Challenging the Standard Model at the LHC, Theory Introduction

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Thanks to Alan Martin, James Stirling and Graeme Watt

Will consider the production of vector bosons, i.e. W^+, W^-, Z and γ^*, top quarks, high-p_T jets and the Standard Model Higgs boson.

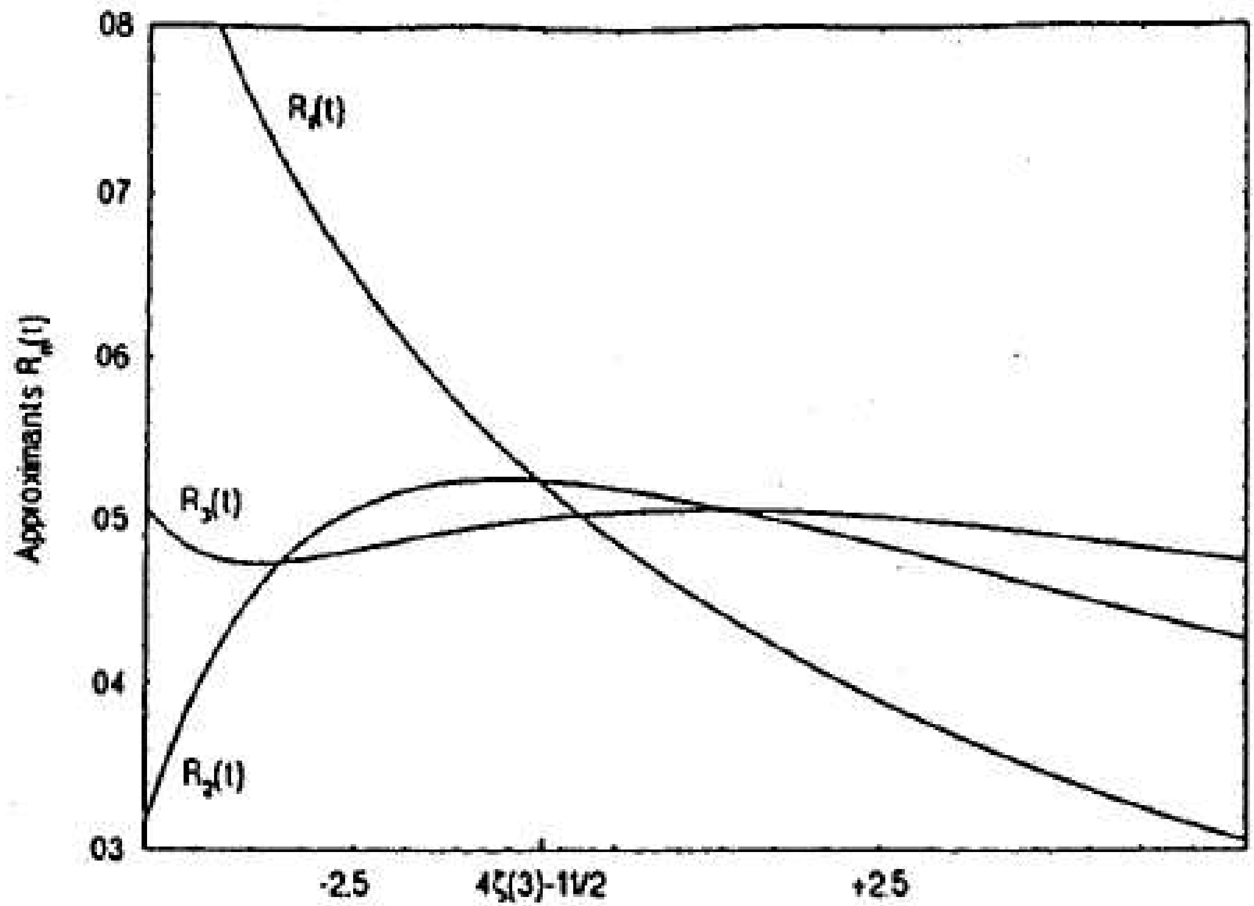
Will concentrate on inclusive cross-sections, but also on more differential distributions, i.e. rapidity y or transverse momentum p_T in some cases.

Some of the above are particles which interact via the electromagnetic and/or weak force, but for all the details of the production rates are influenced by both the electroweak sector of the Standard Model and the strongly interacting part.

In fact in most cases the dominant theoretical uncertainties are associated with the strong interaction, both due to corrections to the final state production cross-section, and also that the final state is always created from the initial state quarks and gluons within the proton.

Hence first consider QCD.

Renormalisation of ultraviolet divergences introduces artificial renormalization scale μ_R on which renormalised couplings, masses, *etc* depend, though dependence disappears (at all orders in physical quantities).



Renormalisation scheme dependence at LO, NLO and NNLO, for ratio of $e^+ + e^- \to \mathrm{hadrons/leptons}$ (Samuel and Surguladze).

Initial State - Parton Distribution Functions (PDFs)

Another complication at the LHC and Tevatron is that the colliding particles are not fundamental.

Hadrons are bound together by the strong force, described QCD.

As seen the strong coupling constant $\alpha_S(\mu^2)$ runs with the energy scale μ^2 of a process, decreasing as μ^2 increases (**asymptotic freedom**), i.e. $\alpha_s(\mu^2)$ is very large if $\mu^2 \sim \Lambda^2_{\rm QCD}$ ($\sim 0.3\rm GeV$), the scale of nonperturbative physics, but $\alpha_s(\mu^2) \ll 1$ if $\mu^2 \gg \Lambda^2_{\rm QCD}$, and perturbation theory can be used.

Because of the strong force it is difficult to perform analytic calculations of scattering processes involving hadronic particles from first principles. However, the weakening of $\alpha_S(\mu^2)$ at higher scales $\rightarrow$ the **Factorization Theorem** – separates processes into nonperturbative **parton distributions** which describe the composition of the proton and can be determined from experiment, and perturbative **coefficient functions** associated with higher scales which are calculated as a power-series in $\alpha_S(\mu_R^2)$.

Factorization introduces another arbitrary scale μ_F^2.

Since both renormalisation scale and factorisation scale dependence diminishes at higher orders, scale variation at fixed order used to *estimate* the *theoretical* uncertainty.

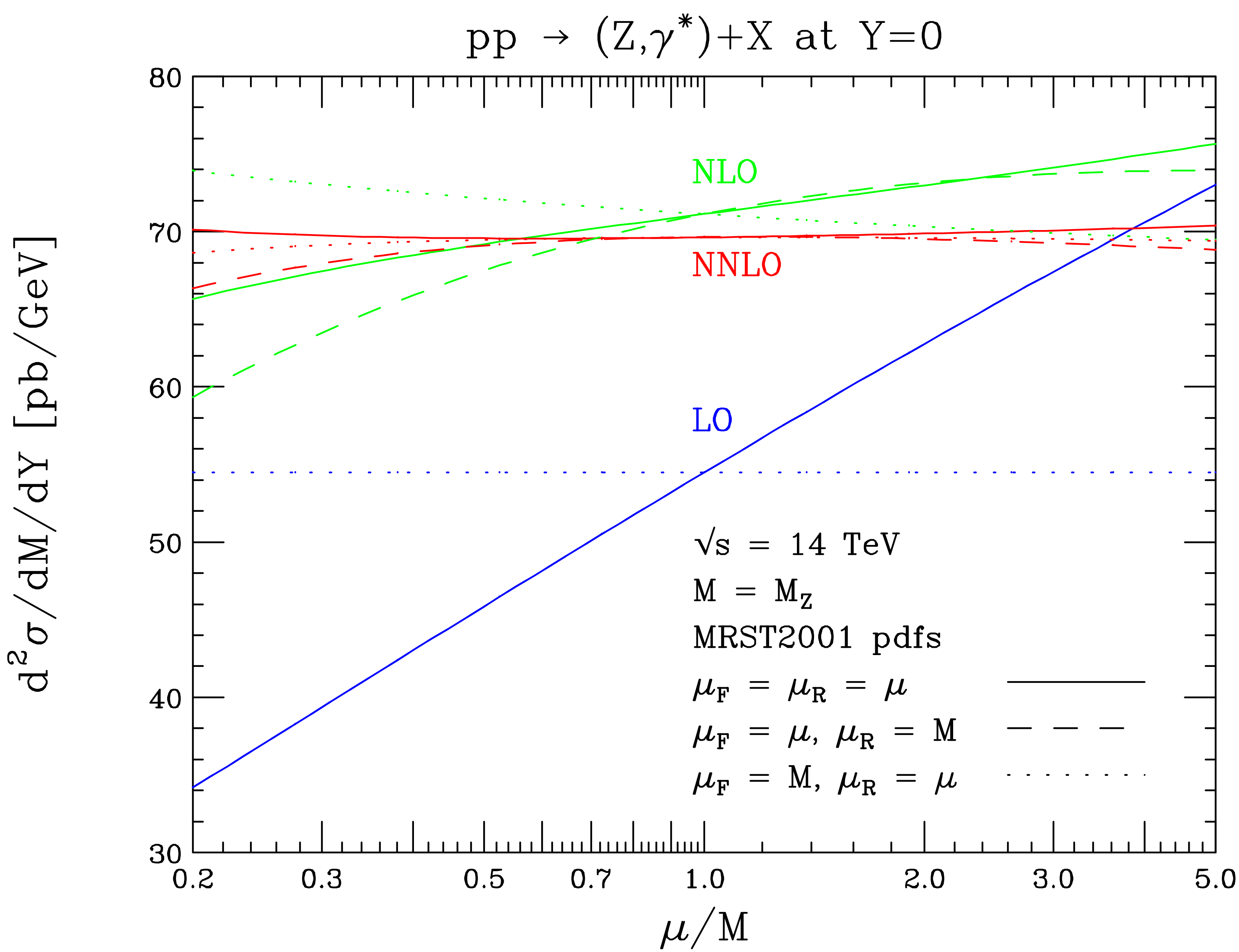
It is seen as a measure of how much allowed variation there is before the unknown higher-order corrections are added.

Usually reasonable, but must be treated with some caution.

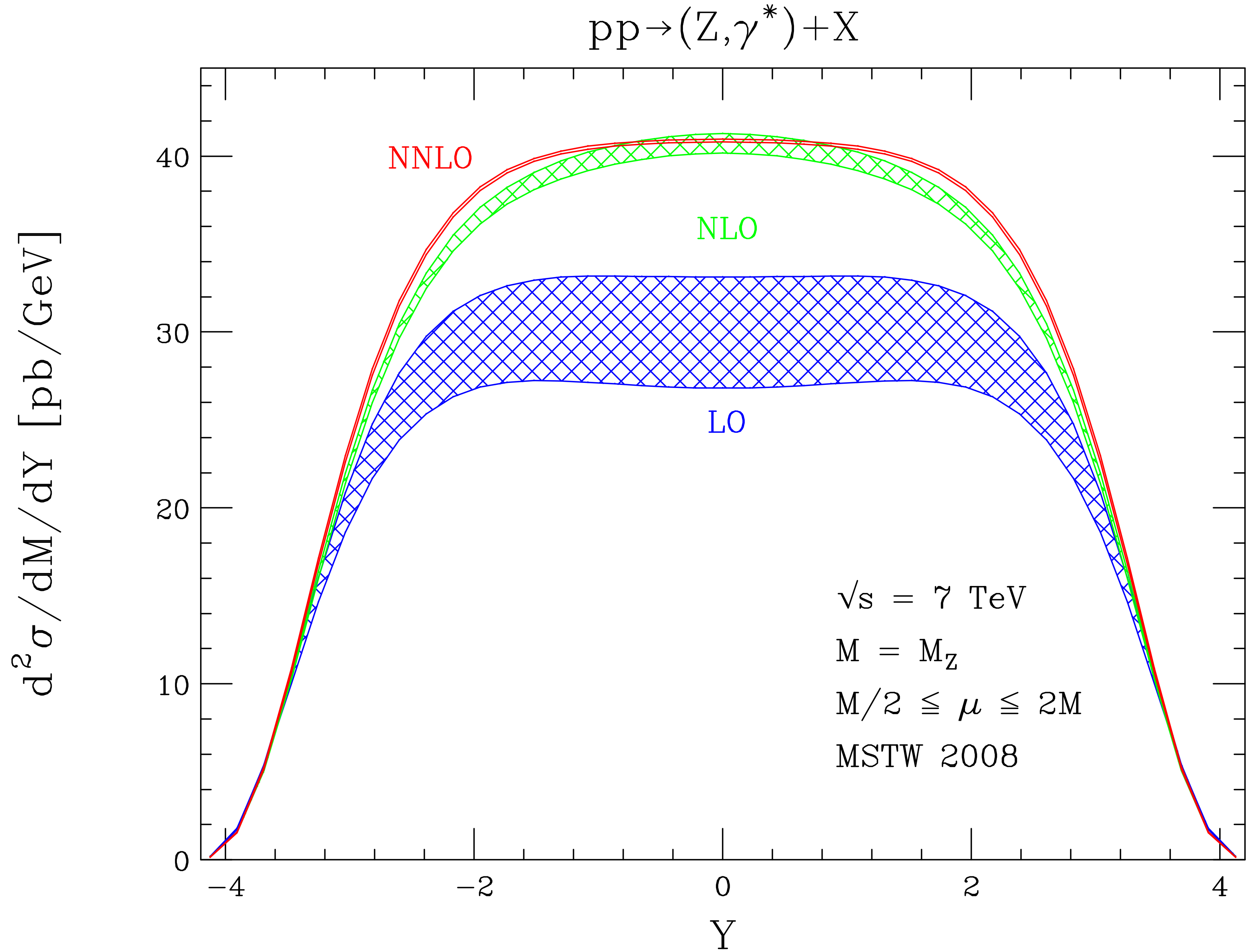
If something fundamentally new appears at higher orders, e.g. a new dependence on energy, scale variation knows nothing of this.

Will see some examples later.

Scale variations of Z production (Anastasiou *et al.*).



With a NNLO correction the scale dependence is postponed to $\mathcal{O}(\alpha_S^4)$.



Based on (Anastasiou *et al.*), from Lance Dixon.

Obtaining PDF sets. – Need many different types for full determination.

- Lepton-proton collider HERA – (DIS) $\rightarrow$ small-x quarks (best below $x \sim 0.05$). Also gluons from evolution (same x), and now $F_L(x, Q^2)$. Also, jets $\rightarrow$ moderate-x gluon.Charged current data some limited info on flavour separation. Heavy flavour structure functions – gluon and charm, bottom distributions and masses.

- Fixed target DIS – higher x – leptons (BCDMS, NMC, ...) $\rightarrow$ up quark (proton) or down quark (deuterium) and neutrinos (CHORUS, NuTeV, CCFR) $\rightarrow$ valence or singlet combinations.

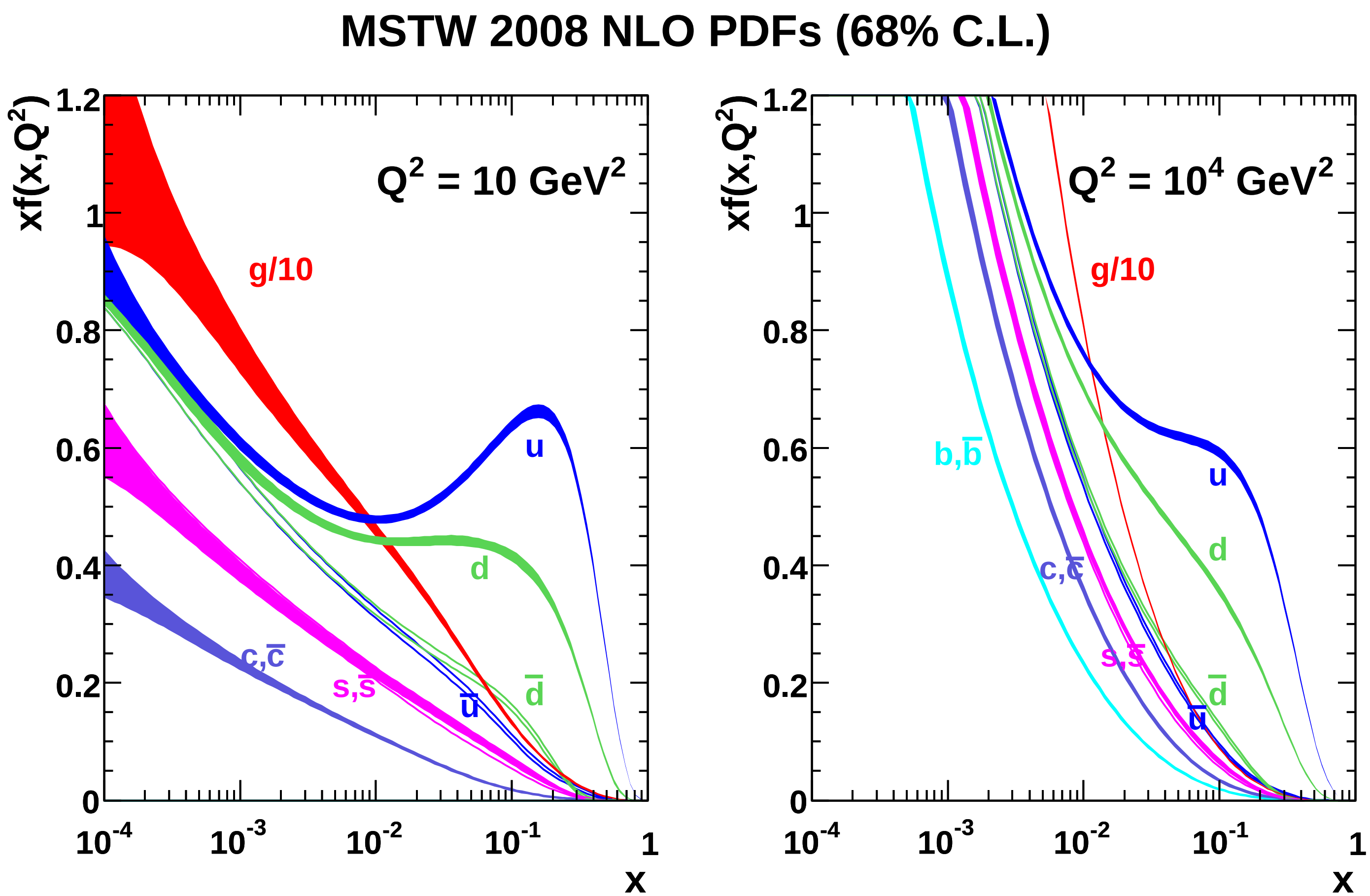
- Di-muon production in neutrino DIS – strange quarks and neutrino-antineutrino comparison $\rightarrow$ asymmetry . Only for $x > 0.01$.

- Drell-Yan production of dileptons – quark-antiquark annihilation (E605, E866) – high-x sea quarks. Deuterium target – $\bar{u}/\bar{d}$ asymmetry.

- High-p_T jets at colliders (Tevatron) – high-x gluon distribution – $x > 0.01$.

- W and Z production at colliders (Tevatron) – different quark contributions to DIS.

This procedure is generally successful and is part of a large-scale, ongoing project. Results in partons of the form shown.



Various choices of PDF – MSTW, CTEQ, NNPDF, Alekhin, HERA, Jimenez-Delgado *et al etc.*. All LHC cross-sections rely on our understanding of these partons.

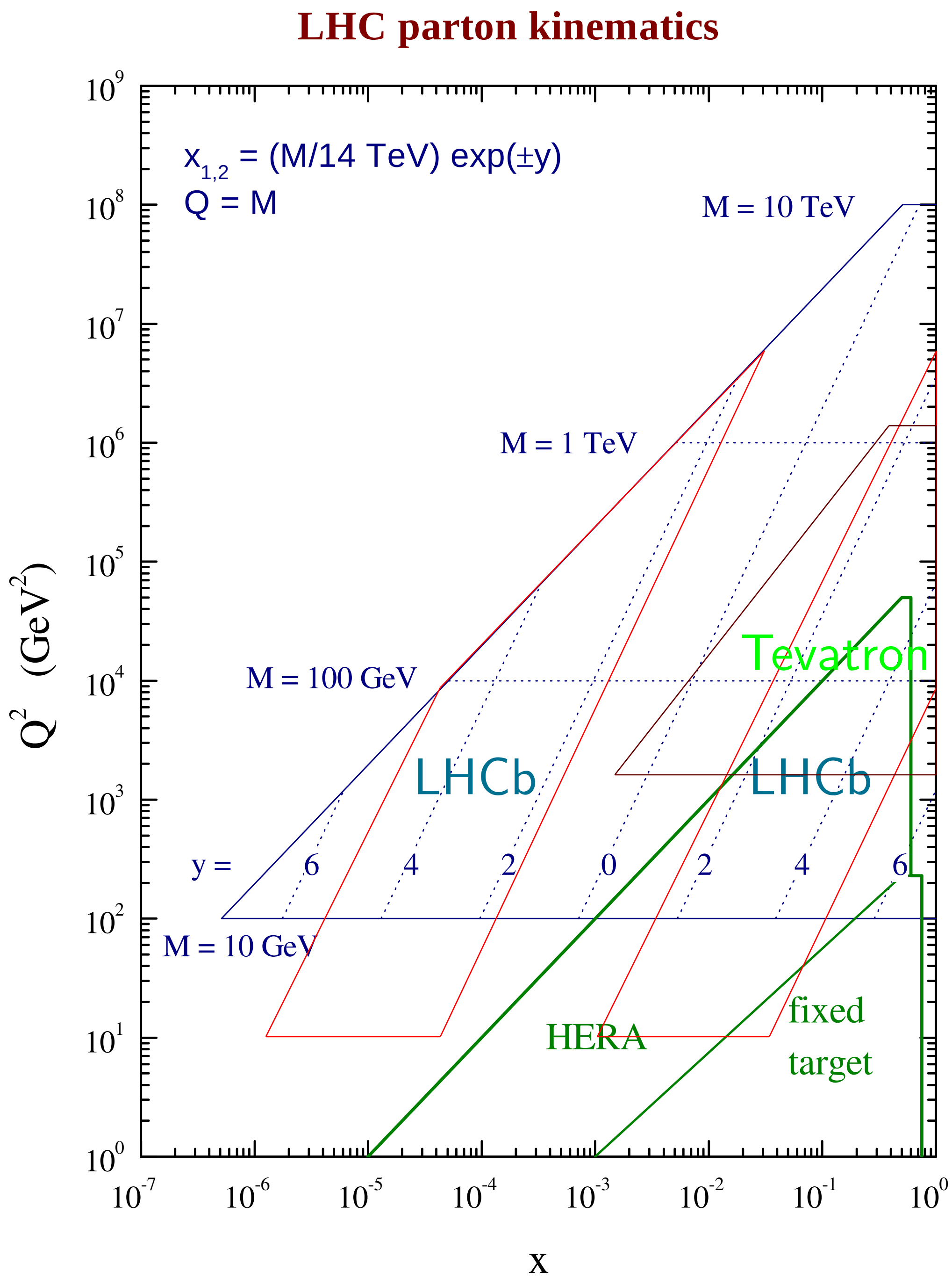
Predictions at the LHC

New kinematic regime.

PDFs mainly extrapolated via evolution rather than measured directly.

High scale and small-x parton distributions are vital for understanding processes at the LHC.

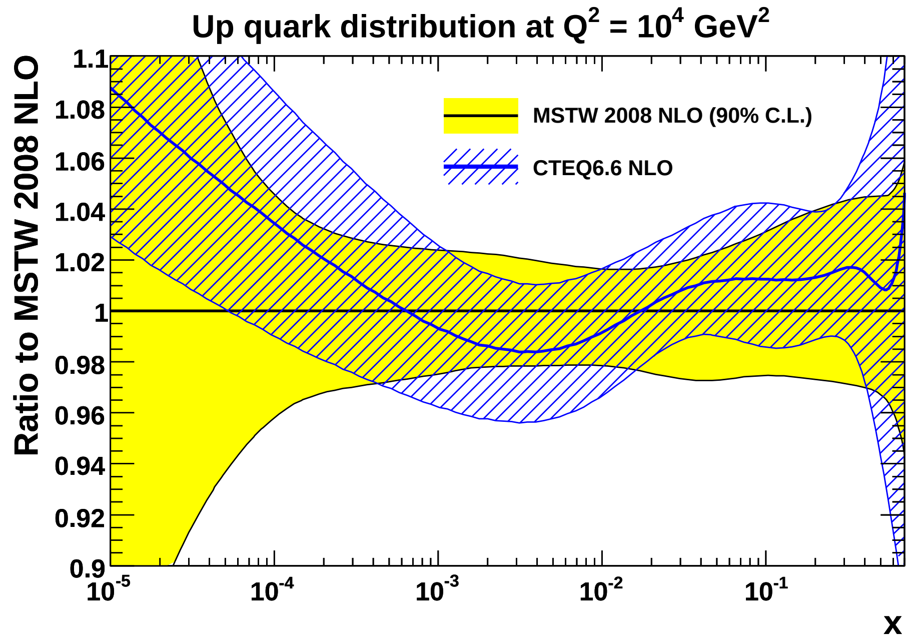
More discrepancy at values of x away from this.

