Spin structure of hadrons from lattice QCD

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Overview

introduction to nucleon spin structure and generalized parton distributions (GPDs)

computation of moments of (G)PDs in lattice QCD

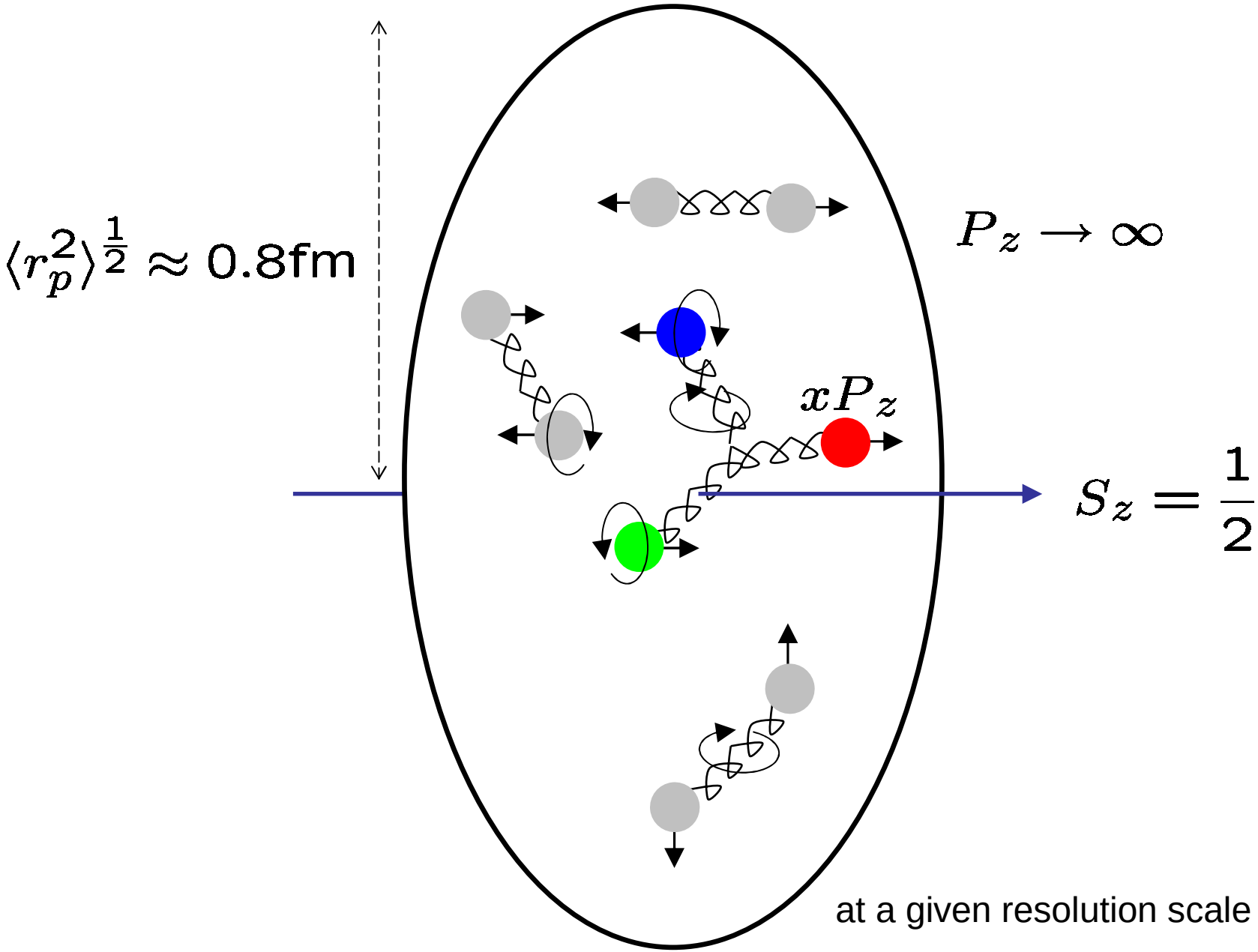
selected lattice results on
- quark spin fraction
- form factors of the energy momentum tensor
- transverse spin densities

relation GPDs ↔ tmdPDFs

implications for asymmetries

summary

Spin structure of the nucleon



at a given resolution scale $\lambda \sim \frac{1}{|Q|}$

Some fundamental sumrules

symmetries → Noether's theorem → conservation laws

QCD energy momentum tensor (and angular momentum density tensor)

$$\langle P'|T^{\mu\nu}|P\rangle = \overline{U}(P')\Big\{\gamma^\mu \overline{P}^\nu A(\Delta^2) + \frac{i\sigma^{\mu\rho}\Delta_\rho \overline{P}^\nu}{2m_N} B(\Delta^2) + \frac{\Delta^\mu \Delta^\nu}{m_N} C(\Delta^2)\Big\} U(P)$$

where $\Delta = P' - P$

momentum sumrule

$$\begin{aligned} 1 &= A_{quark}(0) + A_{gluon}(0) \\ &= \langle x\rangle_q + \langle x\rangle_g \end{aligned}$$

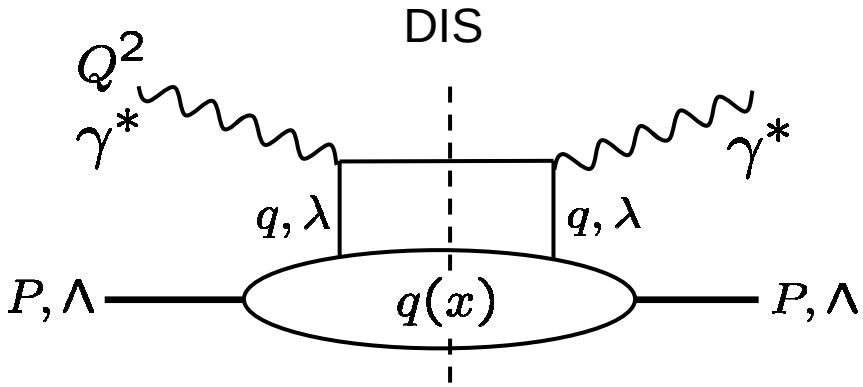
vanishing of the anomalous gravitomagnetic moment

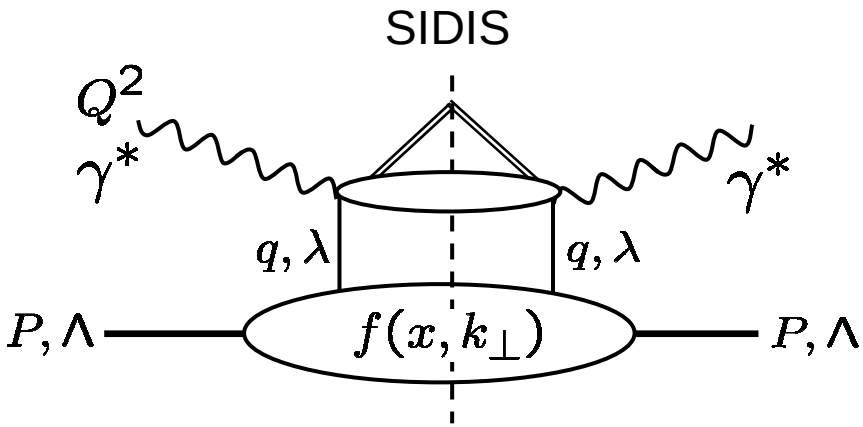
$$0 = B_q(0) + B_g(0)$$

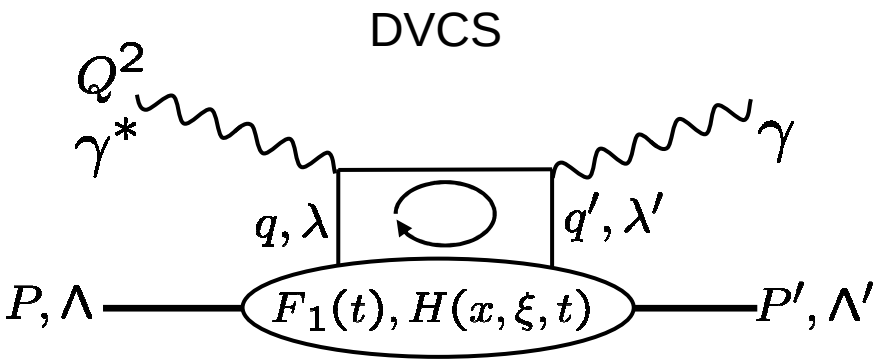
(Jaffe,Ji-) nucleon spin sumrule

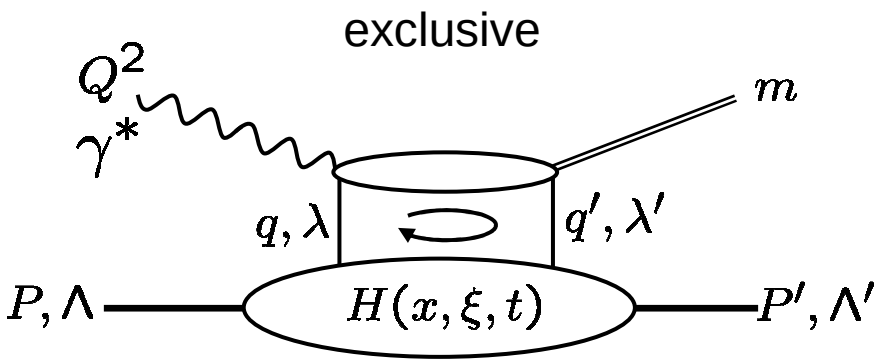
$$\begin{aligned} \frac{1}{2} = S_z &= \frac{1}{2}\Big(A_{quark}(0) + A_{gluon}(0) + B_q(0) + B_g(0)\Big) \\ &= \frac{1}{2}\Delta\Sigma_q + L_q + J_g = \frac{1}{2}\Delta\Sigma_q + L_q + \Delta G + L_g \end{aligned}$$

