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Right unitarity triangles and tri-bimaximal mixing

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Tri-bimaximal lepton mixing

$$\text{PMNS} \quad \approx \quad U_{TB} \quad \equiv \quad \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

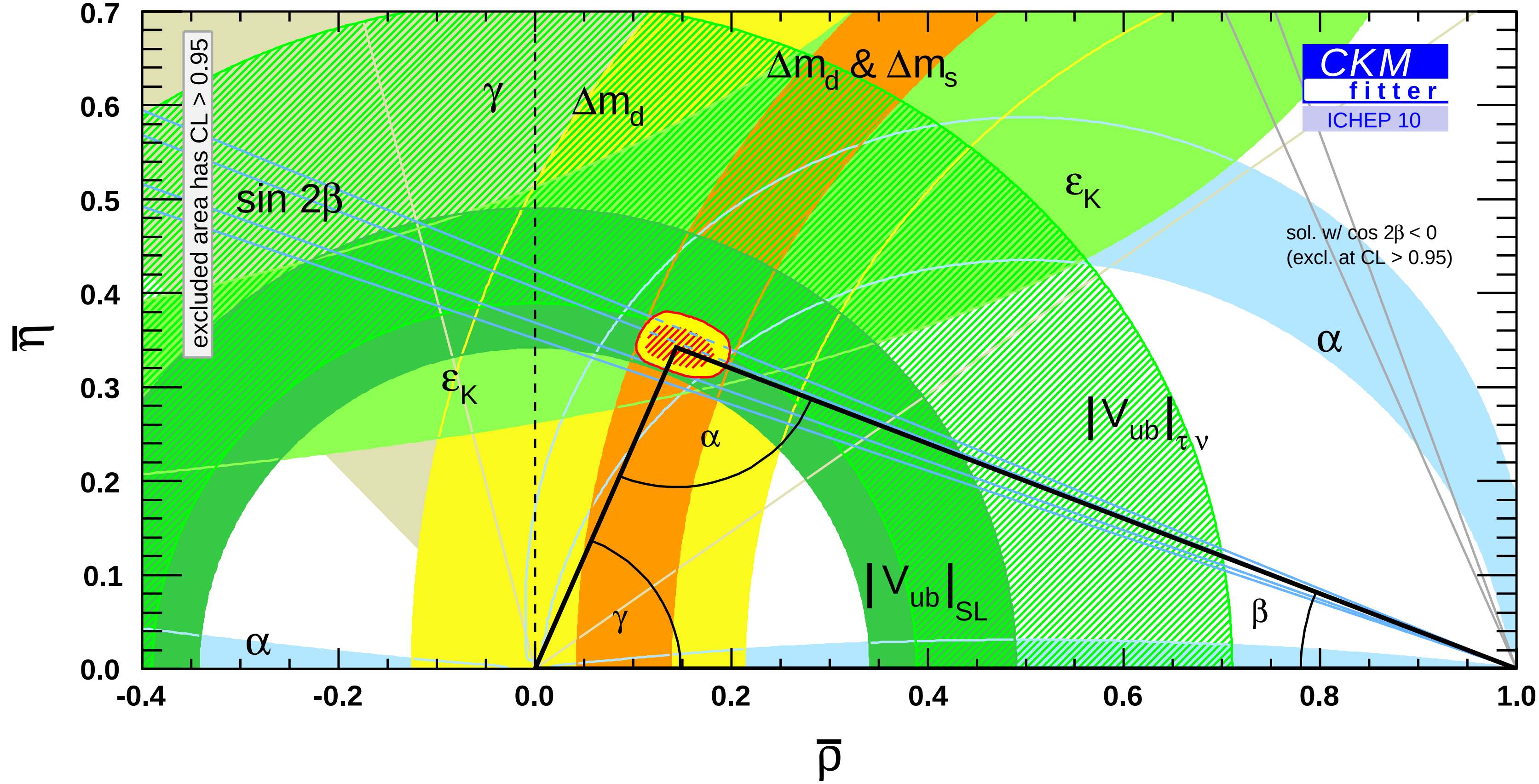
$\Rightarrow$

PMNS-angles	tri-bimax.	1σ exp.
$\sin^2\theta_{12}$:	$\frac{1}{3}$	$0.297 - 0.329$
$\sin^2\theta_{23}$:	$\frac{1}{2}$	$0.45 - 0.57$
$\sin^2\theta_{13}$:	0	$0.004 - 0.019$

