

# Is $m_h$ sensitive to the Majorana scale in a MSSM-seesaw model ?

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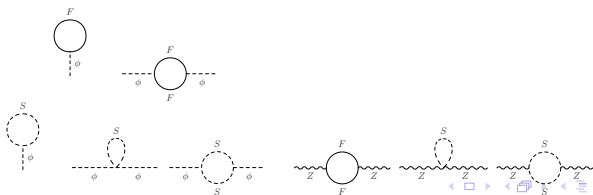
## CALCULATION

- **1 loop** radiative corrections to the lightest Higgs mass of the **MSSM-seesaw** model for **1 generation**  $\nu - \tilde{\nu}$ . (3 gen work in progress)
- 1 loop corrected  $M_h, M_H \rightarrow$  poles of the propagator matrix  $\rightarrow$  solution of the eq:

$$\left[ p^2 - m_{h \text{ tree}}^2 + \hat{\Sigma}_{hh}(p^2) \right] \left[ p^2 - m_{H \text{ tree}}^2 + \hat{\Sigma}_{HH}(p^2) \right] - \left[ \hat{\Sigma}_{hH}(p^2) \right]^2 = 0$$

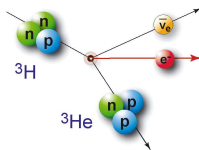
$$\hat{\Sigma}_{hh}(p^2) = \Sigma_{hh}(p^2) + \delta Z_{hh}(p^2 - m_{h, \text{tree}}^2) - \delta m_h^2$$

- New corrections neutrino/sneutrino sector  $\rightarrow \Delta m_h^{\nu/\tilde{\nu}} = M_h^{\nu/\tilde{\nu}} - M_h$



# Why Majorana neutrinos?

- neutrino oscillations  $\Rightarrow$  at least two massive neutrinos
- tritium beta decay exp. (Mainz, Troitsk)  $\Rightarrow m_{\nu_e} < 2.3 \text{ eV}$  (95% C.L.)



- simplest way to explain  $\nu$  masses introduction of  $\nu_R$ 
  - Dirac terms  $\mathbf{m}_D \bar{\nu}_L \nu_R$
  - Majorana terms  $\mathbf{m}_M \nu_R^c \nu_R$  allowed
- Majorana masses violate lepton number, possible explanation of BAU via Leptogenesis
- If heavy Majorana  $\nu$ , one can have large  $Y_\nu$  couplings  $\rightarrow$  phenomenologically relevant
  - Dirac  $\Rightarrow Y_\nu \sim O(10^{-12})$  Majorana  $\Rightarrow$  up to  $Y_\nu \sim O(1)$

# Why radiative corrections to $m_h$ due to Majorana $\nu$ ?

- Indirect search of new physics via radiative corrections is a powerful tool for heavy particles that cannot be produced directly (such as LFV)
- $m_h$  precision observable  $\rightarrow$  prospects in precision measurements on SM-like Higgs boson mass
  - LHC  $\sim 0.2$  GeV
  - ILC  $\sim 0.05$  GeV
- In the MSSM, higher order corrections are crucial
  - $m_{h,\text{tree}} < M_Z$  (free parameter in the SM)
  - Leading higher order correction  $\rightarrow$  Yukawa sector
$$\Delta m_h^2 \sim G_\mu m_t^4 \log \frac{m_t^2}{m_\nu^2}$$
  - 2-loop corrections:  $m_h < 135$  GeV
- Majorana neutrinos can have large  $Y_\nu \simeq Y_t$   
 $\rightarrow$  Do they affect  $\Delta m_h$ ?
- In the SM:

$$V_{\text{eff},\nu,\text{heavy}} \simeq \frac{1}{64\pi^2} m_M^2 Y_\nu^2 H^2 \left( \log \frac{m_M^2}{\mu^2} - \frac{3}{2} + \dots \right)$$

# Seesaw type I

$$-\mathcal{L}_\nu = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}. m_D = Y_\nu v_2; v_2 = v \sin \beta$$

$$m_{\nu, N} = \frac{1}{2} \left( m_M \mp \sqrt{m_M^2 + 4m_D^2} \right) \xrightarrow{m_D < m_M} \begin{cases} m_\nu \sim -\frac{m_D^2}{m_M} \text{ (light)} \\ m_N \sim m_M \text{ (heavy)} \end{cases}$$



