WIMP-Nucleus Spin-Dependent Elastic Scattering Simplified

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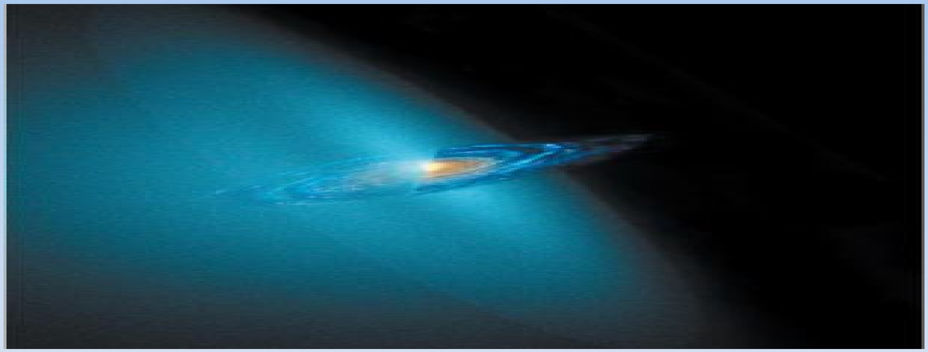
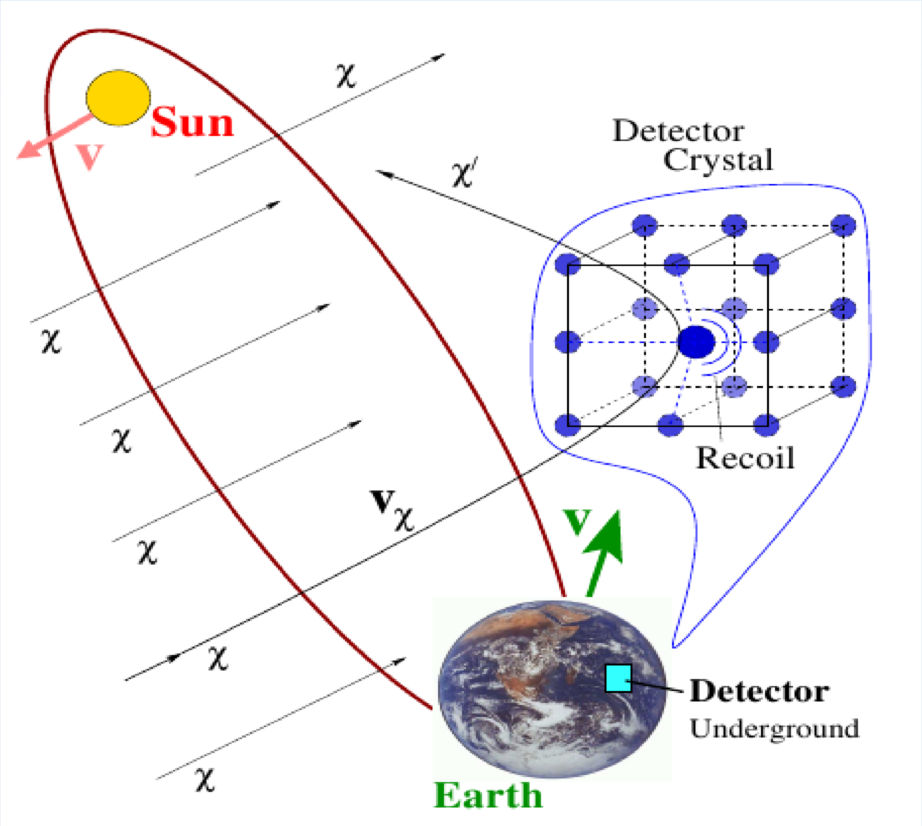
Based on:

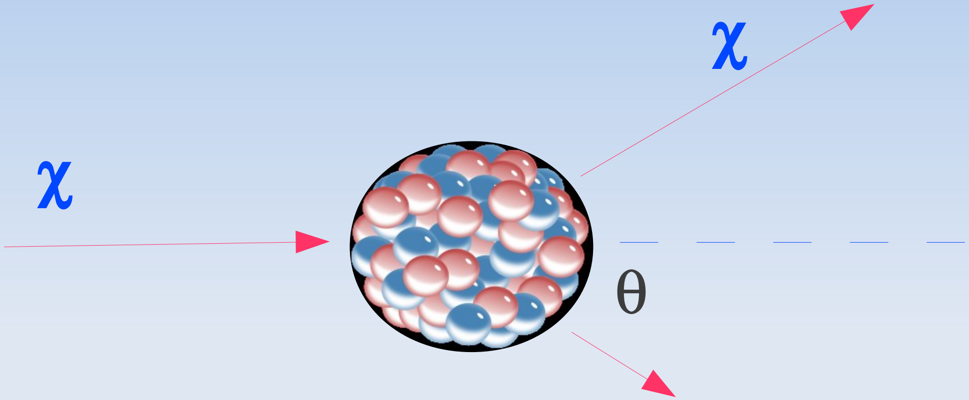
M. C., J.D. Vergados **and M. E. Gomez**
"Scheme for the extraction of WIMP-nucleon scattering cross sections from total event rates"

Phys. Rev. D 83, 075010 (2011), arXiv:1011.6108 [hep-ph]

WIMP-nucleus elastic scattering





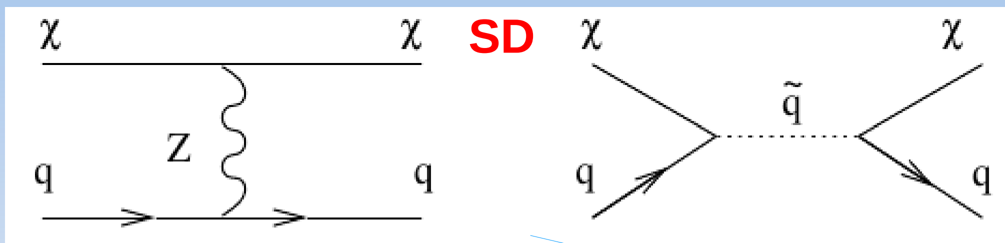


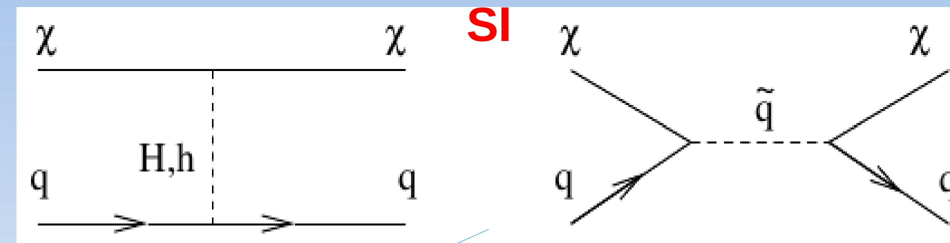
$$q = |\vec{q}| = 2\mu_A v_\chi \cos\theta$$

$$E_R = \frac{q^2}{2m_A}$$

Non relativistic WIMP scatters elastically with nuclei. The recoil of the nucleus deposits a tiny amount of energy in the detector: recoil energies are from few to 100 keV

Neutralino-nucleon cross sections





$$\mathcal{L}_{eff} = g_q(\bar{\chi}\gamma^\mu\gamma^5\chi)(\bar{q}\gamma_\mu\gamma^5 q) + h_q(\bar{\chi}\chi)(\bar{q}q)$$

Axial vector interaction: Spin-spin interaction in the non relativistic limit

$$a_p = \sum_{q=u,d,s} g_q \Delta q^{(p)}$$

$$a_n = \sum_{q=u,d,s} g_q \Delta q^{(n)}$$

Scalar interaction, Spin Independent

$$\lambda^{(p)} = \sum_q h_q f_q^{(p)} \simeq \lambda^{(n)} = \sum_q h_q f_q^{(n)} \equiv c_0$$

