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On universal and non-universal goldstino couplings to matter

work in progress with

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Outline

- Non-linear SUSY and constrained superfields.
- UV versus effective low-energy lagrangians :

- General Kahler potential and generalized chiral constraints.

- Yukawas and generalized chiral constraints.

- Leading-order low-energy lagrangians.

- Using the KS constraints.

- Heavy gauginos and higgsinos.*
- Conclusions

Large literature on SUSY non-linear realizations and low-energy goldstino interactions

- Volkov-Akulov, Ivanov-Kapustinov, Siegel, Samuel-Wess, Clark and Love...

- Casalbuoni, Dominicis, de Curtis, Feruglio, Gatto; Luty, Ponton; Brignole, Feruglio, Zwirner; Brignole,Casas, Espinosa, Navarro; Komargodski and Seiberg; Antoniadis,Tuckmantel; Antoniadis, E.D., Ghilencea, Tziveloglou ; (see talk P. Tziveloglou)

Most phenomenological studies based on a component formalism, tedious computations. We use a constrained superfield formalism, faster computations.

1. Non-linear SUSY and constrained superfields.

In supergravity, the gravitino Ψ_μ becomes massive by absorbing a spin 1/2 fermion, the goldstino G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

Goldstino is part of a multiplet $X = (x, G, F_X)$. The gravitino mass is

$$m_{3/2} \sim \frac{F_X}{M_P} \ ,$$

whereas the sgoldstino mass m_x depends on the microscopic theory.

In a SUSY theory well below the scale of SUSY breaking $E << \sqrt{f}$, SUSY is non-linearly realized.

There is always one light fermion in the effective theory, the goldstino G, of mass

$$m_G \sim \frac{f}{M_P}$$

In the decoupling limit $M_P, m_x \to \infty$, the transverse polarizations of the gravitino and the sgoldstino x decouple; goldstino couplings to matter scale as $1/f$.

There are two cases of goldstino couplings to matter :

i) Non-SUSY matter spectrum (ex: SM...)

$$E \; << \; m_{sparticles} \; , \; \sqrt{f}$$

$\rightarrow$ non-linear SUSY in the matter sector.

ii) SUSY matter multiplets : $(\tilde{q}, q)$, etc.

$$m_{sparticles} \leq E << \sqrt{f}$$

$\rightarrow$ linear SUSY matter sector coupled to the goldstino : new MSSM couplings, correction to the higgs potential (see talk P. Tziveloglou) .

We will consider $\sqrt{f} \sim \; m_{sparticles} \; >>$TeV, so will be in case i) above .

There are various formalisms developed over the years. Here we are using the superfield approach of Siegel, Casalbuoni et al., Komargodski and Seiberg. The Goldstino G can be described by a chiral superfield X, with the constraint

$$X^2 \ = \ 0 \ .$$

The constraint is solved by

$$X \ = \ \frac{GG}{2\,F_X} + \sqrt{2}\,\theta G + \theta\theta \ F_X \ .$$

Here F_X is an auxiliary field to be eliminated via its field equations.

After eliminating F_X, the Volkov-Akulov lagrangian is then given by

$$\mathcal{L}_X = \int d^4\theta \; X^\dagger X + \left\{ \int d^2\theta \, f \; X + h.c. \right\}$$

$$= det \; (E_\mu^a) \; , \quad \text{where} \quad E_\mu^a = e_\mu^a + (\frac{i}{2f^2} G\sigma^a \partial_\mu \bar{G} + h.c.)$$

is the VA "vierbein". In the standard VA prescription, couplings to matter proceed as in gravity :

$$G^{\mu\nu} \; T_{\mu\nu,M} \; = \; g^{\mu\nu} \; T_{\mu\nu,M} \; + \; (\frac{i}{2f^2} G\sigma^\mu \partial^\nu \bar{G} + h.c.) \; T_{\mu\nu,M}$$

Volkov-Akulov and the SUSY constrained formalism are not obviously equivalent if coupling to other (super) fields, due to F_X.

Non-linear matter $\rightarrow$ additional constraints (KS) :

- Heavy scalars : $XQ_i = 0$: eliminates the complex scalars. We get

$$Q_i = \frac{1}{F_X}(\Psi_i - \frac{F_i}{2F_X}G)G + \sqrt{2}\theta\Psi_i + \theta^2 F_i$$

Obs: $X^2 = XQ_i = 0$ uniquely determines the solutions. However, there are other constraints :

$$Q_i \; Q_j \; Q_k \; = \; 0 \; ,$$

where are "redundant", but not obvious consequence of the others.

