Seesaw in the bulk

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Based on the work with
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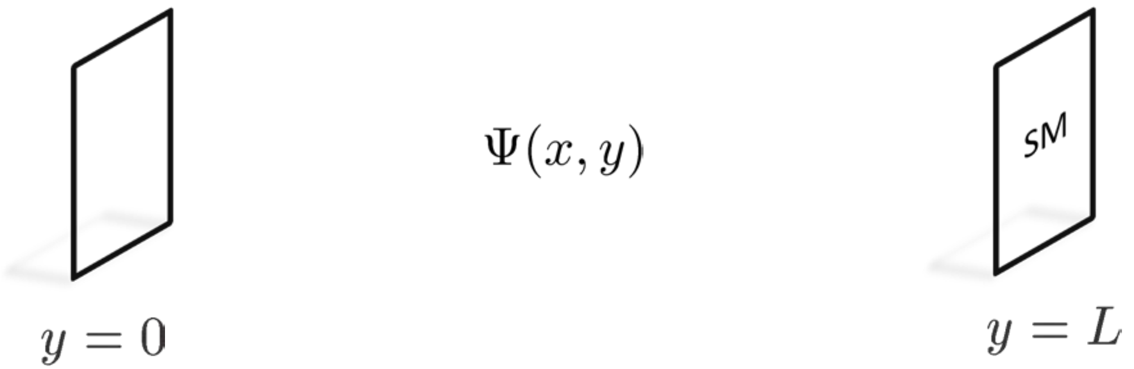
The bulk right-handed neutrinos

[Dienes,Dudas,Gherghetta,1999]

[Arkani-Hamed,Dimopoulos,Dvali,March-Russell,1999]



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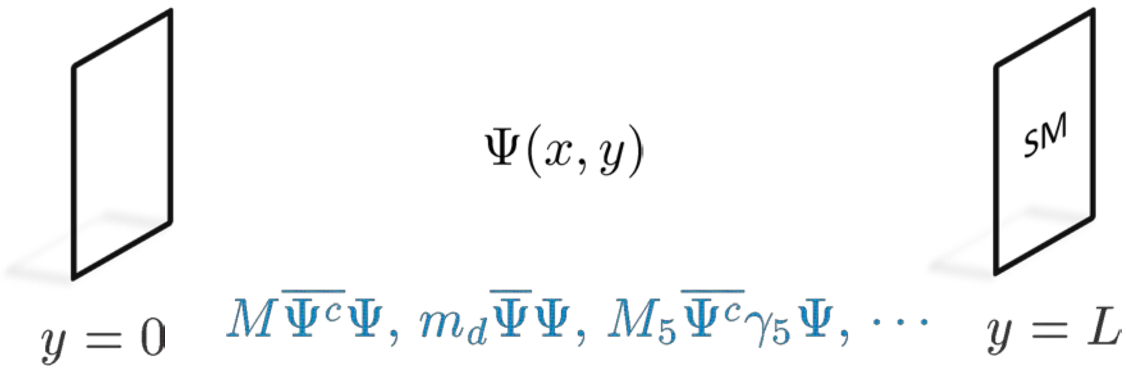

- The mass terms (Majorana, Dirac, Lorentz-violating …)

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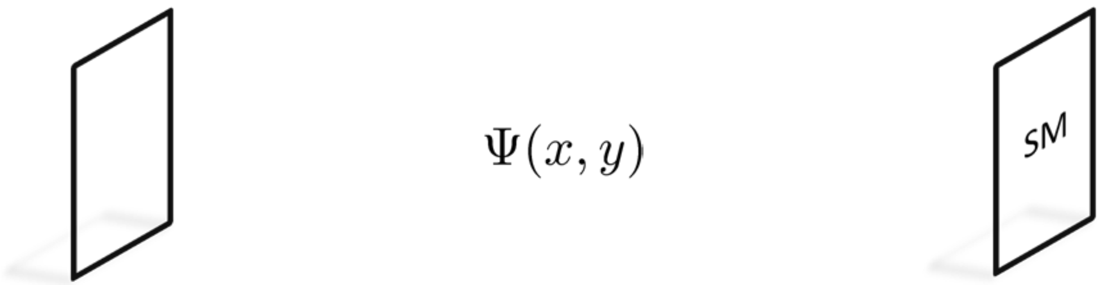

$$\Psi(x,y)$$

$y=0$ $\quad M\overline{\Psi^c}\Psi,\ m_d\overline{\Psi}\Psi,\ M_5\overline{\Psi^c}\gamma_5\Psi,\ \cdots \quad$ $y=L$

$$ds^2=\rho(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

- The mass terms (Majorana, Dirac, Lorentz-violating …)
- The bulk geometry (Flat, Warped, Others …)

