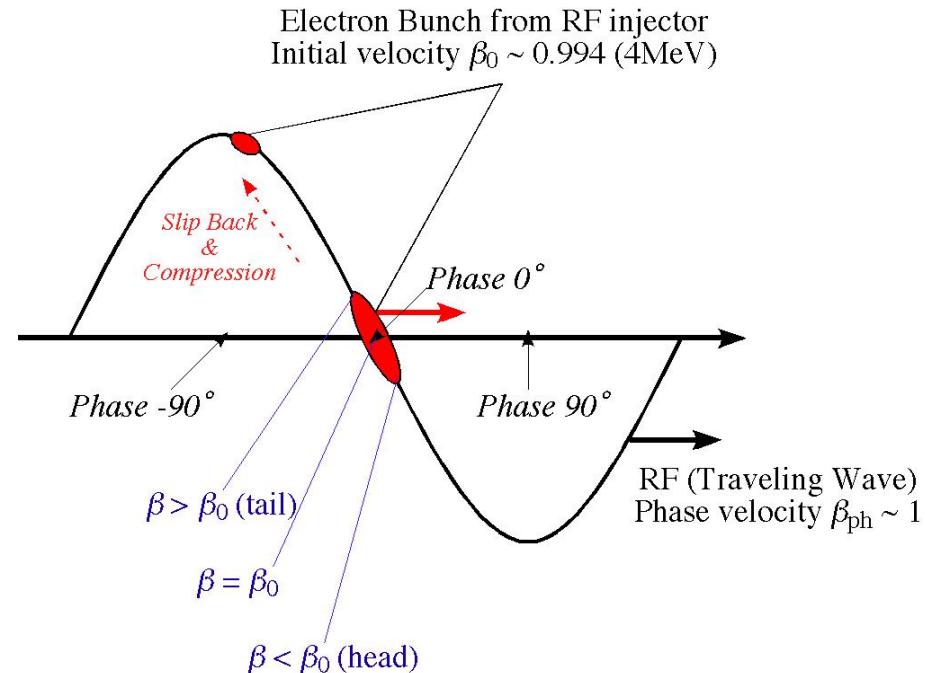
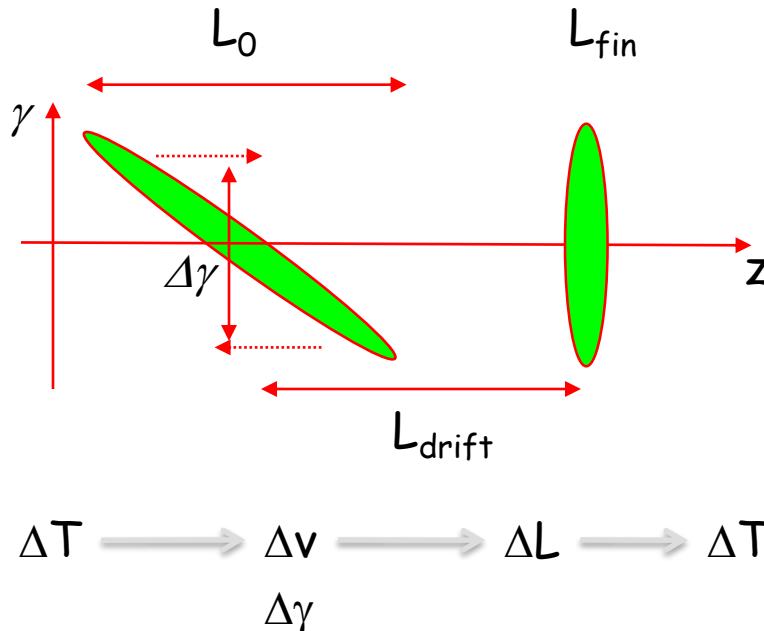
“the Fourier transform of the autocorrelation function is the power spectrum”

$$I(\omega) \propto \int_{-\infty}^{\infty} I(\delta) \cos\left(\frac{\omega\delta}{c}\right) d\delta$$



