Distance between QFTs as a measure of Lorentz violation

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1. Motivations & review of LV

2. Distance between QFTs
    - Definition & properties
    - Distance from a symmetry-preserving surface
    - Simple examples

3. Lorentz-violating QED & experimental bounds
    - Low-energy LV QED
    - QED-subsector of High-energy LV Standard Model

4. Conclusions

Based on arXiv:1105.4209 [hep-ph], DB and D. Anselmi

Lorentz violations

Lorentz invariance is one of the best tested symmetries of Nature.
However, several authors argued that the Lorentz invariance may be broken above some very high energy scale $\Lambda_L \rightarrow$ small effects at low energies.

- At low energies the effects of Lorentz violations are described by an effective quantum field theory (SME) [Kostelecký et al.];
- Explicit symmetry breaking $\Rightarrow$ many independent parameters:

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- define a distance $d(\mathcal{L}, \mathcal{L}')$ between two Lagrangians $\mathcal{L}$ and $\mathcal{L}'$;
- calculate the distance $d(\mathcal{L}, S)$ between a given theory $\mathcal{L}$ and the surface S of symmetry-preserving theories with $\delta_i = 0$.

Distance between QFTs

Consider a small perturbation

$$\mathcal{L} + \mathrm{d}\mathcal{L} = \mathcal{L}(\lambda) + \sum_I \mathrm{d}\lambda^I O_I = \mathcal{L}(\lambda + \mathrm{d}\lambda).$$

Define the infinitesimal distance at energy $E = 1/\hat{x}$ as

$$\mathrm{d}\ell^2 = 2\pi^4 \hat{x}^8 \langle \mathrm{d}\mathcal{L}(x) \mathrm{d}\mathcal{L}(0)^\dagger \rangle \equiv \sum_{IJ} \mathrm{d}\lambda^I g_{IJ}(\hat{x}, \lambda) \mathrm{d}\lambda^{J*},$$

where $x^\mu = (\hat{x}, 0)$. Reflection positivity $\Rightarrow \mathrm{d}\ell \geq 0$.

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Finite distance between two Lagrangian theories:

$$d(\mathcal{L}_1, \mathcal{L}_2) = \min_{\gamma_{12}} \int_{\gamma_{12}} \mathrm{d}\ell,$$

where γ_{12} is a path in parameter space connecting $\mathcal{L}_1$ and $\mathcal{L}_2$.

Some properties & issues

- d has all the properties of a distance in parameter-space;
- the distance is RG-invariant [Zamolodchikov '86];
- d depends on the energy scale $E = 1/\hat{x}$;
- terms $\propto$ field equations do not contribute to the distance;
- d depends on total derivatives in the Lagrangian;
- d depends on reparametrizations of the fields and the coordinates.

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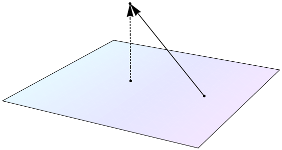
- d depends on reparametrizations of the fields and the coordinates.

To eliminate unwanted dependencies we can either:

- minimize the distance over reparametrizations,
- choose some convention to fix a universal form for the Lagrangians.

Distance from the Lorentz-invariant surface



LV operator $O_i \sim (O_i + \text{Lorentz-invariant terms})$.
$\Rightarrow$ include also Lorentz-invariant perturbations:

$$\mathcal{L}_{LI}(\lambda) + \sum_i \zeta^i O_i + \sum_a \xi^a O_a^{LI} = \mathcal{L}_{LI}(\lambda') + \sum_i \zeta^i O_i$$

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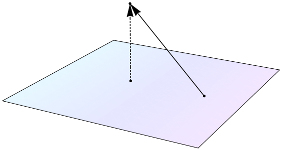

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Minimizing $d\ell$ w.r. to ξ^a gives the distance to the Lorentz surface $\{\zeta = 0\}$:

$$d_L^2 = \min_{\xi_a} \left[\zeta^i g_{ij} \zeta^j + 2\zeta^i g_{ia} \xi^a + \xi^a g_{ab} \xi^b\right] \equiv \zeta^i \gamma_{ij} \zeta^j,$$

where the reduced metric is

$$\gamma_{ij} = g_{ij} - g_{ia} h^{ab} g_{bj}, \qquad h^{ac} g_{cb} = \delta^a_b.$$

