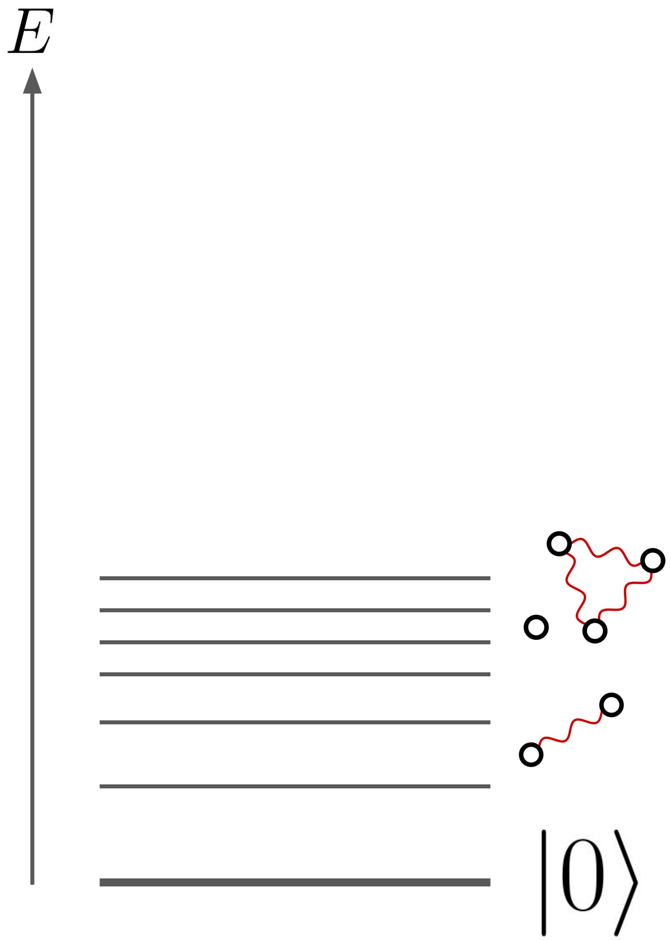
Thermalization and Chaos in 1+1d QFTs

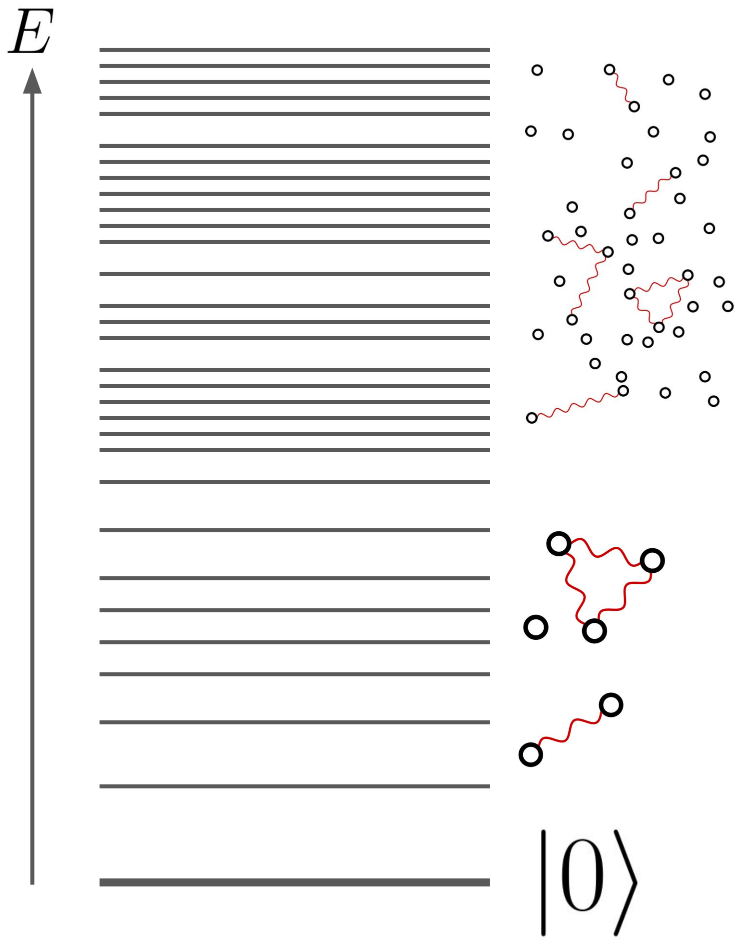
Luca Delacrétaz | U Chicago

Analytic: 2105.02229
Numeric: 2206.09xxx
and more…

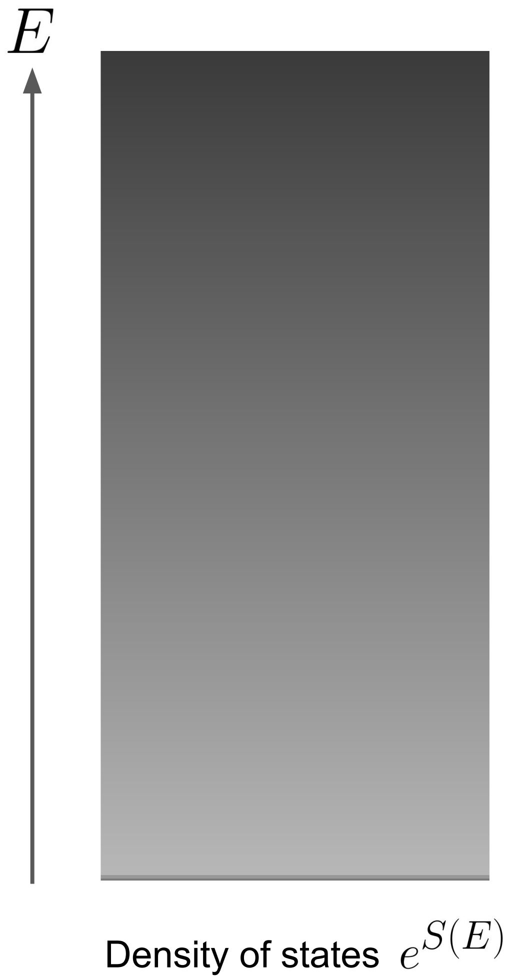
With:
Ami Katz, Liam Fitzpatrick, Matt Walters

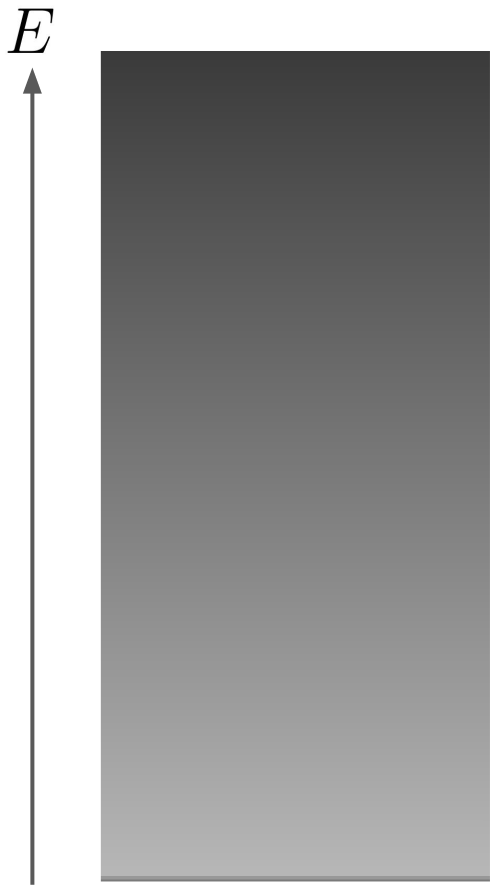


- The vacuum and low lying excitations have played a central role in QFT in the 20th century.



