# Minimal Flavour Violation with hierarchical squark masses

#### M. Farina

(Scuola Normale Superiore)



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Based on arXiv:1011.0730 [hep-ph] with R. Barbieri, E. Bertuzzo, P. Lodone and D. Zhuridov

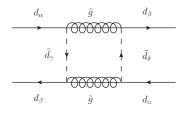
# What we are talking about

$$\mathcal{L} = \frac{g}{\sqrt{2}} W_{\mu}^{+} \bar{u} V \gamma^{\mu} d + h.c.$$

$$d_{\alpha}$$
  $u_{\gamma}$   $d_{\beta}$   $W$   $W$   $d_{\beta}$   $d_{\beta}$   $d_{\beta}$   $d_{\beta}$   $d_{\beta}$ 

$$\mathcal{A}_{\alpha\beta}^{\Delta F=2} \propto (V_{t\alpha}^* V_{t\beta})^2 \ \langle \overline{M} | (\bar{d}_{L\alpha} \gamma_{\mu} d_{L\beta})^2 | M \rangle$$







## The NP Flavour Problem

New Physics flavour effects via a generic effective-theory approach

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum \frac{c_i^{(d)}}{\Lambda^{(d-4)}} O_i^{(d)}(\text{SM fields}).$$

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Operator	Bounds on $\Lambda$ in TeV $(c_{ij}=1)$		Bounds on $c_{ij}$ ( $\Lambda=1$ TeV)		Observables
	Re	lm	Re	lm	
$(\bar{s}_L \gamma^\mu d_L)^2$	$9.8 \times 10^{2}$	$1.6 \times 10^{4}$	$9.0 \times 10^{-7}$	$3.4 \times 10^{-9}$	$\Delta m_K$ ; $\epsilon_K$
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	$1.8 \times 10^{4}$	$3.2 \times 10^{5}$	$6.9 \times 10^{-9}$	$2.6 \times 10^{-11}$	$\Delta m_K$ ; $\epsilon_K$
$(\bar{b}_L \gamma^\mu d_L)^2$	$5.1 \times 10^{2}$	$9.3 \times 10^{2}$	$3.3 \times 10^{-6}$	$1.0 \times 10^{-6}$	$\Delta m_{B_d}$ ; $S_{\psi K_S}$
$(\bar{b}_Rd_L)(\bar{b}_Ld_R)$	$1.9 \times 10^{3}$	$3.6 \times 10^{3}$	$5.6 \times 10^{-7}$	$1.7 \times 10^{-7}$	$\Delta m_{B_d}; S_{\psi K_S}$
$(\bar{b}_L \gamma^\mu s_L)^2$	$1.1 \times 10^{2}$		$7.6 \times 10^{-5}$		$\Delta m_{B_S}$
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With no Yukawa the global flavour symmetry in the quarks sector is

$$U(3)_Q \otimes U(3)_{u_R} \otimes U(3)_{d_R}$$

- Minimal Flavour Violation ⇒ flavour violation completely determined by the structure of the ordinary Yukawa couplings.
- SM like CKM suppression in leading  $\Delta F=2$  and  $\Delta F=1$  FCNC amplitudes:

$$\mathcal{A}_{\alpha\beta}^{\Delta F=2}|_{MFV} = (V_{t\alpha}^* V_{t\beta})^2 \mathcal{A}_{SM}^{(\Delta F=2)} (1 + \epsilon^{\Delta F=2}) .$$

$$\begin{array}{c|cccc} \text{Operator} & \text{Bound on } \Lambda & \text{Observables} \\ \hline \frac{1}{2} (\overline{Q}_L Y^u Y^u ^\dagger \gamma_\mu Q_L)^2 & 5.9 \, \text{TeV} & \epsilon_K, \Delta m_{B_A}, \Delta m_{B_A} \\ \end{array}$$

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In principle SUSY suffers the NP flavour problem.

