Accurate estimate of the relic density and of the kinetic decoupling in non thermal dark matter models

Giorgio Arcadi (SISSA and INFN Trieste)

Based on G.A. and Piero Ullio, arXiv: 1104.3591

Planck 2011, Lisboa 1st June 2011

Plan of the talk

- 1) Numerical evaluation of the Boltzmann equation for non thermal production of dark matter:
- General framework
- Specific framework: G2-MSSM
- 2) Kinetic equilibrium and decoupling in the G2- MSSM:
- System of coupled Boltzmann equations for a set of coannihilating particles with standard assumptions for the computation of the relic density relaxed.
- Computation of kinetic decoupling temperature of a system of coannihilating particles.

Non thermal production of DM

when annihilations become inefficient, if there are no further entropy in $\mathsf{General}~\mathsf{Frame}$ by SM background particles and this is balanced by DM pair and then at Tt.f.o., when neutral then at T the temperature evolves as T ≈ 3/8 and the universe expansion rate as H ∞ T 4 [21, 24]. A standard approximatio is however to treat the decay of the field and the the theorem and the products as instantaneous products as i

In case that non thermal production is efficient there is balance between production and annihilations. The relic density resembles the standard case with the difference that freeze-out $\frac{1}{1000}$ contracts the statius of the reheating temperature. mat non thermal production is emittent the In case that non thermal production is emclent there is balance
between production and annihilations. The relic density enc dens
"se that" y and pair and pair and pair and pair and constant there is belowed. in case that non thermal production is efficient there is balance resembles the standard case with the difference that freeze-out $\frac{1}{10-06}$ $\frac{1}{2}$ referred the reduction is efficient there is belange Th case that non thermal production is emelent there is balance
between production and annihilations. The relic density α α , β for β and α annihilations. The relic density decrease in the community decrease in the set of α and β a decuis at al duile

$$
\Omega^{NT}_{\chi}h^2 \simeq \frac{T_{t.f.o.}}{T_{\rm RH}}\,\Omega^{T}_{\chi}h^2
$$

and can be used to estimate the relic density for $\mathcal{L}^{\mathcal{L}}$

"σv# . (18)
"σv# . (18)
"σv# . (18) - (18) - (18) - (18) - (18) - (18) - (18) - (18) - (18) - (18) - (18) - (18) - (18) - (18) - (18)

Efficient non thermal production favours candidates with high
annihilation cross section. annihilation cross section. $\mathcal{L}_{\mathcal{A}}$ ^ρc(T0) ^Yχ(TRH) [∝]

Po $T_{\text{Ramp} \text{normal}}$ \blacksquare $\$ <u>Fermi anomalies.</u> In fact the Choice of the MIP pair and the WIMP pair and the Choice of the Choice of the WIMP pair and the WIMP

Non thermal production and the MSSM

µ[GeV]

Non thermal production has impact on the expectations for mass and composition neutralino LSP.

Pure gaugino (Wino), or pure higgsino states compatible with cosmology.

 $\frac{100}{100}$ 1000 100 1000 $B_x = 0.25$ T_{BH}=120 GeV $T_{RH}=81 GeV$ T_{RH}^{H} =37 GeV $T_{BH}=20 GeV$ $T_{\rm RH}$ =6 GeV T_{RH}≟2.5 GeV $F_{\text{RH}}=1$ GeV

 M_1 [GeV]

Specific framework: G2-MSSM $f(x) = \frac{1}{2}$ is tipically 3 to 4 orders of magnitude smaller than the final Yy, hence not contributing significantly significantly significantly significantly significantly significantly significantly significantly signi

- Wino LSP with mass O(100-500) GeV.
- Almost degenerate chargino NLSP (mass splitting O(200) MeV). at high energy. A relic density companies is obtained for Lighter than about the cosmological measurements is o
A lighter than about the contract of Lighter than about the cosmological measurements is obtained for Lighter - Annost degenerate chargino in the range spilling $O(Z)$
- Pure Bino with mass O(I) LSP mass
- Gluino with mass O(0.5-1) TeV.
- Other superpartners with mass of the order of the gravitino mass. $\sum_{i=1}^{\infty}$ treatment implemented the relic density calculation may be calculated the remaining \mathbf{g}_R

Our results are based on the assumptions done in B. Acharya et al. arXiv:0801.0478

