D_{14} - A Symmetry for Quarks and Leptons

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H/Meloni; quarks only: Blum/H

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Outline

- Observations: fermion masses and mixings
- Properties of the group D_{14}
- Quarks and D_{14}: prediction of θ_C
  (Blum/H ('09))
- D_{14} also for leptons
  (H/Meloni (to appear))
- Conclusions

Observations: Fermion Masses and Mixings

	Mass at M_Z	in units of $m_t(M_Z)$
u	(1.7 ± 0.4) MeV	λ^8
c	(0.62 ± 0.03) GeV	λ^4
t	(171 ± 3) GeV	1

	Mass at M_Z	in units of $m_b(M_Z)$
d	(3.0 ± 0.6) MeV	λ^4
s	(54 ± 8) MeV	λ^2
b	(2.87 ± 0.03) GeV	1

	Mass at M_Z	in units of $m_\tau(M_Z)$
e	$(0.486570161 \pm 0.000000042)$ MeV	$\lambda^{4\div5}$
μ	$(102.7181359 \pm 0.0000092)$ MeV	λ^2
τ	$1.74624^{+0.00020}_{-0.00019}$ GeV	1

Observations: Fermion Masses and Mixings

- Mild hierarchy among light neutrino masses
  - Two known mass squared differences Δm^2_{21} and $|\Delta m^2_{31}|$ ($2\,\sigma$)

$$\Delta m^2_{21} = \left(7.59^{+0.44}_{-0.37}\right) \cdot 10^{-5}\ \mathrm{eV}^2 \quad \text{and} \quad |\Delta m^2_{31}| = \left(2.40^{+0.24}_{-0.22}\right) \cdot 10^{-3}\ \mathrm{eV}^2$$

  - Cosmological data give upper bound on m_0

$$\sum m_i \lesssim 0.7\ \mathrm{eV} \qquad (2\,\sigma)$$

  - The bounds on m_β and $|m_{ee}|$ also constrain m_0

$$m_\beta \leq 2.2\,\mathrm{eV} \quad \text{and} \quad |m_{ee}| \leq (0.2 ... 1)\,\mathrm{eV}$$

- Normal (NH) & inverted hierarchy (IH) still allowed

Observations: Fermion Masses and Mixings

- The mixing pattern is very peculiar

$$\sin^2(\theta^l_{12}) = 0.318^{+0.042}_{-0.028}\ ,\ \ \sin^2(\theta^l_{23}) = 0.50^{+0.13}_{-0.11}\ \ \text{and}\ \ \sin^2(\theta^l_{13}) \leq 0.039$$

$$\theta^l_{12} = (34.3^{+2.5}_{-1.7})^\circ\ ,\ \ \theta^l_{23} = (45.0^{+7.5}_{-6.4})^\circ\ \ \text{and}\ \ \theta^l_{13} \leq 11.4^\circ \qquad (2\,\sigma)$$

compare to quark sector

$$\theta^q_{12} \approx 13^\circ\ ,\ \ \theta^q_{23} \approx 2.4^\circ\ \ \text{and}\ \ \theta^q_{13} \approx 0.21^\circ$$

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  $\theta^q_{12} \approx 13^\circ\ ,\quad \theta^q_{23} \approx 2.4^\circ\quad$ and $\quad \theta^q_{13} \approx 0.21^\circ$

- One special mixing pattern: $\mu\tau$ symmetry

$$\sin^2(\theta^l_{23}) = \frac{1}{2}\ ,\quad \sin^2(\theta^l_{13}) = 0$$

$$\Rightarrow U_{PMNS} = \begin{pmatrix} \cos(\theta^l_{12}) & \sin(\theta^l_{12}) & 0 \\ -\frac{\sin(\theta^l_{12})}{\sqrt{2}} & \frac{\cos(\theta^l_{12})}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin(\theta^l_{12})}{\sqrt{2}} & -\frac{\cos(\theta^l_{12})}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Group Theory of D_{14}