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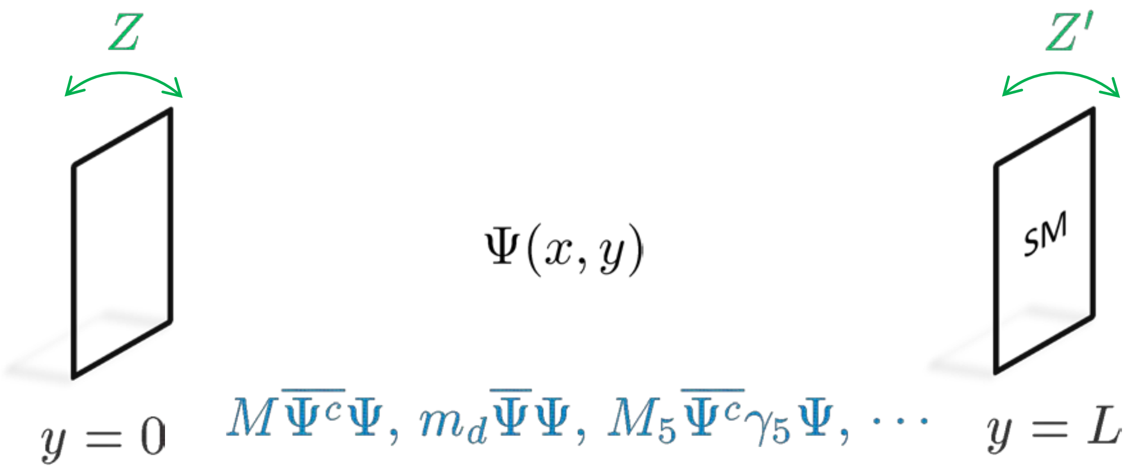

$$M\overline{\Psi^c}\Psi,\ m_d\overline{\Psi}\Psi,\ M_5\overline{\Psi^c}\gamma_5\Psi,\ \cdots$$

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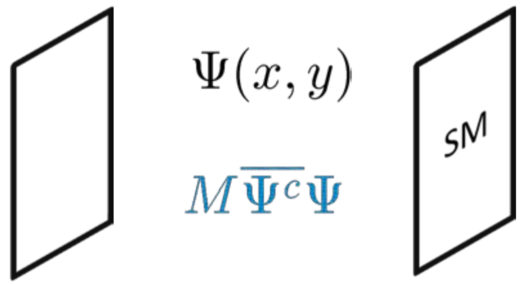
- The mass terms (Majorana, Dirac, Lorentz-violating …)
- The bulk geometry (Flat, Warped, Others …)
- The boundary conditions for the bulk fields

Seesaw in five dimensions

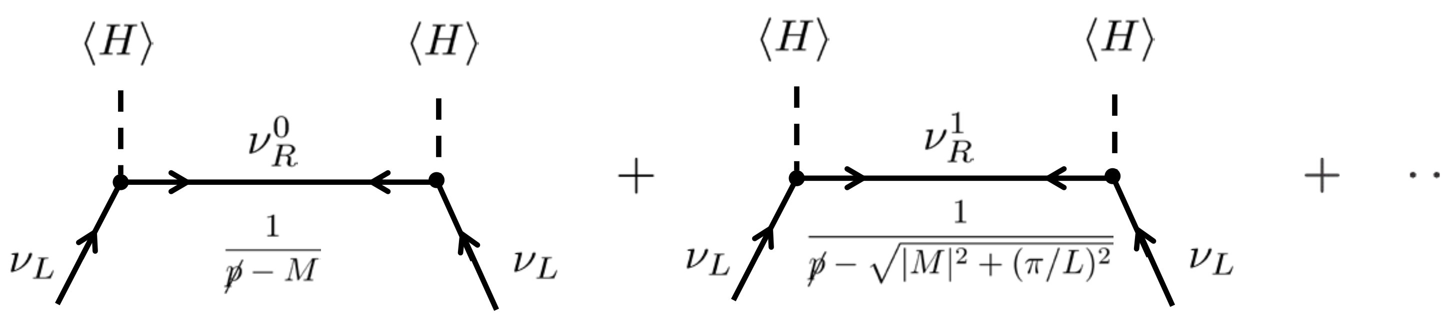
[Dienes,Dudas,Gherghetta,99]
[Lukas, Ramond, Romanino, Ross, 00]