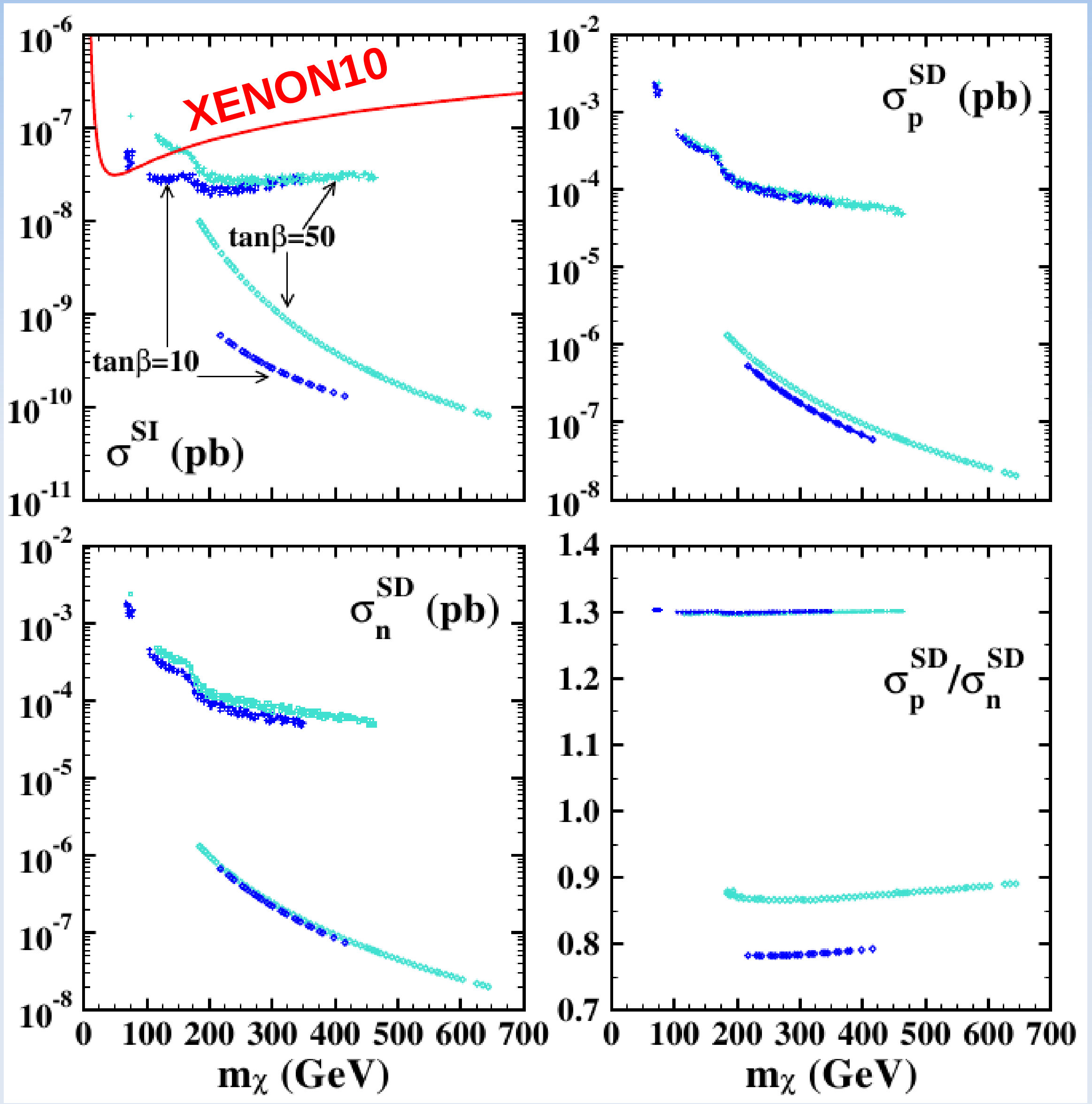
f's and Δ's factors require inputs from non perturbative QCD

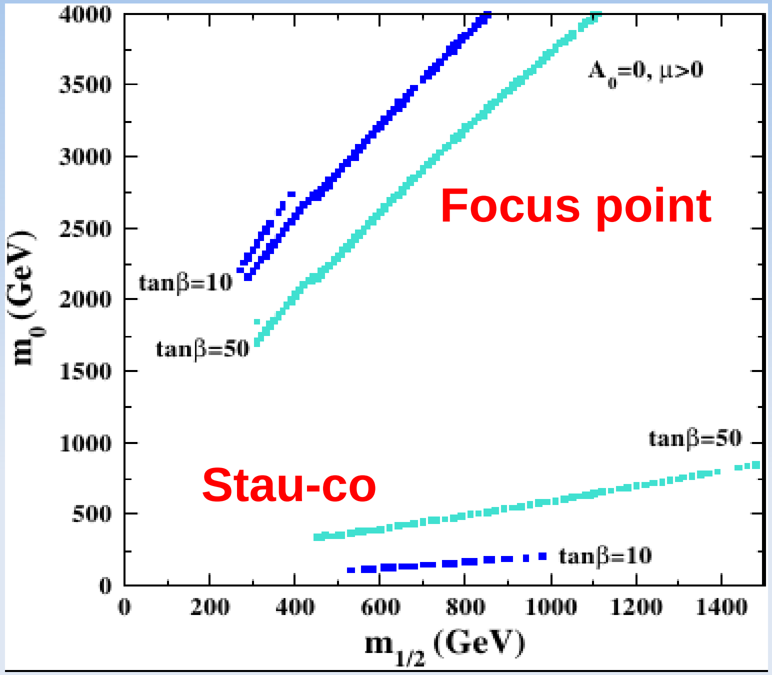
$$\sigma^{SD}_{p,n} = \frac{3\mu_p^2}{\pi}|a_{p,n}|^2$$