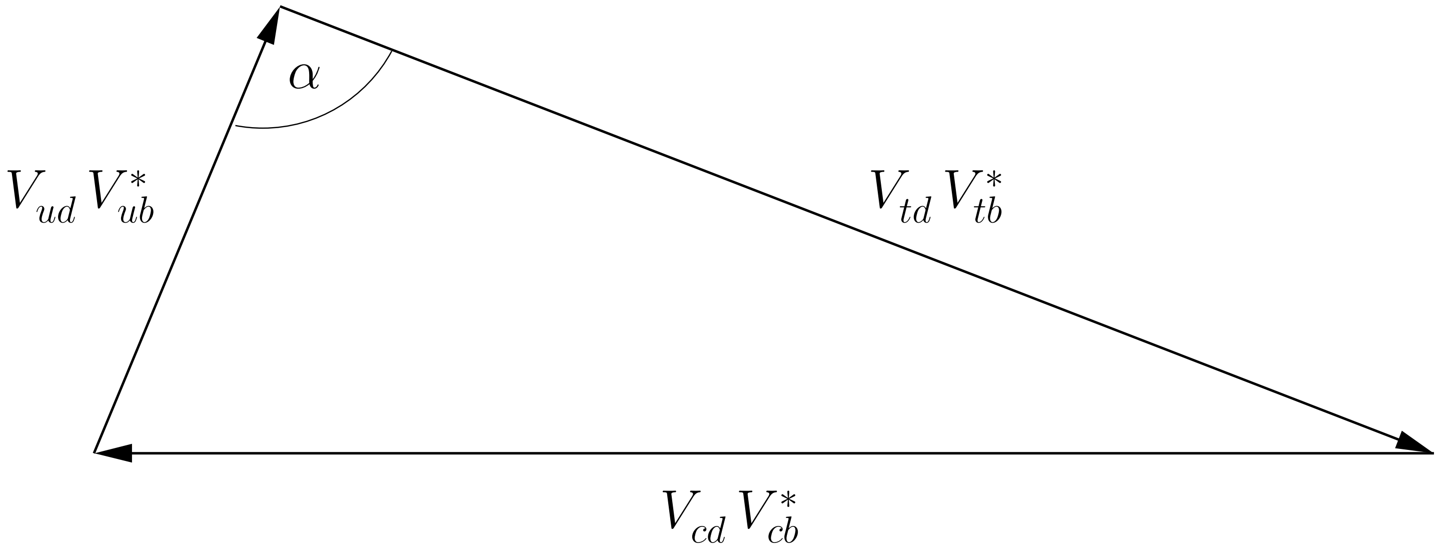
Schwetz, Tórtola, Valle (2011)

$\rightarrow$ motivation for non-Abelian discrete family symmetry, e.g. S_4

Right-angled CKM unitarity triangle



The angle α



$$
\begin{aligned}
\alpha &= \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \\
&= \arg\left(\frac{c_{23}c_{13}(c_{12}c_{23}s_{13}e^{i\delta} - s_{12}s_{23})}{c_{12}c_{13}s_{13}e^{i\delta}}\right) \\
&\approx \arg\left(1 - \frac{\theta_{12}\theta_{23}}{\theta_{13}}e^{-i\delta}\right)
\end{aligned}
$$

Phase sum rule

- CKM matrix $V = V_{u_L} V_{d_L}^\dagger$
- $V_{u_L}^\dagger = U_{23}^{u_L} U_{13}^{u_L} U_{12}^{u_L}$ from M_u and $V_{d_L}^\dagger = U_{23}^{d_L} U_{13}^{d_L} U_{12}^{d_L}$ from M_d
- consider case where $\theta_{13}^u = \theta_{13}^d = 0$

$$\begin{aligned}
\theta_{12} e^{i\delta_{12}} &\approx \theta_{12}^d e^{i\delta_{12}^d} - \theta_{12}^u e^{i\delta_{12}^u} \\
\theta_{23} e^{i\delta_{23}} &\approx \theta_{23}^d e^{i\delta_{23}^d} - \theta_{23}^u e^{i\delta_{23}^u} \\
\theta_{13} e^{i\delta_{13}} &\approx -\theta_{12}^u e^{i\delta_{12}^u} (\theta_{23}^d e^{i\delta_{23}^d} - \theta_{23}^u e^{i\delta_{23}^u})
\end{aligned}$$

$$\delta = \delta_{13} - \delta_{12} - \delta_{23}$$

$$\downarrow$$

$$\begin{aligned}
\alpha &\approx \arg\left(1 - \frac{\theta_{12}\theta_{23}}{\theta_{13}} \cdot \frac{e^{i\delta_{12}} e^{i\delta_{23}}}{e^{i\delta_{13}}}\right) \\
&\approx \arg\left(1 + \frac{\theta_{12}^d e^{i\delta_{12}^d} - \theta_{12}^u e^{i\delta_{12}^u}}{\theta_{12}^u e^{i\delta_{12}^u}}\right) \\
&\approx \delta_{12}^d - \delta_{12}^u
\end{aligned}$$