- Bunch length manipulation
- Ballistic Compression
- Magnetic Compression

# Short bunches by Ballistic/Velocity Compression

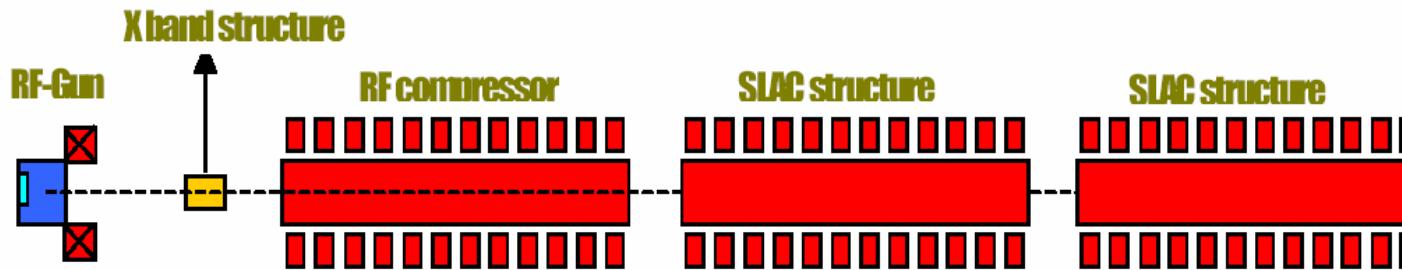


Provide a correlated velocity spread enough to produce,  
in a drift of length  $L_{drift}$  a *path difference* equal to  $\Delta L$

$$\Delta L = \left\lfloor \frac{L_{drift}}{\bar{g}^2} \right\rfloor \frac{Dg}{\bar{g}}$$

P. Piot *et al*, PRSTAB 6 (2003) 033503  
S.G. Anderson *et al*, PRSTAB 8 (2005) 014401

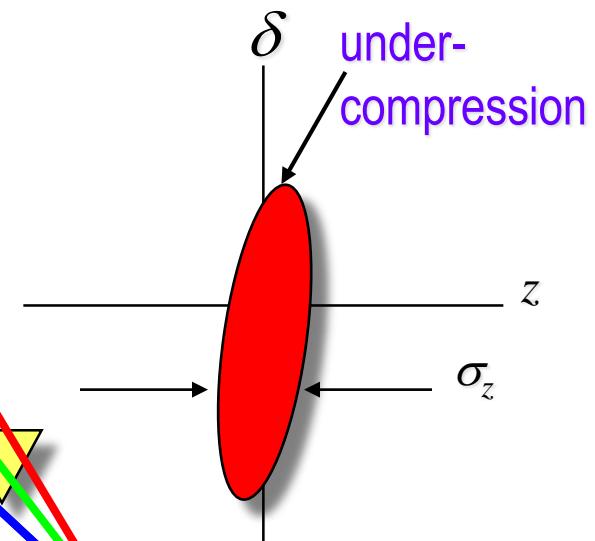
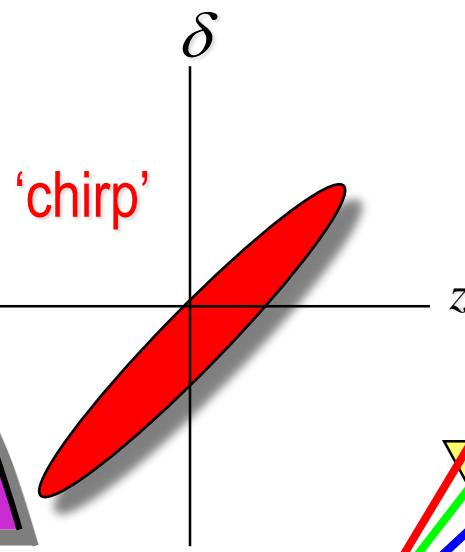
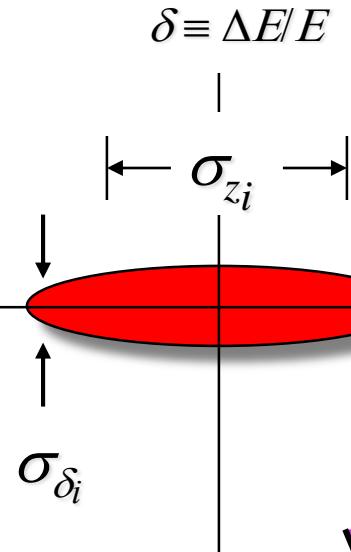
# Short bunches by Ballistic Compression