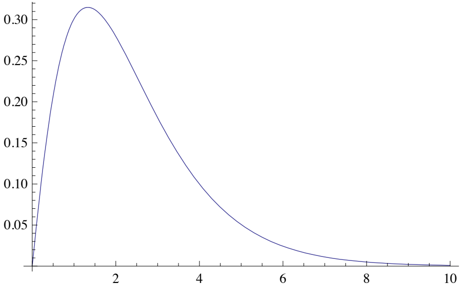
Example I: finite distance between massive scalar fields

Consider a scalar field with Lagrangian, and perturbation $m \to m + \mathrm{d}m$,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2, \qquad \mathrm{d}\mathcal{L} = m\mathrm{d}m\phi^2.$$

The distance reads

$$\mathrm{d}\ell = \frac{1}{2}K_1(m\hat{x})m^2\hat{x}^3\mathrm{d}m, \qquad d(m_1,m_2) = \frac{1}{2}\int_{m_1\hat{x}}^{m_2\hat{x}} u^2 K_1(u)\mathrm{d}u.$$



- $d(m_1,m_2) \to 0$ both in the UV and in the IR,
- $d(m,0) \to 0$ as $\hat{x} \to 0$,
- $d(m,0) \to 1$ as $\hat{x} \to \infty$.

Plot of $\mathrm{d}\ell/\mathrm{d}u$ as a function of $u = m\hat{x}$.

Example II: minimization over tangent displacements

Consider N free, Lorentz-violating scalar fields

$$\mathcal{L} = \frac{1}{2}\sum_{I=1}^{N}(\partial_\mu \phi_I)^2 - \sum_{I=1}^{N} \mathrm{d}\epsilon_I (\nabla_i \phi_I)^2 \equiv \mathcal{L}_0 + \mathrm{d}\mathcal{L}.$$

The distance between $\mathcal{L}$ and $\mathcal{L}_0$ reads

$$\mathrm{d}\ell^2 = 3\sum_I (\mathrm{d}\epsilon_I)^2.$$

If all $\mathrm{d}\epsilon_I$'s are equal, $\mathrm{d}\epsilon_I = \mathrm{d}a \Rightarrow$ the theory is Lorentz-invariant.
Thus we write $\mathrm{d}\epsilon_I = \mathrm{d}a + \mathrm{d}\tilde{\epsilon}_I$, and we minimize over $\mathrm{d}a$, getting

$$\mathrm{d}\ell_r^2 = \sum_{IJ} \mathrm{d}\tilde{\epsilon}_I \gamma_{IJ} \mathrm{d}\tilde{\epsilon}_J = \frac{3}{N}\sum_{I<J}(\mathrm{d}\tilde{\epsilon}_I - \mathrm{d}\tilde{\epsilon}_J)^2, \quad \gamma_{IJ} = 3\delta_{IJ} - \frac{3}{N} 1_{IJ}.$$

At the minimum, $\mathrm{d}\epsilon_I = \mathrm{d}\tilde{\epsilon}_I - \frac{1}{N}\sum_J \mathrm{d}\tilde{\epsilon}_J$, which satisfies $\sum_I \mathrm{d}\epsilon_I = 0$.

Lorentz-violating QED at low energies

Standard Model Extension, QED and CPT-even sector [Kostelecký et al.]

$$\mathcal{L} = \mathcal{L}_{QED} - \frac{1}{4} k_F^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} \bar{\psi} \Big(i c^{\mu\nu} \gamma_\mu \overleftrightarrow{D}_\nu + i d^{\mu\nu} \gamma_5 \gamma_\mu \overleftrightarrow{D}_\nu + H^{\mu\nu} \sigma_{\mu\nu} \Big) \psi.$$

Minimization over unphysical parameters

- Tangent displacements $\propto F^2, F\tilde{F} \Rightarrow (k_F)^{\mu\nu}{}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} k_F^{\mu\nu\rho\sigma} = 0$.
- Spinor reparametrizations $\psi \to \psi + i\omega^{\mu\nu}\sigma_{\mu\nu}\psi \Rightarrow$ symmetric c and d.
- $\mathrm{d}\ell$ does not depend on $c^\mu{}_\mu$ and $d^\mu{}_\mu \Rightarrow$ we can set $c^\mu{}_\mu = d^\mu{}_\mu = 0$.
- Reparametrizations $x^\mu \to x^\mu + \mathrm{d}a^\mu{}_\nu x^\nu$ and $A_\mu \to A_\mu - \mathrm{d}a^\nu{}_\mu A_\nu$
  $\Rightarrow (k_F)^\alpha{}_{\mu\alpha\nu} + c_{\mu\nu} - \frac{1}{4} g_{\mu\nu} c^\alpha{}_\alpha = 0$.

