

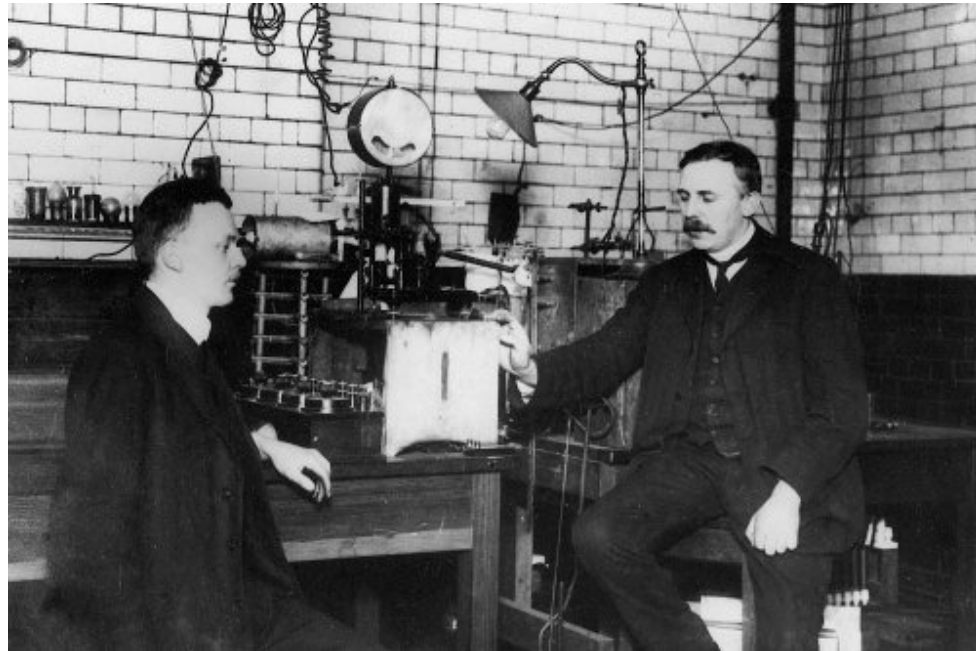


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# Rutherford scattering

## and the Discovery of the nucleus



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Rutherford (1911)

Philos. Mag., Series 6. **21** (125): 669–688.

Geiger & Marsden (1913)

Philos. Mag., Series 6. **25** (148): 604–623.

**BACKGROUND**

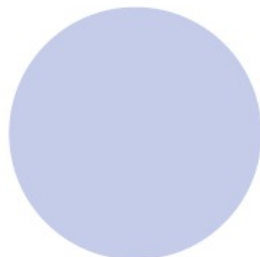
# History of the atom model

- 1803, **Dalton** atom model: indivisible balls of different types
- 1897, **Thomson**'s discovery of the electron  
⇒ 1904, (Kelvin &) Thomson's "plum pudding" model:
- 1904, **Nagaoka**'s "Saturn" model: nucleus with ring
- 1909-1913, **Geiger–Marsden** experiments  
⇒ 1911, **Rutherford**'s "nuclear" model: nucleus with satellites
- 1913, **Bohr**'s "planetary" model with discrete and stable orbits
- 1919, Rutherford showed  $H^+$  (proton) is present in other nuclei
- 1926, **Schrödinger**'s quantum mechanical wave equation
- 1932, **Chadwick**'s discovery of the neutron
- 1964, **Gell-Mann-Zweig**'s quark model  
1968, SLAC deep inelastic scattering  
⇒ 1969, **Feynman**'s parton model: quarks, gluons with pdf's

# A HISTORY OF THE ATOM: THEORIES AND MODELS

How have our ideas about atoms changed over the years? This graphic looks at atomic models and how they developed.

## SOLID SPHERE MODEL



### JOHN DALTON



1803

Dalton drew upon the Ancient Greek idea of atoms (the word 'atom' comes from the Greek 'atomos' meaning indivisible). His theory stated that atoms are indivisible, those of a given element are identical, and compounds are combinations of different types of atoms.

+ RECOGNISED ATOMS OF A PARTICULAR ELEMENT DIFFER FROM OTHER ELEMENTS

- ATOMS AREN'T INDIVISIBLE - THEY'RE COMPOSED FROM SUBATOMIC PARTICLES

## PLUM PUDDING MODEL



### J. J. THOMSON



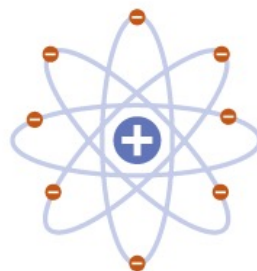
1904

Thomson discovered electrons (which he called 'corpuscles') in atoms in 1897, for which he won a Nobel Prize. He subsequently produced the 'plum pudding' model of the atom. It shows the atom as composed of electrons scattered throughout a spherical cloud of positive charge.

+ RECOGNISED ELECTRONS AS COMPONENTS OF ATOMS

- NO NUCLEUS; DIDN'T EXPLAIN LATER EXPERIMENTAL OBSERVATIONS

## NUCLEAR MODEL



### ERNEST RUTHERFORD



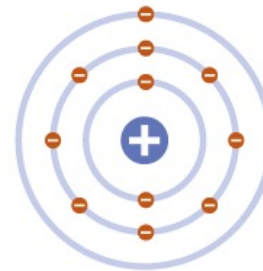
1911

Rutherford fired positively charged alpha particles at a thin sheet of gold foil. Most passed through with little deflection, but some deflected at large angles. This was only possible if the atom was mostly empty space, with the positive charge concentrated in the centre: the nucleus.

+ REALISED POSITIVE CHARGE WAS LOCALISED IN THE NUCLEUS OF AN ATOM

- DID NOT EXPLAIN WHY ELECTRONS REMAIN IN ORBIT AROUND THE NUCLEUS

## PLANETARY MODEL



### NIELS BOHR



1913

Bohr modified Rutherford's model of the atom by stating that electrons moved around the nucleus in orbits of fixed sizes and energies. Electron energy in this model was quantised; electrons could not occupy values of energy between the fixed energy levels.

+ PROPOSED STABLE ELECTRON ORBITS; EXPLAINED THE EMISSION SPECTRA OF SOME ELEMENTS

- MOVING ELECTRONS SHOULD EMIT ENERGY AND COLLAPSE INTO THE NUCLEUS; MODEL DID NOT WORK WELL FOR HEAVIER ATOMS

## QUANTUM MODEL



### ERWIN SCHRÖDINGER



1926

Schrödinger stated that electrons do not move in set paths around the nucleus, but in waves. It is impossible to know the exact location of the electrons; instead, we have 'clouds of probability' called orbitals, in which we are more likely to find an electron.

+ SHOWS ELECTRONS DON'T MOVE AROUND THE NUCLEUS IN ORBITS, BUT IN CLOUDS WHERE THEIR POSITION IS UNCERTAIN

+ STILL WIDELY ACCEPTED AS THE MOST ACCURATE MODEL OF THE ATOM

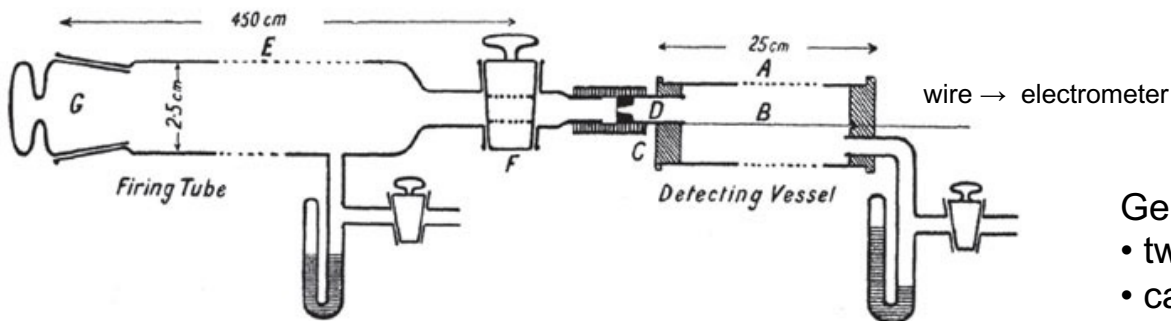


# **METAL FOIL EXPERIMENTS**

## **1909-1910**

# Build up to the Rutherford model

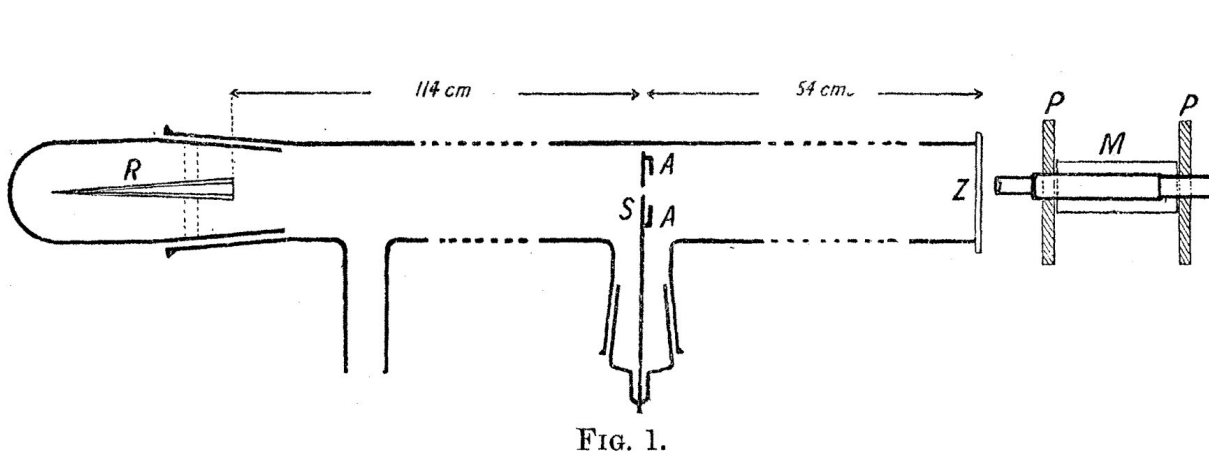
- **1899, Rutherford discovered alpha particles**
  - radiated from radium, uranium samples
  - scintillates on fluorescent screen (e.g. phosphorescent zinc-sulphide)
    - ⇒ count flashes by eye with microscope in a dark room
  - ionizes air
    - ⇒ apply electric field to create electric pulse (Townsend discharge)
    - ⇒ early version Geiger alpha particle counter (1908)
- **measure  $\alpha$ 's charge-to-mass ratio  $q/m$  to confirm whether it is  $\text{He}^{2+}$  with  $+2e$** 
  - measure number of  $\alpha$ 's with Geiger's counter to cross-check scintillation method
    - ⇒ noticed unreliable ionization measurements:
      - large deflections due to scattering in air !
    - ⇒ Rutherford asked Geiger to study  $\alpha$  scattering in metal



