

# Dynamics of the critical fluctuations in heavy-ion collisions

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QGP France - 4th of May 2022

Grégoire Pihan

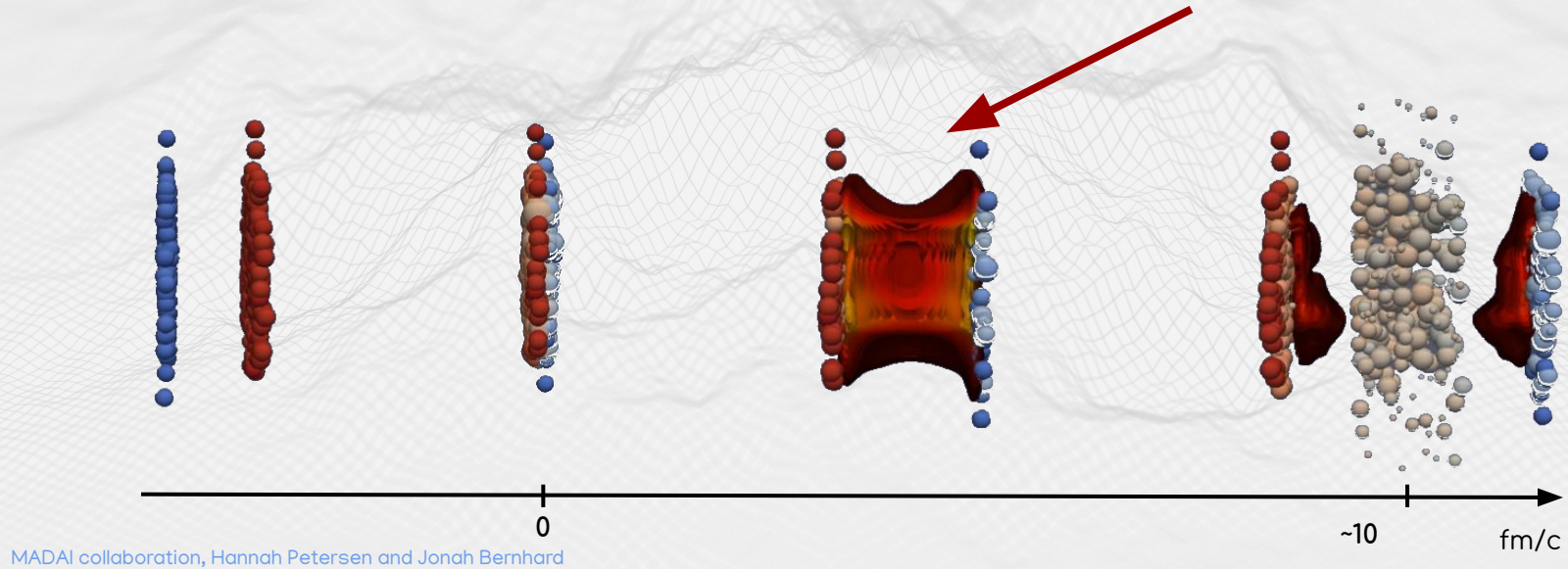
3rd Phd student in the Subatech theory group



Supervisors : Taklit Sami,  
Marlene Nahrgang,  
Marcus Bluhm.

# Introduction

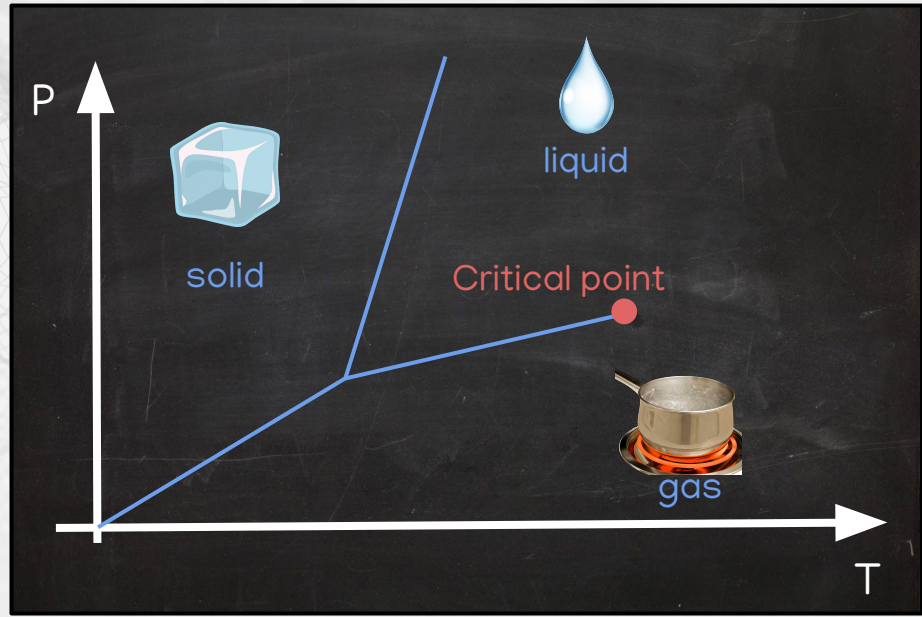
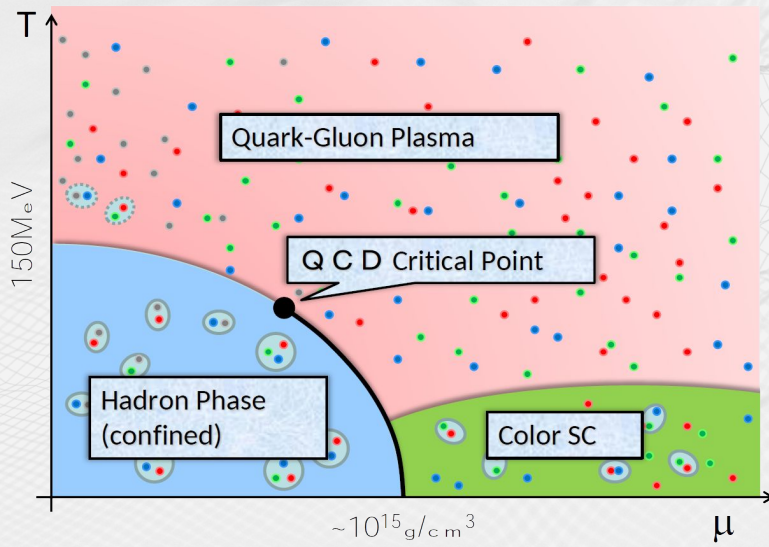
Deconfined state of strongly interacting matter : **The quark-gluon plasma.**



The matter inside nuclei can be found in various phasis !

