Max-Planck-Institut für Kernphysik

Radiative Symmetry Breaking of the Minimal Left-Right Symmetric Model

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with

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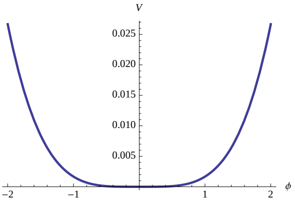
The Coleman-Weinberg Mechanism

- Question: What is the nature of massless scalar QED?
  - $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\varphi)^* D^\mu\varphi - \frac{\lambda}{6}|\varphi|^4$
- Answer(CW '73):

  ``Massless scalar electrodynamics does not stay massless, nor does it remain electrodynamics; both the scalar meson and the photon acquire a mass as a result of radiative corrections''

spontaneous symmetry breaking by radiative corrections

$V(\varphi) = \frac{\lambda}{6}|\varphi|^4$



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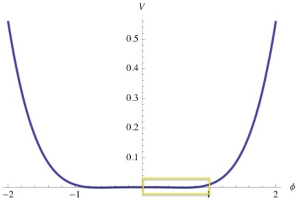

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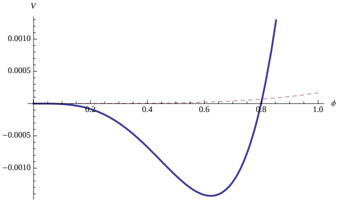

The Coleman-Weinberg Mechanism

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- breakdown of conformal symmetry $\rightarrow$ *conformal anomaly* due to the introduction of mass scale in regularization
- if $\lambda(\mu) \gtrsim e^2(\mu)$: minimum requires cancellation between tree-level and one-loop term $\rightarrow$ spurious minimum @ $\lambda \ln \frac{\varphi^2}{M\mu^2} = -\frac{32}{3}\pi^2 + \mathcal{O}(\lambda)$
- if $\lambda(\mu) \sim e^4(\mu)$: one-loop potential trustworthy at minimum, minimum condition yields $0 = \frac{\partial}{\partial\varphi}V_{eff}(\varphi = \langle\varphi\rangle) = \left(\frac{\lambda(\langle\varphi\rangle)}{6} - \frac{11e^4(\langle\varphi\rangle)}{16\pi^2}\right)\langle\varphi\rangle^3 \rightarrow$ *dimensional transmutation*
- scalar gets one-loop mass: $m_S^2 = \frac{3e^4}{4\pi^2}\langle\varphi\rangle^2$
- $\lambda(\mu) \sim e^4(\mu)$ can always be achieved by small change of RG scale

[see also M. Sher, Phys. Rept. 179 (1989), 273-418.]

CW beyond the SM

Radiative symmetry breaking attractive possibility,

but it does not work in the Standard Model:

- $V = \frac{1}{4}\lambda\phi^4 + \frac{1}{64\pi^2}\left(\frac{3}{16}\left(3g^4 + 2g^2g'^2 + g'^4\right) - 3y_t^4\right)\ln\frac{\phi^2}{M^2}$
  large top mass gives large negative contribution to effective potential $\rightarrow$ potential destabilized
- Higgs mass calculable: $m_H \lesssim 10\,GeV$ (for $m_t < m_Z$)

SM has to be extended.

To overcome the large negative contribution by the top quark, one needs a boson with considerable coupling to the Higgs. Several possibilities have been discussed in the literature:

- adding a singlet scalar[Hempfling 1996;Meissner, Nicolai(MN) 2007; Espinosa,Quiros 2007; Chang, Ng, Wu, 2007; Foot,Kobakhidze, Volkas, 2007; Foot, Kobakhidze, McDonald, Volkas,2007;Hambye, Tytgat, 2007; Meissner, Nicolai 2008, Pilaftsis 2010]
- extended gauge symmetries, extra U(1) factors[Iso, Okada, Orikasa, 2009]

CW beyond the SM

In gauge or singlet scalar extensions of the SM, a technical difficulty arises due to the multi-dimensional Higgs sector.