Geiger's 1908 counter

- two electrodes in a glass tube
- can only measure small deflections

# Metal foil experiment (1908)



- "radium emanation": radon gas
- radium source (R)
  - slit (S) of 0.9 mm
  - slot (AA) for metal foil
  - phosphorescent screen (Z)
  - microscope (M)

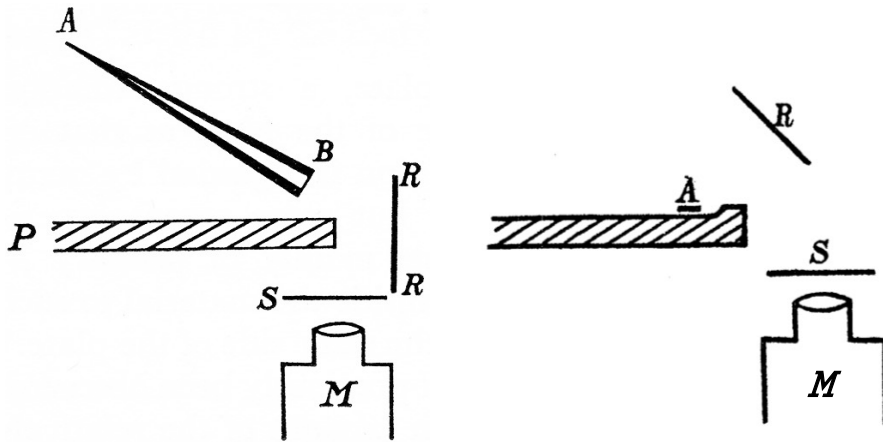
## 1908 Geiger experiment: deflection by scattering ?

1. without air (pumped out) ⇒ neat patch of light on screen
2. with air ⇒ more diffuse image
3. with metal foil in AA ⇒ even more spread out

⇒ matter, as well air, can scatter alpha particles !

BUT: only small angles of deflections can be measured

# Metal foil experiment (1909)



- radium source in glass tube (AB), or (A) naked
- metal foil, or “reflector” (R)
- phosphorescent screen (S)
- lead plate (P) protecting (S) from source
- microscope (M)

## 1909 Geiger-Marsden experiment: large angles ?

1. pointing away tube (AB)  $\Rightarrow$  still backscattering from air molecules
2. pointing tube (AB) at foil (R)  $\Rightarrow$  increase in backscattered  $\alpha$ 's
3. different metal foils (Al, Au, ...)  $\Rightarrow$  higher atomic mass reflects more  $\alpha$ 's

$\Rightarrow$  significantly large angles of scattering of  $\alpha$ 's through matter !

BUT: could not accurately measure number of  $\alpha$ 's to get angular distribution



# Metal foil experiment (1910)

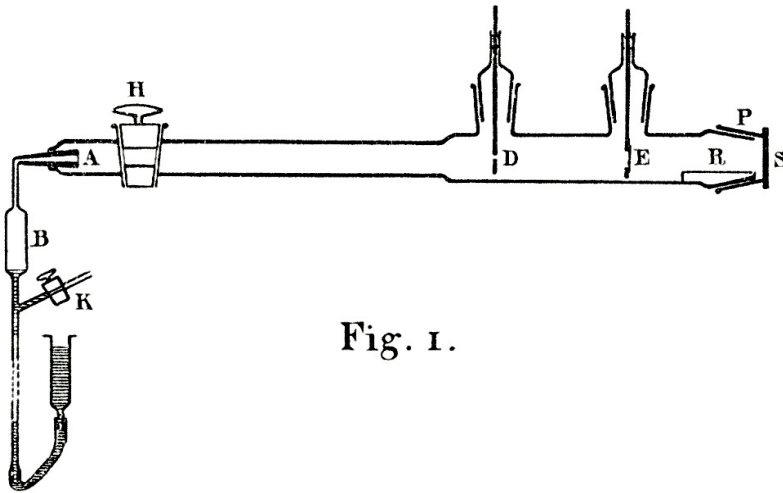


Fig. 1.

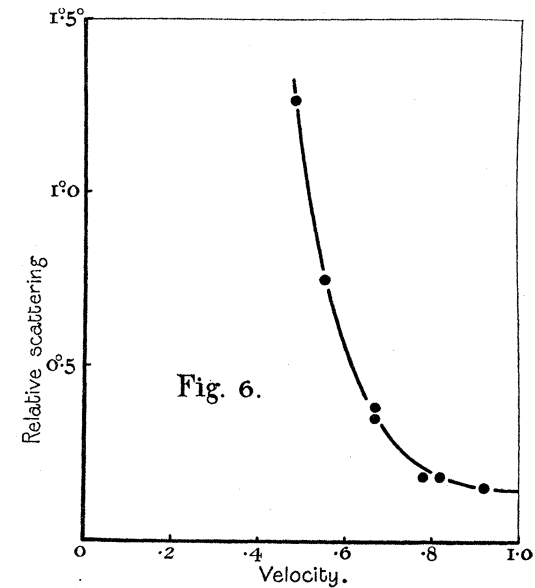
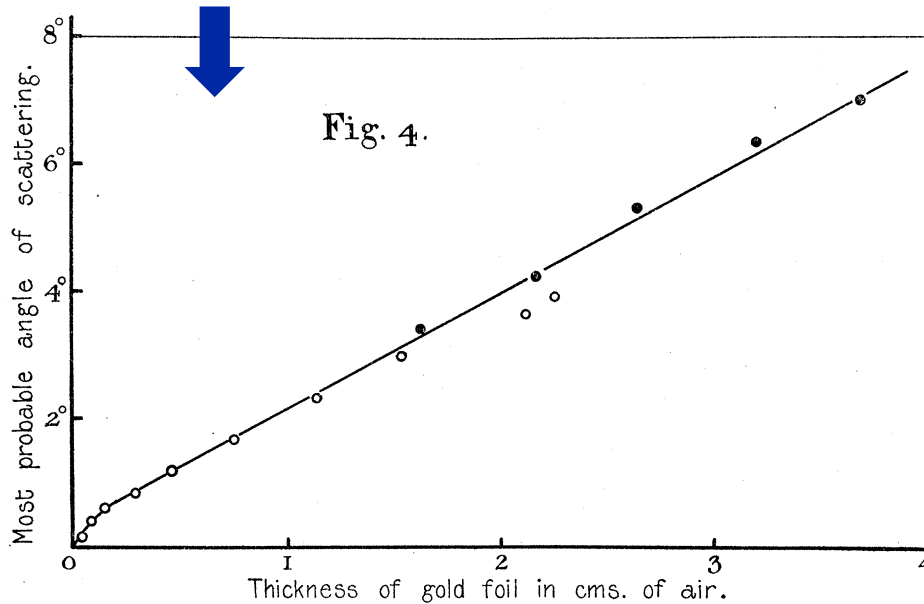
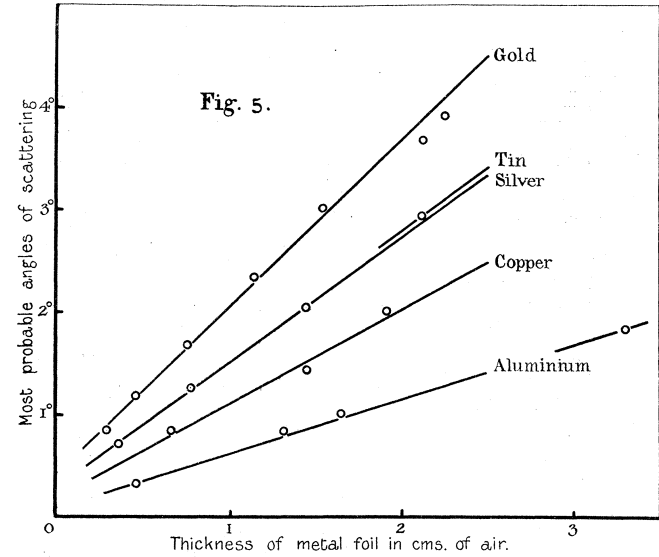
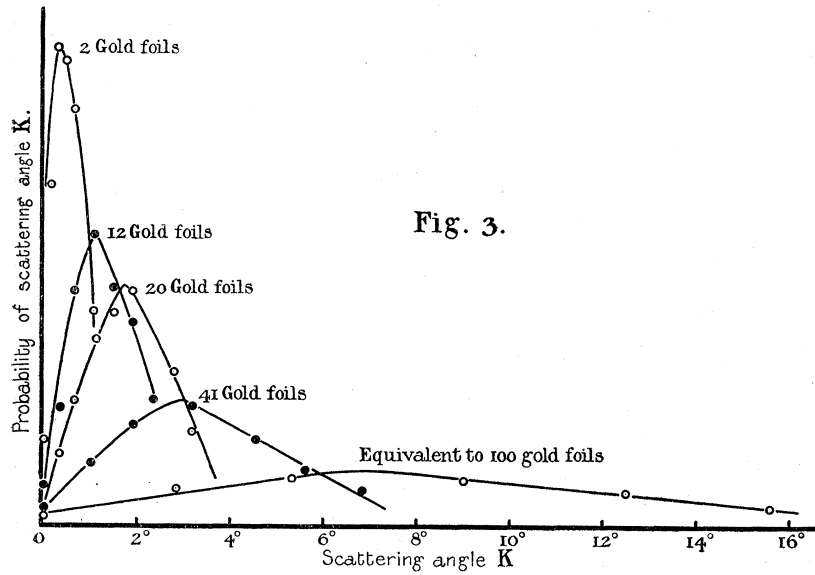
- pressure pump with mercury
- radon gas source in bulb (B)
- narrow circular opening (D),  $< 1$  mm  
⇒ narrow beam
- slots at (D), (E) for metal foil
- phosphorescent screen (S)
- microscope with vertical millimeter scale  
⇒ measure angles precisely