If  $m_M \sim 10^{14}$  GeV one can get  $m_\nu \sim 0.1$  eV with  $Y_\nu \sim Y_t \sim \mathcal{O}(1)$

SUSY preserving terms + SOFT susy breaking terms

$$W_{MSSM+\nu\tilde{\nu}} = \epsilon_{ij} \left[ \mu H_1^i H_2^j + Y_\nu \hat{H}_2^i \hat{L}^j \hat{N} \right] + \frac{1}{2} \hat{N} m_M \hat{N}; \hat{N} = (\tilde{\nu}_R^*, (\nu_R)^c)$$

$$V_{\text{soft}}^{\tilde{\nu}} = m_L^2 \tilde{\nu}_L^* \tilde{\nu}_L + m_R^2 \tilde{\nu}_R^* \tilde{\nu}_R + (Y_\nu A_\nu H_2^2 \tilde{\nu}_L \tilde{\nu}_R^* + m_M B_\nu \tilde{\nu}_R \tilde{\nu}_R + \text{h.c.}) .$$

New interactions in  $\mathcal{L}_{\tilde{\nu}H}$  with respect to the Dirac case

$$\mathcal{L}_{\tilde{\nu}H} = \begin{cases} -\frac{g_D m_M}{2M_W \sin \beta} [(\tilde{\nu}_L \tilde{\nu}_R + \tilde{\nu}_L^* \tilde{\nu}_R^*)(H \sin \alpha + h \cos \alpha)] \\ -i \frac{g_D m_M}{2M_W \sin \beta} [(\tilde{\nu}_L \tilde{\nu}_R - \tilde{\nu}_L^* \tilde{\nu}_R^*) A \cos \beta] \\ + \text{usual int. terms } \tilde{f} \tilde{f} h_i, \tilde{f} \tilde{f} h_i h_i \end{cases}$$

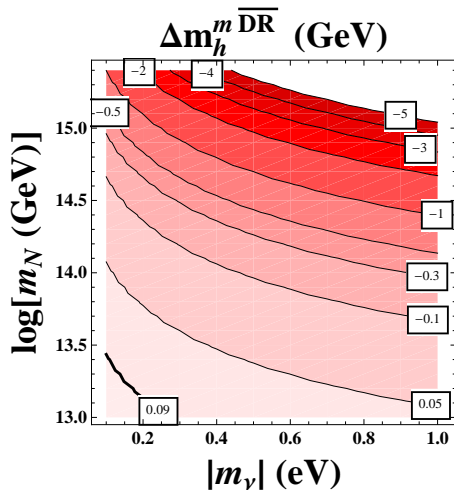
seesaw limit:  $m_M \gg$  all the other scales involved  
 2 light  $\tilde{\nu}$  and 2 heavy  $\tilde{\nu}$

$$m_{\tilde{\nu}_+, \tilde{\nu}_-}^2 = m_L^2 + \frac{1}{2} M_Z^2 \cos 2\beta \mp 2m_D^2 (A_\nu - \mu \cot \beta - B_\nu) / m_M$$

$$m_{\tilde{N}_+, \tilde{N}_-}^2 = m_M^2 \pm 2B_\nu m_M + m_R^2 + 2m_D^2 .$$

# Contourplot of $\Delta m_h^{m\overline{\text{DR}}}$ as a function of $m_N$ and $|m_\nu|$

$$A_\nu = B_\nu = m_{\tilde{L}} = m_{\tilde{R}} = 10^3 \text{ GeV}, \tan \beta = 5, M_A = \mu = 200 \text{ GeV}$$