Structure of quark mass matrices

- LR convention for mass matrices

$$M_u = \begin{pmatrix} a_u & b_u & 0 \\ * & c_u & d_u \\ * & * & e_u \end{pmatrix} \qquad M_d = \begin{pmatrix} a_d & i\,b_d & 0 \\ * & c_d & d_d \\ * & * & e_d \end{pmatrix}$$

$$\theta_{ij}^u \sim \frac{{M_u}^{ij}}{{M_u}^{jj}} \qquad \theta_{ij}^d \sim \frac{{M_d}^{ij}}{{M_d}^{jj}}$$

- zero 1-3 mixing in both up and down sector
- real parameters a, b, c, d, e
- imaginary 1-2 element in M_d
- $\delta_{12}^u = 0$ and $\delta_{12}^d = \frac{\pi}{2} \quad \rightarrow \quad \boxed{\alpha \approx \frac{\pi}{2}}$

Real and imaginary flavon alignment

- CP conservation before family symmetry breaking
- get phases of flavon VEVs through terms like

$$P\left(\phi^2 - M^2\right) \quad \rightarrow \quad \langle\phi\rangle = \pm M$$

$$P\left(\phi^2 + M^2\right) \quad \rightarrow \quad \langle\phi\rangle = \pm iM$$

$$P\left(\frac{\phi^4}{M^2} - M^2\right) \quad \rightarrow \quad \langle\phi\rangle = i^k M$$

- useful 'shaping' symmetries: Z_4 and Z_2
- two steps of getting flavon alignment

  (i) fix *direction* of alignment $\rightarrow$ $W_{\text{flavon}}^{(i)}$

  (ii) fix *phase* of flavon VEV $\rightarrow$ $W_{\text{flavon}}^{(ii)}$

An $SU(5) \times S_4$ family symmetry model

matter	T_3	T	F	N	H_5	$H_{\overline{5}}$	$H_{\overline{45}}$
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Antusch, King,
Luhn, Spinrath (2011)

flavons	ϕ_2^u	$\phi_{1'}^u$	ϕ_3^d	$\widetilde{\phi}_3^d$	ϕ_2^d	$\widetilde{\phi}_2^d$	ϕ_1^ν	ϕ_2^ν	$\phi_{3'}^{\prime\,\nu}$	ξ_1	$\widetilde{\xi}_{1'}$
$SU(5)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}$	$\mathbf{1}'$

$$\langle \phi_1^\nu \rangle \sim \lambda^4 M \qquad \langle \phi_2^\nu \rangle \sim \lambda^4 M \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{3'}^\nu \rangle \sim \lambda^4 M \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\langle \phi_2^u \rangle \sim \lambda^4 M \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \langle \phi_{1'}^u \rangle \sim \lambda^3 M$$

$$\langle \phi_3^d \rangle \sim \lambda^2 M \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \langle \widetilde{\phi}_3^d \rangle \sim \lambda^3 M \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \quad \langle \phi_2^d \rangle \sim \lambda M \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \langle \widetilde{\phi}_2^d \rangle \sim \lambda^3 M \begin{pmatrix} i \\ i \end{pmatrix}$$

Yukawa superpotential & mass matrices

$$W_\nu \;=\; F N H_5 + N\left(\phi_1^\nu + \phi_2^\nu + \phi_{3'}^\nu\right) N$$

- trivial Dirac neutrino Yukawa
- tri-bimaximal neutrino mixing from heavy right-handed neutrinos

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$$W_u \;=\; T_3 T_3 H_5 + \frac{1}{M} T T \phi_2^u H_5 + \frac{1}{M^2} T T (\phi_{1'}^u)^2 H_5 + \frac{1}{M^3} T T (\phi_3^d)^2 \phi_1^\nu H_5$$

$$W_d \;=\; \frac{1}{M} F T_3 \phi_3^d H_{\overline{5}} + \frac{1}{M^2} (F \widetilde{\phi}_3^d)_1 (T \phi_2^d)_1 H_{\overline{45}} + \frac{1}{M^2} (F \phi_3^d)_2 (T \widetilde{\phi}_2^d)_2 H_{\overline{5}}$$

