Axino effective interactions and thermal production of axino dark matter

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Outline

- Motivation and introduction
- Effective interactions of the axion supermultiplet
- Thermal production of axino in the early Universe
- Conclusion

Motivation and Introduction

* Gauge hierarchy problem

$$m_{higgs}^2 \ll \Lambda^2 \quad (\sim M_{GUT}^2 \quad \text{or} \quad M_{Planck}^2)$$

Without SUSY $\quad m_{higgs}^2 = m_{bare}^2 - \frac{\lambda_t^2}{8\pi^2}\Lambda^2 + \ldots$

With SUSY $\quad m_{higgs}^2 = m_{soft}^2 \left(1 - \frac{3\lambda_t^2}{8\pi^2} \ln \frac{\Lambda^2}{m_{soft}^2} + \ldots\right)$

* Strong CP problem

$$|\bar{\theta}| = |\theta_{QCD} + \mathbf{Arg\,Det}(\lambda_u \lambda_d) + \ldots| \lesssim 10^{-9}$$

With $U(1)_{PQ}$ and axion $\quad V_{axion} = -f_\pi^2 m_\pi^2 \sqrt{\frac{m_u^2 + m_d^2 + 2m_u m_d \cos(a/F_a + \bar{\theta})}{(m_u + m_d)^2}}$

➔ dynamical relaxation of theta: $\left\langle \frac{a}{F_a} + \bar{\theta} \right\rangle = 0$

Peccei and Quinn

SUSY + $U(1)_{PQ}$ (axion) Nilles and Raby

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SUPERSYMMETRY AND THE STRONG *CP* PROBLEM*

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We show how supersymmetric grand unified theories provide a natural solution to the strong *CP* problem. Such theories contain an additional global $U(1)_{PQ}$ chiral (Peccei–Quinn) symmetry which is conserved up to color anomalies. Hence the QCD θ angle is irrelevant. Moreover the PQ symmetry is spontaneously broken at the grand unified scale. As a result the standard Weinberg-Wilczek axion has a mass

$$m_a \sim \frac{\Lambda_{QCD}^2}{\Lambda_{GUM}} \sim 10^{-10}\,\text{eV}$$

which effectively decouples from ordinary matter.

$U(1)_{PQ}$ might solve not only the strong CP problem, but also another puzzle of SUSY model, **the mu-problem.** Kim and Nilles

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THE μ-PROBLEM AND THE STRONG *CP*-PROBLEM

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We investigate a possible connection of a solution of the strong *CP*-problem and the generation of a mass term μ in the low energy Higgs superpotential of supersymmetric models. This possibility comes from the fact that both supersymmetry and the Peccei–Quinn symmetry (to give an acceptable invisible axion) are broken at the same scale.

Axion scale generated by SUSY breaking: $F_a = \langle X \rangle \sim \sqrt{m_{3/2} M_{Planck}}$

mu-term generated by $U(1)_{PQ}$ breaking:

$$\frac{1}{M_{Planck}} X^2 H_u H_d \;\Rightarrow\; \mu \sim \frac{F_a^2}{M_{Planck}} \sim m_{3/2} \sim m_{soft}$$

Supersymmetric axion model necessarily contains the superpartners of axion, the axino and saxion, which can have a variety of cosmological implications.

Axino can be a good candidate for cold (or warm) DM.

Rajagopal, Turner, Wilczek; Covi, Kim, Roszkowski

Thermal production of axinos in the early Universe:

Covi, Kim, Kim, Roszkowski; Brandenburg, Steffen; Strumia

Previous works on thermal axino production are based on

$$\mathcal{L}_{\rm eff} = \frac{1}{32\pi^2 F_a}\int AW^aW^a + h.c \quad (A = s + ia + \theta\tilde{a} + F^A)$$
$$= \frac{1}{32\pi^2 F_a}\left[aG^a_{\mu\nu}\tilde{G}^{a\mu\nu} + i\tilde{a}\sigma^{\mu\nu}\gamma_5\tilde{\lambda}^a G^a_{\mu\nu} - \tilde{a}\tilde{\lambda}^a D^a + ...\right]$$

However, a simple use of this effective interaction alone can lead to highly overestimated axino production rate, in particular for a class of models realizing the Kim-Nilles solution of the mu-problem. Bae, KC, Im

For correct result, we need more careful analysis incorporating other effective interactions of the axion supermultiplet.