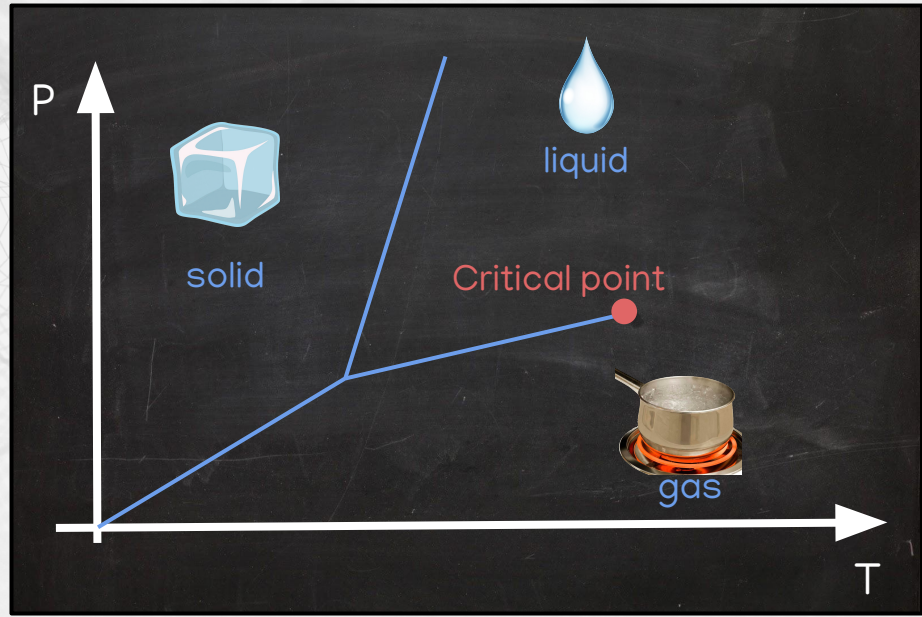
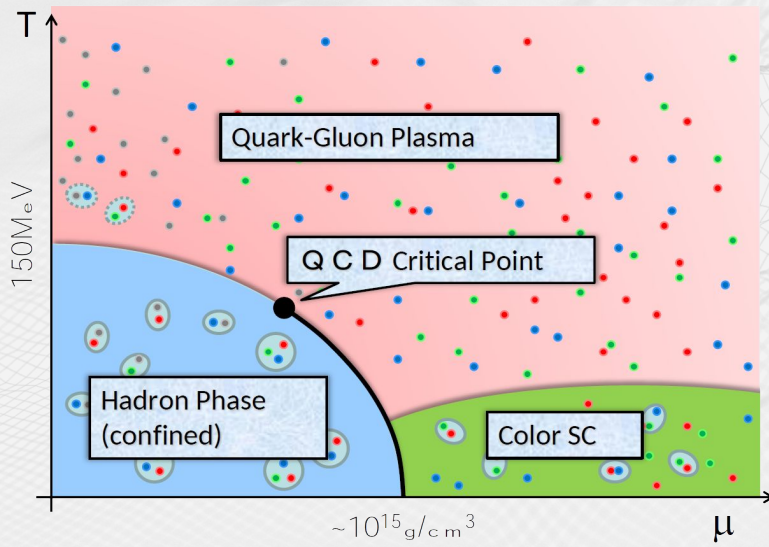
# Introduction

The QCD phase diagram



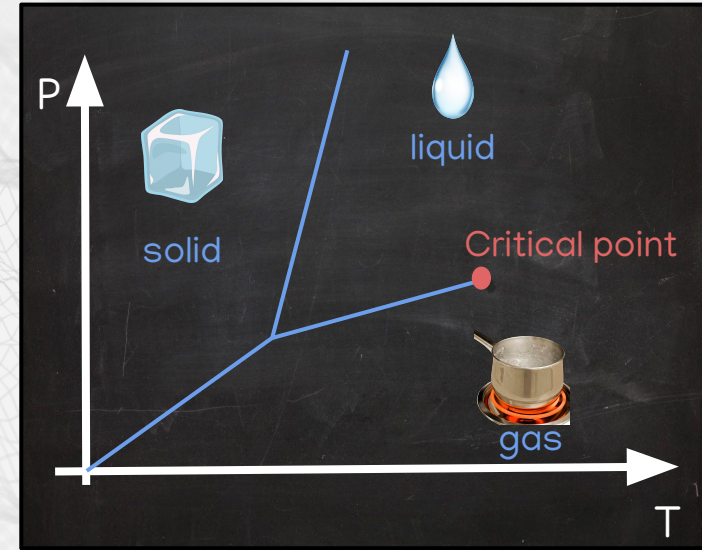
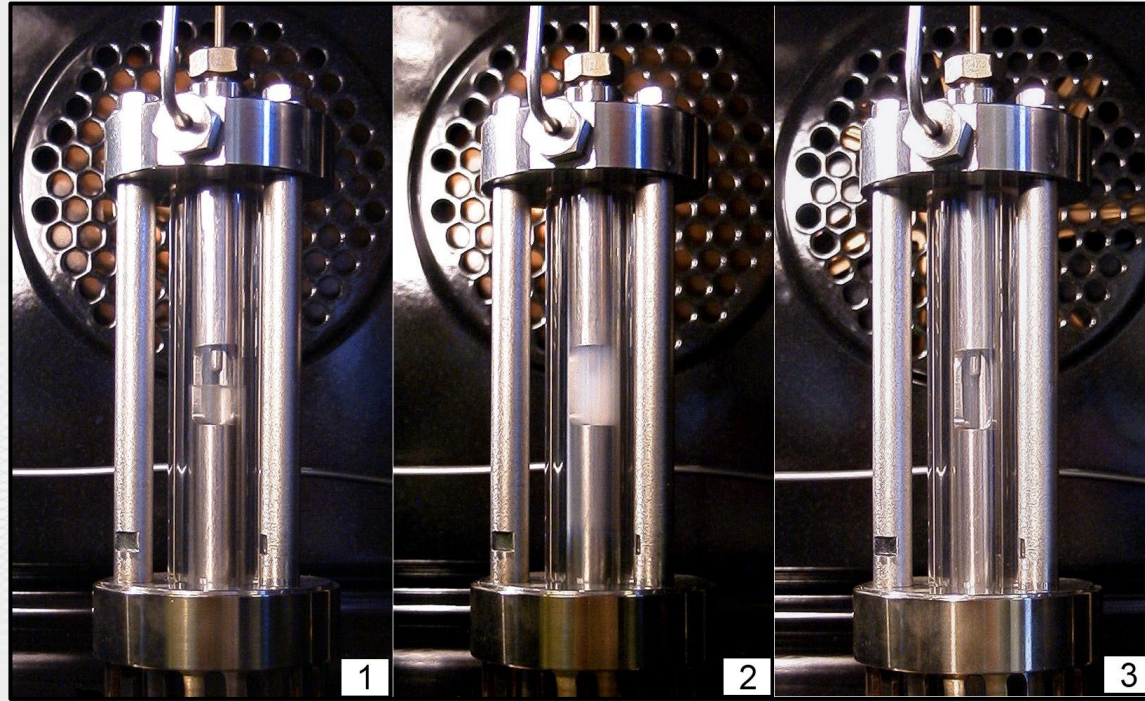
# Introduction