Typical processes



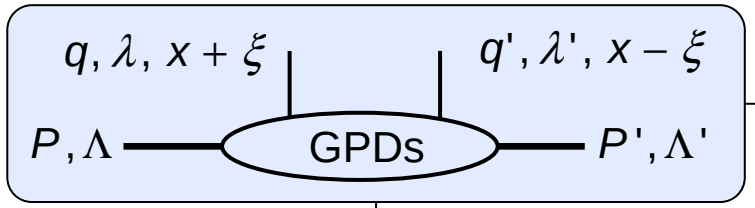






Brief introduction to GPDs

Müller,Robaschik,Geyer, Dittes,Horejsi,1994
Ji, 1997, Radyushkin, 1997



$t = (\Delta \equiv P' - P)^2 \mathrel{\hat{=}}$ momentum transfer squared

$\xi = -n \cdot \Delta / 2 \mathrel{\hat{=}}$ longitudinal momentum transfer

two unpolarized and two polarized generalized distributions at twist 2

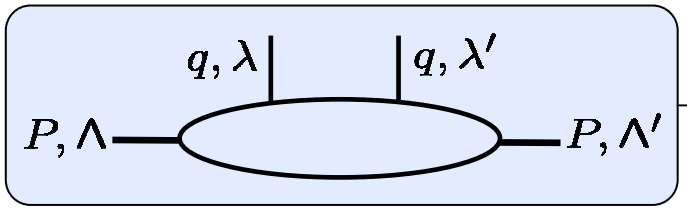
$H(x,\xi,t), E$
$\tilde{H}, \tilde{E}$

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\frac{\eta n}{2}) \gamma^\mu \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \bar{U}(P') \left(\gamma^\mu \underline{H(x,\xi,\Delta^2)} + i \frac{\sigma^{\mu\nu}\Delta_\nu}{2M} \underline{E(x,\xi,\Delta^2)} \right) U(P)$$

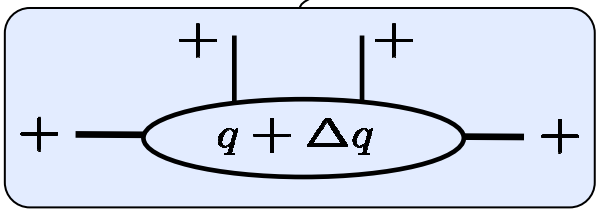
vector

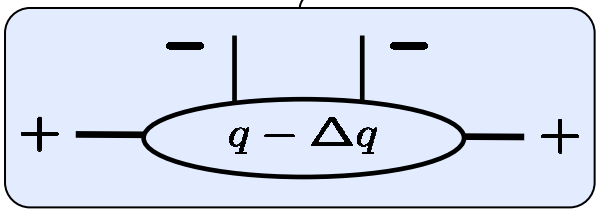
see talk by P. Kroll at 18:30 today and the GPD session on Wednesday

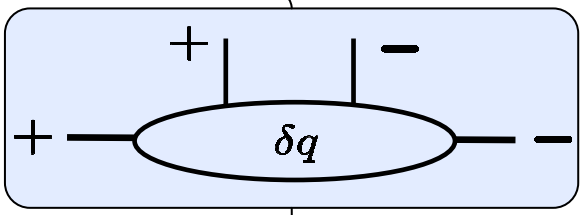
Transversity and spin density matrices



number of <u>independent</u> quark-nucleon helicity amplitudes→







transversity/tensor/ quark helicity-flip

spin density matrix

$$
\begin{array}{cc}
 & (\Lambda, \lambda) \text{ helicities} \\
(\Lambda', \lambda') & \begin{array}{cccc} (+,+) & (-,+) & (+,-) & (-,-) \end{array} \\
\text{helicities} \begin{array}{c} (+,+) \\ (-,+) \\ (+,-) \\ (-,-) \end{array} & \begin{pmatrix} q+\Delta q & 0 & 0 & 2\delta q \\ 0 & q-\Delta q & 0 & 0 \\ 0 & 0 & q-\Delta q & 0 \\ 2\delta q & 0 & 0 & q+\Delta q \end{pmatrix}
\end{array}
$$

Soffer-bound

$$q + \Delta q \geq 2|\delta q|$$

see transverse spin session on Wednesday

Tensor GPDs

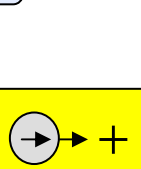
four transversity / tensor / quark helicity flip GPDs at twist 2
(M. Diehl, EPJ C19, 2001)

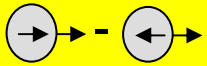
$H_T, \bar{E}_T, \tilde{H}_T, \tilde{E}_T$

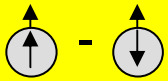
$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\frac{\eta n}{2}) \sigma^{\mu\nu} \gamma_5 \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \bar{U}(P') \Big\{ \sigma^{\mu\nu} \gamma_5 \Big(\underline{H_T(x,\xi,\Delta^2)} - \frac{t}{2m^2} \tilde{H}_T(x,\xi,\Delta^2) \Big)$$
$$+ \frac{\in^{\mu\nu\alpha\beta} \Delta_\alpha \gamma_\beta}{2m} \underline{\bar{E}_T(x,\xi,\Delta^2)} - \frac{\Delta^{[\mu} \sigma^{\nu]\alpha} \gamma_5 \Delta_\alpha}{2m^2} \underline{\tilde{H}_T(x,\xi,\Delta^2)} + \frac{\in^{\mu\nu\alpha\beta} \bar{P}_\alpha \gamma_\beta}{m} \underline{\tilde{E}_T(x,\xi,\Delta^2)} \Big\} U(P)$$

Some basic properties of GPDs

forward limit

$H(x,0,0) = q(x) \mathrel{\hat{=}} 1/2\big($  $\big)$

$\tilde{H}(x,0,0) = \Delta q(x) \mathrel{\hat{=}}$ 

$H_T(x,0,0) = \delta q(x) = h_1(x) \mathrel{\hat{=}}$ 

„local“ limit

$$\int dx H(x,\xi,t) = F_1(t),$$

$$\int dx \tilde{H}(x,\xi,t) = g_A(t),$$

$$\int dx H_T(x,\xi,t) = g_T(t) \quad \text{etc.}$$

form factors of energy momentum tensor

$$\int dx\, x H(x,\xi,t) = A_{20}(t) + (-2\xi)^2 C_{20}(t)$$

$$\int dx\, x E(x,\xi,t) = B_{20}(t) - (-2\xi)^2 C_{20}(t)$$

higher Mellin moments

$$\int dx\, x^{n-1} H(x,\xi,t) = A_{n0}(t) + \ldots$$

$$\int dx\, x^{n-1} E(x,\xi,t) = B_{n0}(t) + \ldots$$

Generalized form factors in Lattice QCD

Mellin moments

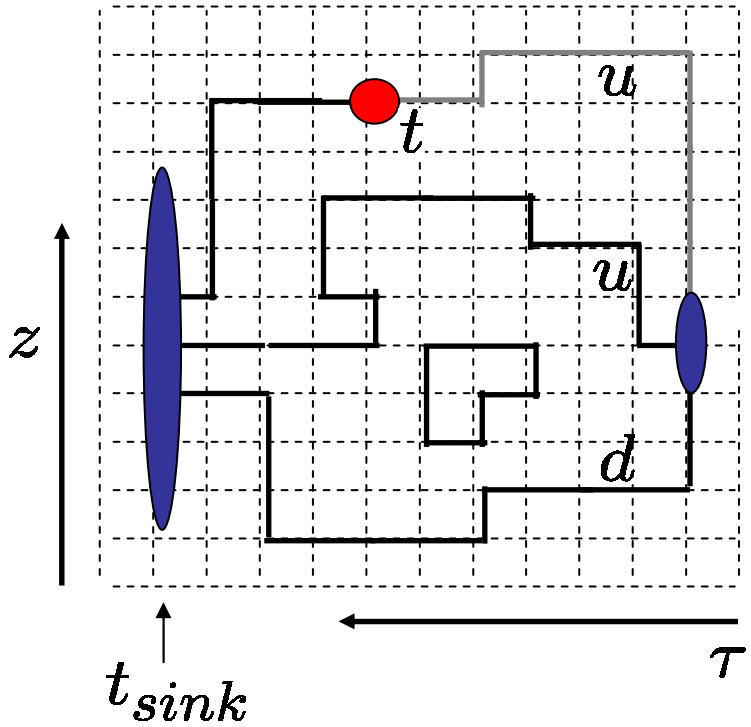
$$\langle P', \Lambda' | \bar{q}(0) \, [\Gamma_{Dirac} D^{\mu_1} D^{\mu_2} \ldots] \, q(0) | P, \Lambda \rangle$$
$$\propto \Delta\Sigma, \delta q, \langle x \rangle, F_1(t), A_{20}(t), B_{20}(t), A_{T20}(t), \ldots$$