For fect relic density obtained for relieating temperatures between four fiev and F de Correct relic density obtained for reheating temperatures between 100 MeV and 1 GeV. h shows the corresponding value of \mathcal{L} my for this specific value of \mathcal{L}

the DM relic density from the 7-year WMAP dataset. Right panel: models with relic density equal to the central value from Until now we have assumed that DM is kinetic equilibrium during production and arter freeze-out (important for coannihilations). The region violation of the plane above the plane dashed line would conrespond to models with α . The models with α the pow We have assumed that DM is kinetic equilibrium during production and after n now we have assumed and DTT is kniede equilibrium during production and area. freeze-out (important for coannihilations). Until now we have assumed that DM is kinetic equilibrium during production and after

 f_{α} is the f_{α} for the final order than the final year than the final f_{α} or f_{α} and contributing significantly significantly significantly significantly significantly significantly significantly significa order to get me Dark matter is produced out-of-equilibrium and then must have efficient interactions in $\frac{1}{\sqrt{1-\frac{1$ order to get into thermal equilibrium.

Non thermal DM might not be in thermal squilibrium because it can be produced at Given the real the real the real time and the regime applies of the call be produced at temperatures close or below kinetic decoupling temperature. Given the reason of the real in the region of the regime application of the region of the proportional to my and inverse lying and to my and inverse lying α proportional DPT implies not be in the mail equilibrium because it can be produced at Non thermal DM might not be in thermal equilibrium because it can be produced at

Kinotic decoupling temperature can be altered in the non-standard cosmology find: $\mathbb{E}[\mathcal{S}^{(1)}]$ 3 $\mathbb{E}[\mathcal{S}^{(2)}]$ 3 $\mathbb{E}[\mathcal{S}^{(1)}]$ 3 $\mathbb{E}[\mathcal{S}^{(1)}]$ 3 $\mathbb{E}[\mathcal{S}^{(2)}]$ 3 $\mathbb{E}[\mathcal{S}^{(1)}]$ "3
"3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3
"3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" "3/2" $\frac{1}{2}$ ^Ωχh² [∝] **3/20 !!! CITC !!** m3/² D1/² Kinetic decoupling temperature can be altered in the non-standard cosmology. Giorgio Arcadi

Energy loss processes 1888 and masses is ∆my " 160 MeV is \sim 160 MeV in the decay chain of the moduli with the moduli with the package PYTHIA \sim few sample benchmark models in the G2-MSSM, assuming a stable wind-like chargino, and found energy distributions \mathbb{R}

it proceeds via a Z boson or a slepton exchange and the corresponding amplitudes are suppressed, respectively, by Two-particle system: charginos and neutralinos. k allowed, the dominant effect is the inelastic scattering into the into the charged Wino, which is mediated by a second term of \sim **NIFU boushing and the two effects making a neutralino production** $\frac{1}{\text{Area}^2}$ if the inelastic scattering itself and the fact that the fact that the produced chargino will effect that the produced chargino will effect that the produced chargino will effect the produced chargino will effect the prod **relativistic neutralinos, the inelastic scattering rate inequality rate in inerally loss rate in inelastic scattering** $\frac{1}{2}$ of the the thermal bath particles in the initial state and the phase space of the out-scattered particles. The expressions of the out-scattered particles in the expressions of the expressions of the expressions of the exp ν in otiq oquilibrium ostablished efter one dustion Kinetic equilibrium established after production if: **possibility which has been investigated, e.g., in Refs. as a first rule of thumb, the energy dependent is efficient of thumb, the energy dependent of the energy dependent of thumb, the energy dependent of the energy depe** \mathbf{W}_{\bullet} relative the times the times the time interval the process is active, which we indicate \mathbf{F}_{\bullet}

$$
\left(-\frac{1}{E}\frac{dE}{dt}\right)\cdot\Delta\tau>1
$$

$$
\left(-\frac{dE}{dt}\right)_{\chi^0 \to \chi^{\pm}} = \sum_{(a,b)} \frac{16\tilde{g}_{Wab} G_{\rm F}^2}{\pi^3} \exp\left(-\frac{m_{\chi} \Delta m_{\chi}}{2ET}\right) T^5 \left(\frac{E}{m_{\chi}}\right)^3 \left(8\frac{ET}{m_{\chi}} + \Delta m_{\chi}\right)^{\frac{1}{0.001}}.
$$