- even for the minimal extension of a real singlet scalar field, $V = \frac{\lambda_1}{4}(\Phi^\dagger\Phi)^2 + \frac{\lambda_2}{2}\varphi^2\Phi^\dagger\Phi + \frac{\lambda_3}{4}\varphi^4$, the effective potential quickly becomes cumbersome to deal with

$$V^{(1)}_{\mathrm{eff,s}} = \frac{1}{64\pi^2}\left[3\left(\lambda_1\Phi^2+\lambda_2\varphi^2\right)^2\left(\ln\frac{\lambda_1\Phi^2+\lambda_2\varphi^2}{\overline{\mu}^2}\right) + F_+^2\left(\ln\frac{F_+}{\overline{\mu}^2}\right) + F_-^2\left(\ln\frac{F_-}{\overline{\mu}^2}\right)\right]$$
$$-\frac{6}{32\pi^2}g_t^4(\Phi^\dagger\Phi)^2\ln\frac{\Phi^\dagger\Phi}{\overline{\mu}^2} - \frac{1}{32\pi^2}g_M^4\varphi^4\ln\frac{\varphi^2}{\overline{\mu}^2}\ .$$

with $F_\pm =$

$$\frac{1}{4}\left[(3\lambda_1+\lambda_2)\,\Phi^\dagger\Phi + (3\lambda_2+\lambda_2)\,\varphi^2 \pm \sqrt{\left((3\lambda_1-\lambda_2)\,\Phi^\dagger\Phi - (3\lambda_1-\lambda_2)\,\varphi^2\right)^2 + 16\lambda_2^2\Phi^2\varphi^2}\right]$$

[MN 07]

- two approaches:
  - numerical minimization of one-loop potential
  - use the renormalization group to get back to an analytical treatment(Gildener-Weinberg)

Method of Gildener and Weinberg(1976)

- Key observation: One-loop effect only relevant where tree-level contribution is small ($\lambda \sim e^4$)
- Use Renormalization Group to enforce a single condition on generic potential $V_0 = \frac{1}{24} f_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l$:

GW condition

$$\min_{N_i N_i = 1} \left(f_{ijkl}(\mu_{GW}) N_i N_j N_k N_l \right)\Big|_{N_i = n_i} = 0.$$

at the scale μ_{GW}, the scalar potential has a tree level *flat direction* $\Phi_i = n_i \varphi$.

- If the scalar couplings in all other directions in field space are sufficiently large $\mathcal{O}\left(e^2\right) \to$ back to the CW potential in flat direction

$$\delta V(n\varphi) = A\varphi^4 + B\varphi^4 \ln \frac{\varphi^2}{\mu_{GW}^2}$$

with $A = \frac{1}{64\pi^2 \langle\varphi\rangle^4} \sum_i n_i M_i^4(n\langle\varphi\rangle) \left(\ln \frac{M^2(n\langle\varphi\rangle)}{\langle\varphi\rangle^2} - c_i \right)$, $B = \frac{1}{64\pi^2 \langle\varphi\rangle^4} \sum_i n_i M_i^4(n\langle\varphi\rangle)$

Method of Gildener and Weinberg(1976)

- GW condition

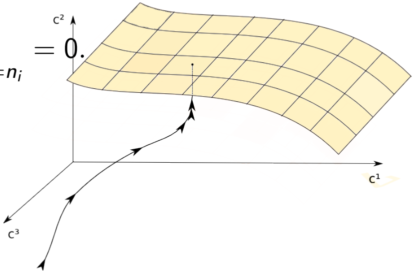
$$\min_{N_i N_i=1} \left(f_{ijkl}(\mu_{GW}) N_i N_j N_k N_l\right)\Big|_{N_i=n_i} = 0.$$

defines hypersurface in space of couplings

- symmetry breaking occurs once condition is fulfilled

$$\ln \frac{\langle\varphi\rangle^2}{\mu_{GW}^2} = -\frac{1}{2} - \frac{A}{B}.$$

- light scalar in flat direction
$m_S^2 = \frac{1}{8\pi^2\langle\varphi\rangle^2}\left(\mathrm{tr}M_S^4 + 3\mathrm{tr}M_V^4 - 4\mathrm{tr}M_D^4\right)$



- CHECK:
  - can flat direction be reached?
  - can large hierarchy be obtained?