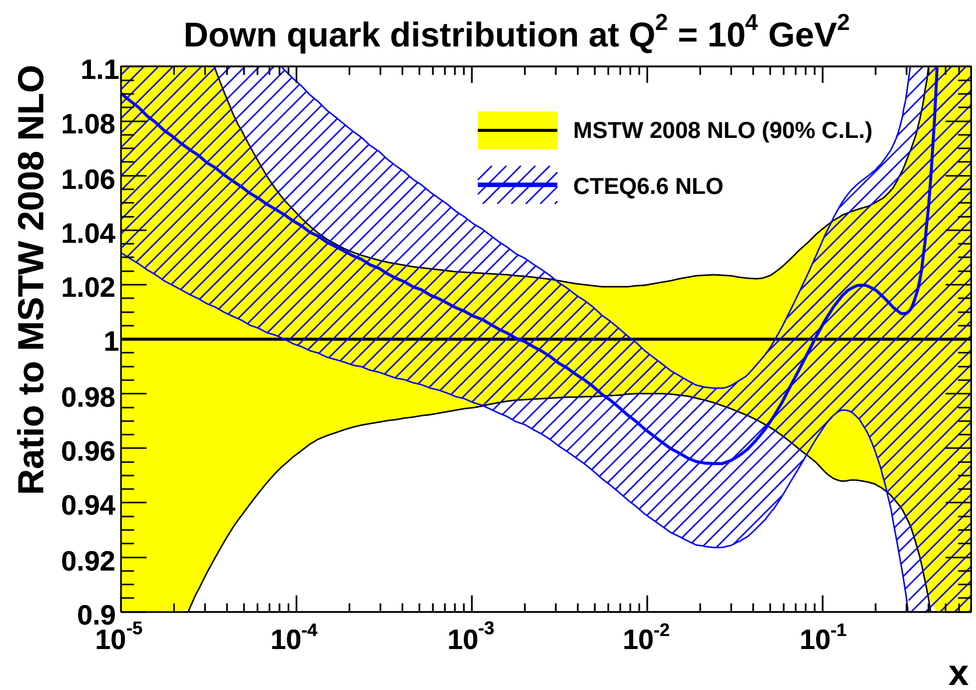


Uncertainty on MSTW u and d distributions, along with CTEQ6.6.

Reasonable agreement between groups.

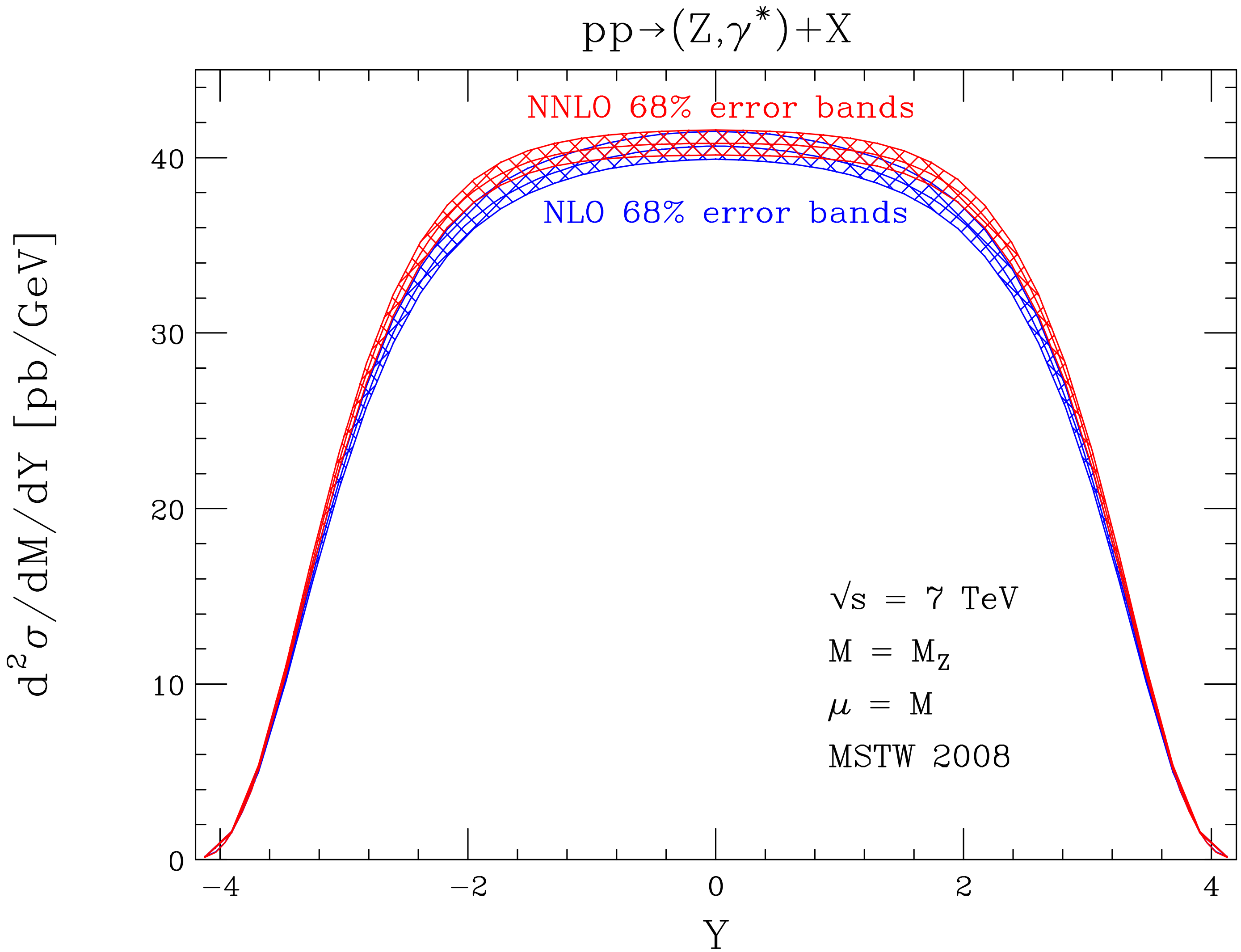
Central rapidity $x = 0.006$ is ideal for uncertainty in W, Z (Higgs?) at the LHC.





Uncertainty due to PDFs

Greater than NNLO scale dependence.



Based on (Anastasiou *et al.*), from Lance Dixon.

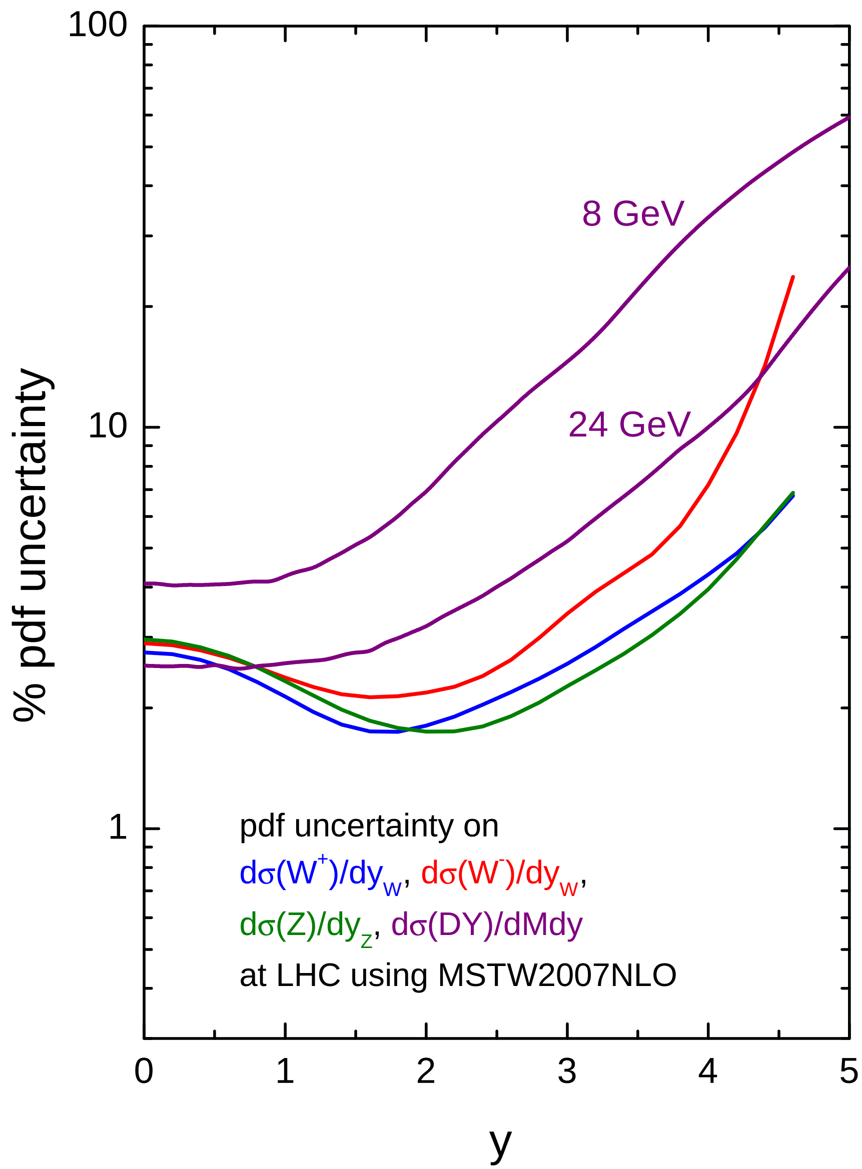
W, Z **uncertainty – more details**

Uncertainty on $\sigma(Z)$ and $\sigma(W^+)$ grows at high rapidity.

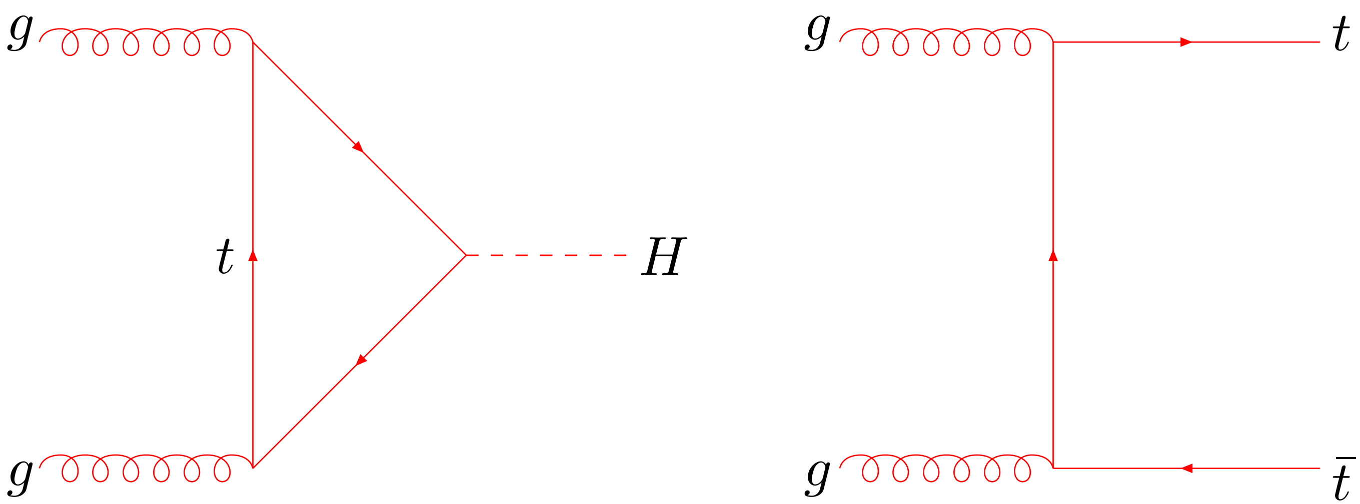
Uncertainty on $\sigma(W^-)$ grows more quickly at very high y – depends on less well-known down quark.

Uncertainty on $\sigma(\gamma^\star)$ is greatest as y increases. Depends on partons at very small x.

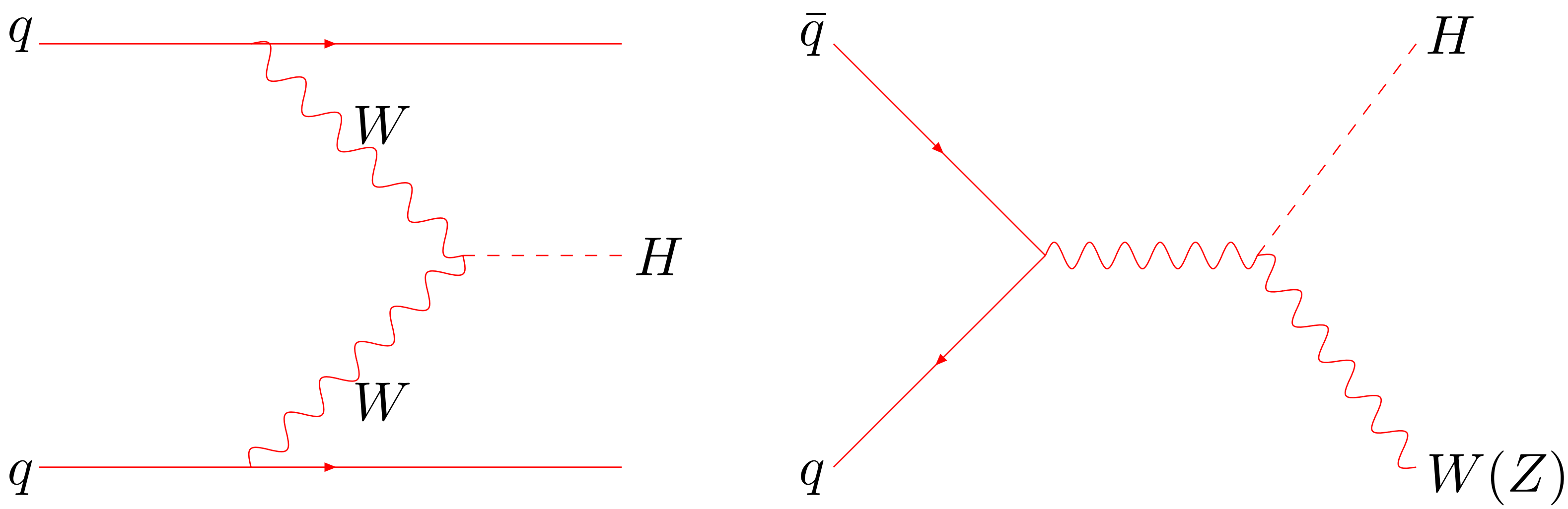
Lots of interest in LHCb range.



Dominant Higgs production mechanism, gluon-gluon fusion via top quark loop. Very similar to top production (at the LHC)

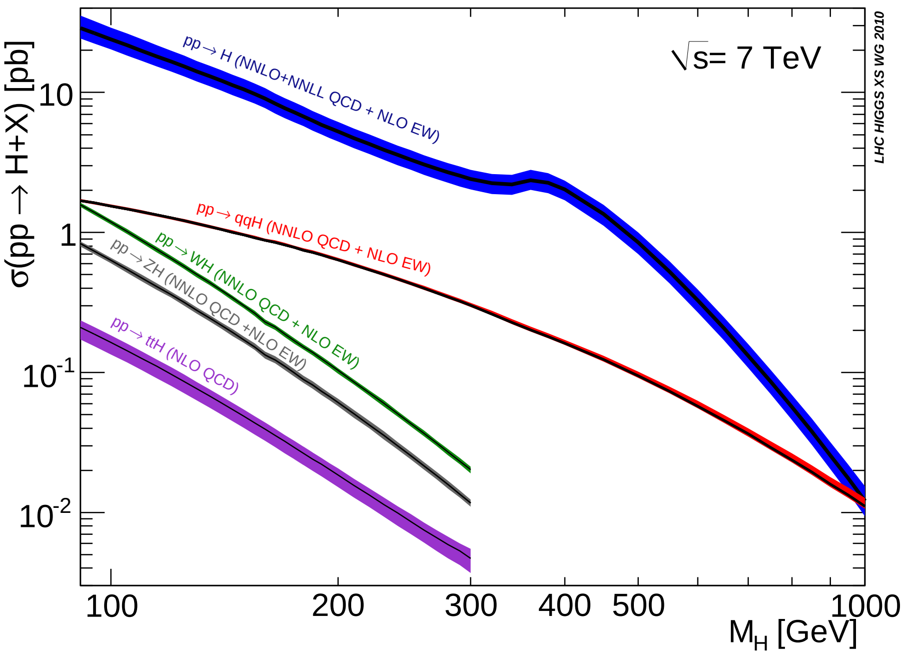


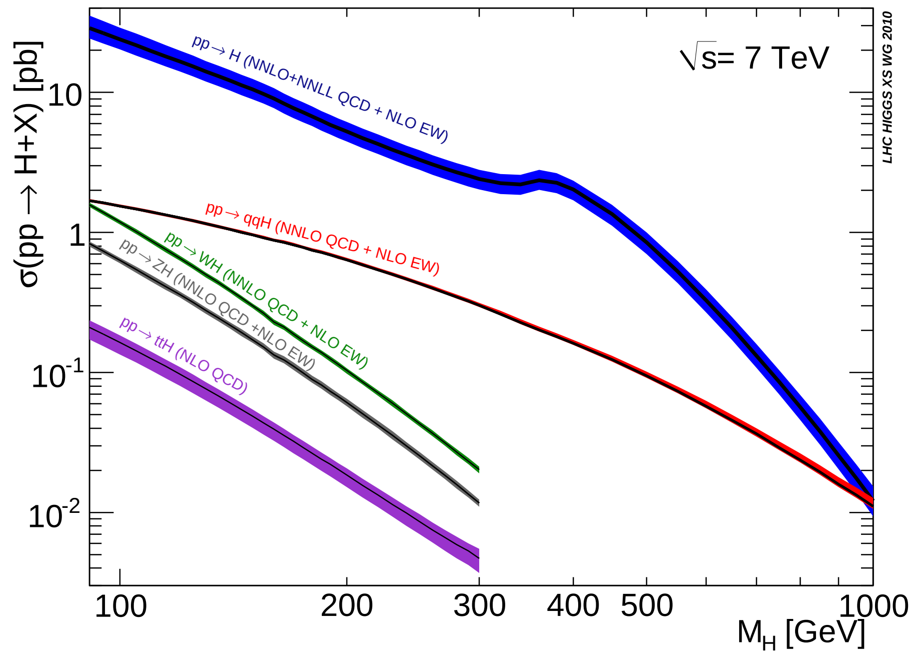
Also large Higgs contributions from vector boson fusion and associated production with $W(Z)$.



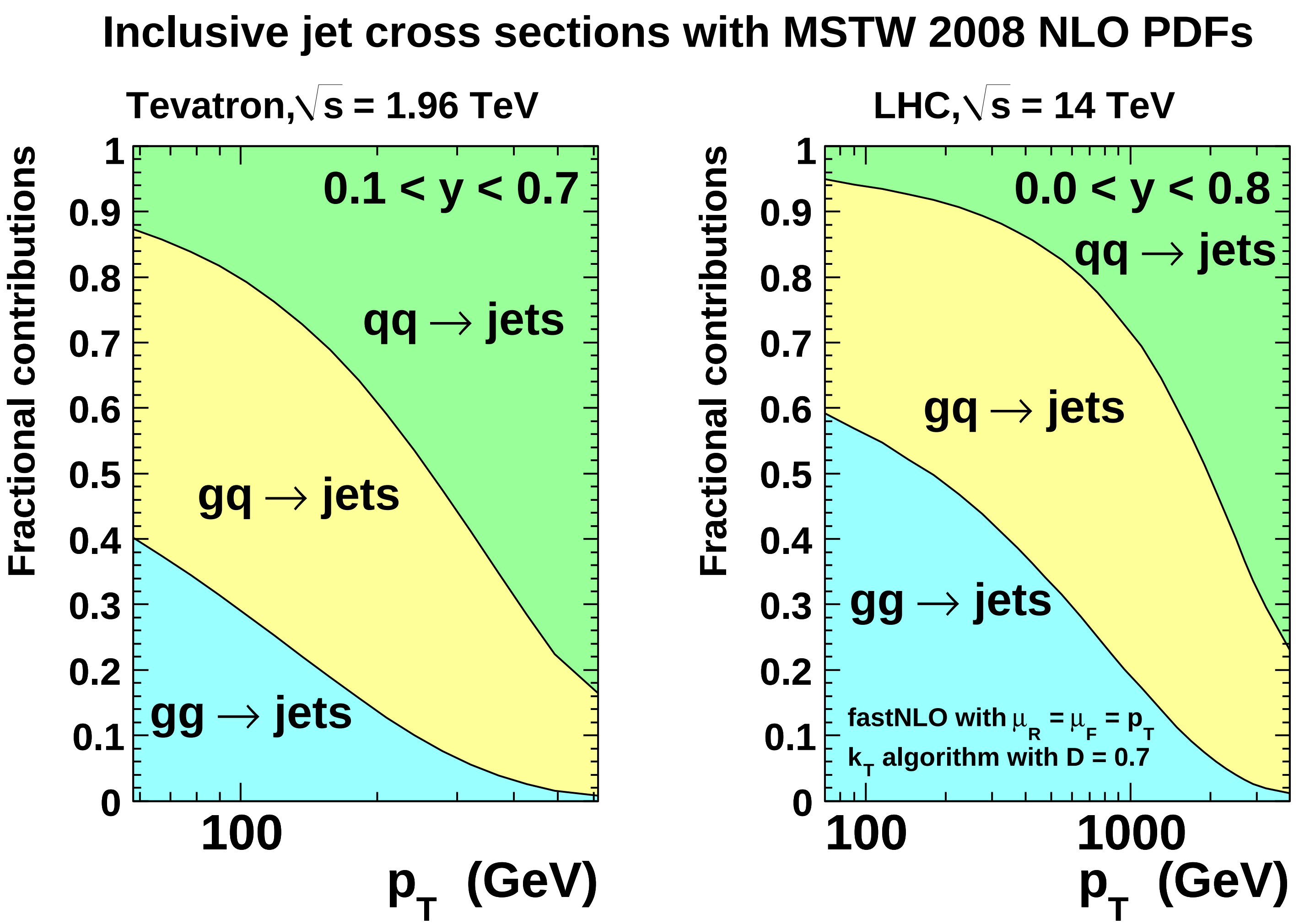
For gluon-gluon fusion cross-section known at NNLO (Harlander, Harlander and Kilgore, Anastasiou and Melnikov, Catani *et al* and Ravindran *et al*) (in large m_t limit). The associated production is known at NNLO (Brein *et al*). There are NLO codes for VBF (VBFNLO – Arnold *et al*, and HAWK – Denner *et al*) and approx. NNLO VBF results are known Bolzoni *et al*).

For given PDF set Higgs cross-sections usually dominated by theory (scale) uncertainties (particularly dominant $gg \rightarrow H$ mechanism).





Plots from Handbook of LHC Higgs Cross Sections



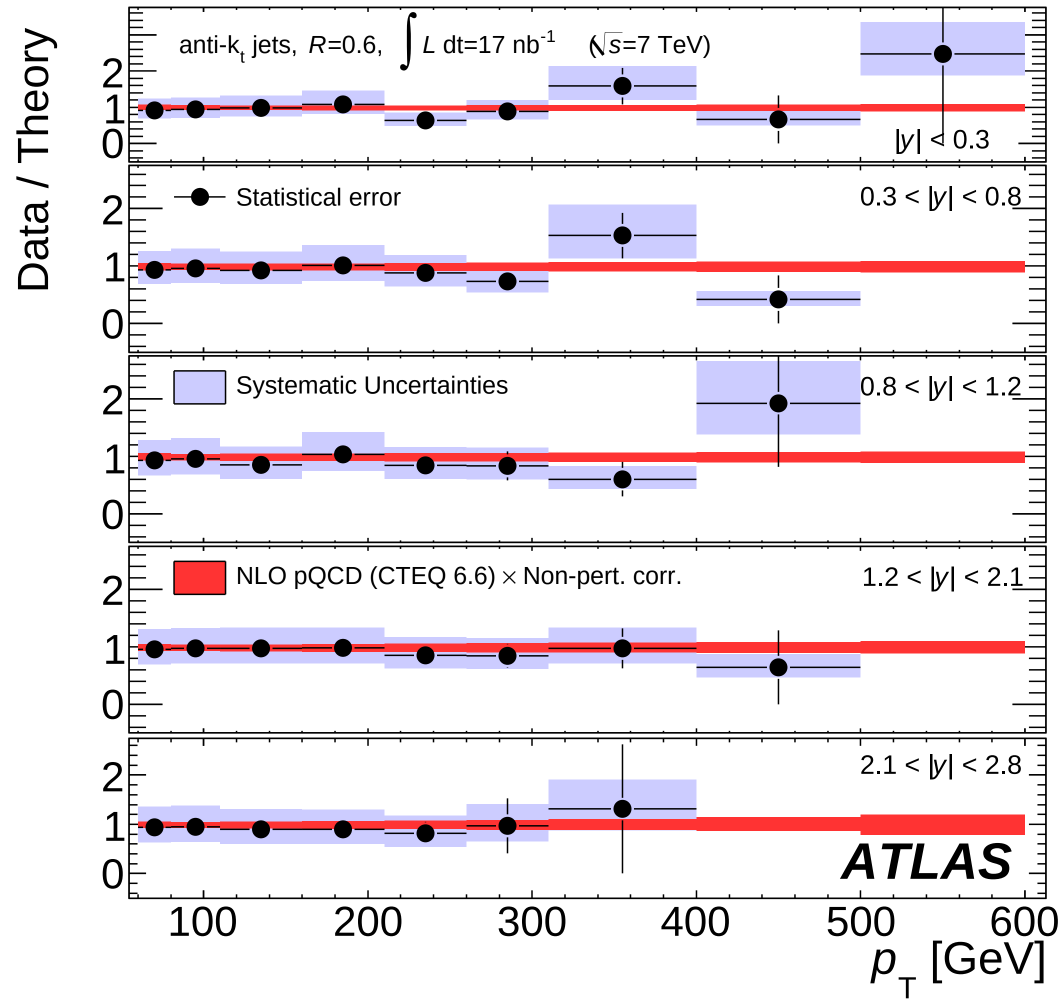
Gluon probed directly by Tevatron jet production.