$$
\left(-\frac{dE}{dt}\right)_{\chi^{\pm}}=\frac{\pi\alpha^2T^2}{3}\Lambda
$$

 100 " 0 +e+/-->" +/-+# 1e+06 !/H " +/-+#->" +e+/- respectively, of 103.5 and 300 GeV, obtained for DXⁱ = 16, δ = −3.5 and δ = −3 and gravitino masses of 107 and $^{\prime\prime}$ -Neutralinos: energy loss through inelastic $\overbrace{\hspace{1.5cm} }^{I^{\text{et+10}}}$ scatterings into charginos. Elastic scatterings $\frac{1}{2}$ models we plot ratios of scattering and decay rates Γ, or of relative energy loss rates −1/E · dE/dt, to the Universe instead suppressed. m² expansion rate H; in the panels on the right-hand side, results are shown for relativistic particles, expansion of the right-hand side, \sim -Charginos: efficient energy loss through \sum_{100} **PINGULIATITIOS. GHET BY TOSS LITTOUSH THETASLIC** $\overline{\mathsf{d}}$.

1 − m2 − m2 − m2
1 − m2 − m2 − m2 − m2 − m2

#−1/2 " Text Construction" |
|-
| 1/2 " Text Construction" |

 10^{16} MeV 2^{16} MeV 2^{16} MeV 2^{16} MeV 2^{16} MeV 2^{16}

 $\mathcal{L}^{\text{max}}_{\text{max}}$ of $\mathcal{L}^{\text{max}}_{\text{max}}$, $\mathcal{L}^{\text{max}}_{\text{max}}$

#

 1 ϵ energy losses, the appropriate timescale ϵ is the shortest between the chargino lifetime and the timescale for electromagnetic interactions. omagnetic interactions.

"

scattering rate. More quantitatively, for the two processes, the two

Boltzmann equations for the number densities neutralino temperature will scale instead as Tχ0 ω.
Tale $\overline{}$ $\mathcal{L} = \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \math$

System of two coupled equations:

$$
\left(\partial_t - H\mathbf{p} \cdot \nabla_\mathbf{p}\right) f_{\chi^0}(p, t) = \frac{1}{E} \hat{\mathbf{C}}_{\chi^0}[f_{\chi^0}, f_{\chi^\pm}] \qquad \left(\partial_t - H\mathbf{p} \cdot \nabla_\mathbf{p}\right) f_{\chi^\pm}(p, t) = \frac{1}{E} \hat{\mathbf{C}}_{\chi^\pm}[f_{\chi^0}, f_{\chi^\pm}]
$$

neutralino temperature will scale instead as ^Tχ⁰ [∝] ^T ².

Dark matter istantaneously thermalized after production as soon neutralino inelastic scatterings are efficient. Dark matter istantaneously thermalized after production as soon neutralino inelastic
exottoring evolutions, afficient \mathbf{C} standards for the collisional operator, embedding all interactions involving neutralinos and charginos, namely \mathbf{C} the neutralino source from chargino decays. Integrating these equation over phase space one obtains two equations $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ re efficient. −∆m^χ

annihilation and scattering processes, as well as the production of neutralinos and chargino from moduli decays and chargino In the non relativistic limit and at the leading order expansion in T/M and ΔM/M: n relativistic lir

$$
\frac{dn_{\chi^0}}{dt} + 3 H n_{\chi^0} = \left(\tilde{\Gamma}_{\chi^0 \leftrightarrow \chi^{\pm}} + \Gamma_{\chi^{\pm}}\right) \left[g_{\chi^0} n_{\chi^{\pm}} - g_{\chi^{\pm}} n_{\chi^0} \exp\left(-\frac{\Delta m_{\chi}}{T}\right)\right] - \langle \sigma v \rangle_{\chi^0 \chi^0} \left[n_{\chi^0}^2 - (n_{\chi^0}^{eq})^2\right]
$$

$$
- \langle \sigma v \rangle_{\chi^0 \chi^{\pm}} \left[n_{\chi^0 \chi^{\pm}} - n_{\chi^0}^{eq} n_{\chi^{\pm}}^{eq}\right]
$$

