On Gauginos and Super-Yang-Mills Theory

Jean-Pierre Derendinger

Albert Einstein Center for Fundamental Physics,
Institute for Theoretical Physics, Bern University

PLANCK 2011
From the Planck Scale to the Electroweak Scale, Lissabon

In honour of Hans-Peter Nilles

UNIVERSITÄT BERN

First: condensates do not generically break SUSY

Phys. Lett. 112B (1982) 455 (3 pages, 15 refs. !)

IS SUPERSYMMETRY AFRAID OF CONDENSATES?

H. Peter NILLES
CERN, Geneva, Switzerland

Received 16 February 1982

Condensates might appear abundantly in strongly interacting supersymmetric Yang–Mills theories. We argue that they do not break supersymmetry dynamically. A toy model is used to illustrate the mechanism.

- Mostly motivated by supersymmetric technicolour: which symmetries (chiral, susy) can be broken by condensates in confining (technicolour) supersymmetric theories ?
- At scales lower than the confinement scale Λ, ' t Hooft anomaly matching as main tool. Indices in some cases.

Second: gaugino condensates may break local SUSY

Phys. Lett. 115B (1982) 193 (4 pages, 14 refs.)

DYNAMICALLY BROKEN SUPERGRAVITY AND THE HIERARCHY PROBLEM

Hans Peter NILLES
CERN, Geneva, Switzerland

Received 24 May 1982

We propose a scenario for grand unified models based on local supersymmetry. We give arguments that condensates of strongly interacting gauge theories might break local supersymmetry. The gravitino mass induces mass splittings in the low energy theory and allows us to understand a hierarchy of $M_{\mathrm{p}} = 10^{19}$ GeV to $M_{\mathrm{W}} = 10^{2}$ GeV naturally.

- In an effective field theory (similar to Veneziano-Yankielowicz) setup, coupled to supergravity . . .
- Full $N = 1$ supergravity couplings, where this result is apparent, not available yet.

Third: gauginos condensates may break SUSY in superstrings

Phys. Lett. 155B (1985) 65 (6 pages, 17 refs.)

ON THE LOW ENERGY $d = 4$, $N = 1$ SUPERGRAVITY THEORY EXTRACTED FROM THE $d = 10$, $N = 1$ SUPERSTRING ☆

J.P DERENDINGER, L.E. IBÁÑEZ [1]

CERN, Geneva, Switzerland

and

H.P. NILLES

Department of Theoretical Physics, University of Geneva, Geneva, Switzerland

Received 8 March 1985

- Heterotic $E_8 \times E_8$ on Calabi-Yau, an exotic (Princeton) novelty.
- Sources of supersymmetry breaking identified: three-form flux and gaugino condensates.

Third: Made at CERN, February 1985

Geneva, February 1985

Temperature in my office reached 8 degrees C, with full CERN heating . . .

Addendum: Geneva, 1980's: Exclusion principle

	CERN, Geneva	Geneva University
… – 1981	–	J.-P. D.
1981 – 1983	H.-P. N.	J.-P. D.
1983 – 1985	J.-P. D.	H.-P. N.
1985 – …	H.-P. N.	–

Notice: 1984, H.-P. N at UniGE, … Budget crisis at the Physics Institute:

SUPERSYMMETRY, SUPERGRAVITY AND PARTICLE PHYSICS

H.P. NILLES*

Département de Physique Théorique, Université de Genève, 1211 Genève 4, Switzerland

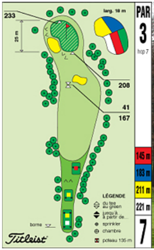
and

CERN, Genève, Switzerland

Received 16 February 1984

A preprint (approx. 160 pages) which everybody wanted !

Common (and simultanous) institutions were later on identified . . .



golf & country club
neuchâtel

INTERNATIONALER GOLF CLUB BONN E.V.
GUT GROSSENBUSCH

Gauginos and super-Yang-Mills

Related works included:

- Veneziano, Yankielowicz, (effective Lagrangian description), 1982.
- Novikov, Shifman, Vainshtein, Zakharov (NSVZ): instanton methods, calculate $\langle\lambda\lambda\rangle$ (one-instanton), 1983-5.
- Amati, Rossi, Veneziano, . . . : dynamical susy breaking, instanton methods, 1984.
- Affleck, Dine, Seiberg, effective Lagrangians, dynamical susy breaking, 1983-4.
- Konishi anomaly, 1983.
- . . .

The first two points used here.

The NSVZ formula

From the instanton calculation of $\langle\lambda\lambda\rangle$, NSVZ obtained an expression for the all-order β function in SYM without, then with matter:

$$\beta(g) = -\frac{g^4}{8\pi^2}\frac{b_0 + 2\gamma(g^2)T(R)}{1-\frac{g^2}{8\pi^2}C(G)} \qquad \mu\frac{d}{d\mu}g^2 = \beta(g^2)$$

$$b_0 = 3\,C(G) - T(R) \qquad \gamma(g) = -\frac{1}{2}\,\frac{d\ln Z(g)}{d\ln\mu}$$

- Anticipated by Jones, for super-Yang-Mills only, using a risky argument based on Ward identities.
- Relation to a perturbative expansion scheme uncertain (still under discussion and controversy). May be considered unimportant.
- Suggests an algebraic formulation of the RG behaviour.