$$\Psi = \begin{pmatrix} \nu_R \\ \nu \end{pmatrix}$$

Neutrino mass after the seesaw

$$M_\nu = \frac{1}{\tanh(ML)}\frac{m^2}{\Lambda}$$



Seesaw with the bulk Dirac mass

$$\mathcal{L} = i\overline{\Psi}\Gamma^M\partial_M\Psi - m_d\theta(y)\overline{\Psi}\Psi - \frac{1}{2}(M\overline{\Psi^c}\Psi + \text{h.c.})$$
$$- \left(\frac{m}{\sqrt{\Lambda}}\overline{\Psi}\nu_L + \text{h.c.}\right)\delta(y-L)$$

Bulk Dirac

The neutrino mass becomes

$$M_\nu = \frac{1}{\Lambda L}\frac{\widetilde{M}L\cosh(\widetilde{M}L) - m_d L\sinh(\widetilde{M}L)}{\sinh(\widetilde{M}L)}\frac{m^\mathrm{T}m}{M^*}, \quad \widetilde{M} = \sqrt{m_d^2 + |M|^2}$$

If $M \ll m_d$ and $\widetilde{M}L \gg 1$,

then $M_\nu \simeq \frac{M}{m_d}\frac{m^\mathrm{T}m}{\Lambda}$.

$\Lambda \approx \text{TeV} \to M/m_d \sim 10^{-10}$

for eV neutrino mass

Inverse seesaw

Whole neutrino mass matrix

$$\mathcal{M} = \begin{array}{c} \\ \nu_L^{\mathrm{T}} \\ \psi_R^{0\,\dagger} \\ \psi_R^{1\,\dagger} \\ \psi_L^{1\,\mathrm{T}} \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} \nu_L & \psi_R^{0\,*} & \psi_R^{1\,*} & \psi_L^{1} & \end{array} \\ \left(\begin{array}{c|cccc} & m_0 & m_1 & & \cdots \\ \hline m_0 & M & & & \cdots \\ m_1 & & M & M_{KK} & \cdots \\ & & M_{KK} & M & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right) \end{array}$$

The zero mode function $\chi^0(L) \sim e^{-m_d L}$ → Zero mode is negligible in seesaw

$$M \ll M_{KK}$$

$$M_\nu \simeq \frac{m_1^2}{\frac{M_{KK}^2}{M}} = \frac{M}{M_{KK}} \frac{m_1^2}{M_{KK}}$$

$$\mathrm{Det}\mathcal{M} = m_1^2 M \cdots = M_\nu M_{KK}^2 \cdots$$

Inverse seesaw

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small

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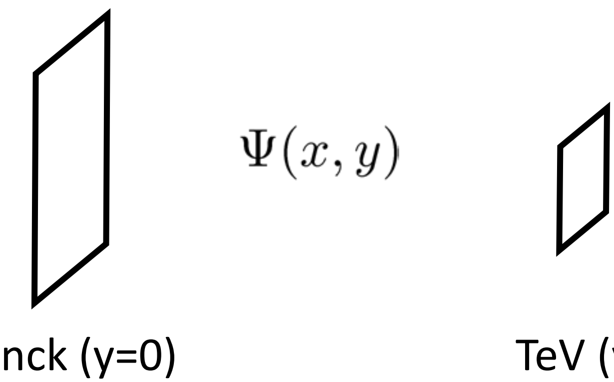
$$\mathrm{Det}\mathcal{M} = m_1^2 M \cdots = M_\nu M_{KK}^2 \cdots$$

On the warped geometry

Randall-Sumdrum background

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$



$$\mathcal{L} = \sqrt{g}\Big[i\overline{\Psi}\not{D}\Psi - m_d\theta(y)\overline{\Psi}\Psi - \Big(\frac{1}{2}M\overline{\Psi^c}\Psi + \frac{m}{\sqrt{\Lambda}}\overline{\Psi}\nu_L\delta(y-L) + \text{h.c.}\Big)\Big]$$

How does the background change the neutrino mass ?

