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Naturally large Yukawa hierarchies

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The SM fermions gauge invariant kinetic term:

$$\sum_{Q,\ell,u,d,e} \bar{\Psi}_i \, \not{D}_i \Psi_i$$

Only five $\not{D}_i$ for 15 fermions.

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No multiplet structure in the spectrum: $\Rightarrow SSB$

(Quarks only) Broken $U(3)^3$ subgroup and scalar fields

$$\mathcal{G}_{\mathcal{B}} = \mathcal{G}/U(1)_{B,Y} = SU(3)_Q \times SU(3)_u \times SU(3)_d \times U(1)_d$$

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Symmetry breaking: Interpret the SM *explicit* breaking as *spontaneous*, driven by a set of scalar "Yukawa fields":

$$Y_u = (3\,, \bar{3},\, 1)_0, \qquad Y_d = (3\,,\, 1,\, \bar{3})_0, \qquad N_d = (1\,, 1,\, 1)_{-1},$$

Assignments such that the effective Yukawa Lagrangian is invariant under $\mathcal{G}$:

$$-\mathcal{L}_Y = \frac{1}{\Lambda} \bar{Q}\, Y_u\, u\, H \;+\; \frac{1}{\Lambda^2} N_d\, \bar{Q}\, Y_d\, d\, \tilde{H}$$

Scalar invariants and TAD parametrization

Singular value decomposition for the non-Abelian fields:

$$Y_u = \mathcal{V}_u^\dagger \, \chi_u \, \mathcal{U}_u \,, \qquad\qquad Y_d = \mathcal{V}_d^\dagger \, \chi_d \, \mathcal{U}_d \,.$$

$\mathcal{V}, \mathcal{U}$ unitary field matrices, $\chi = \mathrm{diag}\,(u_1,\, u_2,\, u_3)$; $u_i \geq 0$.

Relevant objects: $Y \to V_Q Y V_u^\dagger$ and $YY^\dagger \to V_Q (YY^\dagger) V_Q^\dagger$

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$SU(N)$ invariants: Renormalizb. [Wkpd] Non-ren $D > 4$

$$T = \mathrm{Tr}(YY^\dagger) = \textstyle\sum_i u_i^2; \qquad (T^2)$$

$$A = \mathrm{Tr}\left[\mathrm{Adj}\left(YY^\dagger\right)\right] = \tfrac{1}{2}\textstyle\sum_{i\neq j} u_i^2 u_j^2$$

$$\mathcal{D} \equiv e^{i\delta} D = \mathrm{Det}(Y) = e^{i\delta} \textstyle\prod_i u_i$$

$$T_m^n = T\left[(YY^\dagger)^m\right]^n$$

$$A_m^n = A\left[(YY^\dagger)^m\right]^n$$

$$\mathcal{D}_m^n = \mathcal{D}\left[Y^n {Y^\dagger}^m\right]$$

$(\mathcal{D}^*)$; $\quad \delta = \mathrm{Arg}\left(\mathcal{V}^\dagger \mathcal{U}\right)$.

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$(\mathcal{D}^*)$; $\quad \delta = \mathrm{Arg}\left(\mathcal{V}^\dagger \mathcal{U}\right)$. $\quad$ In $SU(2)$: $A = T$; $\quad$ in $SU(3)$: $T_2 = T^2 - 2A$;

in $SU(4)$: $T^3 + 2T_3 - 3T\,T_2 = 6A$; $\qquad$ etc.

Scalar potential and classification of the vacua

$$V = \frac{1}{\Lambda^4}\hat{V} = \lambda\left[T - \frac{m^2}{2\lambda}\right]^2 + \tilde{\lambda}' A + \underbrace{\tilde{\mu}\,\mathcal{D} + \tilde{\mu}^*\,\mathcal{D}^*}_{2\,\mu\,\cos(\phi+\delta)\,D}$$

$$\langle T\rangle = \frac{m^2}{2\lambda}; \quad A\begin{cases}\max: \ \langle\chi\rangle_s = (u,u,u)\\ 0: \quad \langle\chi\rangle_t = (0,0,u_t)\end{cases}; \quad D\begin{cases}\max \ \langle\chi\rangle_s = (u,u,u)\\ 0: \quad \langle\chi\rangle' = (0,u',u')\end{cases}$$