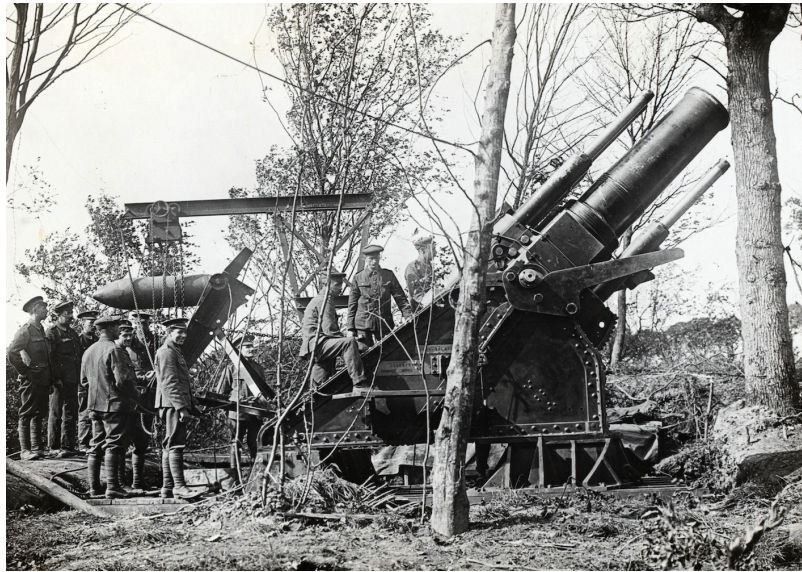
## 1910 Geiger experiment: most probable deflection angle ?

1. increase thickness ⇒ most probable deflection angle increases
2. increase atomic mass ⇒ most probable deflection angle  $\propto$  atomic mass
3. increase velocity ⇒ most probable deflection angle decreases
4. measure backscattering ⇒ very small (1/20,000 for a gold foil)

⇒ Rutherford's idea of a central, positive charge in the atom

# Metal foil experiment (1910): results

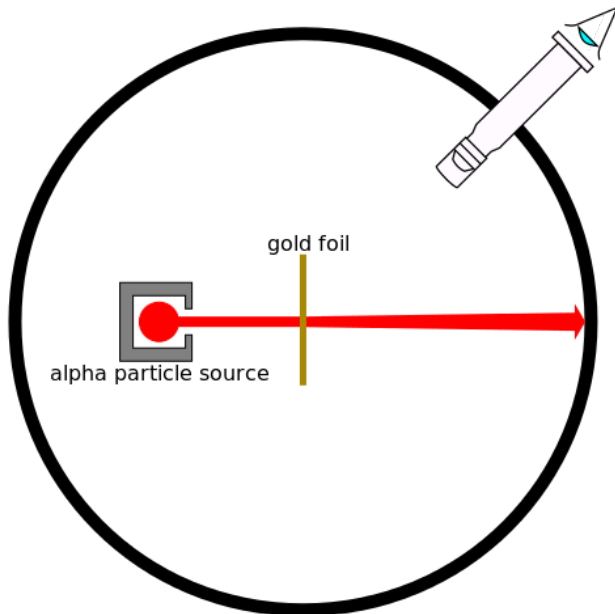
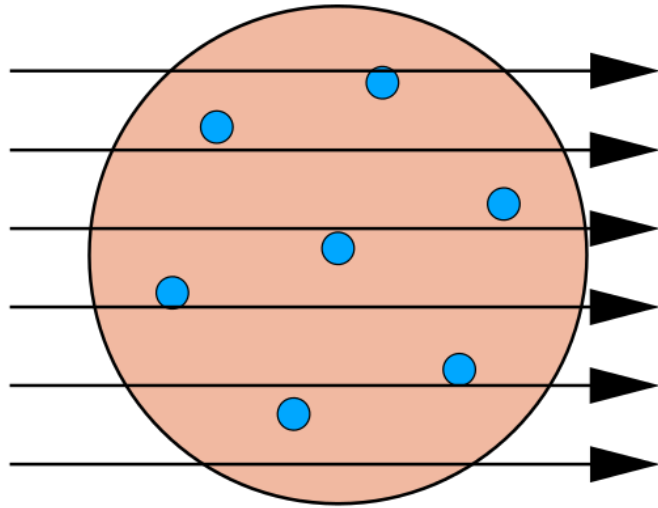




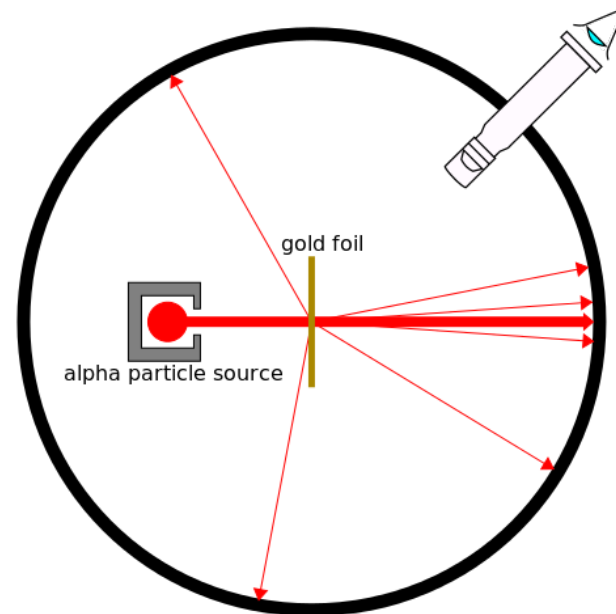
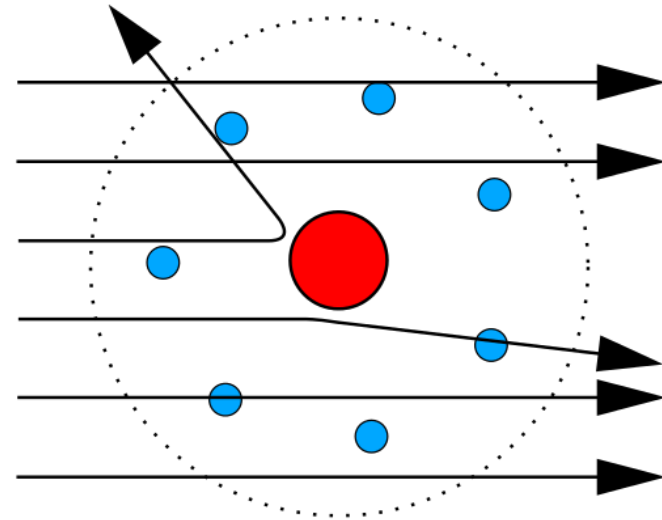
“It was quite the most incredible event that has ever happened to me in my life. **It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.** On consideration, I realized that this scattering backward must be the result of a **single collision**, and when I made calculations I saw that it was **impossible to get anything of that order of magnitude unless you took a system in which the greater part of the mass of the atom was concentrated in a minute nucleus.** It was then that I had the idea of an atom with a minute massive centre, carrying a charge.”

E. Rutherford & J.A. Ratcliffe (1938). *Forty Years of Physics*.

# THOMSON MODEL



# RUTHERFORD MODEL



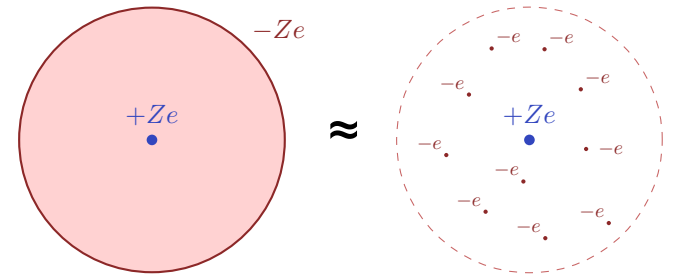
# OBSERVED RESULT

# **RUTHERFORD MODEL**

# Rutherford's 1911 paper

- puts forward new atom model with central charge
- compares to Thomson model
- calculates deflection angle in collision
- calculates change in velocity in collision
- compares one large deflection vs. multiple small angle scatterings (“compound scattering”)
- compares his theory with previous experimental results
  - a) large (“diffuse”) scatter angles of  $\alpha$ 's
  - b) most probable deflection angle  $\propto$  atomic mass
  - c) average scatter angles
  - d)  $\beta$  scattering (Crowther)