- Works well for non ultra-relativistic beam energies
- no Coherent Synchrotron Radiation effect and bend-plane emittance growth
- Longitudinal emittance growth due to RF non linearities

# Short bunches by Magnetic Compression

$$\delta = \Delta E/E$$



$$V = V_0 \sin(kz)$$



**RF Accelerating Voltage**

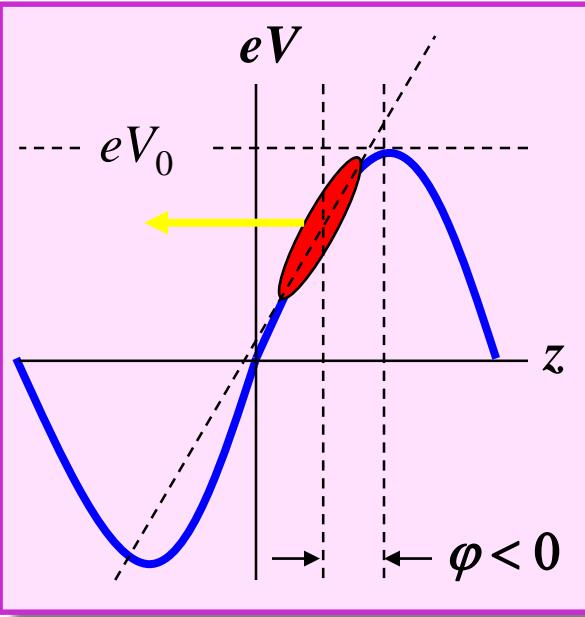
$$\Delta z = R_{56} \delta$$



**Path-Length Energy-Dependent Beamlne**

$$\Delta T \longrightarrow \Delta E \longrightarrow \Delta X \longrightarrow \Delta T$$