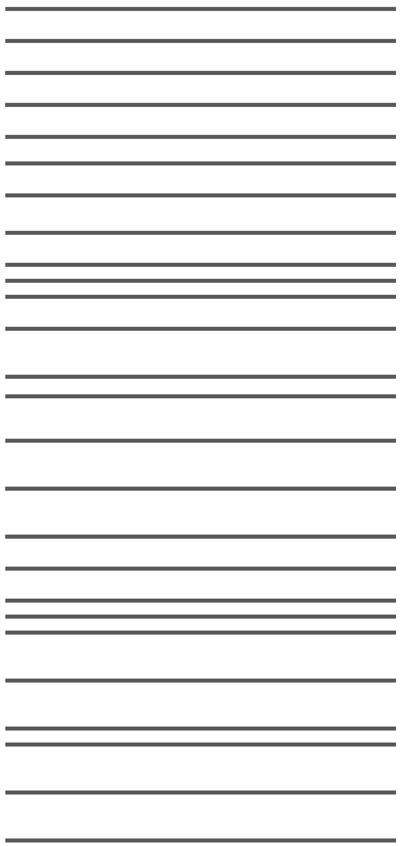
- ★ The vacuum and low lying excitations have played a central role in QFT in the 20th century.
- ★ But QFTs have rich and healthy spectra far beyond their vacuum
- ★ Despite its complexity, the spectrum of excited states has tractable universal features


E
Density of states $e^{S(E)}$

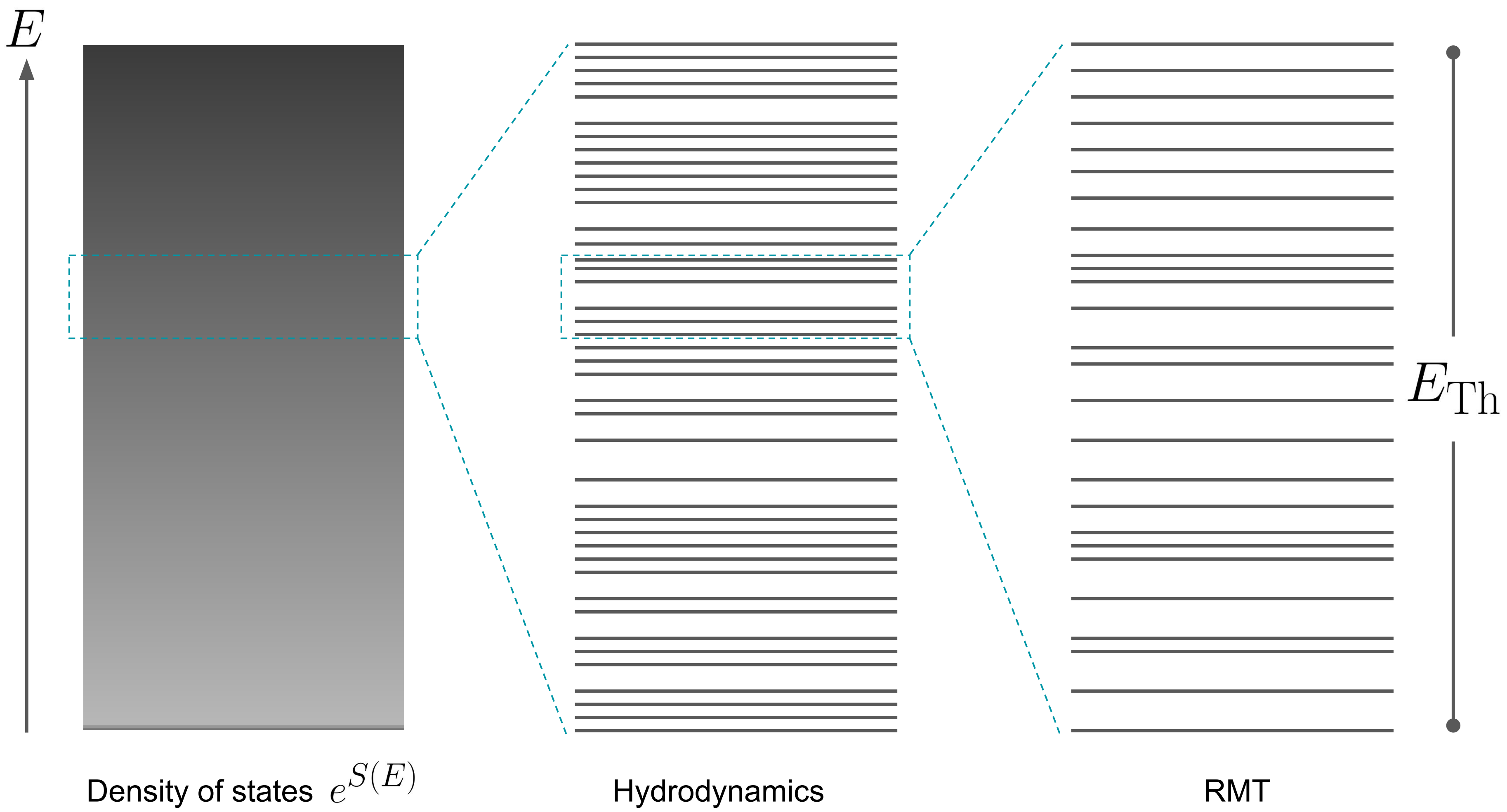


Density of states $e^{S(E)}$

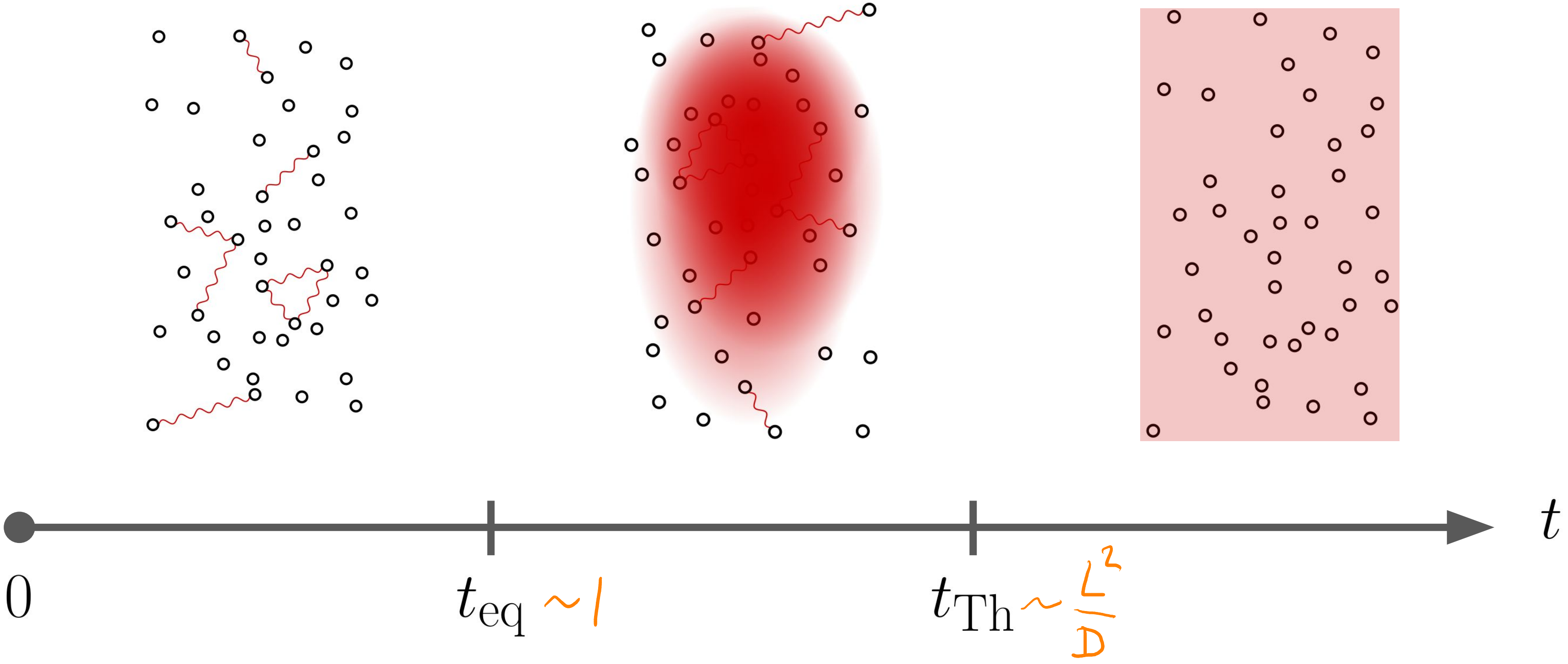
Wigner, Dyson, Mehta
Bohigas Giannoni Schmit ‘84
Berry Tabor ‘87
…
Santos Rigol ‘10
…



RMT


E
E_{Th}
Density of states $e^{S(E)}$
Hydrodynamics
RMT

Timeline of an excited state in an interacting system



Eigenstate thermalization hypothesis

Eigenstates should also thermalize, but they are stationary ⇒ eigenstates are thermal

Deutsch '91 Srednicki '94 Rigol Dunjko Olshanii '08

In particular, they should individually reproduce thermal expectation values of local operators

$$\cancel{\frac{1}{N}\sum_E} \langle E|\mathcal{O}|E\rangle \simeq \langle \mathcal{O}\rangle_\beta$$

Thermalization and chaos beyond weak coupling

- ★ At weak coupling, kinetic theory captures local thermalization. Chaos can also be understood in the semiclassical limit by quantizing periodic orbits [Gutzwiller '87] .