# Rutherford scattering (1): simplified model



- consider **atom** of radius  $R \sim 10^{-10}$  m with
  - **central  $+Ze$  charge**
  - **in a sphere of uniform  $-Ze$  charge**
  - shown later: “corpuscular” electrons will not change result
- electric field and potential *inside* this atom from a distance  $r$  from the center:

$$E = +Ze \frac{1}{r^2} - Ze \frac{r}{R^3}, \quad U = +Ze \left( \frac{1}{r} - \frac{3}{2R} + \frac{3}{2R^3} \right)$$

- consider particle with charge  $q$  is shot *directly* at center N with velocity  $v$   
 $\Rightarrow$  it will come to rest at a **distance  $d^*$**   $\ll R$ :

$$\frac{1}{2}mv^2 = +qZe \left( \frac{1}{d^*} - \frac{3}{2R} + \frac{d^{*2}}{2R^3} \right) \approx \frac{qZe}{d^*}$$

$$\Rightarrow d^* \approx \frac{2qZe}{mv^2}$$



# Rutherford scattering (2): angle of deviation

- consider **particle scattered of central charge** with impact parameter  $b$   
 $\Rightarrow$  **hyperbolic orbit** with central charge in external focus
- conservation of angular momentum & energy:

$$bv_i = |NA|v^*, \quad \frac{mv_i^2}{2} = \frac{mv^{*2}}{2} - \frac{qZe}{|NA|}$$

$$\Rightarrow v^{*2} = v_i^2 \left(1 - \frac{d^*}{|NA|}\right)$$

$$\Rightarrow b^2 = |NA|(|NA| - d^*)$$

- using hyperbola's properties:

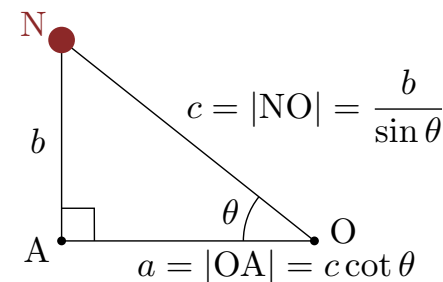
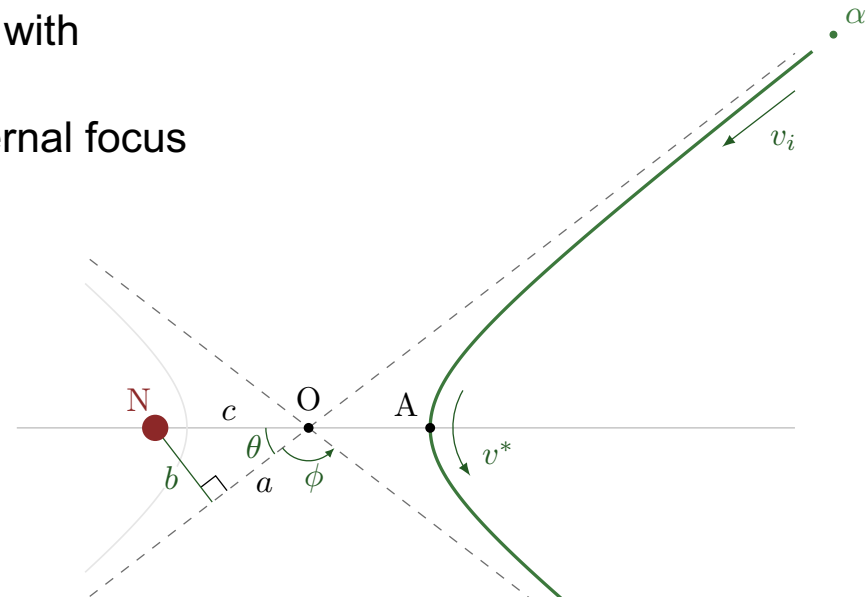
$$d^* = \frac{2qZe}{mv^2} \quad |NA| = |NO| + |OA| = b \cot(\theta/2)$$

$$\Rightarrow d^* = 2b \cot \theta$$

- angle of deviation  $\phi$**  =  $\pi - 2\theta$  becomes

$$\cot(\phi/2) = \frac{2b}{d^*}$$

$$\Rightarrow \text{eccentricity } e \equiv \frac{c}{a} = \frac{1}{\cos \theta}$$





# Rutherford scattering (3): transversing a thin foil

- consider **particle transversing a thin foil of matter** with a thickness  $t$  and  $n$  atoms per unit volume

⇒ number of collisions with atoms of radius  $R$ :

$$\pi R^2 n t$$

⇒ probability  $m$  of entering atom with an impact parameter  $b$ :

$$m = \pi b^2 n t$$

⇒ probability  $dm$  of entering atom with an impact parameter between  $b$  and  $b+db$ :

$$dm = 2\pi n t b db$$

or fraction  $dm$  of particles deflected between  $\phi$  and  $\phi + d\phi$ :

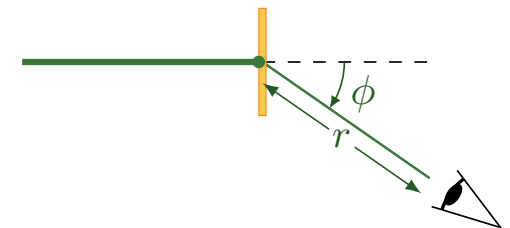
$$dm = \frac{\pi}{4} n t d^2 \cot(\phi/2) \csc^2(\phi/2) d\phi$$

$$\Rightarrow \begin{cases} \rho = \frac{\pi}{4} n t d^2 \cot^2(\phi/2) & \text{fraction deflected with } > \phi \\ \rho = \frac{\pi}{4} n t d^2 [\cot^2(\phi_1/2) - \cot^2(\phi_2/2)] & \text{fraction deflected in } [\phi_1, \phi_2] \end{cases}$$

- consider a **beam of  $L$   $\alpha$ -particles transversing a thin foil**

⇒ number of particles,  $N(\phi)$ , falling into an area  $dA$  at a distance  $r$  of incidence point:

$$\begin{aligned} N(\phi) &= L \frac{dm}{dA} = \frac{L(\pi/4) n t d^2 \cot(\phi/2) \csc^2(\phi/2) d\phi}{2\pi r^2 \sin \phi d\phi} \\ &= \frac{L n t d^2}{16 r^2 \sin^4(\phi/2)} \end{aligned}$$



# Rutherford scattering (4): main result

- number of particles per unit area,  $N$ , scattered at angle  $\phi$  after transversing thin foil:

$$N(\phi) = \frac{Lntd^{*2}}{16r^2 \sin^4(\phi/2)} \quad d^* = \frac{2qZe}{mv^2}$$

⇒ proportional to

- $1/\sin^4(\phi/2)$  or for small  $\phi$  small:  $\phi^{-4}$
- thickness  $t$  of foil (assumed small)
- scattered particle charge  $q^2$  and central charge  $(Ze)^2$
- $1/(mv^2)^2$

- note: if positive charge *were* due to individual, “corpuscular” charges instead,

$$N(\phi) \propto Ze^2$$

so for gold with  $Z \sim 100$ , you would expect 100 times fewer particles at the same scattering angle  $\phi$

- ⇒ the positively charged particles should have very small masses
- ⇒ difficult to find large scattering angles at all !

# Rutherford scattering (5): change of velocity ?

- assume no radiation
- incoming particle  $m$  scatters off atom  $M$ :
  - scattered particle  $\vec{v}_i \rightarrow \vec{v}_f$
  - atom **recoils**  $0 \rightarrow \vec{v}_r$
- conservation of momentum and energy:

$$\begin{cases} (Mv_r)^2 = (mv_i)^2 + (mv_f)^2 - 2m^2v_iv_f \cos \phi \\ Mv_r^2 = mv_i^2 - mv_f^2 \end{cases}$$

defining  $K \equiv M/m$  and  $\rho \equiv v_f/v_i$ :

$$\rho = \frac{\sqrt{K^2 + 2 \cos^2 \phi - 1}}{K + 1}$$

e.g.  $\phi = 90^\circ$  with  $\alpha$ -particles:

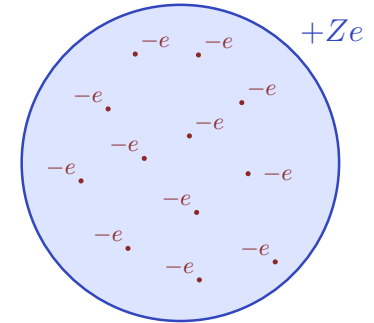
$$\rho = \sqrt{\frac{K - 1}{K + 1}} = \begin{cases} 0.979 & \text{for Au } (K = 197/4) \\ 0.86 & \text{for Al } (K = 27/4) \end{cases}$$

**$\Rightarrow$  change in velocity smaller for larger atomic weight and negligible for  $\beta$  particles !**

# Rutherford scattering (6): effect of individual electrons

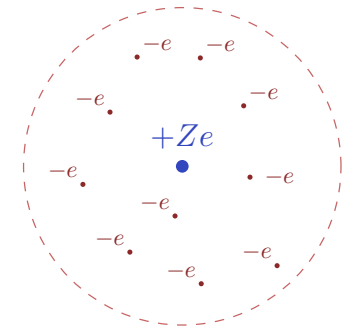
- average scattering angles in **Thomson model**:

$$\left\{ \begin{array}{l} \phi_+ = \frac{\pi d^*}{8 R} \\ \phi_- = \frac{8 d^*}{5 Z R} \sqrt{\frac{3Z}{2}} \end{array} \right. \quad \begin{array}{l} \text{sphere, uniformly charge } +Ze \\ Z \text{ electrons, uniformly distributed} \end{array}$$



- average scattering angles in **Rutherford model**:

$$\left\{ \begin{array}{l} \phi_+ = \frac{3\pi d^*}{8 R} \\ \phi_- = \frac{8 d^*}{5 Z R} \sqrt{\frac{3Z}{2}} \end{array} \right. \quad \begin{array}{l} \text{central charge of } +Ze \\ Z \text{ electrons, uniformly distributed} \end{array}$$



