Accurate estimate of the relic density and of the kinetic decoupling in non thermal dark matter models

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Based on G.A. and Piero Ullio, arXiv: 1104.3591

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Plan of the talk

1) Numerical evaluation of the Boltzmann equation for non thermal production of dark matter:

- General framework
- Specific framework: G2-MSSM

2) Kinetic equilibrium and decoupling in the G2-MSSM:

- System of coupled Boltzmann equations for a set of coannihilating particles with standard assumptions for the computation of the relic density relaxed.
- Computation of kinetic decoupling temperature of a system of coannihilating particles.

Non thermal production of DM

Cosmological moduli

Only gravitational interactions

In general they tend do dominate the energy density of the Universe

Large entropy injection when they decay (reheating phase)

Decoupled from the thermal bath

Deviation from standard cosmology

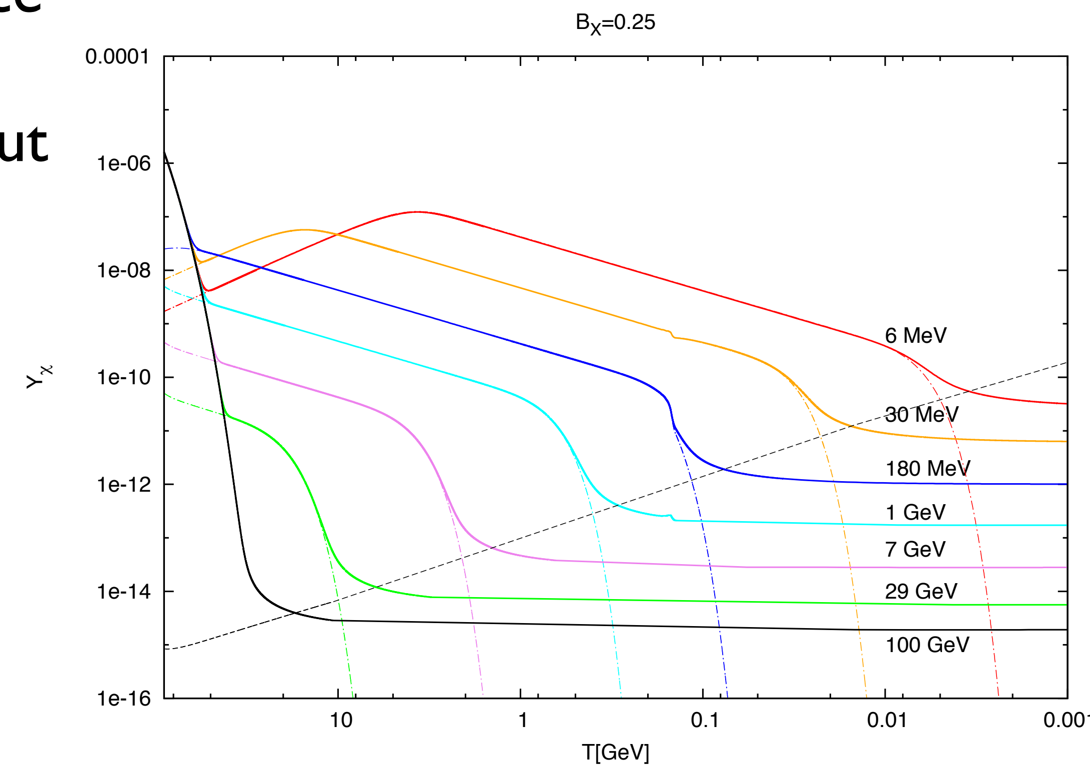
Mass at least O(10-100) TeV (BBN bound)

Dilution of abundances in the thermal bath

Non thermal production of DM:
The moduli decay producing DM either directly or through a cascade decay

General Framework

In case that non thermal production is efficient there is balance between production and annihilations. The relic density resembles the standard case with the difference that freeze-out occurs at around the reheating temperature.

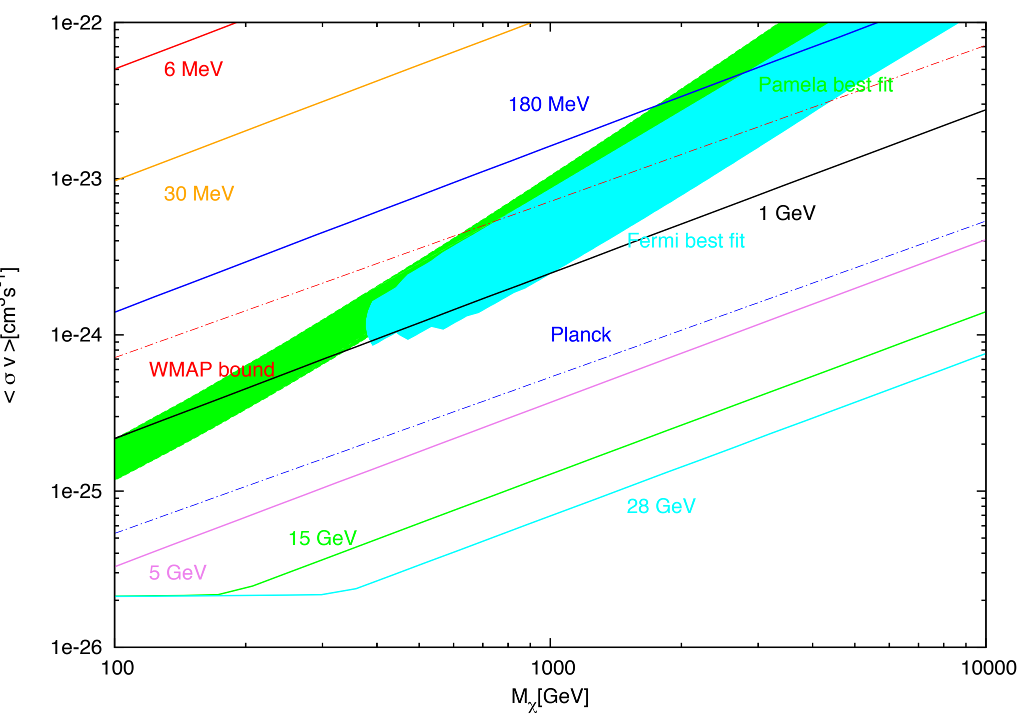


$$n_\chi^{QSE} \equiv \left(\frac{B_X \Gamma_X \rho_X}{m_X \langle \sigma_{\rm eff} v\rangle}\right)^{1/2} \longrightarrow n_\chi^c \simeq \frac{H}{\langle \sigma v\rangle}$$

freeze-out at

$$\Omega_\chi^{NT} h^2 \simeq \frac{T_{t.f.o.}}{T_{\rm RH}}\, \Omega_\chi^T h^2$$

Efficient non thermal production favours candidates with high annihilation cross section.