The Minimal Left-Right Symmetric Model

CW does not work in the SM, it has to be extended. We consider the minimal left-right symmetric model based on the gauge group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$

- Restoration of Parity $Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \overset{P}{\leftrightarrow} Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$
- Natural Neutrino Masses $\nu_R \to L_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$
- Hypercharges explained: $Y = T_R^3 + \frac{1}{2}(B-L)$

$$\begin{array}{c} \textit{Planck Scale Physics} \\ \Downarrow \\ \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times P \\ \Downarrow \langle \chi_R \rangle \sim \text{few TeV} \\ \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \\ \Downarrow \ \langle H \rangle = v/\sqrt{2} \\ \mathrm{U}(1)_{em} \end{array}$$

Scalar Sector of the Minimal LR Model

LR Breaking

- $\chi_R \sim (\mathbf{1}, \mathbf{2}, -1)$, $\langle \chi_R \rangle = \begin{pmatrix} v_R \\ 0 \end{pmatrix}$ conserves $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- $\chi_R \xrightarrow{P} \chi_L \sim (\mathbf{2}, \mathbf{1}, -1)$ dictated by parity, $\langle \chi_L \rangle = \begin{pmatrix} v_L \\ 0 \end{pmatrix}$

Electroweak Symmetry Breaking

- in principle χ_L has the right quantum numbers for EWSB, but Yukawa couplings are not possible as $\overline{Q_L} Q_R \sim (\mathbf{2}, \mathbf{2}, 0)$
- introduce bidoublet $\phi \sim (\mathbf{2}, \mathbf{2}, 0)$,$\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$, which allows

$$\mathcal{L} = \overline{Q_L} \phi Q_R + \overline{Q_L} \tilde{\phi} Q_R$$

with $\tilde{\phi} = \tau_2 \phi^* \tau_2$. Under Parity $\phi \to \phi^\dagger$.

Radiative Left Right Symmetry Breaking

Full Potential cannot be easily analyzed even on tree level

$$\mathcal{V}(\Phi,\Psi) = \frac{\kappa_1}{2}\left(\overline{\Psi}\Psi\right)^2 + \frac{\kappa_2}{2}\left(\overline{\Psi}\Gamma\Psi\right)^2 +$$
$$+\lambda_1\left(\mathrm{tr}\Phi^\dagger\Phi\right)^2 + \lambda_2\left(\mathrm{tr}\Phi\Phi + \mathrm{tr}\Phi^\dagger\Phi^\dagger\right)^2$$
$$+\lambda_3\left(\mathrm{tr}\Phi\Phi - \mathrm{tr}\Phi^\dagger\Phi^\dagger\right)^2 + \lambda_4\left(\mathrm{tr}\Phi\Phi^\dagger\right)\left(\mathrm{tr}\Phi\Phi + \mathrm{tr}\Phi^\dagger\Phi^\dagger\right)$$
$$+\beta_1\overline{\Psi}\Psi\mathrm{tr}\Phi^\dagger\Phi + \beta_2\left(\mathrm{tr}\Phi\Phi + \mathrm{tr}\Phi^\dagger\Phi^\dagger\right)\overline{\Psi}\Psi$$
$$+\mathrm{i}\beta_3\left(\mathrm{tr}\Phi\Phi - \mathrm{tr}\Phi^\dagger\Phi^\dagger\right)\overline{\Psi}\Gamma\Psi + f_1\overline{\Psi}\Gamma[\Phi^\dagger,\Phi]\Psi$$

- $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \sim \mathrm{SO}(4)$: $\Psi = \begin{pmatrix} \chi_L \\ -\mathrm{i}\chi_R \end{pmatrix}$, $\Phi = \begin{pmatrix} 0 & \phi \\ -\tilde{\phi}^\dagger & 0 \end{pmatrix}$, $\Gamma = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$
- Analysis of the full one-loop potential *can only be done numerically*.
- we used the method of Gildener and Weinberg to get back to an analytical treatment.