$$\sigma^{SI} = \frac{\mu_p^2}{\pi}|c_0|^2$$

3 elementary cross sections to be extracted from experiments

CMSSM example





$$\mathrm{WMAP}(3\sigma):\ 0.094 < \Omega h^2 < 0.128$$

$$a_n > 0$$
$$a_p < 0$$
$$\Rightarrow \quad \varrho = \frac{a_p}{a_n} = -1$$

Can the relative sign between the SD amplitudes and the three cross sections be determined experimentally?

WIMP-nucleus SI cross section

1) The structure of the nucleus is important if qR~1: in this case the Zero Momentum Transfer Limit (ZMTL) cross section is not a good approximation, expecially for heavy nuclei

2) The scalar SI interaction is sensible to the mass distribution inside the nucleus

$$\frac{d\sigma^{SI}_{(A)}}{dq^2} = \frac{1}{4(\mu_A v)^2}\frac{\mu_A^2}{\pi}|\lambda^p Z F^Z(q^2) + \lambda^n (A-Z) F^N(q^2)|^2$$

$$\lambda^p \simeq \lambda^n \equiv c_0$$

$$F^Z(q^2) \simeq F^N(q^2) \cong F(q^2)$$

q=0 → 1

$$\sigma^{SI}_{(A)}(0) = \frac{\mu_A^2}{\pi}|c_0|^2 A^2$$

$$\frac{d\sigma^{SI}_{(A)}}{dq^2} = \frac{1}{4(\mu_A v)^2}\sigma^{SI}_{(A)}(0) F^2(q^2)$$

The Particle Physics and the Nuclear Physics degrees of freedom are factorized

WIMP-nucleus SD cross section: standard formalism (1)

Engel PLB 264 (1991)   Engel, Pittel and Vogel IJMP E 1 (1992)

$$\frac{d\sigma^{SD}_{(A)}}{dq^2} \propto |\langle J, M_Z = J| \sum_{i=1}^{A} (a_0 + a_1 \tau_i^3) \vec{\sigma}_i e^{-i\vec{q}\cdot\vec{r}_i} |J, M_Z = J\rangle|^2$$

$$\propto \frac{1}{2J+1} |\langle J|| \sum_{i=1}^{A} (a_0 + a_1 \tau_i^3) \vec{\sigma}_i e^{-i\vec{q}\cdot\vec{r}_i} ||J\rangle|^2$$

Isospin representation

$$a_0 = a_p + a_n$$

$$a_1 = a_p - a_n$$

$$S(q) = a_0^2 S_{00}(q) + a_0 a_1 S_{01}(q) + a_1^2 S_{11}(q)$$

$$S(0) = \frac{2J+1}{\pi} J(J+1) \left[\frac{a_0(\langle \vec{S}_p \rangle + \langle \vec{S}_n \rangle) + a_1(\langle \vec{S}_p \rangle - \langle \vec{S}_n \rangle)}{2J} \right]^2$$