There are three main solutions:

- Degeneracy
- Alignment
- Hierarchy

Naive heavy squark masses limits

Hierarchy only

$$m_{1,2} \gtrsim 500 \ TeV$$

$$m_{1.2} \gtrsim 10 - 20 \ TeV$$

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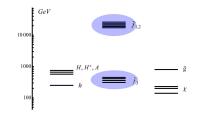
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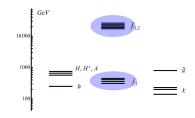
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Special role of the top Yukawa coupling

#### Blending of the three approaches

- Among the squarks, only those that interact with the Higgs system via the top Yukawa coupling are significantly lighter than the others.
- Hierarchy

- With only the up-Yukawa couplings,  $Y_u$ , turned on, but not the down-Yukawa couplings,  $Y_d$ , there is no flavour transition between the different families.
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Alignment

$$U(1)_{\tilde{B}_1} \times U(1)_{\tilde{B}_2} \times U(1)_{\tilde{B}_3} \times U(3)_{d_R}$$

- lacksquare  $Y_d$  is promoted to a non-dynamical spurion field
- $m_{\tilde{Q}}^2, m_{\tilde{u}}^2$  and the A-terms for the charge 2/3 squarks are flavour diagonal (cause of large separation corrections are negligible).
- On the other hand

$$m_{\tilde{d}_R}^2 = m^2 (\mathbf{1} + a Y_d^+ Y_d)$$

- The only other mass matrix that needs to be diagonalized is the d-quark mass matrix,
- The flavour lagrangian depends only on the CKM matrix

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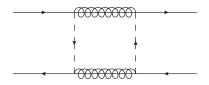
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## The effective lagrangian

• General structure of the  $\Delta F = 2$  effective Lagrangian

$$\mathcal{L}^{\Delta F=2} \propto \xi_k^{\alpha\beta} \xi_j^{\alpha\beta} f_{j,k} \; (\bar{d}_{L\alpha} \gamma_\mu d_{L\beta})^2 + h.c.,$$

where 
$$\xi_j^{\alpha\beta} = V_{j\alpha}V_{j\beta}^*$$



• Using  $\Sigma_i \xi_i^{\alpha\beta} = 0$ 

$$\mathcal{L}^{\Delta F=2} = \mathcal{L}_{33}^{\Delta F=2} + \mathcal{L}_{12}^{\Delta F=2} + \mathcal{L}_{12,3}^{\Delta F=2}$$

- Recalling what happens in MFV, if all FCNC have the form we call this effective Minimal Flavour Violation.
- This turns to

$$\begin{array}{lll} \mathcal{L}_{33}^{\Delta F=2} \leftrightarrow \xi_3^2 & \Rightarrow & \text{Effective MFV} \\ \mathcal{L}_{12}^{\Delta F=2} \leftrightarrow \xi_2^2 & \Rightarrow & \Lambda_{Re} > 9.8 \cdot 10^2 \text{ TeV} \\ \mathcal{L}_{12,3}^{\Delta F=2} \leftrightarrow \xi_2 \xi_3 & \Rightarrow & \Lambda_{Im} > 1.6 \cdot 10^4 \text{ TeV} \end{array}$$

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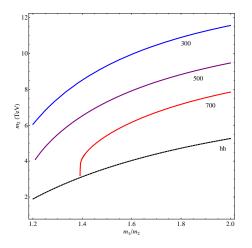
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## Results: Lower Bounds



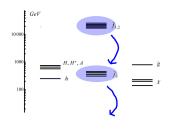
# **QCD** Running

■ The  $\mathcal{L}_{12,3}^{\Delta F=2}$  has a peculiar feature. Being sensitive to two mass scales large logs arise

$$\propto \log \frac{m_h^2}{m_\ell^2}$$

- QCD running and integrating out properly carried in two steps
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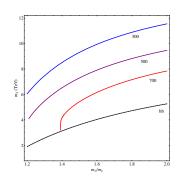
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# Summary

- Minimal Flavour Violation can be compatible with hierarchical sfermions
- Reasonable bounds on heavy squark masses in the case of interest
- Proper QCD effects calculation including a previously neglected effect. Order 20% change from naive calculation.