- D_{14} belongs to the dihedral groups and is the symmetry group of a regular planar 14-gon
- Its order is 28, i.e. it has 28 distinct elements
- It has 4 + 6 real irred. reps., $\underline{\mathbf{1}}_{\mathbf{i}}$, $\mathrm{i} = 1, ..., 4$ and $\underline{\mathbf{2}}_{\mathbf{j}}$, $\mathrm{j} = 1, ..., 6$
- Generator relations of D_{14}

$$\mathrm{A}^{14} = \mathbb{1} \; , \;\; \mathrm{B}^2 = \mathbb{1} \; , \;\; \mathrm{A\,B\,A} = \mathrm{B}$$

- Generators

$$\begin{array}{lcl} \underline{\mathbf{1}}_{\mathbf{1}} & : & \mathrm{A} = 1 \; , \;\; \mathrm{B} = 1 \\ \underline{\mathbf{1}}_{\mathbf{2}} & : & \mathrm{A} = 1 \; , \;\; \mathrm{B} = -1 \\ \underline{\mathbf{1}}_{\mathbf{3}} & : & \mathrm{A} = -1 \; , \;\; \mathrm{B} = 1 \\ \underline{\mathbf{1}}_{\mathbf{4}} & : & \mathrm{A} = -1 \; , \;\; \mathrm{B} = -1 \end{array}$$

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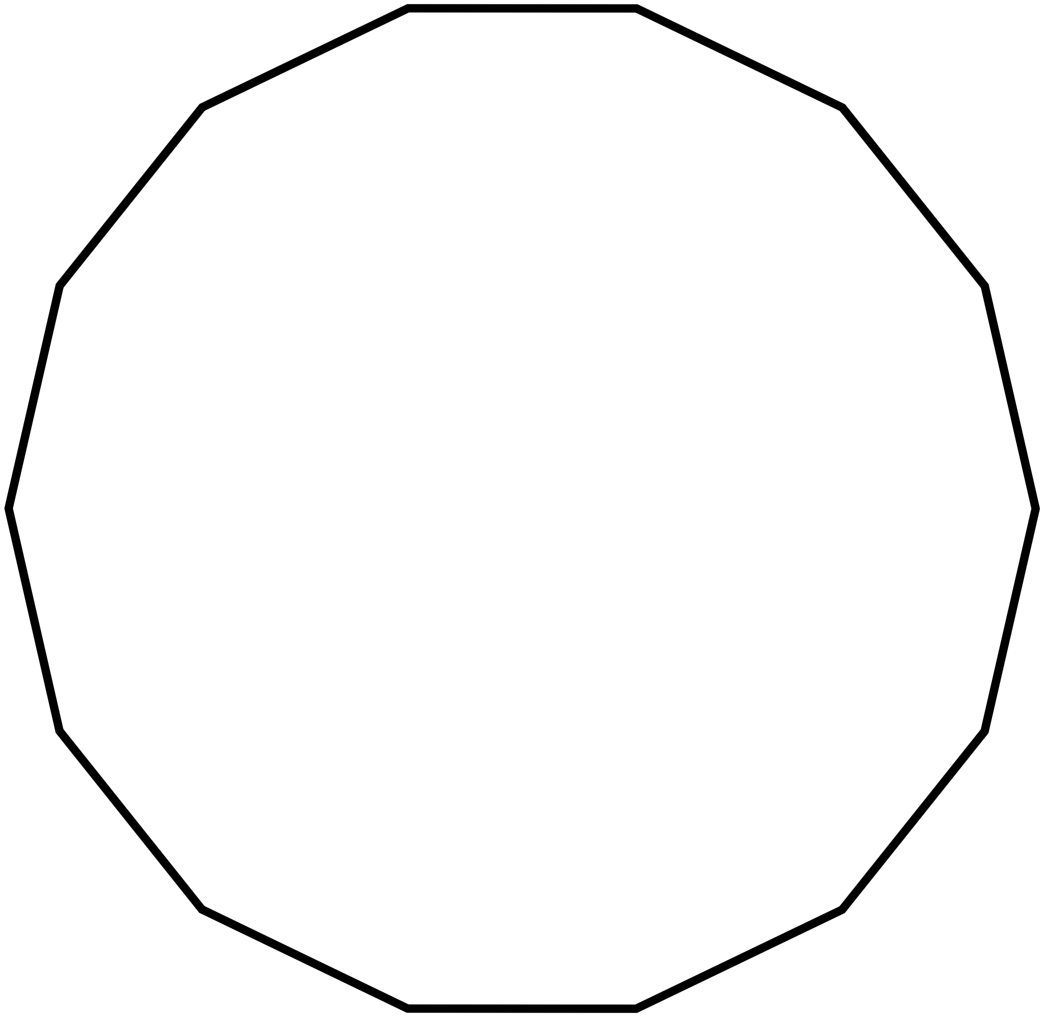
- Generators