$$\sigma^{SD}_{(A)}(0) = \frac{4\mu_A^2}{2J+1} S(0)$$

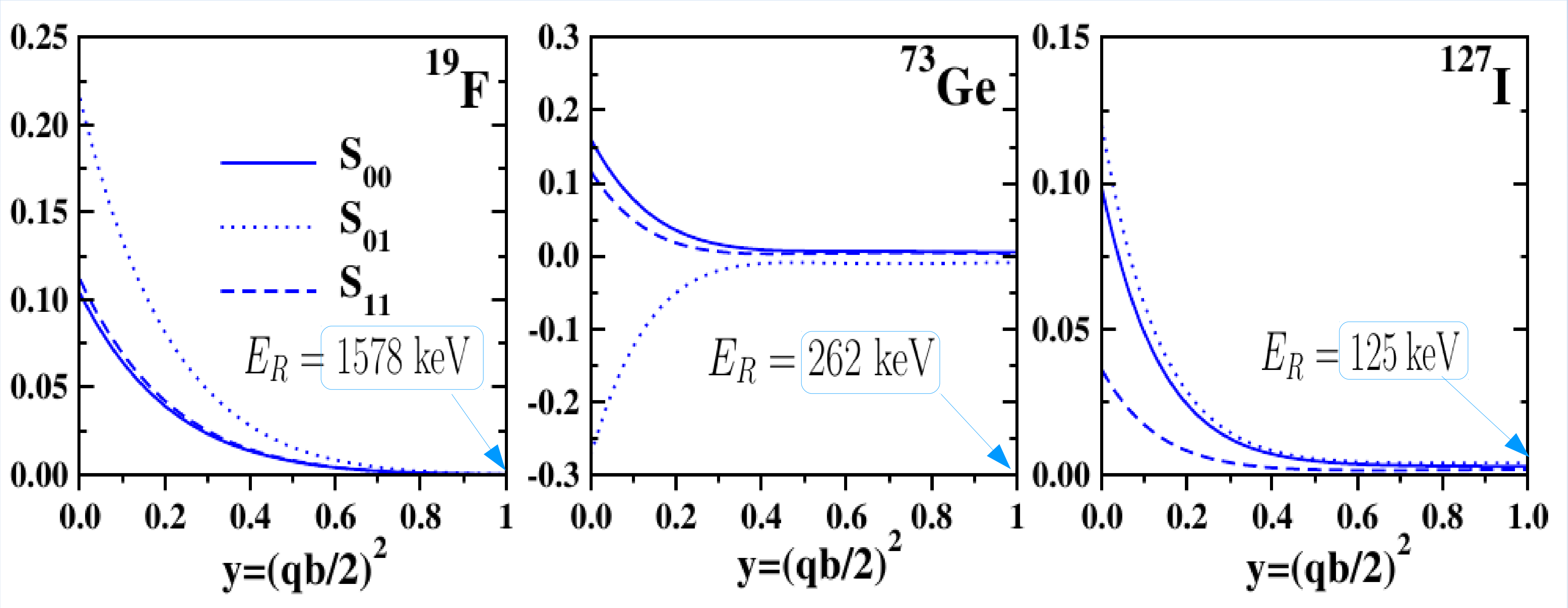
$$\frac{d\sigma^{SD}_{(A)}}{dq^2} = \frac{1}{4(\mu_A v)^2} \sigma^{SD}_{(A)}(0) \frac{S(q)}{S(0)}$$

The Nuclear Physics degrees of freedom are not decoupled from the particle physics ones as in the SI case with the form factor

WIMP-nucleus SD cross section: standard formalism (2)

The structure functions are furnished as polynomial fits to the results of the shell model calculations in terms of the variable $y = (qb/2)^2$

$b = 1\,\text{fm}\, A^{1/6}$



1) The three momentum-dependent structure functions S_{ij} look different
2) The interference term can be negative

WIMP-nucleus cross section: Vergados formalism (1)

$$\frac{d\sigma^{SD}_{(A)}}{dq^2} \propto |\langle J, M_Z = J| \sum_{i=1}^{A} (a_0 + a_1 \tau_i^3) \vec{\sigma}_i e^{-i\vec{q}\cdot\vec{r}_i} |J, M_Z = J\rangle|^2$$

$$\propto \frac{1}{2J+1} |\langle J|| \sum_{i=1}^{A} (a_0 + a_1 \tau_i^3) \vec{\sigma}_i e^{-i\vec{q}\cdot\vec{r}_i} ||J\rangle|^2$$

$$\frac{d\sigma^{SD}_{(A)}}{dq^2} \propto (a_0 \Omega_0(q) + a_1 \Omega_1(q))^2$$

Kosmas and Vergados PRD 55 (1997)
Divari, Kosmas, Vergados, Skouras PRC 61 (2000)

$$a_0^2 \Omega_0^2(0) \left(\frac{\Omega_0(q)}{\Omega_0(0)}\right)^2 + 2a_0 a_1 \Omega_0(0)\Omega_1(0) \left(\frac{\Omega_0(q)\Omega_1(q)}{\Omega_0(0)\Omega_1(0)}\right) + a_1^2 \Omega_1^2(0) \left(\frac{\Omega_1(q)}{\Omega_1(0)}\right)^2$$