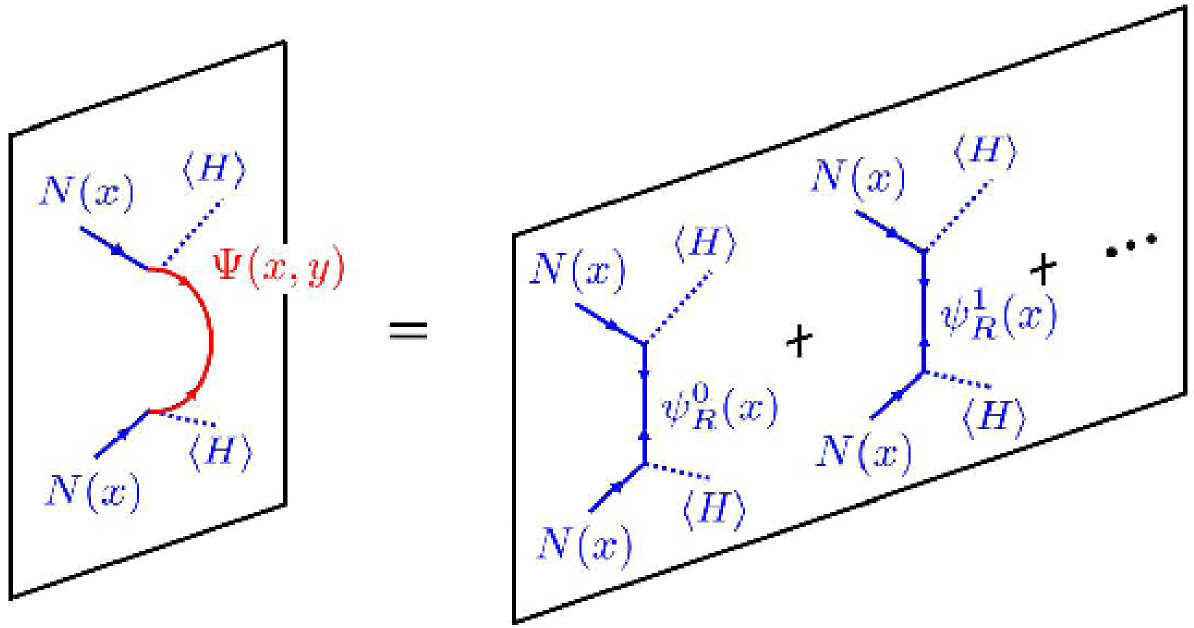
By the usual KK expansion, …

$$\int_0^L dy\, e^{ky} \chi(y)^{n\,\mathrm{T}} M \chi(y)^m \;\approx\; \delta_{nm}$$

$$\mathcal{M} = \begin{array}{c} \\ \nu_L^{\mathrm{T}} \\ \psi_R^{0\,\dagger} \\ \psi_R^{1\,\dagger} \\ \psi_L^{1\,\mathrm{T}} \\ \psi_R^{2\,\dagger} \\ \psi_L^{2\,\mathrm{T}} \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccc} \nu_L & \psi_R^{0\,*} & \psi_R^{1\,*} & \psi_L^{1} & \psi_R^{2\,*} & \psi_L^{2} & \end{array} \\ \left(\begin{array}{c|c|cc|cc|c} & m_0^{\mathrm{T}} & m_1^{\mathrm{T}} & & m_2^{\mathrm{T}} & & \cdots \\ \hline m_0 & -M_{R_{00}}^* & -M_{R_{01}}^* & & -M_{R_{02}}^* & & \cdots \\ \hline m_1 & -M_{R_{01}}^* & -M_{R_{11}}^* & M_{K_1} & -M_{R_{12}}^* & & \cdots \\ & & M_{K_1} & M_{L_{11}} & & M_{L_{12}} & \cdots \\ \hline m_2 & -M_{R_{02}}^* & -M_{R_{21}}^* & & -M_{R_{22}}^* & M_{K_2} & \cdots \\ & & & M_{L_{21}} & M_{K_2} & M_{L_{22}} & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right) \end{array}$$