$$\mathrm{A} = \begin{pmatrix} \mathrm{e}^{\left(\frac{\pi i}{7}\right)\mathrm{j}} & 0 \\ 0 & \mathrm{e}^{-\left(\frac{\pi i}{7}\right)\mathrm{j}} \end{pmatrix} \; , \; \mathrm{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; , \;\; \mathrm{j} = 1, ..., 6$$

Visualization of D_{14}



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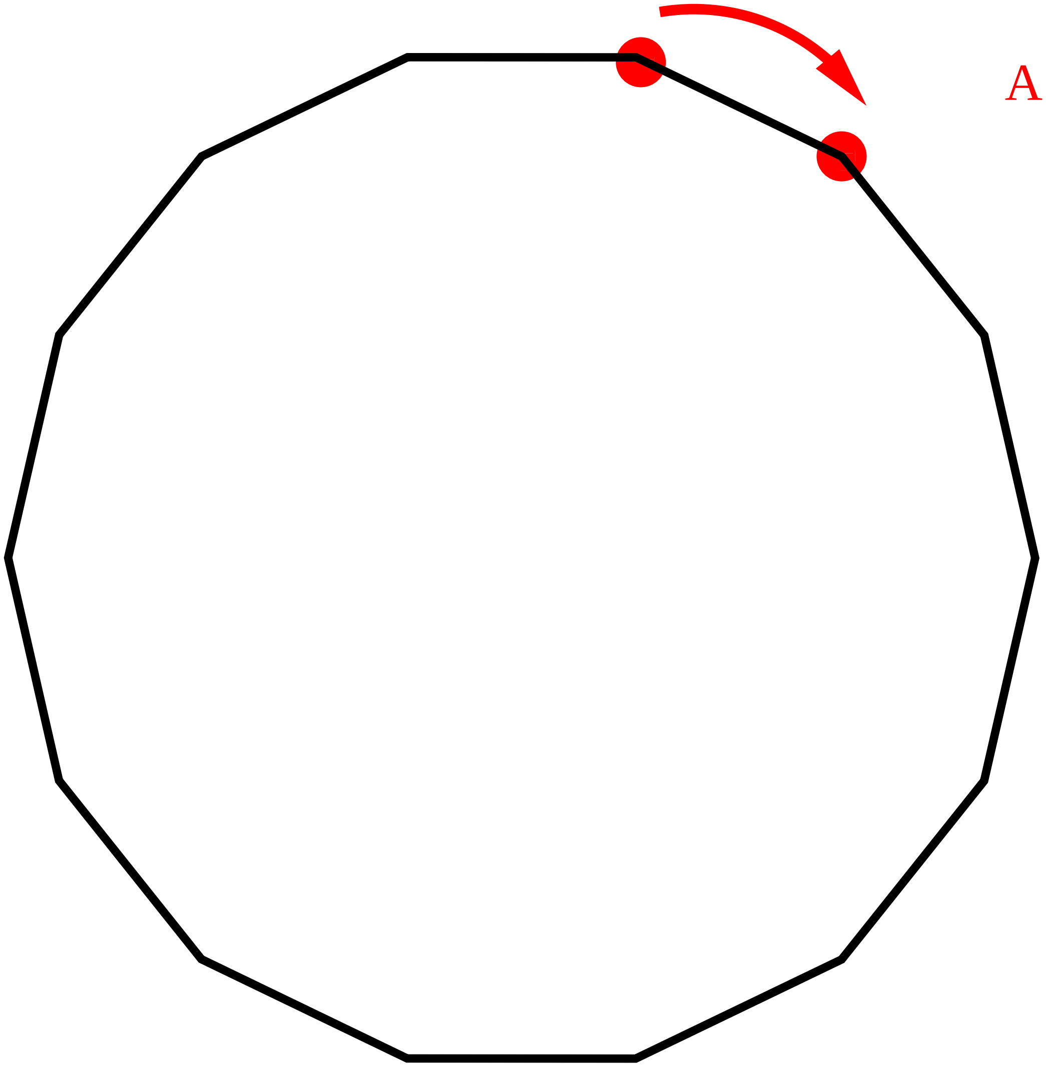