Proton-proton rather than proton-antiproton at LHC leads to more gluon dependence.

Correlated with Higgs production.

In future the LHC jet data will be a good constraint (and test) of the gluon.

At present statistics, and jet energy scale uncertainty not well enough advanced.



Cross-section ratios

Ratios of vector bosons rates are useful. Cancel many experiment uncertainties, and theory – mainly PDF left. Can use

$$R_{Z/W} = \frac{\sigma(Z)}{\sigma(W^+) + \sigma(W^-)} \simeq \frac{Au(\tilde{x}_1)\bar{u}(\tilde{x}_2) + Bd(\tilde{x}_1)\bar{d}(\tilde{x}_2)}{u(x_1)\bar{d}(x_2) + d(x_1)\bar{u}(x_2)} \simeq \frac{Au(\tilde{x}_1) + Bd(\tilde{x}_1)}{u(x_1) + d(x_1),}$$

Where we have used $\bar{u}(\tilde{x}_2) \approx \bar{d}(\tilde{x}_2)$ and ignored small(ish) strange, charm $etc.$ contributions. This is very precisely predicted, but is equal to A plus small corrections.

$$A_{\pm} = \frac{(\sigma(W^+) - \sigma(W^-))}{(\sigma(W^+) + \sigma(W^-))} \simeq \frac{u(x_1)\bar{d}(x_2) - d(x_1)\bar{u}(x_2) + (\bar{d}(x_1)u(x_2) - \bar{u}(x_1)d(x_2))}{u(x_1)\bar{d}(x_2) + d(x_1)\bar{u}(x_2) + (\bar{d}(x_1)u(x_2) + \bar{u}(x_1)d(x_2))}$$

$$\simeq \frac{u_V(x_1) - d_V(x_1)}{u(x_1) + d(x_1)}$$

so is a good test of valence quarks. Alternatively we have

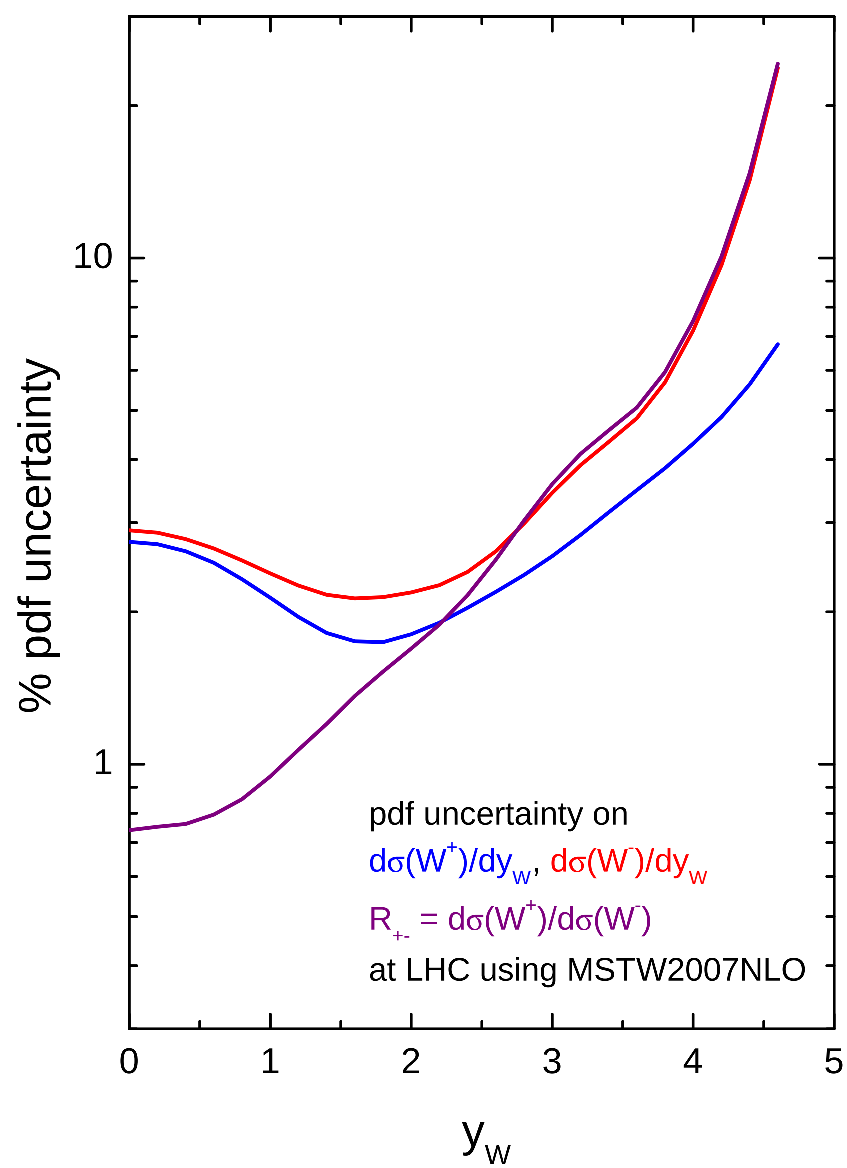
$$R_{\pm} = \frac{\sigma(W^-)}{\sigma(W^+)} \simeq \frac{d(x_1)\bar{d}(x_2)}{u(x_1)\bar{d}(x_2)} \simeq \frac{d(x_1)}{u(x_1)},$$

Uncertainty on $R_{Z/W}$ is very small. Parton combinations highly correlated.

Assumes $\bar{u}(x_2) \approx \bar{d}(x_2)$. Easily checks if this is true.

Uncertainty on $A_\pm$ not strongly y-dependent.

Uncertainty on $R_\pm$ increases strongly at high y.



Lepton Asymmetry

In practice it is leptons seen in final state rather than W and Z. For former this causes complications. For $W^\pm$ only one charged lepton is seen.

Defining angle of lepton in W rest frame

$$\cos^2\theta^* = 1 - 4p_T^2/M_W^2 \quad \rightarrow \quad y_{lep} = y_W \pm 1/2\log((1+\cos\theta^*)/(1-\cos\theta^*))$$

If $p_T = 30\mathrm{GeV}$ – $\cos\theta^* = 0.66$ and $y_{lep} = y_W \pm 0.8$.

From helicity the decay of the lepton from the boson has distribution

$$(1+\cos\theta^*)^2 \quad \text{or} \quad (1-\cos\theta^*)^2.$$

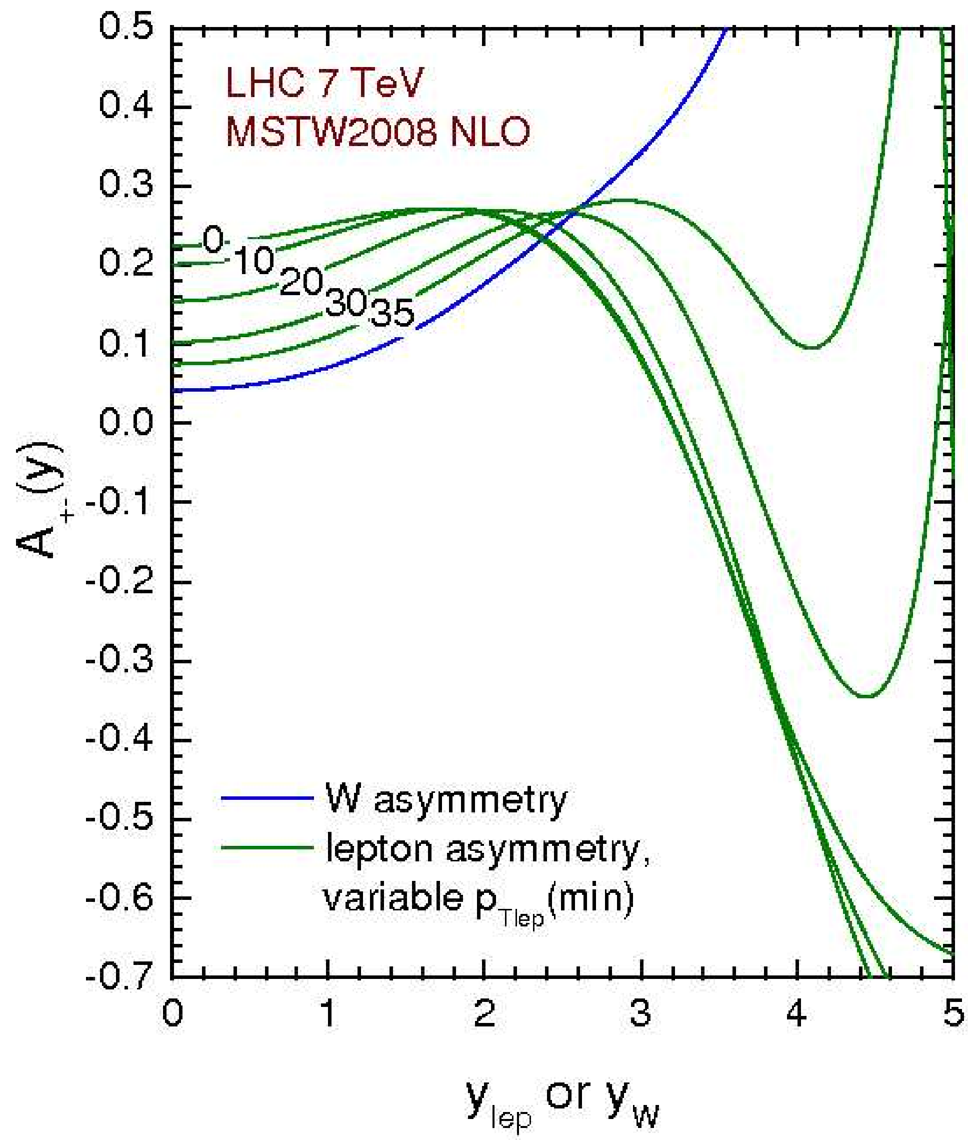
If the former dominates then

$$A_\pm = \frac{(\sigma(l^+) - \sigma(l^-))}{(\sigma(l^+) + \sigma(l^-))} \simeq \frac{\bar{d}(x_1)u(x_2) - d(x_1)\bar{u}(x_2)}{\bar{d}(x_1)u(x_2) + d(x_1)\bar{u}(x_2)}$$

which makes no difference for small y, i.e. $x_1 \approx x_2$ but for $x_1 \gg x_2$ can change the sign of the asymmetry.

The smaller the $p_{T(\min}$ the more effect the decay distribution has.

Ultimately at high enough y or x_1 the dominance of the $u_V(x_1)$ distribution takes over and $A_\pm \to 1$.



How straightforward is it in practice?

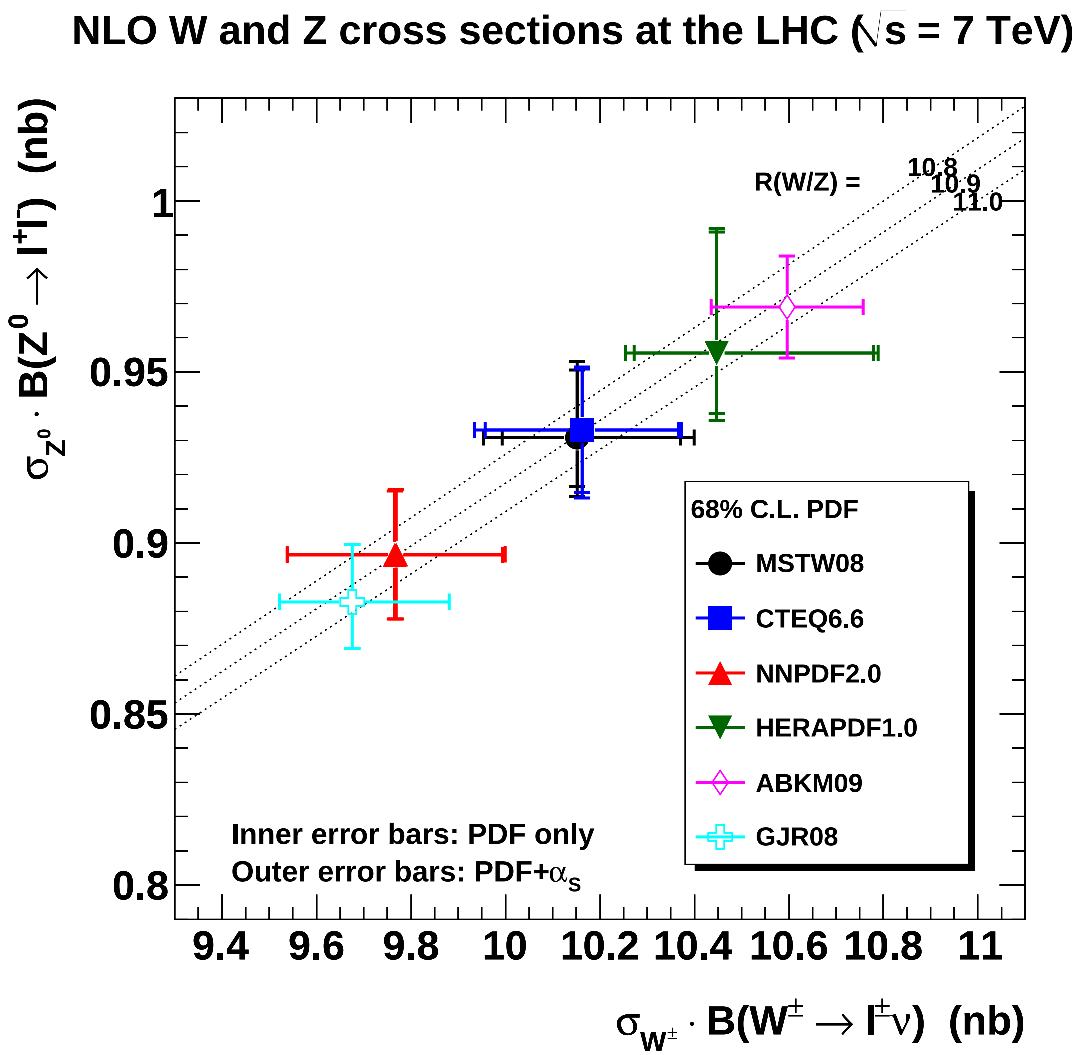
Predictions (Watt) for W and Z cross-sections for LHC with common NLO QCD and vector boson width effects, and common branching ratios, and at 7TeV.

Comparing all groups get significant discrepancies between them even for this benchmark process.

Can understand some of the systematic differences.

Some difference in W/Z ratio.

W, Z total cross-sections best-case scenario.



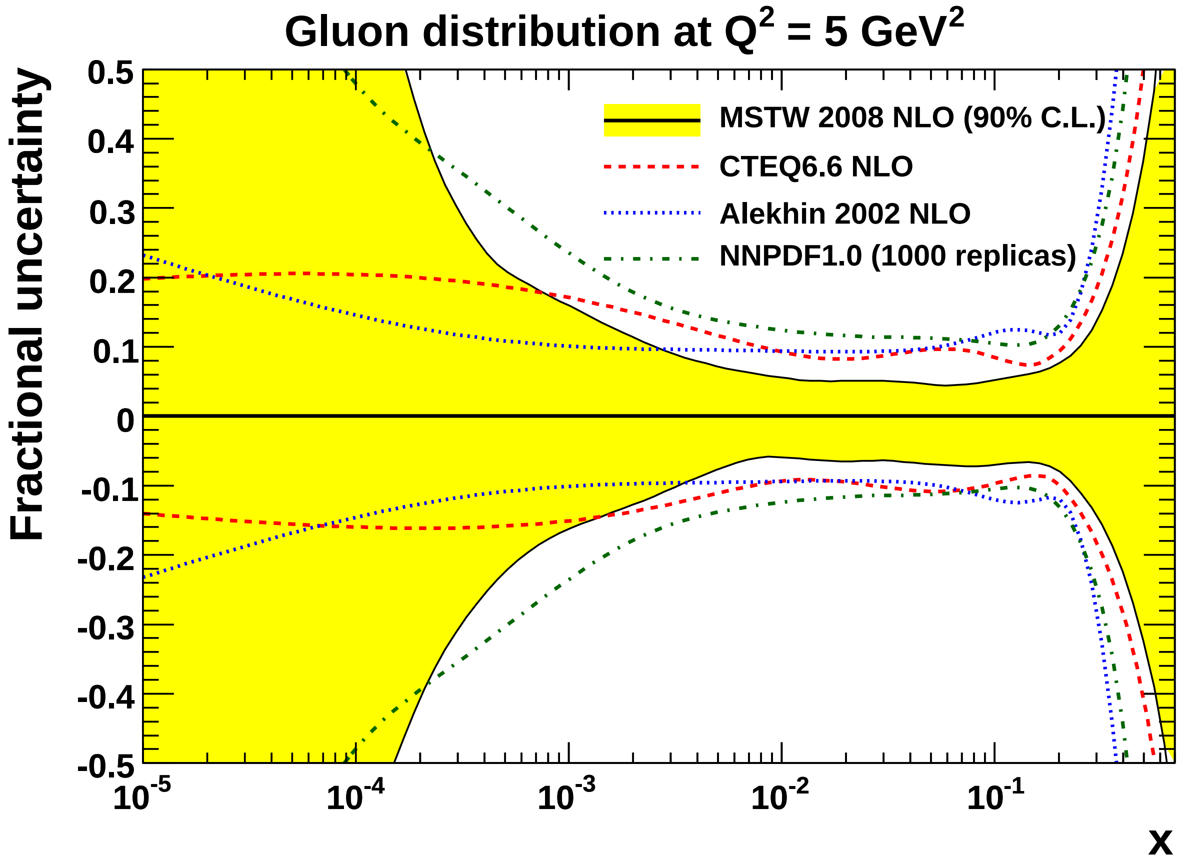
Sources of Variations/Uncertainty

It is vital to consider theoretical/assumption-dependent uncertainties:

- Methods of determining "best fit" and uncertainties.
- Underlying assumptions in procedure, e.g. parameterisations and data used.
- Treatment of heavy flavours.
- PDF and α_S correlations.

Responsible for differences between groups for extraction of fixed-order PDFs.

Gluon Parameterisation - small x – different parameterisations lead to very different uncertainty for small x gluon.

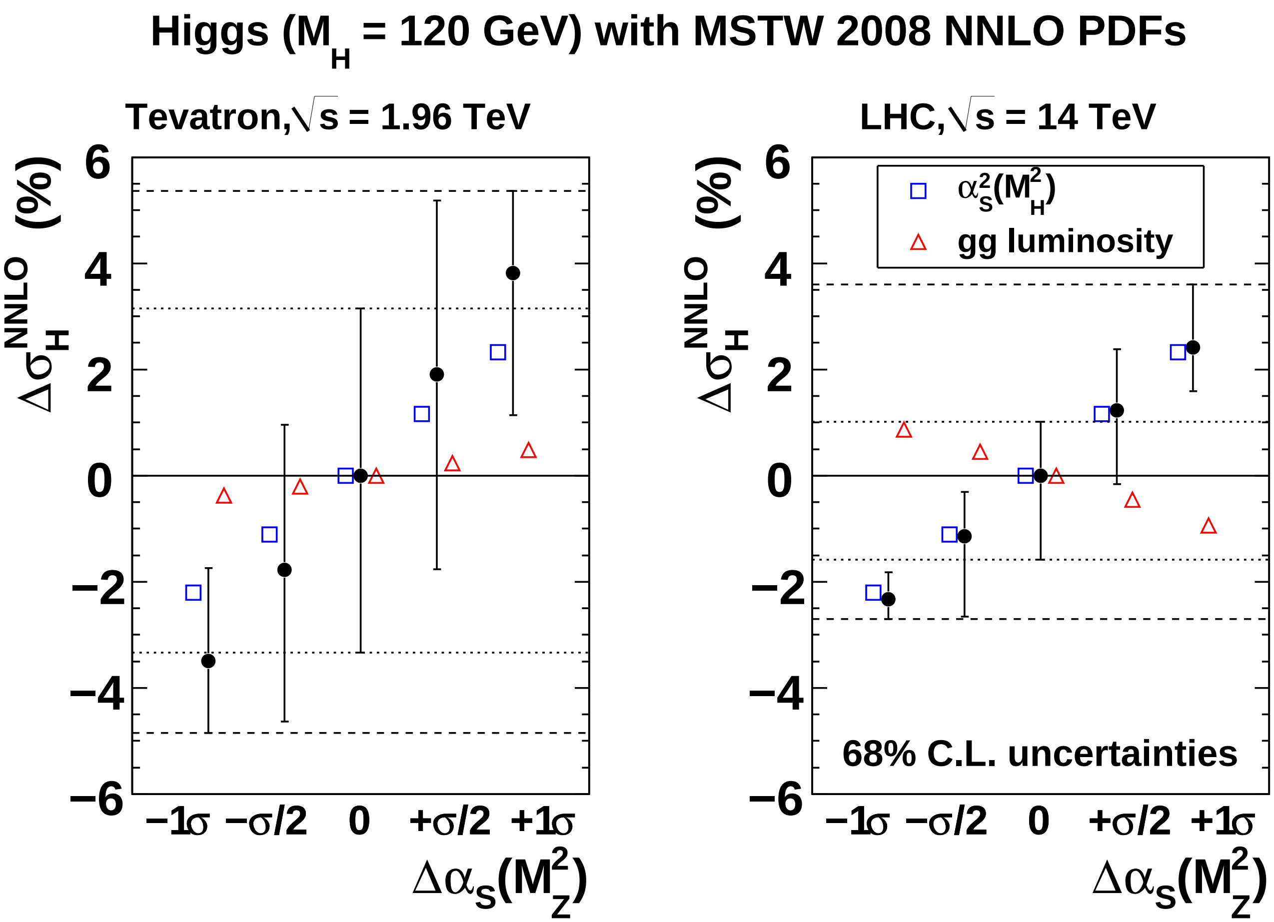


Most assume single power x^{λ} at input $\rightarrow$ limited uncertainty. If input at low Q^2 λ positive and small-x input gluon *fine-tuned* to ~ 0. Artificially small uncertainty.

If $g(x) \propto x^{\lambda \pm \Delta\lambda}$ then $\Delta g(x) = \Delta\lambda \ln(1/x) * g(x)$.

MRST/MSTW and NNPDF more flexible (can be negative) $\rightarrow$ rapid expansion of uncertainty where data runs out.

PDF correlation with α_S. NNLO predictions for Higgs ($120\mathrm{GeV}$) production for different allowed $\alpha_S(M_Z^2)$ values and their uncertainties.



Increases by a factor of $2-3$ (up more than down) at LHC. Direct $\alpha_S(M_Z^2)$ dependence mitigated somewhat by anti-correlated small-x gluon (asymmetry feature of *minor* problems in fit to HERA data). At Tevatron intrinsic gluon uncertainty dominates.

Heavy Quarks. Should use a **General Mass Variable Flavour Number Scheme** (GM-VFNS) rather than **Fixed Flavour Number Scheme** (FFNS) or **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS) to fit structure function data.

Still variation – values of the predicted cross-sections at NLO for Z and a 120 GeV Higgs boson at the Tevatron and the LHC (latter for 14 TeV) as GM-VFNS altered.