$t = \Delta^2$

nucleon source/sink uud

operator insertion at time t

quark-path through lattice

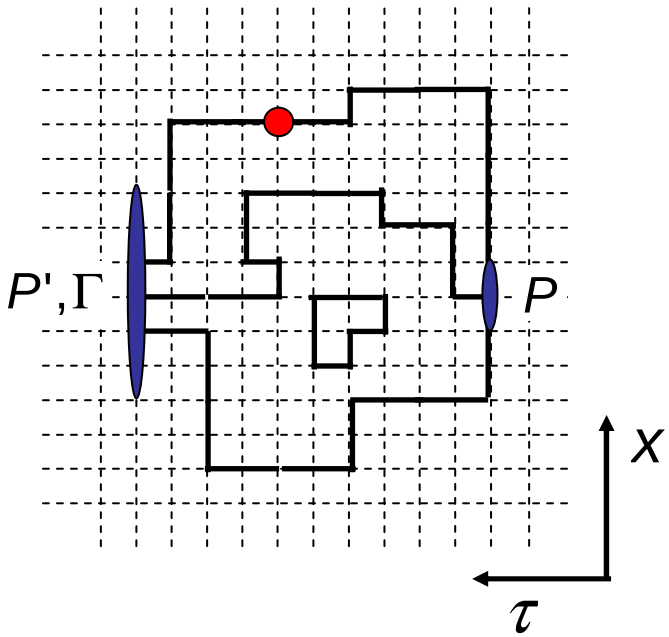


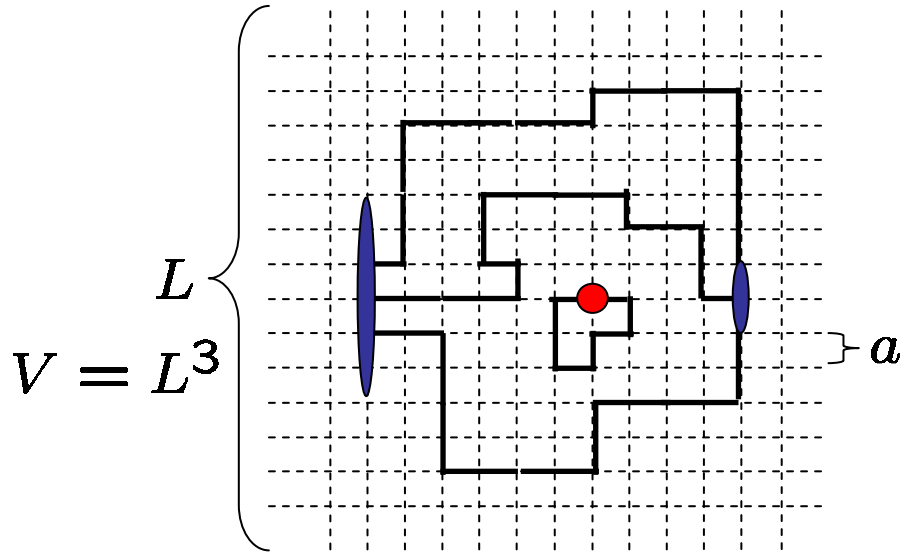
$$\rightarrow \Delta\Sigma^{lat}, \delta q^{lat}, \langle x \rangle^{lat}, F^{lat}{}_1(t), A^{lat}_{20}(t), B^{lat}_{20}(t), A^{lat}_{T20}(t), \ldots$$

Computation of moments of GPDs on the lattice

unquenched (full, dynamical) $\det(D[U]) \neq 1, n_f \neq 0$

disconnected contributions





drop out for u-d

very expensive, not included so far

probably small for helicity flip operators

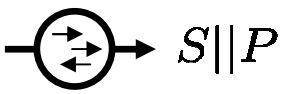
finite lattice spacing

finite volume

large quark masses

Longitudinal spin structure of the nucleon

(in collaboration with LHPC/MILC)



Quark spin fraction from lattice simulation

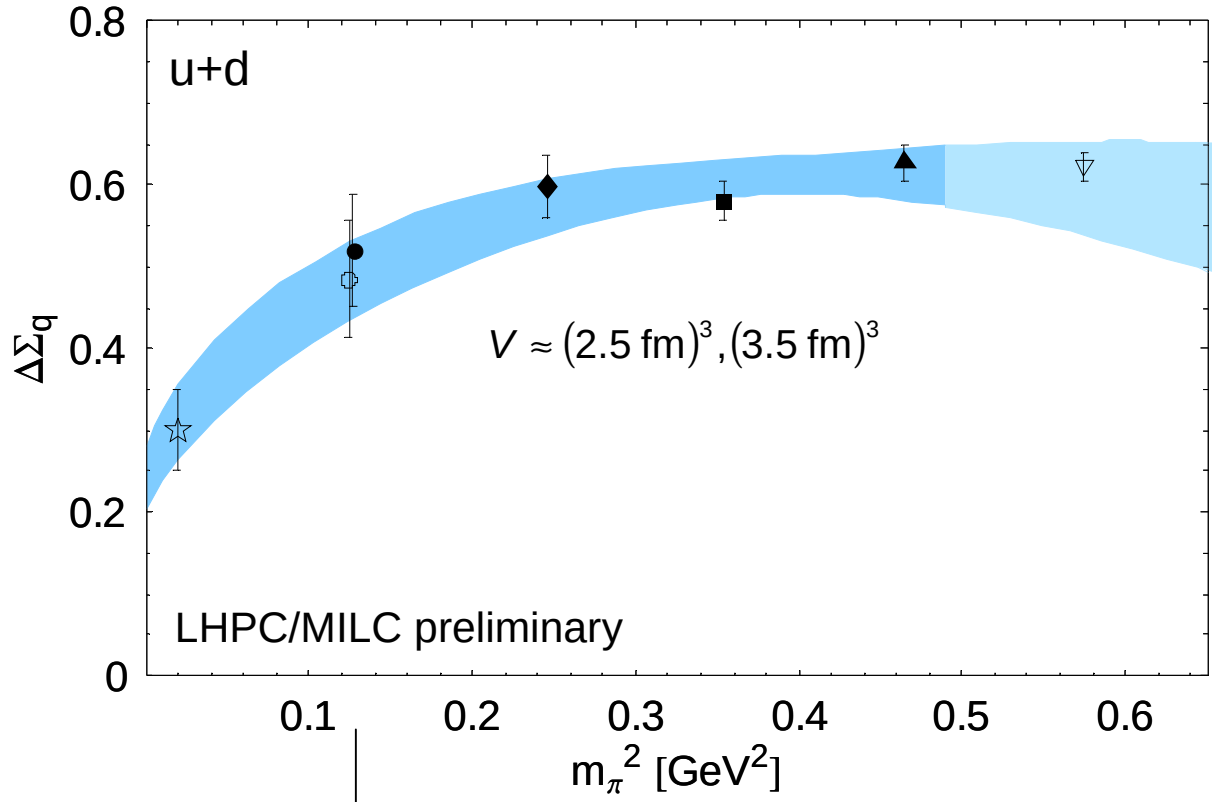
$$\langle P|\bar{q}(0)\gamma_\mu\gamma_5 q(0)|P\rangle = \bar{U}(P)\gamma_\mu\gamma_5 U(P)\,\Delta\Sigma$$

$$\Delta\Sigma = \tilde{A}_{10}(0) = \int_{-1}^{1} dx\,\Delta q(x) = \langle 1\rangle_{\Delta q} + \langle 1\rangle_{\Delta\bar{q}}$$

Gell-Mann-Oakes-Renner
$m_\pi^2 \propto m_q$

chiral extrapolation based on leading order heavy baryon chPT

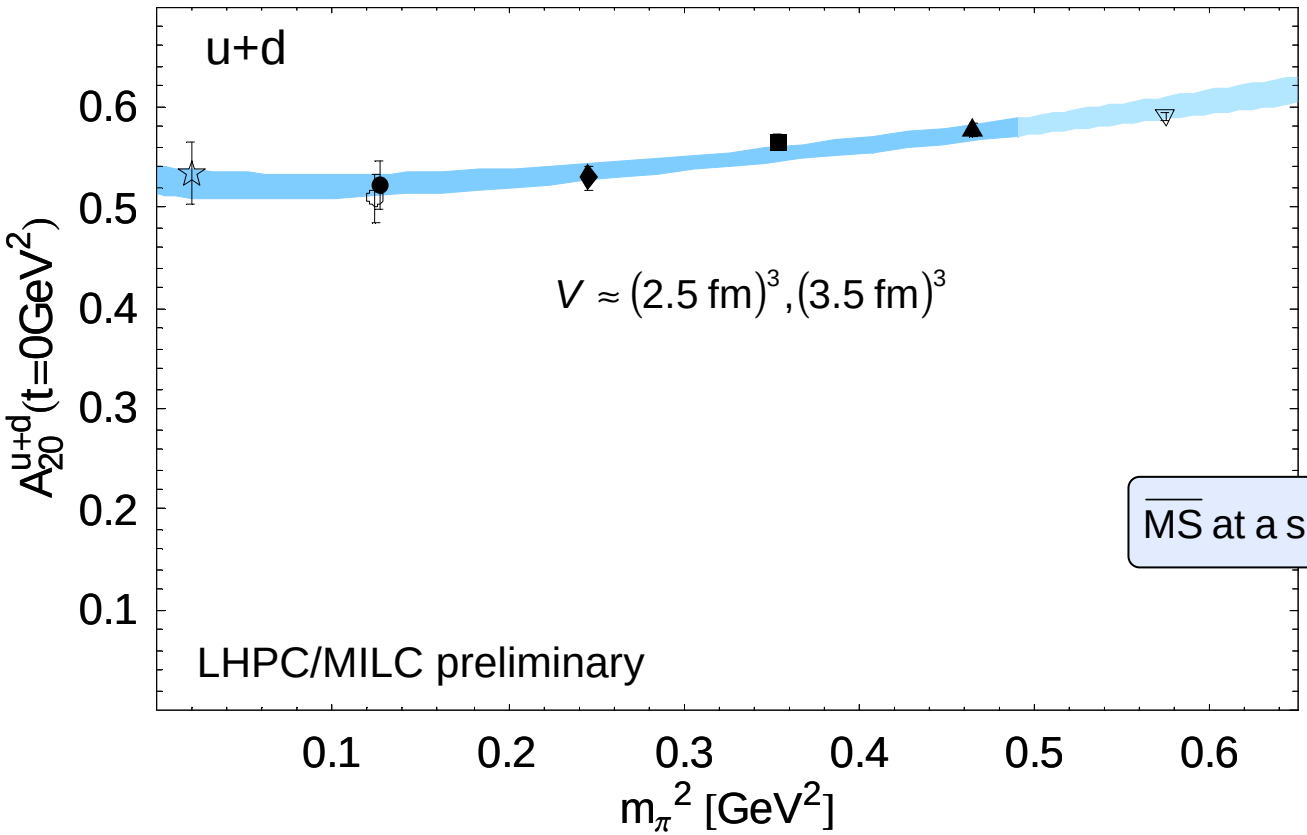
Diehl, Manashov&Schäfer, EPJA 2006



$$\Delta\Sigma_{u+d}^{lat} = 0.50(5) \;@\; m_\pi \approx 350\text{MeV}$$

Isosinglet momentum fraction of quarks in the nucleon

$$\langle P|\bar{q}(0)\gamma^{\{\mu}D^{\nu\}}q(0)|P\rangle = \bar{U}(P)\gamma^{\{\mu}\bar{P}^{\nu\}}U(P)\langle x\rangle \longleftrightarrow \langle x\rangle = A_{20}(0) = \int_{-1}^{1} dx\; x\, q(x) = \langle x\rangle_q + \langle x\rangle_{\bar{q}}$$