The QCD phase diagram



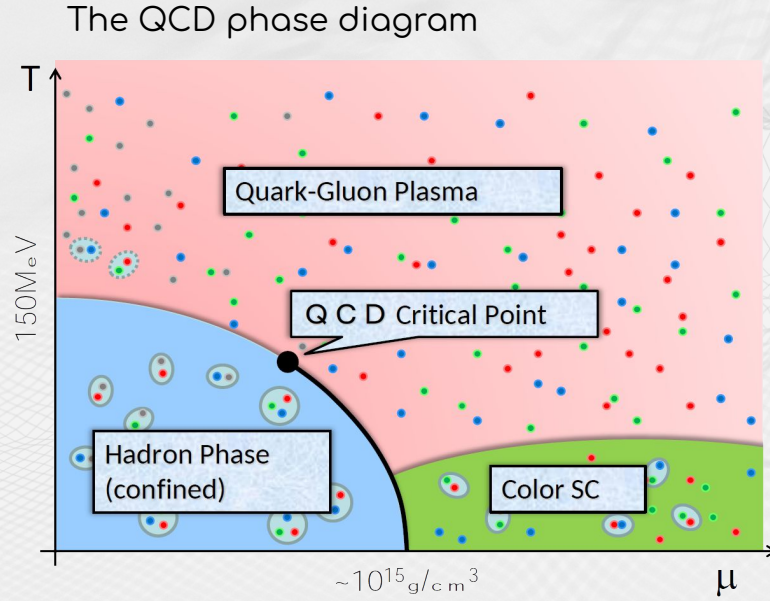
Is there a critical point in the QCD phase diagram ?

# Introduction



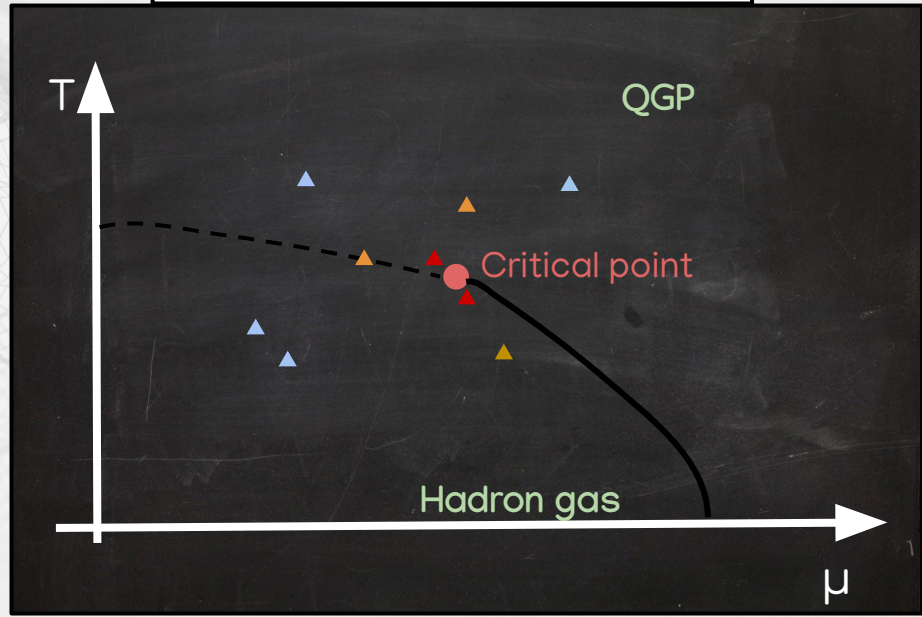
Fluctuations of density are a very relevant probe for criticality !

# Introduction



Density fluctuations

▲ Small    ▲ large    ▲ Very large

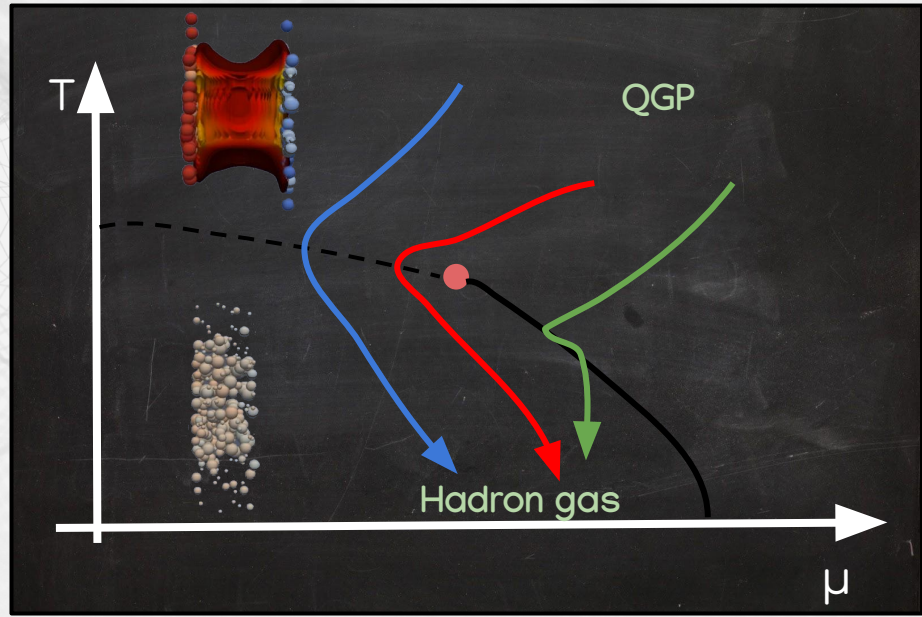
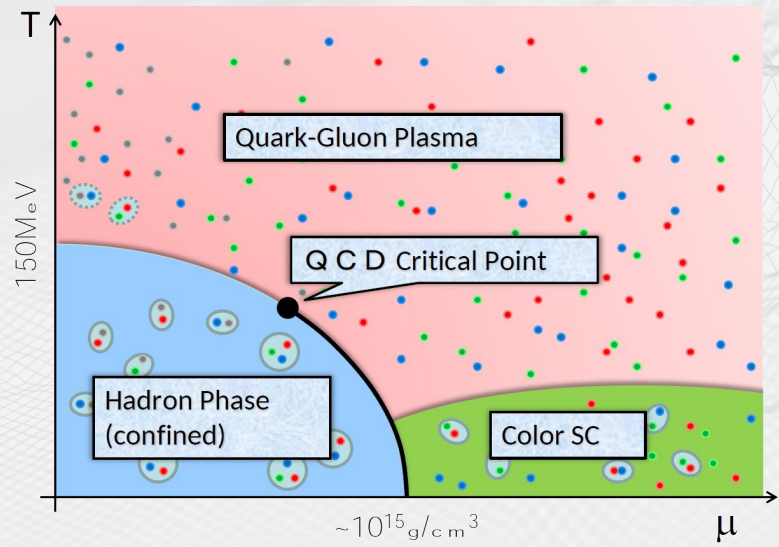


Can we measure the density fluctuations experimentally ?