It is hard to perform seesaw diagonalization

Alternative to the KK expansion ⇒ 5D propagator



The equations for the bulk propagator

$$\left[e^{2k|y|}p^2 - m_d^2 - |M|^2 + \partial_y^2\right] \langle \Psi_L^c(p,y)\overline{\Psi_L}(p,y')\rangle$$
$$-ke^{k|y|}p_\mu\sigma^\mu \langle \Psi_R^c(p,y)\overline{\Psi_R}(p,y')\rangle = iM\delta(y-y')$$

➡ the warped effect is vanishing away at the low-energy limit

The result is

$$M_\nu = \frac{1}{\Lambda' L}\frac{\widetilde{M}L\cosh(\widetilde{M}L) - m_d L\sinh(\widetilde{M}L)}{\sinh(\widetilde{M}L)}\frac{m^{\mathrm{T}}m}{M^*}, \quad \Lambda' = \Lambda e^{-kL}$$

- The neutrino mass is not much affected by the background
- The result is the same for more general metric $ds^2 = \rho(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$
- The seesaw mass can be calculated without knowing the KK expansion

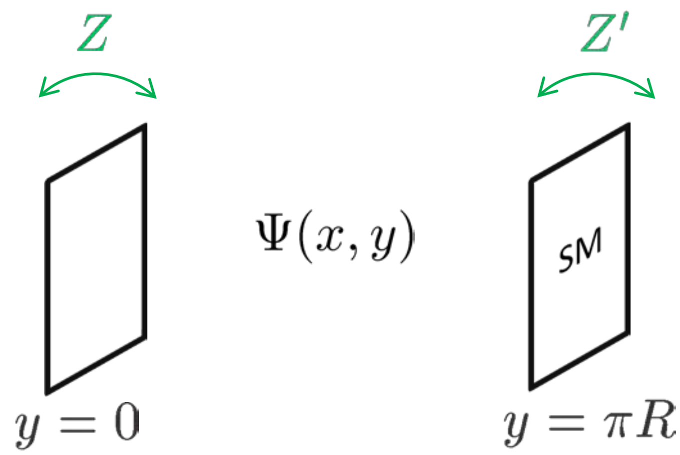
Even with a complicated backgroud where a suitable KK expansion cannot be find, the seesaw neutrino mass is calculable.

Flavor symmetry breaking at the boundaries

General boundary conditions

$$\Psi_i(x,-y) = Z_{ij} \otimes \gamma_5 \Psi_j(x,y)$$
$$\Psi_i(x,-y+2\pi R) = Z'_{ij} \otimes \gamma_5 \Psi_j(x,y)$$

elements of the symmetry



S_3 group $\Rightarrow$ tri-bimaximal mixing is obtained [Haba,AW,Yoshioka,06]

S_4 group 24 elements: $1,\ Q,\ P,\ Q^2,\ PQP^2,\ \cdots,\ QP.$

Irreducible representations: $\underline{1}, \underline{1}', \underline{2}, \underline{3}, \underline{3}'$

Exmaple of the triplet
$$Q = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Example I:

$$Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad Z' = 1$$

KK expansion

$$\Psi_i(x,y) = \begin{pmatrix} \sum_n \chi^n_{R_{ij}}(y)\psi^n_{R_j}(x) \\ \sum_n \chi^n_{L_{ij}}(y)\psi^n_{L_j}(x) \end{pmatrix},$$