- ★ Away from weak coupling, how universal is this? Can we actually observe it? (MBL, scars?)

- ★ Are there universal constraints on these processes and their time scales?

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★ Away from weak coupling, how universal is this? Can we actually observe it? (MBL, scars?)

$\rightarrow$ Numerically study using Hamiltonian truncation

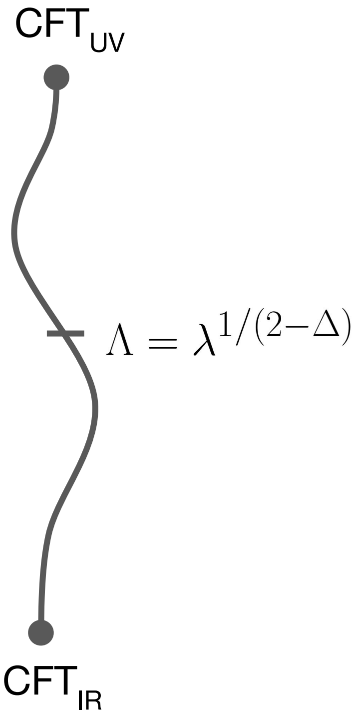
upcoming, LVD Fitzpatrick Katz Walters

★ Are there universal constraints on these processes and their time scales?

$\rightarrow$ Nonperturbative analytic bounds from causality

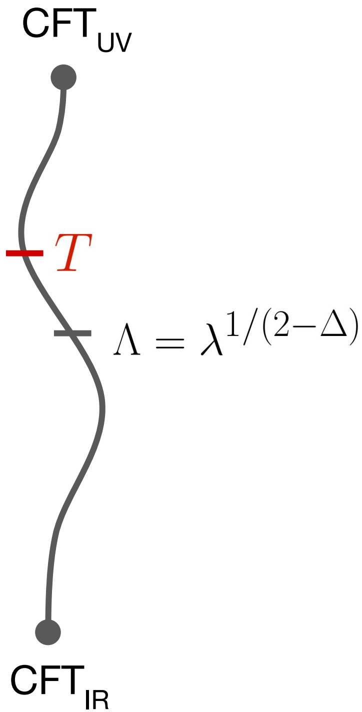
2105.02229 LVD Fitzpatrick Katz Walters

QFTs from CFTs



$$S_{\mathrm{QFT}} = S_{\mathrm{CFT}} + \lambda \int d^2x\, \mathcal{O}$$

QFTs from CFTs



$$S_{\mathrm{QFT}} = S_{\mathrm{CFT}} + \lambda \int d^2x\, \mathcal{O}$$

We will probe the entire RG flow by studying the theory at finite temperature

The equilibrium thermodynamics of a relativistic QFT is fixed by the equation of state $P(T)$

$$\langle T_{\mu\nu}\rangle_\beta = (\varepsilon + P)\,\delta^0_\mu\delta^0_\nu + P\eta_{\mu\nu}\,,$$

$$\varepsilon + P = sT$$

$$dP = sdT$$

It is convenient to work with the dimensionless entropy $s_o(T) \equiv \frac{s(T)}{T}$ $\sim T^d$

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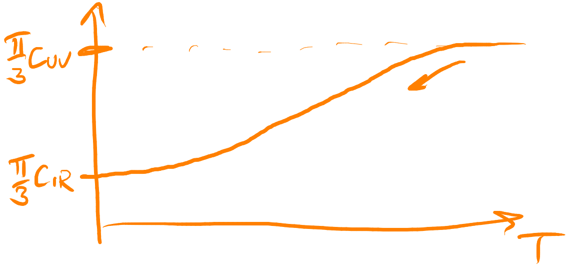
Subluminality of sound implies $s_o(T)$ is a c-function

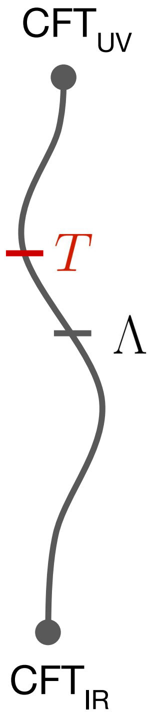
$s_0 = \frac{\pi}{3} c$

$s_0'(T) \geq 0$

$$1 \leq \frac{1}{c_s^2} = \frac{d\varepsilon}{dP} = \frac{Tds}{sdT} = 1 + \frac{Tds_o}{s_o dT}$$



Integrable flows



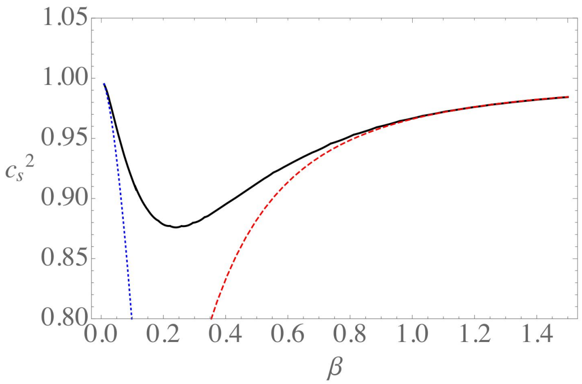
Example:

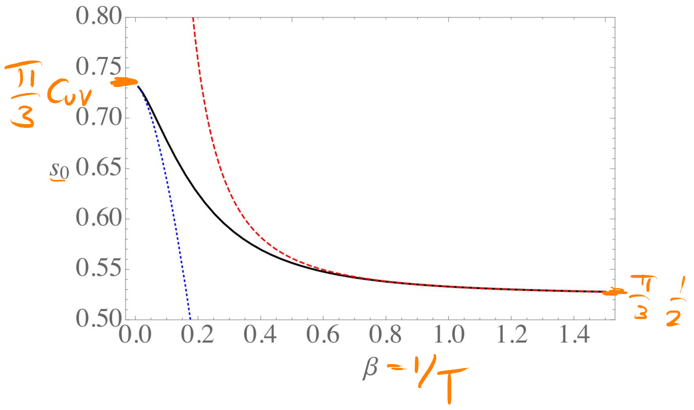
Zamolodchikov ‘91

Tricritical Ising $\rightarrow$ Ising

$$\left(c_{\mathrm{UV}} = \frac{7}{10}\right) \qquad \left(c_{\mathrm{IR}} = \frac{1}{2}\right)$$

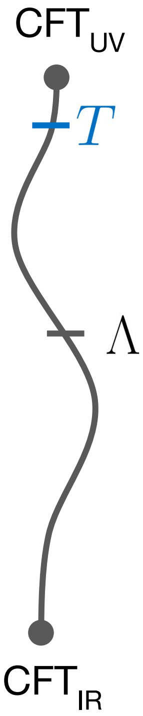
Triggered by relevant deformation $\mathcal{O} = \epsilon' \ (\Delta = \frac{6}{5})$





Non-integrable flows

$$S_{\rm QFT} = S_{\rm CFT} + \lambda \int d^2x\, \mathcal{O}$$



At high temperatures $T \gg \Lambda$,
conformal perturbation theory captures corrections

$$s_o(T) = \frac{\pi}{3} c_{\rm UV} \left[1 - \alpha_\Delta \left(\frac{\lambda}{T^{2-\Delta}} \right)^2 + \cdots \right]$$

$$\alpha_\Delta = \frac{\Gamma(2-\Delta)\Gamma(\frac{\Delta}{2})^2}{\Gamma(\Delta)\Gamma(1-\frac{\Delta}{2})^2} \geq 0$$

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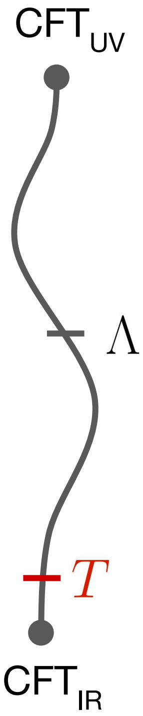

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$\to \dfrac{\partial\phi^2 - m^2\phi^2 - \lambda\phi^4}{\lambda^{1/3}}$