- $\Delta m_h^{m\overline{\text{DR}}} < 0.1 \text{ GeV}$  if  $10^{13} \text{ GeV} < m_M < 10^{14} \text{ GeV}$  (or, equivalently,  $10^{13} \text{ GeV} < m_N < 10^{14} \text{ GeV}$ ) and  $0.1 \text{ eV} < |m_\nu| < 1 \text{ eV}$
- $\Delta m_h^{m\overline{\text{DR}}}$  change to negative sign and grow in size for larger  $m_M$  and/or  $|m_\nu|$  values (up to  $\sim -5 \text{ GeV}$  for  $m_M = 10^{15} \text{ GeV}$  and  $|m_\nu| = 1 \text{ eV}$ )

# The seesaw limit

- expansion of  $\hat{\Sigma}_{hh}^{m\overline{DR}}$  in powers of the seesaw parameter  $\xi = \frac{m_D}{m_M}$

$$\hat{\Sigma}(p^2) = \underbrace{\left(\hat{\Sigma}(p^2)\right)_{m_D^0}}_{\text{gauge-MSSM}} + \underbrace{\left(\hat{\Sigma}(p^2)\right)_{m_D^2} + \left(\hat{\Sigma}(p^2)\right)_{m_D^4} + \dots}_{\text{Yukawa}}$$

- The **gauge contribution** dominates for  $m_M < 10^{12}$  GeV  $\Rightarrow$  contribution of Majorana neutrinos indistinguishable from Dirac neutrinos and from the MSSM without neutrino masses.
- The relevant Yukawa contributions come from the  $\mathcal{O}(m_D^2)$  term**

$$\begin{aligned} \left(\hat{\Sigma}_{hh}^{\overline{DR}}(p^2)\right)_{m_D^2} = & \left(\frac{g^2 m_D^2}{64\pi^2 M_W^2 \sin^2 \beta}\right) \left[1 - \log\left(\frac{m_M^2}{\mu_{\overline{DR}}^2}\right)\right] \left[-2M_A^2 \cos^2(\alpha - \beta) \cos^2 \beta \right. \\ & \left. + 2p^2 \cos^2 \alpha - M_Z^2 \sin \beta \sin(\alpha + \beta) \left(2\left(1 + \cos^2 \beta\right) \cos \alpha - \sin 2\beta \sin \alpha\right)\right] \end{aligned}$$

- growing of  $\Delta m_h^{m\overline{DR}}$  with  $m_M$  ONLY due to  $Y_\nu \propto \frac{\sqrt{m_M |m_\nu|}}{v_2}$



$$h \text{---} \text{---} h = -i\Sigma_{hh}^{\nu_L \nu_R},$$

$$\begin{aligned}
 -i\Sigma_{hh}^{\nu_L \nu_R} &= (-1)(\mu)^{4-D} g_{h\nu_L \nu_R}^2 \int \frac{d^D k}{(2\pi)^D} \text{Tr} \left\{ \left( \frac{i}{\not{k} - m_{\nu_R}} \right) \left( \frac{i}{\not{p} + \not{k} - m_{\nu_L}} \right) \right\} \\
 &= 4 \frac{g^2 m_D^2}{s_\beta^2 4M_W^2} \int \frac{d^D k}{(2\pi)^D} \left\{ \frac{kp + k^2}{(k^2 - m_{m_M}^2) ((p+k)^2)} \right\}
 \end{aligned}$$

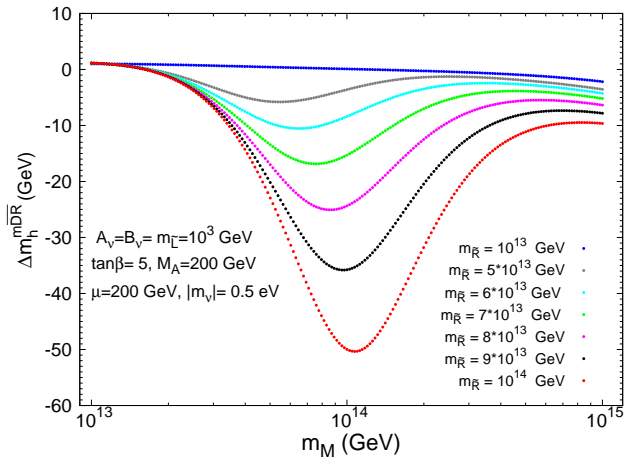
using

$$\int \frac{kp}{[k^2 - m_{m_M}^2] [(k+p)^2]} = \frac{1}{2} \int \frac{1}{k^2 - m_{m_M}^2} - \frac{1}{2} \int \frac{1}{k^2} - \frac{m_{m_M}^2 + p^2}{2} \int \frac{1}{[k^2 - m_{m_M}^2] [(k+p)^2]}$$