$$
\frac{dn_{\chi^{\pm}}}{dt} + 3 H n_{\chi^{\pm}} = \left(\tilde{\Gamma}_{\chi^0 \leftrightarrow \chi^{\pm}} + \Gamma_{\chi^{\pm}}\right) \left[g_{\chi^{\pm}} n_{\chi^0} \exp\left(-\frac{\Delta m_{\chi}}{T}\right) - g_{\chi^0} n_{\chi^{\pm}}\right] - \langle \sigma v \rangle_{\chi^{\pm} \chi^{\pm}} \left[n_{\chi^{\pm}}^2 - (n_{\chi^{\pm}}^{eq})^2\right]
$$

$$
- \langle \sigma v \rangle_{\chi^{\pm} \chi^0} \left[n_{\chi^{\pm}} n_{\chi^0} - n_{\chi^{\pm}}^{eq} n_{\chi^0}^{eq}\right] \underbrace{\left(\sum_{i} \frac{B_{X_i}}{m_{X_i}} \Gamma_{X_i} \rho_{X_i}\right)}_{m_{X_i}} - \sum_{i} \underbrace{\tilde{g}_{Wab} 8 G_{\rm F}^2}_{\tilde{\chi}^0 \leftrightarrow \chi^{\pm}} T^3 \underbrace{\left(\Delta m_{\chi}^2 + 6 \Delta m_{\chi} T + 12 T^2\right)}_{\text{thermal source term for non thermal production}}
$$

transfer in the t-channel is small and the collision term can be computed expanding in its powers, see also [14, 64];

tends to be anticipated. This latter feature was already pointed in the non-thermal production of \mathbb{R}^n TRH σ and the left hand-side refer to non-relativistic particles, E/M σ , while those on the right-hand-side right-hand-side refer to σ (some further details and a sketch of the derivation of this expression is given in Appendix (A)). When including pair could induce the standard temperatures compared temperatures compared to the standard case of Wino DM, however, in case of Wino DM, how tends to be anticipated. This latter feature was already pointed in [65], showing that the non-thermal production could induce the temperature temperatures compared temperatures compared to the standard case of Wino DM, however, i the production and decay or charginos in the moduli decay has always a larger impact. The kinetic decoupling

From the second momentum of the distribution function: is parametrized defining the temperature of the distribution function. From the second momentum of the distribution function: the production and decay or charginos in the moduli decay has always a larger impact. The kinetic decoupling even in case of the internation of the distribution run case.

$$
\int \frac{d^3 p}{(2\pi)^3} \, g_{\chi^0} \, p^2 \, f_{\chi^0}(p,t) \equiv 3 m_\chi \, T_{\chi^0}(t) \, n_{\chi^0}(t)
$$

can be defined a temperature which parametrizes deviations from kinetic equilibrium. <u>can be defined</u> as
Charginos are al The gives are always kept into equilibrium Charginos are always kept into equilibrium by electromagnetic interactions. Only one equation needed: which is a set of the set of an he defined o temperature which perspectives deviations from Linetic equations on Can be denned a temperature winch can be defined a temperature which parametrizes deviations from kinetic equilibrium. from the ratio of the ratio Charginos are always kept into equilibrium by electromagnetic interactions. Only one should find cases in which the standard thermal assumption is invalid at higher temperatures, possibly even close to fair be defined a temperature which parametrizes deviations from Kinetic equinorium. should find cases in which the standard thermal assumption is invalid at higher temperatures, possibly even close to