# Introduction

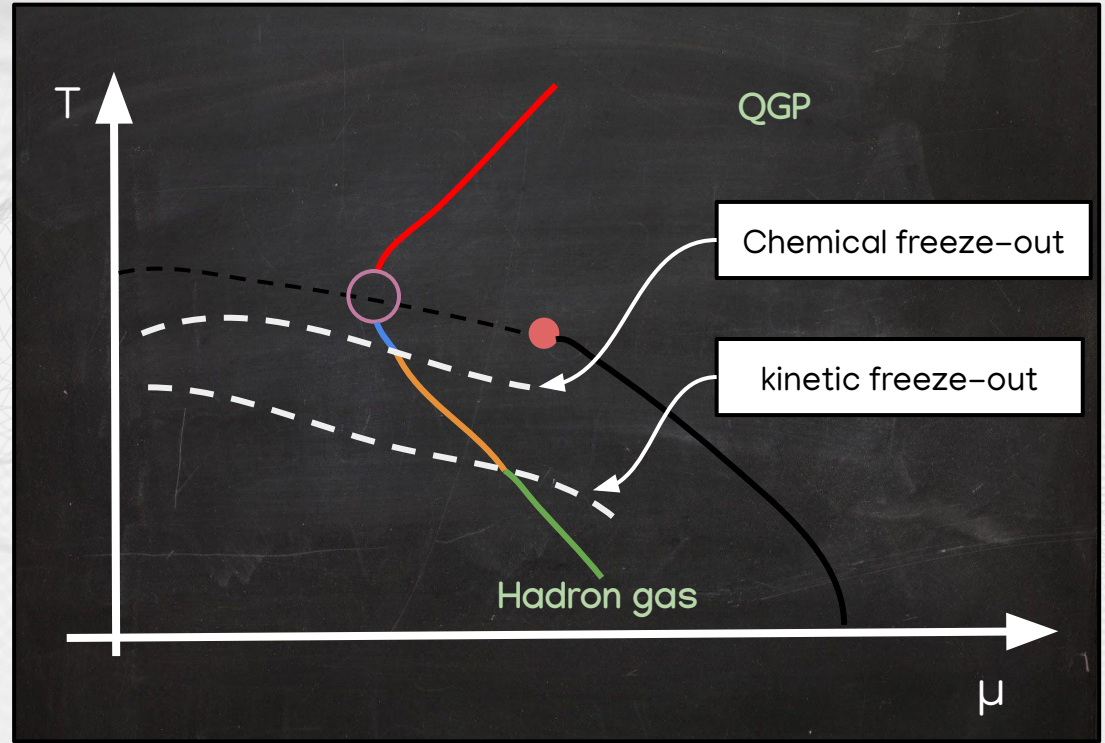
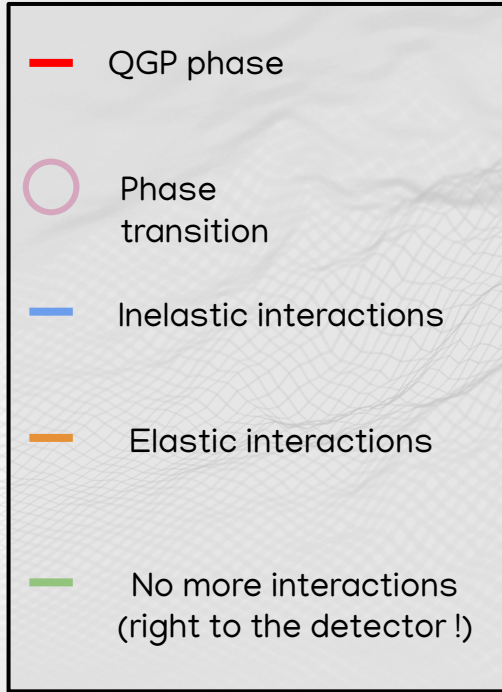
Yes, with ultra-relativistic heavy-ion collisions !

The QCD phase diagram



Can we find the critical point using (UR) heavy-ion collisions ?

# Introduction

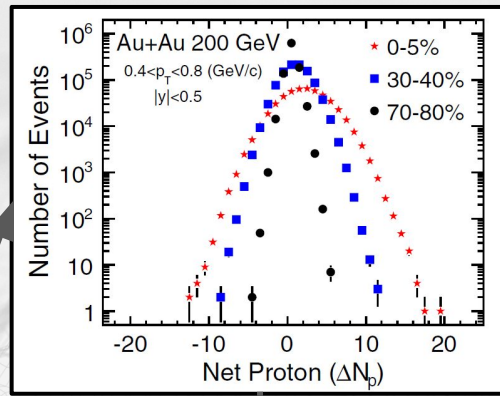
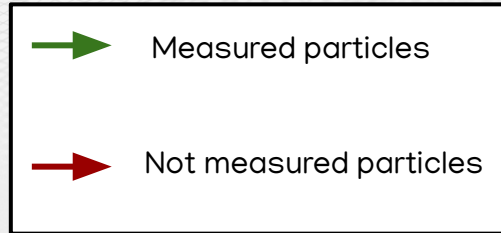
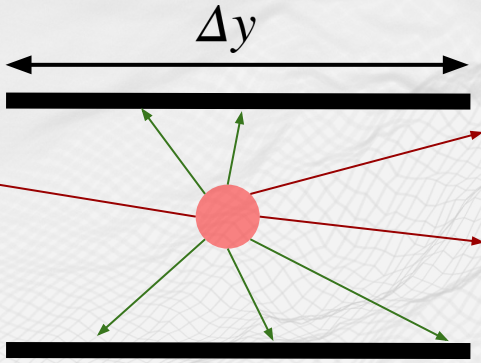


Are we able to measure fluctuations from ○ and ● after freeze-outs ?



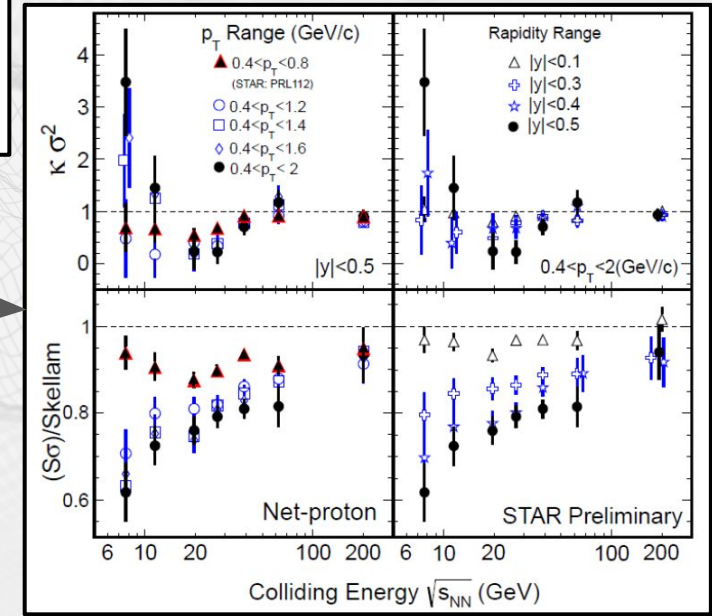
# Introduction

The finite size of detectors



The experimentally measured fluctuations are connected to the density fluctuations at the collision point!