## QCD<sub>1</sub>

#### Usual treatment

Integrating out one obtains an effective lagrangian

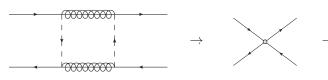
$$\mathcal{L}^{\Delta F=2} = C_1(m_h)Q_1 + h.c.$$

with 
$$Q1=Q_1=(\overline{d}^{\alpha}\gamma^{\mu}P_Ls^{\alpha})\,(\overline{d}^{\beta}\gamma_{\mu}P_Ls^{\beta})$$

QCD corrections taken into account corrections using ADM formalism

$$\frac{dC_1}{d\log\mu} = \Gamma C_1,$$

$$C_1(\mu) = U(\mu, m_h)C_1(m_h)$$





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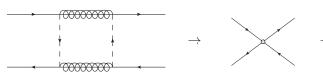
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## QCD 2

Large Logs and  $\Delta F = 1$  operators

■ The  $\mathcal{L}_{12,3}^{\Delta F=2}$  has a peculiar feature. Being sensitive to two mass scales large logs arise

$$C_1 \propto \log \frac{m_h^2}{m_\ell^2}$$

■ The new ingredient is the mixing between  $\Delta F = 2$  and new  $\Delta F = 1$  operators

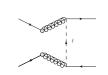


## QCD3

#### Improved running

■ The RGE for  $C_1$  now has the form

$$\frac{dC_1}{d\log\mu} = \frac{\alpha_s}{2\pi} \left( \gamma_1 C_1 + \xi_3^{ds} \hat{\gamma}_{g1} \hat{C}_g \right),$$



In conclusion

$$C_1(m_l) = \left(\frac{\alpha_s(m_l)}{\alpha_s(m_h)}\right)^{\gamma_1/b_0} C_1(m_h) + \xi_3^{ds} \hat{\gamma}_{g1} A B_D A^{-1} \hat{C}_g(m_h) ,$$

$$(B_D)_{kk} = \frac{1}{\gamma_k - \gamma_1} \left[ \left(\frac{\alpha_s(m_l)}{\alpha_s(m_h)}\right)^{\gamma_k/b_0} - \left(\frac{\alpha_s(m_l)}{\alpha_s(m_h)}\right)^{\gamma_1/b_0} \right] ,$$

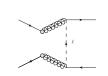
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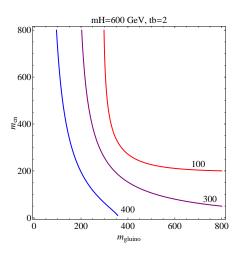
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# Light-Light Case



#### **CKM Matrix**

Wolfenstein parametrization

$$V \simeq$$

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

$$\lambda = 0.2257 \pm 0.0010, \quad A = 0.814 \pm 0.022,$$

$$\rho = +0.135^{+0.031}_{-0.016}, \quad \eta = +0.349 \pm 0.017.$$

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$$\mathcal{L}_{12,3}^{\Delta F=2} = 2(\xi_2^{\alpha\beta} \xi_3^{\alpha\beta}) (f_{3,2} - f_{3,1} + f_{1,1} - f_{1,2}) Q_1^{\alpha\beta} + h.c.$$

$$\xi_2^{ds} \xi_3^{ds} \sim \lambda^6 + i\lambda^6$$

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$$\mathcal{L}_{12,3}^{\Delta F=2} = 2(\xi_2^{\alpha\beta} \xi_3^{\alpha\beta}) (f_{3,2} - f_{3,1} + f_{1,1} - f_{1,2}) Q_1^{\alpha\beta} + h.c.$$

$$\xi_2^{ds} \xi_3^{ds} \sim \lambda^6 + i\lambda^6$$

Recalling what happens in MFV, if all FCNC have the form

$$\mathcal{A}_{\alpha\beta}^{\Delta F=2}|_{MFV}=(\xi_3^{\alpha\beta})^2\mathcal{A}_{\alpha\beta}^{\Delta F=2}|_{SM}(1+\epsilon^{\Delta F=2})$$

we call this effective Minimal Flavour Violation.

$$\mathcal{L}^{\Delta F=2} = \mathcal{L}_{33}^{\Delta F=2} + \mathcal{L}_{12}^{\Delta F=2} + \mathcal{L}_{12,3}^{\Delta F=2}$$

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 $\Delta F = 1$  case

Recalling what happens in MFV, if all FCNC have the form

$$\mathcal{A}_{\alpha\beta}^{\Delta F=1,s}|_{MFV} = (V_{t\alpha}^* V_{t\beta}) \ \mathcal{A}_{\alpha\beta}^{\Delta F=1,s}|_{SM} (1 + \epsilon^{\Delta F=1,s})$$

we call this effective Minimal Flavour Violation.