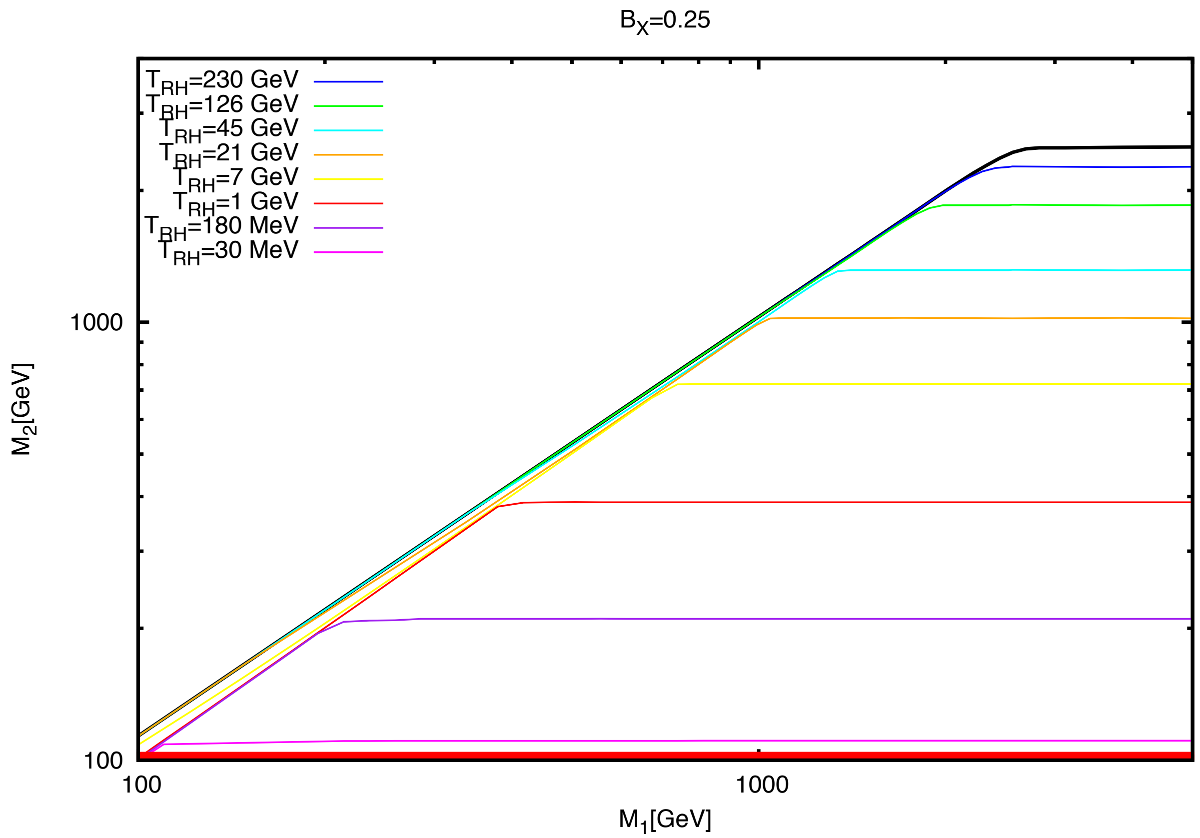


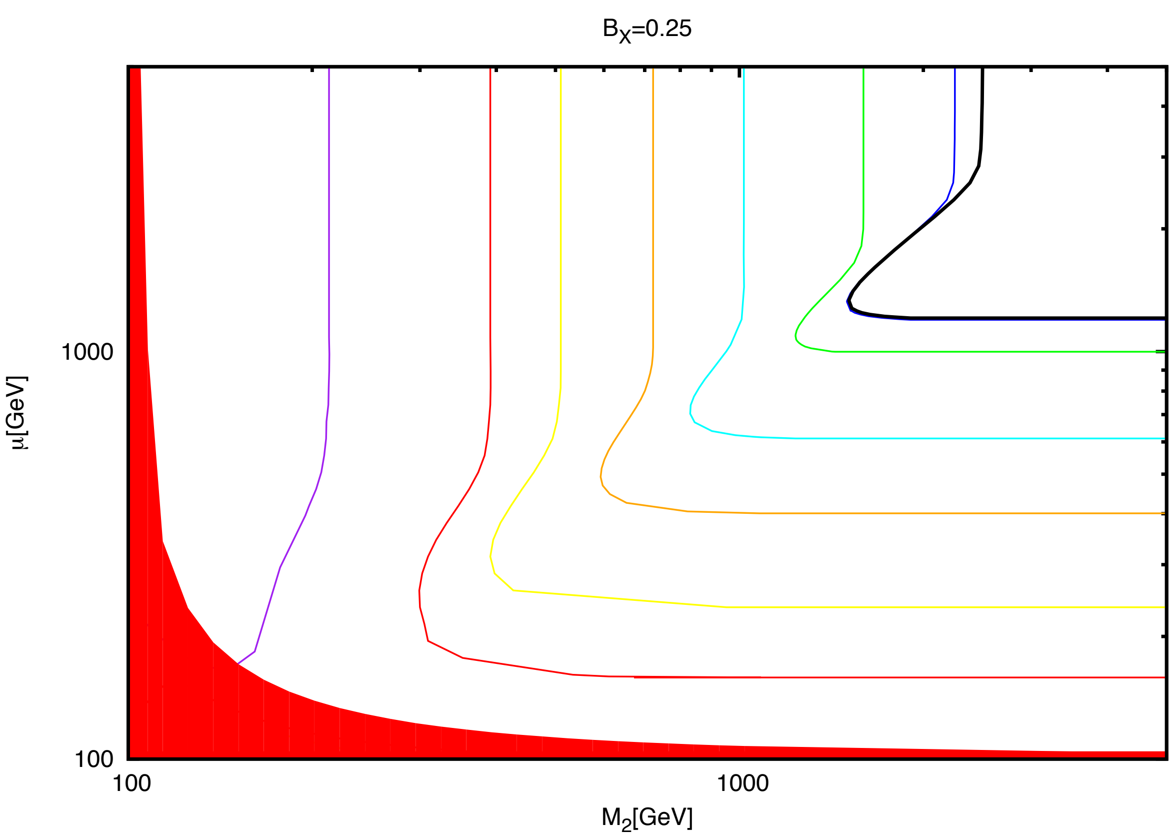
Possible explanation to Pamela and Fermi anomalies.