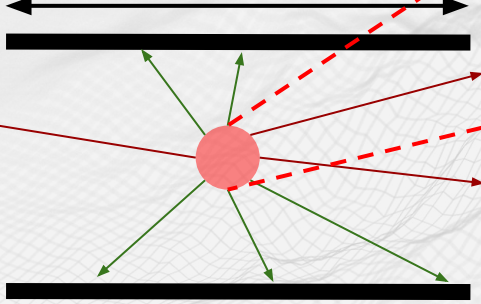
STAR Collab. ~2010



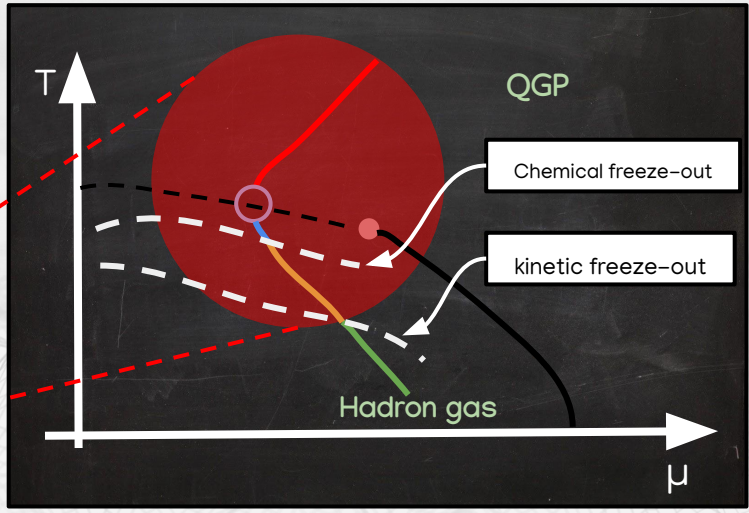
# Introduction

The finite size of detectors

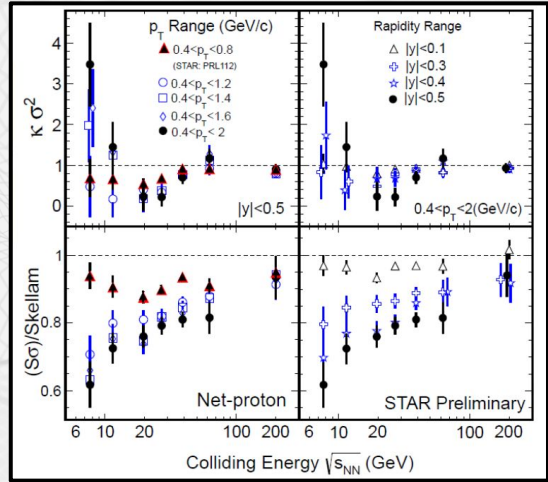
$$\Delta y$$



All collisions steps are in the circle !



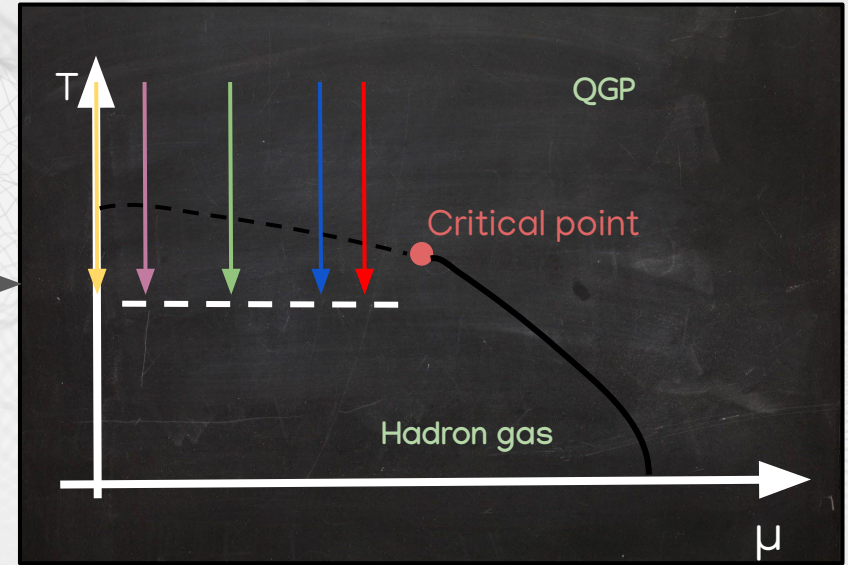
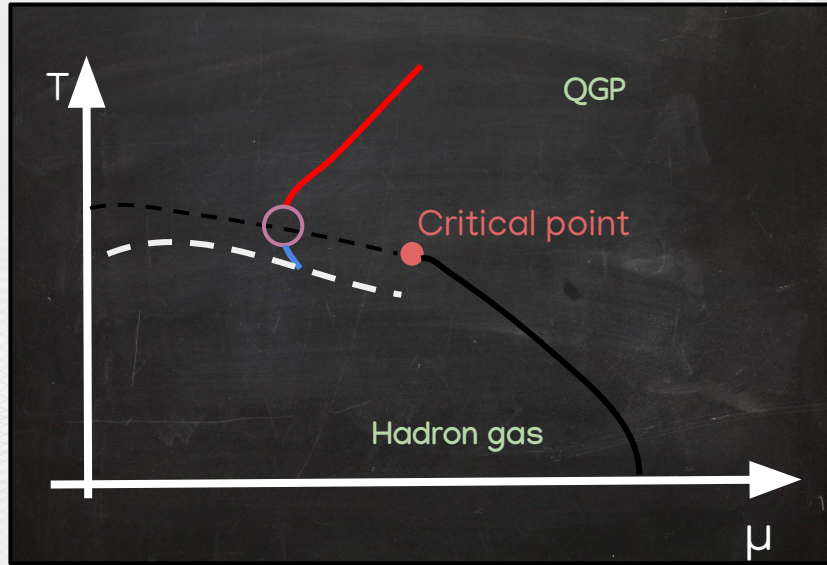
Are the measured fluctuations only related to the **critical point** ?



STAR Collab. ~2010

What are the impact of the dynamics on the fluctuations in heavy-ion collisions ?

# Introduction



Study the dynamics of net-baryon density critical fluctuations along different trajectories !

$u^\mu$  : 4-velocity of flow

$$\Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu$$

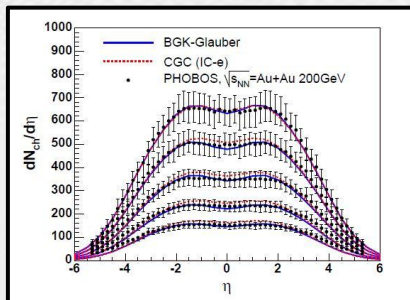
Fluctuating fluid dynamics model

Net-baryon number conservation

$$\partial_{;\mu} N_B^\mu = 0$$

Bjorken's ansatz

$$u^\mu = \left( \frac{t}{\sqrt{t^2 - z^2}}, \frac{z}{\sqrt{t^2 - z^2}} \right)$$



B. B. Back et al. Phys. Rev. Lett. 91, 052303

$$N_B^\mu = n_B u^\mu + \kappa T \Delta^{\mu\nu} \partial_{;\nu} \left\{ \frac{\mu_B}{T} \right\} + \xi^\mu$$

$$\langle \xi^\mu(x) \xi^\nu(x') \rangle = -2\kappa T \delta^{(4)}(x-x') \Delta^{\mu\nu}$$