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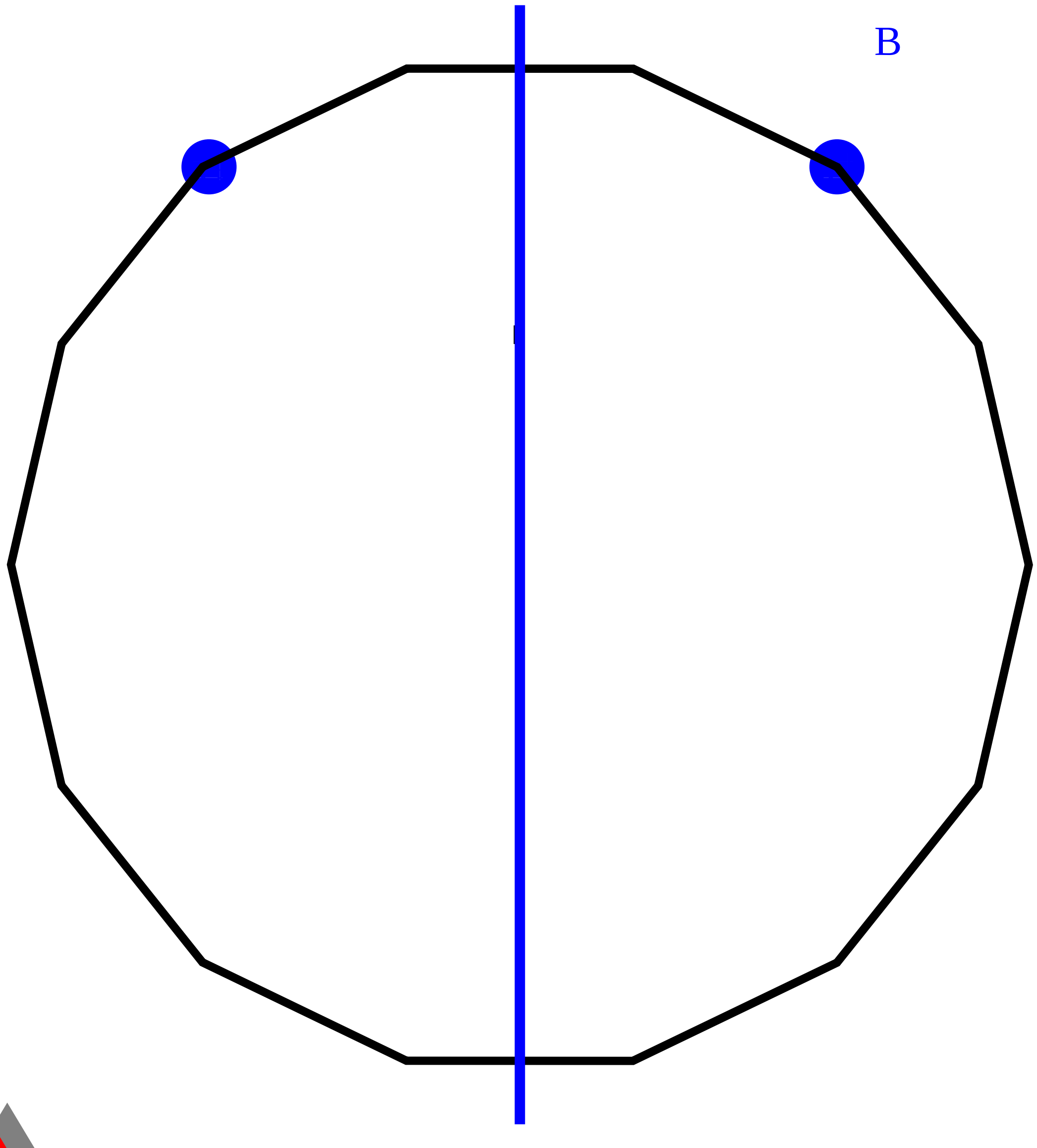

D_{14} Quark Model

(Blum/H ('09))