⇒ combining average scattering angles (as error propagation)

$$\sqrt{\phi_+^2 + \phi_-^2} \approx \frac{b}{2R} \sqrt{5.54 + \frac{15.4}{Z}}$$

- ⇒ **effect of electrons negligible for heavy atoms:**  
 electrons' electric field more evenly distributed in heavy atoms (with larger Z)

# Rutherford scattering (7): compound vs. single scattering

- what is the most likely way to get a large scattering angle ?
  1. multiple small angle scatterings: “compound” scattering
  2. one single large angle scattering
  3. ~~several large angle scatterings~~ → probability vanishingly small !
- average scattering angle after transversing matter of thickness  $t$ :  $\theta_t = \frac{3\pi d^*}{8} \sqrt{\pi n t}$
- **probabilities  $p$  a particle gets scattered at an angle greater than  $\phi$ :**

$$\begin{cases} p_c = e^{-\phi^2/\theta_t^2} & \text{by compound scattering} \\ p_s = \frac{\pi}{4} d^{*2} n t \cot^2(\phi/2) & \text{by single scattering} \end{cases}$$

⇒ comparing probabilities:

$$p_s \ln p_c = -0.181 \phi^2 \cot^2(\phi/2) \approx -0.72$$

$$\Rightarrow \text{e.g. } \begin{cases} p_c = 0.24 & \text{if } p_s = 0.50 \\ p_c = 0.0004 & \text{if } p_s = 0.10 \end{cases}$$

⇒ **any scattering angle is always more probable due to one collision, and the less likely a scattering angle, the more probable it is due to a single collision**

# Rutherford scattering (8): effect of atomic weight

- assuming

$$Ze \propto A$$

we know from the main result that number of scattered particles at  $\phi$ :

$$N(\phi) \propto ntA^2$$

- compare to stopping power of an  $\alpha$ -particle in an atom (Bragg):

$$-\left\langle \frac{dE}{dX} \right\rangle \propto \frac{1}{\sqrt{A}}$$

**$\Rightarrow N(\phi)/A^{3/2}$  should be constant**

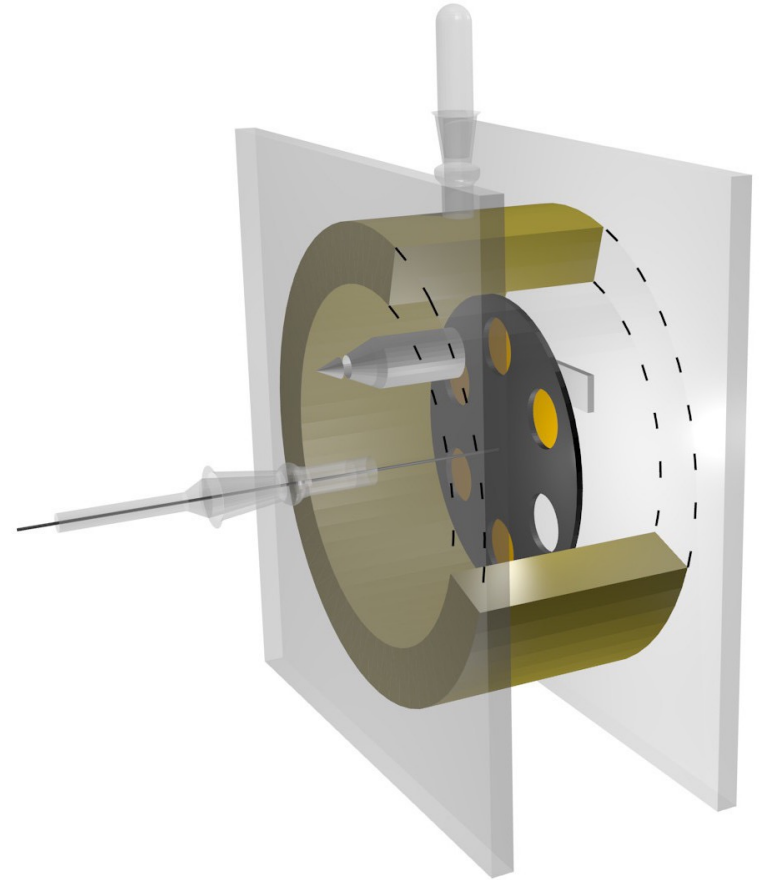
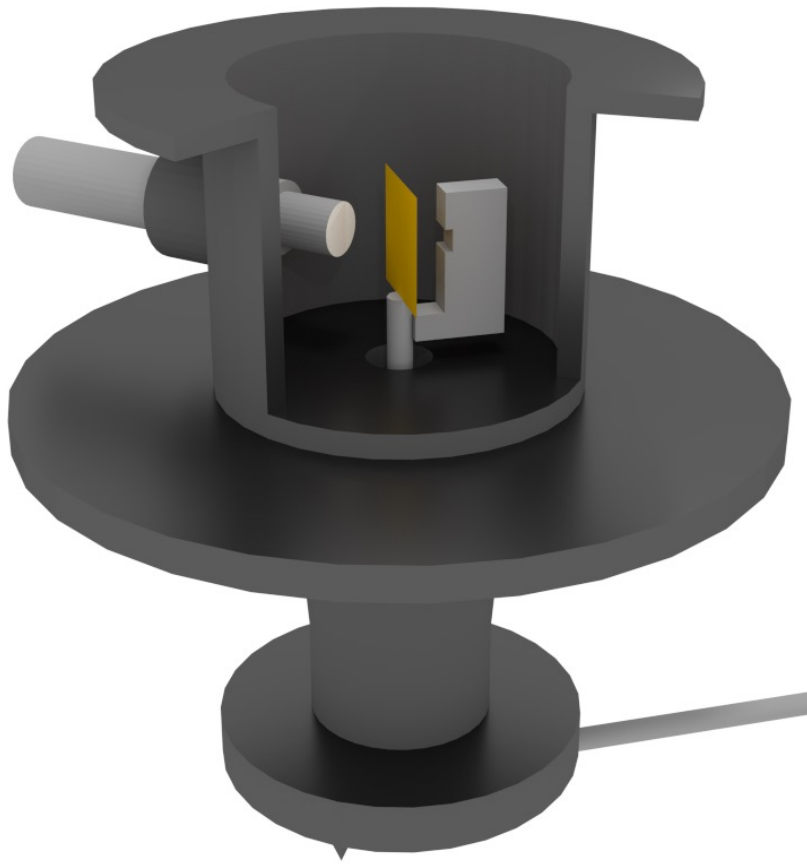
- previous experimental result by Geiger:
- using previous calculations and Geiger's results, one finds  $Z \sim 100$  for gold (actual value  $Z = 79$ )

Metal.	Atomic weight.	$z.$	$z/A^{3/2}.$
Lead .....	207	62	208
Gold .....	197	67	242
Platinum .....	195	63	232
Tin .....	119	34	226
Silver .....	108	27	241
Copper .....	64	14.5	225
Iron .....	56	10.2	250
Aluminium ...	27	3.4	243
Average			233

# **METAL FOIL EXPERIMENTS**

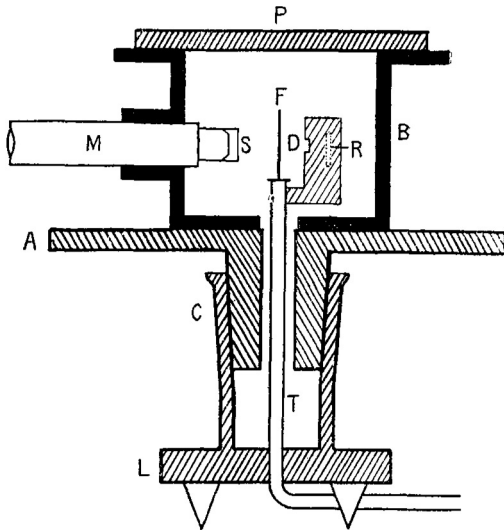
## **1913**

# Metal foil experiment (1913): in 3D

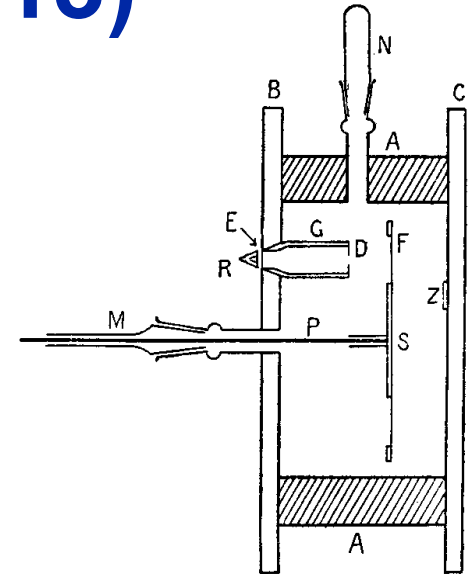




# Metal foil experiment (1913)



- rotatable metal box (B)
- tube (T)
- radium source (R)
- diaphragm (D)
- metal foil (F)
- phosphorescent screen (S)
- microscope (M)



## 1913 Geiger-Marsden experiment: test Rutherford's predictions !

1. vary scattering angle  $1/\sin^4(\phi/2)$  up to  $150^\circ$
2. vary foil thickness
3. vary atomic mass
4. vary  $\alpha$ 's velocity
5. measure angular distribution

⇒ all in accordance with Rutherford's "nuclear" model !

# Metal foil experiment (1913): results

TABLE I.—Variation of Scattering with Angle. (Example of a set of measurements.) Silver Foil. Time elapsed since filling of emanation tube, 51 hours. Correction for decay, 0.683.