2. UV versus effective low-energy lagrangians

The constraints should be understood as IR consequences of UV dynamics generating SUSY breaking and large superpartner masses. It was argued (Komargodski-Seiberg) that the superfield constraints are unique and independent of high-energy physics. Ex :

$$\begin{aligned} W &= f\ X\ , \\ K &= X^\dagger X + Q^\dagger Q - \frac{c_x}{\Lambda^2}(X^\dagger X)^2 - \frac{c_q}{\Lambda^2}(X^\dagger X)(Q^\dagger Q) \end{aligned}$$

For $c_i = 0$ we get an O'R model, $F_X = -f$ and X is a flat direction. $c_i > 0$ stabilize $\langle X \rangle = \langle Q \rangle = 0$.

The fermions stays massless $\rightarrow$ non-linear SUSY at low-energy. The low-energy lagrangian is obtained by "integrating-out" the scalars:

$$\mathcal{L} = -f^2 + |F_X + f|^2 - \frac{c_x}{\Lambda^2} |2xF_X - GG|^2 - \frac{c_q}{\Lambda^2} |qF_X + xF_q - G\Psi_q|^2 + \text{derivative terms}$$

Field eqs. for X, q give

$$x = \frac{GG}{2F_X} \quad , \quad q = \frac{1}{F_X}(\Psi_q - \frac{F_q G}{2F_X})G$$

i.e. the previous superfield constraints, independently of c_i. Are these constraints unique, independent of the high-energy theory ?

- General Kahler potential and generalized chiral constraints

Let's add another UV correction to the Kahler potential

$$\Delta K \;=\; -\frac{c_3}{\Lambda^2}(Q^\dagger Q)^2 \;-\; \frac{c_4}{\Lambda^2}(X^\dagger)^2 Q^2$$

- c_3 is not protected by any symmetry.

In this case, we find $(\Psi_i = G, \Psi_q)$

$$X = a_{ij}\Psi_i\Psi_j + \sqrt{2}\theta G + \theta^2 F_X$$

$$Q = b_{ij}\Psi_i\Psi_j + \sqrt{2}\theta\Psi_q + \theta^2 F_q$$

where a_{ij}, b_{ij} are easily calculated as functions of ϵ_a, F_i.

Here $\quad X^2 \neq 0, XQ \neq 0.$

Nonetheless we find the cubic constraints

$$X^3 \;=\; X^2 Q \;=\; X Q^2 \;=\; Q^3 \;=\; 0 \qquad (1)$$

Interestingly, the solution of (1) is not unique, it depends on two free parameters. It can be parameterized as

$$X = \frac{GG}{2F_X} - \frac{c_1}{2F_X}(F_q G - F_X \Psi_q)^2 \ ,$$
$$Q = \frac{\Psi_q \Psi_q}{2F_q} - \frac{c_2}{2F_q}(F_q G - F_X \Psi_q)^2 \ .$$

• Non-uniqueness of the solutions of the constraints reflect the UV sensitivity of the low-energy lagrangian.

• Previous constraints recovered if $c_x, c_q >> c_3, c_4$. Notice that c_x, c_q determine the scalar masses

$$m_x^2 = \frac{4c_x f^2}{\Lambda^2} \quad , \quad m_q^2 = \frac{c_q f^2}{\Lambda^2} \ .$$

• The higher-order constraints $\leftrightarrow$ UV sensitivity come because we don't take the limit $m_{sparticles} >> f$ that KS used. This limit would ask for $c_x, c_q >> 1$, not easy to justify.

Our new results change low-energy actions for $m_{sparticles} \lesssim f$.