- Flavor group $G_F = D_{14} \times Z_3 \times U(1)_{FN}$
- Framework: MSSM
- Quarks transform non-trivially under G_F
- MSSM Higgs doublets $h_{u,d}$ are singlets under G_F
- Necessity of gauge singlets (flavons) transforming under G_F

$$\{\psi^u_{1,2}, \chi^u_{1,2}, \xi^u_{1,2}, \eta^u\} \quad \text{and} \quad \{\psi^d_{1,2}, \chi^d_{1,2}, \xi^d_{1,2}, \eta^d, \sigma\}$$

- FN field θ is only charged under $U(1)_{FN}$
- Structure of Yukawa couplings

$$\frac{\theta^2}{\Lambda^4}(Q_D u^c \chi^u \xi^u) h_u \quad \text{and} \quad \frac{1}{\Lambda} Q_3 (b^c \eta^d) h_d$$

Setup of Quark Model

Field	Q_D	Q_3	u^c	c^c	t^c	d^c	s^c	b^c	$h_{u,d}$
D_{14}	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$
Z_3	1	1	1	1	1	ω^2	ω^2	ω^2	1
$U(1)_{FN}$	0	0	2	0	0	1	1	0	0

Field	$\psi^u_{1,2}$	$\chi^u_{1,2}$	$\xi^u_{1,2}$	η^u	$\psi^d_{1,2}$	$\chi^d_{1,2}$	$\xi^d_{1,2}$	η^d	σ	θ
D_{14}	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$
Z_3	1	1	1	1	ω	ω	ω	ω	ω	1
$U(1)_{FN}$	0	0	0	0	0	0	0	0	0	-1

Leading Order in Up Quark Sector

no ϕ

(33) $$Q_3\, t^c\, h_u$$

$1\,\phi$

(13), (23) $$\frac{1}{\Lambda}(Q_D \psi^u) t^c h_u$$

(32) $$\frac{1}{\Lambda} Q_3 (c^c \eta^u) h_u$$

$2\,\phi$

(11), (21) $$\frac{\theta^2}{\Lambda^4}(Q_D u^c \chi^u \xi^u) h_u + \frac{\theta^2}{\Lambda^4}\left(Q_D u^c (\xi^u)^2\right) h_u + \frac{\theta^2}{\Lambda^4}(Q_D \psi^u \eta^u u^c) h_u$$

(12), (22) $$\frac{1}{\Lambda^2}(Q_D c^c \chi^u \xi^u) h_u + \frac{1}{\Lambda^2}\left(Q_D c^c (\xi^u)^2\right) h_u + \frac{1}{\Lambda^2}(Q_D \psi^u)(\eta^u c^c) h_u$$

Leading Order in Down Quark Sector

$$\boxed{1\,\phi}$$

$$(33) \qquad \frac{1}{\Lambda} Q_3 (b^c \eta^d) h_d$$

$$(32) \qquad \frac{\theta}{\Lambda^2}\, Q_3\, s^c \sigma h_d$$

$$(12),\ (22) \qquad \frac{\theta}{\Lambda^2}\, (Q_D \psi^d)\, s^c h_d$$

Vacuum Structure

Up quark sector $\langle \eta^u \rangle \neq 0$

$$\begin{pmatrix} \langle \psi_1^u \rangle \\ \langle \psi_2^u \rangle \end{pmatrix} = v^u \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \langle \chi_1^u \rangle \\ \langle \chi_2^u \rangle \end{pmatrix} = w^u \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \langle \xi_1^u \rangle \\ \langle \xi_2^u \rangle \end{pmatrix} = z^u \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Down quark sector $\langle \eta^d \rangle \neq 0$ and $\langle \sigma \rangle \neq 0$

$$\begin{pmatrix} \langle \psi_1^d \rangle \\ \langle \psi_2^d \rangle \end{pmatrix} = v^d \begin{pmatrix} \mathrm{e}^{-\frac{\pi i k}{7}} \\ 1 \end{pmatrix}, \begin{pmatrix} \langle \chi_1^d \rangle \\ \langle \chi_2^d \rangle \end{pmatrix} = w^d \mathrm{e}^{\frac{\pi i k}{7}} \begin{pmatrix} \mathrm{e}^{-\frac{2\pi i k}{7}} \\ 1 \end{pmatrix},$$

$$\begin{pmatrix} \langle \xi_1^d \rangle \\ \langle \xi_2^d \rangle \end{pmatrix} = z^d \mathrm{e}^{\frac{2\pi i k}{7}} \begin{pmatrix} \mathrm{e}^{-\frac{4\pi i k}{7}} \\ 1 \end{pmatrix}$$