At low temperatures $T \ll \Lambda$,

$\partial\phi^2$ $\qquad$ $\partial\phi^4$

$$\text{QFT} = \text{CFT}_{\text{IR}} + \sum_i \lambda_i \int \mathcal{O}_i + \lambda_{T\bar{T}} \int T\bar{T} + \cdots$$

$J\bar{T}$

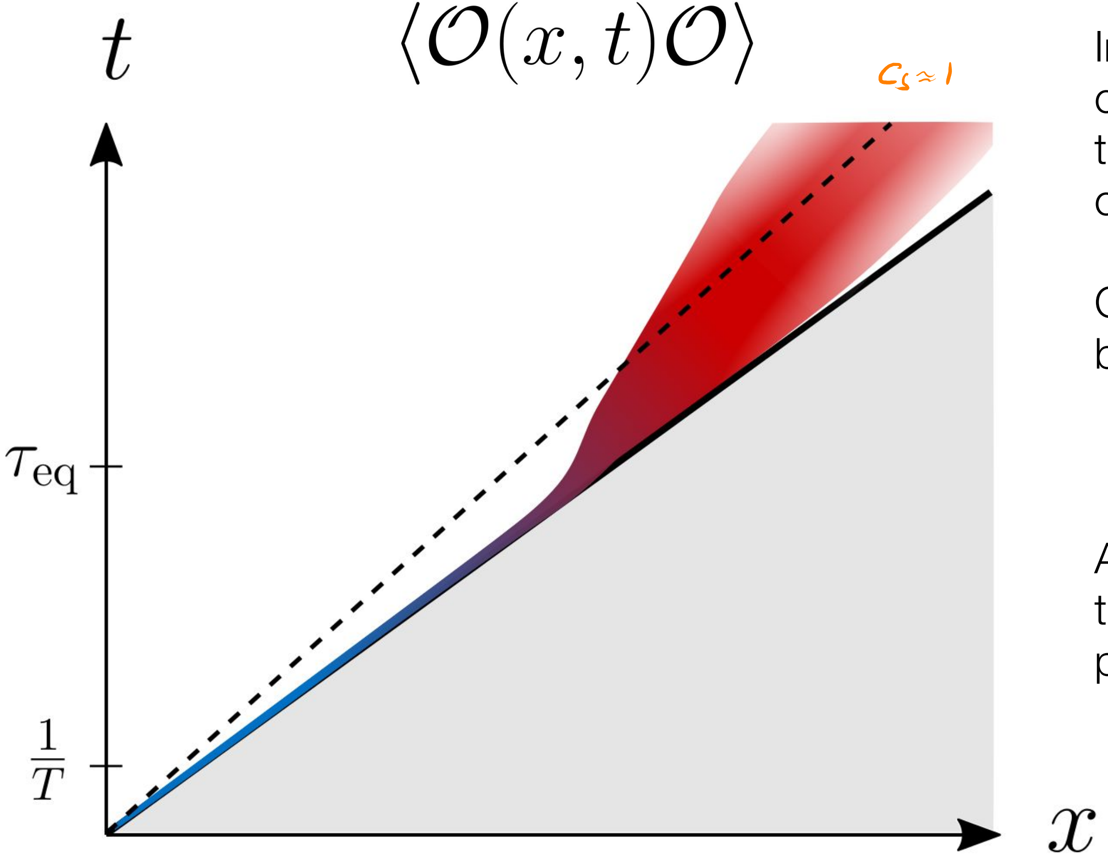
$$s_o(T) = \tfrac{\pi}{3} c_{\text{IR}} \left[1 + \lambda_{T\bar{T}} c_{\text{IR}} T^2 + \cdots \right]$$

Adams et al ... '03

causality $\Leftrightarrow$ $\lambda_{T\bar{T}} > 0$ (*)

(*) if no operator with $2 \leq \Delta < 3$

Causality constraints on thermalization



In 1+1d QFTs, hydrodynamics cannot emerge too soon, or the sound mode would spread outside the lightcone.

Can prove conjectured "Planckian bound"

Sachdev '99
Zaanen '04

$$\tau_{\rm eq} \gtrsim \frac{\hbar}{T}$$

At high and low temperatures, thermalization must be parametrically slower

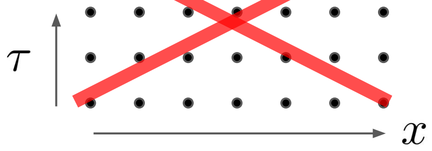
$$\tau_{\rm eq} \gtrsim \frac{1}{T}\frac{1}{c_{\rm UV}}\left(\frac{T}{\Lambda}\right)^{2(2-\Delta)}$$

II. Numerics

Observing thermalization and chaos of QFTs numerically

- Need to access real time dynamics, at late times $t \gg \beta$

  $\longrightarrow$ ~~Monte-Carlo on a Euclidean lattice~~



β

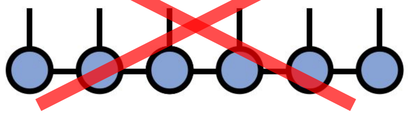
$\tau < \beta$

$\downarrow$

$\cancel{t > \beta}$

- High energy "thermal" states are highly entangled

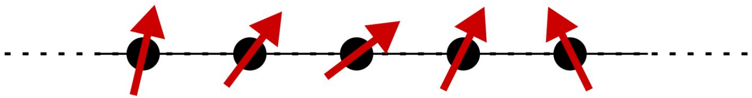
  $\longrightarrow$ ~~Matrix-product states~~



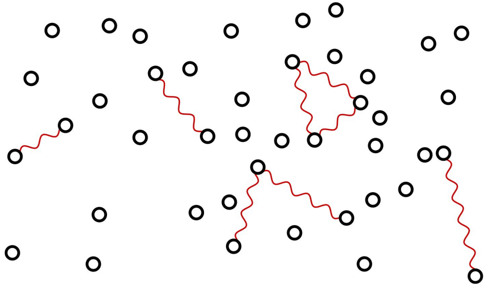
- Basically need to solve theory (exact diagonalization) $^{(*)}$

Observing thermalization and chaos of QFTs numerically

The lattice



The continuum



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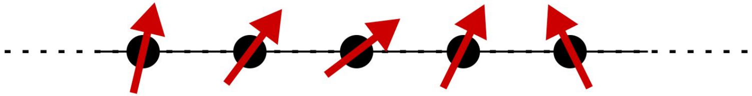


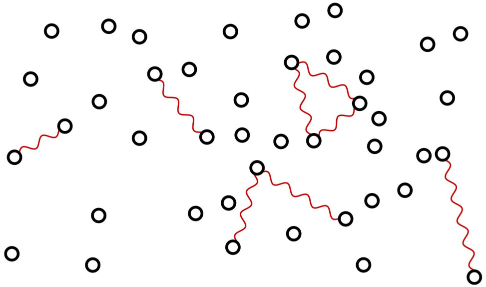
Lack of translation invariance

Even if irrelevant, Umklapp processes dissipate momentum at finite temperature

Need hierarchy $t_{\mathrm{eq}} \ll t_{\mathrm{Umklapp}}$

(nevertheless, thermalization of lattice interesting in its own right! e.g. Huse Oganesyan Mukerjee ‘06)