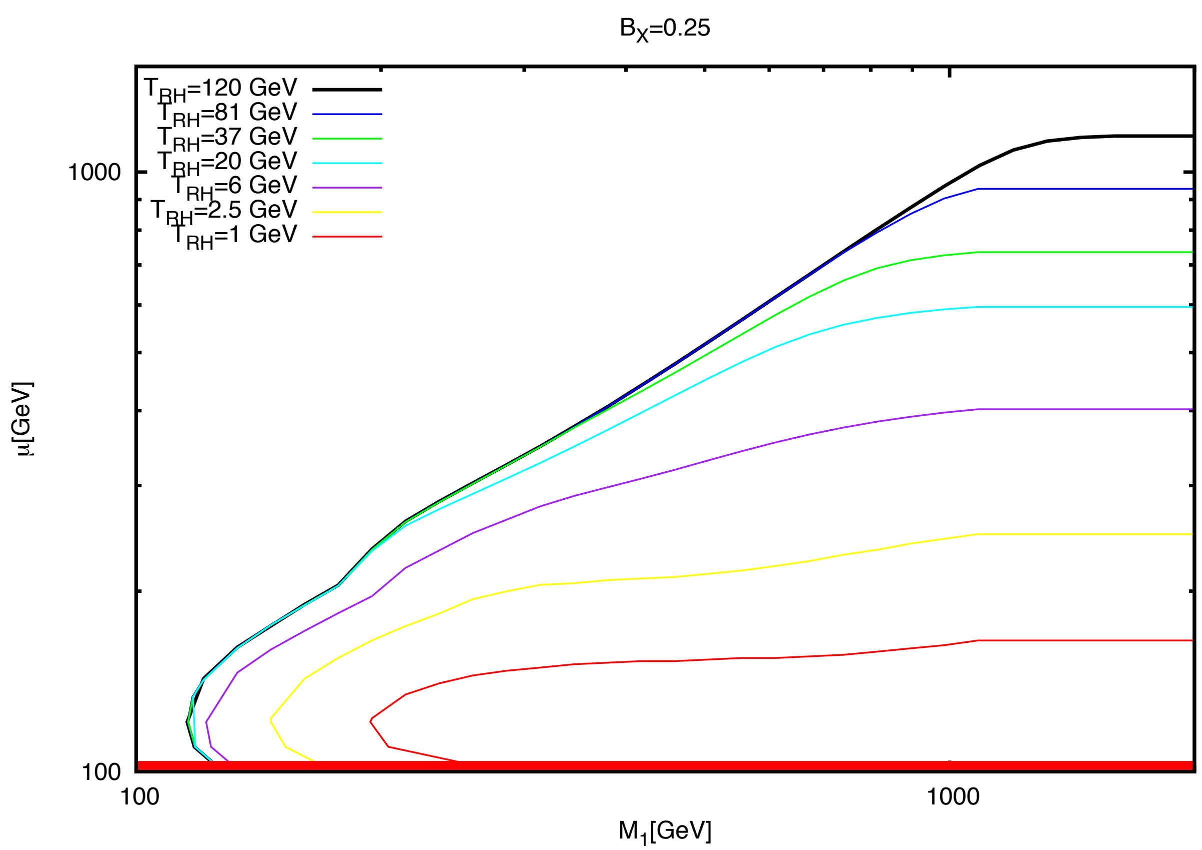
Non thermal production and the MSSM

Non thermal production has impact on the expectations for mass and composition neutralino LSP.

Pure gaugino (Wino), or pure higgsino states compatible with cosmology.







Specific framework: G2-MSSM

String M-theory

Heavy moduli m=O(100) m3/2
Meson field m=O(1) m3/2
Light moduli m=O(1) m3/2

Effective sugra

$$\mu = z_{eff} m_{3/2}$$
$$A \simeq 1.48\, m_{3/2}$$
$$m_0 = m_{3/2}$$
$$tan\beta \simeq 1.5 - 2$$

$$\Omega_\chi h^2 \propto \frac{[F(\delta)]^3 \left(m_{3/2}\right)^{3/2}}{D_{X_i}^{1/2}}$$