PDF set	Tev		LHC	(14 TeV)
	σ_Z (nb)	σ_H(pb)	σ_Z (nb)	σ_H(pb)
MSTW08	7.207	0.7462	59.25	40.69
GMvar1	$+0.3\%$	-0.5%	$+1.1\%$	$+0.2\%$
GMvar2	$+0.7\%$	-1.1%	$+3.0\%$	$+1.5\%$
GMvar3	$+0.1\%$	-0.3%	$+1.1\%$	$+0.8\%$
GMvar4	$+0.0\%$	-0.1%	-0.4%	-0.2%
GMvar5	-0.1%	-0.1%	-0.5%	-0.3%
GMvar6	$+0.3\%$	-0.4%	$+1.6\%$	$+0.8\%$
GMvaropt	$+0.3\%$	-1.5%	$+2.0\%$	$+0.4\%$
ZM-VFNS	-0.7%	-1.2%	-3.0%	-3.1%

Little more than 1% variation at Tevatron in σ_Z. Up to $+3\%$ and -0.5% variation in σ_Z at the LHC. About half as much in σ_H due to higher average x sampled.

The values of the predicted cross-sections at NNLO.

PDF set	Tev		LHC	(14 TeV)
	σ_Z (nb)	σ_H(pb)	σ_Z (nb)	σ_H(pb)
MSTW08	7.448	0.9550	60.93	50.51
GMvar1	$+0.1\%$	-0.5%	$+0.1\%$	-0.2%
GMvar2	$+0.3\%$	-0.8%	$+0.5\%$	$+0.1\%$
GMvar3	$+0.4\%$	-0.1%	$+0.5\%$	$+0.7\%$
GMvar4	$+0.0\%$	-0.2%	$+0.1\%$	-0.1%
GMvar5	$+0.1\%$	-0.3%	-0.2%	-0.2%
GMvar6	$+0.1\%$	-0.9%	$+0.3\%$	-0.2%
GMvaropt	$+0.4\%$	-0.2%	$+0.6\%$	$+0.8\%$
GMvarmod	-0.2%	-0.4%	-1.4%	-1.0%
GMvarmod′	$+0.0\%$	-0.7%	$+0.0\%$	$+0.1\%$

Maximum variations of order 1% at LHC. High-x gluon leads to 1% on σ_H at Tevatron.

Much improved stability compared to NLO.

Uncertainties due to m_c and m_b

Add uncertainties in quadrature with PDF parameter and α_S combined uncertainty.

LHC, $\sqrt{s} = 7$ TeV	$B_{\ell\nu} \cdot \sigma^W$	$B_{\ell^+\ell^-} \cdot \sigma^Z$	σ^H
Central value	10.47 nb	0.958 nb	15.50 pb
PDF only uncertainty	$^{+1.7\%}_{-1.6\%}$	$^{+1.7\%}_{-1.5\%}$	$^{+1.1\%}_{-1.6\%}$
PDF+α_S uncertainty	$^{+2.5\%}_{-1.9\%}$	$^{+2.5\%}_{-1.9\%}$	$^{+3.7\%}_{-2.9\%}$
PDF+α_S+$m_{c,b}$ uncertainty	$^{+2.7\%}_{-2.2\%}$	$^{+2.9\%}_{-2.4\%}$	$^{+3.7\%}_{-2.9\%}$

LHC, $\sqrt{s} = 14$ TeV	$B_{\ell\nu} \cdot \sigma^W$	$B_{\ell^+\ell^-} \cdot \sigma^Z$	σ^H
Central value	21.72 nb	2.051 nb	50.51 pb
PDF only uncertainty	$^{+1.7\%}_{-1.7\%}$	$^{+1.7\%}_{-1.6\%}$	$^{+1.0\%}_{-1.6\%}$
PDF+α_S uncertainty	$^{+2.6\%}_{-2.2\%}$	$^{+2.6\%}_{-2.1\%}$	$^{+3.6\%}_{-2.7\%}$
PDF+α_S+$m_{c,b}$ uncertainty	$^{+3.0\%}_{-2.7\%}$	$^{+3.1\%}_{-2.8\%}$	$^{+3.7\%}_{-2.8\%}$

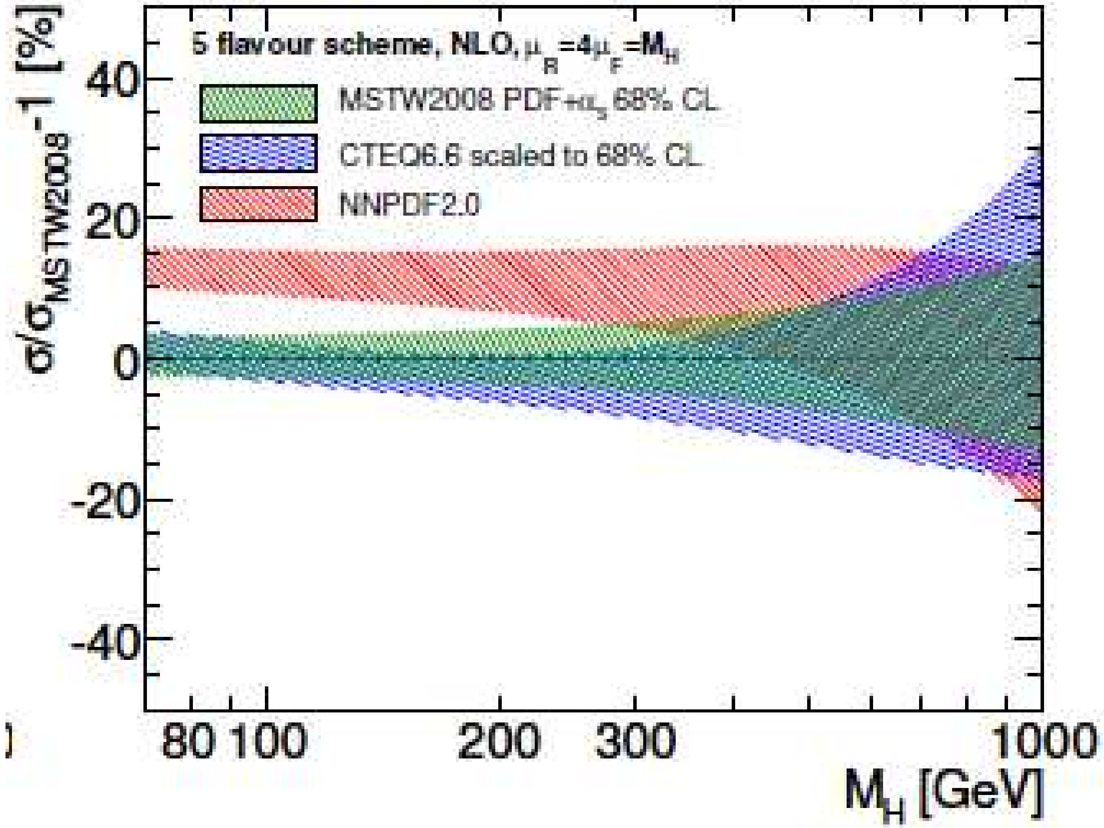
NNLO predictions for W, Z and Higgs ($M_H = 120$ GeV) total cross sections or 7 TeV LHC and 14 TeV LHC. Similar results in HERAPDF study Cooper-Sarkar.

α_S uncertainties more important, particularly for Higgs. Mass uncertainties significant, but least important of three effects, particularly for Higgs.

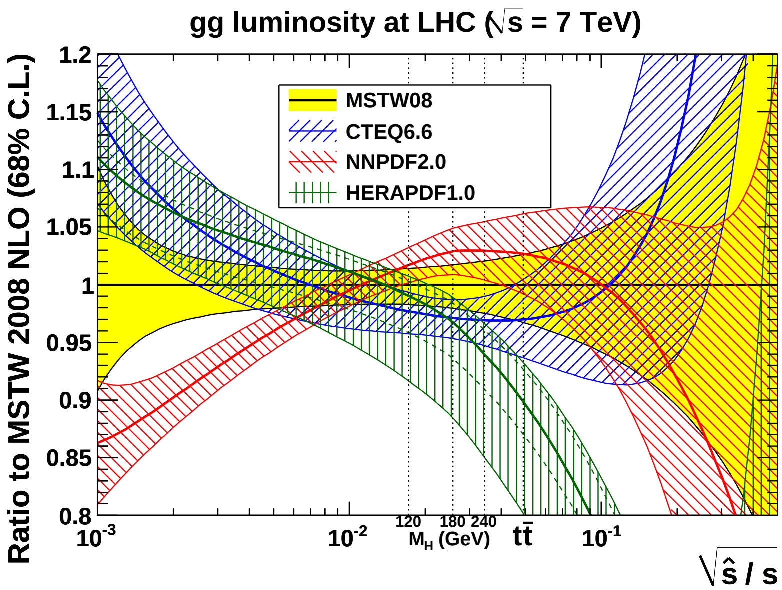
But consider BSM example for Higgs by Warsinsky at recent Higgs-LHC working group meeting.

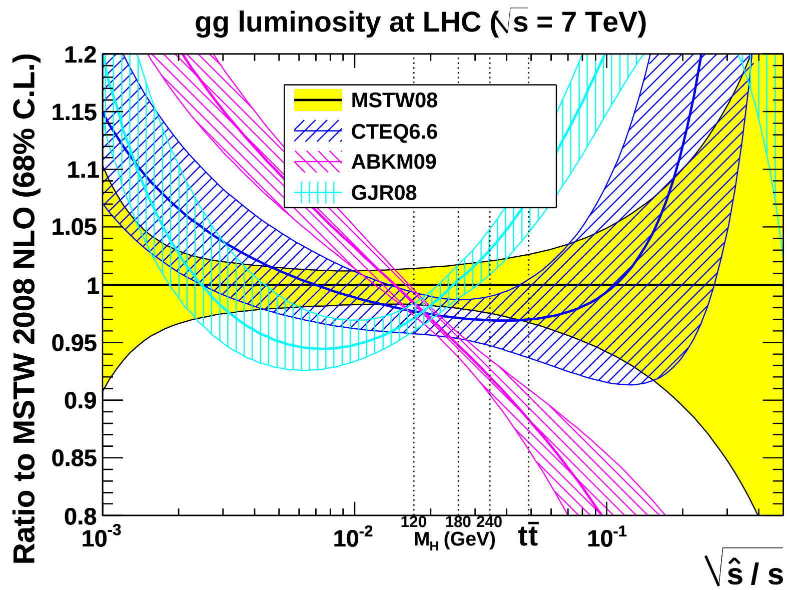
m_b values bring CTEQ and MSTW together but exaggerate NNPDF difference.

Couplings have assumed common mass value.



Predictions by various groups - parton luminosities – NLO. Plots by G. Watt.



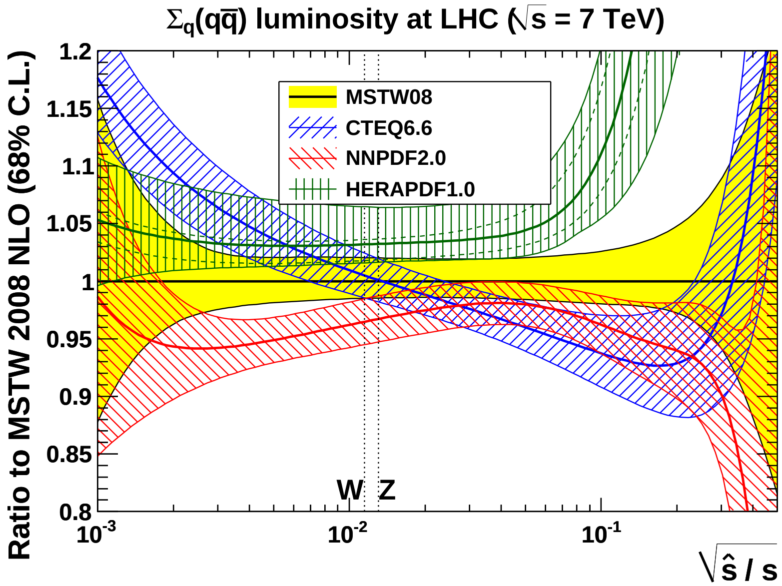


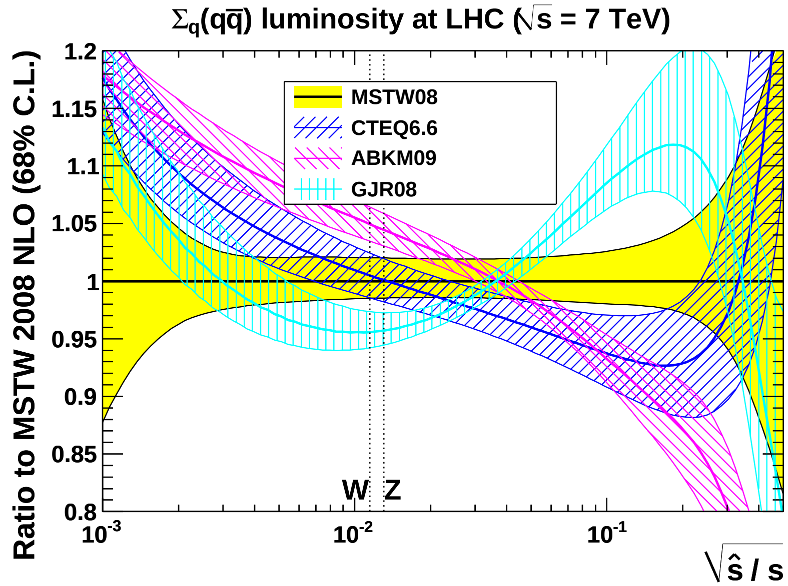
$$\frac{dL_{ij}}{d\hat{s}dy} = \frac{1}{s}\frac{1}{1+\delta_{ij}}[f_i(x_1)f_j(x_2) + f_i(x_1)f_j(x_2)] \quad \text{and integrate over } y$$

Cross-section for $t\bar{t}$ almost identical in PDF terms to 450GeV Higgs.

Also $H + t\bar{t}$ at $\sqrt{\hat{s}/s} \sim 0.1$.

Clearly some distinct variation between groups. Much can be understood in terms of previous differences in approaches.



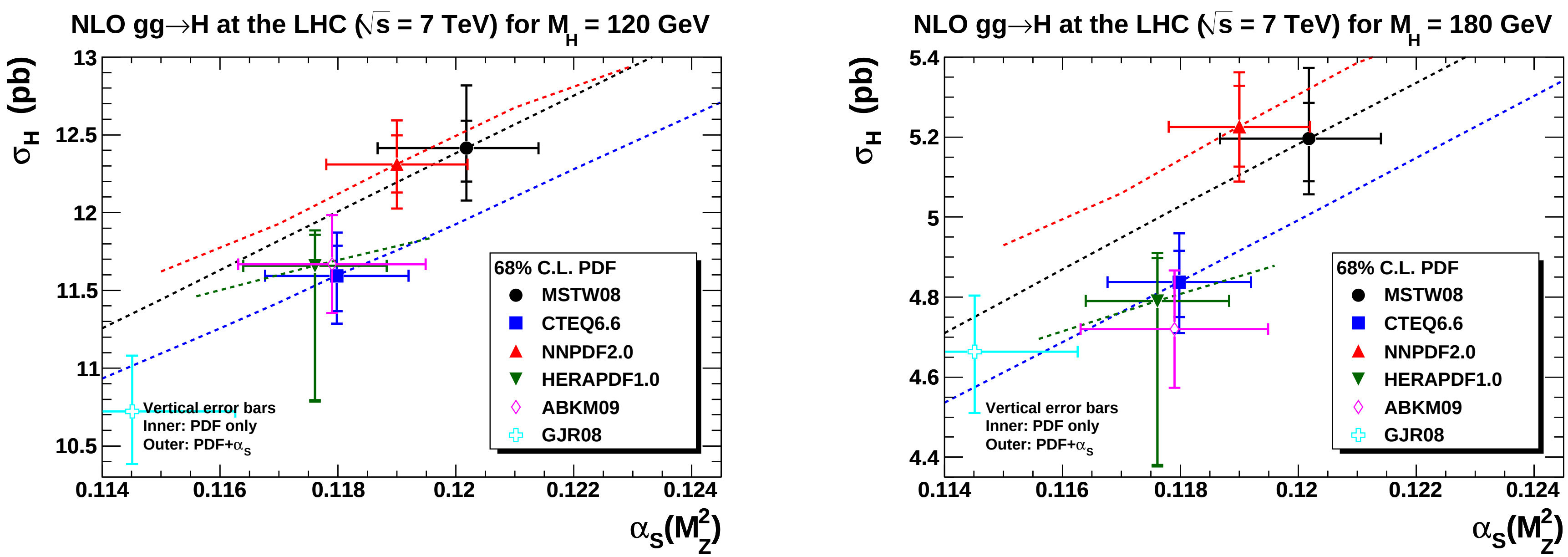


Many of the same general features for quark-antiquark luminosity. Some differences mainly at higher x.

Canonical example W, Z production, but higher $\hat{s}/s$ relevant for WH or vector boson fusion.

All plots and more at http://projects.hepforge.org/mstwpdf/pdf4lhc

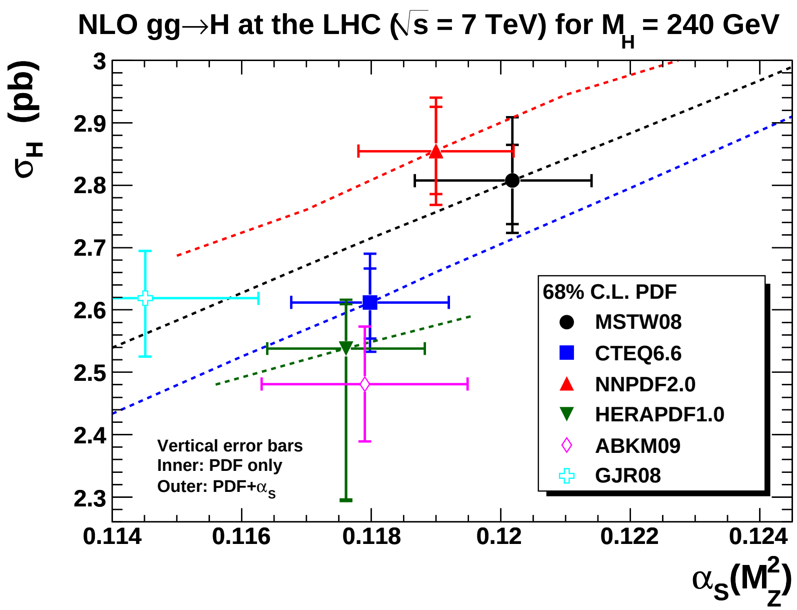
Variations in Cross-Section Predictions – NLO

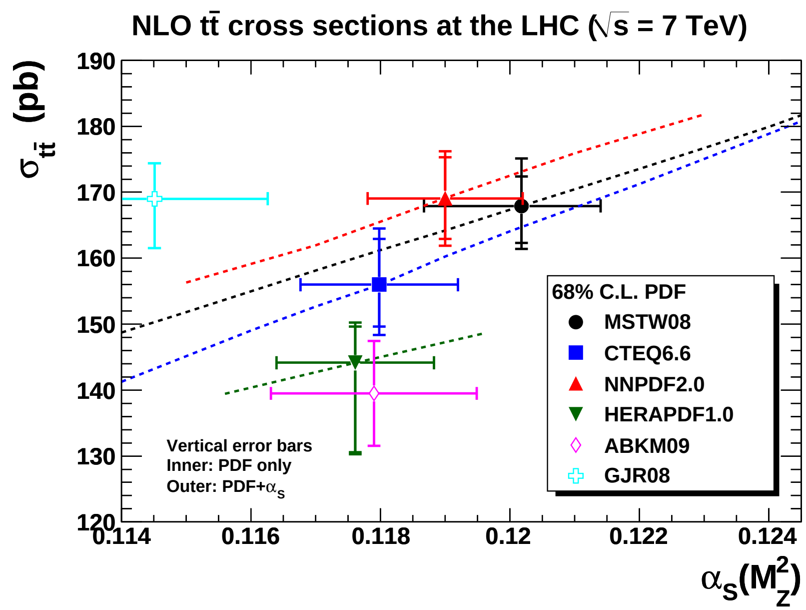


Dotted lines show how central PDF predictions vary with $\alpha_S(M_Z^2)$.

Plots based on PDF4LHC benchmark criteria, but from extensive independent study by G. Watt.

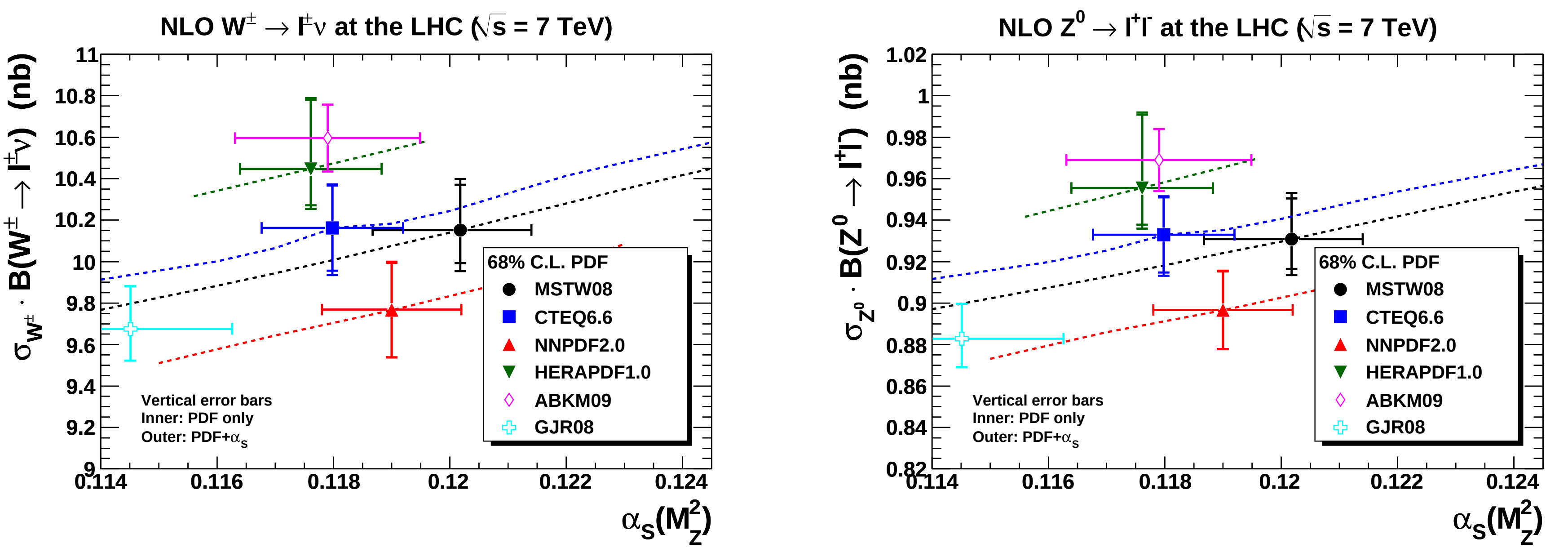
Clearly much more variation in predictions than uncertainties claimed by individual groups.





Excluding GJR08 amount of difference due to $\alpha_S(M_Z^2)$ variations $3-4\%$.

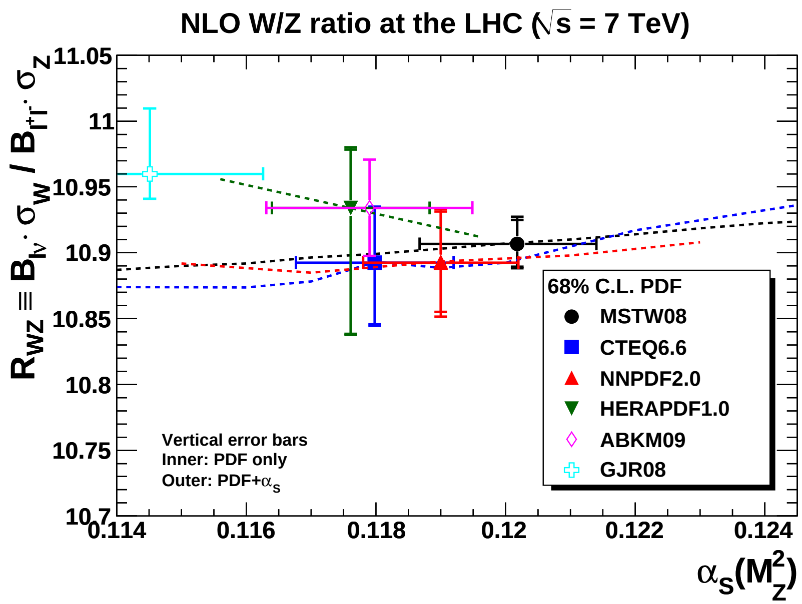
CTEQ6.6 now heading back towards MSTW08 and NNPDF2.0.