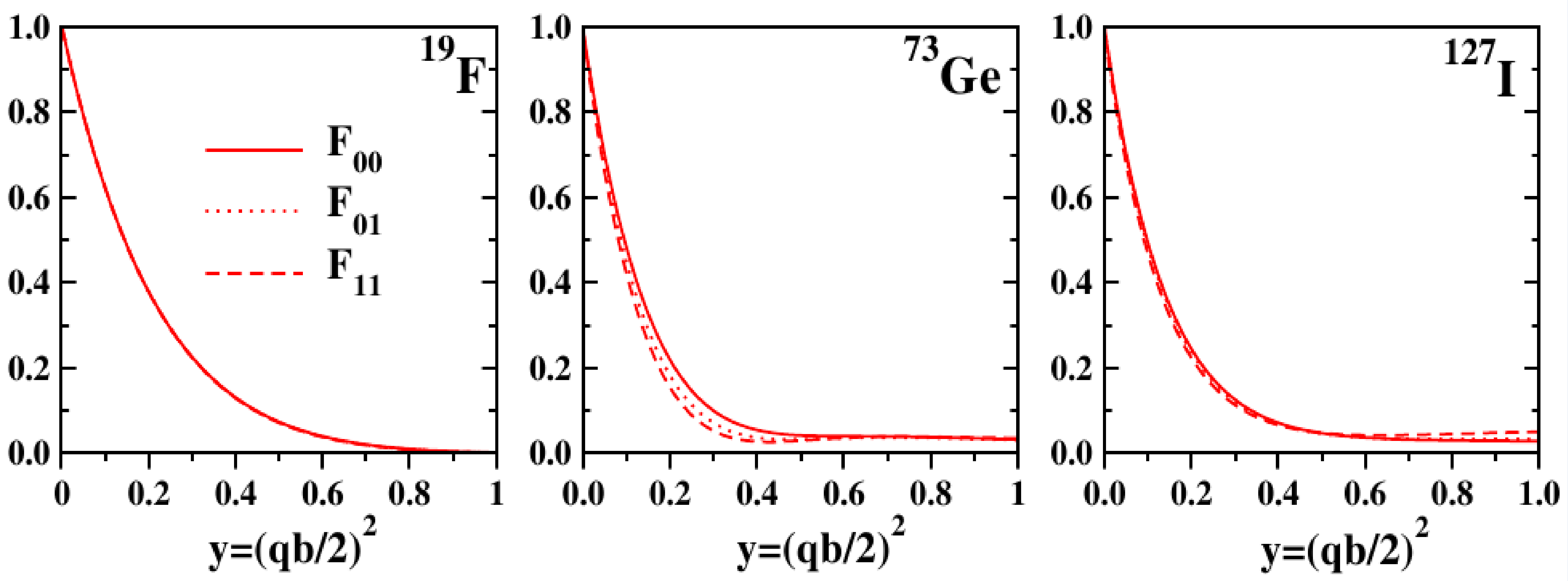
$F_{00}(q)$ **$F_{01}(q)$** **$F_{11}(q)$**

WIMP-nucleus SD crosssection: connection between the two formalisms

$$S_{00}(q) = \frac{2J+1}{16\pi}\Omega_0^2 \; F_{00}(u)$$
$$S_{01}(q) = \frac{2J+1}{8\pi}\Omega_0\Omega_1 \; F_{01}(u)$$
$$S_{11}(q) = \frac{2J+1}{16\pi}\Omega_1^2 \; F_{11}(u)$$

$$\frac{S_{00}(q)}{S_{00}(0)} = F_{00}(u)$$
$$\frac{S_{01}(q)}{S_{01}(0)} = F_{01}(u)$$
$$\frac{S_{11}(q)}{S_{11}(0)} = F_{11}(u)$$

$$u = q^2b^2/2$$



Advantages of the normalized functions

1) $$F_{00}(u) \simeq F_{01}(u) \simeq F_{11}(u)$$

One only needs one structure function, that by definition is normalized to one at zero momentum transfer

2) $$\frac{d\sigma^{SD}_{(A)}}{du} = \frac{1}{2(\mu_A b v)^2}\sigma^{SD}_{(A)}(0)F_{11}(u)$$

The particle physics degrees of freedom are factorized from the nuclear physics ones. The cross section has the same form as in the spin-independent case