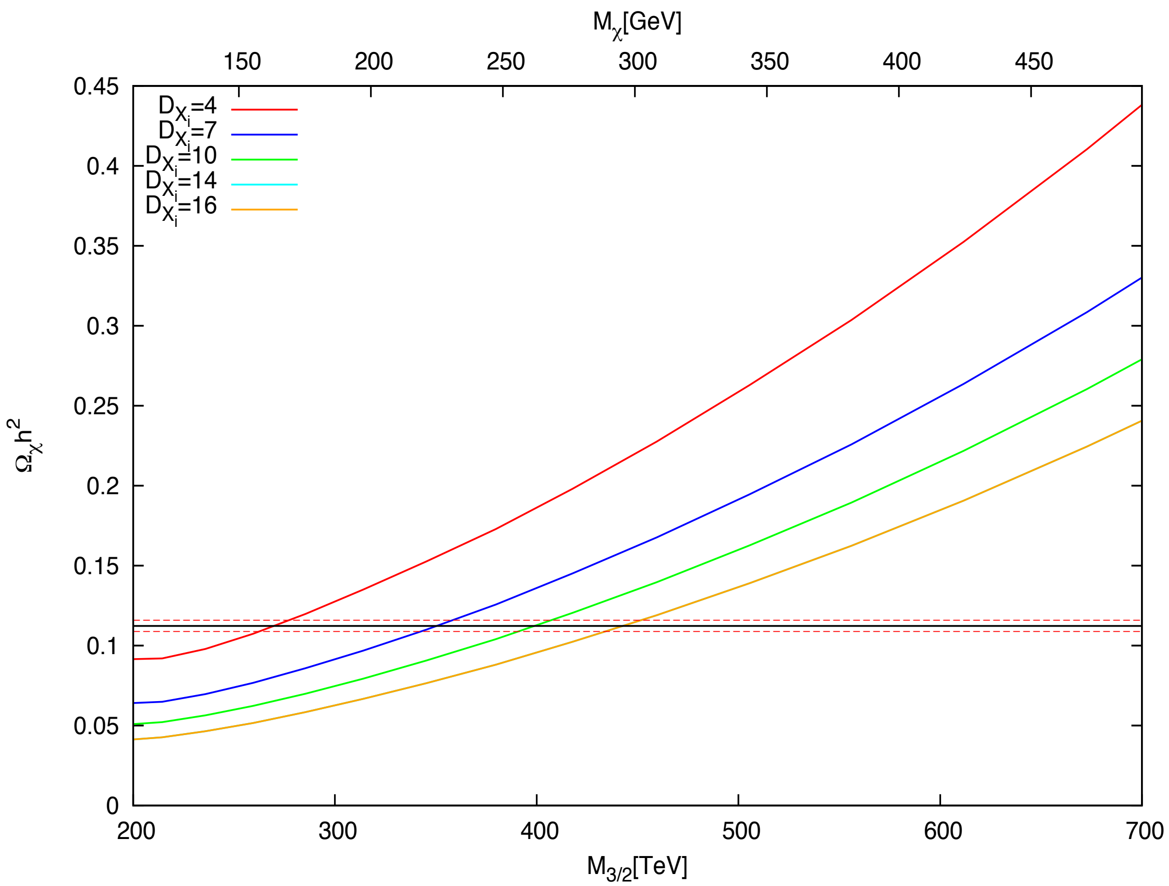
Non universal gaugino masses

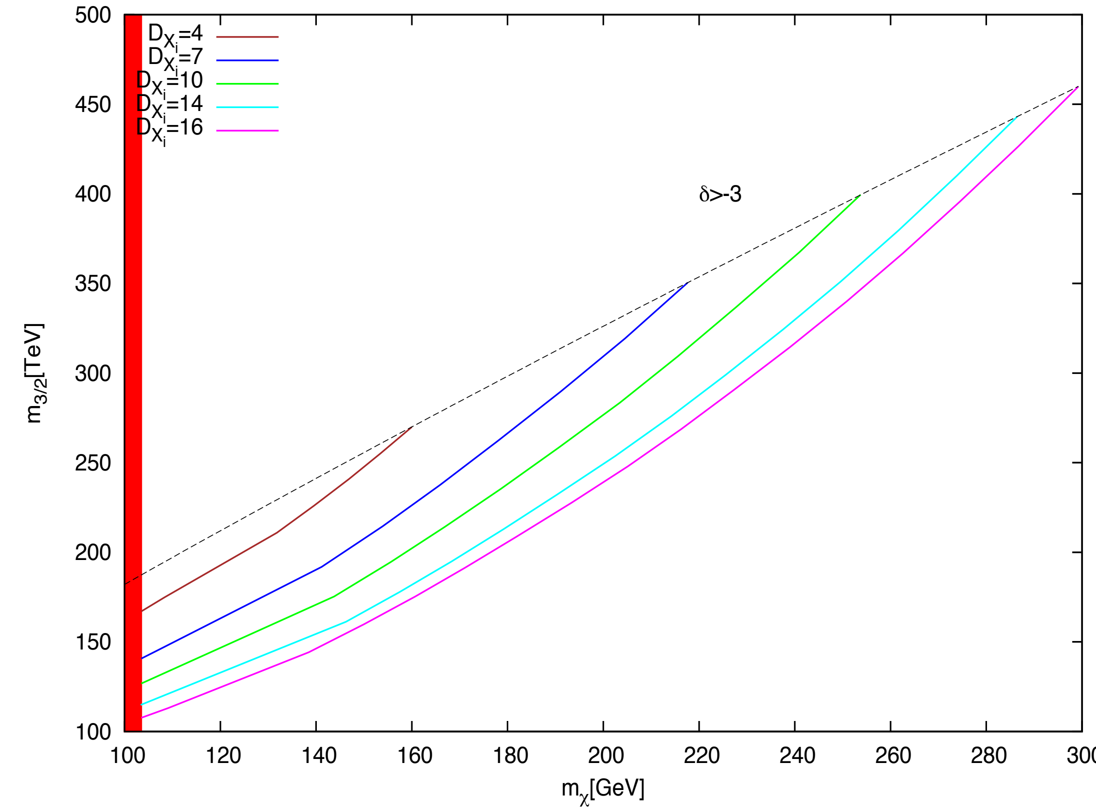
$$M_a = (f_a \alpha_{GUT} - \varepsilon\eta) m_{3/2}$$
$$\eta = 1 - \alpha_{GUT}\delta$$

O(10-100) TeV gravitino mass

- Wino LSP with mass O(100-500) GeV.
- Almost degenerate chargino NLSP (mass splitting O(200) MeV).
- Pure Bino with mass O(1) LSP mass
- Gluino with mass O(0.5-1) TeV.
- Other superpartners with mass of the order of the gravitino mass.

Our results are based on the assumptions done in B. Acharya et al. arXiv:0801.0478





Correct relic density obtained for reheating temperatures between 100 MeV and 1 GeV.

Until now we have assumed that DM is kinetic equilibrium during production and after freeze-out (important for coannihilations).

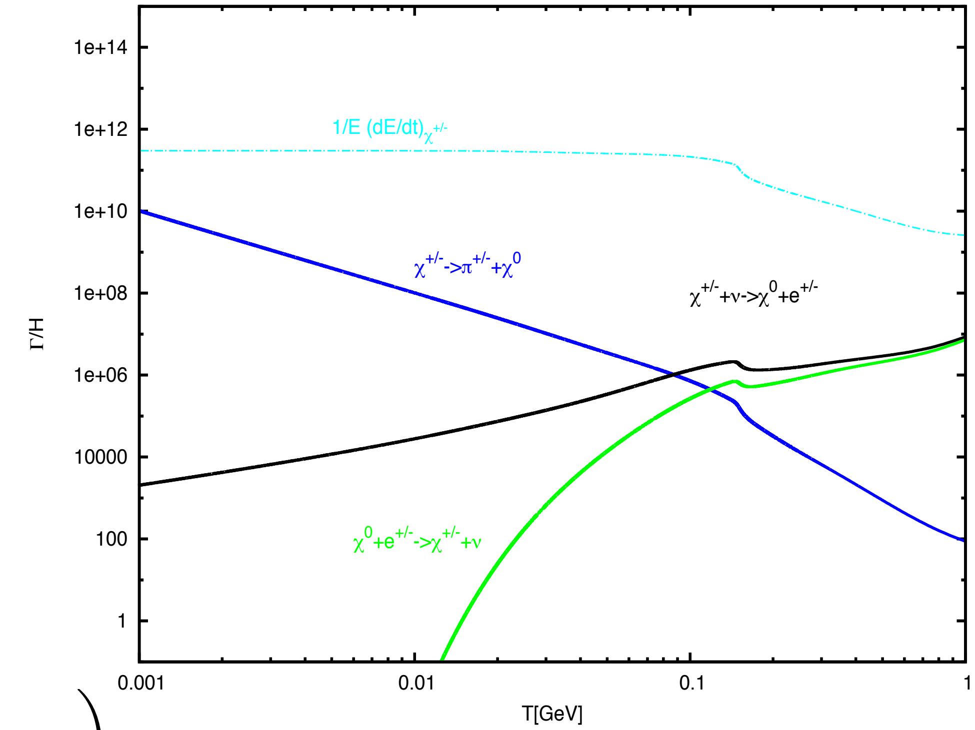
Dark matter is produced out-of-equilibrium and then must have efficient interactions in order to get into thermal equilibrium.

Non thermal DM might not be in thermal equilibrium because it can be produced at temperatures close or below kinetic decoupling temperature.

Kinetic decoupling temperature can be altered in the non-standard cosmology.

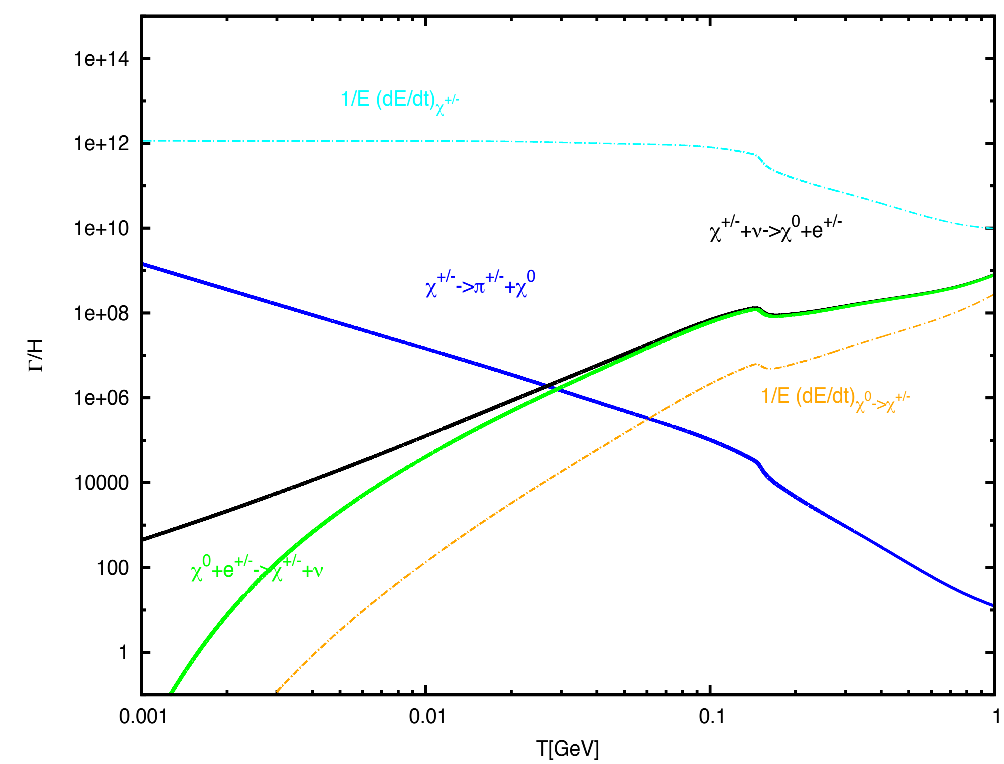
Two-particle system:charginos and neutralinos. Kinetic equilibrium established after production if:

$$\left(-\frac{1}{E}\frac{dE}{dt}\right)\cdot\Delta\tau > 1$$



$$\left(-\frac{dE}{dt}\right)_{\chi^0\to\chi^\pm} = \sum_{(a,b)} \frac{16\tilde{g}_{Wab}\,G_F^2}{\pi^3}\exp\left(-\frac{m_\chi \Delta m_\chi}{2ET}\right) T^5 \left(\frac{E}{m_\chi}\right)^3 \left(8\frac{ET}{m_\chi}+\Delta m_\chi\right)$$

$$\left(-\frac{dE}{dt}\right)_{\chi^\pm} = \frac{\pi\alpha^2 T^2}{3}\Lambda$$



-Neutralinos: energy loss through inelastic scatterings into charginos. Elastic scatterings instead suppressed.
-Charginos: efficient energy loss through electromagnetic interactions.

Boltzmann equations for the number densities

System of two coupled equations:

$$(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}})\, f_{\chi^0}(p,t) = \frac{1}{E}\,\hat{\mathbf{C}}_{\chi^0}[f_{\chi^0}, f_{\chi^\pm}] \qquad (\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}})\, f_{\chi^\pm}(p,t) = \frac{1}{E}\,\hat{\mathbf{C}}_{\chi^\pm}[f_{\chi^0}, f_{\chi^\pm}]$$

Dark matter istantaneously thermalized after production as soon neutralino inelastic scatterings are efficient.

In the non relativistic limit and at the leading order expansion in T/M and ΔM/M:

$$\frac{dn_{\chi^0}}{dt} + 3\,H\,n_{\chi^0} = \left(\widetilde{\Gamma}_{\chi^0\leftrightarrow\chi^\pm} + \Gamma_{\chi^\pm}\right)\left[g_{\chi^0} n_{\chi^\pm} - g_{\chi^\pm} n_{\chi^0}\exp\left(-\frac{\Delta m_\chi}{T}\right)\right] - \langle\sigma v\rangle_{\chi^0\chi^0}\left[n_{\chi^0}^2 - (n_{\chi^0}^{eq})^2\right]$$
$$-\langle\sigma v\rangle_{\chi^0\chi^\pm}\left[n_{\chi^0} n_{\chi^\pm} - n_{\chi^0}^{eq} n_{\chi^\pm}^{eq}\right]$$

$$\frac{dn_{\chi^\pm}}{dt} + 3\,H\,n_{\chi^\pm} = \left(\widetilde{\Gamma}_{\chi^0\leftrightarrow\chi^\pm} + \Gamma_{\chi^\pm}\right)\left[g_{\chi^\pm} n_{\chi^0}\exp\left(-\frac{\Delta m_\chi}{T}\right) - g_{\chi^0} n_{\chi^\pm}\right] - \langle\sigma v\rangle_{\chi^\pm\chi^\pm}\left[n_{\chi^\pm}^2 - (n_{\chi^\pm}^{eq})^2\right]$$
$$-\langle\sigma v\rangle_{\chi^\pm\chi^0}\left[n_{\chi^\pm} n_{\chi^0} - n_{\chi^\pm}^{eq} n_{\chi^0}^{eq}\right] + \sum_i \frac{B_{X_i}}{m_{X_i}}\Gamma_{X_i}\rho_{X_i}$$

$$\widetilde{\Gamma}_{\chi^0\leftrightarrow\chi^\pm} = \sum_{(a,b)} \frac{\tilde{g}_{Wab} 8 G_{\rm F}^2}{\pi^3} T^3 \left(\Delta m_\chi^2 + 6\Delta m_\chi T + 12 T^2\right)$$