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_u \qquad\qquad M_d \sim \begin{pmatrix} 0 & i\,\lambda^5 & 0 \\ i\,\lambda^5 & \lambda^4 & 2\,\lambda^4 \\ 0 & 0 & \lambda^2 \end{pmatrix} v_d$$

- $\delta_{12}^u = 0$ and $\delta_{12}^d = \frac{\pi}{2} \quad \rightarrow \quad \alpha \approx \frac{\pi}{2}$
- $\theta_{23} \approx \theta_{23}^d \approx 2\frac{m_s}{m_b}$

Prediction for the lepton sector

- charged lepton sector

$$M_e \sim \begin{pmatrix} 0 & i\,\lambda^5 & 0 \\ i\,\lambda^5 & -3\,\lambda^4 & 0 \\ 0 & -6\,\lambda^4 & \lambda^2 \end{pmatrix} v_d$$

- $\theta^e_{12} \sim \frac{\lambda}{3}$ and $\delta^e_{12} = \pm\frac{\pi}{2}$
- neutrino sector real and of tri-bimaximal form
- charged lepton contributions to PMNS mixing

$$\sin^2\theta_{12} \approx \frac{1}{3} \qquad \sin^2\theta_{23} \approx \frac{1}{2} \qquad \theta_{13} \approx \frac{\lambda}{3\sqrt{2}} \approx 3^\circ$$

$$\delta_{\text{PMNS}} \approx \pm\left(\frac{\pi}{2} - \frac{\lambda}{3}\right) \approx \pm 86^\circ$$

- maximal leptonic CP oscillation phase

Flavon VEVs – direction

$$
\begin{aligned}
W^{(i)}_{\text{flavon}} = \; & Y^\nu_2 \zeta_1^{Y^\nu_2} \frac{1}{M} (\phi^\nu_1\phi^\nu_2 + \phi^\nu_2\phi^\nu_2 + \phi^\nu_{3'}\phi^\nu_{3'}) + Z^\nu_{3'} \zeta_1^{Z^\nu_{3'}} \frac{1}{M} (\phi^\nu_1\phi^\nu_{3'} + \phi^\nu_2\phi^\nu_{3'} + \phi^\nu_{3'}\phi^\nu_{3'}) \\
& + X^d_1 \zeta_1^{X^d_1} \frac{1}{M} (\phi^d_2)^2 \qquad + Y^d_2 \zeta_1^{Y^d_2} \frac{1}{M^3} (\phi^d_2)^2 (\phi^d_3)^2 \\
& + Y^{du}_2 \zeta_1^{Y^{du}_2} \frac{1}{M} \phi^d_2 \phi^u_2 \qquad + X^{\nu d}_{1'} \zeta_1^{X^{\nu d}_{1'}} \frac{1}{M} \phi^\nu_2 \widetilde{\phi}^d_2 \, . \\
& + \widetilde{X}^d_1 \zeta_1^{\widetilde{X}^d_1} \frac{1}{M^2} \phi^d_2 \phi^d_3 \widetilde{\phi}^d_3 + \widetilde{X}^{\nu d}_{1'} \zeta_1^{\widetilde{X}^{\nu d}_{1'}} \frac{1}{M^3} \left[(\phi^d_3)^2 \right]_{3'} \phi^\nu_{3'} \widetilde{\phi}^d_3
\end{aligned}
$$

- each driving field has its associated auxiliary field ζ_1
- first line gives, among others, $\langle \phi^\nu_2 \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$
\langle \phi^\nu_2 \rangle \langle \widetilde{\phi}^d_2 \rangle \Big|_{\mathbf{1}'} = \langle \phi^\nu_{2,1} \rangle \langle \widetilde{\phi}^d_{2,2} \rangle - \langle \phi^\nu_{2,2} \rangle \langle \widetilde{\phi}^d_{2,1} \rangle = 0 \quad \longrightarrow \quad \langle \widetilde{\phi}^d_2 \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}
$$