Milne coordinates

Boost-invariance = no dependence on  $y$

proper-time

$$\tau = \sqrt{t^2 - z^2}$$

spatial rapidity

$$y = \frac{1}{2} \ln \left( \frac{t+z}{t-z} \right)$$

$$\partial_\tau n_B(\tau, y) = \partial_y \left\{ \frac{\kappa T}{\tau} \partial_y \left\{ \frac{\mu_B}{T} \right\} \right\} + \partial_y \xi^y$$

$$\langle \xi^y(\tau, y) \xi^y(\tau', y') \rangle = 2 \frac{\kappa T}{\tau} \delta(\tau - \tau') \delta(y - y')$$

# Stochastic diffusion equation

Assumptions

$$\partial_\tau n_B(\tau, y) = \partial_y \left\{ \frac{\kappa T}{\tau} \partial_y \left\{ \frac{\mu_B}{T} \right\} \right\} + \partial_y \xi^y$$
$$\langle \xi^y(\tau, y) \xi^y(\tau', y') \rangle = 2 \frac{\kappa T}{\tau} \delta(\tau - \tau') \delta(y - y')$$

Constant diffusion coefficient  $D$

$$\kappa T = D$$

The baryochemical potential is given by a Ginzburg–Landau type free-energy functional

$$\mu_B = \frac{\delta \mathcal{F}}{\delta n_B}$$

$$\partial_\tau n_B = D(\tau) \partial_y^2 \left\{ \frac{\delta \mathcal{F} / T}{\delta n_B} \right\} + \partial_y \xi^y$$

$$\langle \xi^y(\tau, y) \xi^y(\tau', y') \rangle = 2 D(\tau) \delta(\tau - \tau') \delta(y - y')$$

$$D(\tau) = \frac{D}{\tau}$$

The free-energy functional encodes the critical physics

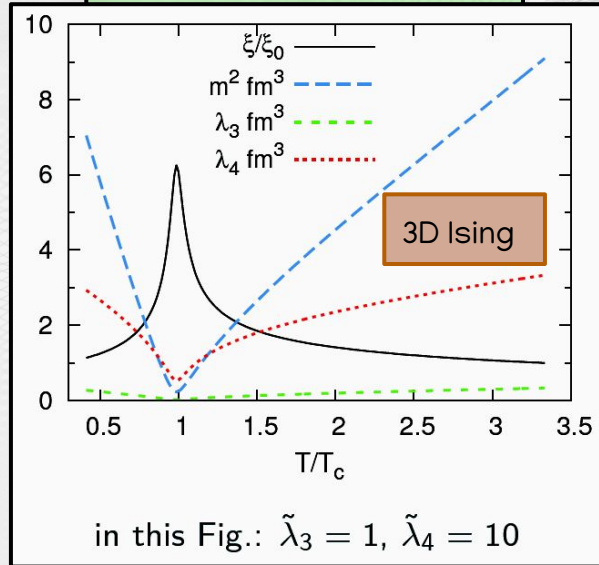
The diffusion coefficient is a tool to study the impact of the dynamics on the fluctuations

# Free-energy functional

$$\mathcal{F}(\tau) = \int dV \left\{ \frac{n_B^2}{2\chi_2(\tau)} + \frac{K(\tau)}{2} (\partial_y n_B)^2 + \frac{n_B^4}{24\chi_4(\tau)} \right\}$$

$$T(\tau) = T_i \left( \frac{\tau_0}{\tau} \right)$$

Critical contribution



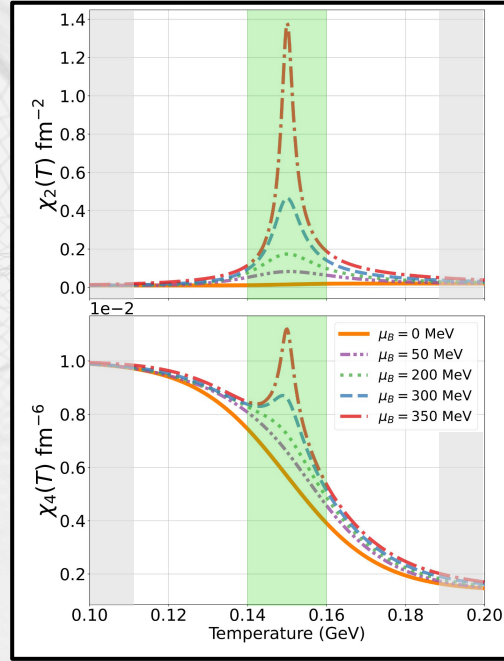
Non-critical contribution

$$\frac{\chi_2^B}{s} T^3$$

Lattice QCD

$$\chi_n^{\text{reg}}(T) = \chi_{0,n}^{\text{H}} + (\chi_{0,n}^{\text{QGP}} - \chi_{0,n}^{\text{H}}) S(T)$$

	Hadron gas	QGP
$\chi_2$	$0.01 \text{ fm}^{-2}$	$0.02 \text{ fm}^{-2}$
$\chi_4$	$0.01 \text{ fm}^{-6}$	$0.01 \text{ fm}^{-6}$



