



# Graph Generative Networks

PARTICLE CLOUD GENERATION WITH MPGANs

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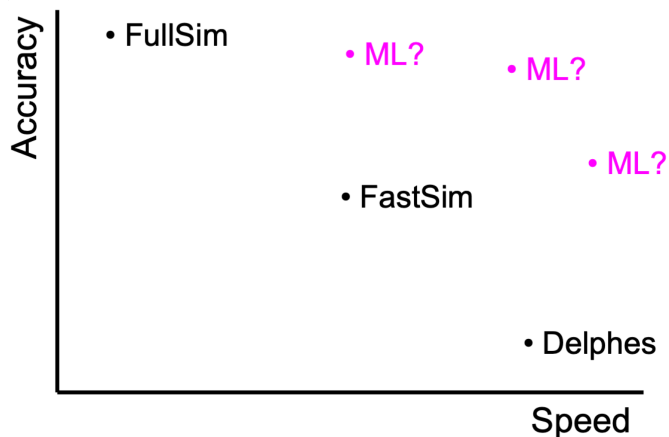
# Overview

LHC collision events always present hadronic jets.

- ❑ State of the art: ME generation + hadronisation + simulation.
- ❑ Full simulation: based on GEANT.
- ❑ Parameterised sims: Delphes, FastSim, ...

Machine learning-based alternative.

- ❑ Hadronic jets generation using a generative neural network.
- ❑ Possible applications:
  - Train on simulated jets: better simulation (substitute FastSim).
  - Train on real reconstructed jets: better hadronization + simulation.



# Motivation

HL-LHC jump in computing needs is well-known by now.

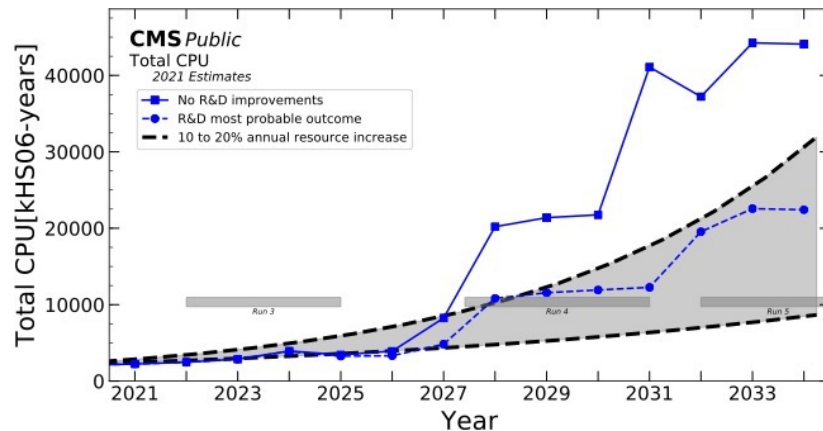
- ❑ Luminosity: 10x increase.
- ❑ High-granularity detectors.

Graphs are a more (semantically) powerful data structure than arrays.

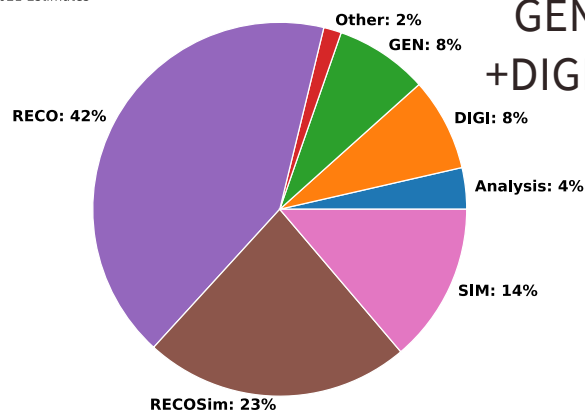
- ❑ This includes ND-arrays (images).

Data are a good match for graph architecture descriptions.

- ❑ Irregular geometry.
- ❑ Intrinsically unordered.