Flat Directions of LR Potential

- LR breaking due to potential $\mathcal{V} = \frac{\kappa_1}{2}\left(\overline{\Psi}\Psi\right)^2 + \frac{\kappa_2}{2}\left(\overline{\Psi}\Gamma\Psi\right)^2$ in unitary gauge

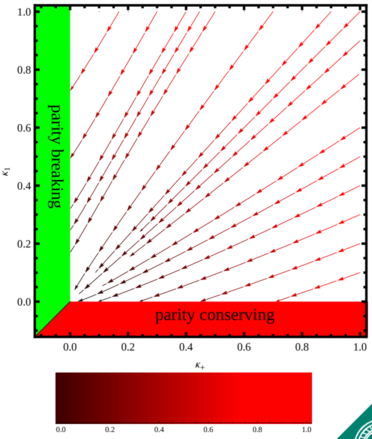
$$\chi_{L/R} = \begin{pmatrix} n_{1/2} \\ 0 \end{pmatrix} \varphi$$

GW condition

$$\min_{n_1^2 \leq 1} \frac{1}{8}\varphi^4 \left(\kappa_1 + \kappa_2(-1+2n_1^2)^2\right) = 0$$

- Flat Directions:
  - $\kappa_2 > 0$: $n_1 = n_2 = \frac{1}{\sqrt{2}}$, $\kappa_1 = 0$ $\langle\chi_L\rangle = \langle\chi_R\rangle \to P\checkmark$
  - $\kappa_2 < 0$: $n_1 = 0$, $n_2 = 1$, $\kappa_+ = \kappa_1 + \kappa_2 = 0$ $\langle\chi_L\rangle = 0 \to \not{P}$
- large hierarchy can be created

RG evolution (no gauge contribution)



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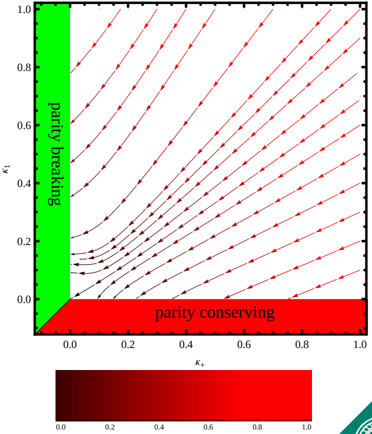

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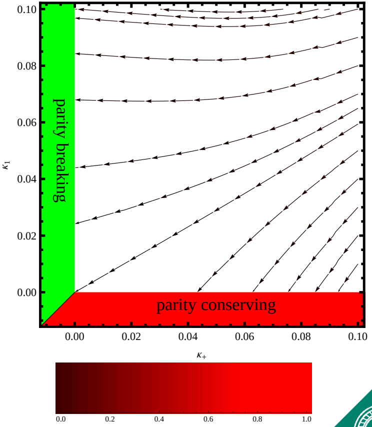
- large hierarchy can be created

RG evolution (gauge contribution dominates)



Combined LR+EW Symmetry Breaking

All flat directions can be given analytically for the simplified potential

$$\mathcal{V}(\Phi,\Psi) = \frac{\kappa_1}{2}\left(\overline{\Psi}\Psi\right)^2 + \frac{\kappa_2}{2}\left(\overline{\Psi}\Gamma\Psi\right)^2 + \lambda_1\left(\mathrm{tr}\Phi^\dagger\Phi\right)^2 + \lambda_2\left(\mathrm{tr}\Phi\Phi + \mathrm{tr}\Phi^\dagger\Phi^\dagger\right)^2$$
$$+ \lambda_3\left(\mathrm{tr}\Phi\Phi - \mathrm{tr}\Phi^\dagger\Phi^\dagger\right)^2 + \beta_1\overline{\Psi}\Psi\mathrm{tr}\Phi^\dagger\Phi + f_1\overline{\Psi}\Gamma[\Phi^\dagger,\Phi]\Psi$$

which respects the $\mathbb{Z}_4$ symmetry $\Phi \to \mathrm{i}\Phi$.