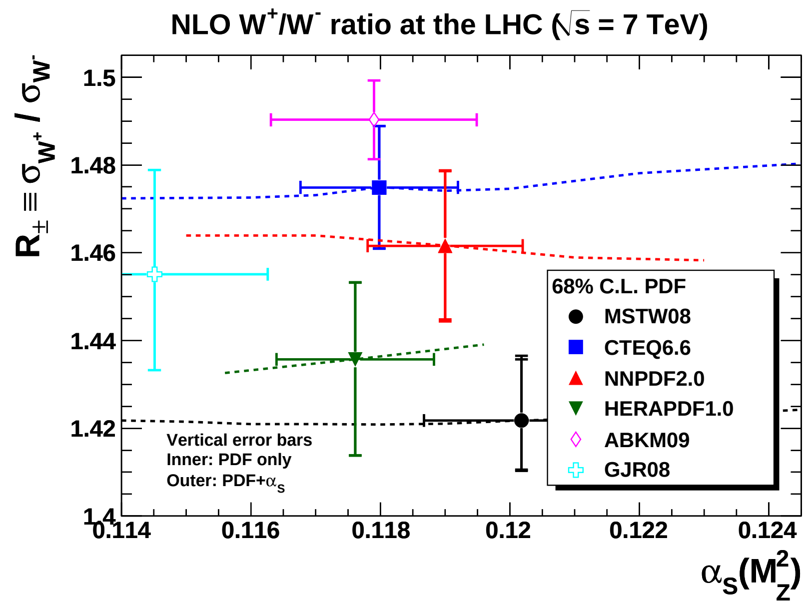


$W^+ + W^-$ cross-section. $\alpha_S(M_Z^2)$ dependence now more due to PDF variation with $\alpha_S(M_Z^2)$.

Again variations somewhat bigger than individual uncertainties.

Roughly similar variation for $\hat{s}$ up to a few times higher.





For W/Z values consistent but uncertainties vary. Largely due to strange uncertainty.

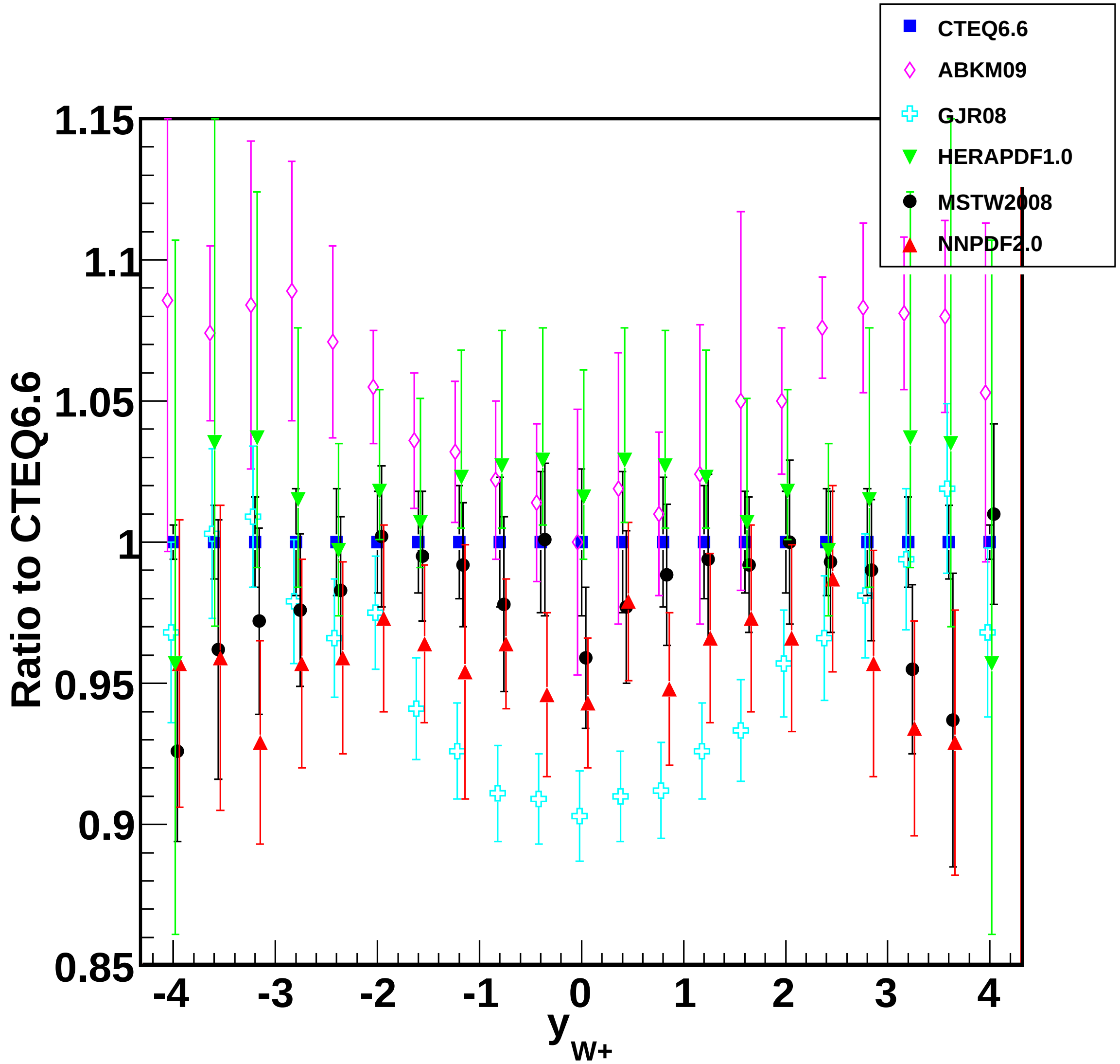
Quite a variation in ratio for W^+/W^-. Shows variations in flavour and quark-antiquark decompositions.

All plots and more at http://projects.hepforge.org/mstwpdf/pdf4lhc

Differences also clear in rapidity distributions.

Plot from PDF4LHC Interim Report.

Shape discriminating even if normalisation will be difficult for a while.

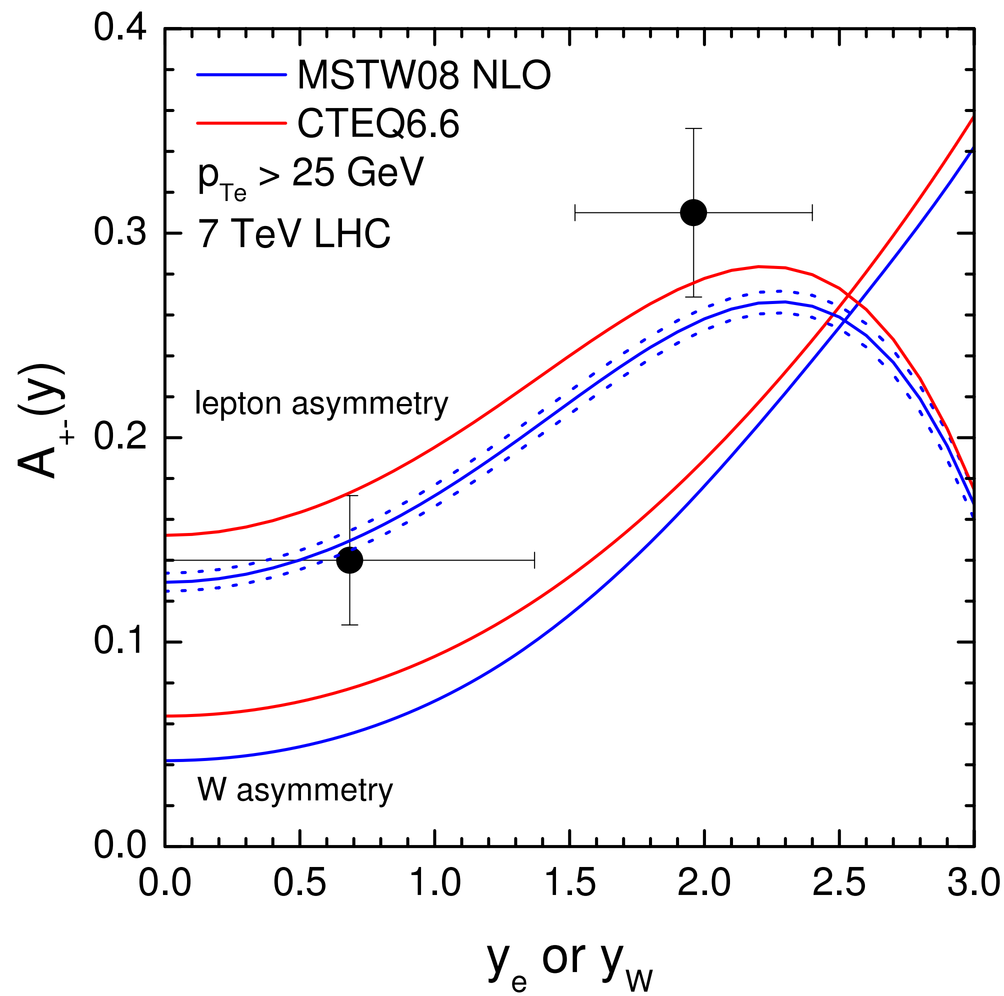


Translates into some significant differences in the more different W-asymmetry predictions.

MSTW08 and CTEQ6.6 about the biggest discrepancy at low y.

HERAPDF diverges from others at highest y.

Possibly first real discriminating power from LHC measurements.



Deviations In predictions clearly much more than uncertainty claimed by each.

In some cases clear reason why central values differ, e.g. lack of some constraining data, though uncertainties then do not reflect true uncertainty.

Sometimes no good understanding, or due to difference in procedure which is simply a matter of disagreement, e.g. gluon parameterisation at small x affects predicted Higgs cross-section.

What is true uncertainty for use in setting limits? Task asked of PDF4LHC group.

Interim recommendation take envelope of $global$ sets, MSTW, CTEQ NNPDF (check other sets) and take central point as uncertainty.

Not very satisfactory, but not clear what would be an improvement, especially as a general rule.

Usually not a big disagreement, and factor of about 2 expansion of MSTW uncertainty.

PDF uncertainty larger than that from single group, but only by a reasonable factor usually ≤ 2. Can be much more if all PDFs used.

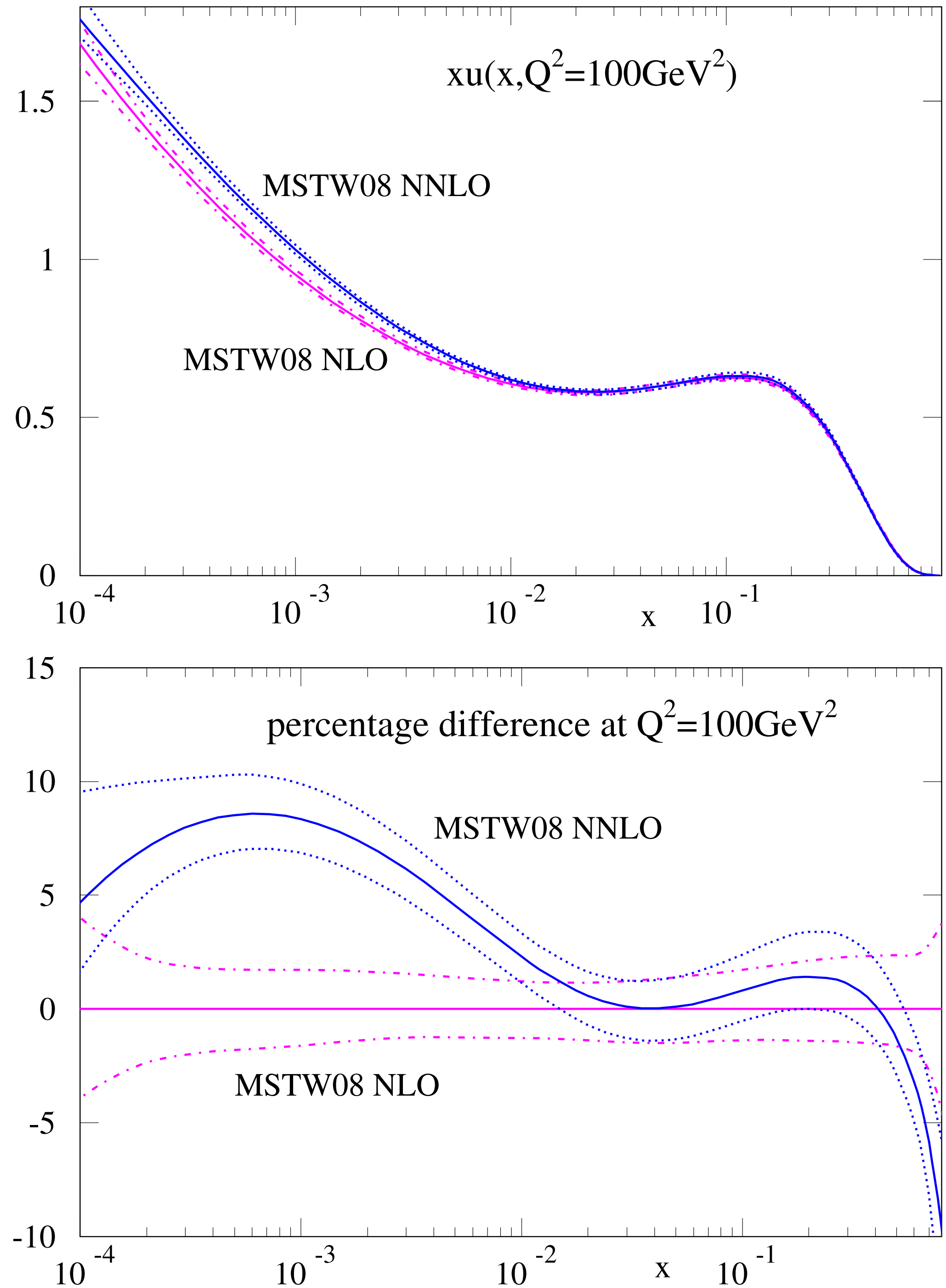
Other sources of Uncertainty.

Also other sources which (mainly) lead to inaccuracies common to all fixed-order extractions.

- QED and Weak (comparable to NNLO ?) ($\alpha_s^3 \sim \alpha$). Sometime enhancements.
- Standard higher orders (NNLO – some sets available here.)
- Resummations, e.g. small x ($\alpha_s^n \ln^{n-1}(1/x)$), or large x ($\alpha_s^n \ln^{2n-1}(1-x)$) or equivalently summations in high-energy limit and threshold limit.
- low Q^2 (higher twist), saturation.

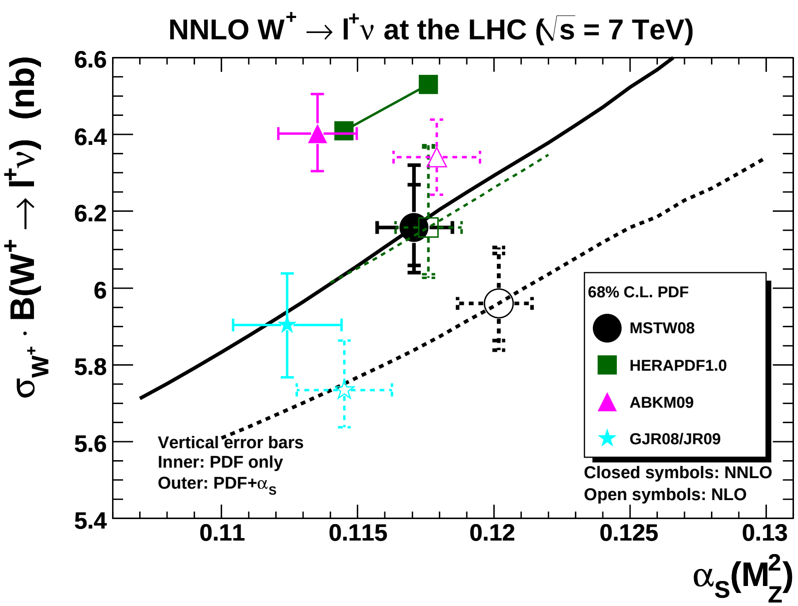
Stability order-by-order.

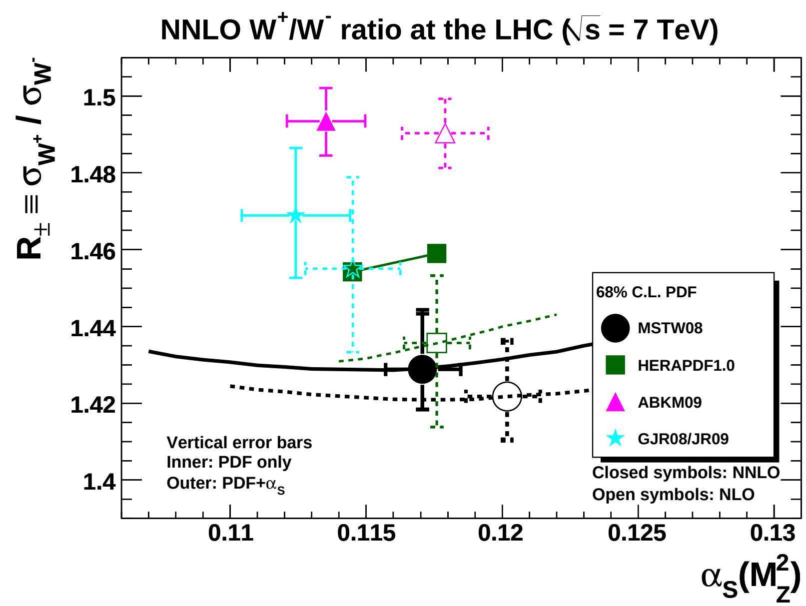
Systematic difference between PDF defined at NLO and at NNLO.



Consideration of NNLO – Benchmark results

Plots from not yet published results by G. Watt



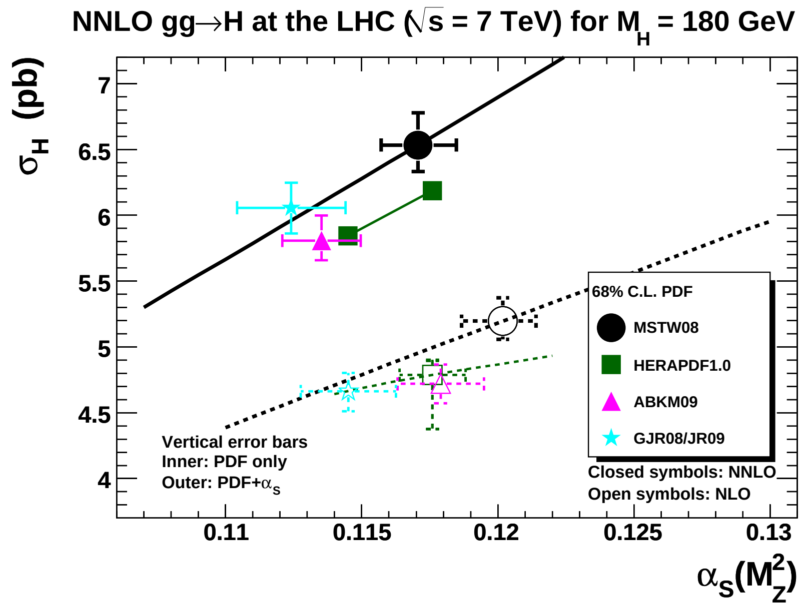


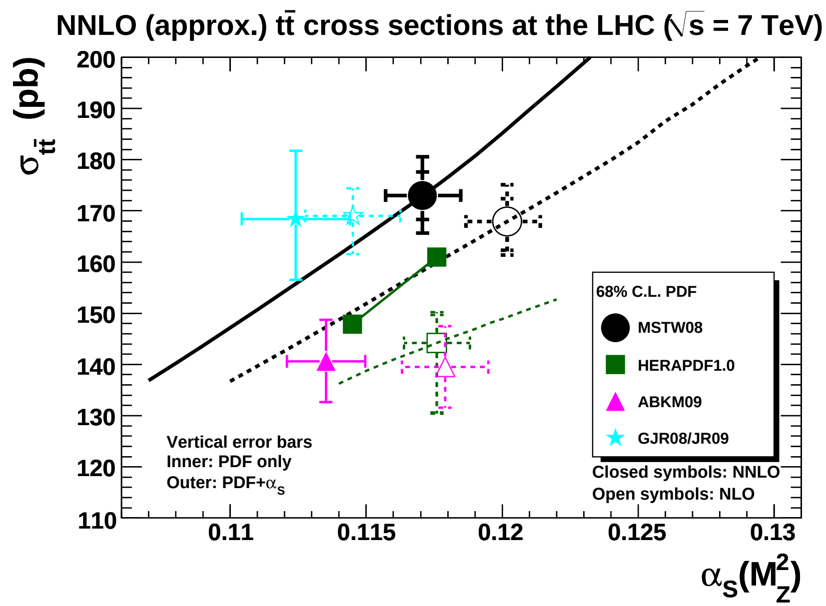
Differences between PDF sets very similar as at NLO, whereas differences from theoretical choices should diminish differences.

More stability if NNLO α_S lower than at NLO.

NNLO corrections have minor effect on asymmetries.

Plots from not yet published results by G. Watt





Differences between groups significant at NNLO, and similar to NNLO – parton luminosity compassion similar at NLO to NNLO.

Approx NNLO using HATHOR - (Aliev *et al*), includes scale-dependent parts and large threshold corrections at NNLO. Hence some theoretical uncertainty, but NNLO corrections not large at LHC.

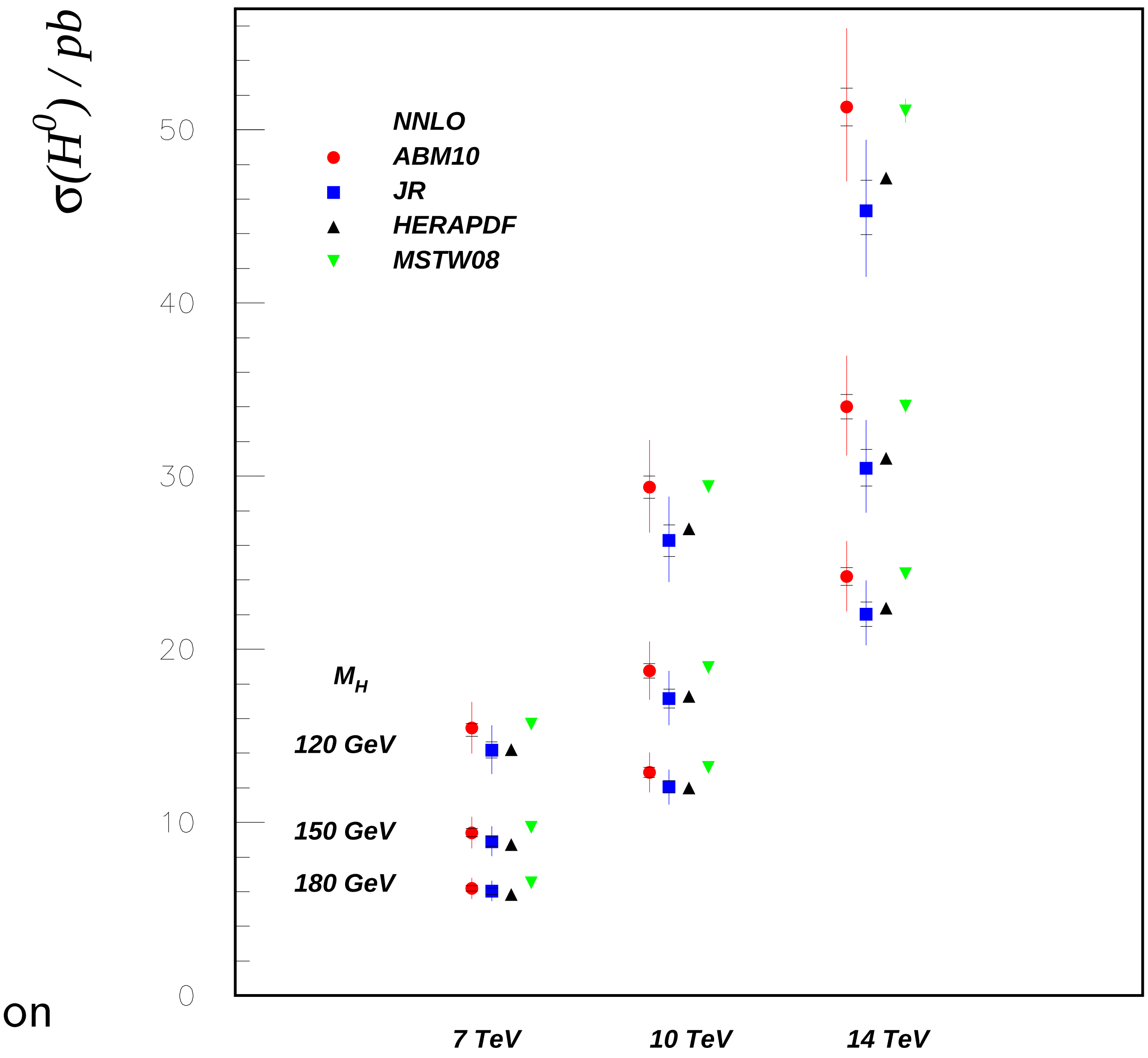
Top cross-section measurement potential discriminator of PDF sets, and correlated to Higgs predictions.