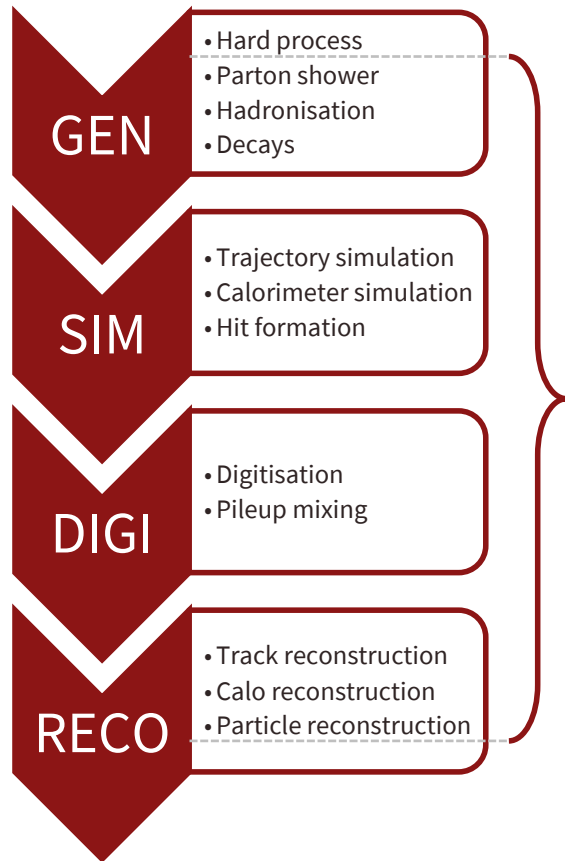


**CMS Public**  
Total CPU HL-LHC (2029/No R&D Improvements) fractions  
2021 Estimates



GEN+SIM  
+DIGI = 30%

# Jet Simulation in Excruciating Detail



Goal: substitute some/all of these steps by a Generative Model.

Proposal: *Particle Cloud Generation with Message Passing Generative Adversarial Networks*, Kansal et. al.

❑ <https://arxiv.org/abs/2106.11535>

❑ Standard dataset: JetNet  
<https://zenodo.org/record/6302454>

# Refresher: Generative Adversarial Networks

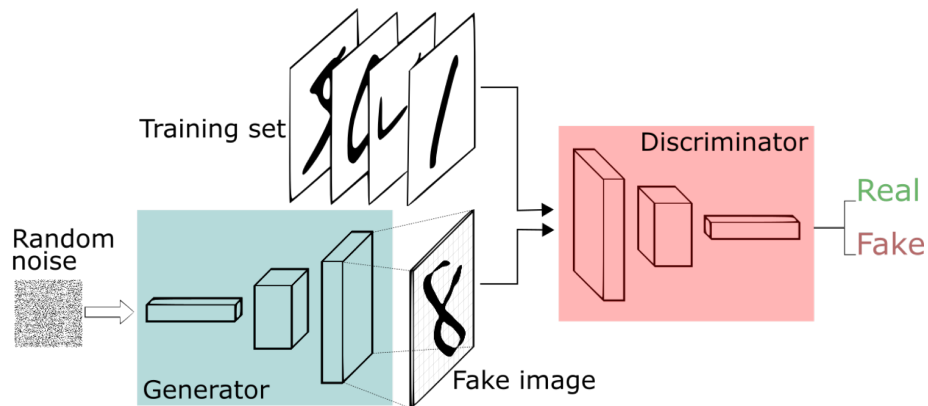
2 neural networks in a zero-sum minimax game.

## Generator $G$ :

- Learns data distribution.
- Generate samples from random noise.

## Discriminator $D$ :

- Estimates probability it is real or generated sample.



<http://thesecatsdonotexist.com/>



$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

# The Message-Passing GAN (MPGAN)

Key insight:

**high-level jet features are global.**

Design choices:

- Fully connected graph architecture.
- Message-passing algorithms.
  - Aggregation to compute a function of the entire graph.
- Concatenation of all particle features.
  - Preservation of the global structure.
- Residual connection to previous particle features.

Definitions:

- $N$ -particle cloud at iteration  $t$ :

$$J^t = \{p_1^t, \dots, p_N^t\}$$

- Particle with features  $h_i^t$ .
- Message-passing algorithm:

$$m_{ij}^{t+1} = f_e^{t+1} (h_i^t \oplus h_j^t)$$

$$h_i^{t+1} = f_n^{t+1} \left( h_i^t \oplus \sum_{j \in J} m_{ij}^{t+1} \right)$$

- $f_{(e,n)}^{t+1}$  implemented as multilayer perceptrons with 3 FC layers.