(∂^t [−] ^H^p · [∇]p) ^fχ⁰ (p, t) = ¹ ^E ^C^ˆ ^χ⁰ [fχ⁰ , fχ[±]] (40) (∂^t [−] ^H^p · [∇]p) ^fχ[±] (p, t) = ¹ ^E ^C^ˆ ^χ[±] [fχ⁰ , fχ[±]] , where Cˆ stands for the collisional operator, embedding all interactions involving neutralinos and charginos, namely annihilation and scattering processes, as well as the production of neutralinos and chargino from moduli decays and the neutralino source from chargino decays. Integrating these equation over phase space one obtains two equations dTχ⁰ dt + 2HTχ⁰ ⁼ \$! Γ"χ0↔χ[±] + Γχ[±] # gχ⁰ nχ[±] nχ⁰ ' (T − Tχ⁰) (43) (the derivation of this equation is also sketched in the appendix). The numerical solution of the problem proceeds now analogously to what done so far. After the appropriate change of variables, the system in Eq. (41) replaces Eq. (7) in the system of Eq. (14). The explicit solution for nχ⁰ (t) and nχ[±] (t) are then implemented in Eq. (43) to find Tχ⁰ (t). Our first application is to the G2-MSSM models singled out in the previous Section as cosmologically favored. As we had guessed in the analysis we performed at the level of energy loss and scattering rates and shown graphically in Fig. 6, the departure from kinetic equilibrium tends to be at a temperature sensibly lower than the nominal reheating temperatures for these models (which are of the order of 100 MeV or larger). The numerical solution indeed shows that the ratio nχ[±] /nχ⁰ tends to follow very closely the ratio of the thermal equilibrium number densities neq the whole phase of DM production in the moduli decays, as well as at later times. The solution of the equation for the neutralino temperature shows that kinetic equilibrium is maintained up to a temperature of the order of 10 MeV, independently of the neutralino mass since, in the non-relativistic limit, the inelastic scattering rate (which together with chargino electromagnetic interactions enforces the equilibrium) depends only on the chargino-neutralino mass the chemical freeze out temperature; in those cases there should be a sizable change in the relic abundance as well and the formalism we developed would be suitable for an accurate computation of the relic density for such case. 1e-30 1e-25 1e-20 1e-15 1e-10 1e-05 1 0.001 0.01 0.1 n!+/-/n!0 T(GeV) TR=5 MeV TR=8 MeV TR=10 MeV TR=15 MeV TR=20 MeV thermal distribution 0 0.005 0.01 0.015 0.02 Standard decoupling the chemical freeze out temperature; in those cases there should be a sizable change in the relic abundance as well and the formalism we developed would be suitable for an accurate computation of the relic density for such case. 1e-30 1e-25 1e-20 1e-15 1 0.001 0.01 0.1 n!+/-/n!0 TR=5 MeV TR=8 MeV TR=10 MeV TR=15 MeV TR=20 MeV thermal distribution 0.2 0.4 0.6 0.8 1 0 0.005 0.01 0.015 0.02 T!0/T T[GeV] TR=5 MeV TR=8 MeV TR=10 MeV TR=15 MeV TR=20 MeV Standard decoupling

Giorgio Arcadi Planck 2011 splitting which is essentially the same over the same over the whole range of selected models. The transition b the regime T α T α takes place on relatively short timescales; since at 10 MeV non-thermal production has become α FIG. 7. Left panel: ratio of the chargino number density over the neutralino number density for several values of TRH. Right FIG. 7. Left panel: ratio of the chargino number density over the neutralino number density for several values of TRH. Right

Conclusions

Systematic and numerically accurate approach to non-thermal dark matter generation models.

We have analized the impact of non thermal dark matter in cosmic ray physics and LHC phenomenology.

Solution of Boltzmann equations for a general set of coannihilating particles without assuming kinetic equilibrium.

Developement of a formalism for computation of kinetic decoupling temperature of a set of coannihilating particles.

Reference: arXiv:1104.3591

Backup Slides

decay into a lighter will not state with the state will not encounter a case of this case of this case of this kind in explicit models and it would just models and it would just models and it would just models and it would like a condensation; provided that matter fluid; provided that energy is initially stored in \mathbb{R} a phase of matter domination lasting until the field decays, with the transition that needs to be transition t
The transition that needs to be transition that needs to be transition to be transition to be treated as a con T otrace the number density of the ζ states, especially when two or more or more of the ζ are nearly degenerate in mass ζ (coannihilation), one should refer to a system of coupled Boltzmann equations for non-thermal DIYI allow to implement Eq. (7) to the first is kinetic equilibrium for each species \mathbb{R} , namely that the scattering \mathbb{R} **Processes are efficient and make the phase space of the phase space space space space space space space space**

shape of the corresponding thermal equilibrium phase space density, namely ^fa(ka, t) = ^C(t) · ^f eq

$$
\frac{dn_{\psi_j}}{dt}+3Hn_{\psi_j}=\sum_i\frac{B_{\psi_j,X_i}}{m_{X_i}}\Gamma_{X_i}\rho_{X_i}-\Gamma_{\psi_j}n_{\psi_j}
$$

$$
\frac{d\rho_{X_i}}{dt} + 3H\rho_{X_i} = -\Gamma_{X_i}\rho_{X_i}
$$

#

n2

χ − (nequest)
γ − (nequest)
γ − (nequest)