Similar study published by Alekhin *et al*.

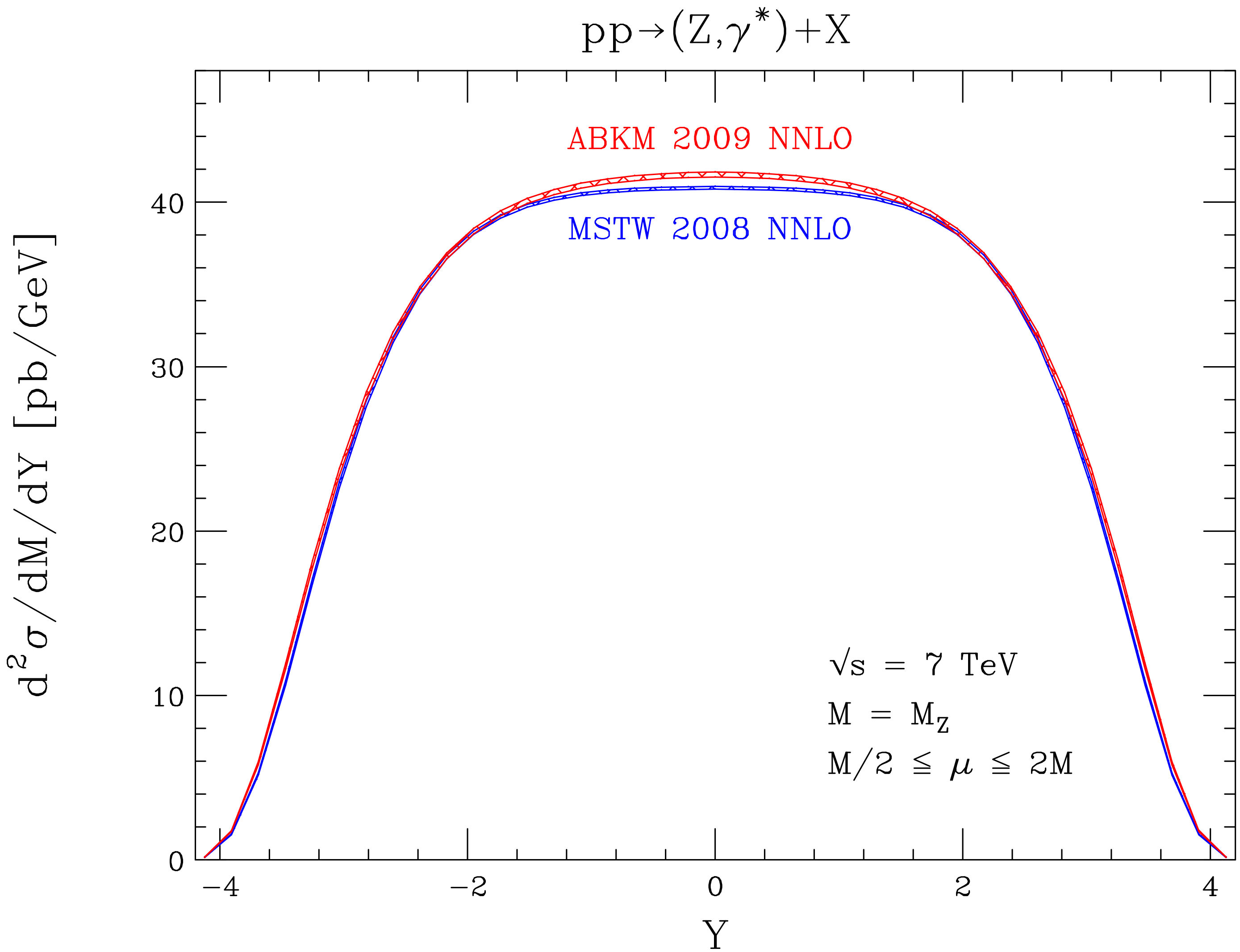
Difficult to compare PDF uncertainties meaningfully in this plot, but size of scale uncertainty illustrated for some sets.

Dominates Higgs cross-section, but very highly correlated between PDFs, i.e. overlap in uncertainties when included not strictly "agreement".

Consider full PDF uncertainty and then theory (scale) uncertainty separately. Correlation depends on process, but hopefully low.



Differences in rapidity distributions evident at NNLO.

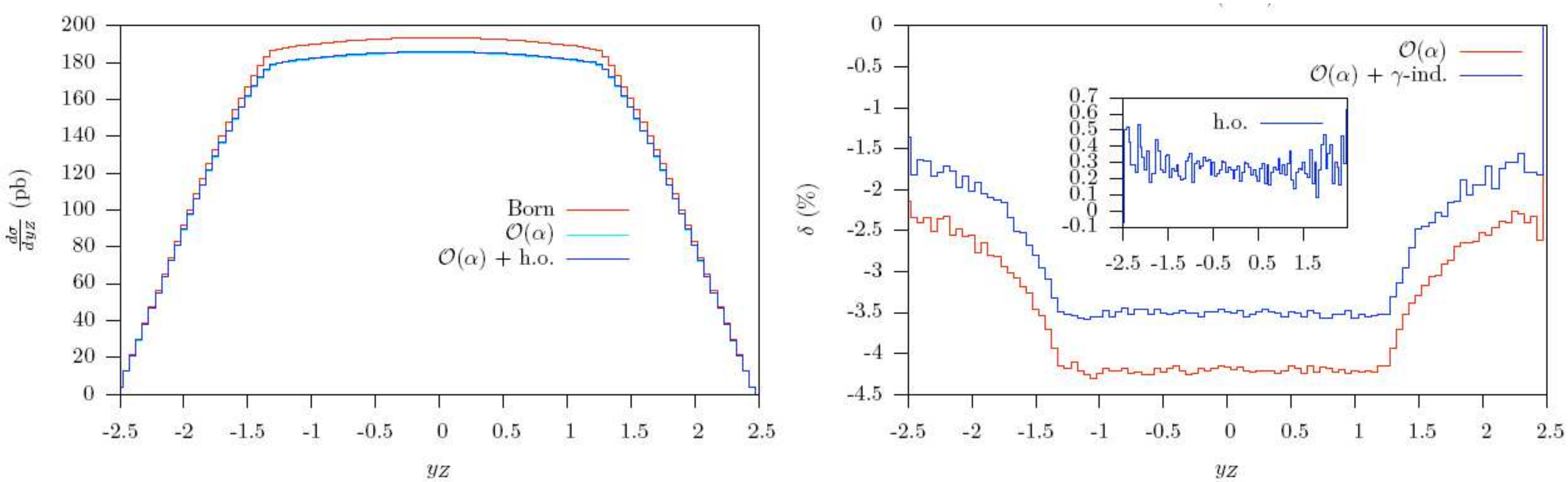


Based on (Anastasiou *et al.*), from Lance Dixon.

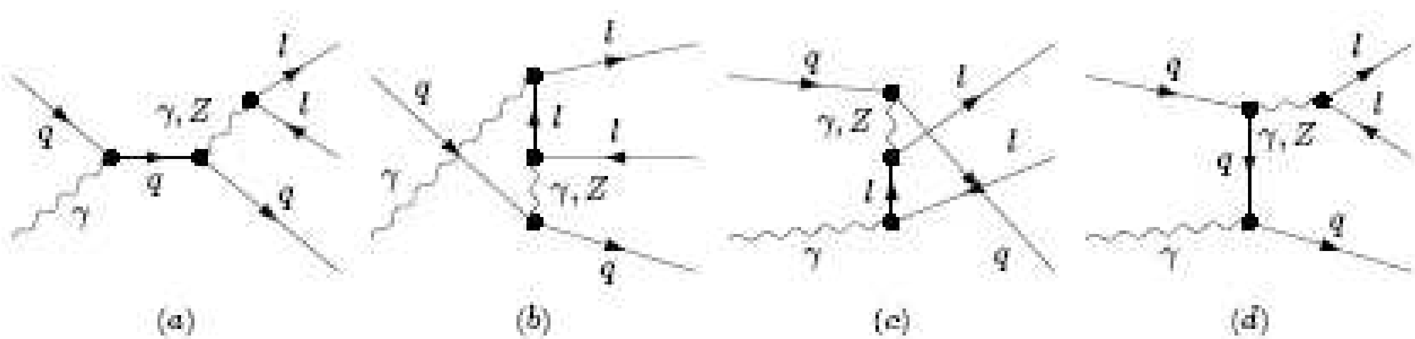
Differences bigger than uncertainties.

Electroweak corrections

Typically a few percent, e.g. Calone Calame *et al* who look at Drell-Yan processes.



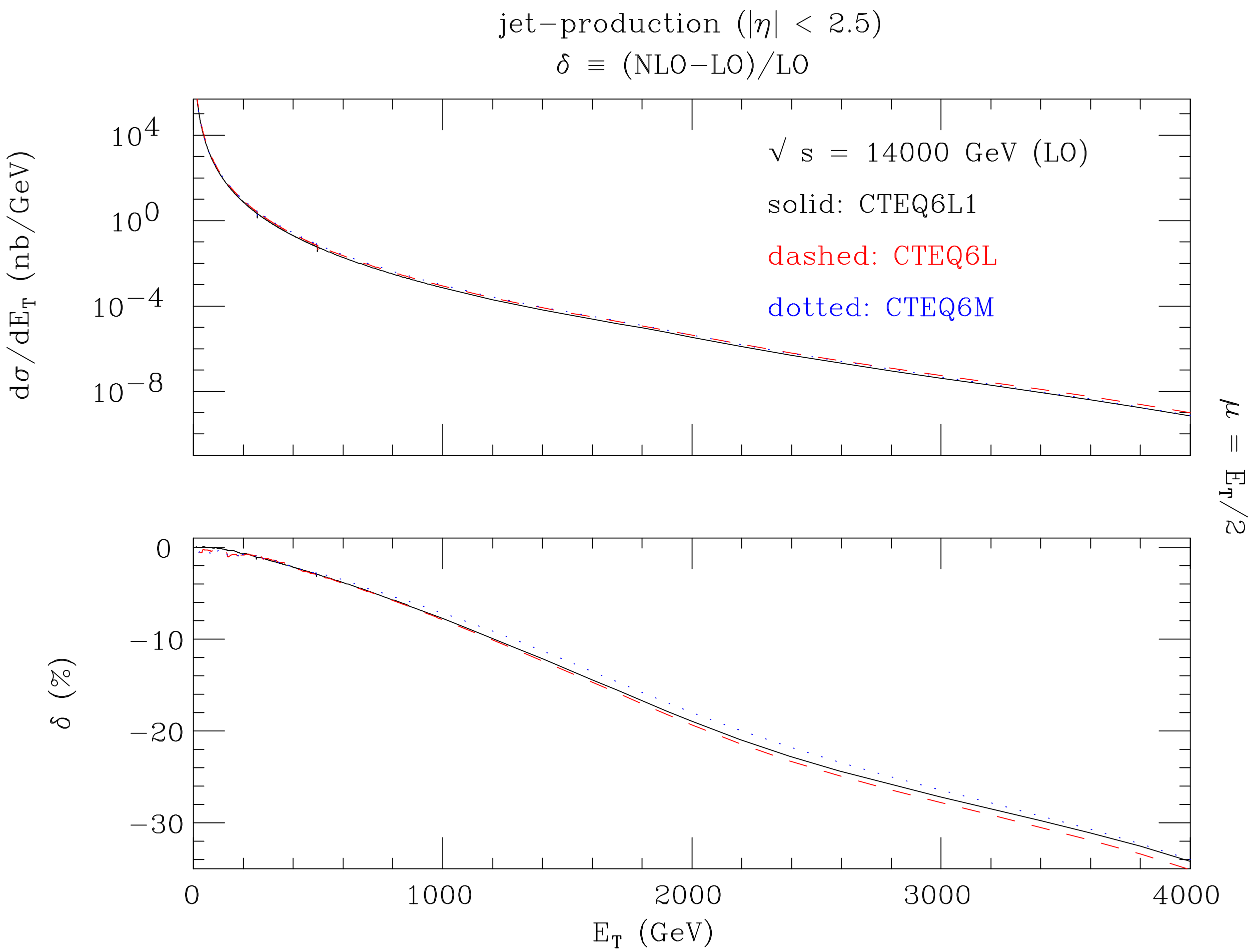
Also consider photon-induced processes. Requires the photon distribution of the proton. Currently only one QED-corrected pdf (MRST2004) set (leads to automatic isospin violation - reduces NuTeV anomaly).



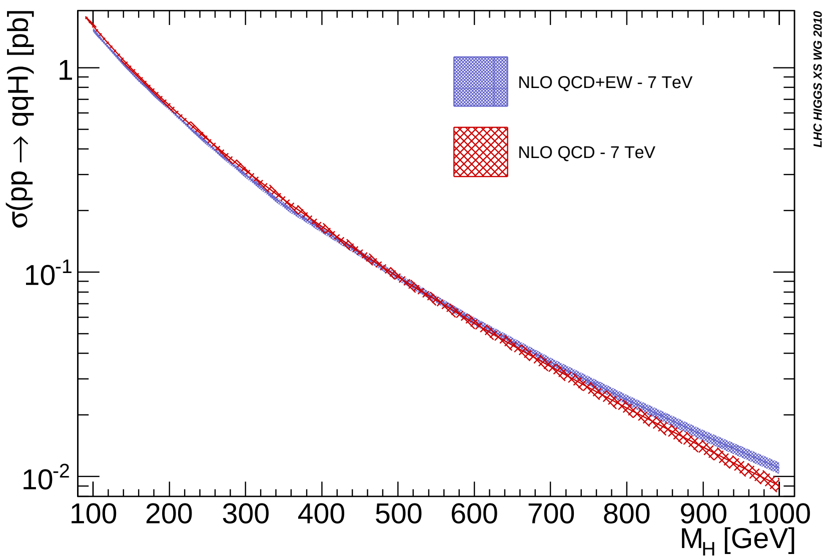
Can also be a couple of percent (here in opposite direction).

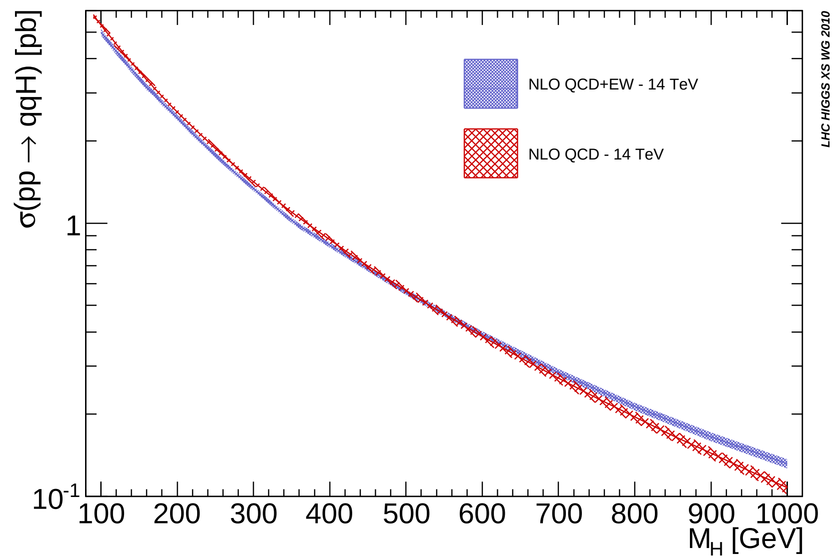
Large Electroweak corrections

Jet cross-section a major example – calculation by Moretti, Nolten, Ross, goes like $(1 - \frac{1}{3} C_F \frac{\alpha_W}{\pi} \log^2(E_T^2/M_W^2))$.



Big effect at LHC energies – $\log^2(E_T^2/M_W^2)$ a very large number. Up to 30%. Bigger than NLO QCD. Other examples.





Effect of electroweak corrections to Higgs production from vector boson fusion.

Plots from Vector Boson Fusion section of Handbook of LHC Higgs Cross Sections.

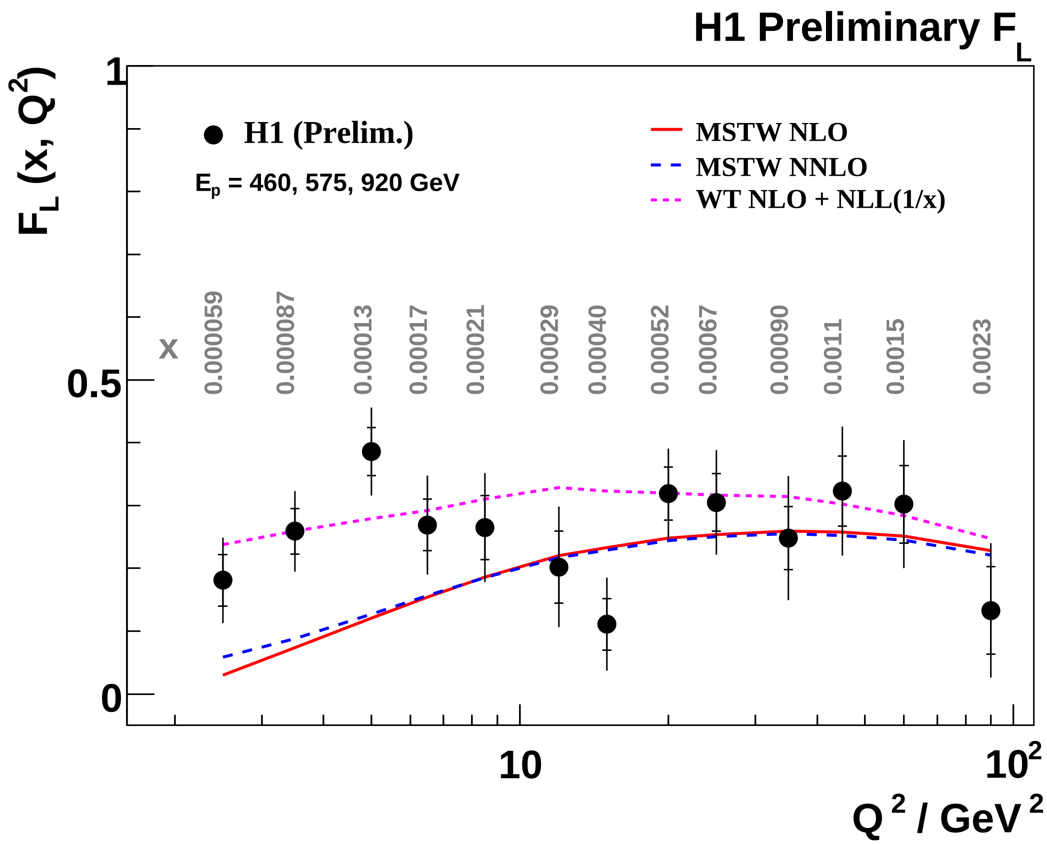
Small-x Theory

At each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e. $P_{ij}(x, \alpha_s(Q^2)), C_i^P(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x)$.

Summed using BFKL equation (and a lot of work – Altarelli-Ball-Forte, Ciafaloni-Colferai-Salam-Stasto and White-RT)

Comparison to H1 prelim data on $F_L(x, Q^2)$ at low Q^2, only within White-RT approach, suggests resummations may be important.

Could possibly give a few percent effect on Higgs cross sections.

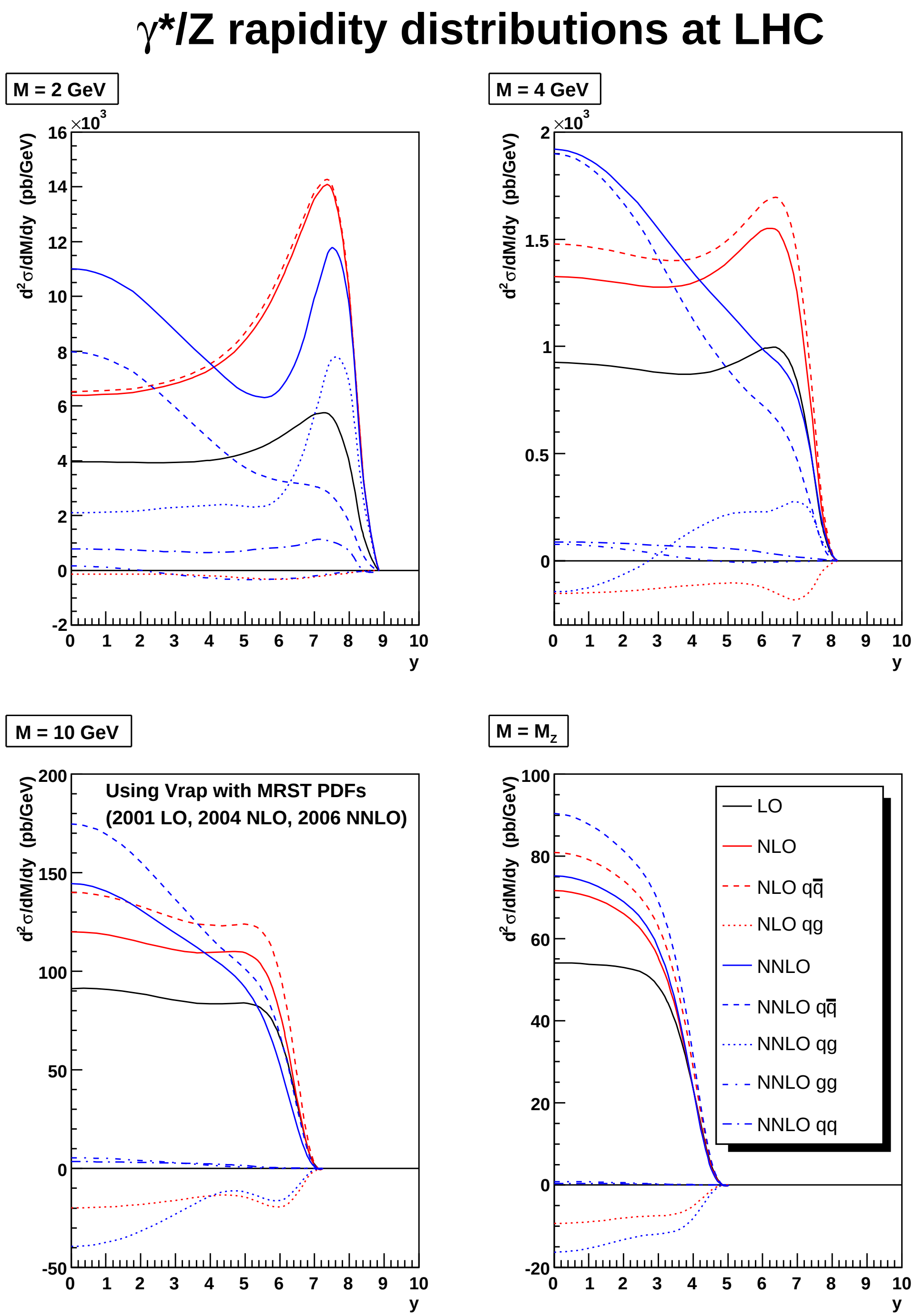


The region of large theory corrections is lower M^2 and high y.

However, this assumes perturbative prediction of Drell-Yan production is reliable.

As seen very large change in prediction from order to order, particularly for low M and high y.

Problem with perturbative stability. This is due to both partons and cross-sections.



Conclusions

We can calculate to NLO or NNLO in QCD and sometimes include electroweak corrections. Also need PDFs.

One can determine the parton distributions and predict cross-sections at the LHC, and the fit quality using NLO or NNLO QCD is fairly good.

Various ways of looking at *experimental* uncertainties on PDFs. Uncertainties $\sim 1-5\%$ for most LHC quantities. Major uncertainty in vector boson production. Ratios, e.g. W^+/W^- tight, and hopefully early constraint on partons.

Effects from input assumptions e.g. selection of data fitted, cuts and input parameterisation can shift central values of predictions significantly. Also affect size of uncertainties. Want balance between freedom and sensible constraints. Complete heavy flavour treatments essential in extraction and use of PDFs. α_S and PDFs heavily correlated.

Errors from higher orders estimated using scale variations, which should often be reasonable. Can dominate, e.g. Higgs. Resummation effects also potentially large. At LHC measurement at high rapidities, e.g. W, Z would be useful in testing understanding of QCD, and particularly quantities sensitive to low x at low scales, e.g. low mass Drell-Yan.

Comparison to Standard Model predictions at the LHC far from a straightforward procedure. Lots of theoretical issues to consider for real precision. Relatively few cases where Standard Model discrepancies will not require some significant input from QCD, PDF and electroweak physics to determine real significance.