- Yukawas and generalized chiral constraints

Yukawas (R-parity violating couplings in MSSM) increase the order of the monomial chiral constraints. Simplest example

$$K = X^\dagger X + Q^\dagger Q - \frac{c_x}{\Lambda^2}(X^\dagger X)^2 - \frac{c_q}{\Lambda^2}(Q^\dagger Q)(X^\dagger X) \ ,$$
$$W = f\ X\ + \frac{\lambda}{3}Q^3 \ .$$

In this case the integration of the heavy scalars leads to low-energy fields of the form

$$X = a_{ij}\,\psi_i\psi_j + a_2\ (\bar{F}_q\,\bar{\psi}_X - \bar{F}_X\,\bar{\psi}_q)^2\ + \sqrt{2}\ \theta\ \psi_X\ +\ \theta^2\ F_X\ ,$$
$$Q = b_{ij}\,\psi_i\psi_j + b_2\ (\bar{F}_q\,\bar{\psi}_X - \bar{F}_X\,\bar{\psi}_q)^2\ + \sqrt{2}\ \theta\ \psi_q\ +\ \theta^2\ F_q$$

By Grassmann variable arguments one can check that in this case we obtain quartic constraints

$$X^4 = X^3 Q = X^2 Q^2 = X Q^3 = Q^4 = 0 \, .$$

3. Leading-order low-energy lagrangians

Consider N superfields (quarks and/or leptons for MSSM) plus the goldstino superfield X. We add the minimal high-energy Kahler potential needed to decouple all scalars and add also an R-parity violating coupling, denoted generically λ_{ijk} below.

$$K = X^\dagger X + Q_i^\dagger Q^i - \frac{m_x^2}{4f^2}(X^\dagger X)^2 - \frac{m_i^2}{f^2}(Q_i^\dagger Q^i)(X^\dagger X) \ ,$$
$$W = f\ X + \frac{1}{3}\lambda_{ijk}Q^i Q^j Q^k \ .$$

By integrating-out the $N+1$ heavy scalars of mass $4m_X^2, m_i^2$ we get higher-order chiral constraints.

We use here a more pragmatic approach, by expanding the solution in F_i/F_X. First order in the expansion :

$$x = \frac{GG}{2F_X} \ , \ q_i = \frac{G\psi_i}{F_X} - \frac{1}{m_i^2}\lambda_{ijk}\bar{\psi}_j\bar{\psi}_k \ .$$

The low-energy lagrangian, up to four-fermion fields :

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{kin}} - \frac{1}{4f^2}\bar{G}^2\Box G^2 - \frac{1}{f^2}(\bar{G}\bar{\psi}_i)\Box(G\psi_i) + \frac{1}{m_i^2}(\lambda_{ijk}\bar{\psi}_j\bar{\psi}_k)(\bar{\lambda}_{imn}\psi_m\psi_n) - \frac{2}{m_i^2 f}\ (\bar{\lambda}_{ijk}\psi_j\psi_k)\Box(G\psi_i) - \frac{3}{m_i^4}\ (\lambda_{ijk}\bar{\psi}_j\bar{\psi}_k)\Box(\bar{\lambda}_{imn}\psi_m\psi_n) - f^2 \ .$$

The terms $(\bar{\lambda}_{ijk}\psi_j\psi_k)\Box(G\psi_i)$ are R-parity violating, lead to $qq \to qG$ processes, LHC relevance ? (detailed study of $qq \to GGg, GG\gamma$ by Brignole,Feruglio,Zwirner)

As expected, non-derivative terms involving the goldstino canceled. The first line contains universal goldstino couplings, whereas the second and third lines describe model-dependent couplings
(previous work: Brignole,Feruglio,Zwirner).

• For $m_i^2 \lesssim f$, the model-dependent couplings are as important at low-energy as the universal couplings of the goldstino to matter.

• Pragmatic question : is it possible to write the same low-energy action by using the KS constraints ?

• If yes, what is the most convenient formalism to write general low-energy SM actions with non-linear SUSY ?

Using the KS constraints.

Previous action can also be written using KS constraints

$$\mathcal{L} \;=\; \int d^4\theta \Big(X^\dagger X + Q_i^\dagger Q_i \;+ \frac{\lambda_{ijk}\lambda_{imn}}{m_i^2} Q_i^\dagger Q_j^\dagger Q_m Q_n$$
$$+ \frac{\lambda_{ijk}}{m_i^2} Q_i Q_j D^2 Q_k + \frac{\lambda_{ijk}\lambda_{imn}}{m_i^4} Q_i^\dagger Q_j^\dagger \Box (Q_m Q_n) \Big)$$
$$+ (\int d^2\theta f X + \text{h.c.}) \; . \qquad (2)$$

- Arbitrary coefficients in (2) (ex. $\epsilon_{ijmn} Q_i^\dagger Q_j^\dagger Q_m Q_n$) do not correspond to a simple UV theory $\rightarrow$ "swampland"?
- The operators in red seem irrelevant. However, they give contributions similar to the universal couplings in blue for $m_i^2 \sim f$.