- $\mathbb{Z}_4$ potential sufficiently simple, flat directions can be analytically obtained
- has to be broken by down-type Yukawas,
- can view it as approximate symmetry of scalar potential

Flat Directions of $\mathbb{Z}_4$ Potential

$$\langle \chi_{L/R} \rangle = \begin{pmatrix} v_{L/R} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} n_{1/2} \\ 0 \end{pmatrix} \langle \varphi \rangle, \quad \langle \phi \rangle \quad = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{\mathrm{i}\alpha} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} n_3 & 0 \\ 0 & n_4 e^{\mathrm{i}\alpha} \end{pmatrix} \langle \varphi \rangle .$$

	GW condition	$\frac{n_1^2}{n_2^2}$	$n_1^2 + n_2^2$	$\frac{n_3^2}{n_4^2}$	$n_3^2 + n_4^2$	α	$\frac{n_3^2+n_4^2}{n_1^2+n_2^2}$
$\mathrm{Ia}_{\not P}$ Ia_{P}	$\left.\begin{array}{c}\kappa_+ \\ \kappa_1\end{array}\right\} = \frac{\beta_1^2}{2(\lambda_1+4\lambda_2)} - \frac{f_1^2}{32\lambda_2}$	0 1	$\frac{2(\lambda_1+4\lambda_2)}{2\lambda_1+8\lambda_2-\beta_1}$	$\frac{4(2\beta_1+f_1)\lambda_2+f_1\lambda_1}{4(2\beta_1-f_1)\lambda_2-f_1\lambda_1}$	$\frac{-\beta_1}{2\lambda_1+8\lambda_2-\beta_1}$	0	$\frac{-\beta_1}{2\lambda_1+8\lambda_2}$
$\mathrm{Ib}_{\not P}$ Ib_{P}	$\left.\begin{array}{c}\kappa_+ \\ \kappa_1\end{array}\right\} = \frac{\beta_1^2}{2(\lambda_1-4\lambda_3)} + \frac{f_1^2}{32\lambda_3}$	0 1	$\frac{2(\lambda_1-4\lambda_3)}{2\lambda_1-8\lambda_3-\beta_1}$	$\frac{4(2\beta_1+f_1)\lambda_3-f_1\lambda_1}{4(2\beta_1-f_1)\lambda_3+f_1\lambda_1}$	$\frac{-\beta_1}{2\lambda_1-8\lambda_3-\beta_1}$	$\frac{\pi}{2}$	$\frac{-\beta_1}{2\lambda_1-8\lambda_3}$
Ic	$\lambda_1 = -4\lambda_2$	$\frac{0}{0}$	0	1	1	0	∞
Id	$\lambda_1 = 4\lambda_3$	$\frac{0}{0}$	0	1	1	$\frac{\pi}{2}$	∞
$\mathrm{IIa}_{\not P}$ IIa_{P}	$\left.\begin{array}{c}\kappa_+ \\ \kappa_1\end{array}\right\} = \frac{(2\beta_1-f_1)^2}{8\lambda_1}$	0 1	$\frac{4\lambda_1}{-2\beta_1+f_1+4\lambda_1}$	∞	$\frac{2\beta_1-f_1}{2\beta_1-f_1-4\lambda_1}$	–	$\frac{f_1-2\beta_1}{4\lambda_1}$
$\mathrm{IIb}_{\not P}$ IIb_{P}	$\left.\begin{array}{c}\kappa_+ \\ \kappa_1\end{array}\right\} = \frac{(2\beta_1+f_1)^2}{8\lambda_1}$	0 1	$\frac{4\lambda_1}{-2\beta_1-f_1+4\lambda_1}$	0	$\frac{2\beta_1+f_1}{2\beta_1+f_1-4\lambda_1}$	–	$\frac{-f_1-2\beta_1}{4\lambda_1}$
$\mathrm{IIc}_{\not P}$ IIc_{P}	$\left.\begin{array}{c}\kappa_+ \\ \kappa_1\end{array}\right\} = 0$	0 1	1	$\frac{0}{0}$	0	–	0
IId	$\lambda_1 = 0$	$\frac{0}{0}$	0	0	1	–	∞
IIe	$\lambda_1 = 0$	$\frac{0}{0}$	0	∞	1	–	∞

GW: $\frac{\partial V}{\partial n_i} = 0$, $\sum_{i=1}^4 n_i^2 = 1$. Solutions of type Ia and Ib are related by $(\alpha, \lambda_2, \lambda_3) \to (\pi - \alpha, -\lambda_3, -\lambda_2)$, IIa and IIb are related by $(n_3^2, n_4^2, f_1) \to (n_4^2, n_3^2, -f_1)$

Flat Directions of $\mathbb{Z}_4$ Potential

The phenomenologically favourable flat direction requires

$$\boxed{\mathrm{IIa}_{\not{P}}: \ \kappa_+ = \frac{(f_1 - 2\beta_1)^2}{8\lambda_1}}$$

and gives $v_L = \kappa' = 0$ and

$$\frac{\kappa^2}{v_R^2} = \frac{f_1 - 2\beta_1}{4\lambda_1}$$

- no additional fine-tuning needed apart from usual $\kappa^2 \ll v_R^2$ common to LR potentials [Despande et al., 1990]

Can this GW conditon be reached by the RG flow?