# MPGAN: Generator and Discriminator

Two approaches to MP generator:

- ❑ Direct cloud generation.
  - $N$  particles.
  - $L$  random features.
- ❑  $Z$ -dimensional latent vector.
  - FC-layer to expand to a  $(N \times L)$ -dimensional matrix.

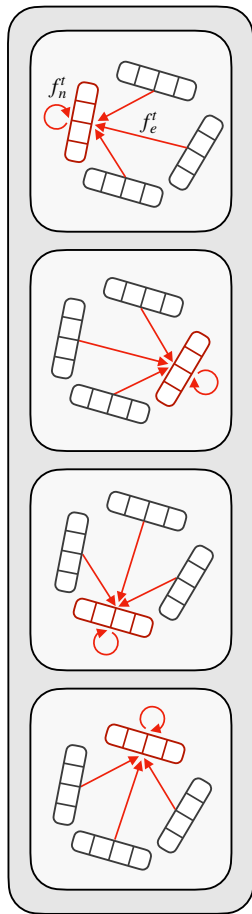
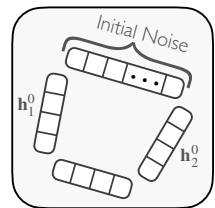
Other baseline generators:

- ❑ rGAN (FC): Achlioptas et. al. 2018
- ❑ TreeGAN : Shu, Park et. al. 2019
- ❑ GraphCNN: Valsesia et. al. 2019

Two approaches for discriminator:

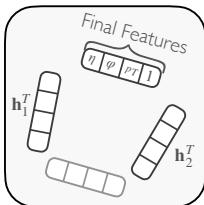
- ❑ Baseline is PointNet-Mix: Wang et. al. 2021
  - Symmetry function (MaxPool) for unordered datasets.
  - Combination of local and global information (segmentation).
  - Alignment to ensure symmetries (rotation, translations).
- ❑ MP discriminator
  - Similar structure to generator.

# MP Generator

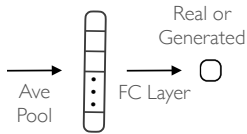
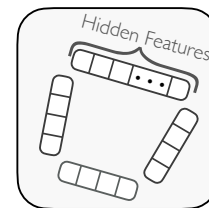


Real Particle Cloud

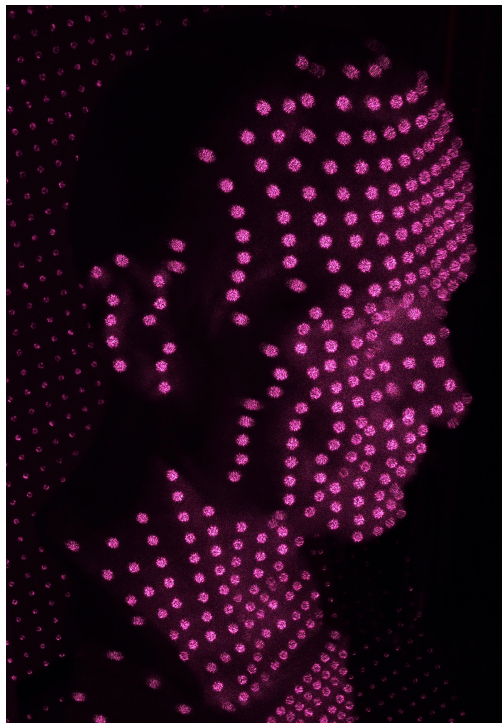
Generated Particle Cloud



# MP Discriminator



# Metrics (1) – MMD and Coverage



From the computer-vision field.

- ❑ Widely used to evaluate 3D object point clouds.

Let  $\{X\}$  a real set and  $\{Y\}$  a generated set.

- ❑ Match each element in  $\{Y\}$  to the closest element in  $\{X\}$ .
  - “Closest”: smaller energy-mover’s distance (EMD).
- ❑ Minimum Matching Distance (MMD) is the average EMD.
  - Minimizing MMD ensures *quality*.
- ❑ Coverage is the fraction of matched elements in  $\{Y\}$ .
  - Maximizing coverage ensures *diversity*.

# Metrics (2) – Fréchet Particle Net Distance

Derived from Fréchet Inception Distance.