^χ)

i

2\$

+!

complicate the notation). The Boltzmann equation for the ψ^j number density is:

$$
\frac{dn_X}{dt} + 3 H n_X = -\langle \sigma_{\text{eff}} v \rangle \left[n_X^2 - (n_X^{eq})^2 \right] + \sum_i \frac{B_{X_i}}{m_{X_i}} \Gamma_{X_i} \rho_{X_i} + \sum_j B_{\psi_j} \Gamma_{\psi_j} n_{\psi_j}
$$

$$
\frac{d\rho_R}{dt} + 3H(\rho_R + p_R) \simeq \sum_i \left(1 - \frac{\sum_j B_{\psi_j, X_i} \langle E_{\psi_j, X_i} \rangle + B_{X_i} m_X}{m_{X_i}}\right) \Gamma_{X_i} \rho_{X_i} + \sum_j \left(\langle E_{\psi_j} \rangle - m_{\chi} B_{\psi_j}\right) \Gamma_{\psi_j} n_{\psi_j} + m_{\chi} \langle \sigma_{\text{eff}} v \rangle \left[n_{\chi}^2 - (n_{\chi}^{eq})^2\right] .
$$

BXⁱ

 $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2$

 $\mathcal{M}(\mathcal{M})$ is a subset of $\mathcal{M}(\mathcal{M})$

nange or varia **)|** $\overline{}$ t $s(\mathbf{T} \mid \mathbf{r}) = \mathbf{r} \cdot \mathbf{r}$ numerical solution, it is more convenient to use as independent variable, rather than the time than the rescale Change of variables: <u>Following</u> of the inverse of an energy. For an energy of an energy of an energy of an energy of an energy and the inverse of an energy. For an energy and the inverse of an energy. For an energy of an use as dependent variables the dimensionless quantities: Change of variables:

$$
\xi_{X_i} \equiv \frac{\rho_{X_i} a^3}{\Lambda}, \qquad N_{\psi_j} \equiv n_{\psi_j} a^3 \qquad \text{and} \qquad N_{\chi} \equiv n_{\chi} a^3
$$

$$
\frac{d\xi_{X_i}}{dA} = -\frac{A^{1/2}a_I^{3/2}}{\mathcal{H}}\Gamma_{X_i}\xi_{X_i}
$$
\n
$$
\frac{dN_{\psi_j}}{dA} = \frac{A^{1/2}a_I^{3/2}}{\mathcal{H}}\left(\Lambda \sum_i \frac{B_{\psi_j,X_i}}{m_{X_i}}\Gamma_{X_i}\xi_{X_i} - \Gamma_{\psi_j}N_{\psi_j}\right)
$$
\n
$$
\frac{dN_X}{dA} = -\frac{\langle \sigma_{\text{eff}}v \rangle}{A^{5/2}a_I^{3/2}\mathcal{H}}\left[N_{\chi}^2 - (N_{\chi}^{eq})^2\right] + \frac{A^{1/2}a_I^{3/2}}{\mathcal{H}}\left(\Lambda \sum_i \frac{B_{X_i}}{m_{X_i}}\Gamma_{X_i}\xi_{X_i} + \sum_j B_{\psi_j}\Gamma_{\psi_j}N_{\psi_j}\right)
$$
\n
$$
\frac{dT}{dA} = \left(1 + \frac{T}{4g_{\text{eff}}}\frac{dg_{\text{eff}}}{dT}\right)^{-1} \left\{-\frac{h_{\text{eff}}}{g_{\text{eff}}}\frac{T}{A} + \frac{h_{\text{eff}}}{3g_{\text{eff}}s(T)}\frac{1}{A^{5/2}a_I^{3/2}\mathcal{H}}\left[\sum_j \left(\langle E_{\psi_j}\rangle - m_{\chi}B_{\psi_j}\right)\Gamma_{\psi_j}N_{\psi_j} + \Lambda \sum_i \left(1 - \frac{\sum_j B_{\psi_j,X_i}\langle E_{\psi_j,X_i}\rangle + B_{X_i}m_{\chi}}{m_{X_i}}\right) \Gamma_{X_i}\xi_{X_i} + \frac{m_{\chi}\langle \sigma_{\text{eff}}v\rangle}{A^3a_I^3}\left[N_{\chi}^2 - (N_{\chi}^{eq})^2\right]\right\}
$$