Results for Quarks at Leading Order

Assume $\frac{\langle \Phi^u \rangle}{\Lambda} \approx \epsilon$, $\frac{\langle \Phi^d \rangle}{\Lambda} \approx \epsilon$, $t = \frac{\langle \theta \rangle}{\Lambda} \approx \epsilon \approx \lambda^2 \approx 0.04$

then $\mathcal{M}_u$ and $\mathcal{M}_d$ read

$$\mathcal{M}_u = \begin{pmatrix} -\alpha^u_1 \, t^2 \, \epsilon^2 & \alpha^u_2 \, \epsilon^2 & \alpha^u_3 \, \epsilon \\ \alpha^u_1 \, t^2 \, \epsilon^2 & \alpha^u_2 \, \epsilon^2 & \alpha^u_3 \, \epsilon \\ 0 & \alpha^u_4 \, \epsilon & y_t \end{pmatrix} \langle h_u \rangle$$

$$\mathcal{M}_d = \begin{pmatrix} 0 & \alpha^d_1 \, t \, \epsilon & 0 \\ 0 & \alpha^d_1 \, \mathrm{e}^{-\pi i k/7} \, t \, \epsilon & 0 \\ 0 & \alpha^d_2 \, t \, \epsilon & y_b \, \epsilon \end{pmatrix} \langle h_d \rangle$$

Results for Quarks at Leading Order

Quark Masses

$$m_u^2 : m_c^2 : m_t^2 \quad \sim \quad \epsilon^8 : \epsilon^4 : 1 \,, \quad m_d^2 : m_s^2 : m_b^2 \sim 0 : \epsilon^2 : 1 \,,$$
$$m_b^2 : m_t^2 \quad \sim \quad \epsilon^2 : 1 \quad \text{for} \quad \text{small} \quad \tan\beta$$

CKM matrix

$$|V_{CKM}| = \begin{pmatrix} |\cos(\frac{k\,\pi}{14})| & |\sin(\frac{k\,\pi}{14})| & 0 \\ |\sin(\frac{k\,\pi}{14})| & |\cos(\frac{k\,\pi}{14})| & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \mathcal{O}(\epsilon^4) & \mathcal{O}(\epsilon^2) \\ \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon) \\ \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon^2) \end{pmatrix}$$

$$\Downarrow$$

$k = 1$ or $k = 13$ leads to $|V_{ud}| \approx 0.97493$

Experimental value: $|V_{ud}|_{\rm exp} = 0.97419^{+0.00022}_{-0.00022}$

Completion: Add Leptons

(H/Meloni (to appear))

Goals :

- Add leptons in minimal way
- Predict $\mu\tau$ symmetry in the lepton sector
- Do not disturb quark sector

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Solution

Field	L_1	L_D	e^c	μ^c	τ^c	ν_1^c	ν_D^c	$\chi_{1,2}^e$
D_{14}	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$
Z_7	1	1	ω_7^5	ω_7^5	ω_7^5	ω_7^4	ω_7^4	ω_7^2

Neutrino Sector

Majorana mass matrix for right-handed neutrinos

$$\boxed{1\,\phi}$$

$$(11) \qquad \nu_1^c \nu_1^c \sigma$$

$$(23) \qquad (\nu_D^c \nu_D^c)\sigma$$

leads to

$$\mathcal{M}_R = \begin{pmatrix} \alpha_1^M & 0 & 0 \\ 0 & 0 & \alpha_2^M \\ 0 & \alpha_2^M & 0 \end{pmatrix} \epsilon\,\Lambda$$