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(1): $\tilde{\lambda}' = -\lambda' < 0$: $\Rightarrow$ $A_{\max}$, $\langle\chi\rangle_s$, $D_{\max}$, $\langle\delta\rangle = \pi - \phi$

(2): $\tilde{\lambda}' = \lambda' > 0$: $\Rightarrow \begin{cases} \frac{\mu^2}{m^2} > \mathcal{F}(\frac{\lambda'}{\lambda}): \quad D_{\max}, \ \langle\chi\rangle_s; \quad \langle\delta\rangle = \pi - \phi \\ \frac{\mu^2}{m^2} < \mathcal{F}(\frac{\lambda'}{\lambda}): \quad A = 0, \ \langle\chi\rangle_t, \ D = 0, \ \langle\delta\rangle = ? \end{cases}$

V admits vacua with $\langle\chi\rangle_t = (0,0,u_t)$!

Lifting the vanishing entries: $(0,0,1) \to (\epsilon_1, \epsilon_2, 1)$

With all parameters $\mathcal{O}(1)$, adding $T^n_m,\ A^n_m, D^n_m,\ (A-\Lambda^2)^2$ etc. can only yield: $\langle\chi\rangle \sim (1,1,1);\ \ (0,1,1);\ \ (0,0,1)$

If we add a Log corrections to 3- and 4-points interactions:

$$V_{\mathcal{D}} \longrightarrow V_{\mathcal{D}} + V_{\mathcal{D}}^{(1)} = 2\mu\cos(\phi+\delta)\, D(1+\underline{c_D\, \log D})$$

$$\langle D\rangle = e^{-\left(\frac{1}{c_D}+1\right)} \equiv \epsilon_D$$

$$\langle\delta\rangle = -\phi$$

$$V_A \longrightarrow V_A + V_A{}^{(1)} = \lambda'\, A(1+\underline{c_A\, \log A})$$

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The system has real solutions if $\epsilon_D < \frac{\epsilon_A}{2}$:

$$\langle\chi\rangle \simeq \left(\frac{\epsilon_D^2}{\epsilon_A}, \epsilon_A, 1\right)$$

$$c_A \sim -\left(\log\langle A_u\rangle\right)^{-1} \approx 0.10$$

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$$\frac{N}{32\pi^2} \approx 0.06$$

Why three families ?

I do not know, but the observed hierarchy implies $\mathbf{n_f = 3}$:

$\mathbf{n_f > 4}$:	$\mathrm{Dim}(A),\ \mathrm{Dim}(\mathcal{D}) > 4$	non-ren
$\mathbf{n_f = 4}$:	$\mathrm{Dim}(A) = 6$	non-ren
$\mathbf{n_f = 2}$:	$\lambda \left(\mathcal{D} \pm \frac{\mu^2}{2\lambda}\right)^2$	Log terms are not relevant.

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Coupling the up and down quark sectors

The only relevant term is: $A_{ud} \equiv T_u \cdot T_d - T_{ud}$

where: $T_{ud} = \mathrm{Tr}(Y_u Y_u^\dagger Y_d Y_d^\dagger) \;=\; \mathrm{Tr}\left(\mathscr{V}^\dagger \chi_u^2 \mathscr{V} \chi_d^2\right)$

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Let us rewrite:

$$+A_{ud} = \sum_{ij} \left(1 - |\mathscr{V}_{ij}|^2\right) u_i^2 d_j^2 \xrightarrow{\langle\ldots\rangle} + \underbrace{\left(1 - \langle|\mathscr{V}_{tb}|^2\rangle\right)}_{\text{at minimum: } O(\epsilon)} \times \mathcal{O}(1) \; + \; \mathcal{O}(\epsilon)$$

$\langle|\mathscr{V}_{tb}|^2\rangle$ is driven to values exponentially close to 1.
The family structure for the quarks emerges.

Conclusions

1. The SM fermions are likely to belong triplets of a Spontaneously Broken fundamental symmetry.
2. The renormalizable potential of the "Yukawa fields" Y_u, Y_d, Y_e admits vacua with singular values $\chi \sim (0,0,1)$.
3. Exponents of the negative inverse of loop coefficients $\exp(-1/c_{\text{loop}})$ are of about the right size to account for the fermion mass hierarchies.
4. The vev of the Y_u-Y_d mixed field matrix approaches a diagonal form, when the same ordering for the singular values χ_u and χ_d is taken.
5. The MFV hypothesis is automatically realized.