M. Nahrgang, M. Bluhm Phys. Rev. D 102, 094017

M. Nahrgang, M. Bluhm, T. Schöfer, S.A Bass Phys. Rev. D 99, 116015

Asakawa, Heinz, Müller PhysRevLett.85.2072

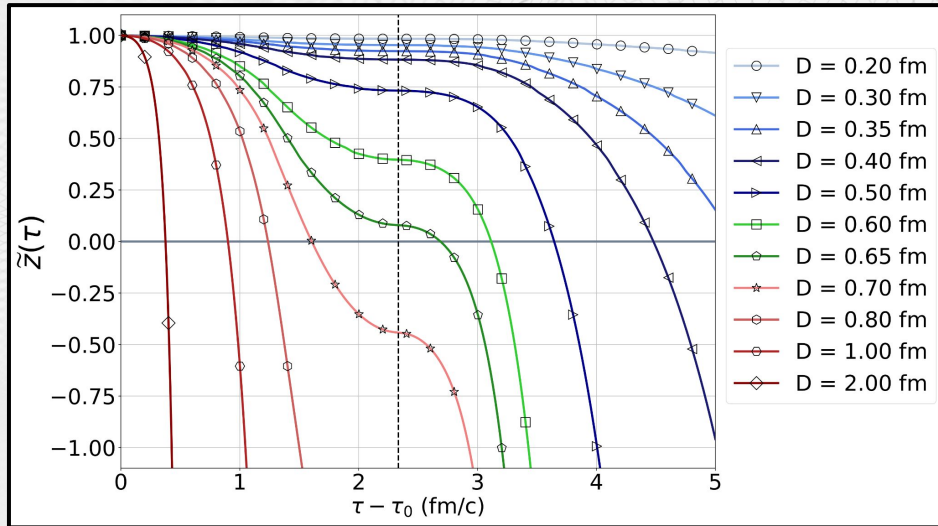
M. Cheng et al Phys. Rev. D 79, 074505

A.Bozakov et al Phys. Rev. D 95, 054504

# The diffusion coefficient

In the **scaling region**

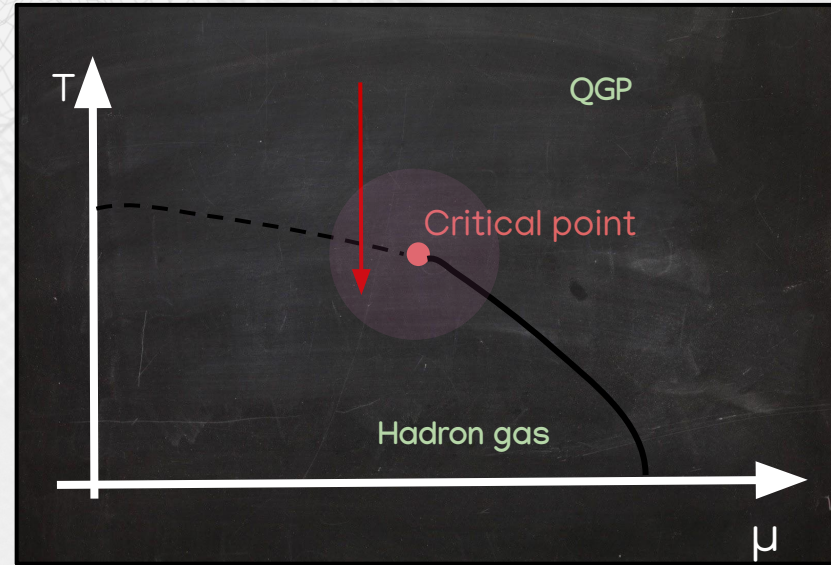
$$D \propto \frac{1}{\tau_r}$$



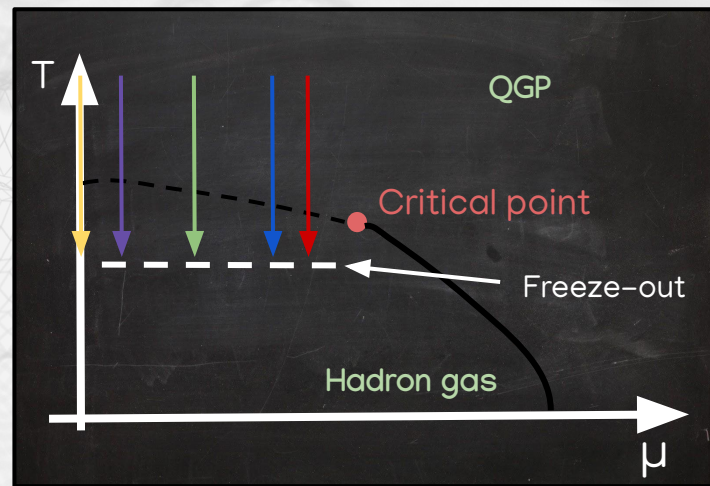
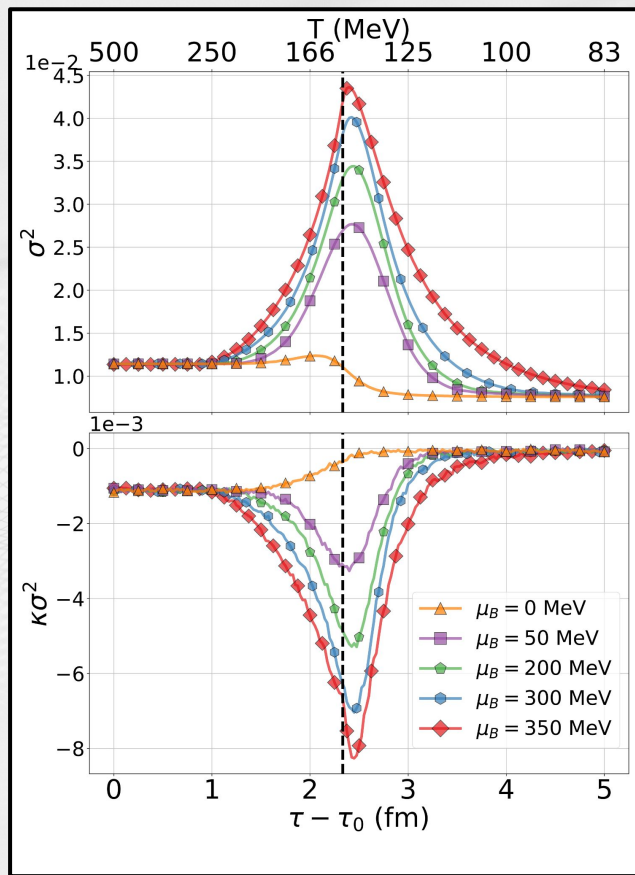
GP, Bluhm, Kitazawa, Nahrgang, CPOD2021 Proceeding arXiv:2111.14466 [nucl-th]

Diffusion coefficient

$$\partial_\tau n_B = D(\tau) \partial_y^2 \left\{ \frac{\delta \mathcal{F}/T}{\delta n_B} \right\} + \partial_y \xi^y$$



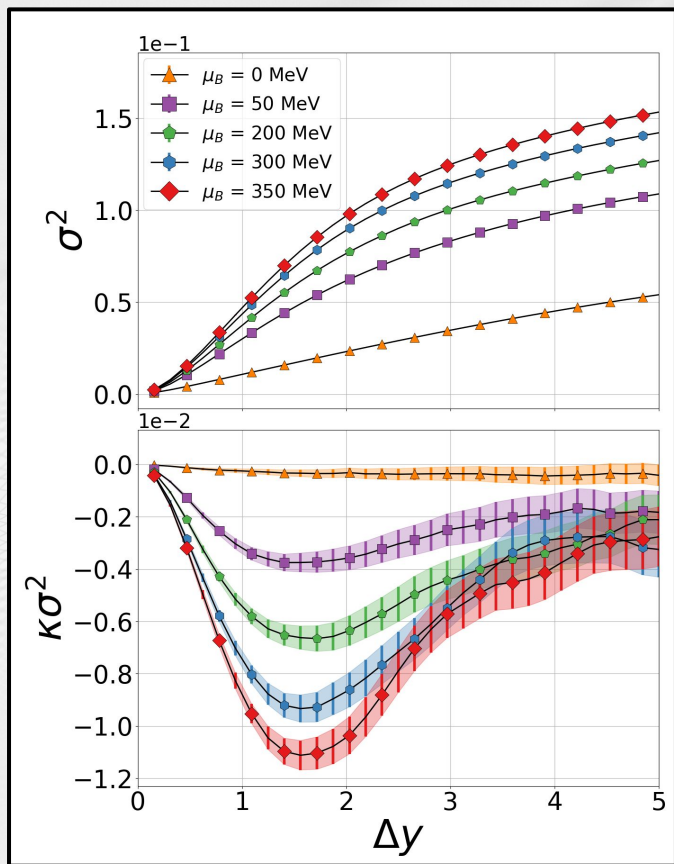
# time-dependence of variance and the kurtosis