Axino effective interactions Bae, KC, Im

Axion effective interactions:

$$\mathcal{L}_{\rm eff} = \frac{C_W}{32\pi^2}\frac{a}{F_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + y_Q\frac{\partial_\mu a}{2F_a}\bar{Q}\gamma^\mu\gamma_5 Q - m_Q\bar{Q}\left[\exp\left(ix_Q\gamma_5\frac{a}{F_a}\right)\right]Q$$

$U(1)_{PQ}$: $\quad a \to a + \alpha F_a, \quad Q \to \exp\left(-\frac{i\alpha x_Q\gamma_5}{2}\right)Q$

$$J^\mu_{PQ} = F_a\partial^\mu a + x_Q\bar{Q}\gamma^\mu\gamma_5 Q$$

$$\partial_\mu J^\mu_{PQ} = \frac{C_{PQ}}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad (C_{PQ} = C_W + x_Q)$$

Change of field basis: $\quad Q \to \exp\left(-iz_Q\gamma_5\frac{a}{2F_a}\right)Q$

➔ Reparameterization of the Wilsonian axion couplings

$$C_W \to C_W + z_Q, \qquad y_Q \to y_Q + z_Q, \qquad x_Q \to x_Q - z_Q$$

For low energy effective lagrangian of axion, one often chooses the field basis for which all fields except the axion are invariant under $U(1)_{PQ}$, and the PQ anomaly is encoded entirely in C_W : Georgi, Kaplan, Randall

$$\mathbf{C_{PQ} = C_W \;, \qquad x_Q = 0}$$

$$\mathcal{L}_{\rm eff} = \frac{C_W}{32\pi^2}\frac{a}{F_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + y_Q\frac{\partial_\mu a}{2F_a}\bar{Q}\gamma^\mu\gamma_5 Q - m_Q\bar{Q}\left[\exp\left(i x_Q\gamma_5\frac{a}{F_a}\right)\right]Q$$

All observables derived from $L_{\rm eff}$ should be invariant under

$$C_W \to C_W + z_Q, \qquad y_Q \to y_Q + z_Q, \qquad x_Q \to x_Q - z_Q$$

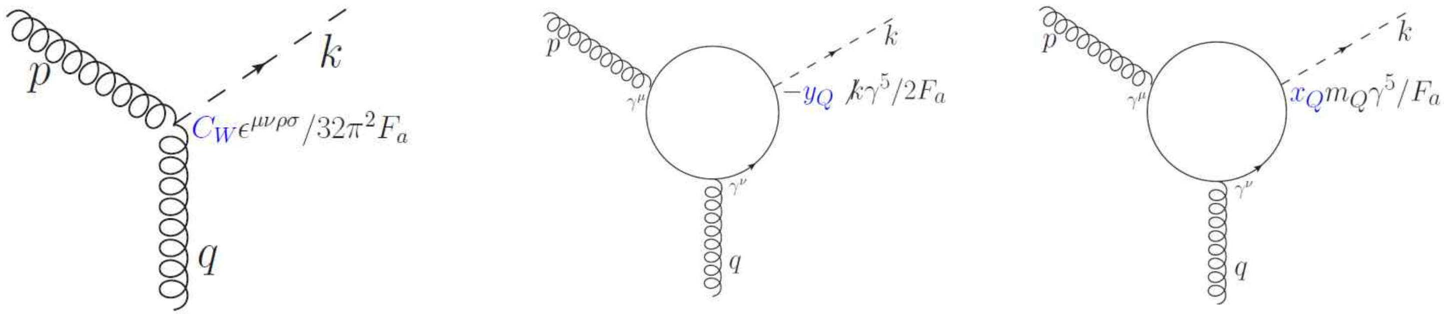
* PQ-breaking axion potential:

$$V_{\rm axion} = \begin{cases} \Lambda^4_{\rm QCD}\cos\left(C_{PQ}\, a/F_a + \bar{\theta}\right) & \text{for} \quad m_Q \gg \Lambda_{\rm QCD} \\ m_Q\Lambda^3_{\rm QCD}\cos\left(C_{PQ}\, a/F_a + \bar{\theta}\right) & \text{for} \quad m_Q \ll \Lambda_{\rm QCD} \end{cases}$$