Neutrino Sector

Dirac neutrino mass matrix

$$\boxed{1\,\phi}$$

$$(12),(13) \qquad \frac{1}{\Lambda}(L_1 \nu_D^c \xi^u) h_u$$

$$(21),(31) \qquad \frac{1}{\Lambda}(L_D \nu_1^c \chi^u) h_u$$

$$(23),(32) \qquad \frac{1}{\Lambda}(L_D \nu_D^c \psi^u) h_u$$

leads to

$$\mathcal{M}_\nu^D = \begin{pmatrix} 0 & \alpha_1^D & \alpha_1^D \\ \alpha_2^D & 0 & \alpha_3^D \\ -\alpha_2^D & \alpha_3^D & 0 \end{pmatrix} \epsilon \, \langle h_u \rangle$$

Neutrino Sector

Light neutrino mass matrix

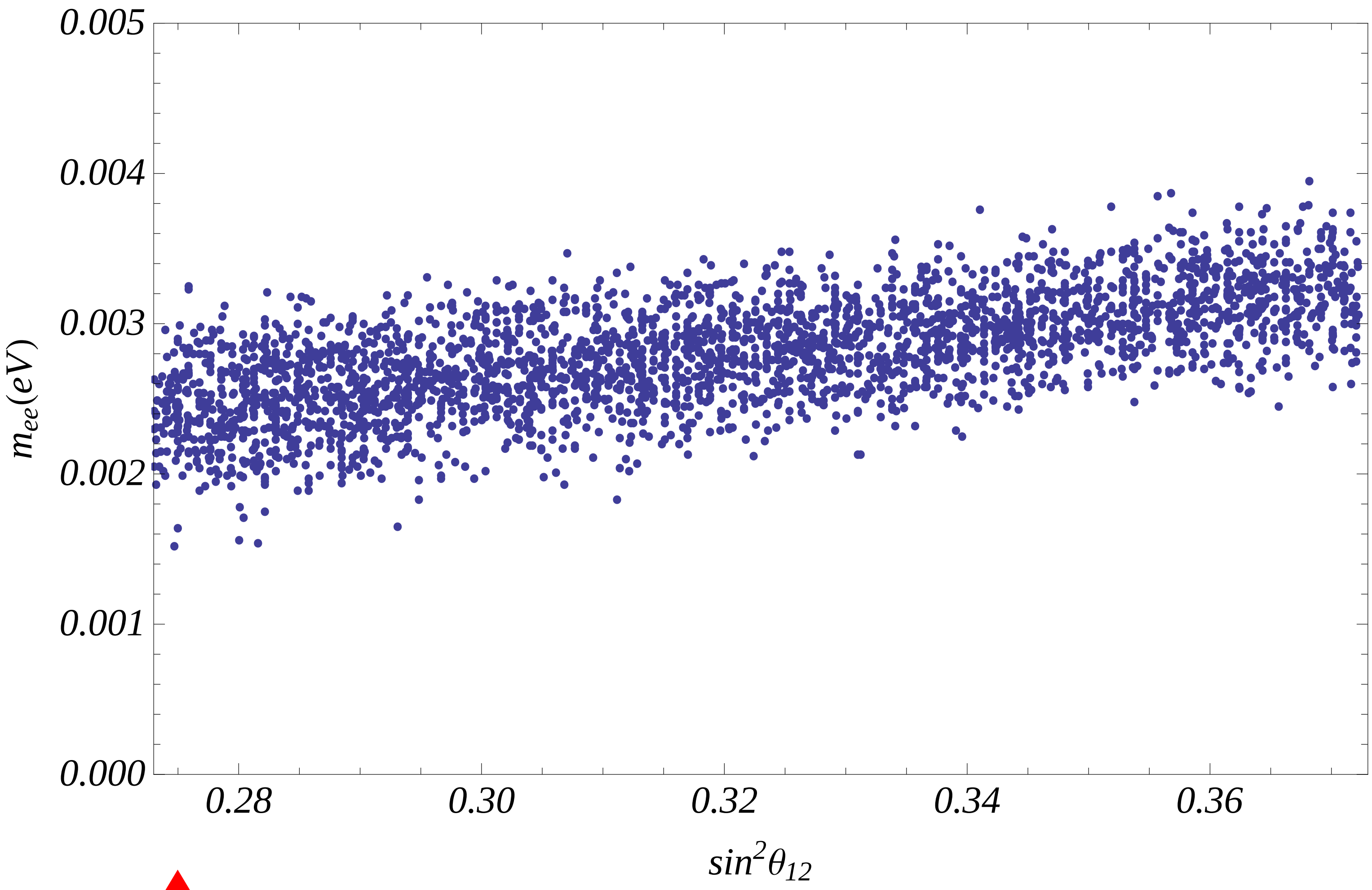
$$\mathcal{M}_\nu = \begin{pmatrix} 2x^2/v & x & x \\ x & z & v-z \\ x & v-z & z \end{pmatrix} \epsilon \langle h_u \rangle^2 / \Lambda$$