3) $$t^{SD} = \int_{u_{min}}^{u_{max}} du\, F_{11}(u) \int_{v_{min}(u)}^{v_{max}} d^3\vec{v} \frac{v}{\sqrt{\langle v^2\rangle}} \frac{f(\vec{v})}{2(\mu_A b v)^2}$$

$$t^{SI} = \int_{u_{min}}^{u_{max}} du\, |F(u)|^2 \int_{v_{min}(u)}^{v_{max}} d^3\vec{v} \frac{v}{\sqrt{\langle v^2\rangle}} \frac{f(\vec{v})}{2(\mu_A b v)^2}$$

To get the total cross section one has to integrate over the recoil energy and the velocity distribution function. Two very similar integrals

An application: cross sections from total rates

$$R = \frac{\rho_l}{m_\chi} \frac{1}{A m_p} \sqrt{\langle v^2 \rangle} \left(\sigma^{SI}_{(A)}(0)\ t^{SI} + \sigma^{SD}_{(A)}(0)\ t^{SD} \right)$$

$$\Omega_{p,n} = 2\sqrt{\frac{J+1}{J}} \langle \vec{S}_{p,n} \rangle$$

$$\sigma^{SI}_{(A)}(0) = \left(\frac{\mu_A}{\mu_p} \right)^2 A^2 \sigma^{SI}$$

$$\sigma^{SD}_{(A)}(0) = \left(\frac{\mu_A}{\mu_p} \right)^2 \frac{1}{3} \left(\Omega^A_p \sqrt{\sigma^{SD}_p} + \varrho \Omega^A_n \sqrt{\sigma^{SD}_n} \right)^2$$

DAMA, KIMS COUPP, ANAIS

$$^{127}\mathrm{I} \Longleftrightarrow A_1:\ \Omega^{127}_p = 0.731\ \Omega^{127}_n = 0.177$$

COUPP, PICASSO, SIMPLE

$$^{19}\mathrm{F} \Longleftrightarrow A_2:\ \Omega^{19}_p = 1.646\ \Omega^{19}_n = -0.030$$

CDMS, COGENT EDELWEISS

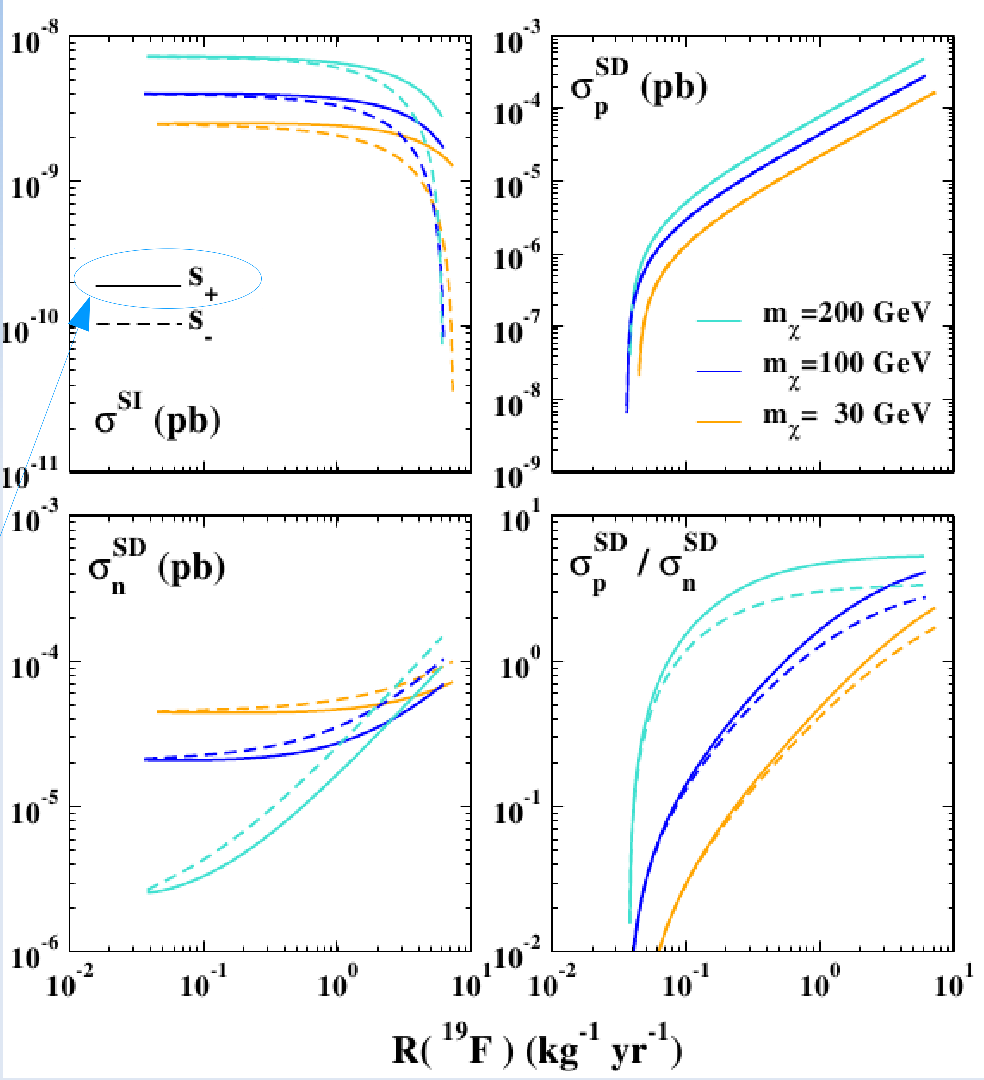
$$^{73}\mathrm{Ge} \Longleftrightarrow A_3:\ \Omega^{73}_p = 0.066\ \Omega^{73}_n = 0.836$$