- $d_F$  between gaussians fitted to the activations of FC layer of the ~~Inception v3~~ ParticleNet, Qu and Gouskos (2019) network in response to real / generated samples.

$$d_F^2 = |\mu_X - \mu_Y|^2 + \text{tr}(\Sigma_X + \Sigma_Y - 2(\Sigma_X \Sigma_Y)^{1/2})$$

Sensitive to quality.

- Directly measures the high-level features (in the “last layer”).

Sensitive to diversity.

- A lack of diversity would lead to differing supports.

# Metrics (3) – 1-Wasserstein Distance $W_1$

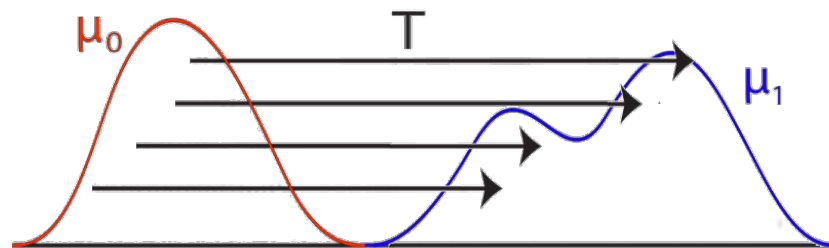
$$W_1(\mu, \nu) = \left( \inf_{\gamma \in \Gamma(\mu, \nu)} \int_{M \times M} d(x, y) \, d\gamma(x, y) \right)$$

$W_1$  (real, generated).

- Close to what we usually do when comparing “data and simulation”.

High-level distributions.

- $\eta^{\text{rel}}, \phi^{\text{rel}}, p_{\text{T}}^{\text{rel}}$ , relative jet mass.
- Alternative: the energy-flow polynomials.
  - Basically n-particle energy correlations.
  - Basis for IRC-safe jet observables.



Conceptually equivalent to FPND.

- But in terms of interpretable physical quantities.

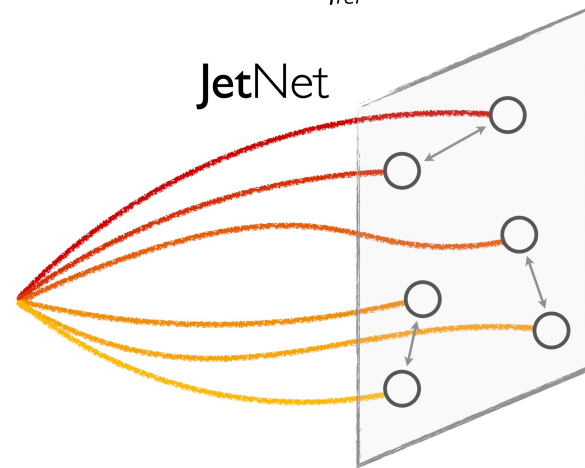
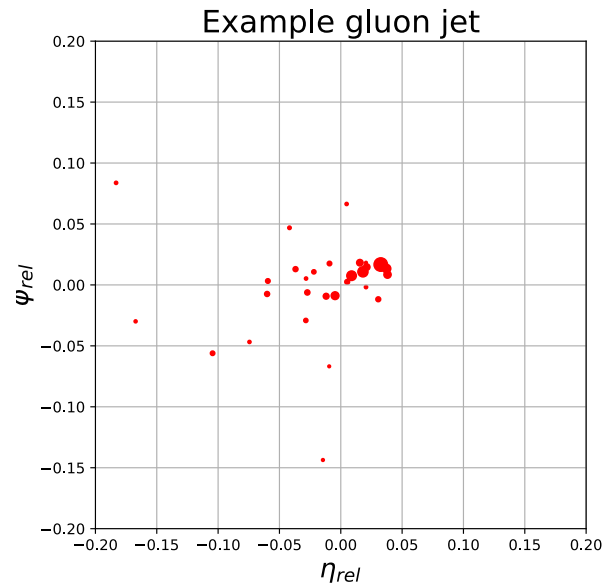
# The JetNet Dataset

A proposed benchmark for LHC jet studies.