$$
\mathcal{H} \equiv (a_I A)^{3/2} H = \left(\frac{\Lambda \sum_i \xi_{X_i} + \rho_R(T) A^3 a_I^3 + m_{\chi} N_{\chi} + \sum_j \langle E_{\psi_j} \rangle N_{\psi_j}}{3M_{PL}^2}\right)^{1/2}
$$

a

The collisional operator for the inelastic scatterings is: while complematioperator for the molastic seated migo is. The collisional operator for the inelastic scatterings is:

$$
\frac{\hat{C}_{\chi^0,\mathrm{is}}}{E}[f_{\chi^0},f_{\chi^{\pm}}] = \sum_{(a,b)} \tilde{g}_{Wab} g_{\chi^{\pm}} \int \frac{d^3k}{(2\pi)^3 2k} \int \frac{d^3k'}{(2\pi)^3 2k'} \int \frac{d^3p'}{(2\pi)^3 2E'} \frac{|\bar{M}|_{ab}^2}{2E} (2\pi)^4 \delta^4(P' + K' - P - K) \cdot [f_b(k')(1 - f_a(k)) f_{\chi^{\pm}}(p') - f_a(k)(1 - f_b(k')) f_{\chi^0}(p)] ;
$$

Thanks to its efficients interactions dark matter loses essentially all its kinetic energy. The collisional operators can be taken in the non relativistic limit and expanded in powers of $\Delta M/M$ and T/M. expanded in powers of $ΔM/M$ and T/M. Thanks to its emicients interactions dark matter loses essentially all its kinetic Expanded in powers of ΔVUT and TVU .

$$
\frac{\hat{C}_{\chi^0,\rm is}[f_{\chi^0}]}{E} = \sum_{(a,b)} \frac{\tilde{g}_{Wabg_\chi\pm}}{256\pi^5 E E'} \int \frac{d^3k}{k} f(k) \int \frac{d^3k'}{k'} |\bar{M}|^2_{ab} \delta(E' + k' - E - k)
$$
\n
$$
\left[\left(f_{\chi^\pm}(p) e^{\frac{\Delta m_\chi}{T}} - f_{\chi^0}(p) \right) - \left(\frac{\Delta m_\chi v^2}{2T} f_{\chi^\pm}(p) + \frac{\mathbf{q} \cdot \mathbf{v}}{v} \frac{df_{\chi^\pm}}{dp} + \frac{\mathbf{q} \cdot \mathbf{v}}{T} f_{\chi^\pm}(p) \right) e^{\frac{\Delta m_\chi}{T}}
$$
\n
$$
+ \left(\frac{q^2}{2m_\chi T} f_{\chi^\pm}(p) + \frac{(\mathbf{q} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{v})}{2T^2} f_{\chi^\pm}(p) + \frac{(\mathbf{q} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{v})}{vT} \frac{df_{\chi^\pm}}{dp} + \frac{(\mathbf{q} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{v})}{2v^2} \Delta_{\mathbf{p}} f_{\chi^\pm} \right) e^{\frac{\Delta m_\chi}{T}}
$$
\n
$$
+ \frac{1}{2} \left(\frac{q^2}{v} - \frac{3(\mathbf{q} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{v})}{v^3} \right) \frac{df_{\chi^\pm}}{dp} e^{\frac{\Delta m_\chi}{T}}
$$