# Short bunches by Magnetic Compression



$$E(z) = E_0 + eV_0 \cos(\varphi + 2\pi z/\lambda)$$

$$\delta \equiv \frac{\Delta E}{E} \approx \dots$$

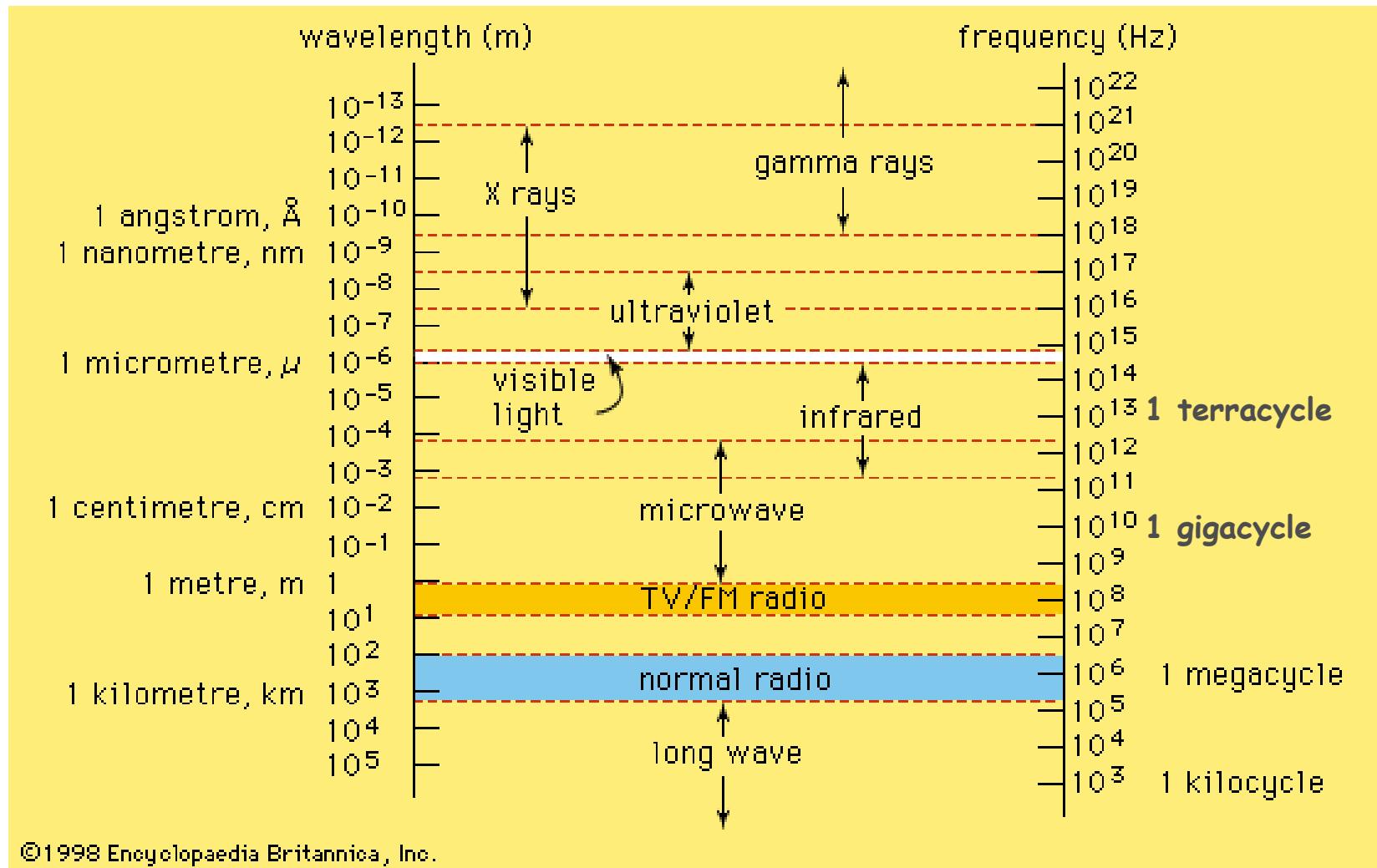
$$\delta_0 \frac{E_0}{E} + \left(1 - \frac{E_0}{E}\right) \left[ \frac{\cos(\varphi + \Delta\varphi) - (2\pi z/\lambda) \sin(\varphi + \Delta\varphi)}{\cos(\varphi)} - 1 \right]$$

$$k(\varphi) \equiv \frac{\partial \delta}{\partial z} = -\frac{2\pi}{\lambda} \left(1 - \frac{E_0}{E}\right) \frac{\sin(\varphi + \Delta\varphi)}{\cos(\varphi)} \quad \text{'chirp'}$$

final bunch length and energy spread...

$$\sigma_z = \sqrt{(1 + kR_{56})^2 \sigma_{z0}^2 + R_{56}^2 \sigma_\delta^2 E_0^2 / E^2} \quad , \quad \sigma_\delta = \sqrt{k^2 \sigma_{z0}^2 + \sigma_\delta^2 E_0^2 / E^2}$$

# Bunch Frequency Spectrum



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# Coherent Synchrotron Radiation in Magnetic Chicane

- Powerful radiation generates energy spread in bends
- Energy spread breaks achromatic system
- Causes emittance growth (short bunch worse)