Nonzero PQ anomaly coefficient $C_{PQ} = C_W + x_Q$ is essential for the axion potential solving the strong CP problem. (C_{PQ} = axionic domain wall number)

* PQ-invariant 1PI axion-gluon-gluon amplitude:

$$\mathcal{A}_{\rm 1PI}[\mathrm{a(k), g(p), g(q)}] = \frac{\mathrm{C_{1PI}}}{32\pi^2 \mathrm{F_a}} \delta^4(\mathrm{k+p+q}) \epsilon_{\mu\nu\rho\sigma} \mathrm{k}^\mu \mathrm{p}^\nu \epsilon_1^\rho \epsilon_2^\sigma$$



C_W y_Q x_Q

$$\mathrm{C_{1PI}} = (\mathrm{C_W - y_Q}) + (\mathrm{x_Q + y_Q})\, \mathrm{L(p, q; m_Q)}$$

$$L(p,q;m_Q) = \int_0^1 dx \int_0^{1-x} dy \frac{2m_Q^2}{m_Q^2 - [p^2 x(1-x) + q^2 y(1-y) + 2(p\cdot q)xy]}$$

$$\mathrm{C_{1PI}} = \begin{cases} \mathrm{C_W - y_Q} + \mathcal{O}\left(\mathrm{m_Q^2/p^2}\right) & \text{for} \quad |\mathrm{p}^2| \gg \mathrm{m_Q^2}, \quad \mathrm{k^2 = q^2 = 0} \\ \mathrm{C_W + x_Q} + \mathcal{O}\left(\mathrm{p^2/m_Q^2}\right) & \text{for} \quad |\mathrm{p}^2| \ll \mathrm{m_Q^2}, \quad \mathrm{k^2 = q^2 = 0} \end{cases}$$

(Recall $C_{PQ} = C_W + x_Q$ at every energy scales.)

In some sense, the Wilsonian axion coupling **C_W**, the PQ anomaly coefficient **C_{PQ}**, and the 1PI axion-gluon-gluon amplitude **C_{1PI}** look similar to each other, but they are basically different.

* **C_W** = field-basis-dependent lagrangian parameter

* **PQ-breaking C_{PQ}:**
   1) field-basis-independent constant which is a true measure of the explicit breaking of $U(1)_{PQ}$ by the QCD anomaly
   2) has a common value at every energy scales
   3) exactly determined at 1-loop.

* **PQ-conserving C_{1PI}:**
   1) field-basis-independent, and generically contains non-local piece
   2) can have different values at different energy scales
   3) receives higher order corrections
   4) determines the axion production by gluons at each energy scale

UV completion with linearly realized $U(1)_{PQ}$:

$$\mathcal{L}_{UV} = \partial_\mu X \partial^\mu X^* - \frac{1}{4g_a^2} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{32\pi^2} \bar{\theta} G^{a\mu\nu} \tilde{G}^a_{\mu\nu} + i\bar{Q}\gamma^\mu D_\mu Q + \left(\kappa \frac{X^n}{M_{Planck}^{n-1}} \bar{Q}_R Q_L + h.c \right) - \lambda \left(|X|^2 - F_a^2 \right)^2$$

Q = the heaviest PQ-charged and gauge-charged fermion which becomes massive as a consequence of spontaneous PQ-breaking

* KSVZ (Kim, Shifman, Vainshtein, Zakharov) model : Q = exotic quark, n = 1
* SUSY DFSZ (Dine, Fischler, Srednicki, Zhitnitsky) model realizing the Kim-Nilles solution of the mu-problem:

Q = Higgsinos, n > 1 , m_Q = Higgs mu-parameter

➔ In SUSY DFSZ model, m_Q = weak scale << F_a.

It is also possible that m_Q << F_a in KSVZ model.

In the limit m_Q ➔ 0, axion is decoupled from the gauge-charged sector.
(Axion becomes a Goldstone boson of anomaly-free U(1) symmetry.)