using the expression of the one loop integral  $B_0(p^2, m_{m_M}, 0)$  in the limit  $p \ll m_M$

$$-i\Sigma_{hh}^{\nu_L \nu_R} \simeq \frac{g^2 p^2 m_D^2}{16\pi^2 M_W^2 s_\beta^2} \left( 1 - \log \frac{m_M^2}{\mu^2} - \frac{p^2}{m_M^2} \right)$$

# $\Delta m_h^{m\overline{\text{DR}}}$ dependence on $m_M$ for different $m_{\tilde{R}}$



- The corrections are independent of  $m_{\tilde{R}}$  when  $m_{\tilde{R}} < 10^{13}$  GeV
- For  $m_{\tilde{R}} \geq 10^{13}$  GeV  $\Rightarrow \Delta m_h^{m\overline{\text{DR}}}$  can be very big reaching its maximum at  $m_{\tilde{R}} = m_M$

# Conclusions

- The MSSM Higgs sector is **sensitive** to the heavy **Majorana scale** via radiative corrections due to the big Yukawa couplings as it happens in LFV observables such as  $\tau \rightarrow \mu\gamma$ .
- The radiative corrections to  $m_h$  can be relevant when  $m_M > 10^{13}$  GeV, bigger than the anticipated experimental precision (LHC-0.2 GeV, ILC-0.05 GeV)  $\Rightarrow$  they should be taken into account
- These corrections are **negative**  $\Rightarrow$  they push down the lightest Higgs mass.

# Backup frames



# Effective potential approach

The Coleman-Weinberg MSSM effective potential (in the  $\overline{DR}$  scheme which is the supersymmetric version of the  $\overline{MS}$  with dimensional reduction):

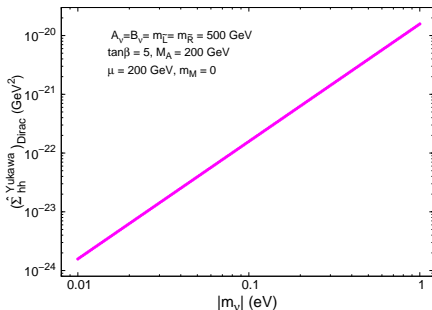
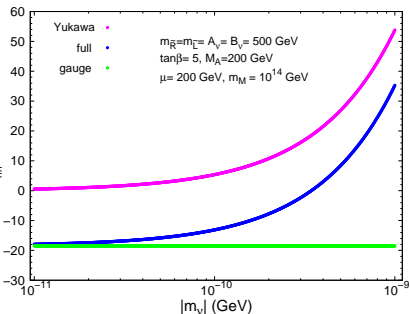
$$V(v_1, v_2; \mu) = V_0(v_1, v_2) + \frac{1}{64\pi^2} \text{Str} M^4 \left[ \log \left( \frac{M^2}{\mu^2} \right) - \frac{3}{2} \right]$$

$\text{Str} = \sum_f (-1)^{2S_f} N_{C_f} N_{S_f}$ .  $N_{C_f}, N_{S_f} \rightarrow$  colour, spin degrees of freedom

$$\left. \frac{\partial V_{\text{eff}}}{\partial \phi} \right|_{\phi=v} = 0, \quad \left. \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} \right|_{\phi=v} = m_h^2,$$

$$\begin{aligned} \Delta m_h^{\nu/\bar{\nu}} &\simeq -\frac{g^2 m_D^4}{32\pi^2 M_W^2 m_M^2} \left( m_{\bar{R}}^2 \left( 1 + 2 \log \frac{m_M^2 + m_{\bar{R}}^2}{\mu^2} \right) - 2m_M^2 \log \frac{m_M^2 + m_{\bar{R}}^2}{m_M^2} \right) \\ &\propto \frac{m_D^4}{m_M^2} \quad \text{if } m_{\bar{R}} \ll m_M \end{aligned}$$