$$
\begin{split} &\frac{\hat{\mathbf{C}}_{\chi_{0},\rm is}\left[f_{\chi^{0}},f_{\chi^{\pm}}\right]}{E}=\sum\frac{2G_{\rm F}^{2}\tilde{g}_{Wab}g_{\chi\pm}}{\pi^{3}}\left\{\left[4T^{3}\left(\Delta m_{\chi}^{2}+6\Delta m_{\chi}T+12T^{2}\right)\left(1+\frac{\Delta m_{\chi}}{m_{\chi}}\right)\right.\\&\left.\left.-2\Delta m_{\chi}^{2}T^{2}\left(\Delta m_{\chi}+2T\right)\frac{p}{m_{\chi}}+\frac{2}{3}T\left(\Delta m_{\chi}^{4}+3\Delta m_{\chi}^{3}T+32\Delta m_{\chi}^{2}T^{2}+114\Delta m_{\chi}T^{3}+144T^{4}\right)\frac{p^{2}}{m_{\chi}^{2}}\right]\left(f_{\chi^{\pm}}-f_{\chi^{0}}e^{-\frac{\Delta m_{\chi}}{T}}\right)\\&-\frac{8}{3}\Delta m_{\chi}T^{3}\left(\Delta m_{\chi}^{2}+6\Delta m_{\chi}T+12T^{2}\right)\left(\frac{p^{2}}{Tm_{\chi}^{2}}f_{\chi\pm}+\frac{\mathbf{p}\cdot\nabla_{\mathbf{p}}f_{\chi^{\pm}}}{m_{\chi}}\right)\\&+\frac{2}{3}T^{3}\left(\Delta m_{\chi}^{4}+10\Delta m_{\chi}^{3}T+60\Delta m_{\chi}^{2}T^{2}+240\Delta m_{\chi}T^{3}+480T^{4}\right)\left(\Delta_{\mathbf{p}}f_{\chi^{\pm}}+\frac{\mathbf{p}\cdot\nabla_{\mathbf{p}}f_{\chi^{\pm}}}{m_{\chi}T}+\frac{3}{m_{\chi}T}f_{\chi^{\pm}}\right)\\&-2T^{2}\Delta m_{\chi}\left(\Delta m_{\chi}^{2}+6\Delta m_{\chi}T+12T^{2}\right)\frac{p^{2}}{m_{\chi}^{2}}f_{\chi^{\pm}}\right\} \end{split}
$$

$$
\begin{split} &\frac{\hat{\mathbf{C}}_{\chi^{\pm},\mathrm{is}}\left[f_{\chi^{0}},f_{\chi^{\pm}}\right]}{E}=\sum\frac{2G_{\mathrm{F}}^{2}\tilde{g}_{Wab}g_{\chi^{0}}}{\pi^{3}}\left\{\left[4T^{3}\left(\Delta m_{\chi}^{2}+6\Delta m_{\chi}T+12T^{2}\right)\left(1-\frac{\Delta m_{\chi}}{m_{\chi}}\right)\right.\\ &\left.\left.-2\Delta m_{\chi}^{2}T^{2}\left(\Delta m_{\chi}+2T\right)\frac{p}{m_{\chi}}+\frac{2}{3}T\left(\Delta m_{\chi}^{4}-4\Delta m_{\chi}^{3}T-10\Delta m_{\chi}^{2}T^{2}+30\Delta m_{\chi}T^{3}+144T^{4}\right)\frac{p^{2}}{m_{\chi}^{2}}\right]\left(f_{\chi^{0}}e^{-\frac{\Delta m_{\chi}}{T}}-f_{\chi^{\pm}}\right)\\ &+\frac{8}{3}\Delta m_{\chi}T^{3}\left(\Delta m_{\chi}^{2}+6\Delta m_{\chi}T+12T^{2}\right)\left(\frac{p^{2}}{Tm_{\chi}^{2}}f_{\chi^{0}}+\frac{\mathbf{p}\cdot\nabla_{\mathbf{p}}f_{\chi^{0}}}{m_{\chi}}\right)e^{-\frac{\Delta m_{\chi}}{T}}\\ &+\frac{2}{3}T^{3}\left(\Delta m_{\chi}^{4}+10\Delta m_{\chi}^{3}T+60\Delta m_{\chi}^{2}T^{2}+240\Delta m_{\chi}T^{3}+480T^{4}\right)\left(\Delta_{\mathbf{p}}f_{\chi^{0}}+\frac{\mathbf{p}\cdot\nabla_{\mathbf{p}}f_{\chi^{0}}}{m_{\chi}T}+\frac{3}{m_{\chi}T}f_{\chi^{0}}\right)e^{-\frac{\Delta m_{\chi}}{T}}\\ &+2T^{2}\Delta m_{\chi}\left(\Delta m_{\chi}^{2}+6\Delta m_{\chi}T+12T^{2}\right)\frac{p^{2}}{m_{\chi}^{2}}f_{\chi^{0}}e^{-\frac{\Delta m_{\chi}}{T}}\right\} \end{split}
$$