The continuum



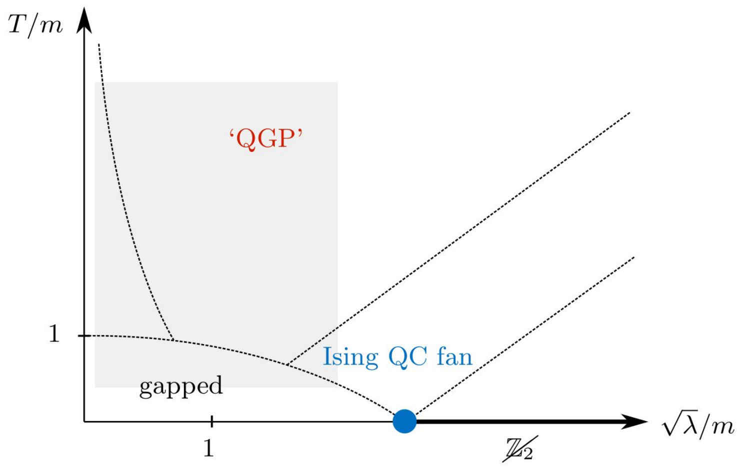
Need to truncate the Hilbert space

Case study:

$$S = \int d^2x\, (\partial\phi)^2 - m^2\phi^2 - \lambda\phi^4$$

In the regime

$$T \;\; \gtrsim \;\; m, \sqrt{\lambda} \;\; \gg \;\; 1/L$$



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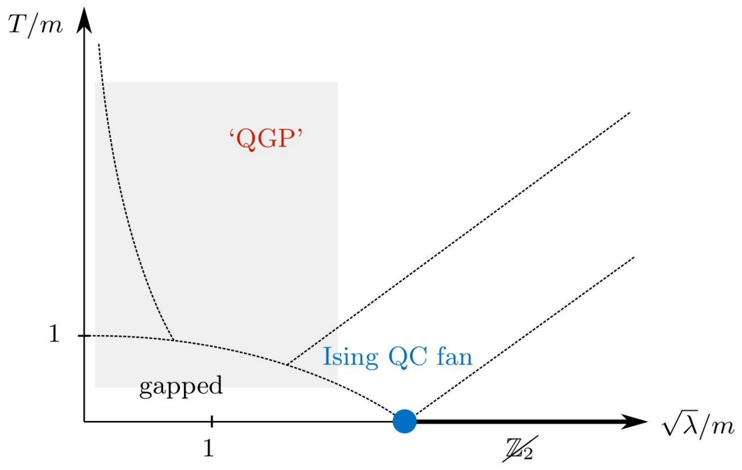
- Finite coupling has small effect on equilibrium thermodynamics… $\lambda^{1/3}$

- …but major effect on out-of-equilibrium physics!



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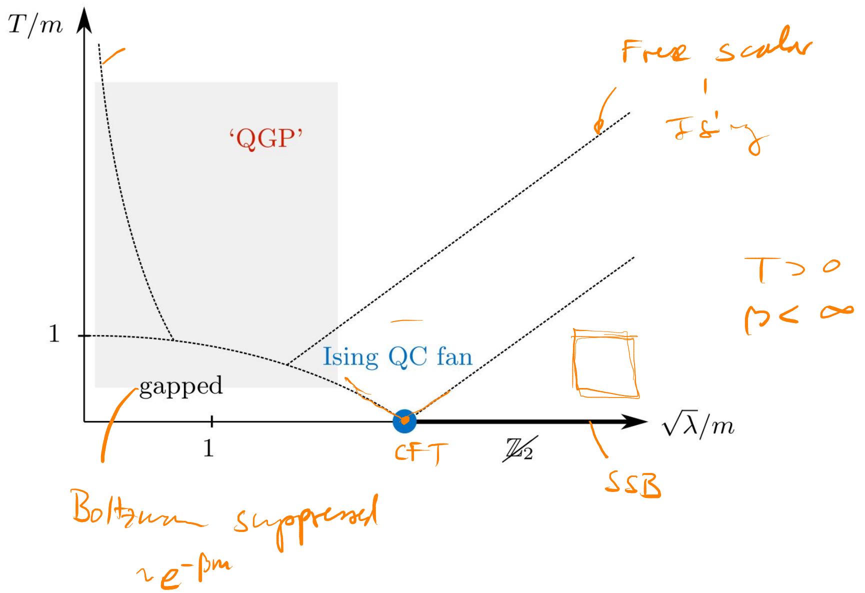
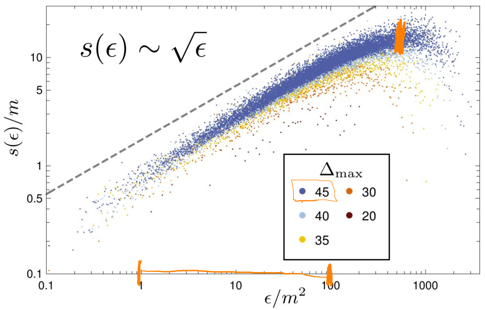
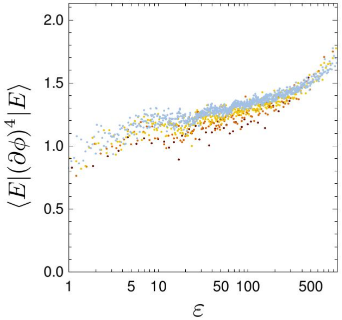


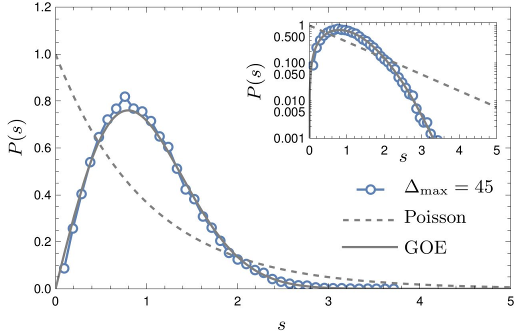
Cardy-like density of states $E = L T^2$



Eigenstate thermalization



RMT universality for $\lambda > 0$



Hamiltonian truncation methods for QFT

TCSA(-inspired)

Yurov Zamolodchikov … Hogervorst Rychkov van Rees Vitale Elias-Miro Tsvelik Bajnok Lajer Cohen Tilloy Farnsworth Houtz Luty …

Including out-of-equilibrium studies: Konik Brandino Mussardo Kukuljan Sotiriadis Takács Rakovszky Mestyán Collura Kormos Srdinšek Prosen Robinson James

Today: Lightcone Conformal Truncation [see Liam Fitzpatrick's bootcamp lectures]

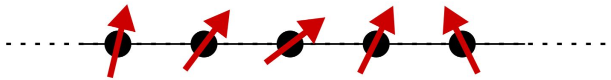
Katz Tavares Xu Anand Fitzpatrick Khandker Walters Xin Henning Chen Karateev

(other methods include: discrete lightcone quantization)

Volume of states

Lightcone quantization evades the orthogonality catastrophe → infinite volume (which also allows for exact boost symmetry)

But there is still an IR cutoff: excited states have a finite volume



'small' operator: $\cdots \otimes \mathbb{1} \otimes \sigma_x \otimes \sigma_z \otimes \mathbb{1} \otimes \mathbb{1} \otimes \cdots$

'big' operator: $\cdots \otimes \sigma_x \otimes \sigma_x \otimes \sigma_z \otimes \sigma_x \otimes \sigma_y \otimes \cdots$

$$\sim \quad |\phi\partial\partial\phi\phi\partial\partial\partial\phi\cdots\partial\phi\rangle$$

Truncation keeps states with $\leq \Delta_{\max}$ fields and derivatives ⇒ expect $L \sim \Delta_{\max}/E$

(this lattice analogy is actually used, Krylov basis, e.g. Parker Cao Scaffidi Altman '18)

Volume of states

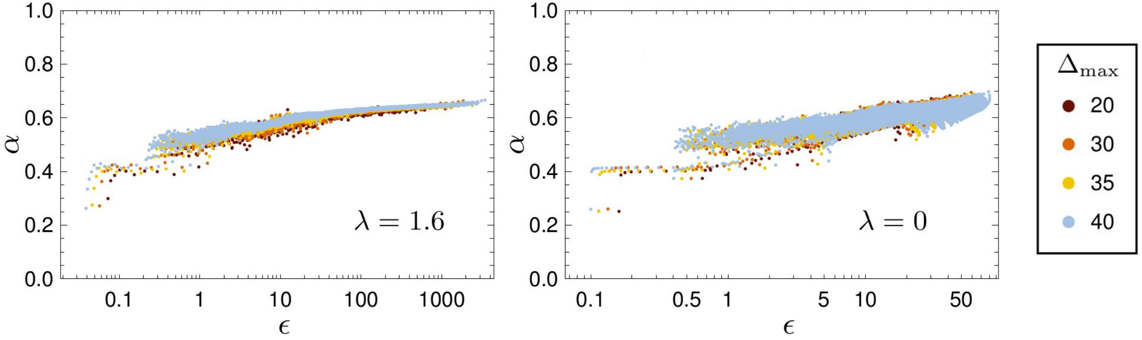
To measure the volume of an eigenstate, we cannot use $\langle E, P|\mathcal{O}(t,x)|E,P\rangle$

Volume of states

To measure the volume of an eigenstate, we cannot use $\langle E, P | \mathcal{O}(t,x) | E, P \rangle$