- $\mu\tau$ symmetric neutrino mixing
- θ_{12}^ν is given by $\tan(\theta_{12}^\nu) = \sqrt{2} \left| \frac{x}{v} \right|$
- Normal ordering with $m_1 = 0$ is predicted and

$$m_2^2 = \frac{(|v|^2 + 2|x|^2)^2}{|v|^2} \left(\frac{\epsilon \langle h_u \rangle^2}{\Lambda} \right)^2 \quad , \quad m_3^2 = |v - 2z|^2 \left(\frac{\epsilon \langle h_u \rangle^2}{\Lambda} \right)^2$$

- Additional relation $|m_{ee}| = m_2 \sin^2(\theta_{12}^\nu) = \sqrt{\Delta m_{21}^2} \sin^2(\theta_{12}^\nu)$

Neutrino Sector



Charged Lepton Sector

Alignment of new flavon $\chi^e_{1,2}$

$$\begin{pmatrix} \langle \chi^e_1 \rangle \\ \langle \chi^e_2 \rangle \end{pmatrix} = v^e \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

for which just one additional driving field σ^{0e} is needed

$$w_{f,l} = a_l \, \sigma^{0e} \, \chi^e_1 \, \chi^e_2$$

Nota bene: Also this alignment preserves a Z_2 subgroup of D_{14}, because $\underline{\mathbf{2}}_{\mathbf{2}}$ is unfaithful.

Yukawa Operators for Charged Leptons

$\boxed{1\,\phi}$

$$(3\alpha) \qquad \frac{1}{\Lambda}(L_D \chi^e)\,\alpha^c\,h_d$$

$\boxed{2\,\phi}$

$$(2\alpha) \qquad \frac{1}{\Lambda^2}(L_D \chi^e \xi^u)\,\alpha^c\,h_d$$

$\boxed{3\,\phi}$

$$(1\alpha) \qquad \frac{1}{\Lambda^3}(L_1 \chi^e \psi^u \xi^u)\alpha^c h_d$$

$$(1\alpha) \qquad \frac{1}{\Lambda^3}(L_1 \eta^u)(\chi^e \chi^u)\alpha^c h_d$$

Charged Lepton Mass Matrix

For $\frac{v^e}{\Lambda} \approx \epsilon \approx \lambda^2$ we get

$$\mathcal{M}_e = \begin{pmatrix} \alpha_1^e\,\epsilon^3 & \alpha_2^e\,\epsilon^3 & \alpha_3^e\,\epsilon^3 \\ \alpha_4^e\,\epsilon^2 & \alpha_5^e\,\epsilon^2 & \alpha_6^e\,\epsilon^2 \\ \alpha_7^e\,\epsilon & \alpha_8^e\,\epsilon & \alpha_9^e\,\epsilon \end{pmatrix} \langle h_d \rangle$$

- Charged lepton masses $m_e : m_\mu : m_\tau \sim \epsilon^2 : \epsilon : 1$
- Charged lepton mixing angles $\theta^e_{12} \sim \epsilon$, $\theta^e_{13} \sim \epsilon^2$, $\theta^e_{23} \sim \epsilon$

Lepton mixings are nearly $\mu\tau$ symmetric

$$\sin^2(\theta^l_{23}) = \frac{1}{2} + \mathcal{O}(\epsilon)\ ,\ \ \sin(\theta^l_{13}) = \mathcal{O}(\epsilon)\ ,\ \ \sin^2(\theta^l_{12}) = \mathcal{O}(1)$$

Conclusions

- D_{14} for quarks and leptons through minimal extension of the existing quark model
- Prediction of Cabibbo angle and now also of $\mu\tau$ symmetry in lepton sector
- Other mixing angles naturally of correct size
- All fermion mass hierarchies are explained
- Normal ordering in the neutrino sector with $m_1 = 0$ at leading order

Thanks.