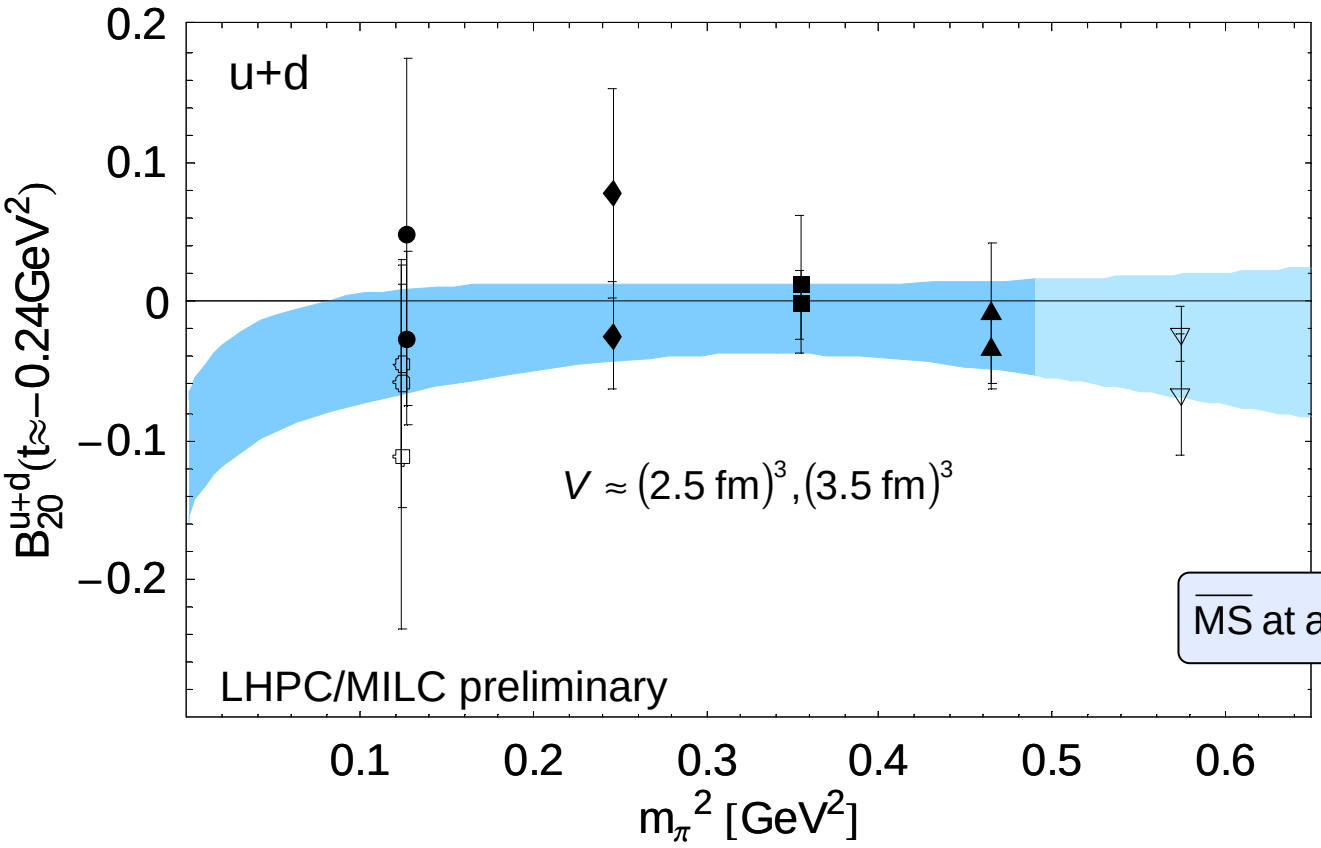
$\overline{\mathrm{MS}}$ at a scale of $Q^2 = 4\ \mathrm{GeV}^2$

chiral extrapolation based on covariant chPT O(p²) — Dorati, Gail, Hemmert 2007

Isosinglet $B_{20}(t)$ generalized form factor

anomalous gravitomagnetic moment of quarks

$$\langle P'|\bar{q}(0)\gamma^{\{\mu}D^{\nu\}}q(0)|P\rangle = \bar{U}(P')\left\{\ldots - i\frac{\Delta_\rho \sigma^{\rho\{\mu}\bar{P}^{\nu\}}}{2m_N}B_{20}(t) + \ldots\right\}U(P) \longleftrightarrow B_{20}(t) = \int_{-1}^{1} dx\; x\, E(x,0,t)$$