The decoupling of axion in the limit $m_Q \rightarrow 0$ should reveal in physical 1PI amplitudes at energy scales above m_Q.

$$C_{1PI} = \begin{cases} \mathcal{O}\left(m_Q^2/p^2\right) & \text{for} \quad m_Q < p < F_a \\ nN_Q + \mathcal{O}\left(p^2/m_Q^2\right) & \text{for} \quad p < m_Q \end{cases}$$

But this decoupling is not manifest in the commonly used field-basis for which $C_{PQ} = C_W$, which is achieved by making an appropriate axion-dependent field redefinition of Q :

$$\mathcal{L}_{\rm eff} = \frac{C_W}{32\pi^2}\frac{a}{F_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + y_Q\frac{\partial_\mu a}{2F_a}\bar{Q}\gamma^\mu\gamma_5 Q - m_Q\bar{Q}\left[\exp\left(ix_Q\gamma_5\frac{a}{F_a}\right)\right]Q$$

$$C_W = C_{PQ} = nN_Q, \qquad y_Q = n, \qquad x_Q = 0$$

($C_{1PI} = O(m_Q^2/p^2)$ is due to the cancellation between C_W and y_Q .)

The suppression by " m_Q^2/p^2 " of C_{1PI} in the energy range $m_Q < p < F_a$ is a generic feature of axion models which have a UV completion in which $U(1)_{PQ}$ is linearly realized in the standard manner.

Generalization to SUSY axion model is straightforward:

$$\mathcal{L}_{\rm UV} = \int d^4\theta \sum_I \Phi_I^* \Phi_I + \int d^2\theta \left(\kappa \frac{X^n}{M_{\rm Planck}^{n-1}} Q Q^c + ... \right)$$

$$\mathcal{L}_{\rm eff} = \int d^2\theta \left[\frac{C_W}{32\pi^2} \frac{A}{F_a} W^{a\alpha} W_\alpha^a + \exp\left(\frac{(x_Q + x_{Q^c})A}{F_a} \right) m_Q Q Q^c \right]$$
$$+ \int d^4\theta \frac{A + A^*}{F_a} (y_Q Q^* Q + y_{Q^c} Q^{c*} Q^c) \qquad (A = s + ia + \theta\tilde{a} + F^A)$$

Q = the heaviest PQ-charged and gauge-charged matter field in the model with m_Q which can be far below F_a

1PI axino-gluino-gluon amplitude showing the decoupling in the limit $m_Q \rightarrow 0$

$$\mathcal{A}_{\rm 1PI}[\tilde{a}(k), g(p), \tilde{g}(q)] = \frac{C_{\rm 1PI}}{32\pi^2 F_a} \delta^4(k + p + q) \bar{\tilde{a}}(k) \sigma_{\mu\nu} \gamma_5 \tilde{g}(q) \epsilon^\mu p^\nu$$

$$C_{\rm 1PI} = \begin{cases} \mathcal{O}\left(m_Q^2/p^2\right) & \text{for} \quad m_Q < p < F_a \\ nN_Q + \mathcal{O}\left(p^2/m_Q^2\right) & \text{for} \quad p < m_Q \end{cases}$$

Thermal production of axinos Bae, KC, Im

$$\mathcal{L}_{\rm eff} = \int d^2\theta \left[\frac{C_W}{32\pi^2}\frac{A}{F_a}W^{a\alpha}W^a_\alpha + \exp\left(\frac{(x_Q + x_{Q^c})A}{F_a}\right) m_Q Q Q^c\right] + \int d^4\theta \frac{A + A^*}{F_a}(y_Q Q^* Q + y_{Q^c} Q^{c*} Q^c)$$

* The AWW-coupling does not involve any suppression by m_Q , while the physical 1PI axino-gluino-gluon amplitude C_{1PI} is suppressed by m_Q^2/p^2 at energy scale $p > m_Q$.

➔ The analysis using the AWW-coupling alone for the axino production at $T > m_Q$ gives a highly overestimated production rate.

* At $T > m_Q$, axinos are produced dominantly by the processes involving Q , whose amplitude is suppressed only by a single power of m_Q/T.

* At $T \ll m_Q$, axinos are produced dominantly by the gluon multiplet with $C_{1PI} = O(1)$, and then the previous analysis using the AWW-coupling alone (in the basis with $C_W = C_{PQ}$) can be simply applied to determine the axino production rate.