Renormalization Group Evolution of Scalar Potential

$$\beta_{\beta_1} = \frac{1}{256\pi^2}\left[-4\beta_1\left(-8\beta_1 + 6g_1^2 + 27g_2^2 - 2(20\kappa_1 + 4\kappa_2 + 40\lambda_1 + 32\lambda_2 - 32\lambda_3 + T_2)\right) + 24f_1^2 + 9g_2^4\right]$$

$$\beta_{f_1} = \frac{f_1}{64\pi^2}\left[16\beta_1 - 6g_1^2 - 27g_2^2 + 8\kappa_1 + 8\kappa_2 + 16(\lambda_1 - 4\lambda_2) + 64\lambda_3 + 2T_2\right]$$

$$\beta_{\kappa_1} = \frac{1}{512\pi^2}\left[\kappa_1\left(-96g_1^2 - 144g_2^2 + 576\kappa_1 + 384\kappa_2\right) + 192\kappa_2^2 + 256\beta_1^2 + 128f_1^2 + 24g_1^4 + 12g_1^2g_2^2 + 9g_2^4\right]$$

$$\beta_{\kappa_2} = \frac{1}{512\pi^2}\left[\kappa_2\left(-96g_1^2 - 144g_2^2 + 512\kappa_1 + 384\kappa_2\right) + 128f_1^2 + 12g_1^2g_2^2 + 9g_2^4\right]$$

$$\beta_{\lambda_1} = \frac{1}{128\pi^2}\left[\lambda_1\left(-72g_2^2 + 256\,(\lambda_1 + \lambda_2 - \lambda_3) + 8T_2\right) + 1024\left(\lambda_2^2 + \lambda_3^2\right) + 32\beta_1^2 + 8f_1^2 + 9g_2^4 - 4T_4\right]$$

$$\beta_{\lambda_2} = \frac{1}{512\pi^2}\left[\lambda_2\left(-288g_2^2 + 768\lambda_1 + 3072\lambda_2 + 1024\lambda_3 + 32T_2\right) - 8f_1^2 + 3g_2^4 + 2T_4\right]$$

$$\beta_{\lambda_3} = \frac{1}{256\pi^2}\left[\lambda_3(-144g_2^2 + 384\lambda_1 - 512\lambda_2 - 1536\lambda_3 + 16T_2) + 4f_1^2 - 3g_2^4 - T_4\right]$$

$$\beta_{Y_{\mathbb{L}}^-} = \frac{1}{64\pi^2}\left[(-6g_1^2 - 9g_2^2)Y_{\mathbb{L}}^- + Y_{\mathbb{L}}^- T_2 + 4Y_{\mathbb{L}}^{-\,3}\right]$$

$$\beta_{Y_{\mathbb{Q}}^+} = \frac{1}{64\pi^2}\left[(-\frac{2}{9}g_1^2 - 9g_2^2 - 32g_3^2)Y_{\mathbb{Q}}^+ + Y_{\mathbb{Q}}^+ T_2 + 4Y_{\mathbb{Q}}^{+\,3}\right]$$

where we have used $T_2 = \mathrm{tr}(Y_{\mathbb{L}}^{-\,2} + 3Y_{\mathbb{Q}}^{+\,2})$ $\;T_4 = \mathrm{tr}(Y_{\mathbb{L}}^{-\,4} + 3Y_{\mathbb{Q}}^{+\,4})$

Flat Directions of Full Potential

Phenomenologically preferred flat direction

$IIa_{\not P}$: $v_L = \kappa' = 0$; $\quad \frac{\kappa^2}{v_R^2} = \frac{f_1 - 2\beta_1}{4\lambda_1}$

There is a set of couplings that survives up to the Planck scale:

@ TeV scale

$m_h = 120\,\mathrm{GeV}\,,$

$v_R = 7\,\mathrm{TeV}\,,$

$\lambda_2 = -\lambda_3 = 0.001\,,$

$\kappa_1 = 0.2\,,\ \ \beta_1 = 0.01\ \Rightarrow$

$\lambda_1 \approx 0.119\,,\ \ \kappa_2 \approx -0.200$

and $f_1 = 2.16 \cdot 10^{-2}$

$\Longrightarrow$

@ Planck scale

$\lambda_1 \approx 0.204$, $\lambda_2 \approx 0.00494$,

$\lambda_3 \approx -0.0120$, $\kappa_1 \approx 0.336$,

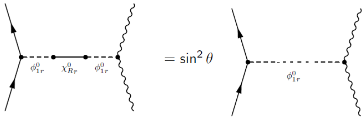
$\kappa_2 \approx -0.100$, $\beta_1 \approx 0.031$,

$f_1 \approx 0.0118$

Generally one finds $m_h < 180\,\mathrm{GeV}$ and $m_i < v_R$ for the other scalars, since large scalar couplings generally lead to Landau poles before the Planck scale.

Some Phenomenology

- gauge boson masses within LHC reach: $M_{Z_R} = 3828\,\mathrm{GeV}$, $M_{W_R^\pm} = 3168\,\mathrm{GeV}$
- phenomenology as in usual low scale LR models[talk today at 18:30 by Nemevsek], radiative symmetry breaking gives:
- light scalar spectrum: scalon $m_S = 690\,\mathrm{GeV}$ & Higgs $m_h = 120\,\mathrm{GeV}$
- $s = n_2\ \chi^0_{R\,r} + n_3\ \phi^0_{1\,r} = \cos\theta\ \chi^0_{R\,r} + \sin\theta\ \phi^0_{1\,r}$



- Angle can also be obtained from $\tan\theta = \frac{\kappa}{v_R} \approx \frac{m(W_L^\pm)}{m(W_R^\pm)}$.
- advantage over singlet extensions

Conformal Invariance as a possible Solution to the Hierarchy Problem ?

- Higgs mass term μ^2 is the only dimensionful parameter in the SM, $\mu^2 = 0 \rightarrow$ SM conformally invariant at tree-level
- classical conformal invariance as reason for existence of small mass scales in nature? [K.A. Meissner, H. Nicolai(MN),Phys.Lett.B648:312-317,2007]
  - logarithmic running of dimensionless couplings breaks scale invariance (*conformal anomaly*)
  - quadratic divergences break the classical conformal symmetry in 'hard way' and are a spurious result of using the wrong regulator[See also: W. Bardeen, FERMILAB-CONF-95-391-T, FERMILAB-CONF-95-377-T]
  - dimensional regularization should be used, as it breaks conformal symmetry in the least possible way [MN, Phys.Lett.B660:260-266,2008]
  - $v^2 \ll M_{Pl}^2$ can be naturally explained by logarithmic running
- BUT: requires direct embedding into gravity:
  - no intermediate scales(GUTs, see-saw etc.)
  - requires a finite theory of quantum gravity, toy example has been studied [MN,Phys.Rev.D80:086005,2009]

Conclusions

- Conformal Symmetry might be an economical alternative to low-energy SUSY in addressing the hierarchy problem, but
  - origin of conformal symmetry out of a non-conformal theory of gravity?
  - generation of hierarchy well explained by logarithmic running but
  - stabilization of hierarchy relies on dynamics of quantum gravity
- Coleman-Weinberg breaking of left-right symmetry is possible and can be falsified by
  - survivability of theory up to the Planck scale
  - measuring mass and mixing of *scalon*
- Little Hierarchy between LR and EW scales is not explained

Backup Slides

Low Scale LR Symmetry Breaking & FCNCs

- Fermion masses are generated by

$$\mathcal{L}_{\rm Yuk} = -Y_u^{+\,ij}\overline{Q_L}_i\phi Q_{Rj} - Y_d^{-\,ij}\overline{Q_L}_i\tilde{\phi} Q_{Rj} + \text{h.c.}$$

which give $M_u = \frac{\kappa}{\sqrt{2}}Y_u^+$ and $M_d = \frac{\kappa}{\sqrt{2}}Y_d^-$ with $Q_R \stackrel{P}{\rightarrow} -\mathrm{i}Q_R$.