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$$\mathrm{d}\ell^2_{QED} = \sum_{\mu\nu=0}^{3} \Big[\frac{2}{3} (\tilde{k}^{\mu\nu} - 2c^{\mu\nu})^2 + 4(d^{\mu\nu})^2 + \hat{x}^2 (H^{\mu\nu})^2 \Big] + 2 \sum_{i,j=1}^{3} (\tilde{\kappa}^{ij}_{e+})^2 + (\tilde{\kappa}^{ij}_{o-})^2 ,$$

where $\tilde{k}^{\mu\nu} = (k_F)^{\mu\alpha\nu}{}_\alpha$, and $\tilde{\kappa}_{e+}, \tilde{\kappa}_{o-}$ are combinations of k_F.

Experimental bounds

CPT-even parameters are fully measured only for the electron.

Coefficient	Sensitivity
$c^{\mu\nu}$	$10^{-14} \div 10^{-17}$
$d^{\mu\nu}$	$10^{-16} \div 10^{-25}$
$H^{\mu\nu}$	10^{-26} GeV

Coefficient	Sensitivity
$\tilde{\kappa}_{e+}, \tilde{\kappa}_{o-}$	10^{-32}
$\tilde{k}^{\mu\nu}$	$10^{-17} \div 10^{-13}$

Upper bounds for some coefficients of SME [Kostelecký, Russell, arxiv:0801.0287].

- Measurements on vacuum birefringence $\Rightarrow \tilde{\kappa}_{e+}, \tilde{\kappa}_{o-}$ neglegible;
- Electron: bounds on $d^{\mu\nu}$ are at least a few orders of magnitude stronger than those on $c^{\mu\nu}$;
- Main contribution from the photon parameter $\tilde{\kappa}_{o+}^{XY} \leq 10^{-13}$;

$$\mathrm{d}\ell_{QED} \lesssim \sqrt{10^{-26} + 5 \cdot 10^{-39} (\hat{x}\mathrm{GeV})^2} \sim 10^{-13}.$$

- For $\hat{x} \leq 1/m_e$ the H-contribution is neglegible;

Higher derivatives

LV operators of dimension higher than 4 can be added to the Lagrangian; they are renormalizable by weighted power-counting [Anselmi, Halat '07].

$$\mathcal{L}_0 = -\frac{1}{4}F^2, \qquad \mathrm{d}\mathcal{L} = -\frac{\delta_2}{4}(F^{ij})^2 - \frac{\tau_1}{4\Lambda_L^2}(\partial_k F^{ij})^2 - \frac{\tau_0}{4\Lambda_L^4}(\partial_k \partial_l F^{ij})^2,$$

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$$\mathrm{d}\ell^2 = \frac{3}{2}\delta_2^2 - 24\frac{\delta_2\tau_1}{\hat{x}^2\Lambda_L^2} + 480\frac{\delta_2\tau_0}{\hat{x}^4\Lambda_L^4} + 384\frac{\tau_1^2}{\hat{x}^4\Lambda_L^4} - 3840\frac{\tau_1\tau_0}{\hat{x}^6\Lambda_L^6} + 505920\frac{\tau_0^2}{\hat{x}^8\Lambda_L^8}.$$

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Present limits: $\tau_0/\Lambda_L^4 \leq 10^{-24}\mathrm{GeV}^{-4}$, $\tau_1/\Lambda_L^2 \leq 10^{-21}\mathrm{GeV}^{-2}$.

$$\mathrm{d}\ell^2 \leq 6\times 10^{-28}(1+10^{-6}q^2+10^{-8}q^4+10^{-14}q^6+10^{-15}q^8), \quad q = (\hat{x}\mathrm{GeV})^{-1}.$$

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$\mathrm{d}\mathcal{L}$ is small up to $E \sim \Lambda_L \gtrsim 10^6$ GeV, where $\mathrm{d}\ell^2 \lesssim 5\times 10^5$.
$\mathrm{d}\ell$ is of order 1 at $E \sim \Lambda_L/6$.

Conclusions

- We defined a distance in the parameter space of quantum field theories in a RG-invariant way, and studied its properties.
- The distance has some unpleasant features, such as dependencies on unphysical parameters, which can be eliminated either with a minimization, or fixing some prescription.
- When used to quantify the amount of violation of a symmetry, the distance collects all the parameters of the violation in a single quantity that vanishes when the symmetry is restored.
- We calculated the distance from the Lorentz surface in the LV extensions of QED, and found that when higher derivatives are included, they may lower the scale at which the effects of the Lorentz violation become important.

Some reference...

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