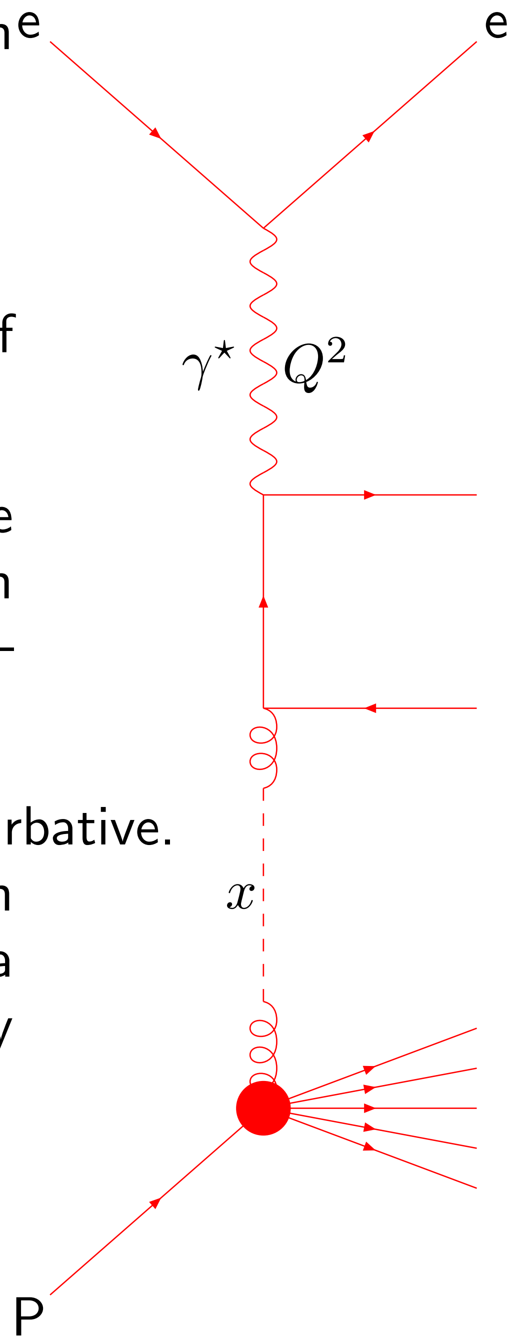
Hadron scattering with an electron factorizes.

Q^2 – Scale of scattering

$x = \frac{Q^2}{2P\cdot q}$ – Momentum fraction of parton.

μ_F – factorisation scale, i.e. scale at which PDF and hard cross-section separated (terms of $\ln(\mu_F^2/m_g^2)$ – infrared divergences).

The partons are intrinsically nonperturbative. However, once μ_F^2 is large enough they do evolve with μ_F^2 in a perturbative manner determined by splitting functions $P_{ij}(x, \alpha_s(\mu_R^2))$

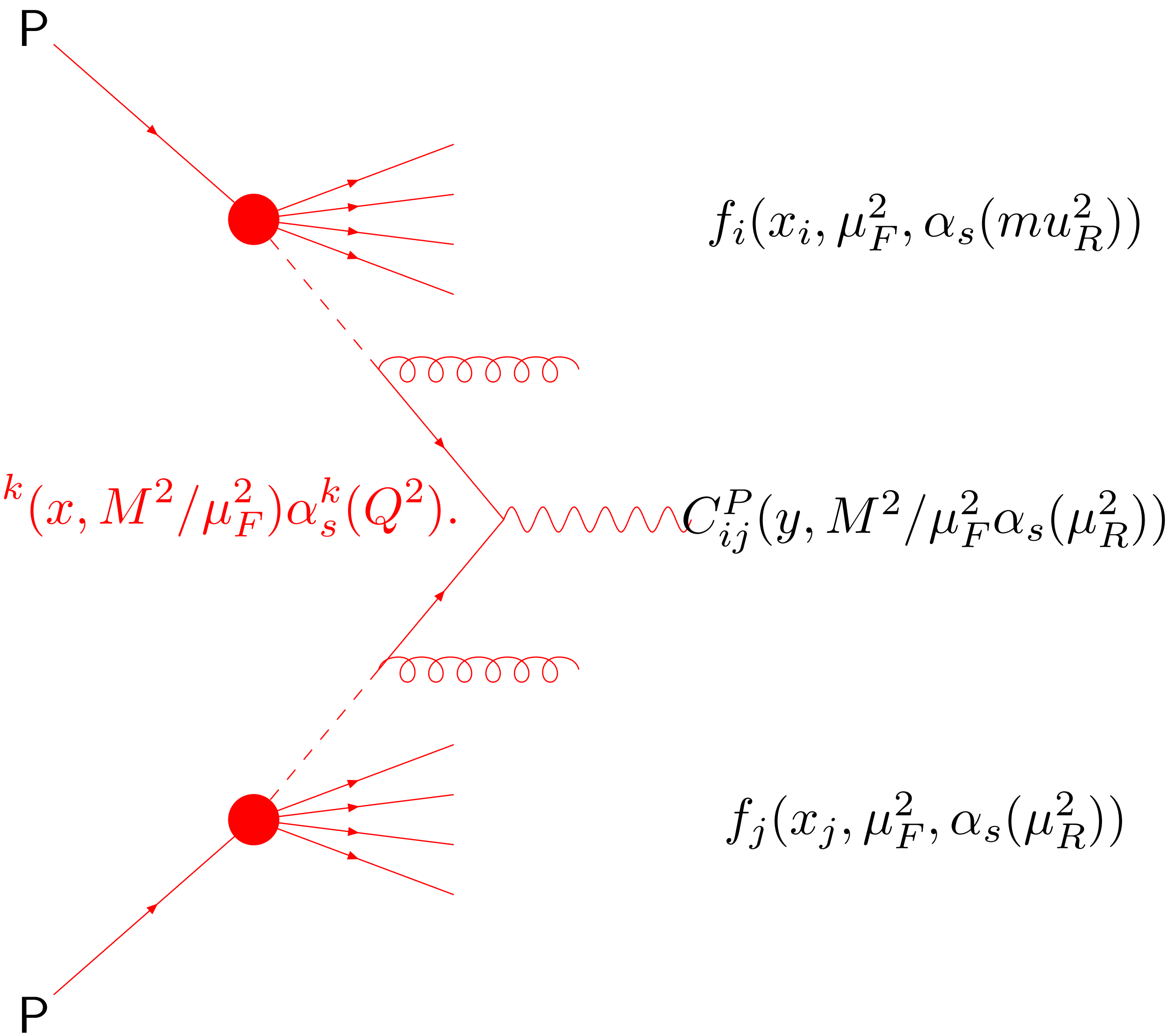


perturbative
calculable
coefficient function
$C_i^P(x, Q^2/\mu_F^2, \alpha_s(\mu_R^2))$

nonperturbative
incalculable
parton distribution
$f_i(x, \mu_F^2, \alpha_s(\mu_F^2))$

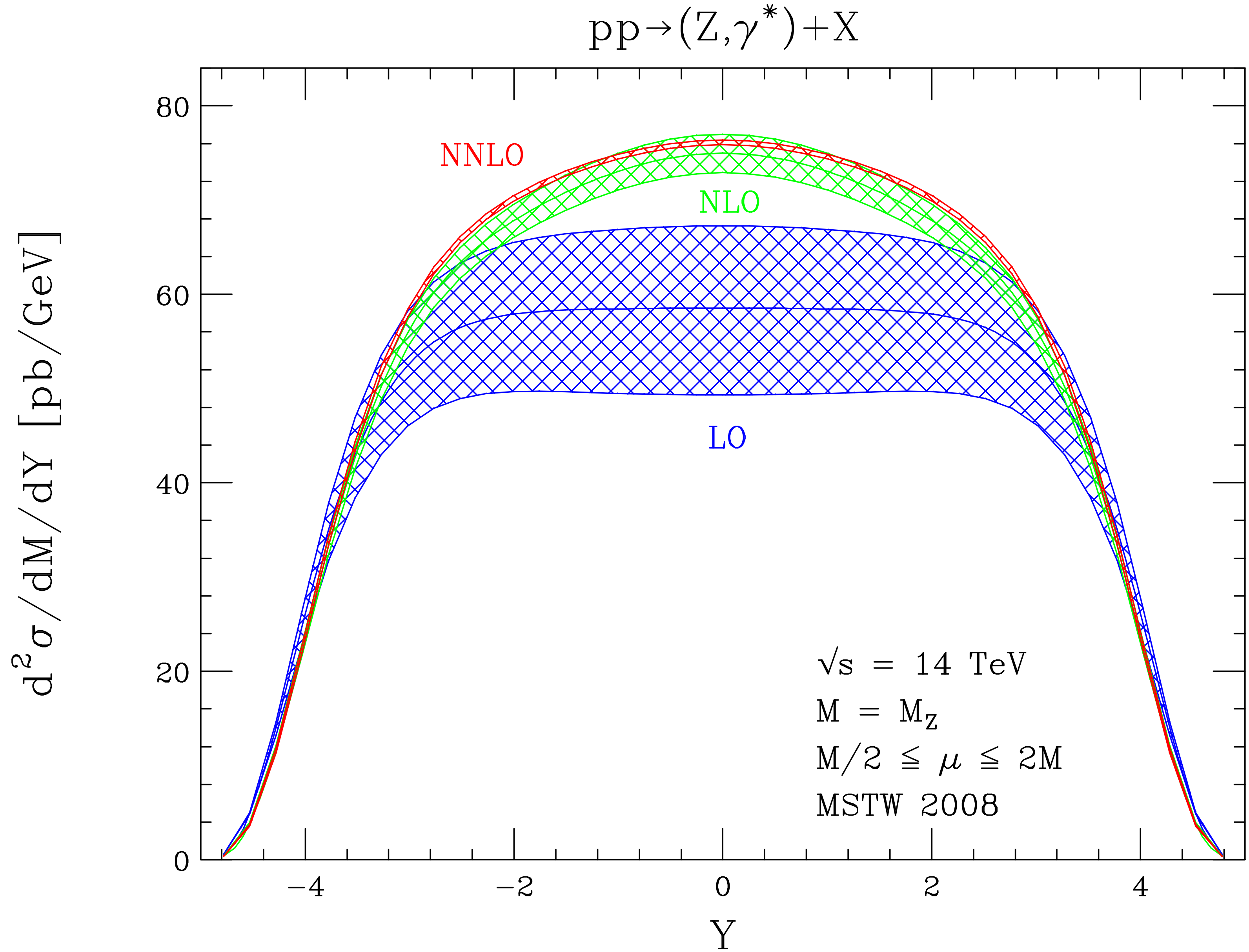
The coefficient functions $C_i^P(x, M^2/\mu_F^2, \alpha_s(\mu_R^2))$ are process dependent (new physics) but are calculable as a power-series in $\alpha_s(\mu_R^2)$.

$$C_i^P(x, M^2/\mu_F^2, \alpha_s(\mu_R^2)) = \sum_k C_i^{P,k}(x, M^2/\mu_F^2)\alpha_s^k(Q^2).$$



Since the parton distributions $f_i(x, \mu_F^2, \alpha_s(\mu_R^2))$ are process-independent, i.e. **universal**, once they have been measured at one experiment, one can predict many other scattering processes.

However, μ_F is a new source of uncertainty at finite order.



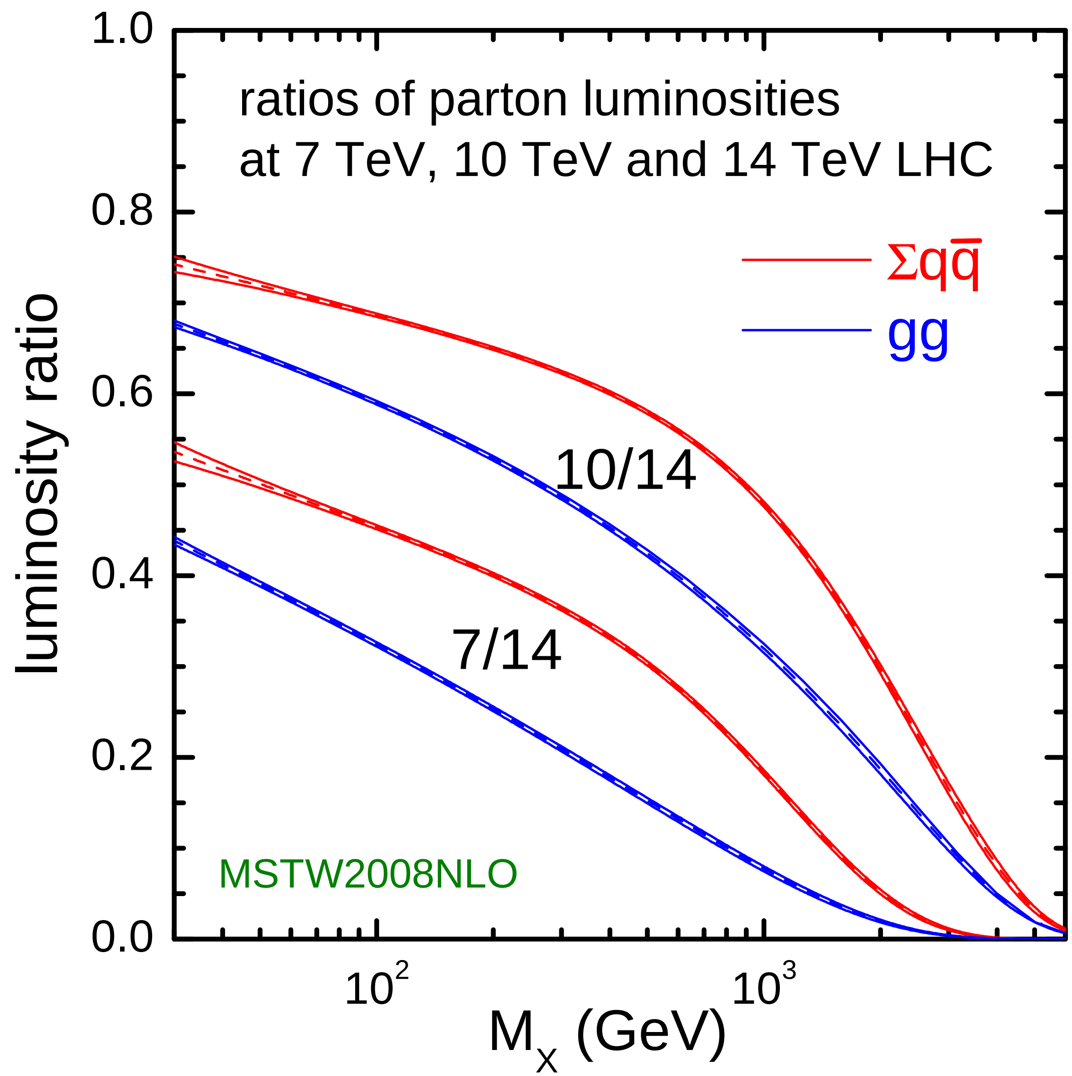
Based on (Anastasiou *et al.*), from Lance Dixon.

Initial Running

Of course, the LHC has started running at 7 TeV rather than the full 14 TeV.

Reduces rapidity range by $\ln 2$.

Roughly $30 - 50\%$ the full cross-sections for most standard model (including light Higgs) processes.



Interplay of LHC and QCD/

Make predictions for all processes, both SM and BSM, as accurately as possible given current experimental input and theoretical accuracy.

Check against well-understood processes, e.g. central rapidity W, Z production (luminosity monitor), lowish-E_T jets, ...

Compare with predictions with more uncertainty and lower confidence, e.g. high-E_T jets, high rapidity bosons or heavy quarks .....

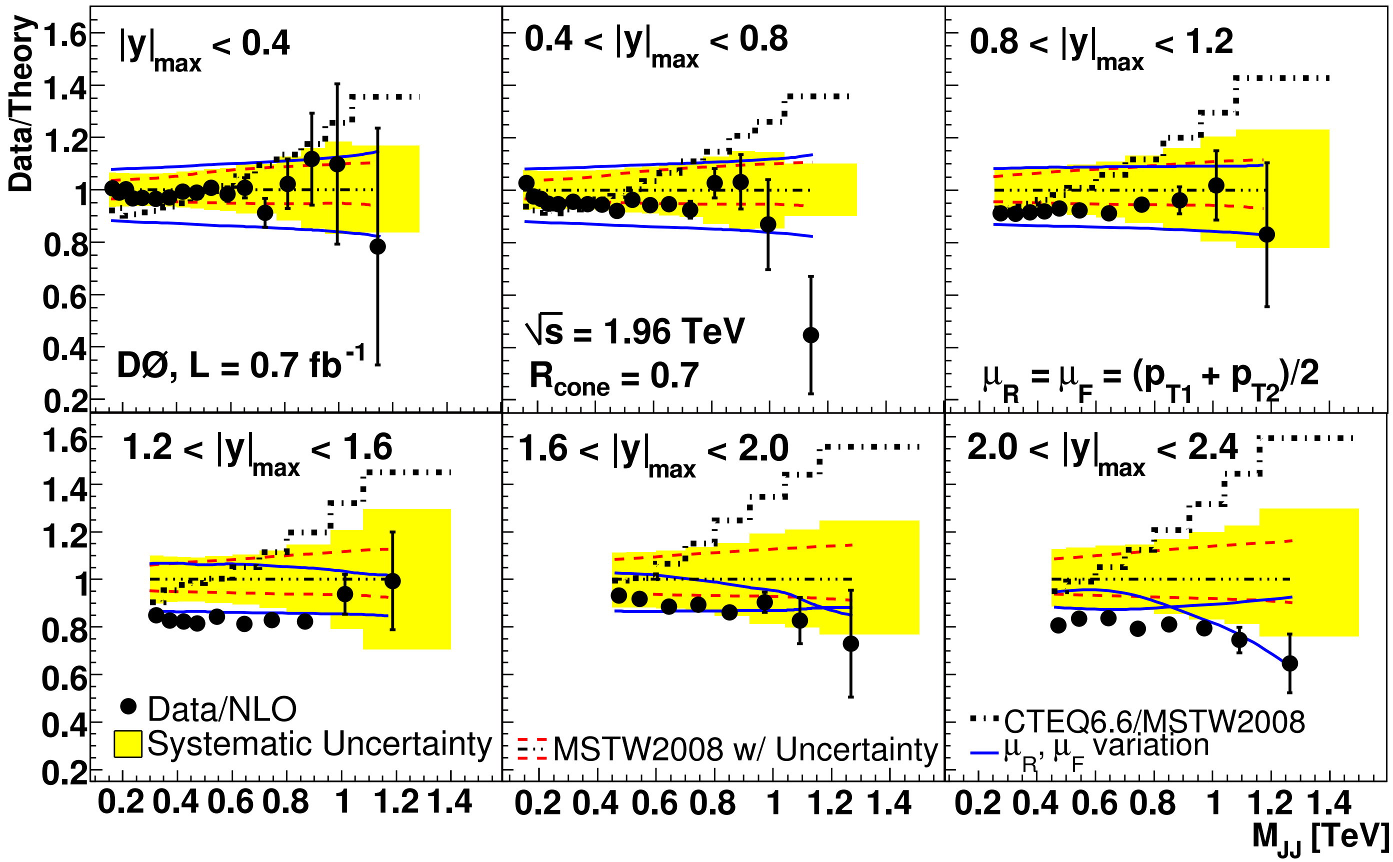
Improve uncertainty on parton distributions by improved constraints, and check understanding of theoretical uncertainties, and determine where NNLO, electroweak corrections, resummations $etc.$ needed.

Make improved predictions for both background and signals with improved partons and Standard Model theory.

Spot new physics from deviations in these predictions. As a nice by-product improve our understanding of the Standard Model considerably.

Remainder of talk describes this process in more detail.

For jet production probed at the Tevatron scale and PDF uncertainty similar (and both similar to data *systematic* uncertainty)



Mixture of quark-quark, quark-gluon and gluon gluon induced processes.

Quarks known from DIS so constrains gluon mainly.

Other Ratios

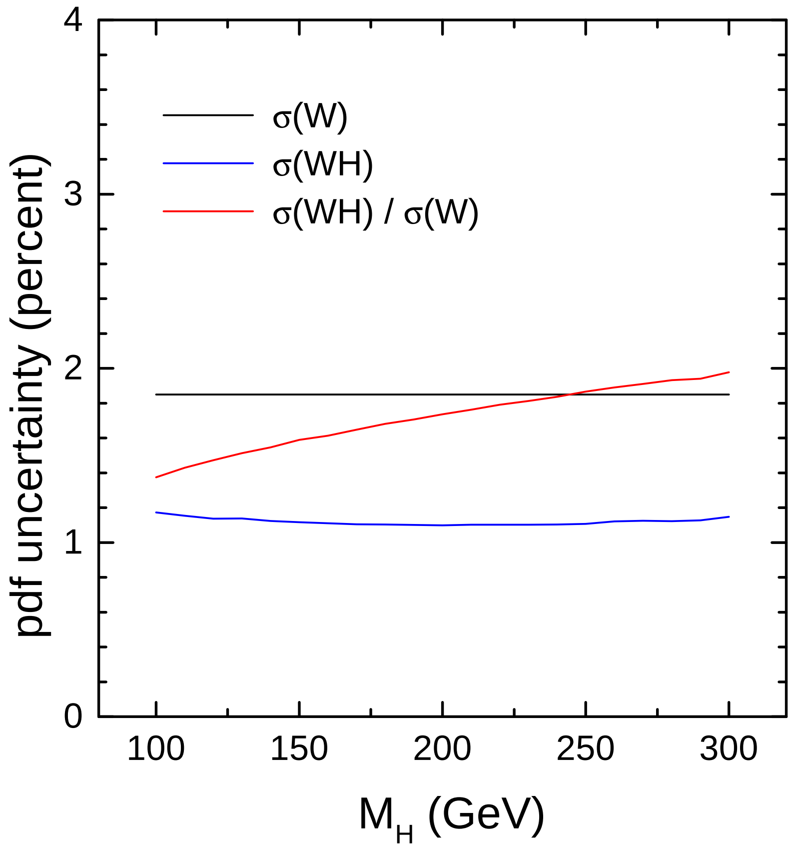
Could $\sigma(W)$ or $\sigma(Z)$ be used to calibrate other cross-sections, e.g. $\sigma(WH)$, $\sigma(Z')$?

$\sigma(WH)$ more precisely predicted because it samples quark pdfs at higher x, and scale, than $\sigma(W)$.

However, ratio shows no improvement in uncertainty, and can be worse.

Partons in different regions of x are often anti-correlated rather than correlated, partially due to sum rules.

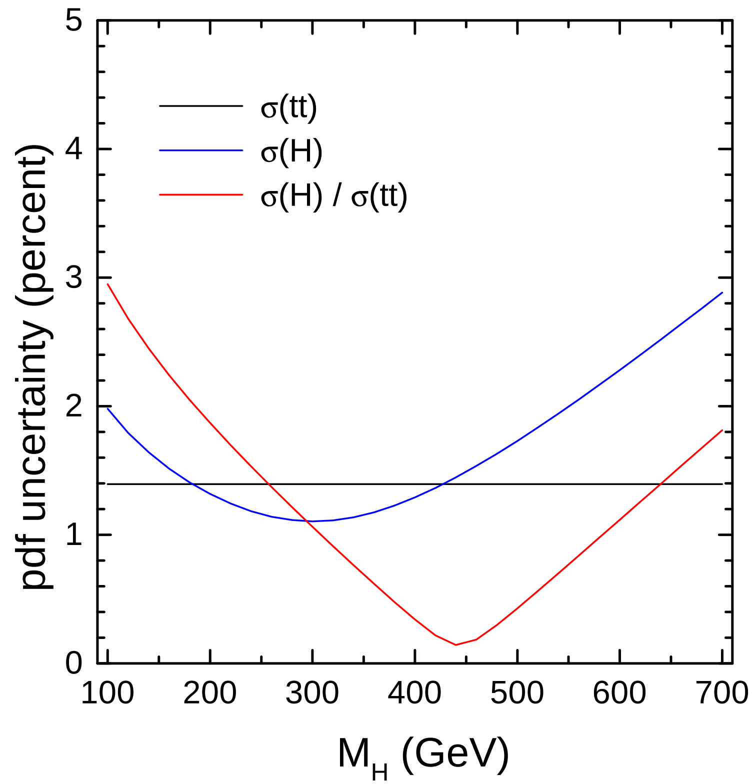
pdf uncertainties on W, WH cross sections at LHC (MRST2001E)



No obvious advantage in using $\sigma(t\bar{t})$ as a calibration SM cross-section, except maybe for very particular, and rather large, M_H.

$\sigma(t\bar{t})$ very similar indeed to $450\mathrm{GeV}$ Higgs.

pdf uncertainties on top, (gg→) H
cross sections at LHC (MRST2001E)



Different PDF sets

- MSTW08 – fit all previous types of data. Most up-to-date Tevatron jet data. Not most recent HERA combination of data. PDFs at LO, NLO and NNLO.