Angle $\phi$ .	Scintillations per minute.				$\frac{1}{\sin^4 \phi/2}$	$N \times \sin^4 \phi/2$ .
	Without foil.	With foil.	Corrected for effect without foil.	Corrected for decay, N.		
150...	0.2	4.95	4.75	6.95	1.15	6.0
135 ..	2.6	8.3	5.7	8.35	1.38	6.1
120...	3.8	10.3	6.5	9.5	1.79	5.3
105...	0.6	10.6	10.0	14.6	2.53	5.8
75...	0.0	28.6	28.6	41.9	7.25	5.8
60...	0.3	69.2	68.9	101	16.0	6.3

Variation of Scattering with Thickness for Different Atomic Weights

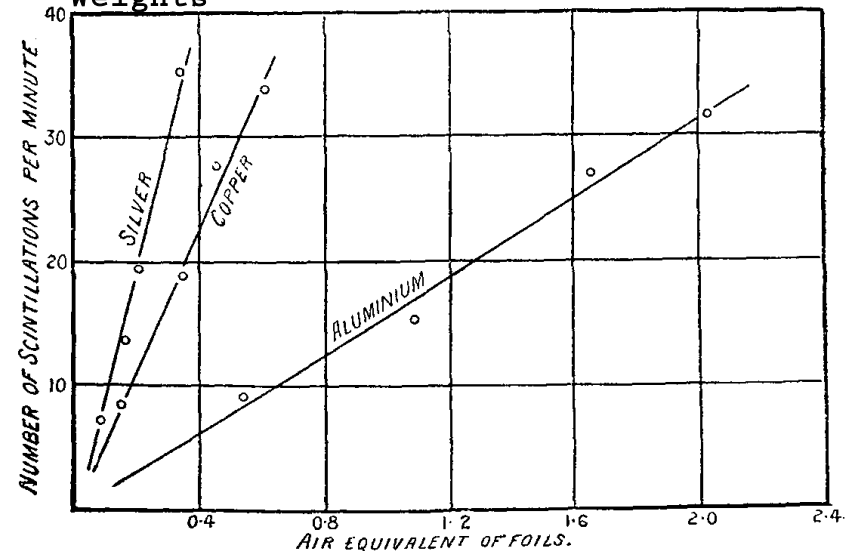


TABLE IV.

Variation of Scattering with Atomic Weight. (Example of a set of measurements.)

I. Substance.	II. Atomic weight, A.	III. Air equivalent in cm.	IV. Number of scintillations per minute corrected for decay.	V. Number N of scintillations per cm. air equivalent.	VI. $A^{3/2}$ .	VII. $N \times A^{2/3}$ .
Gold .....	197	.229	133	581	2770	0.21
Tin .....	119	.441	119	270	1300	0.21
Silver .....	107.9	.262	51.7	198	1120	0.18
Copper .....	63.6	.616	71	115	507	0.23
Aluminium..	27.1	2.05	71	34.6	141	0.24

TABLE VII.

Variation of Scattering with Velocity.

I. Number of sheets of mica.	II. Range R of $\alpha$ particles after leaving mica.	III. Relative values of $1/v^4$ .	IV. Number N of scintillations per minute.	V. $Nv^4$ .
0	5.5	1.0	24.7	25
1	4.76	1.21	29.0	24
2	4.05	1.50	33.4	22
3	3.32	1.91	44	23
4	2.51	2.84	81	28
5	1.84	4.32	101	23
6	1.04	9.22	255	28

# Metal foil experiment (1913): summary

## *Summary.*

The experiments described in the foregoing paper were carried out to test a theory of the atom proposed by Prof. Rutherford, the main feature of which is that there exists at the centre of the atom an intense highly concentrated electrical charge. The verification is based on the laws of scattering which were deduced from this theory. The following relations have been verified experimentally:—

(1) The number of  $\alpha$  particles emerging from a scattering foil at an angle  $\phi$  with the original beam varies as  $1/\sin^4 \phi/2$ , when the  $\alpha$  particles are counted on a definite area at a constant distance from the foil. This relation has been tested for angles varying from  $5^\circ$  to  $150^\circ$ , and over this range the number of  $\alpha$  particles varied from 1 to 250,000 in good agreement with the theory.

(2) The number of  $\alpha$  particles scattered in a definite direction is directly proportional to the thickness of the scattering foil for small thicknesses. For larger thicknesses the decrease of velocity of the  $\alpha$  particles in the foil causes a somewhat more rapid increase in the amount of scattering.

(3) The scattering per atom of foils of different materials varies approximately as the square of the atomic weight. This relation was tested for foils of atomic weight from that of carbon to that of gold.

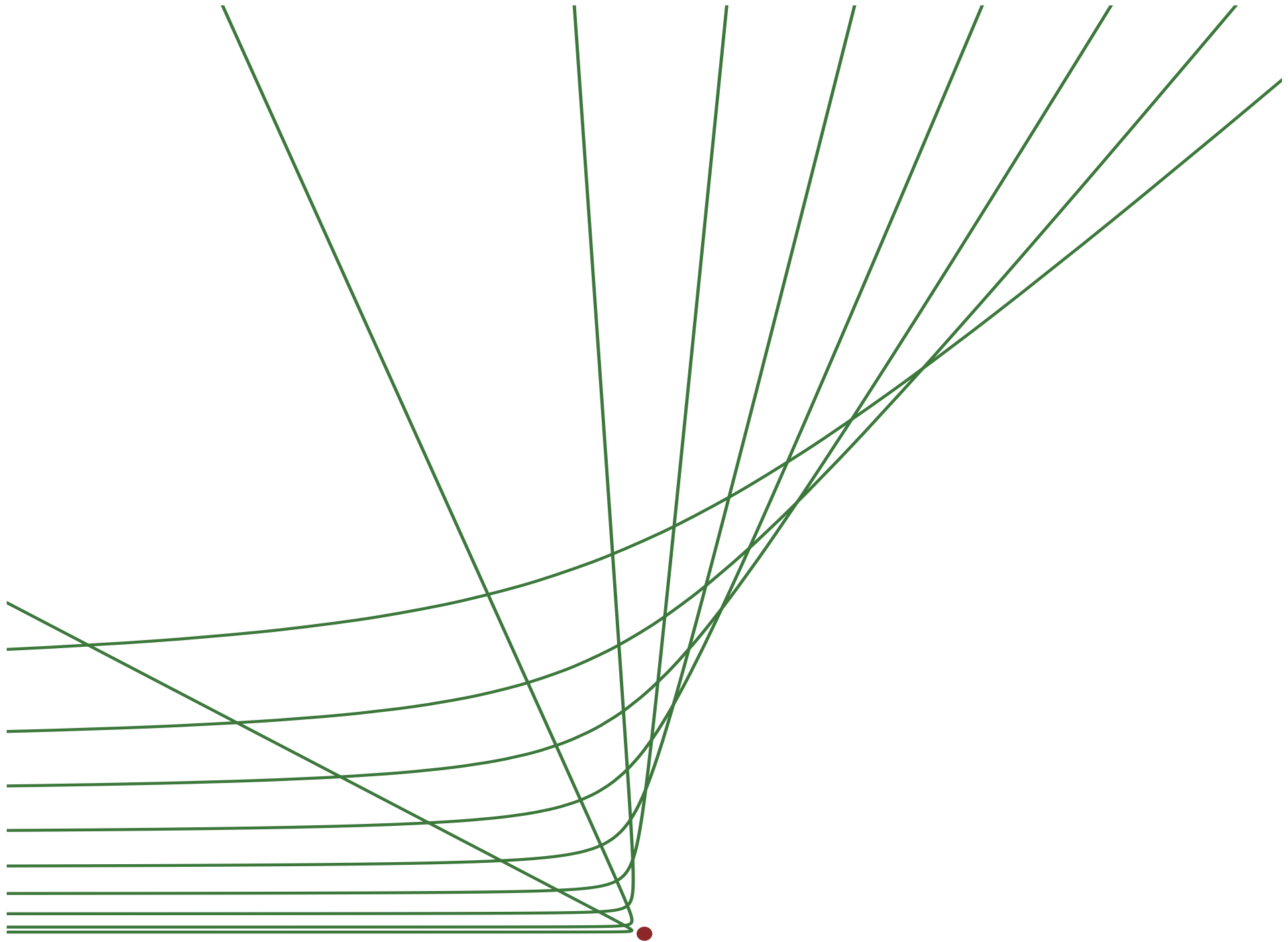
(4) The amount of scattering by a given foil is approximately proportional to the inverse fourth power of the velocity of the incident  $\alpha$  particles. This relation was tested over a range of velocities such that the number of scattered particles varied as 1 : 10.

(5) Quantitative experiments show that the fraction of  $\alpha$  particles of Ra C, which is scattered through an angle of  $45^\circ$  by a gold foil of 1 mm. air equivalent ( $2.1 \times 10^{-5}$  cm.), is  $3.7 \times 10^{-7}$  when the scattered particles are counted on a screen of 1 sq. mm. area placed at a distance of 1 cm. from the scattering foil. From this figure and the foregoing results, it can be calculated that the number of elementary charges composing the centre of the atom is equal to half the atomic weight.