$$\chi^n_R(y) = \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \cos\left(n\frac{y}{R}\right) & 0 & 0 \\ 0 & \frac{-1}{\sqrt{2}}\cos\left[\left(n-\frac{1}{2}\right)\frac{y}{R}\right] & \frac{1}{\sqrt{2}}\cos\left(n\frac{y}{R}\right) \\ 0 & \frac{1}{\sqrt{2}}\cos\left[\left(n-\frac{1}{2}\right)\frac{y}{R}\right] & \frac{1}{\sqrt{2}}\cos\left(n\frac{y}{R}\right) \end{pmatrix}$$

$$\chi^n_L(y) = \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \sin\left(n\frac{y}{R}\right) & 0 & 0 \\ 0 & \frac{-1}{\sqrt{2}}\sin\left[\left(n-\frac{1}{2}\right)\frac{y}{R}\right] & \frac{1}{\sqrt{2}}\sin\left(n\frac{y}{R}\right) \\ 0 & \frac{1}{\sqrt{2}}\sin\left[\left(n-\frac{1}{2}\right)\frac{y}{R}\right] & \frac{1}{\sqrt{2}}\sin\left(n\frac{y}{R}\right) \end{pmatrix}$$

Majorana mass matrix after the seesaw

$$M_\nu = \begin{pmatrix} A & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}(A+B) & \frac{1}{\sqrt{2}}(A-B) \\ 0 & \frac{1}{\sqrt{2}}(A-B) & \frac{1}{\sqrt{2}}(A+B) \end{pmatrix}, \quad \begin{cases} A = \frac{1}{\Lambda R}\frac{|M|R}{\tanh(\pi|M|R)}\frac{m^2}{M^*} \\ B = \frac{1}{\Lambda R}|M|R\tanh(\pi|M|R)\frac{(m^c)^2}{M} \end{cases}$$

- only one mixing angle
- degenerate masses

Example II: $Z = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad Z' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

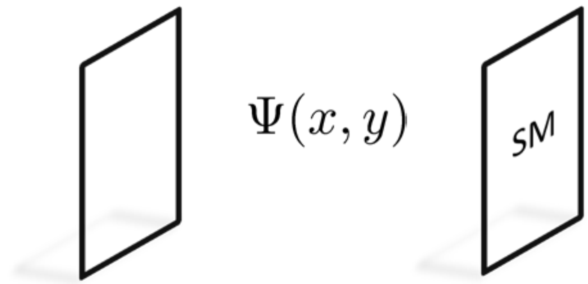
The result is

$$M_\nu = \frac{1}{\Lambda R}\Bigg[\overbrace{\frac{s|M|R}{c+1/2}\frac{m^2}{M^*}}^{m_1} \begin{pmatrix} \frac{4}{6} & \frac{-2}{6} & \frac{-2}{6} \\ \frac{-2}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{-2}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} + \overbrace{\frac{|M|R}{\tanh(\pi|M|R)}\frac{m^2}{M^*}}^{m_2} \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$
$$- \underbrace{\frac{s|M|R}{c+1/2}\frac{(m^c)^2}{M}}_{m_3} \begin{pmatrix} & & \\ & \frac{1}{2} & \frac{-1}{2} \\ & \frac{-1}{2} & \frac{1}{2} \end{pmatrix} - \cancel{\frac{|M|R}{c+1/2}\frac{m m^c}{|M|}} \begin{pmatrix} & \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{2} & \\ \frac{-1}{2} & & \frac{1}{2} \end{pmatrix} \Bigg]$$

$c \equiv \cosh(2\pi|M|R)$, $s \equiv \sinh(2\pi|M|R)$

- S_4 is completely broken
- $m^c = 0 \to$ tri-bimaximal mixing with inverted hierarchy
- $MR \gg 1 \to$ tri-bimaximal

Summary



We have explored ``bulk seesaw”

- Inverse seesaw (bulk Dirac mass)
- Geometry free nature of the neutrino mass
- Flavor symmetry breaking at the boundaries