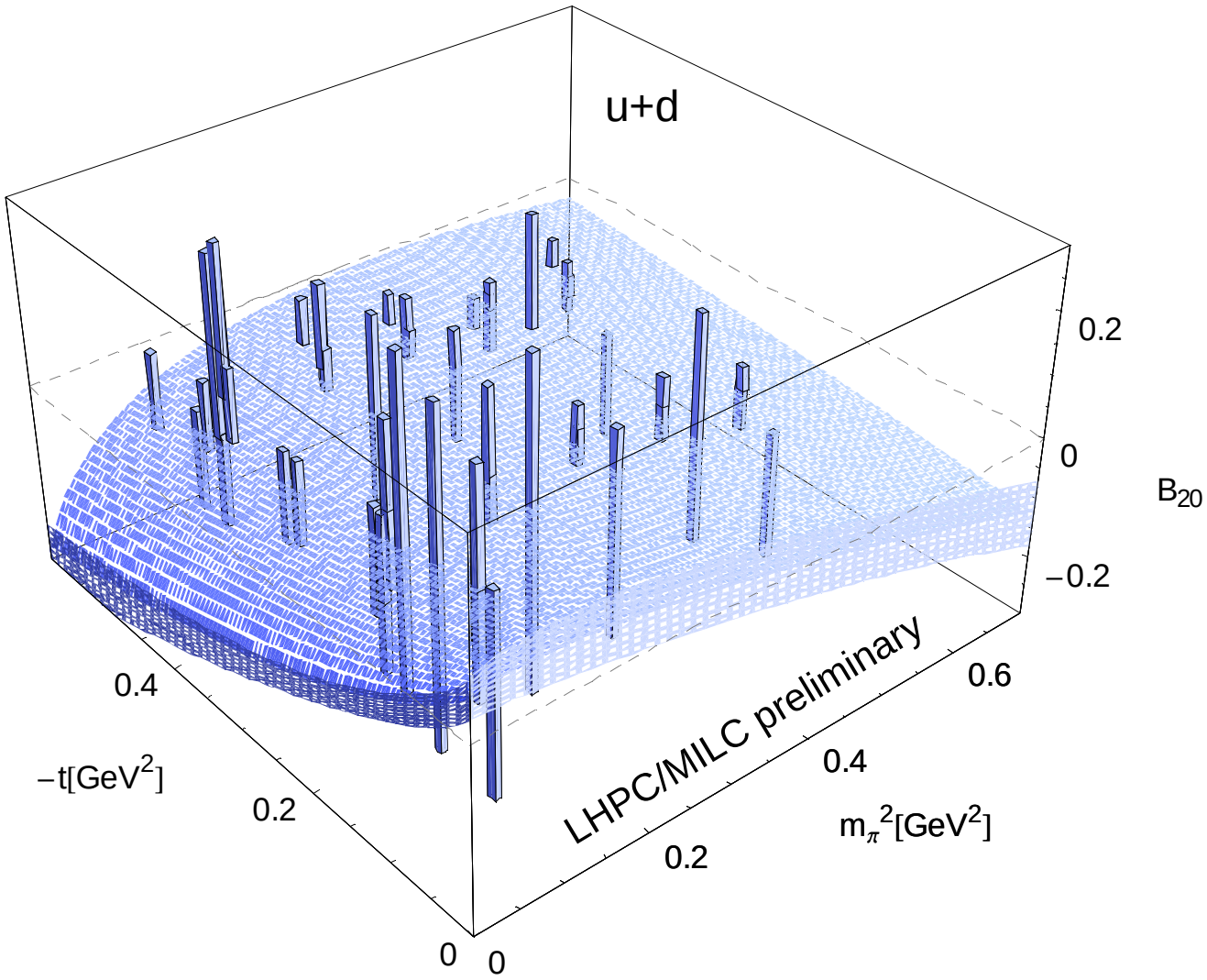


$\overline{\text{MS}}$ at a scale of $Q^2 = 4\text{ GeV}^2$

$$B_{20}^{extr.}(0) = -0.09 \pm 0.06$$

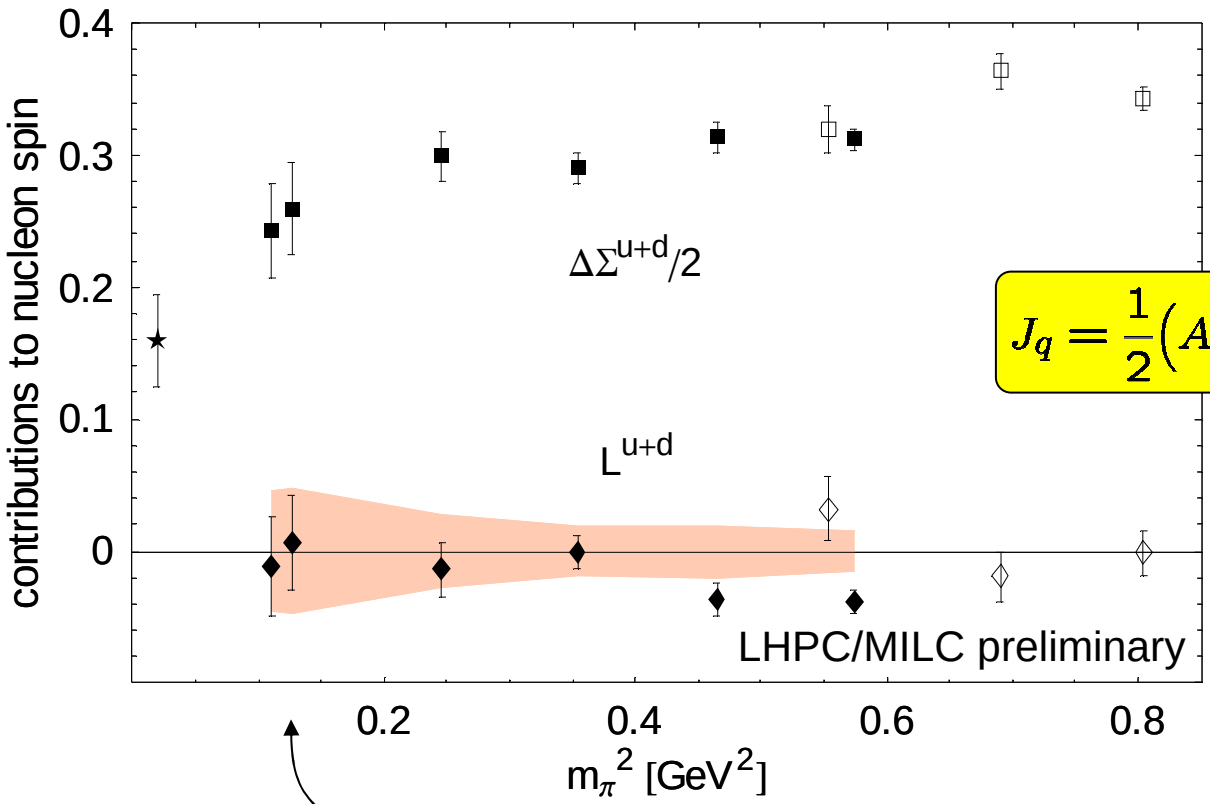
Ph. Hägler, COMPASS, 2007

Anomalous gravitomagnetic form factor of quarks



chiral extrapolation based on heavy baryon chPT O(q²) — Diehl, Manashov, Schäfer, EPJC 2006

Quark spin and OAM contributions to the nucleon spin



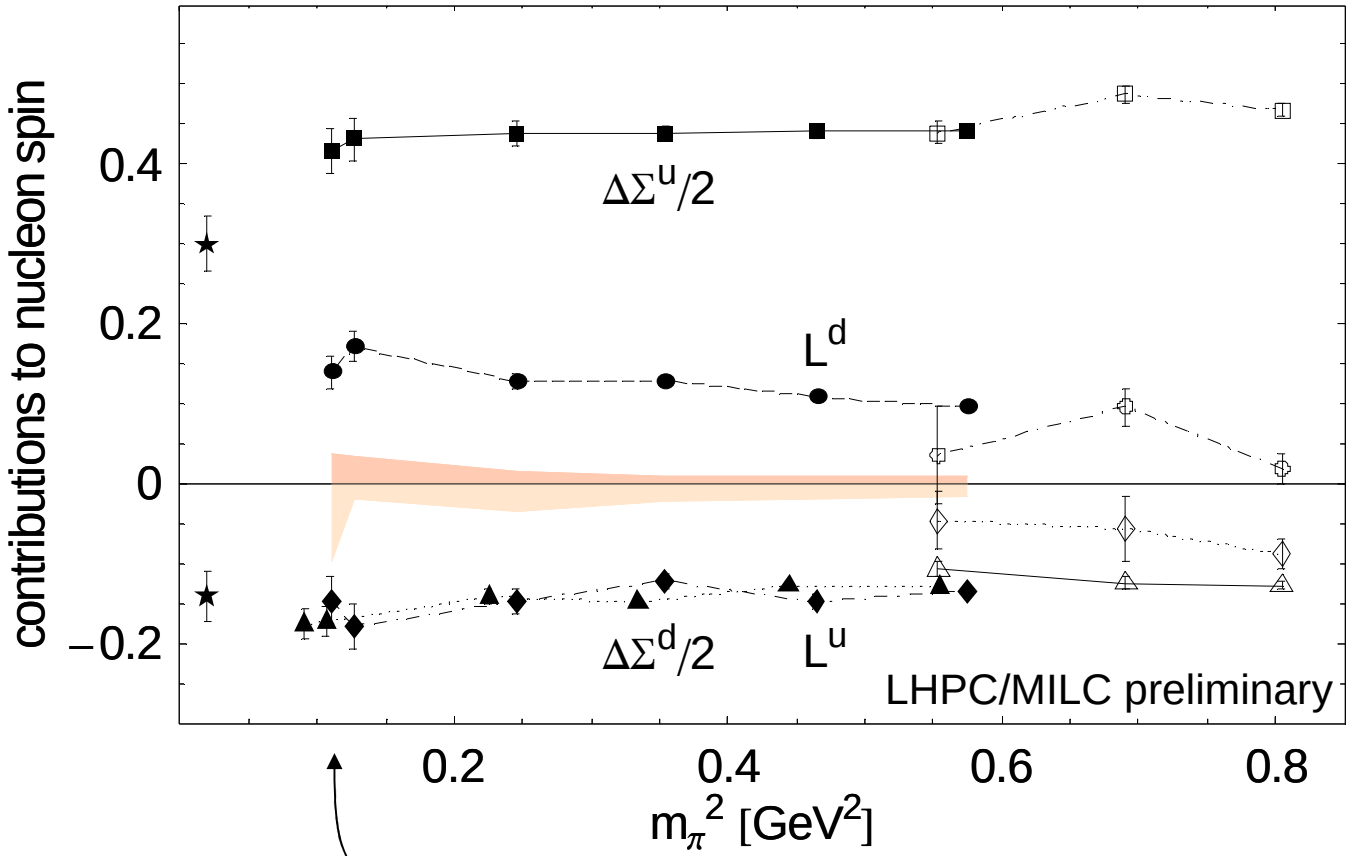
$$J_q = \frac{1}{2}\left(A_q + B_q\right) = \frac{1}{2}\Delta\Sigma_q + L_q$$

at $m_\pi \approx 350\,\text{MeV}$

$$L_q^{u+d} = J_q^{u+d} - \frac{1}{2}\Delta\Sigma^{u+d} = \frac{1}{2}\left(\langle x\rangle^{u+d} + B_{20}^{u+d}(0) - \Delta\Sigma^{u+d}\right) \approx 0$$

$$B_{20}^{u+d}(0) \approx 0 \text{ and } \Delta\Sigma^{u+d} \approx \langle x\rangle^{u+d}$$

Quark spin and OAM contributions to the nucleon spin



at $m_\pi \approx 350\,\text{MeV}$

$L_q^d \approx -L_q^u \approx 20-30\,\%$ of $\frac{1}{2}$

$J_q^u \approx 0.24 \pm 0.01$

$J_q^d \approx -0.02 \pm 0.01$

at $m_{\pi,phys} = 140$ MeV

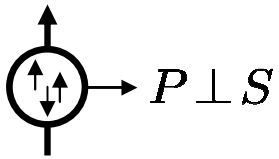
$J_q^{extr.,u} \approx 0.22 \pm 0.02$

$J_q^{extr.,d} \approx 0.00 \pm 0.02$

Ph. Hägler, COMPASS, 2007

Transverse spin structure of the nucleon

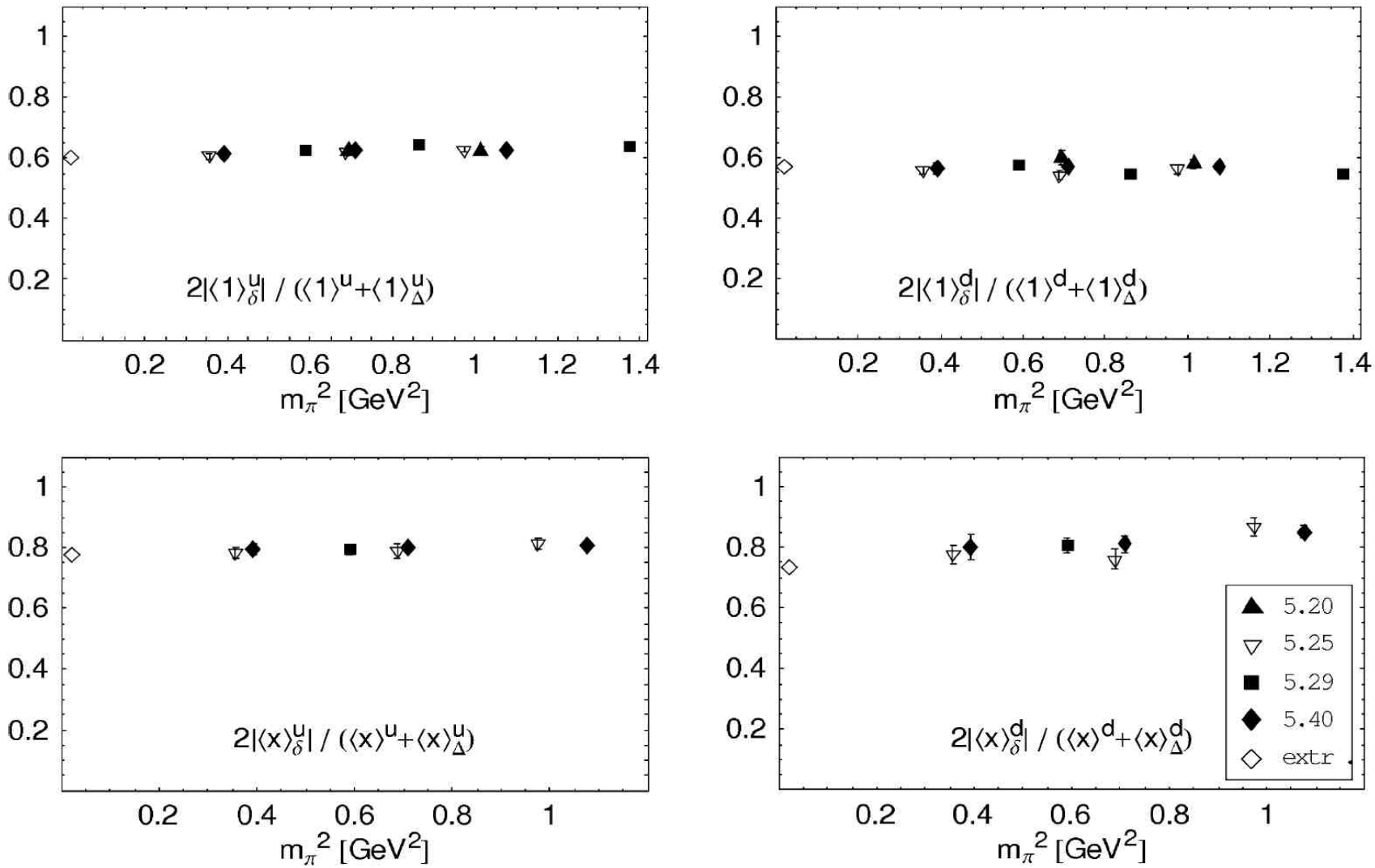
(in collaboration with QCDSF/UKQCD)