• Using  $\Sigma_i \xi_i^{\alpha\beta} = 0$ 

$$\mathcal{L}^{\Delta F=1} = \mathcal{L}_{31}^{\Delta F=1} + \mathcal{L}_{21}^{\Delta F=1}$$

$$\mathcal{L}_{31}^{\Delta F=1} = \Sigma_s \Sigma_{\alpha \neq \beta} \xi_3^{\alpha \beta} (f_3^{(s)} - f_1^{(s)}) Q_{(s)}^{\alpha \beta} + h.c.$$

$$\mathcal{L}_{21}^{\Delta F=1} = \Sigma_s \Sigma_{\alpha \neq \beta} \xi_2^{\alpha \beta} (f_2^{(s)} - f_1^{(s)}) Q_{(s)}^{\alpha \beta} + h.c.$$





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$$\xi_3^{os} \sim \lambda^2 + \imath \lambda^4$$

$$\mathcal{L}_{21}^{\Delta F=1} = \sum_{s} \sum_{\alpha \neq \beta} \xi_2^{\alpha \beta} (f_2^{(s)} - f_1^{(s)}) Q_{(s)}^{\alpha \beta} + h.c.$$



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$$\xi_2^{bs} \sim \lambda^2$$

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$$\mathcal{L}_{31}^{\Delta F=1} = \Sigma_s \Sigma_{\alpha \neq \beta} \xi_3^{\alpha \beta} \widehat{(f_3^{(s)} - f_1^{(s)})} Q_{(s)}^{\alpha \beta} + h.c., \qquad \xi_3^{bs} \sim \lambda^2 + i\lambda^4$$

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$$\mathcal{L}^{\Delta F=1} = \mathcal{L}_{31}^{\Delta F=1} + \mathcal{L}_{21}^{\Delta F=1}$$

$$\mathcal{L}_{31}^{\Delta F=1} = \Sigma_{s} \Sigma_{\alpha \neq \beta} \xi_{3}^{\alpha \beta} \widehat{(f_{3}^{(s)} - f_{1}^{(s)})} Q_{(s)}^{\alpha \beta} + h.c., \qquad \xi_{3}^{bs} \sim \lambda^{2} + i\lambda^{4}$$

$$\mathcal{L}_{21}^{\Delta F=1} = \Sigma_{s} \Sigma_{\alpha \neq \beta} \xi_{2}^{\alpha \beta} (f_{2}^{(s)} - f_{1}^{(s)}) Q_{(s)}^{\alpha \beta} + h.c. \qquad \xi_{2}^{bs} \sim \lambda^{2}$$

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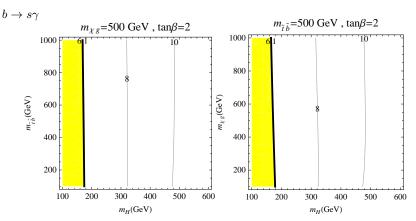
• Using  $\Sigma_i \xi_i^{\alpha\beta} = 0$ 

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Process dominated by charged Higgses exchanges. So that for

$$m_{H^+} \gtrsim 200~GeV$$

20 Aply a very weak constraint on the squarks masses is present.

What about less restrictive flavour symmetries?

$$U(1)_{\tilde{B}_{1}} \times U(1)_{\tilde{B}_{2}} \times U(1)_{\tilde{B}_{3}} \times U(1)_{d_{R_{3}}} \times U(2)_{d_{R}}$$

$$\Pi_{i=1}^{3} U(1)_{\tilde{B}_{i}} \times U(1)_{d_{R_{i}}}$$

Now U cannot be transformed away

$$m_{\tilde{d}_R}^2 = m^2 (\mathbf{1} + aY_d^+ Y_d),$$

The flavour Lagrangian gets extra terms

$$\Delta \mathcal{L}_{FC} = -\sqrt{2} \frac{g'}{3} \tilde{d}_R^* U \,\overline{\tilde{B}} \, d_R + \sqrt{2} \, g_3 \,\tilde{d}_R^* \, \lambda^b \, U \,\overline{\tilde{g}}{}^b \, d_R + h.c.$$