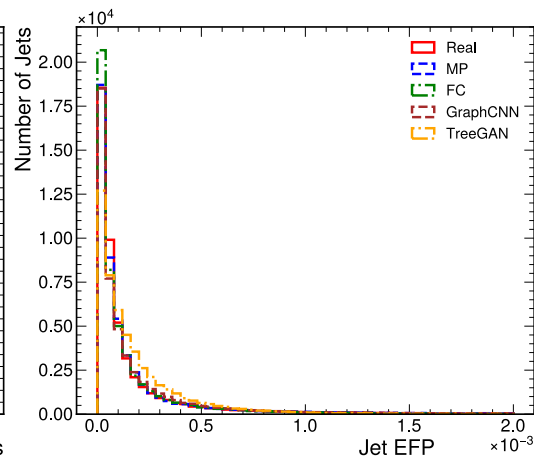
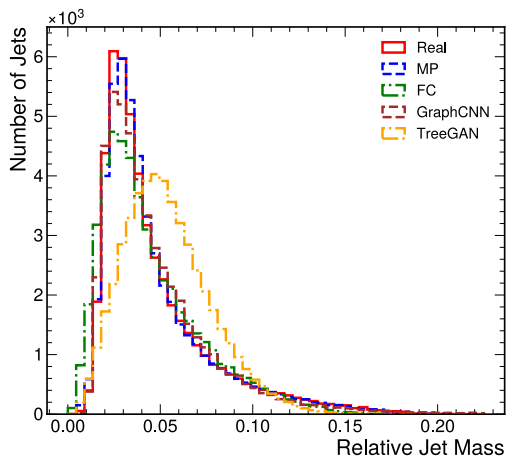
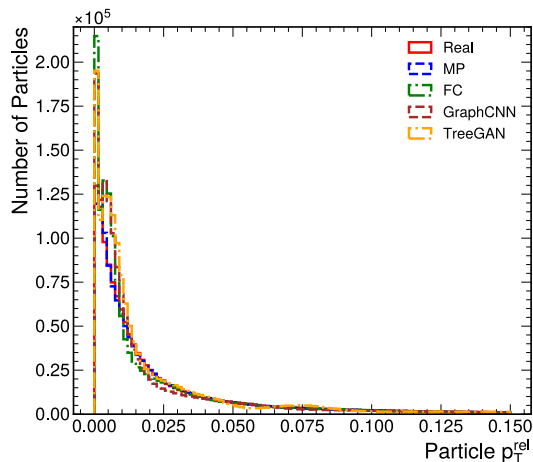
- ❑ Three classes: gluon (g), top quark (t), light quark (q).
- ❑  $\sim 175\text{k}$  jets / class.
- ❑ Up to 30 particles / jet.
  - Mask for missing particles.
- ❑  $[N, 30, 4]$  frame.
  - $[\eta^{\text{rel}}, \phi^{\text{rel}}, p_{\text{T}}^{\text{rel}}, \text{mask}]$

Associated JetNet library

- ❑ <https://github.com/jet-net/JetNet>
- ❑ “Common standardized PyTorch-based datasets, evaluation metrics, and loss functions for working with jets using ML.”

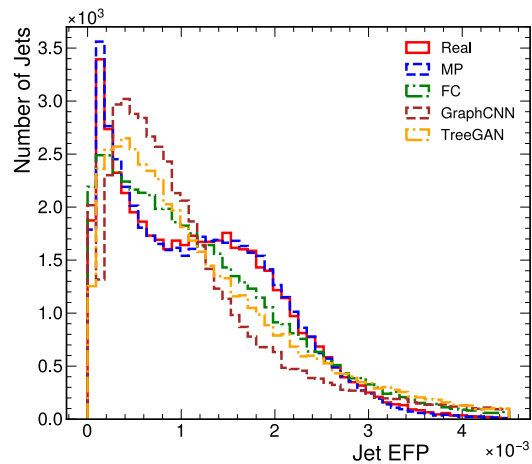
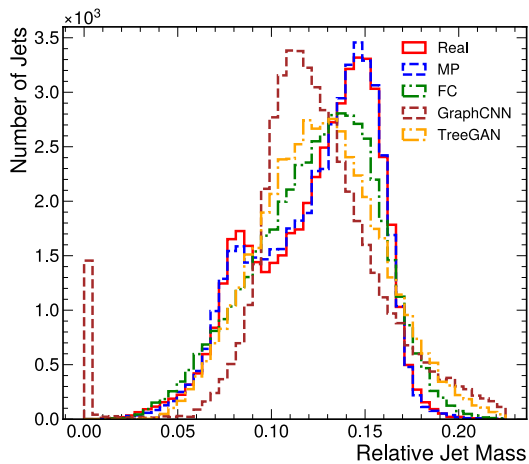
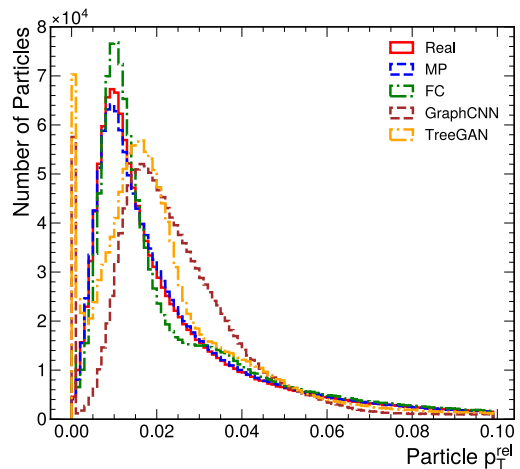