Instead, measure matrix element between boosted state $\langle E, P | \mathcal{O}(t,x) | E, P' \rangle$

$$\langle E, q | T_{00} | E, 0 \rangle = E^2 \left(1 - \left(\underbrace{\alpha \frac{\Delta_{\text{max}}}{E}}_{L} \right)^2 q^2 + \cdots \right)$$



Volume of states

In the thermodynamic limit, the expectation values of local operators are intrinsic

$$\langle \mathcal{O} \rangle_\beta = f_{\mathcal{O}}(\beta) = f_{\mathcal{O}}(\varepsilon)$$

$$\varepsilon = \frac{E}{L} = \frac{E^2}{\Delta}$$

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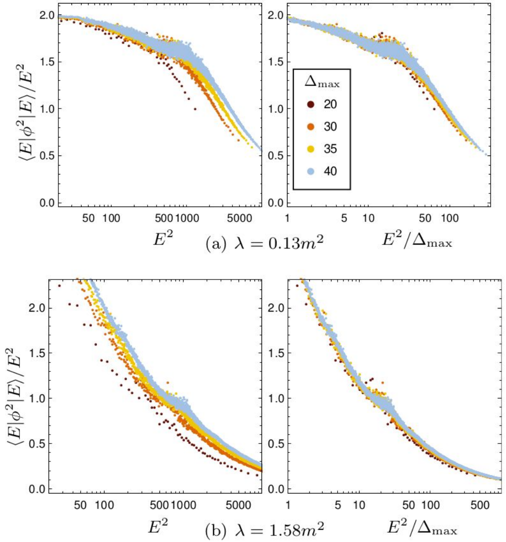

(a) $\lambda = 0.13m^2$

(b) $\lambda = 1.58m^2$

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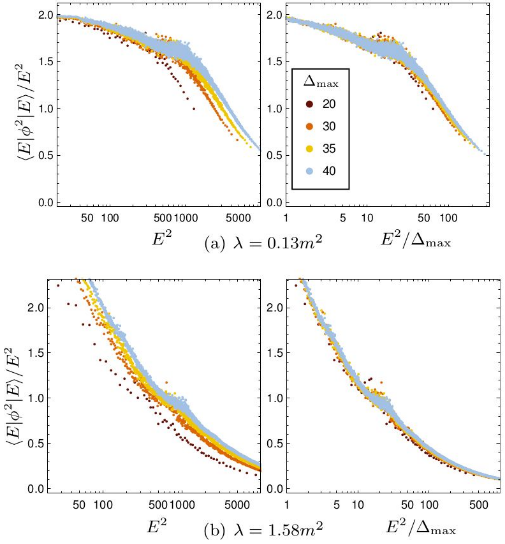
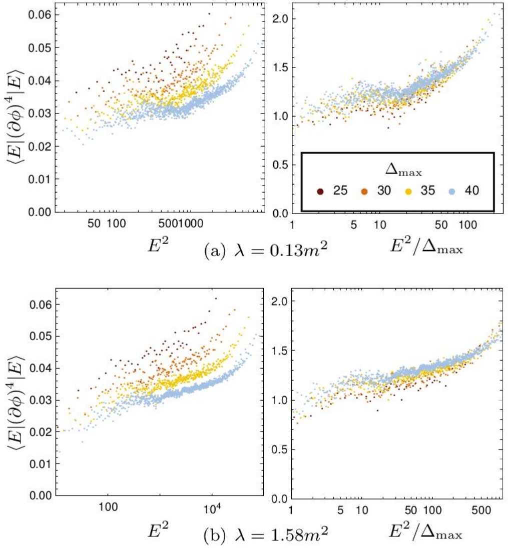


$\mathcal{O} = (\partial\phi)^4$



Eigenstate thermalization

$$S = \int d^2x\,(\partial\phi)^2 - m^2\phi^2 - \lambda\phi^4$$

At high temperatures $T \gg m, \sqrt{\lambda} \gg 1/L$, thermal expectation values are approximately that of the UV free scalar, e.g.:

$$\langle(\partial\phi)^4\rangle_\beta = 3\varepsilon^2 \qquad T^4 \qquad\qquad \langle(\partial^2\phi)^2\rangle_\beta = \frac{24\pi}{5}\varepsilon^2$$

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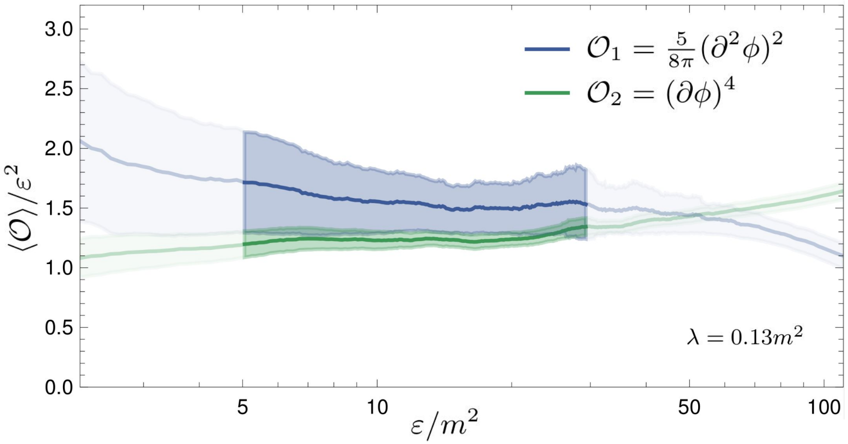
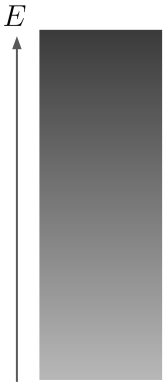
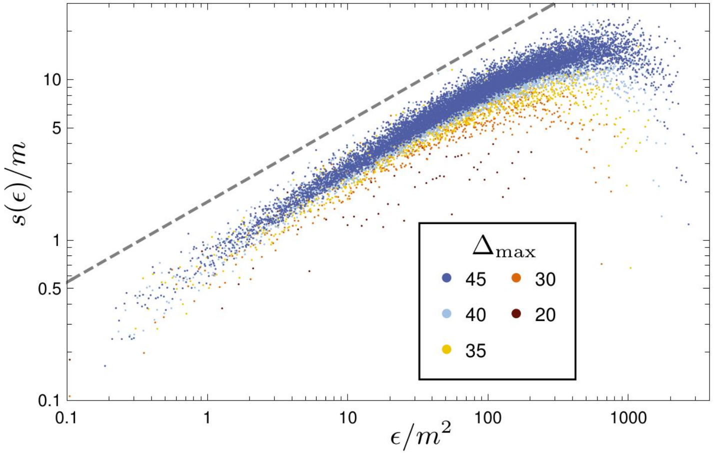


Thermodynamics from the spectrum



Density of states $e^{S(E)}$ from average eigenvalue spacing

Cardy-like growth $s(\epsilon) \sim \sqrt{\epsilon}$



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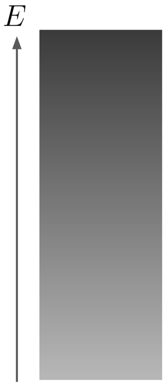

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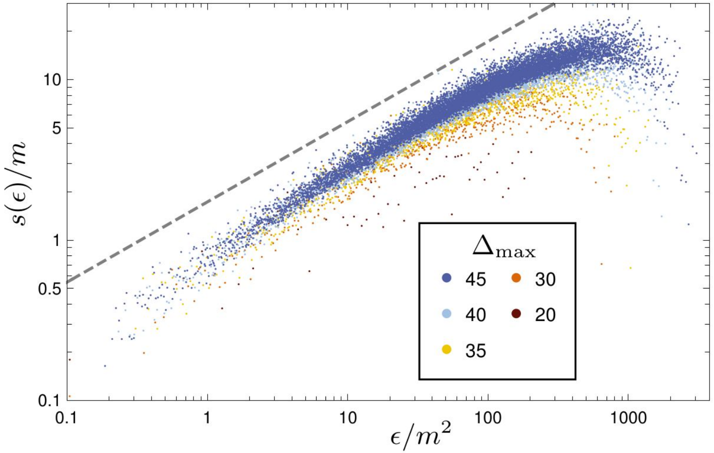
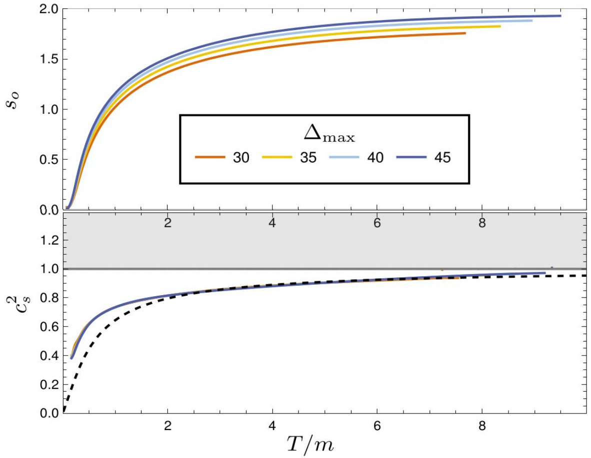
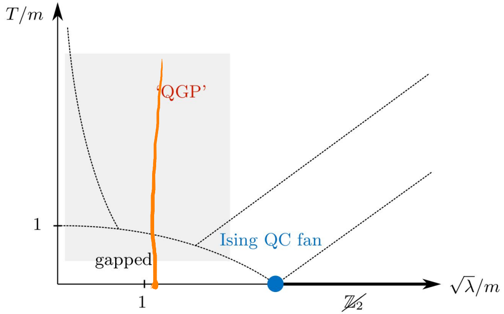