Axino production per unit spacetime volume :

$$\mathcal{L}_{\rm eff} = \int d^2\theta \left[\frac{C_W}{32\pi^2} \frac{A}{F_a} W^{a\alpha} W^a_\alpha + \exp\left(\frac{(x_Q + x_{Q^c})A}{F_a} \right) m_Q Q Q^c \right] + \int d^4\theta \frac{A + A^*}{F_a} (y_Q Q^* Q + y_{Q^c} Q^{c*} Q^c)$$

$$\mathcal{A}_{1PI}[\tilde{a}(k), g(p), \tilde{g}(q)] = \frac{C_{1PI}}{32\pi^2 F_a} \delta^4(k+p+q) \bar{\tilde{a}}(k) \sigma_{\mu\nu} \gamma_5 \tilde{g}(q) \epsilon^\mu p^\nu$$

Previous result using the effective AWW-coupling alone :

$$\Gamma_{\tilde{a}}(C_W) \sim \frac{C_W^2 g^6 T^6}{(32\pi^2 F_a)^2} \quad \text{for all} \quad T < F_a$$

Correct production rate using the 1PI amplitude C_{1PI} :

$$\Gamma_{\tilde{a}}(C_{1PI}) \sim \begin{cases} g^6 m_Q^4 T^2 \ln^4(T^2/m_Q^2)/(32\pi^2 F_a)^2 & \text{for} \quad m_Q < T < F_a \\ g^6 T^6/(32\pi^2 F_a)^2 & \text{for} \quad T \ll m_Q \end{cases}$$

Production by the matter multiplet Q :

$$\Gamma_{\tilde{a}}(y_Q, x_Q) \sim \frac{(x_Q + y_Q)^2 g^2 m_Q^2 T^4}{F_a^2} \quad \text{for} \quad m_Q < T < F_a$$