External source term for non thermal production

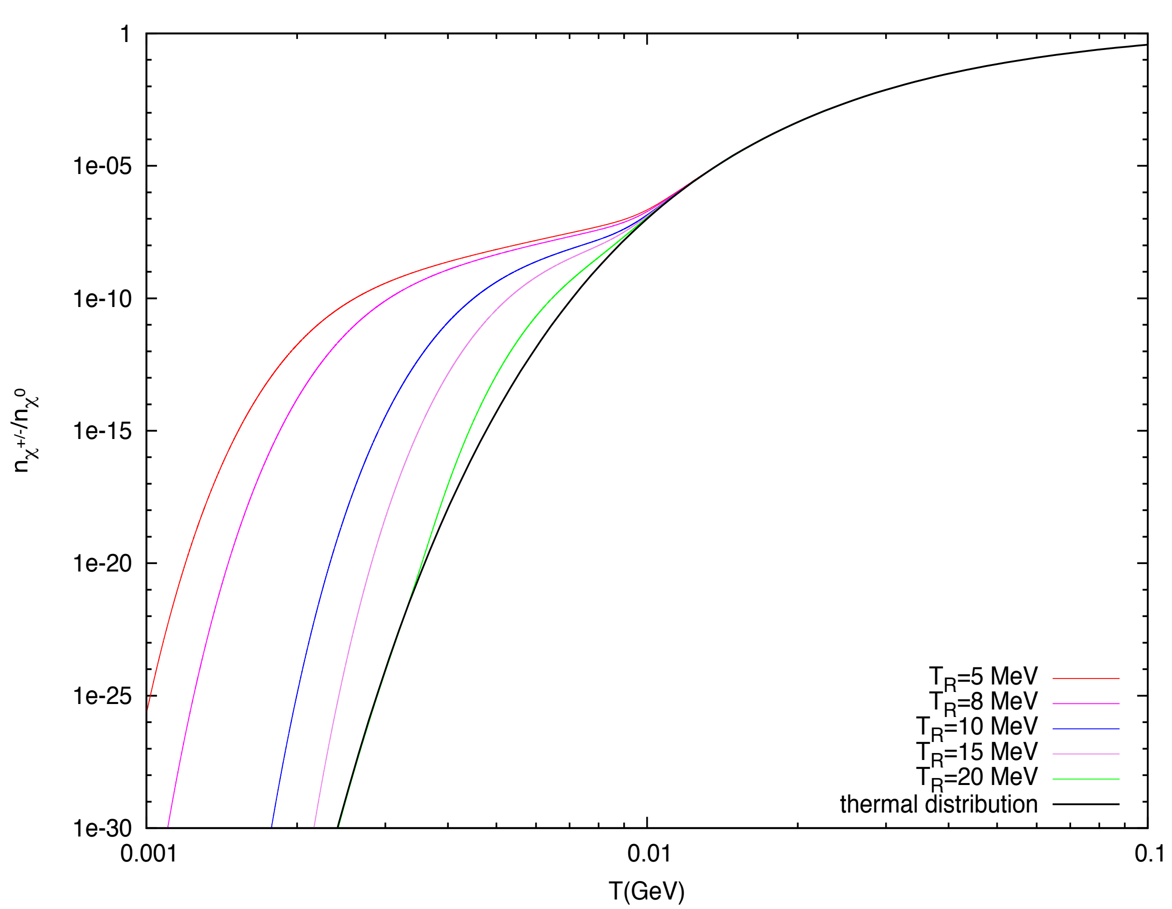
Equations for kinetic decoupling

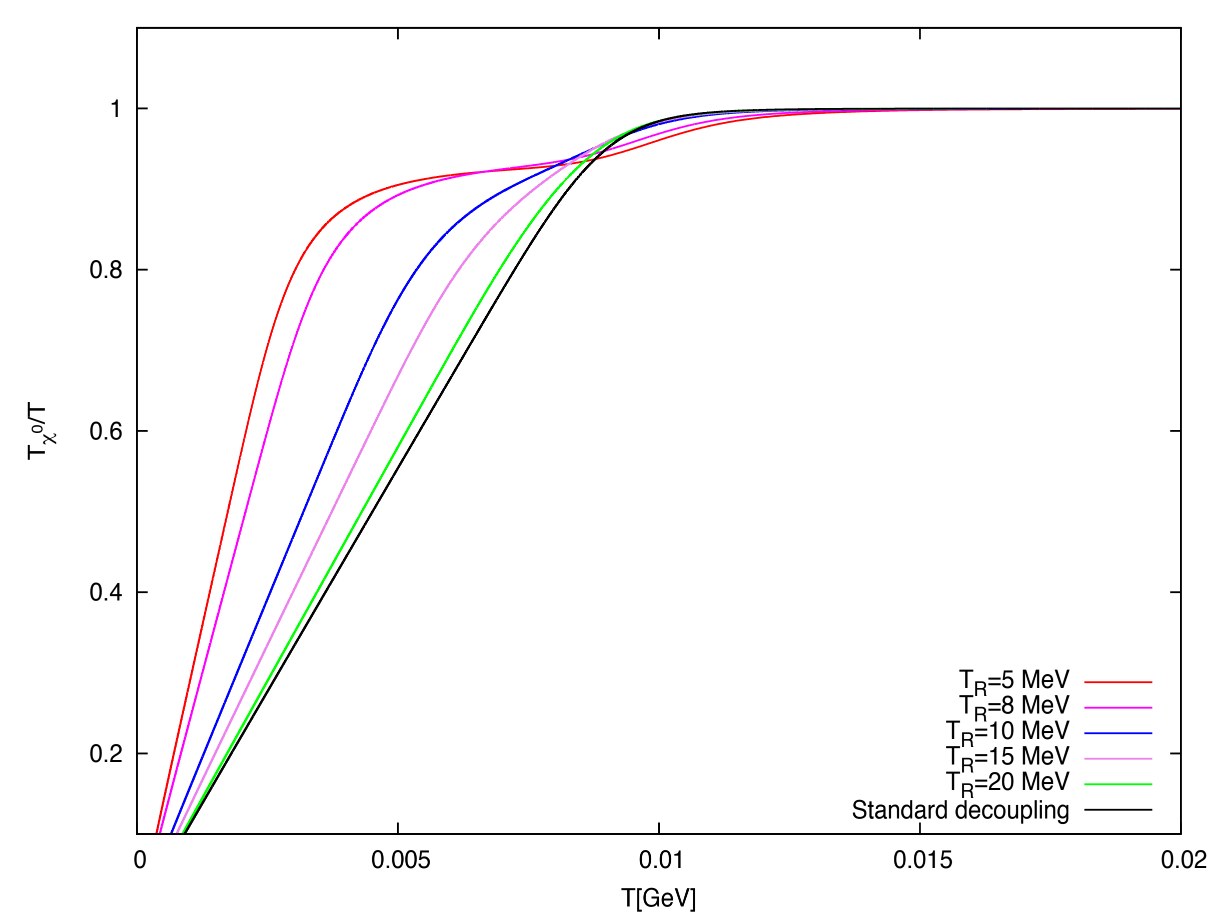
From the second momentum of the distribution function:

$$\int \frac{d^3p}{(2\pi)^3} g_{\chi^0}\, p^2\, f_{\chi^0}(p,t) \equiv 3 m_\chi\, T_{\chi^0}(t)\, n_{\chi^0}(t)$$

can be defined a temperature which parametrizes deviations from kinetic equilibrium. Charginos are always kept into equilibrium by electromagnetic interactions. Only one equation needed:

$$\frac{dT_{\chi^0}}{dt} + 2HT_{\chi^0} = \left[\left(\widetilde{\Gamma}_{\chi^0 \leftrightarrow \chi^\pm} + \Gamma_{\chi^\pm}\right) g_{\chi^0} \frac{n_{\chi^\pm}}{n_{\chi^0}}\right] (T - T_{\chi^0})$$





Conclusions

Systematic and numerically accurate approach to non-thermal dark matter generation models.

We have analized the impact of non thermal dark matter in cosmic ray physics and LHC phenomenology.

Solution of Boltzmann equations for a general set of coannihilating particles without assuming kinetic equilibrium.

Developement of a formalism for computation of kinetic decoupling temperature of a set of coannihilating particles.

Reference:
arXiv:1104.3591

Backup Slides

Boltzmann equations for non-thermal DM

$$\frac{dn_{\psi_j}}{dt} + 3Hn_{\psi_j} = \sum_i \frac{B_{\psi_j,X_i}}{m_{X_i}}\Gamma_{X_i}\rho_{X_i} - \Gamma_{\psi_j}n_{\psi_j}$$

$$\frac{d\rho_{X_i}}{dt} + 3H\rho_{X_i} = -\Gamma_{X_i}\rho_{X_i}$$

$$\frac{dn_\chi}{dt} + 3\,H\,n_\chi = -\langle\sigma_{\rm eff}v\rangle\left[n_\chi^2 - (n_\chi^{eq})^2\right] + \sum_i \frac{B_{X_i}}{m_{X_i}}\Gamma_{X_i}\rho_{X_i} + \sum_j B_{\psi_j}\Gamma_{\psi_j}n_{\psi_j}$$

$$\frac{d\rho_R}{dt} + 3H(\rho_R + p_R) \simeq \sum_i \left(1 - \frac{\sum_j B_{\psi_j,X_i}\langle E_{\psi_j,X_i}\rangle + B_{X_i}\,m_\chi}{m_{X_i}}\right)\Gamma_{X_i}\rho_{X_i} + \sum_j \left(\langle E_{\psi_j}\rangle - m_\chi B_{\psi_j}\right)\Gamma_{\psi_j}n_{\psi_j}$$
$$+m_\chi\langle\sigma_{\rm eff}v\rangle\left[n_\chi^2 - (n_\chi^{eq})^2\right]\ .$$