Lowest two moments of the Soffer-bound from lattice QCD

$$\int_{-1}^{1} dx\, x^{n-1} q(x) = \langle x^{n-1} \rangle$$

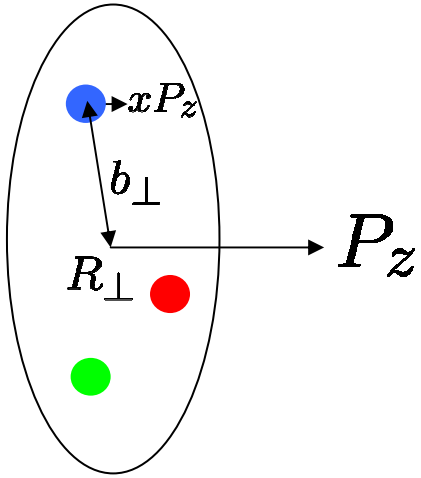
QCDSF/UKQCD PLB 627 (2005)



Transverse spin structure of the nucleon

let‘s add an additional degree of freedom

distance of the quark to the center of momentum of the nucleon in the transverse plane $b_\perp$



$$\mathcal{H}_{(\Lambda',\lambda'),(\Lambda,\lambda)} = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \left\{ P,\Lambda \;\text{—}\; \overset{q,\lambda}{\vert} \; H, \widetilde{H}, H_T, \ldots \; \overset{q',\lambda'}{\vert} \;\text{—}\; P',\Lambda' \right\}$$

where $\Delta_\perp = P'_\perp - P_\perp$
$\xi = 0$

$$\begin{array}{c|c} (\Lambda',\lambda') \backslash (\Lambda,\lambda) & \begin{array}{cccc} (+,+) & (-,+) & (+,-) & (-,-) \end{array} \\ \begin{array}{c} (+,+) \\ (-,+) \\ (+,-) \\ (-,-) \end{array} & \begin{pmatrix} H(b_\perp^2) + \tilde{H}(b_\perp^2) & -ie^{-i\phi}\frac{b}{m}E' & ie^{i\phi}\frac{b}{m}\overline{E}'_T & 2(H_T - \frac{1}{4m^2}\Delta_b \tilde{H}_T) \\ ie^{i\phi}\frac{b}{m}E' & H - \tilde{H} & 2e^{2i\phi}\frac{b^2}{m^2}\tilde{H}''_T & ie^{i\phi}\frac{b}{m}\overline{E}'_T \\ -ie^{-i\phi}\frac{b}{m}\overline{E}'_T & 2e^{-2i\phi}\frac{b^2}{m^2}\tilde{H}''_T & H - \tilde{H} & -ie^{-i\phi}\frac{b}{m}E' \\ 2(H_T - \frac{1}{4m^2}\Delta_b \tilde{H}_T) & -ie^{-i\phi}\frac{b}{m}\overline{E}'_T & ie^{i\phi}\frac{b}{m}E' & H + \tilde{H} \end{pmatrix} \end{array}$$

Diehl / PhH EPJC 2005

where $b_\perp = (b\cos\phi, b\sin\phi)$

Transversely polarized quarks in transversely polarized nucleons

transversity basis

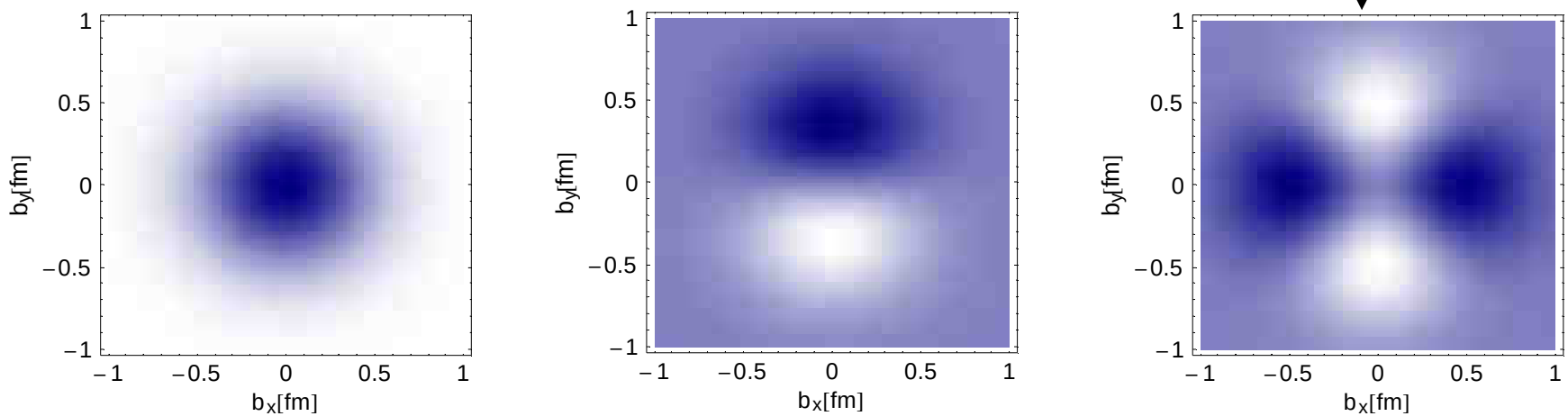
$$\left|P,\Lambda=0,S_\perp=(1,0)\right\rangle=\sqrt{2}^{-1}\left(\left|P,+\right\rangle+\left|P,-\right\rangle\right)$$

obtain probability density interpretation
for the transversity part of the density matrix

multipole-expansion

Diehl / PhH EPJC 2005

$$\langle P^+,0_\perp,S_\perp|\hat{\rho}_T(x,b_\perp;s_\perp)|P^+,0_\perp,S_\perp\rangle$$
$$=\frac{1}{2}\left\{H+s_\perp^iS_\perp^i\left(H_T-\frac{1}{4m^2}\Delta_b\widetilde{H}_T\right)-\epsilon_{ij}S_\perp^ib_\perp^i\frac{1}{m}E'-\epsilon_{ij}s_\perp^ib_\perp^i\frac{1}{m}\overline{E}'_T+s_\perp^i(2b_\perp^ib_\perp^j-b_\perp^2\delta^{ij})S_\perp^j\frac{1}{m^2}\widetilde{H}''_T\right\}$$