- the bidoublet contains two neutral scalars
$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$ of which one, ϕ_2^0 , has non-diagonal couplings in the mass basis
$\mathcal{L}_{FCNC} = \overline{D}_L V^\dagger Y_u^{diag} V D_R \phi_2^0 + \text{h.c.}$,
- this leads to flavour changing neutral currents on tree level

$$\mathcal{L}_{\Delta S=2} = \frac{1}{M^2\kappa^2}\left(\sum_{j=u,c,t} V_{jd}^* m_j V_{js}\right)^2 \left[(\overline{d}\gamma_5 s)^2 - (\overline{d}s)^2\right].$$

- constraint $M = m(\phi_{2r,i}^0) \approx \sqrt{f_1}v_R > 15\,\mathrm{TeV}$

Low Scale LR Symmetry Breaking & FCNCs

- Fermion masses are generated by

$$\mathcal{L}_{\rm Yuk} = -Y_u^{+\,ij}\overline{Q_L}_i \phi Q_{Rj} - Y_d^{-\,ij}\overline{Q_L}_i \tilde{\phi} Q_{Rj} + \text{h.c.}$$

  which give $M_u = \frac{\kappa}{\sqrt{2}} Y_u^+$ and $M_d = \frac{\kappa}{\sqrt{2}} Y_d^-$ with $Q_R \to -\mathrm{i} Q_R$.

- the bidoublet contains two neutral scalars of which one, ϕ_2^0 , has non-diagonal couplings in the mass basis $\mathcal{L}_{FCNC} = \overline{D}_L V^\dagger Y_u^{diag} V D_R \phi_2^0 + \text{h.c.}$,
- FCNC constraint $M = m(\phi^0_{2r,i}) \approx \sqrt{f_1} v_R > 15\,\mathrm{TeV}$
- problem completely independent of radiative breaking mechanism, VEV configuration of the bidoublet
- one way out flavour symmetry: $-\mathcal{L}_t = \lambda_t \left(\overline{Q}_{L,1} + \overline{Q}_{L,2} + \overline{Q}_{L,3}\right) \phi \left(Q_{R,1} + Q_{R,2} + Q_{R,3}\right)$
- other model-building possibilities are discussed in Mohapatra, Guadolni 2010: 1008.1074

Flat Directions of $\mathbb{Z}_4$ Potential

In unitary gauge and assuming that electric charge remains unbroken, the flat directions can be parametrized as

$$\langle\Psi\rangle = \begin{pmatrix} v_L e^{i\theta} \\ 0 \\ v_R \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} n_1 e^{i\theta} \\ 0 \\ n_2 \\ 0 \end{pmatrix} \langle\phi\rangle$$

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} n_3 & 0 \\ 0 & n_4 e^{i\alpha} \end{pmatrix} \langle\phi\rangle.$$

The Gildener-Weinberg prescription then amounts to finding the minimum of classical potential on unit sphere $\sum_{i=1}^{4} n_i^2 = 1$ and then use RGEs to set potential to zero at this point.

Classification of Flat Directions of $\mathbb{Z}_4$ Potential

The various solutions can be classified by

- $$0 = \frac{\partial \mathcal{V}}{\partial \alpha}\Big|_{N_i=n_i} = -8 n_3^2 n_4^2 \sin\alpha\cos\alpha$$

  which implies either

  - $\alpha = 0, \pi$ ('type I') or
  - $\kappa = 0$ or $\kappa' = 0$ ('type II')

- if we assume $n_1 \neq 0 \neq n_2$

  $$0 = \frac{1}{n_1}\frac{\partial \mathcal{V}_D}{\partial N_1}\Big|_{N_i=n_i} - \frac{1}{n_2}\frac{\partial \mathcal{V}_D}{\partial N_2}\Big|_{N_i=n_i} = \kappa_2\left(n_1^2 - n_2^2\right)$$

  for $\kappa_2 \neq 0$, we have either

  - $v_L = 0$ ('$\not{P}$') or
  - $v_R^2 = v_L^2$ ('P').