Deviation from the tri-bimaximal mixing

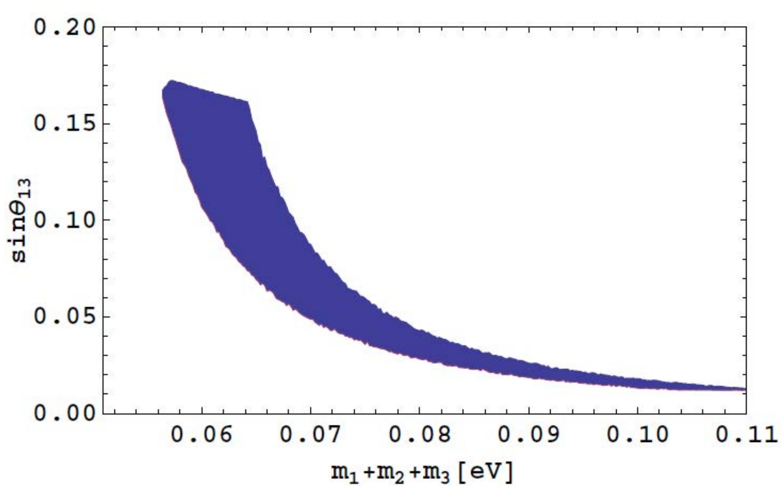
$$M_\nu = \frac{-|M|}{\Lambda} V_{\text{tri-bi}} \begin{pmatrix} \frac{-2s}{2c+1}\frac{m^2}{M^*} & 0 & \frac{\sqrt{3}}{2c+1}\frac{mm^c}{|M|} \\ 0 & \frac{-1}{\tanh(\pi|M|R)}\frac{m^2}{M^*} & 0 \\ \frac{\sqrt{3}}{2c+1}\frac{mm^c}{|M|} & 0 & \frac{2s}{2c+1}\frac{(m^c)^2}{M} \end{pmatrix} V^{\rm T}_{\text{tri-bi}},$$

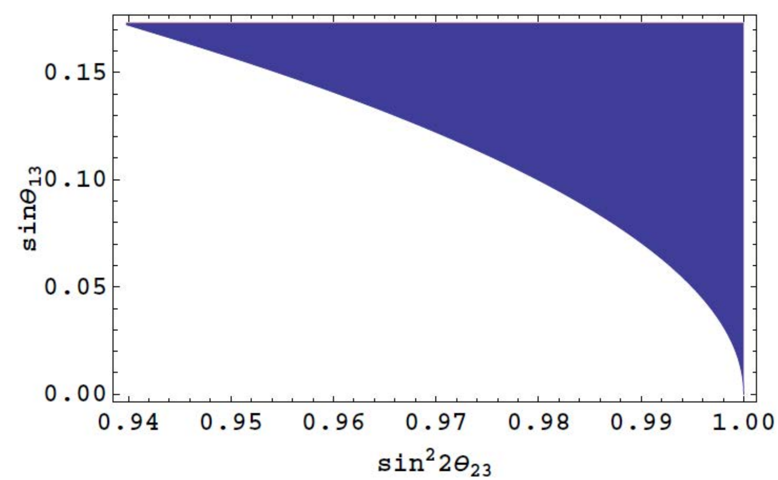
$$U_{e2} = \frac{1}{\sqrt{3}} e^{i\rho}$$

$$U_{e3} = \frac{2i}{\sqrt{6}} \sin\theta e^{i\rho}$$

$$U_{\mu 3} = -i\left(\frac{1}{\sqrt{2}}\cos\theta e^{i\sigma} + \frac{1}{\sqrt{6}}\sin\theta e^{i\rho}\right)$$

3 effective parameters: $|m|^2/\Lambda,\ |M|R,\ |m^c|/|m|$





Charged-lepton sector

For Example

	e_R	(μ_R, τ_R)	(L_e, L_μ, L_τ)	H	(ϕ_1, ϕ_2, ϕ_3)
S_4	$\underline{1}$	$\underline{2}$	$\underline{3}$	$\underline{1}$	$\underline{3}$

$$M_\ell = vY_s \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + vY_d \begin{pmatrix} 0 & 0 & 0 \\ \alpha_1 & \omega^2\alpha_2 & \omega\alpha_3 \\ \alpha_1 & \omega\alpha_2 & \omega^2\alpha_3 \end{pmatrix}, \qquad \alpha_i \equiv \langle\phi_i\rangle/\Lambda$$

$$\alpha_1 v \sim m_e,\ \alpha_2 v \sim m_\mu,\ \alpha_3 v \sim m_\tau$$

$\Rightarrow$ small mixing for the left-handed direction