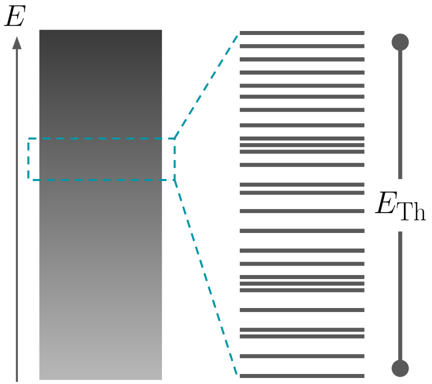


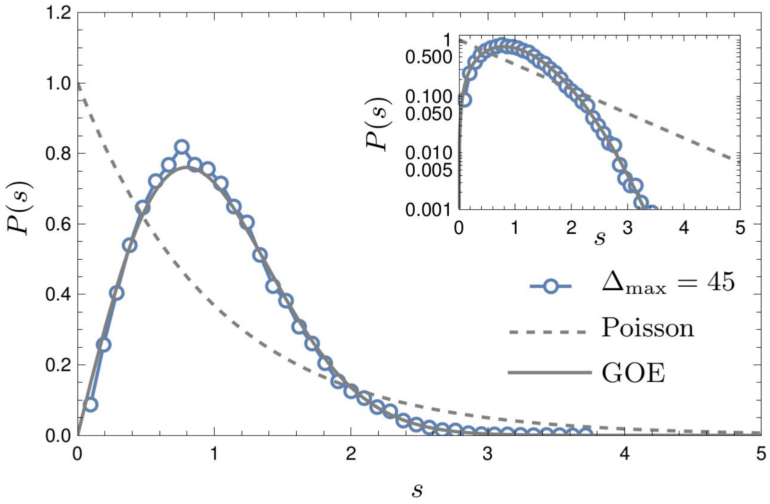
A more refined thermodynamic test is the deviation from Cardy
Consider:

$$s_o(T) \equiv \frac{s(T)}{T} \quad \text{or} \quad c_s^2 = \frac{dP}{d\mathcal{E}}$$

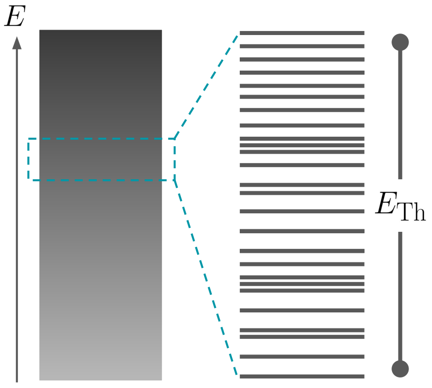


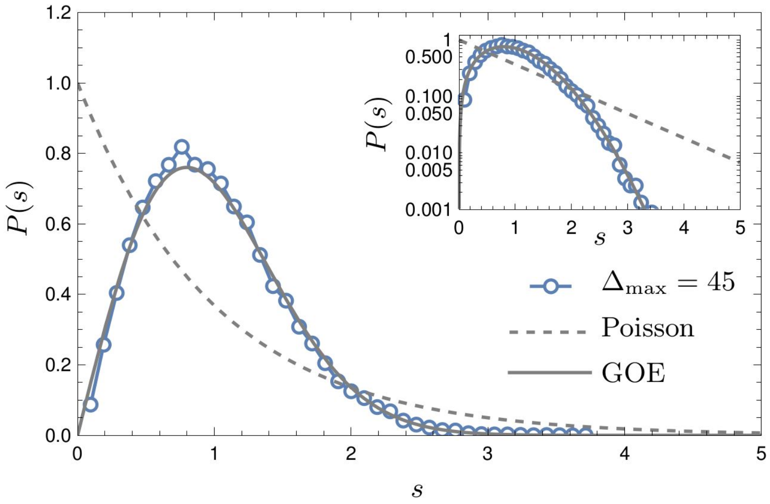
Eigenvalue statistics





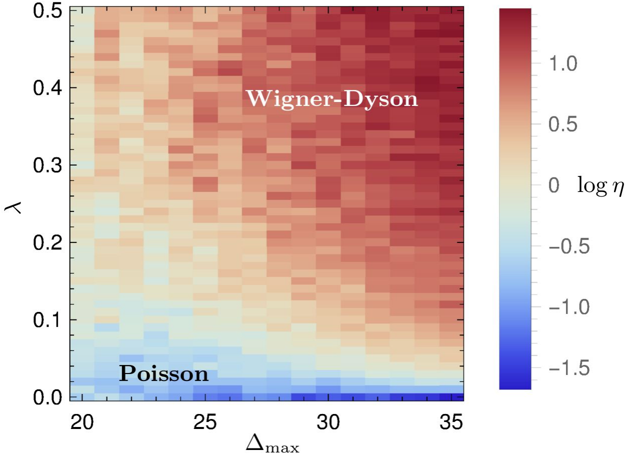
Eigenvalue statistics





Find RMT universality away from the edges of the spectrum, at any $\lambda > 0$

$$\eta = \frac{\|P - P_{\text{Poisson}}\|}{\|P - P_{\text{GOE}}\|}$$



(e.g.: Konik Brandino Mussardo ‘10)

How random are eigenvectors?

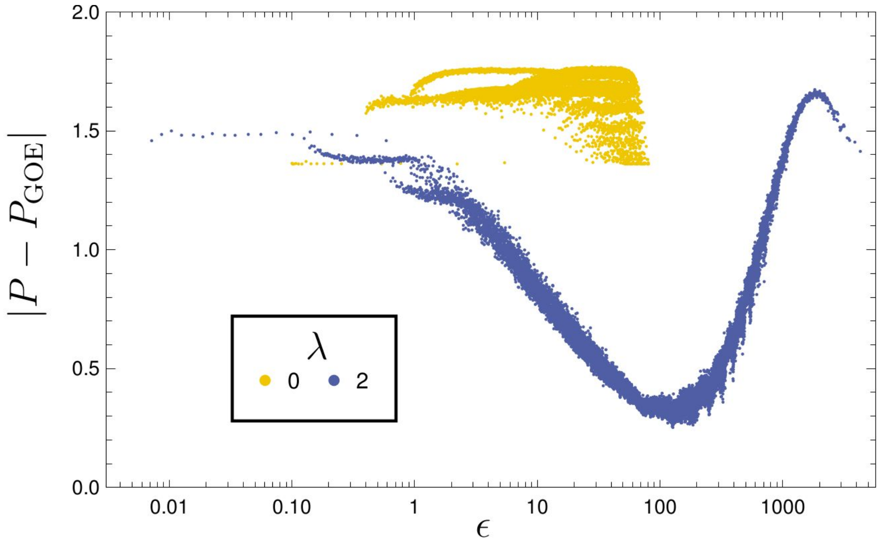
Exact diagonalization gives access to entire eigenvectors $|E\rangle = \sum_i |i\rangle\langle i|E\rangle$