Change of variables:

$$\xi_{X_i} \equiv \frac{\rho_{X_i} a^3}{\Lambda}, \qquad N_{\psi_j} \equiv n_{\psi_j} a^3 \qquad \text{and} \qquad N_\chi \equiv n_\chi a^3$$

$$\frac{d\xi_{X_i}}{dA} = -\frac{A^{1/2} a_I^{3/2}}{\mathcal{H}} \Gamma_{X_i} \xi_{X_i}$$

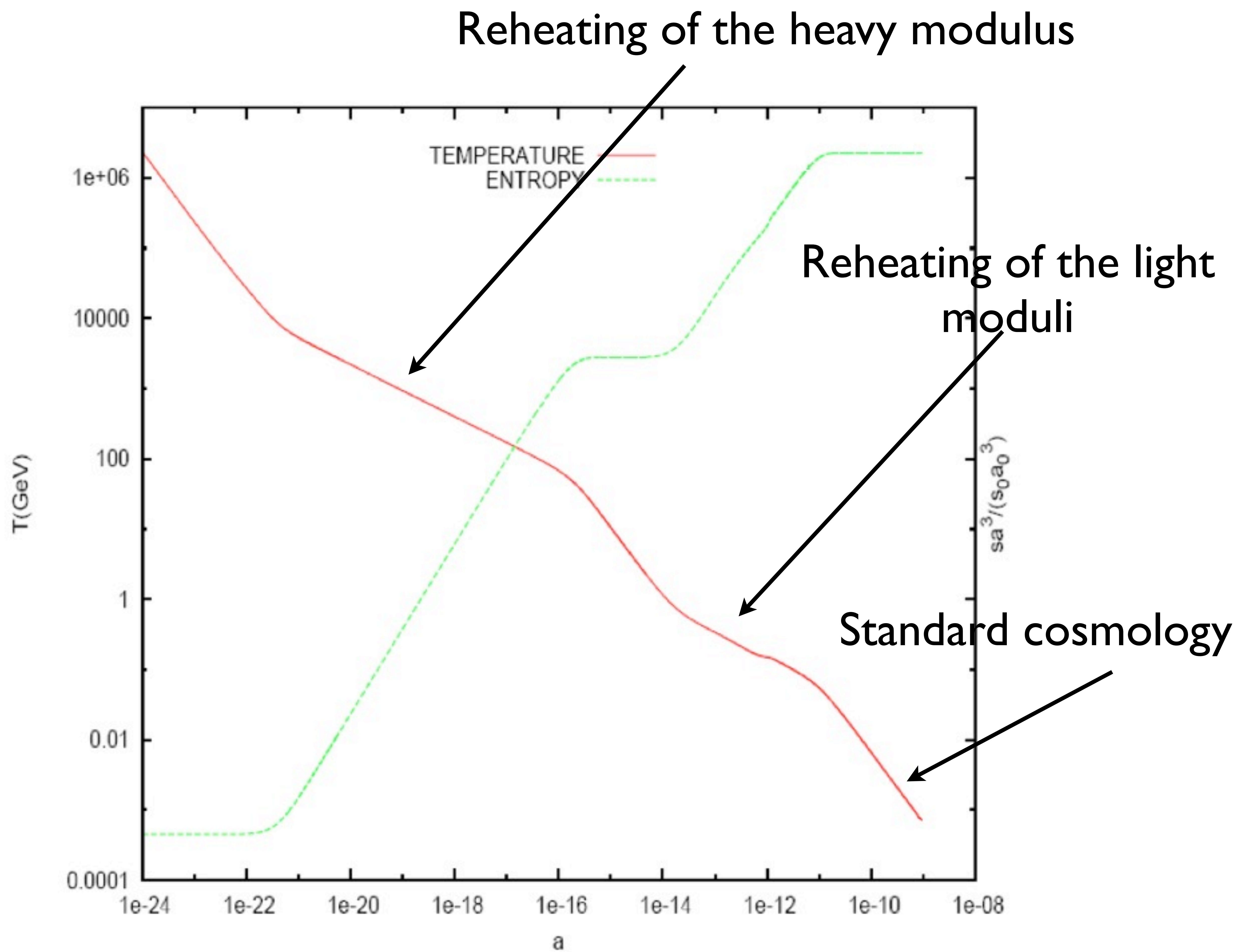
$$\frac{dN_{\psi_j}}{dA} = \frac{A^{1/2} a_I^{3/2}}{\mathcal{H}} \left(\Lambda \sum_i \frac{B_{\psi_j, X_i}}{m_{X_i}} \Gamma_{X_i} \xi_{X_i} - \Gamma_{\psi_j} N_{\psi_j} \right)$$

$$\frac{dN_\chi}{dA} = -\frac{\langle \sigma_{\rm eff} v \rangle}{A^{5/2} a_I^{3/2} \mathcal{H}} \left[N_\chi^2 - (N_\chi^{eq})^2 \right] + \frac{A^{1/2} a_I^{3/2}}{\mathcal{H}} \left(\Lambda \sum_i \frac{B_{X_i}}{m_{X_i}} \Gamma_{X_i} \xi_{X_i} + \sum_j B_{\psi_j} \Gamma_{\psi_j} N_{\psi_j} \right)$$

$$\frac{dT}{dA} = \left(1 + \frac{T}{4 g_{\rm eff}} \frac{d g_{\rm eff}}{dT} \right)^{-1} \left\{ -\frac{h_{\rm eff}}{g_{\rm eff}} \frac{T}{A} + \frac{h_{\rm eff}}{3 g_{\rm eff} s(T)} \frac{1}{A^{5/2} a_I^{3/2} \mathcal{H}} \left[\sum_j \left(\langle E_{\psi_j} \rangle - m_\chi B_{\psi_j} \right) \Gamma_{\psi_j} N_{\psi_j} \right. \right.$$

$$\left. \left. + \Lambda \sum_i \left(1 - \frac{\sum_j B_{\psi_j, X_i} \langle E_{\psi_j, X_i} \rangle + B_{X_i} m_\chi}{m_{X_i}} \right) \Gamma_{X_i} \xi_{X_i} + \frac{m_\chi \langle \sigma_{\rm eff} v \rangle}{A^3 a_I^3} \left[N_\chi^2 - (N_\chi^{eq})^2 \right] \right] \right\}$$

$$\mathcal{H} \equiv (a_I A)^{3/2} H = \left(\frac{\Lambda \sum_i \xi_{X_i} + \rho_R(T) A^3 a_I^3 + m_\chi N_\chi + \sum_j \langle E_{\psi_j} \rangle N_{\psi_j}}{3 M_{PL}^2} \right)^{1/2}$$



Reheating of the heavy modulus
Reheating of the light moduli
Standard cosmology
TEMPERATURE
ENTROPY
1e+06
10000
100
1
0.01
0.0001
T(GeV)
$sa^3/(s_0a_0^3)$
1e-24
1e-22
1e-20
1e-18
1e-16
1e-14
1e-12
1e-10
1e-08
a