The system has two sets of analitical solutions, each solution linked with the relative sign ρ

$$\begin{cases} \sigma^{SI} + \mathcal{R}_{A_1} \left(\Omega^{A_1}_p \sqrt{\sigma^{SD}_p} + \varrho \Omega^{A_1}_n \sqrt{\sigma^{SD}_n} \right)^2 - \mathcal{S}_{A_1} = 0 \\ \sigma^{SI} + \mathcal{R}_{A_2} (\Omega^{A_2}_p)^2 \sigma^{SD}_p - \mathcal{S}_{A_2} = 0 \\ \sigma^{SI} + \mathcal{R}_{A_3} (\Omega^{A_3}_n)^2 \sigma^{SD}_n - \mathcal{S}_{A_3} = 0 \end{cases}$$

Cross sections from rates

$$s_+ : \begin{cases} \sigma_+^{SI} = \dfrac{\mathcal{B}+\sqrt{\mathcal{B}^2-\mathcal{A}\mathcal{C}}}{\mathcal{A}} \\ \sigma_{p,+}^{SD} = \dfrac{\mathcal{S}_{A_2}-\sigma_+^{SI}}{\mathcal{R}_{A_2}(\Omega_p^{A_2})^2} \\ \sigma_{n,+}^{SD} = \dfrac{\mathcal{S}_{A_3}-\sigma_+^{SI}}{\mathcal{R}_{A_3}(\Omega_n^{A_3})^2} \end{cases} \qquad s_- : \begin{cases} \sigma_-^{SI} = \dfrac{\mathcal{B}-\sqrt{\mathcal{B}^2-\mathcal{A}\mathcal{C}}}{\mathcal{A}} \\ \sigma_{p,-}^{SD} = \dfrac{\mathcal{S}_{A_2}-\sigma_-^{SI}}{\mathcal{R}_{A_2}(\Omega_p^{A_2})^2} \\ \sigma_{n,-}^{SD} = \dfrac{\mathcal{S}_{A_3}-\sigma_-^{SI}}{\mathcal{R}_{A_3}(\Omega_n^{A_3})^2} \end{cases}$$



In the case of I, F, Ge, the two sets of solutions are connected with the relative sign between the proton and neutron amplitudes appearing in the equation for I.
We find:

CMSSM

$$\varrho = +1 \Leftrightarrow s_- \qquad \varrho = -1 \Leftrightarrow s_+$$

$$R(^{127}\mathrm{I}) = R(^{73}\mathrm{Ge}) = 1\ \mathrm{kg}^{-1}\ \mathrm{y}^{-1}$$

Summary and Conclusions

- The spin-dependent elastic scattering can be described in terms of only one momentum dependent structure function that by definition is normalized to one at zero momentum transfer.
- The particle physics degrees of freedom are factorized from the nuclear physics ones as in spin-independent scattering with the nuclear form factor.
- The three elementary cross sections, together with the relative sign between the spin-dependent proton and neutron couplings, can be determined analytically employing the previous formalism ones the total event rates are measured (hopefully!) in three suitably chosen nuclei.