For any individual eigenstate, can study distribution of coefficients:

$$P\left(|\langle i|E\rangle|^2\right)$$

No ‘quantum scars’, all states are thermal (resolution to Srdinšek Prosen Sotiriadis ‘19)

nonthermal states were seen in a different regime in Robinson James Konik ‘18

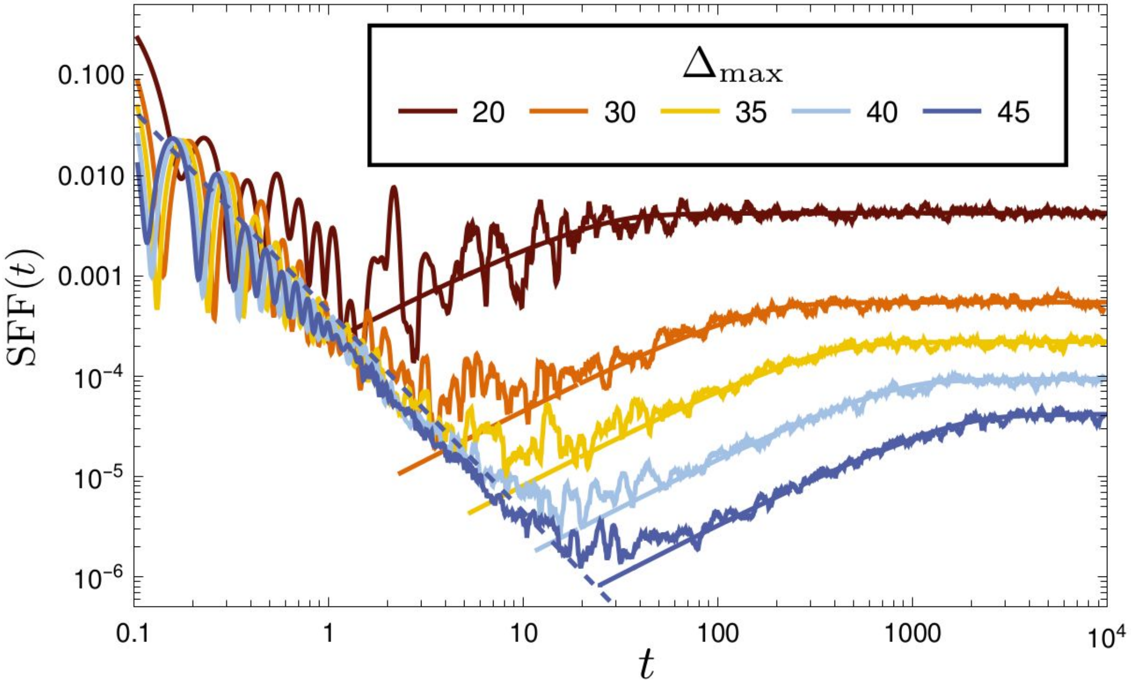


Real time dynamics

$$\langle \mathcal{O}(t)\mathcal{O}\rangle_\beta = \sum_{EE'} e^{i(E-E')t} e^{-\beta E} |\langle E|\mathcal{O}|E'\rangle|$$

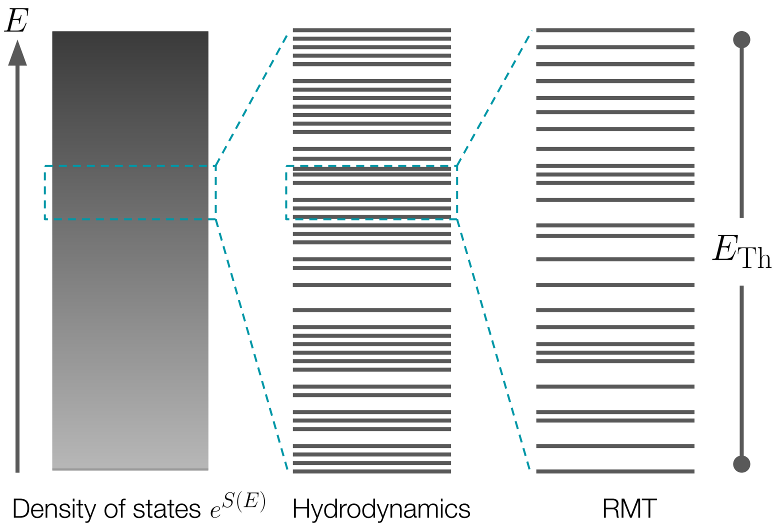
Real time dynamics

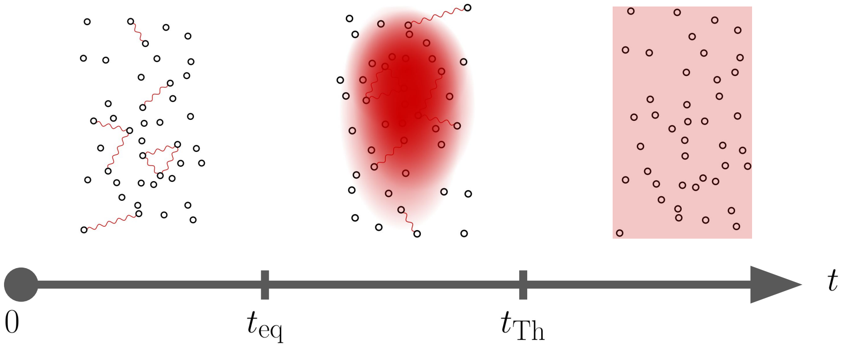
$$\mathrm{SFF}(t) = \sum_{EE'} e^{i(E-E')t} e^{-\beta E} \cancel{|\langle E|\mathcal{O}|E'\rangle|}$$



Hydrodynamics?

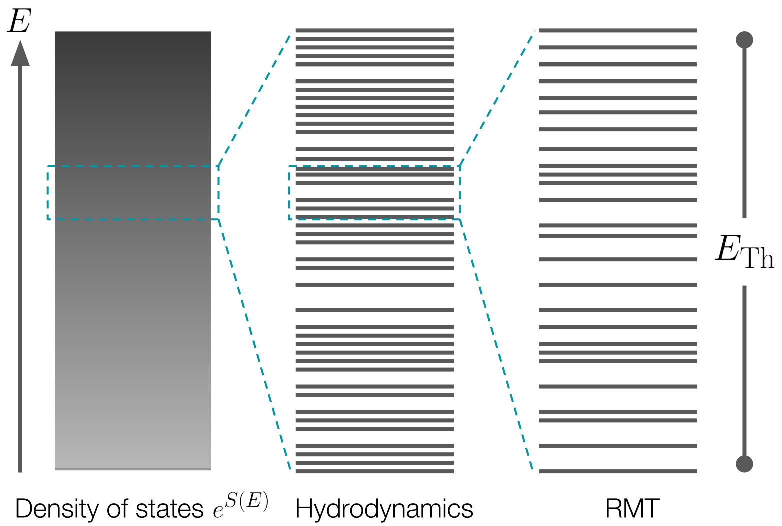
QFTs have much more structure than just RMT

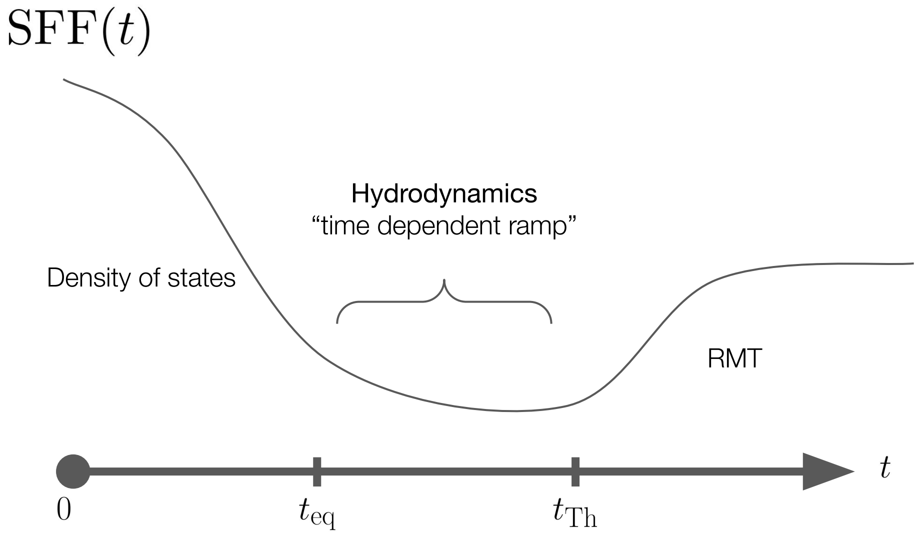




Hydrodynamics?

QFTs have much more structure than just RMT





Friedman Chan De Luca Chalker '19
Swingle Winer '20

Thank you!