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Now U cannot be transformed away

$$m_{\tilde{d}_{P}}^{2} = m^{2}(\mathbf{1} + aY_{d}^{+}Y_{d}),$$

The flavour Lagrangian gets extra terms

$$\Delta \mathcal{L}_{FC} = -\sqrt{2} \frac{g'}{3} \tilde{d}_R^* U \overline{\tilde{B}} d_R + \sqrt{2} g_3 \tilde{d}_R^* \lambda^b U \overline{\tilde{g}^b} d_R + h.c.$$

• We define  $\eta_j^{\alpha\beta}=U_{j\alpha}U_{j\beta}^*$  and we consider

$$\eta_j^{\alpha\beta} = \xi_j^{\alpha\beta} e^{i\phi_j^{\alpha\beta}}$$

Dominance of left right operators due to stronger bounds

$$Q_{4,5} = (\bar{d}_R s_L)(\bar{d}_L s_R)$$

So that in a generic form

$$\Delta \mathcal{L}_{(123,12)}^{\Delta S=2,LR} pprox (\xi_2 \eta_3, \xi_2 \eta_2) \frac{\alpha_s^2}{m_{_L}^2} Q_{4,5}$$

 $\blacksquare \text{ For the symmetry } U(1)_{\tilde{B}_1} \times U(1)_{\tilde{B}_2} \times U(1)_{\tilde{B}_3} \times U(1)_{d_{R_3}} \times U(2)_{d_R}$ 

$$m_h \gtrsim 450 \ TeV \left( \left| \frac{\eta_3}{\xi_3} \right| \sin \phi_3 \right)^{1/2}$$

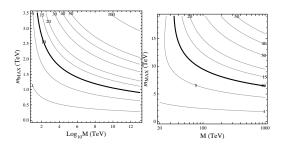
 $\blacksquare$  For the symmetry  $\Pi_{i=1}^3 U(1)_{\tilde{B}_i} \times U(1)_{d_{R_i}}$ 

$$m_h \gtrsim 10^4 \ TeV \left( \left| \frac{\eta_2}{\xi_2} \right| \sin \phi_2 \right)^{1/2}$$

## Non Standard Susy Spectrum

•  $\lambda$  SUSY. This is the NMSSM case with an extra chiral singlet S coupled in the superpotential to the usual Higgs doublets by  $\Delta f = \lambda S H_1 H_2$ , where the upper bound on the lightest scalar is:

$$m_h^2 \le m_Z^2 (\cos^2 2\beta + \frac{2\lambda^2}{g^2 + g'^2} \sin^2 2\beta)$$
 (1)



## QCD 2

Large Logs and  $\Delta F = 1$  operators

■ The  $\mathcal{L}_{12,3}^{\Delta F=2}$  has a peculiar feature. Being sensitive to two mass scales large logs arise

$$C_1 \propto \log \frac{m_h^2}{m_\ell^2}$$

■ The new ingredient is the mixing between  $\Delta F = 2$  and new  $\Delta F = 1$  operators

$$Q_{1}^{g} = \delta^{ab}\delta_{\beta\alpha}(\overline{d}^{\beta}P_{R}\widetilde{g}^{b})(\overline{\widetilde{g}^{a}}P_{L}s^{\alpha})$$

$$Q_{2}^{g} = d^{bac}t_{\beta\alpha}^{c}(\overline{d}^{\beta}P_{R}\widetilde{g}^{b})(\overline{\widetilde{g}^{a}}P_{L}s^{\alpha})$$

$$Q_{3}^{g} = if^{bac}t_{\beta\alpha}^{c}(\overline{d}^{\beta}P_{R}\widetilde{g}^{b})(\overline{\widetilde{g}^{a}}P_{L}s^{\alpha})$$

## QCD<sub>2</sub>

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$$\begin{array}{rcl} Q_1^g & = & \delta^{ab}\delta_{\beta\alpha}(\overline{d}^\beta P_R\widetilde{g}^b)(\overline{\widetilde{g}^a}P_Ls^\alpha) \\ Q_2^g & = & d^{bac}t^c_{\beta\alpha}(\overline{d}^\beta P_R\widetilde{g}^b)(\overline{\widetilde{g}^a}P_Ls^\alpha) \\ Q_3^g & = & if^{bac}t^c_{\beta\alpha}(\overline{d}^\beta P_R\widetilde{g}^b)(\overline{\widetilde{g}^a}P_Ls^\alpha) \ . \end{array}$$