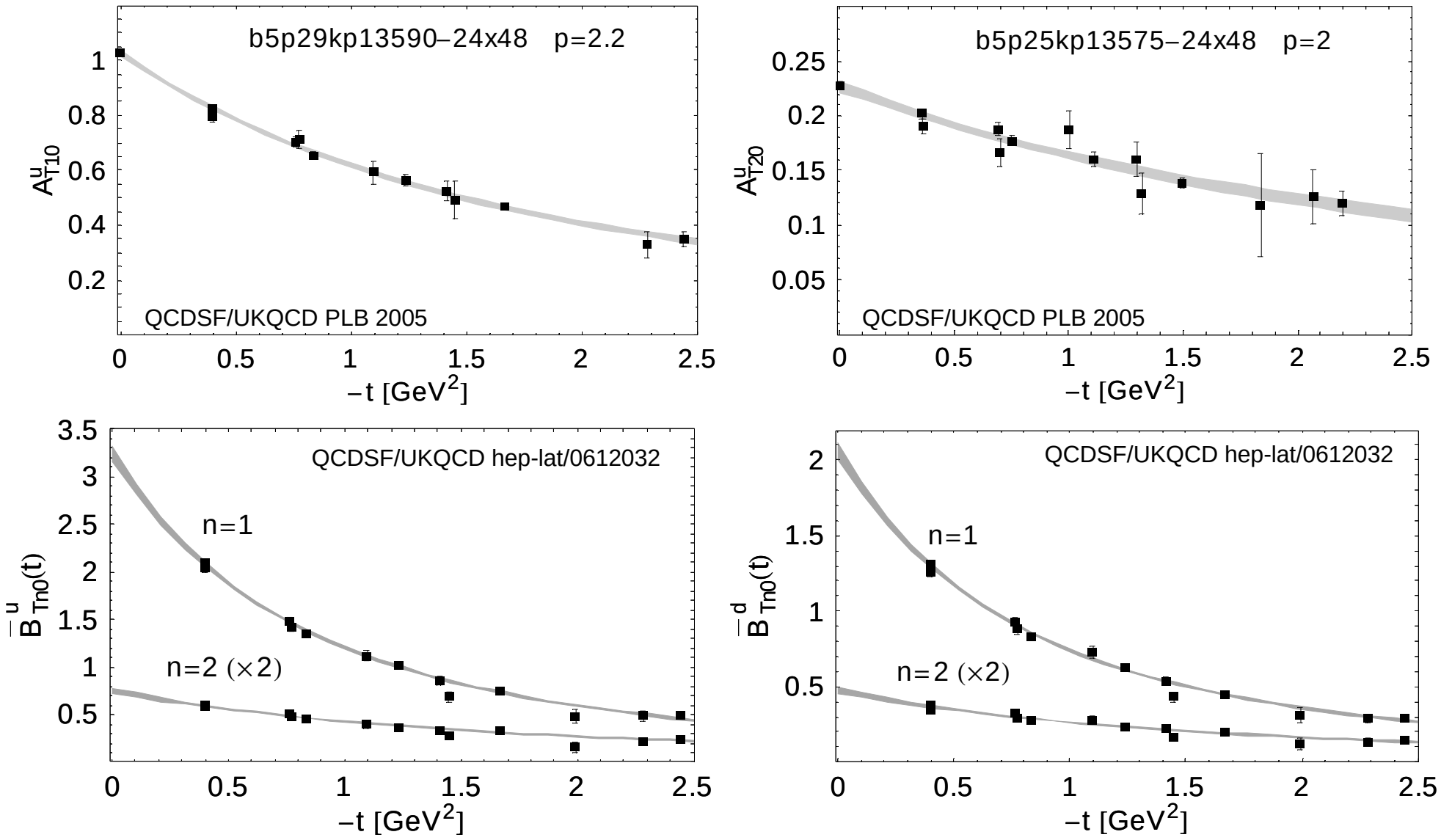


monopole

dipole

quadrupole

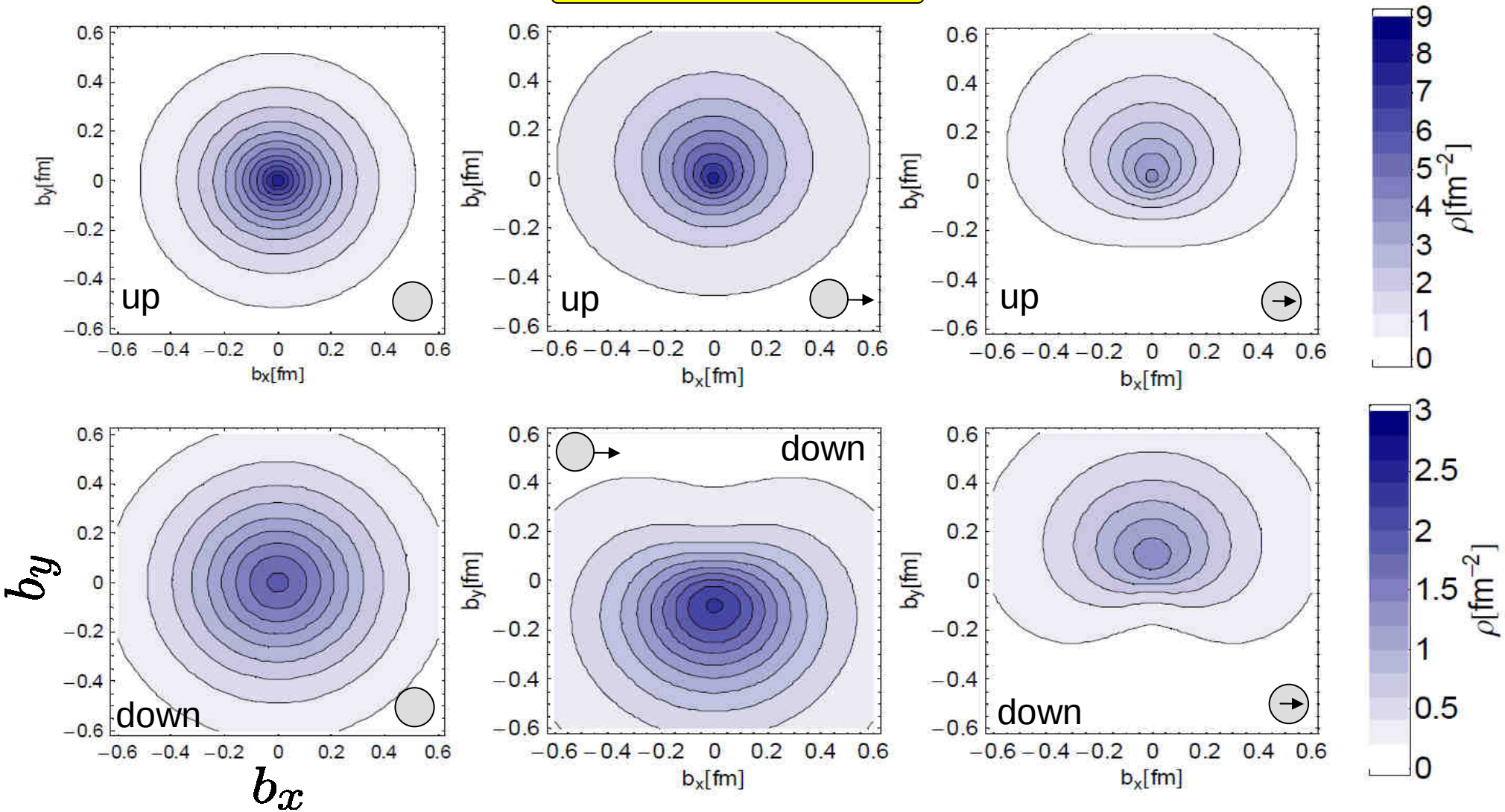
Results for moments of the tensor GPDs



the tensor GFFs $\overline{B}_{Tn0}(t)$ are remarkably large

Lowest n=1 moments of up- and down-quark densities

QCDS/UKQCD hep-lat/0612032



strong deviations from spherical symmetry

Interpretation of the observed deformation patterns

in parts based on M. Burkardts interpretation of distortions

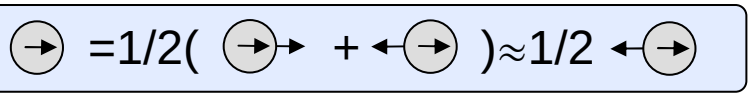
we know from longitudinal polarized quarks/nucleons:

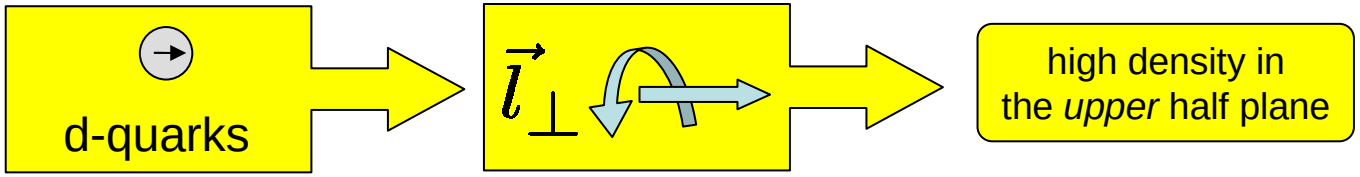
spin of up-quarks is aligned with nucleon spin

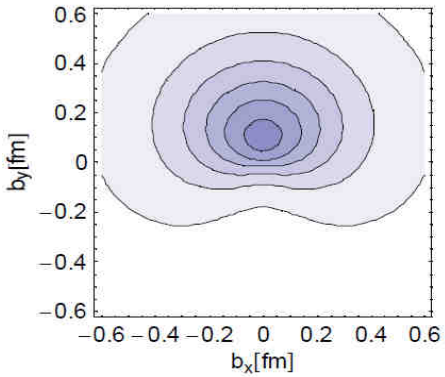
spin of down-quarks is anti-aligned with nucleon spin