Actually, we can probably write any low-energy action by using the KS formalism with the constraints :

$$X^2 = XQ_i = Q_iQ_jQ_k = 0$$

and the field equations for the constrained superfields

$$\frac{1}{4}X\bar{D}^2X^\dagger = fX \quad , \quad \frac{1}{4}Q_iQ_j\bar{D}^2X^\dagger = fQ_iQ_j \ ,$$
$$X\bar{D}^2Q_i^\dagger = 0 \quad , \quad Q_j\bar{D}^2Q_i^\dagger = 0.$$

However, operator dimensions can give wrong intuition about their low-energy relevance.

4.1 KS constraints higgsinos and gauginos

- heavy fermions : $X\bar{H} =$ chiral : eliminates the fermions.

In this case

$$H = h + i\sqrt{2}\theta\sigma^m\partial_m h\frac{\bar{G}}{\bar{F}_X} + \theta^2[-\partial_n(\frac{\bar{G}}{\bar{F}_X})\bar{\sigma}^m\sigma^n\partial_m h\frac{\bar{G}}{\bar{F}_X} + \frac{1}{2\bar{F}_X^2}\bar{G}^2\partial^2 h]$$

In this case there is not anymore an auxiliary field F_h.

- heavy gauginos : $XW_\alpha = 0$ eliminates the gauginos.

The solution is

$$W_\alpha = \frac{1}{\sqrt{2}F_X}(D - i\sigma^{mn}F_{mn})G - \frac{G^2}{2F_X^2}\sigma^m\partial_m\bar{\lambda}$$
$$+(D - i\sigma^{mn}F_{mn})\theta + \theta^2\sigma^m\partial_m\bar{\lambda}\ .$$

- 4.2 Heavy gauginos from UV: constrained vector superfields

Simplest UV lagrangian providing large masses to the sgoldstino scalar and the gaugino :

$$\mathcal{L} \;=\; \int d^4\theta \; \left[X^\dagger X \;-\; \epsilon\,(X^\dagger X)^2\right]$$
$$+\;\left\{ \int d^2\theta \; \left(f X \;+\; \frac{1}{4} W^\alpha W_\alpha \;+\; \frac{M}{f}\; X\; W^\alpha W_\alpha \right) + \text{h.c.} \right\} ,$$

where M is the gaugino mass. The zero-momentum gaugino equation has the solution

$$\lambda \;=\; \frac{i}{\sqrt{2} F_X} (D - i \sigma^{mn} F_{mn}) \; G \; .$$

The corresponding field strength is

$$W_\alpha = \frac{1}{\sqrt{2}F_X}(D - i\sigma^{mn}F_{mn})G + (D - i\sigma^{mn}F_{mn})\theta + \theta^2\sigma^m\partial_m\bar{\lambda} \tag{3}$$

and satisfies

$$X\ W_\alpha = \frac{GG}{2F_X}\ (\sigma^m\partial_m\bar{\lambda})_\alpha\ \theta^2, \qquad X\ W^\alpha\ W_\alpha = 0\ ,$$

where the second equation is the generalized constraint whose unique solution is (3). The KS gauginos are the solution of the implicit eq.

$$\lambda = \frac{i}{\sqrt{2}F_X}(D - i\sigma^{mn}F_{mn})\,G - i\frac{GG}{2F_X^2}\sigma^m\partial_m\bar{\lambda}$$

- The difference between KS solution and (3) is of higher-order in an $1/f$ expansion in the low-energy action.
- In both cases the leading goldstino-gauge field interaction comes from the gaugino kinetic term by using the common terms prop. to $\sigma^{mn}F_{mn}\, G$ in (3).

Conclusions and perspectives

• The couplings of goldstino to matter are not unique. More general couplings easily captured by the constrained superfield formalisms.

• Constraints eliminating superpartners not unique.

• Non-universal goldstino couplings important and dominant for $m^2_{sparticles} \lesssim f$.

• R-parity violating goldstino couplings to matter worth LHC dedicated study : $qq \to qG$.

• Some subtleties for the heavy higgsinos case.

- Most general (four-fermion) SM couplings to goldstino : in progress.

Thank you !