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- [https://en.wikipedia.org/wiki/Geiger–Marsden\\_experiment](https://en.wikipedia.org/wiki/Geiger–Marsden_experiment)
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**EXTRA**

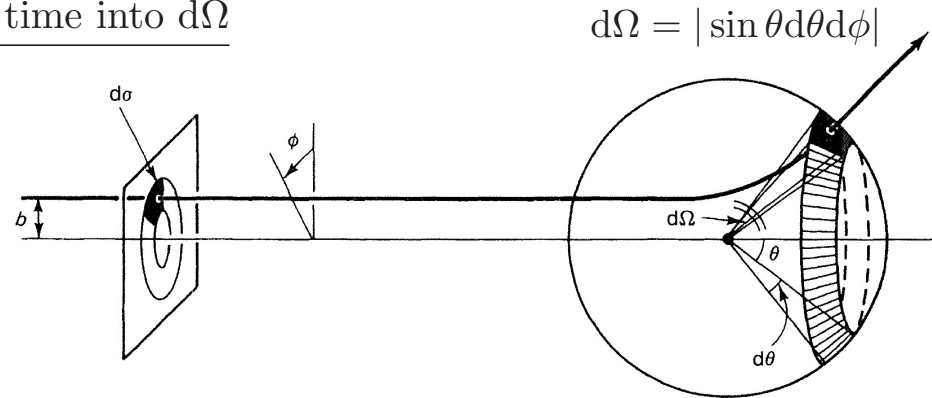


# Rutherford scattering as a cross section

$$d\sigma(\Omega) = \frac{\text{number of particles scattered per unit time into } d\Omega}{\text{beam flux}}$$

$$= |bdbd\phi|$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \frac{db}{d\theta} \right|$$



- **Rutherford scattering:** elastic scattering by charge  $q_1$  off stationary charge  $q_2$ :

$$b = \frac{q_1 q_2}{2E} \cot(\theta/2)$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \left( \frac{q_1 q_2}{4E \sin^2(\theta/2)} \right)^2$$

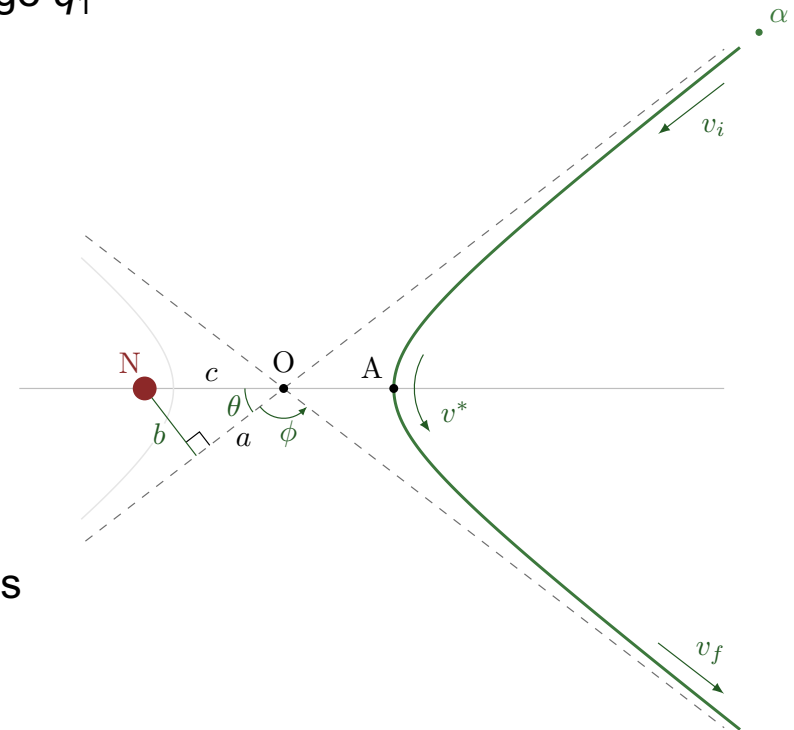
note: total cross section  $\sigma$  will be infinite !

- relativistic version (recoil still neglected):

## Mott scattering

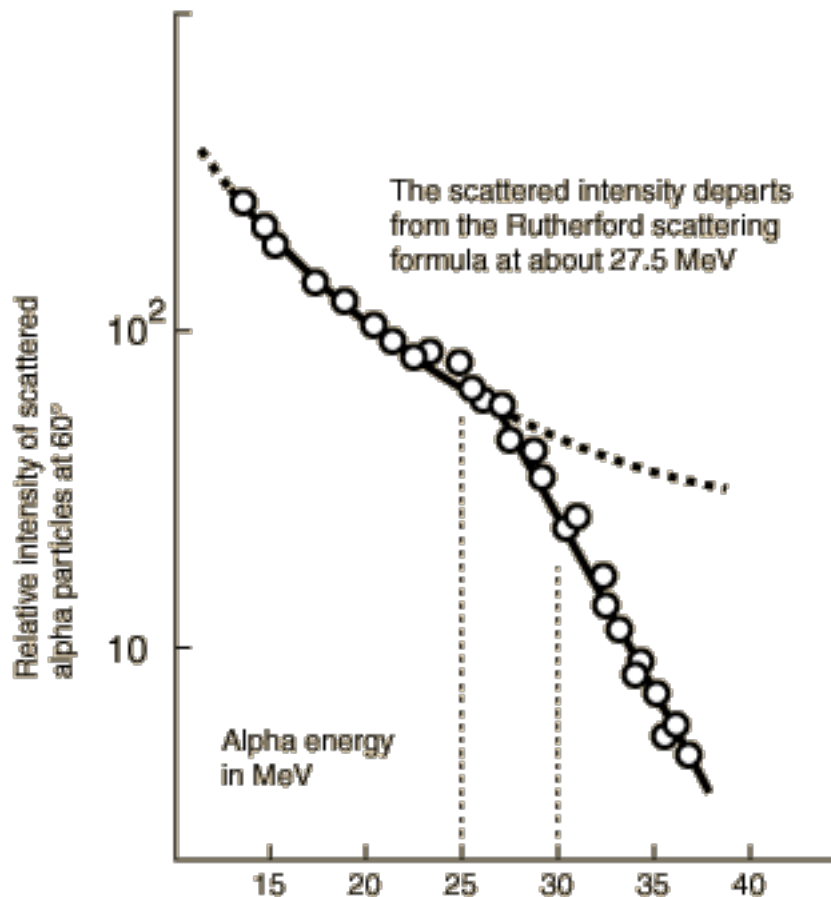
- $e^-p^+ \rightarrow e^-p^+$ , taking into account recoil and proton's substructure with form factors:

## Rosenbluth formula

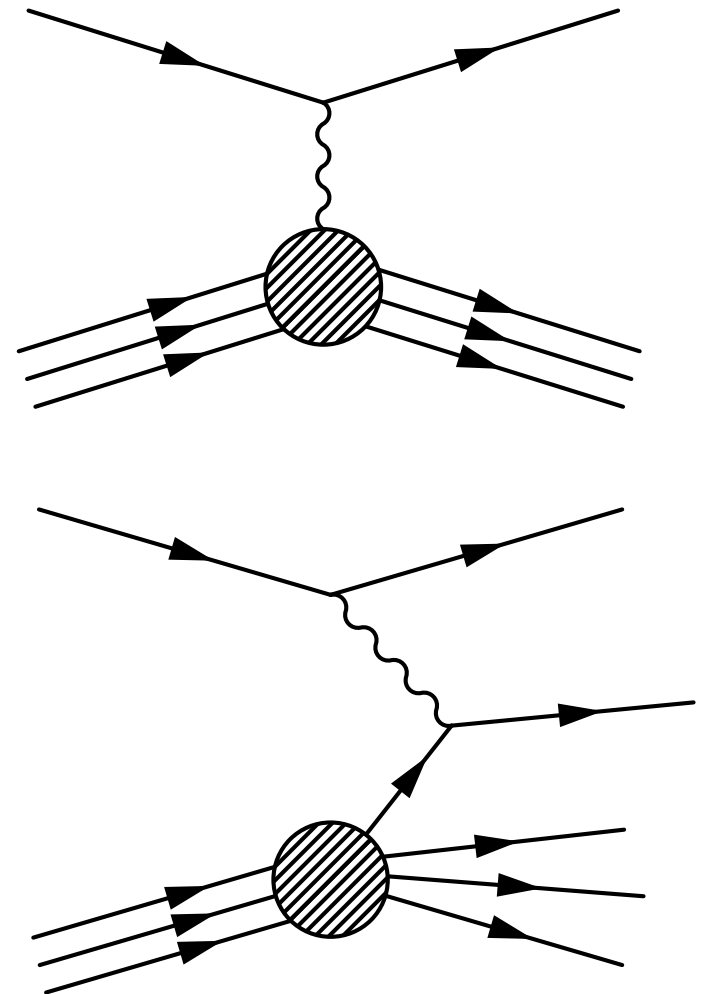


# Deep inelastic scattering

- collide  $\alpha$ -particles or electrons into protons until it breaks to probe its substructure