- CTEQ6.6 – very similar. Not quite as up-to-date on Tevatron data. PDFs at NLO. New – CT10 include HERA combination and more Tevatron data. Little changes.

- NNPDF2.0 – include all except HERA jet data (not strong constraint) and heavy flavour structure functions. Include HERA combined data. PDFs at NLO.

- HERAPDF2.0 – based entirely on HERA inclusive structure functions, neutral and charged current. Use combined data. PDFs at LO, NLO.

- ABKM09 – fit to DIS and fixed target Drell-Yan data. PDFs at NLO and NNLO. (Now prelim results using Tevatron jets).

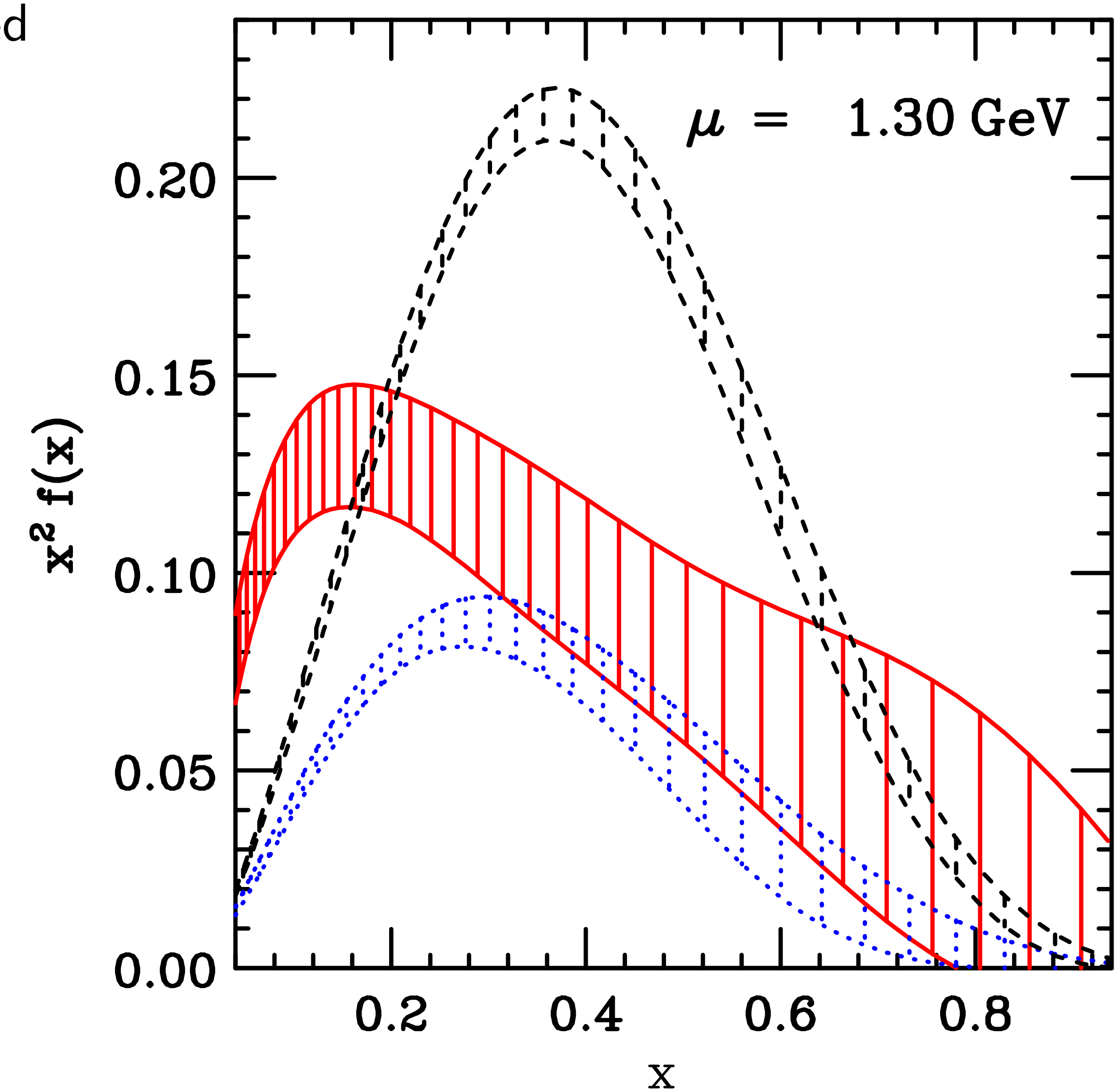
- GJR08 – fit to DIS, fixed target Drell-Yan and Tevatron jet data. PDFs at NLO and NNLO.

Use of HERA combined data instead of original data slight increase in quarks at low x (depending on procedure).

Generally high-x PDFs parameterised so will behave like $(1-x)^\eta$ as $x \to 1$. More flexibility in CTEQ.

Very hard high-x gluon distribution (more-so even than NNPDF uncertainties).

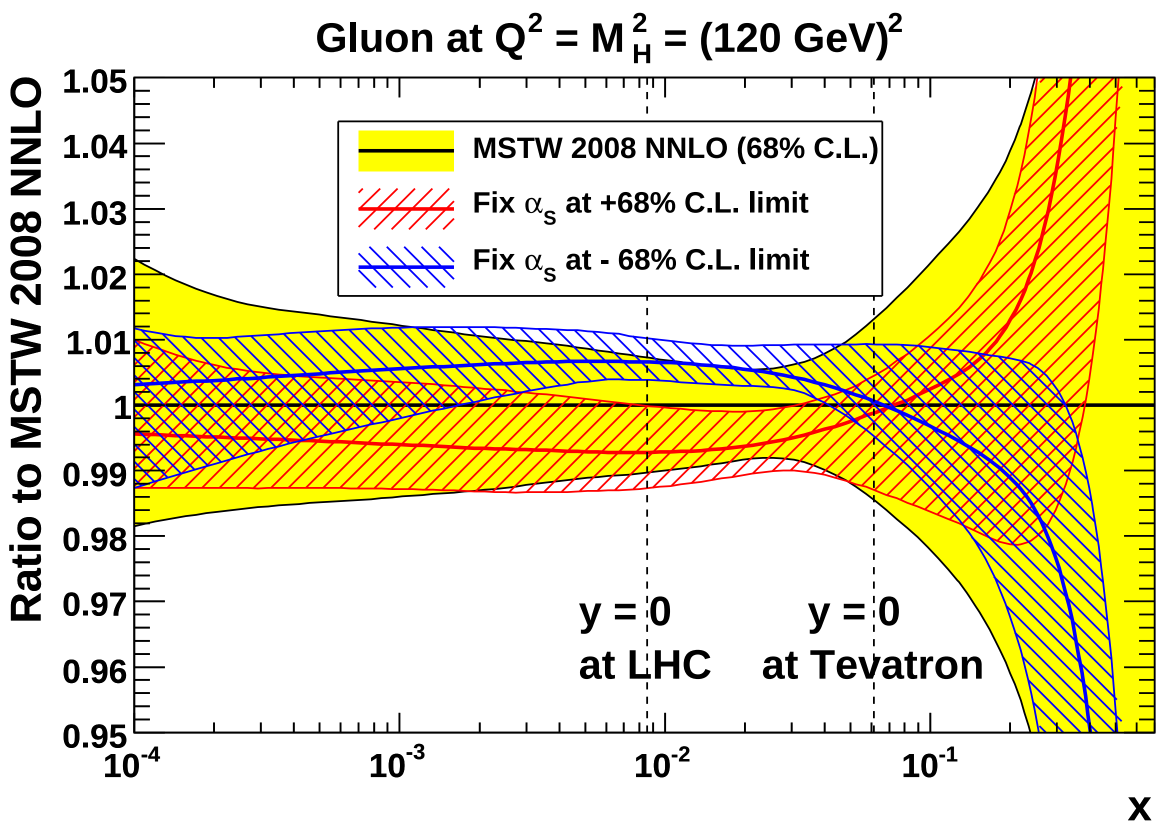
However, is gluon, which is radiated from quarks, harder than the up valence distribution for $x \to 1$?



PDF correlation with α_S.

Can also look at PDF changes and uncertainties at different $\alpha_S(M_Z^2)$. Latter usually only for one fixed $\alpha_S(M_Z^2)$. Can be determined from fit, e.g. $\alpha_S(M_Z^2) = 0.1202^{+0.0012}_{-0.0015}$ at NLO and $\alpha_S(M_Z^2) = 0.1171^{+0.0014}_{-0.0014}$ at NNLO from MSTW.

PDF uncertainties reduced since quality of fit already worse than best fit.



Expected gluon–$\alpha_S(M_Z^2)$ small–x anti-correlation $\rightarrow$ high-x correlation from sum rule.

Heavy Quarks – Essential to treat these correctly. Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x, Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Does not sum $\ln^n(Q^2/m_H^2)$ terms, and not calculated for many processes beyond LO. Still occasionally used. Sometimes final state details in this scheme only.

Alternative, at high scales $Q^2 \gg m_H^2$ heavy quarks like massless partons. Behave like up, down, strange. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Normal assumption in calculations. Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x, Q^2) = C_j^{ZMVF} \otimes f_j^{n_f+1}(Q^2).$$

Need a **General Mass Variable Flavour Number Scheme** (GM-VFNS) interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$. Used by MRST/MSTW and more recently (as default) by CTEQ, and now also more regularly by H1,ZEUS.

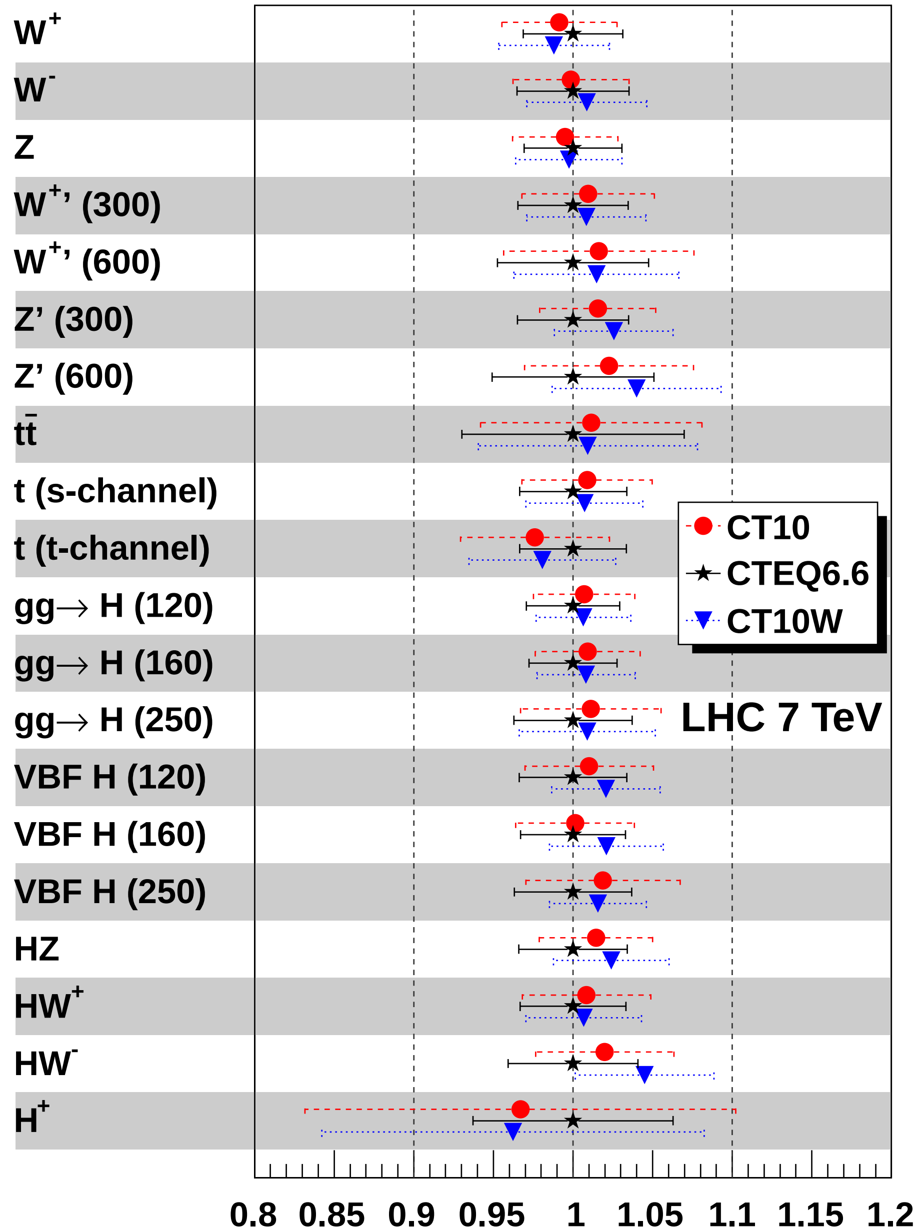
Very Recent Updates

MSTW find new combined HERA data lead to increase in W, Z by couple of $\%$. Less than 1% on Higgs (Tevatron and LHC).

CT10 (right) find change in W, Z very small (probably countered by gluon parameterisation change).

Slight increase in Higgs, $t\bar{t}$ (again probably gluon shape).

NNPDF find prelim GM-VFNS fits bring them closer to MSTW,CTEQ for W, Z.



NNLO splitting functions now known. (Moch, Vermaseren and Vogt). Essentially full NNLO determination of partons now being performed (MSTW, ABKM,GJR,HERA), though heavy flavour not fully worked out in the fixed-flavour number scheme (FFNS) PDFs and jet cross-sections approximate. Improve consistency of fit very slightly, and reduces α_S.

Surely this is best, i.e. most accurate.

Yes, but ...... only know some hard cross-sections at NNLO.

Processes with two strongly interacting particles largely completed

DIS coefficient functions and sum rules

$pp(\bar{p}) \rightarrow \gamma^\star, W, Z$ (including rapidity dist.), H, A^0, WH, ZH.

But for many other final states NNLO not known. NLO still more appropriate.

Consideration of NNLO

Very good evidence that one should use NNLO if possible rather than NLO – many physical cross-sections, particularly $gg \rightarrow H$, not very convergent.

Fewer PDF sets available, can study differences between them better at NLO, but for central prediction need NNLO.

Related to issue of use and uncertainty of $\alpha_S(M_Z^2)$. Noted systematic change in value form fit as one goes from NLO to NNLO. Also highlighted in stability of predictions.

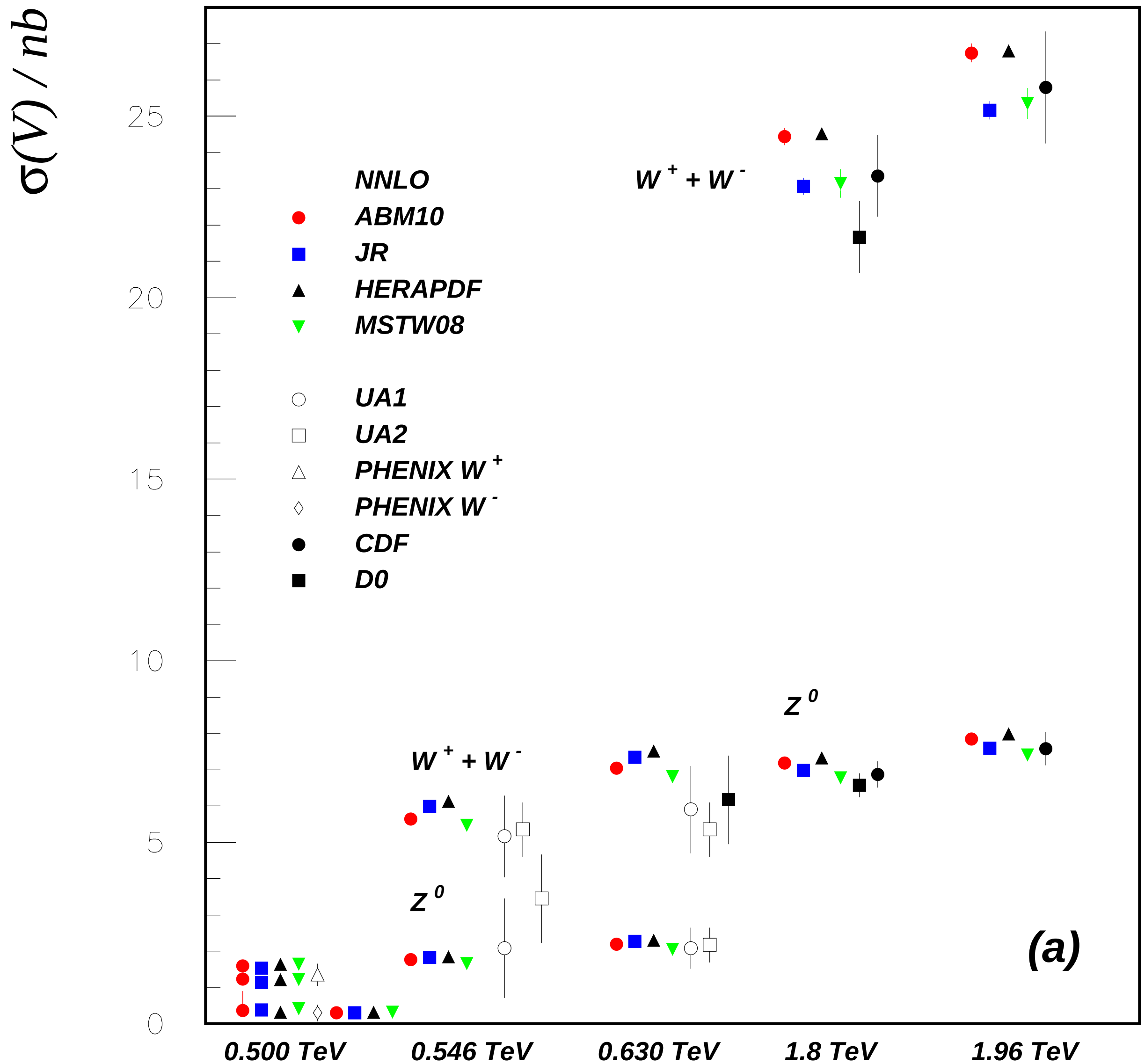
Consider percentage change from NLO to NNLO in MSTW08 predictions for best fit α_S compared to fixed $\alpha_S(M_Z^2) = 0.119$.

	$\sigma_{W(Z)}$ 7TeV	$\sigma_{W(Z)}$ 14TeV	σ_H 7TeV	σ_H 7TeV
MSTW08 best fit α_S	3.0	2.6	25	24
MSTW08 $\alpha_S = 0.119$	5.3	5.0	32	30

$\alpha_S(M_Z^2)$ is not a physical quantity. In (nearly) all PDF related quantities (and many others) shows tendency to decrease from order to order. Noticeable if one has fit at NNLO. Any settling on, or near common $\alpha_S(M_Z^2)$ **has** to take this into account.

Study published by Alekhin *et al*.

Note consistent normalisation difference between CDF and D0.

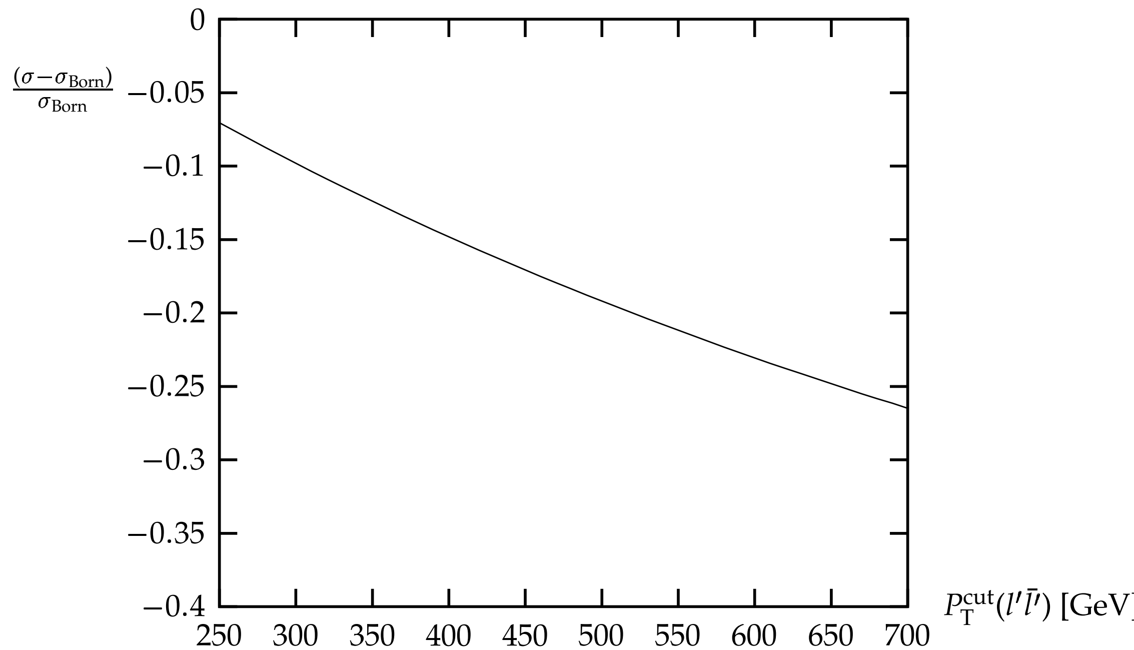


Similar results for corrections to other processes with a hard scale, e.g. Di-boson production (Accomando *et al*).

Plot shows fractional corrections as function of reconstructed Z transverse momentum in WZ production.

Same sort of corrections in large-p_T vector bosons in conjunction with jets (Kühn *et al*, Maina *et al*)...

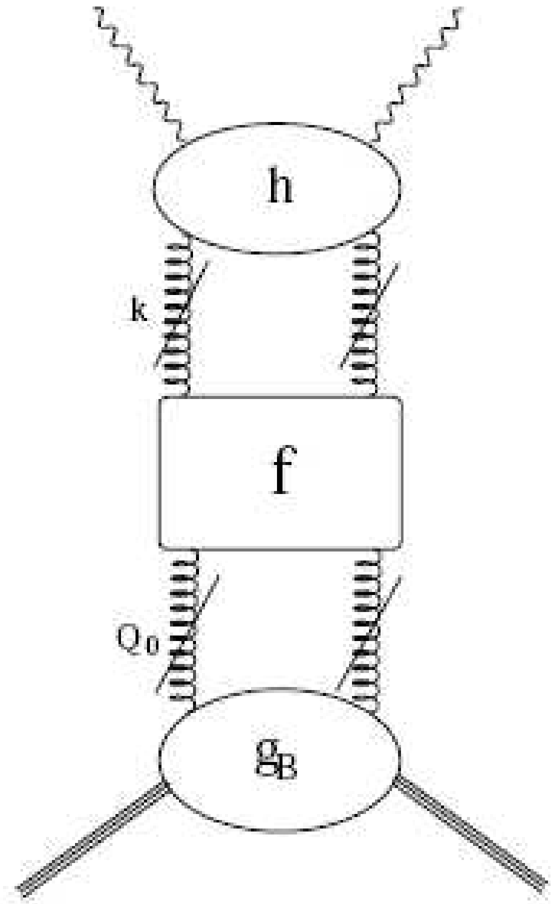
$\ln(s/m_W^2)$ terms can also affect Γ_W extraction from the transverse mass distribution.



Small-x Theory

Reason for this instability – at each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e. $P_{ij}(x, \alpha_s(Q^2)), C_i^P(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x)$.

BFKL equation for high-energy limit



$$f(k^2, x) = f_I(Q_0^2) + \int_x^1 \frac{dx'}{x'} \bar{\alpha}_S \int_0^\infty \frac{dq^2}{q^2} K(q^2, k^2) f(q^2, x),$$

where $f(k^2, x)$ is the unintegrated gluon distribution $g(x, Q^2) = \int_0^{Q^2} (dk^2/k^2) f(x, k^2)$, and $K(q^2, k^2)$ is a calculated kernel known to NLO.

Physical structure functions obtained from

$$\sigma(Q^2, x) = \int (dk^2/k^2)\, h(k^2/Q^2) f(k^2, x)$$

where $h(k^2/Q^2)$ is a calculable impact factor.

The global fits usually assume that this is unimportant in practice, and proceed regardless.

Fits work well at small x, but could improve.

Keeping partons fixed while changing cross-sections (using MRST2006 NNLO partons) also shows part of instability due to partons. Unusual behaviour in very small x partons at NNLO. Due to similar high and low z terms in splitting functions.

Overall most obvious effect – large change in quark-gluon (and quark-quark) contributions at NNLO due to $1/z$ and $\ln(1-z)$ divergences in cross-sections appearing at this order.

Reminiscent of behaviour of $F_L(x, Q^2)$ which has similar large logs.

Cross-section may be sensitive to resummations (high and low z) at lowest M and highest y. In region where measurements can be made?