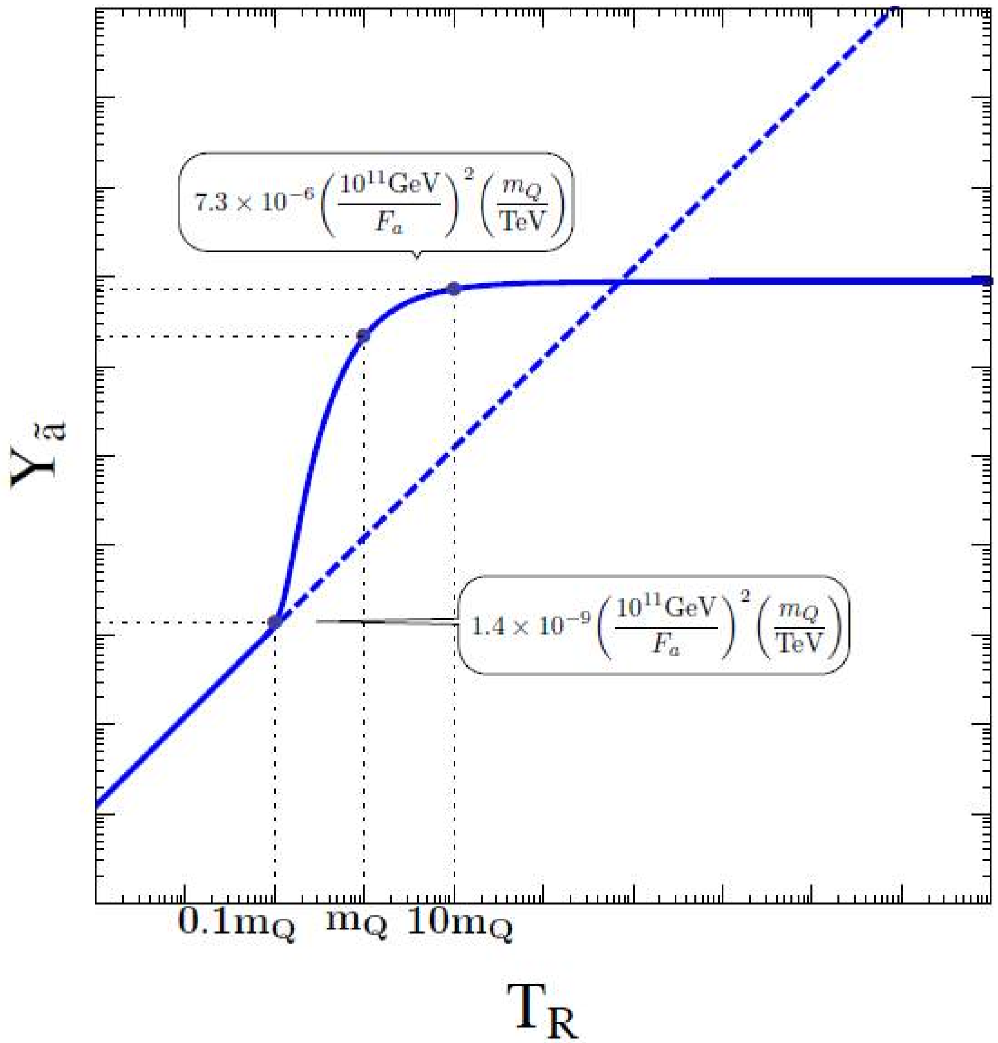
Axino production processes

	Process	$\lvert\mathcal{M}\rvert^2(m_Q < T \ll F_a)$	$\lvert\mathcal{M}\rvert^2(T \ll m_Q)$
A	$g + g \to \tilde{a} + \tilde{g}$	negligible	$4\mathcal{C}_2(s + 2t + 2t^2/s)$
B	$g + \tilde{g} \to \tilde{a} + g$	negligible	$-4\mathcal{C}_2(t + 2s + 2s^2/t)$
C	$\tilde{Q} + g \to \tilde{a} + Q$	$-\mathcal{C}_1(1 + \frac{s-m_Q^2}{t-m_Q^2})$	$2s\mathcal{C}_3$
D	$Q + g \to \tilde{a} + \tilde{Q}$	$\mathcal{C}_1(1 + \frac{t-m_Q^2}{s-m_Q^2})$	$-2t\mathcal{C}_3$
E	$\overline{\tilde{Q}} + Q \to \tilde{a} + g$	$-\mathcal{C}_1\frac{s-m_Q^2}{t-m_Q^2}$	$-2t\mathcal{C}_3$
F	$\tilde{g} + \tilde{g} \to \tilde{a} + \tilde{g}$	negligible	$-8\mathcal{C}_2(s^2 + t^2 + u^2)^2/stu$
G	$Q + \tilde{g} \to \tilde{a} + Q$	$\mathcal{C}_1(4 + \frac{2m_Q^2}{s-m_Q^2} + \frac{2m_Q^2}{t-m_Q^2})$	$-4\mathcal{C}_3(s + s^2/t)$
H	$\tilde{Q} + \tilde{g} \to \tilde{a} + \tilde{Q}$	$\mathcal{C}_1(2 - \frac{t-3m_Q^2}{s-m_Q^2} - \frac{s-3m_Q^2}{t-m_Q^2})$	$-2\mathcal{C}_3(t + 2s + 2s^2/t)$
I	$Q + \overline{Q} \to \tilde{a} + \tilde{g}$	$\mathcal{C}_1(4 + \frac{2m_Q^2}{u-m_Q^2} + \frac{2m_Q^2}{t-m_Q^2})$	$-4\mathcal{C}_3(t + t^2/s)$
J	$\tilde{Q} + \overline{\tilde{Q}} \to \tilde{a} + \tilde{g}$	$\mathcal{C}_1(2 - \frac{t-3m_Q^2}{u-m_Q^2} - \frac{u-3m_Q^2}{t-m_Q^2})$	$2\mathcal{C}_3(s + 2t + 2t^2/s)$

$\mathcal{C}_1 = 8g^2 m_Q^2 \delta^{ab}\delta_{ab}/F_a^2$ $\qquad$ $\mathcal{C}_2 = g^6|f^{abc}|^2/128\pi^4 F_a^2$ $\qquad$ $\mathcal{C}_3 = g^6 \sum_q |T_{ij}^a|^2/128\pi^4 F_a^2$

Relic axino number density :