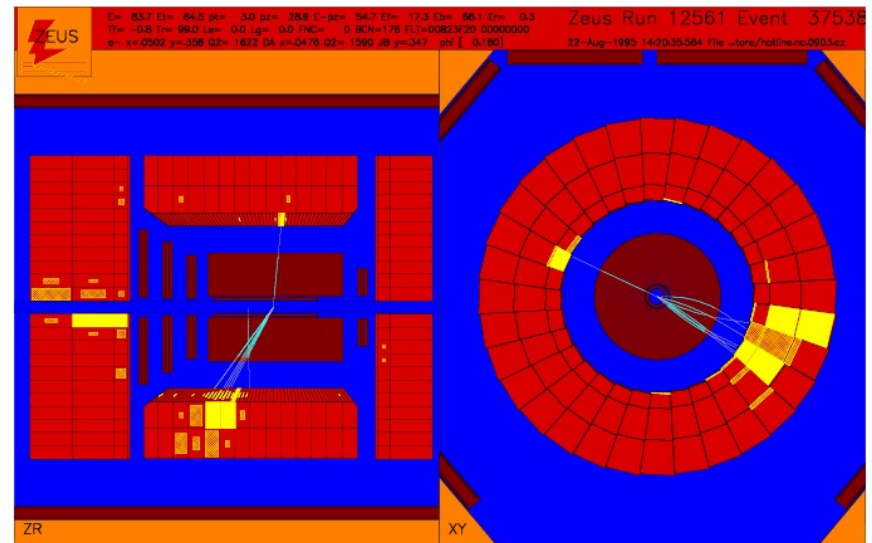
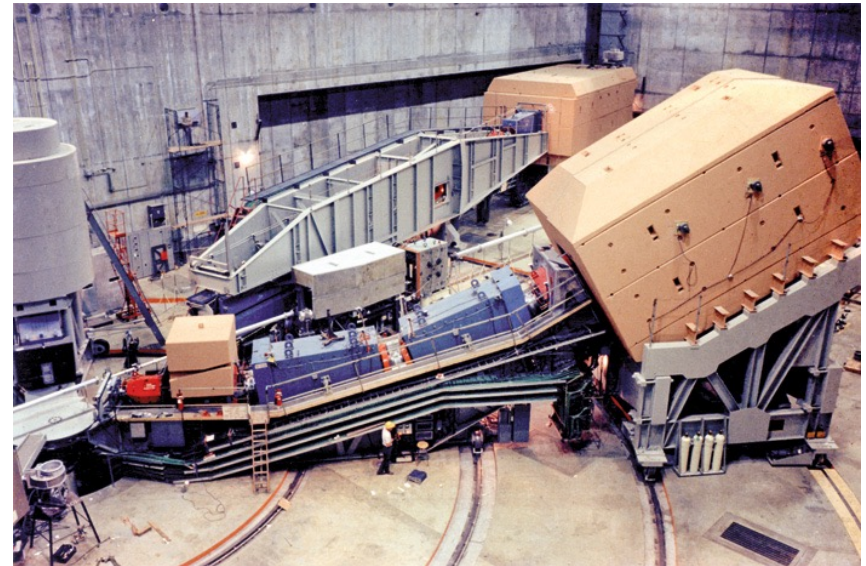
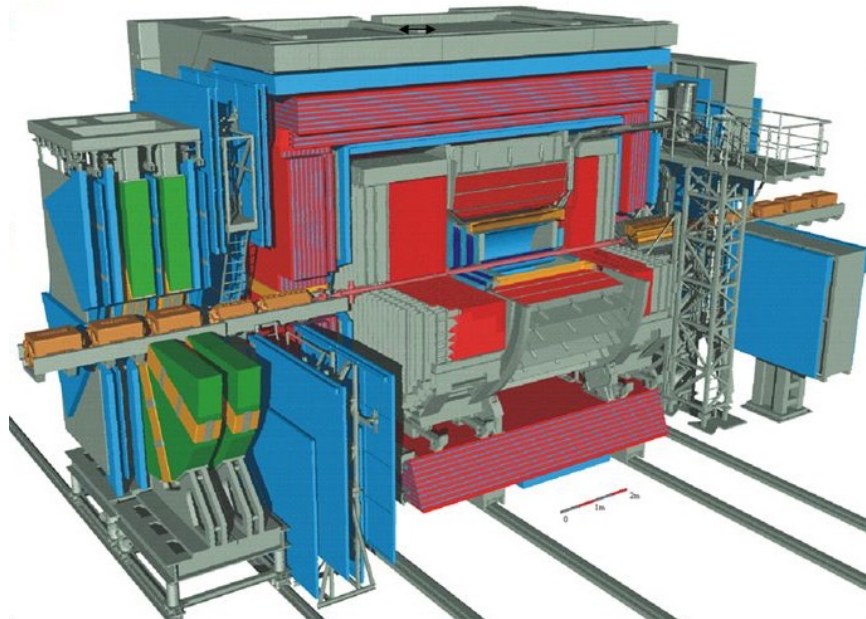
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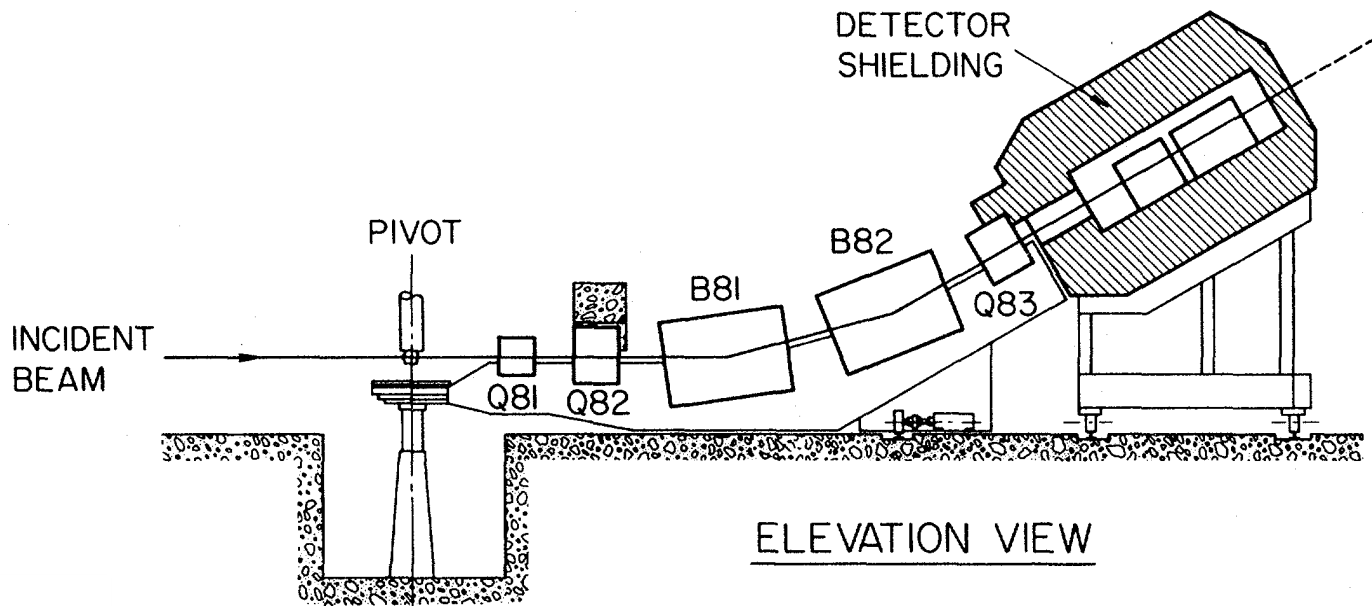




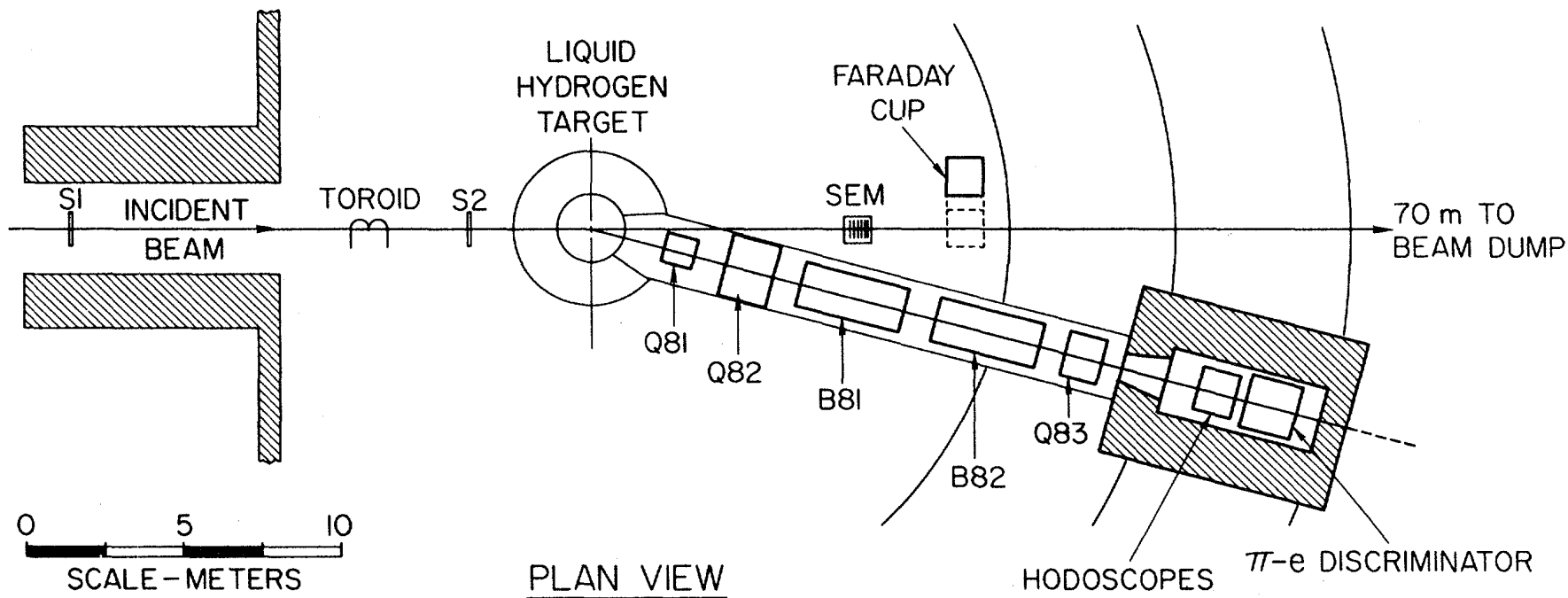
# Deep inelastic scattering - detectors

- 1968, SLAC
  - $e^-$  on fixed target of **H** atoms
- 1992, DESY:
  - HERA collider:
    - 27.5 GeV  $e^\pm$  on 902 GeV  $p^+$
  - ZEUS and H1 detector





ELEVATION VIEW



PLAN VIEW