# Results: Light-quark Jets



Generator	Discriminator	$W_1^M (\times 10^{-3})$	$W_1^P (\times 10^{-3})$	$W_1^{EFP} (\times 10^{-5})$	FPND	COV $\uparrow$	MMD
FC	PointNet	$3.1 \pm 0.2$	$4.5 \pm 0.4$	$2.3 \pm 0.6$	17	0.37	0.028
GraphCNN	PointNet	$4 \pm 1$	$5.2 \pm 0.5$	$50k \pm 100k$	316	0.37	0.031
TreeGAN	PointNet	$10.1 \pm 0.1$	$5.7 \pm 0.5$	$4.1 \pm 0.3$	11	0.47	0.031
MP	MP	<b><math>0.6 \pm 0.2</math></b>	$4.9 \pm 0.5$	<b><math>0.7 \pm 0.4</math></b>	0.35	0.50	0.026
MP-LFC	MP	$0.7 \pm 0.2$	<b><math>2.6 \pm 0.4</math></b>	$0.9 \pm 0.9$	<b>0.08</b>	<b>0.52</b>	<b>0.024</b>

# Results: Top-quark jets



Generator	Discriminator	$W_1^M (\times 10^{-3})$	$W_1^P (\times 10^{-3})$	$W_1^{EFP} (\times 10^{-5})$	FPND	COV $\uparrow$	MMD
FC	PointNet	$2.7 \pm 0.1$	<b><math>1.6 \pm 0.4</math></b>	$7.7 \pm 0.5$	3.9	0.56	0.075
GraphCNN	PointNet	$11.3 \pm 0.9$	$30 \pm 10$	$37 \pm 2$	30k	0.39	0.085
TreeGAN	PointNet	$5.19 \pm 0.08$	$9.1 \pm 0.3$	$16 \pm 2$	17	0.53	0.079
MP	MP	<b><math>0.6 \pm 0.2</math></b>	$2.3 \pm 0.3$	<b><math>2 \pm 1</math></b>	<b>0.37</b>	<b>0.57</b>	<b>0.071</b>
MP-LFC	MP	$0.9 \pm 0.3$	$2.2 \pm 0.7$	<b><math>2 \pm 1</math></b>	0.93	0.56	0.073

# Some Remarks

Remember: end goal is a conditional GAN.

- ❑ Condition on parton  $p_T$  and type.

MPGAN outperforms baseline models on almost every metric.

- ❑ Particularly on high-level metrics.
- ❑ But: the pair (G,D) has to be taken as a team.
- ❑ High  $W_1(\text{mass})$  and EFP scores  $\rightarrow$  correct description of jet substructure.

For light quark jets:

- ❑ Sometimes less than 30 particles...
- ❑ Successful mitigation with masking strategy.

For top quark jets:

- ❑ Bimodal structure (“top-jet” vs. “(W+b)-jet”).
- ❑ MPGAN is the only one that learns it!

# Conclusions and Outlook

HL-LHC data are a perfect match for graph NN approaches.

- ❑ High granularity, very sparse.
- ❑ Irregular geometry.
- ❑ Intrinsically unordered.

Generative models are a good approach to the simulation needs.

- ❑ Simulation will still be 30% of the HL-LHC computing budget.
- ❑ Generative adversarial networks (GANs): frontier of research.

Message-Passing GAN (MPGAN) to simulate high-energy jets.

- ❑ Very good learning of jet high-level features and substructure.