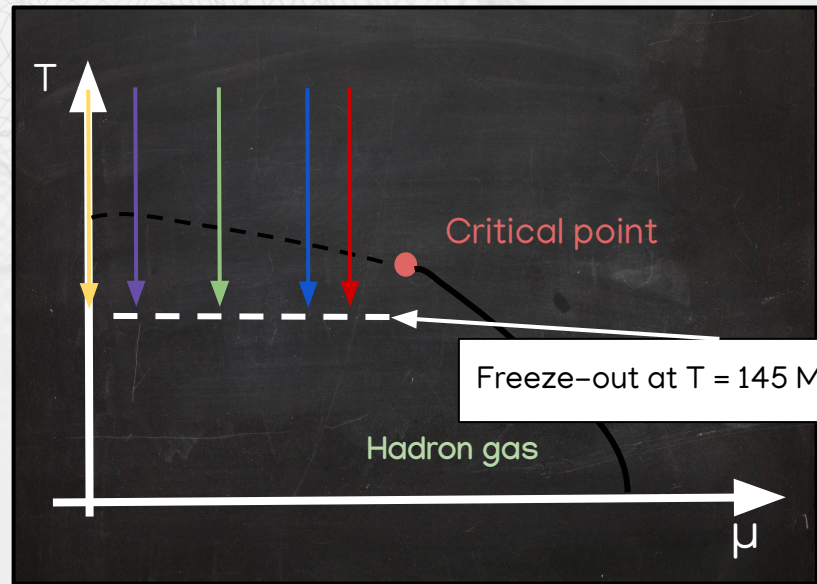
- Enhancement of the fluctuations due to the critical point !
- The signal is very sensitive to the freeze-out temperature.



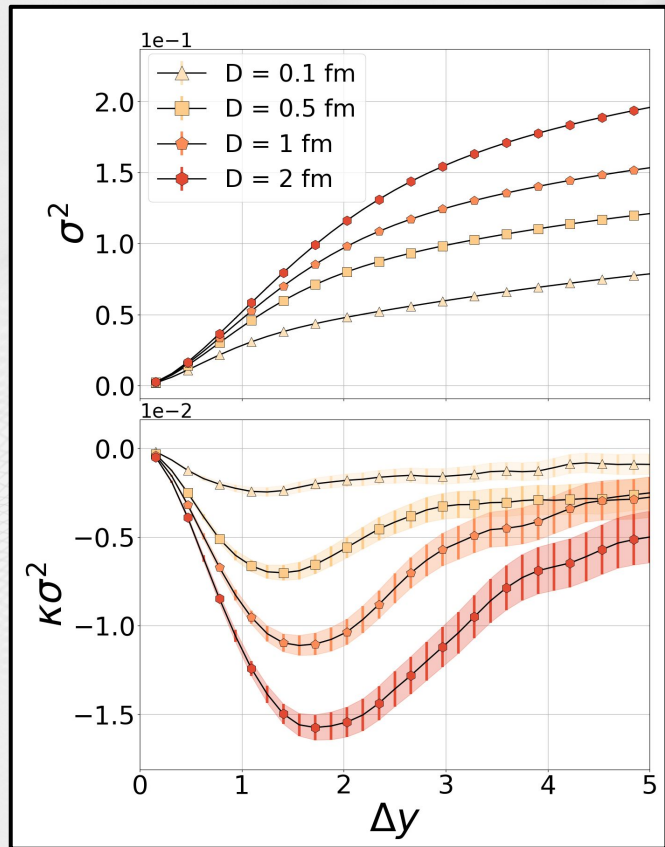
# Rapidity window dependence of the variance and kurtosis



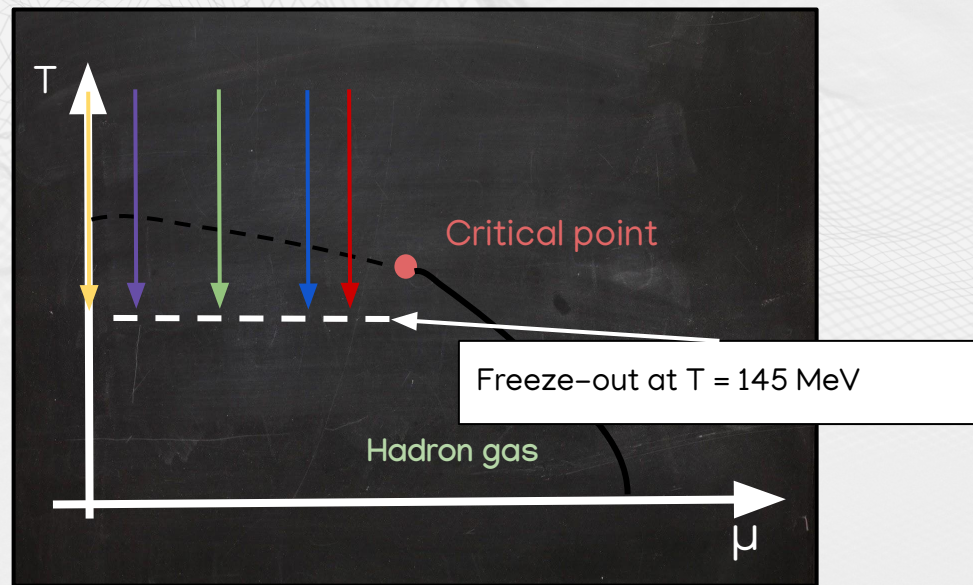
$$n_{B,i}(\Delta y) = \int_{-\Delta y/2}^{\Delta y/2} dy n_{B,i}$$



# Rapidity window dependence of the variance and kurtosis



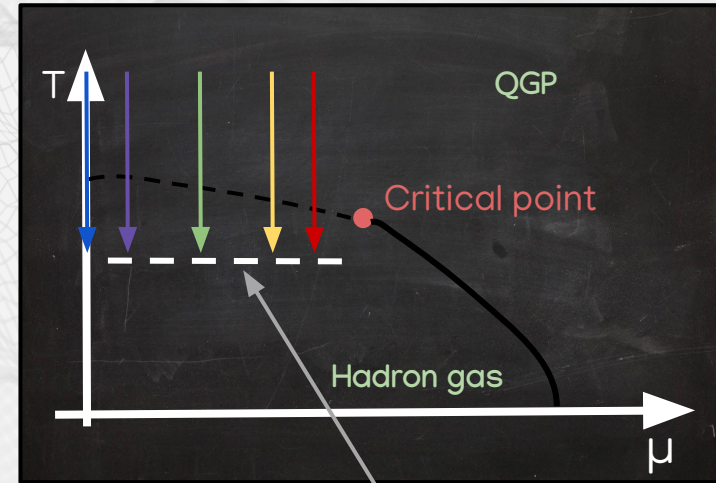
The cumulants are very sensitive to the diffusion length



# Conclusion

- A critical signal can be observed !
- The signal is very sensitive to the freeze-out temperature.
- The signal is very sensitive to diffusion length.

Need more reliable values for the diffusion length and freeze-out parameters.  
Proper particlization is needed !



Freeze-out at  $T = 145$  MeV