The collisional operator for the inelastic scatterings is:

$$\frac{\hat{\mathbf{C}}_{\chi^0,\mathrm{is}}}{E}[f_{\chi^0},f_{\chi^\pm}] = \sum_{(a,b)} \tilde{g}_{Wab} g_{\chi^\pm} \int \frac{d^3k}{(2\pi)^3 2k} \int \frac{d^3k'}{(2\pi)^3 2k'} \int \frac{d^3p'}{(2\pi)^3 2E'} \frac{|\bar{M}|^2_{ab}}{2E} (2\pi)^4 \delta^4(P'+K'-P-K)$$
$$\cdot \left[f_b(k')(1-f_a(k)) f_{\chi^\pm}(p') - f_a(k)(1-f_b(k')) f_{\chi^0}(p) \right] \; ;$$

Thanks to its efficients interactions dark matter loses essentially all its kinetic energy. The collisional operators can be taken in the non relativistic limit and expanded in powers of ΔM/M and T/M.

$$\frac{\hat{\mathbf{C}}_{\chi^0,\mathrm{is}}[f_{\chi^0}]}{E} = \sum_{(a,b)} \frac{\tilde{g}_{Wab} g_{\chi^\pm}}{256\pi^5 E E'} \int \frac{d^3k}{k} f(k) \int \frac{d^3k'}{k'} |\bar{M}|^2_{ab}\, \delta\left(E'+k'-E-k\right)$$
$$\left[\left(f_{\chi^\pm}(p) e^{\frac{\Delta m_\chi}{T}} - f_{\chi^0}(p) \right) - \left(\frac{\Delta m_\chi v^2}{2T} f_{\chi^\pm}(p) + \frac{\mathbf{q}\cdot\mathbf{v}}{v} \frac{df_{\chi^\pm}}{dp} + \frac{\mathbf{q}\cdot\mathbf{v}}{T} f_{\chi^\pm}(p) \right) e^{\frac{\Delta m_\chi}{T}} \right.$$
$$+ \left(\frac{q^2}{2m_\chi T} f_{\chi^\pm}(p) + \frac{(\mathbf{q}\cdot\mathbf{v})(\mathbf{q}\cdot\mathbf{v})}{2T^2} f_{\chi^\pm}(p) + \frac{(\mathbf{q}\cdot\mathbf{v})(\mathbf{q}\cdot\mathbf{v})}{vT} \frac{df_{\chi^\pm}}{dp} + \frac{(\mathbf{q}\cdot\mathbf{v})(\mathbf{q}\cdot\mathbf{v})}{2v^2} \Delta_{\mathbf{p}} f_{\chi^\pm} \right) e^{\frac{\Delta m_\chi}{T}}$$
$$\left. + \frac{1}{2} \left(\frac{q^2}{v} - \frac{3(\mathbf{q}\cdot\mathbf{v})(\mathbf{q}\cdot\mathbf{v})}{v^3} \right) \frac{df_{\chi^\pm}}{dp} e^{\frac{\Delta m_\chi}{T}} \right]$$

$$
\begin{aligned}
\frac{\hat{\mathbf{C}}_{\chi_0,\mathrm{is}}\left[f_{\chi^0},f_{\chi^\pm}\right]}{E} &= \sum \frac{2G_{\mathrm{F}}^2\tilde{g}_{Wab}g_{\chi_\pm}}{\pi^3}\left\{\left[4T^3\left(\Delta m_\chi^2+6\Delta m_\chi T+12T^2\right)\left(1+\frac{\Delta m_\chi}{m_\chi}\right)\right.\right.\\
&-2\Delta m_\chi^2T^2\left(\Delta m_\chi+2T\right)\frac{p}{m_\chi}+\frac{2}{3}T\left(\Delta m_\chi^4+3\Delta m_\chi^3T+32\Delta m_\chi^2T^2+114\Delta m_\chi T^3+144T^4\right)\left.\frac{p^2}{m_\chi^2}\right]\left(f_{\chi^\pm}-f_{\chi^0}e^{-\frac{\Delta m_\chi}{T}}\right)\\
&-\frac{8}{3}\Delta m_\chi T^3\left(\Delta m_\chi^2+6\Delta m_\chi T+12T^2\right)\left(\frac{p^2}{Tm_\chi^2}f_{\chi_\pm}+\frac{\mathbf{p}\cdot\nabla_{\mathbf{p}}f_{\chi^\pm}}{m_\chi}\right)\\
&+\frac{2}{3}T^3\left(\Delta m_\chi^4+10\Delta m_\chi^3T+60\Delta m_\chi^2T^2+240\Delta m_\chi T^3+480T^4\right)\left(\Delta_{\mathbf{p}}f_{\chi^\pm}+\frac{\mathbf{p}\cdot\nabla_{\mathbf{p}}f_{\chi^\pm}}{m_\chi T}+\frac{3}{m_\chi T}f_{\chi^\pm}\right)\\
&\left.-2T^2\Delta m_\chi\left(\Delta m_\chi^2+6\Delta m_\chi T+12T^2\right)\frac{p^2}{m_\chi^2}f_{\chi^\pm}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\hat{\mathbf{C}}_{\chi^\pm,\mathrm{is}}\left[f_{\chi^0},f_{\chi^\pm}\right]}{E} &= \sum \frac{2G_{\mathrm{F}}^2\tilde{g}_{Wab}g_{\chi^0}}{\pi^3}\left\{\left[4T^3\left(\Delta m_\chi^2+6\Delta m_\chi T+12T^2\right)\left(1-\frac{\Delta m_\chi}{m_\chi}\right)\right.\right.\\
&-2\Delta m_\chi^2T^2\left(\Delta m_\chi+2T\right)\frac{p}{m_\chi}+\frac{2}{3}T\left(\Delta m_\chi^4-4\Delta m_\chi^3T-10\Delta m_\chi^2T^2+30\Delta m_\chi T^3+144T^4\right)\left.\frac{p^2}{m_\chi^2}\right]\left(f_{\chi^0}e^{-\frac{\Delta m_\chi}{T}}-f_{\chi^\pm}\right)\\
&+\frac{8}{3}\Delta m_\chi T^3\left(\Delta m_\chi^2+6\Delta m_\chi T+12T^2\right)\left(\frac{p^2}{Tm_\chi^2}f_{\chi^0}+\frac{\mathbf{p}\cdot\nabla_{\mathbf{p}}f_{\chi^0}}{m_\chi}\right)e^{-\frac{\Delta m_\chi}{T}}\\
&+\frac{2}{3}T^3\left(\Delta m_\chi^4+10\Delta m_\chi^3T+60\Delta m_\chi^2T^2+240\Delta m_\chi T^3+480T^4\right)\left(\Delta_{\mathbf{p}}f_{\chi^0}+\frac{\mathbf{p}\cdot\nabla_{\mathbf{p}}f_{\chi^0}}{m_\chi T}+\frac{3}{m_\chi T}f_{\chi^0}\right)e^{-\frac{\Delta m_\chi}{T}}\\
&\left.+2T^2\Delta m_\chi\left(\Delta m_\chi^2+6\Delta m_\chi T+12T^2\right)\frac{p^2}{m_\chi^2}f_{\chi^0}e^{-\frac{\Delta m_\chi}{T}}\right\}
\end{aligned}
$$