assume : $\vec{l}_\perp \parallel \vec{s}_\perp$ for up- and down-quarks

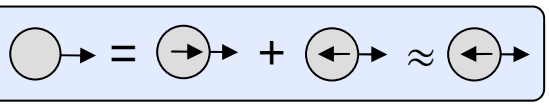
down-quarks in an „unpolarized“ nucleon

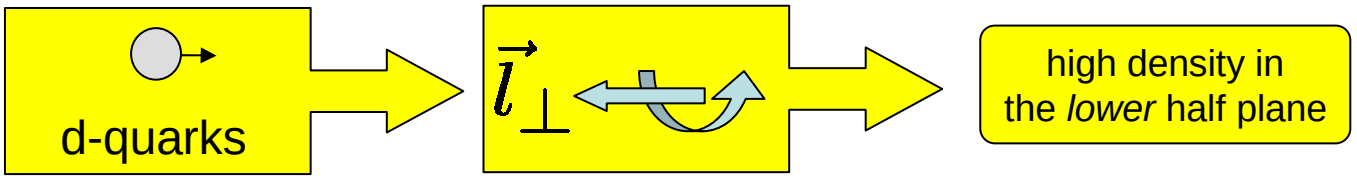


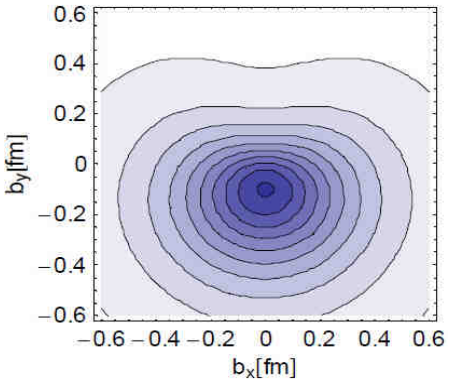




„unpolarized“ down-quarks in a polarized nucleon







Relation between GPDs and tmdPDFs

GPDs

generalized Pauli form factor $E(x, 0, b_\perp)$ ↔ $b_\perp \times S_\perp$

N.N. $\overline{E}_T(x, 0, b_\perp)$ ↔ $b_\perp \times s_\perp$

tmdPDFs

Sivers $f_{1T}^\perp(x, k_\perp)$ ↔ $k_\perp \times S_\perp$

Boer-Mulders $h_1^\perp(x, k_\perp)$ ↔ $k_\perp \times s_\perp$

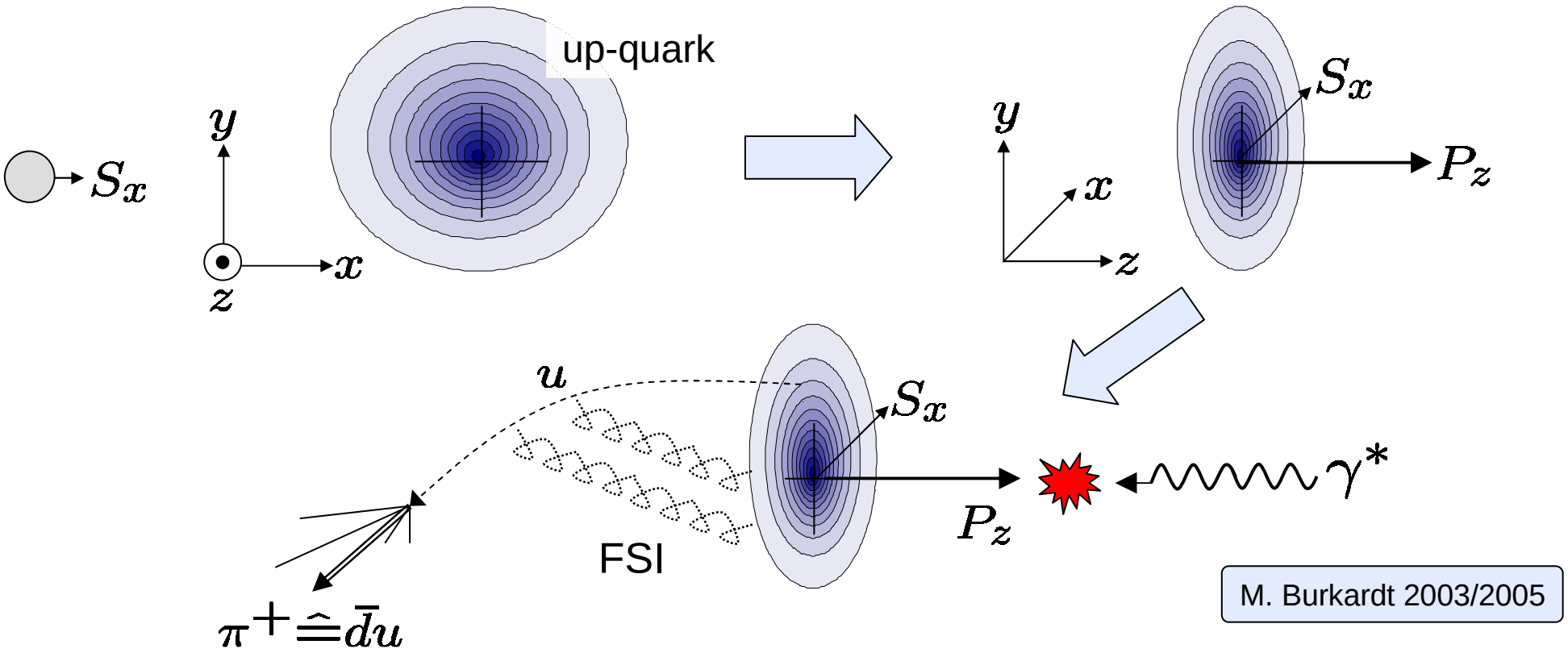
conjecture / hypothesis
Burkardt PRD 72 (2005)

Sivers $f_{1T}^\perp \sim -E$

Boer-Mulders $h_1^\perp \sim -\overline{E}_T$

→ asymmetries

Implications for asymmetries



M. Burkardt 2003/2005

expect sizeable effect with opposite signs for up- and down-quarks (Sivers-effect)

↓

sizeable single spin asymmetries in semi-inclusive deep ineleastic scattering,
seen by HERMES (PRL, 2005)

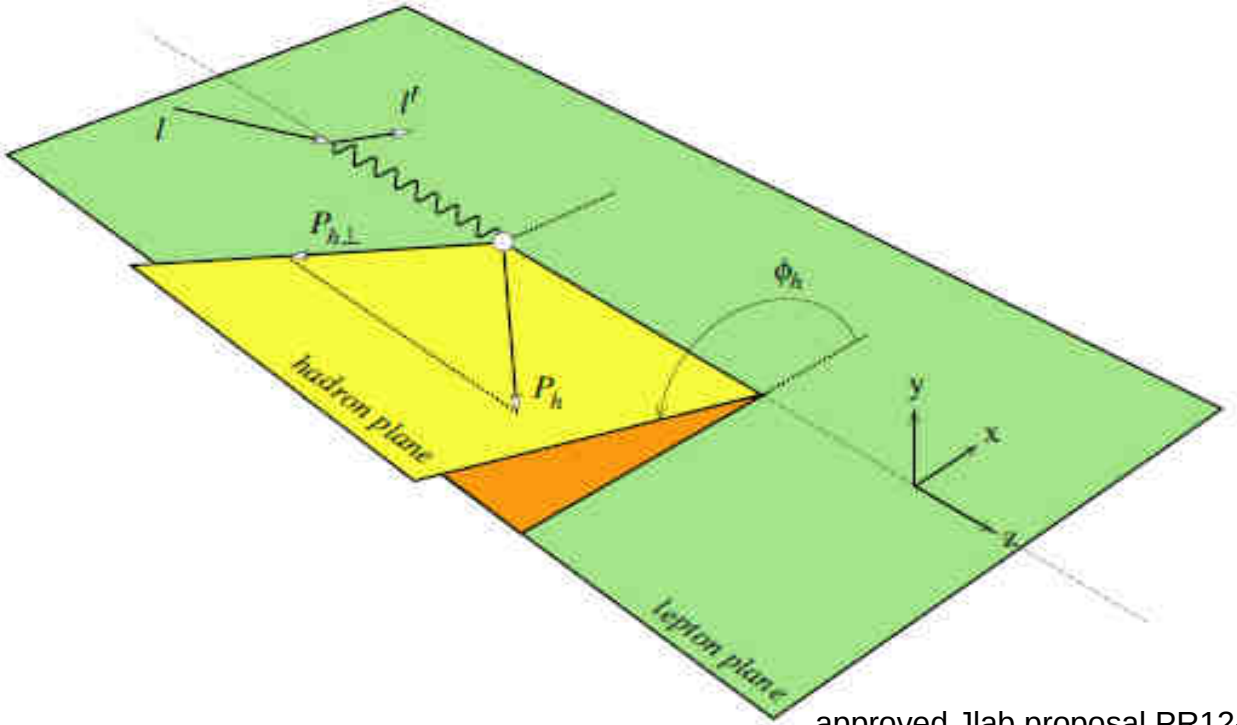
Lattice QCD prediction for azimuthal asymmetries

strongly distorted densities of transversely polarized quarks in an unpolarized nucleon → lead to prediction of a

large Boer-Mulders effect with the **same sign for up and down quarks**

could be observed in

- cos(2Φ) asymmetries in SIDIS (JLab, COMPASS)
- unpolarized Drell-Yan (GSI-FAIR/PANDA)



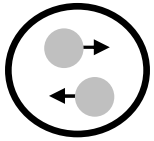
approved Jlab proposal PR12-06-112

Ph. Hägler, COMPASS, 2007

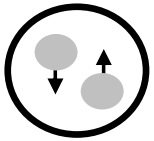
First results on the spin structure of pions from lattice QCD

pion spin structure?

longitudinal spin structure is trivial



$S=0$
$P_z \to \infty$



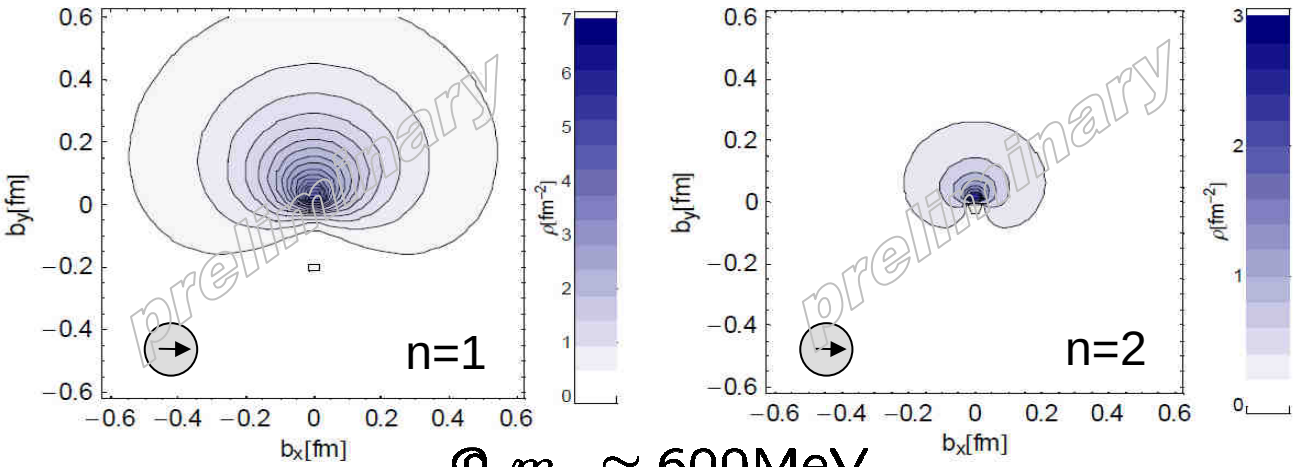
$P_z \to \infty$

what about the transverse spin structure of the pion?

$$\rho_T(x, b_\perp; s_\perp) = \frac{1}{2}\left\{H^\pi(x, b_\perp^2) - \epsilon_{ij} s_\perp^i b_\perp^i \frac{1}{m_\pi} E_T^{\pi\prime}\right\}$$

but is $E_T^{\pi\prime}$ non-zero?

YES



@ $m_\pi \approx 600$MeV

the pion has a very suprising non-trivial transverse spin structure!

Ph. Hägler, COMPASS, 2007

Summary

small total orbital angular momentum of quarks small

separate up- and down-quark OAM contributions sizeable ~20%

at $m_\pi \approx 350\text{MeV}$

up- and down-quark densities strongly distorted for transversely polarized quarks in transversely polarized nucleons

important consequences for asymmetries in SDIS and Drell-Yan (following arguments by M. Burkardt)

unpolarized quarks in a transversely polarized nucleon: significant SSAs (seen by e.g. HERMES, PRL 2005)

transversely polarized quarks in an unpolarized nucleon: significant azimuthal asymmetries („lattice QCD prediction“, to be confirmed by e.g. Jlab, COMPASS, GSI-FAIR/PANDA measurements)

the pion has a non-trivial (transverse) spin structure