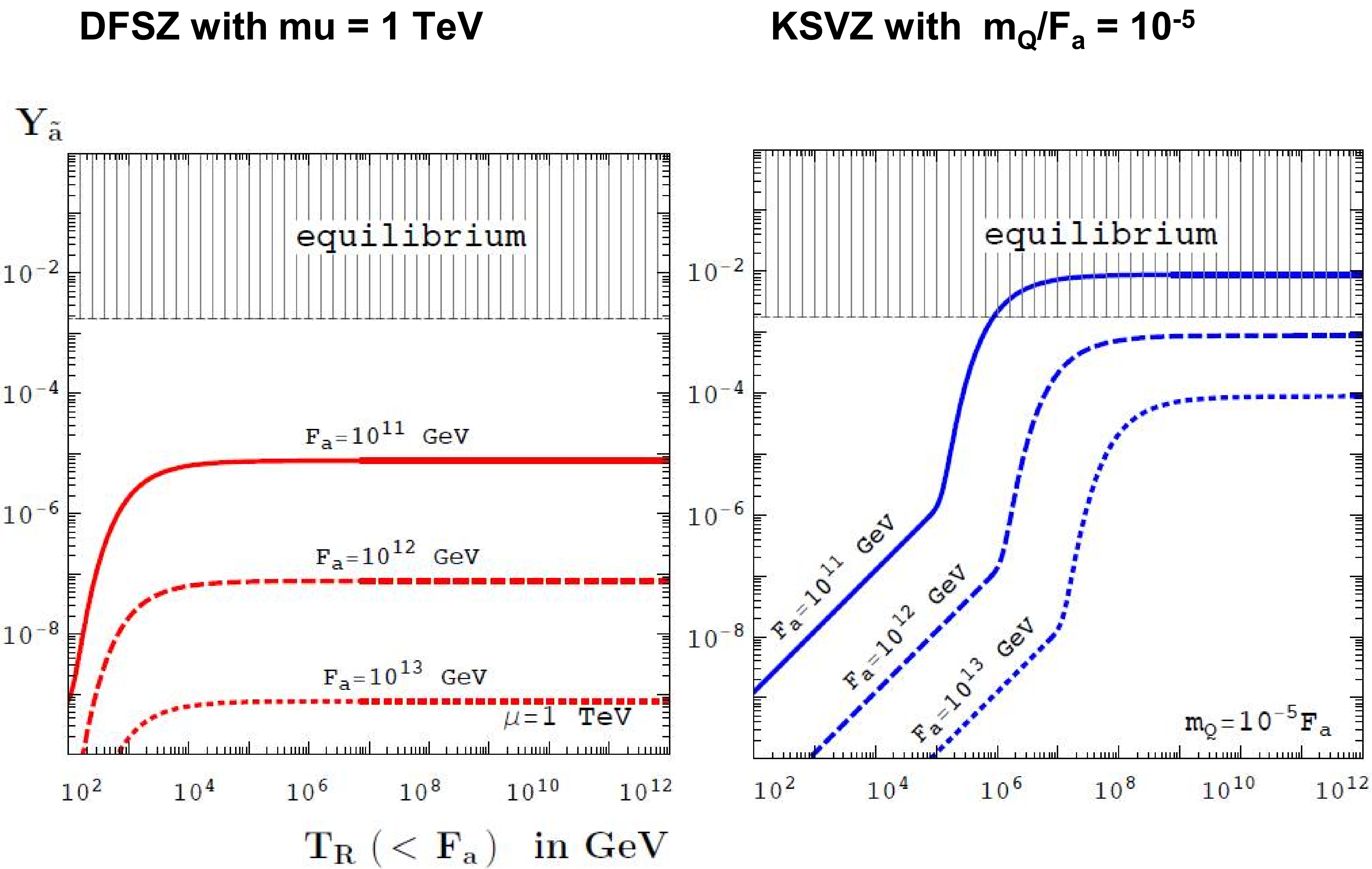
$$Y_{\tilde{a}} = \frac{N_{\tilde{a}}}{\text{entropy}} = \int_{T_0}^{T_R} \frac{dT}{T} \frac{\Gamma_{\tilde{a}}}{s(T)H(T)} \propto T_R^N \qquad N = \begin{cases} 0 & \text{for} \quad T_R \gg m_Q \\ 1 & \text{for} \quad T_R \ll m_Q \end{cases}$$