# Dependence of $\hat{\Sigma}_{hh}^{\overline{m}DR}(\rho^2)$ on $m_\nu \rightarrow$ Majorana versus Dirac



● In both cases  $\hat{\Sigma}_{hh}$  grow with the neutrino mass, due to the  $Y_\nu$  dependence on  $m_\nu$

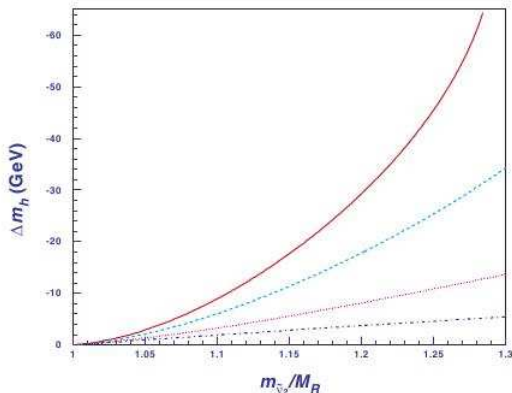
- Dirac case  $\rightarrow Y_\nu = m_\nu/v_2 \rightarrow O(10^{-12})$
- Majorana case  $\rightarrow Y_\nu = m_D/v_2 \sim \sqrt{|m_\nu| m_M}/v_2$

# Renormalization conditions

- OS conditions for the mass counterterms  $\Rightarrow \delta m_{ij} = \text{Re} \Sigma_{ij}(m_{ij}^2)$
- Different schemes adopted for field and  $\tan \beta$  renormalization
  - OS
  - $\overline{\text{DR}}$
  - $m\overline{\text{DR}}$   $\rightarrow [ ]^{\text{div}}$  terms  $\propto \Delta_m \equiv \Delta - \log(m_M^2/\mu_{\overline{\text{DR}}}^2) \rightarrow \mu_{\overline{\text{DR}}} = m_M$ .
- $m\overline{\text{DR}}$   $\rightarrow$  best scheme to minimize higher order corrections  $\rightarrow$  the large logarithms of the heavy scale are avoided



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- Neutrino/sneutrino contribution to  $m_h$ . The solid, dashed, dashed-dotted curve is for  $m_{\bar{R}} = 10^3, 10^7, 10^{11}, 10^{13} \text{ GeV}$ , respectively,  $m_M = 10^{14} \text{ GeV}$
- J. Cao and J. M. Yang, *Phys. Rev. D* **71** (2005) 111701

# Higgs Boson Sector

- The Higgs sector content in the MSSM-seesaw is as in the MSSM

3 neutral bosons :  $h, H$  ( $\mathcal{CP} = +1$ ),  $A$  ( $\mathcal{CP} = -1$ )

2 charged bosons :  $H^+, H^-$

two ind parameters  $\rightarrow \tan \beta = v_2/v_1$  and  $M_A^2 = -m_{12}^2(\tan \beta + \cot \beta)$

$$m_{H,h}^2{}_{\text{tree}} = \frac{1}{2} \left[ M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta} \right]$$

$$m_{h}^2{}_{\text{tree}} \leq M_Z |\cos 2\beta| \leq M_Z \quad m_{h_{\text{SM}}}^2 = \frac{1}{2} \lambda v^2$$

- Higher-order corrections to  $m_h$

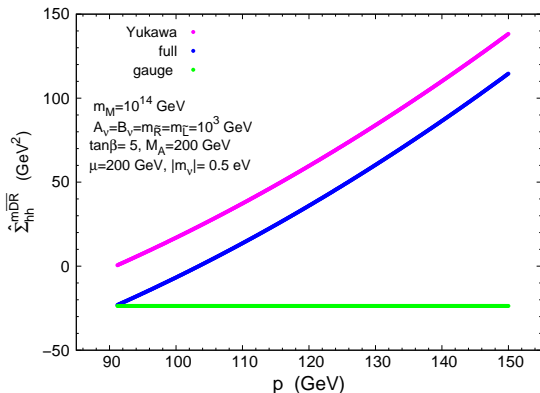
$M_h, M_H \rightarrow$  poles of the propagator matrix  $\rightarrow$  solution of the eq:

$$\left[ p^2 - m_{h}^2{}_{\text{tree}} + \hat{\Sigma}_{hh}(p^2) \right] \left[ p^2 - m_{H}^2{}_{\text{tree}} + \hat{\Sigma}_{HH}(p^2) \right] - \left[ \hat{\Sigma}_{hH}(p^2) \right]^2 = 0$$

$$\hat{\Sigma}_{hh}(p^2) = \Sigma_{hh}(p^2) + \delta Z_{hh}(p^2 - m_{h,\text{tree}}^2) - \delta m_h^2$$

$$\delta m_h^2 = f(\delta M_A^2, \delta M_Z^2, \delta T_H, \delta T_h, \delta \tan \beta)$$

# Dependence of $\hat{\Sigma}_{hh}^{m\overline{DR}}(p^2)$ on $p$



- Strong dependence of  $\hat{\Sigma}_{hh}$  with the external momentum  $\rightarrow$  usual  $p = 0$  approx not valid
- The gauge part is quasi insensitive to  $p \rightarrow \hat{\Sigma}_{hh}^{gauge} \sim p^2 M_Z^2 / m_{SUSY}^2$
- The yukawa part increases with  $p \rightarrow \left( \hat{\Sigma}_{hh}^{m\overline{DR}}(p^2) \right)_{m_D^2} \sim Y_\nu^2 p^2$

# Renormalization conditions

- OS conditions for the mass counterterms

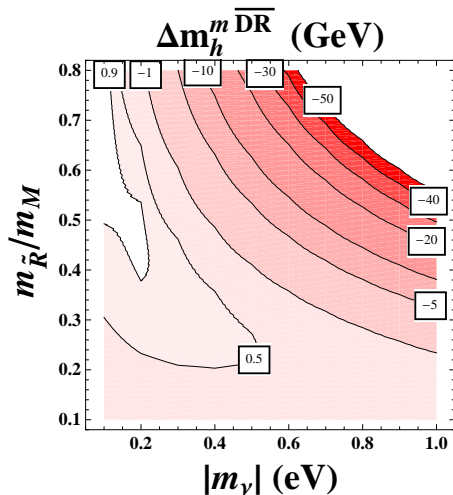
$$\delta M_Z^2 = \text{Re } \Sigma_{ZZ}(M_Z^2), \quad \delta M_W^2 = \text{Re } \Sigma_{WW}(M_W^2), \quad \delta M_A^2 = \text{Re } \Sigma_{AA}(M_A^2). \\ \delta T_h = -T_h, \quad \delta T_H = -T_H.$$

- Different schemes adopted for field and  $\tan \beta$  renormalization
  - OS
  - $\overline{\text{DR}}$
  - $m\overline{\text{DR}}$   $\rightarrow$   $[ ]^{\text{div}}$  terms  $\propto \Delta_m \equiv \Delta - \log(m_M^2/\mu_{\overline{\text{DR}}}^2) \rightarrow \mu_{\overline{\text{DR}}} = m_M.$
- $m\overline{\text{DR}}$   $\rightarrow$  best scheme to minimize higher order corrections  $\rightarrow$  the large logarithms of the heavy scale are avoided

# Contourplot of $\Delta m_h^{m\overline{DR}}$ as a function of $m_{\tilde{R}}/m_M$ and $|m_\nu|$

$$m_M = 10^{14} \text{ GeV},$$

$$A_\nu = B_\nu = m_{\tilde{L}} = 10^3 \text{ GeV}, \tan \beta = 5, M_A = \mu = 200 \text{ GeV}$$



- Very large negative corrections for large  $m_M$  and  $m_{\tilde{R}}$ , of  $\mathcal{O}(10^{14})$  GeV, and  $|m_\nu|$  of  $\mathcal{O}(1)$  eV:

$$\Delta m_h^{m\overline{DR}} \sim -30 \text{ GeV}$$

for  $m_M = 10^{14}$  GeV,

$m_{\tilde{R}}/m_M = 0.7$  and  $|m_\nu| = 0.6$  eV