Flavon VEVs – phase

$$
\begin{aligned}
W_{\text{flavon}}^{(ii)} \;=\;& P_0^{(1)}\,\zeta_1^{P_0^{(1)}}\left[\frac{1}{M}(\xi_1)^2 - m^{(1)}\right] \;+\; P_0^{(2)}\,\zeta_1^{P_0^{(2)}}\left[\frac{1}{M}(\widetilde{\xi}_{1'})^2 - m^{(2)}\right] \\
&+ P_0^{(3)}\,\zeta_1^{P_0^{(3)}}\left[\frac{1}{M}(\phi_1^\nu)^2 - m^{(3)}\right] \\
&+ P_1^{(1)}\,\zeta_1^{P_1^{(1)}}\left[\frac{1}{M}(\phi_{1'}^u)^2 - c^{(1)}\,\xi_1\right] \;+\; P_1^{(2)}\,\zeta_1^{P_1^{(2)}}\left[\frac{1}{M}(\widetilde{\phi}_2^d)^2 + c^{(2)}\,\xi_1\right] \\
&+ P_1^{(3)}\,\zeta_1^{P_1^{(3)}}\left[\frac{1}{M}(\widetilde{\phi}_3^d)^2 - c^{(3)}\,\xi_1\right] \;+\; P_1^{(4)}\,\zeta_1^{P_1^{(4)}}\left[\frac{1}{M^2}(\phi_2^d)^2\phi_2^\nu - c^{(4)}\,\xi_1\right] \\
&+ \widetilde{P}_{1'}^{(1)}\,\zeta_1^{\widetilde{P}_{1'}^{(1)}}\left[\frac{1}{M^2}(\phi_3^d)^2\phi_2^\nu - \widetilde{c}^{(1)}\,\widetilde{\xi}_{1'}\right] \\
&+ \widetilde{P}_{1'}^{(2)}\,\zeta_1^{\widetilde{P}_{1'}^{(2)}}\widetilde{\zeta}_1^{\widetilde{P}_{1'}^{(2)}}\,\frac{1}{M}\left[\frac{1}{M^4}\phi_2^u(\phi_2^d)^4 - \widetilde{c}^{(2)}\,\widetilde{\xi}_{1'}\right]
\end{aligned}
$$

- auxiliary flavon fields ξ_1 and $\widetilde{\xi}_{1'}$ enter the stage
- signs of (real) parameters such that all flavon VEVs real except for $\widetilde{\phi}_2^d$

Need for messengers

- most general set of 'shaping' symmetries for given superpotential

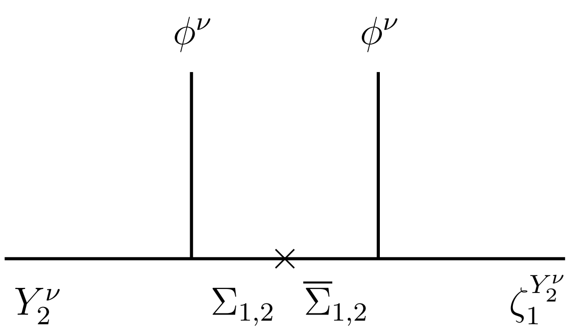
  $\longrightarrow$ two Z_4s and five Z_2s

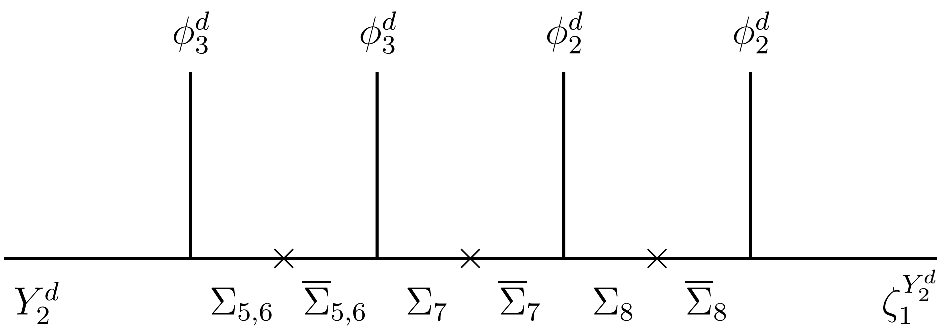
- determine all effectively allowed terms

- any dangerous ones ? – YES, in W_u as well as in $W_{\text{flavon}}^{(i)+(ii)}$

- e.g. $Y_2^\nu \zeta_1^{Y_2^\nu}$ and $Y_2^d \zeta_1^{Y_2^d}$ have identical quantum numbers

  $(\phi^\nu)^2$ $\quad$ $(\phi_2^d)^2(\phi_3^d)^2$





- it is mandatory to formulate a complete messenger sector

Conclusion

- family symmetry models
  - tri-bimaximal lepton mixing
  - $\alpha \approx 90^\circ$
- spontaneous CP violation
- possibility to predict leptonic CP violation
- controlling effective operators
  - Z_4 and Z_2 'shaping' symmetries not powerful enough
  - construct a messenger sector
- try to find a more efficient model . . .

Thank you