T_R = reheat temperature

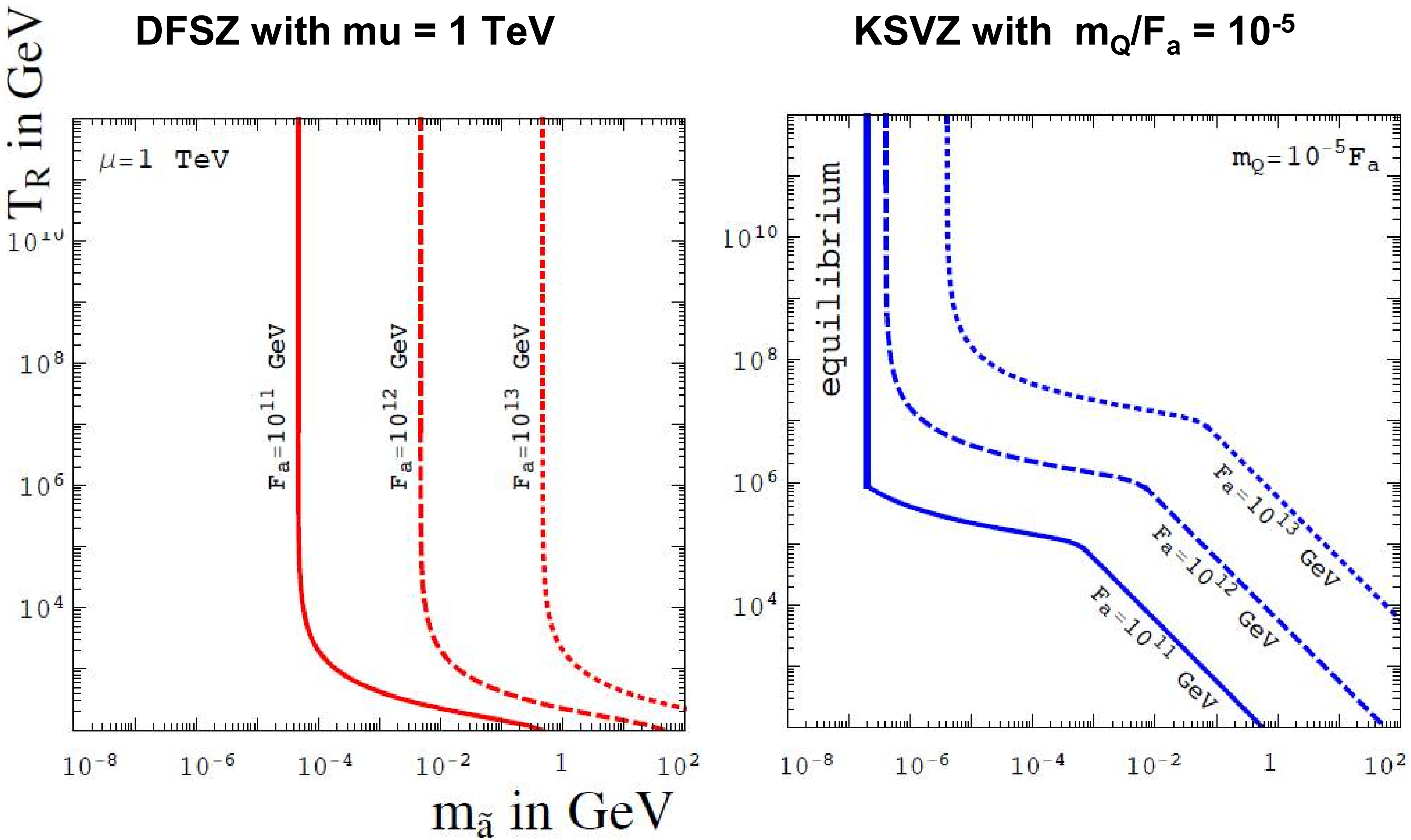
m_Q = Higgsino mass in DFSZ model (exotic quark mass in KSVZ model)

Relic axino number density



Axino mass vs Reheat temperature for $\Omega_{\tilde{a}} h^2 = 0.11$

DFSZ with mu = 1 TeV **KSVZ with $m_Q/F_a = 10^{-5}$**



High T_R can be allowed for wider range of the axino mass.

Conclusion

* Combining SUSY with axion is a very compelling idea, which might solve the gauge hierarchy problem, the strong CP problem, and the mu-problem altogether in an elegant manner.

* The fermionic superpartner of axion, the axino, can have a variety of cosmological implications, in particular it can be a good DM candidate.

* Thermal production of axinos requires more careful analysis incorporating all relevant effective interactions together, which correctly reveals the decoupling feature in the limit $m_Q /T \rightarrow 0$, which is a feature of generic axion model having a UV completion with linearly realized PQ symmetry.

* Compared to the previous analysis using only one particular type of effective interaction, the correct analysis can give very different axino production rates at high temperature, which allows high T_R for wider range of the axino mass, in particular for the DFSZ model realizing the Kim-Nilles solution of the mu-problem.