Effective Lagrangian for gaugino condensates

Veneziano and Yankielowicz:

- Describe the condensate with a chiral superfield U:

$$W^\alpha W_\alpha \quad \Longrightarrow \quad \langle W^\alpha W_\alpha \rangle \quad \Longrightarrow \quad U\,, \qquad \overline{D}_\alpha U = 0$$

- Write an effective Lagrangian:

$$\mathcal{L} = \int d^2\theta d^2\overline{\theta}\, \mathcal{K}(U, \overline{U}) + \int d^2\theta\, W(U) + \int d^2\overline{\theta}\, \overline{W}(\overline{U})$$

- Match symmetries and (mixed gauge-global) anomalies of the microscopic (SYM) theory: leads to superpotential

$$W(U) = \frac{1}{4\, g^2}\, U + \frac{A}{4}\left[U \ln(U/\mu^3) - U\right] \qquad \text{What is } A \text{ ?}$$

- Classical global symmetries of super-Yang-Mills are $U(1)_R$ and scale invariance (actually: super-Poincaré extends to superconformal).

Anomaly matching:

- Canonical R symmetry, one-loop only (Adler-Bardeen th.)

$$A = \frac{b_0}{24\pi^2} \qquad\qquad b_0 = 3C(G) - T(R)$$

- Scale anomaly $\sim \beta$ function: all-order.
- One number A for two different anomalies. Mismatch, incomplete.

Then: solve for U:

$$\frac{dW(U)}{dU} = 0 \qquad \Longrightarrow \qquad \langle U \rangle = \mu^3 \exp\left(-\frac{24\pi^2}{b_0\, g^2}\right)$$

SUSY unbroken of course. RG-invariance: $\langle U \rangle$ is μ–independent:

$$\beta(g^2) = -\frac{b_0\, g^4}{8\pi^2} \qquad \text{One-loop only}$$

Gauge coupling field

Couplings are background values of fields / superfields (following Seiberg). Use then supersymmetry constraints on effective Lagrangians to derive algebraic information on the effective dynamics.

- Successful for $N = 2$ and moduli spaces of $N = 1$ theories with matter parameterized by holomorphic gauge invariants
  (Buccella, Ferrara, Savoy, JPD, 1982)
- An abundant, somewhat chaotic, literature.
- Borrow from work with Ferrara, Kounnas, Zwirner (1991) / Burgess, Quevedo, Quiros (1995) / Ambrosetti, Arnold, Hartong (2011).

Couple a real scalar field (and its supermultiplet) to $N = 1$ SYM Lagrangian:

$$-\frac{1}{4}\frac{1}{g^2}F_{\mu\nu}F^{\mu\nu} + \dots$$

$g^2 \sim$ function of a real scalar field

- In $N = 2$: special Kähler geometry, holomorphic prepotential, ...
- In $N = 1$ however: nothing particular ...

Gauge coupling field, VY approach

VY: use a chiral S as gauge coupling field: microscopic theory is

$$\mathcal{L}_{SYM} = \frac{1}{4}\int d^2\theta\, S\, WW + \frac{1}{4}\int d^2\overline{\theta}\, \overline{S}\, \overline{WW} + \int d^2\theta d^2\overline{\theta}\, \mathcal{K}(S+\overline{S})$$

Since $\mathrm{Im}\, WW|_{\theta\theta} = F\wedge F + \ldots = d(\omega+\ldots)$: axionic shift symmetry $S \longrightarrow S + ic$: important, perturbation theory does not depend on the vacuum angle $\theta \sim \mathrm{Im}\, S$ (couples to $F\wedge F$).

Effective description of gaugino condensate, matching R anomaly:

$$\frac{1}{4} SWW \quad \Longrightarrow \quad W_{eff.} = \frac{1}{4}\left(SU + \frac{b_0}{24\pi^2}\left[U \ln(U/\mu^3) - U \right] \right)$$

Axionic shift symmetry incorrectly broken by the replacement
$WW \rightarrow \langle WW \rangle = U$

Except if $U = \overline{DD}\, V$, V *real superfield.*

i.e. if $\mathrm{Im}\, U|_{\theta\theta} = \partial^\mu V_\mu$, as for the microscopic WW.

Gauge coupling field, VY approach / 2

Considering the gauge coupling S as a background field, minimum of the potential (solving $f_U = 0$) gives:

$$\frac{dW_{eff.}}{dU} = 0 \qquad \Longrightarrow \qquad U = \langle WW \rangle = \mu^3 \exp\left(-\frac{24\pi^2}{b_0} S\right)$$

- Connects the value of the vacuum angle θ and R–symmetry anomalous breaking:

$$\mathrm{Arg}\,\langle \lambda\lambda \rangle = -\frac{24\pi^2}{b_0}\,\mathrm{Im}\,S$$

- R–symmetry anomaly matching correct, one-loop only, in agreement with Adler-Bardeen theorem.
- Scale-anomaly matching incorrect.
- There is a simple way out ...

Gauge coupling field / 2

$N = 1$: the most general gauge kinetic (local) Lagrangian with a gauge coupling field is

$$\mathcal{L}_{SYM} = \int d^2\theta d^2\overline{\theta}\, \mathcal{F}(L - 2\Omega) \qquad L \text{ real linear: } \quad \overline{DD}L = 0$$

Ω: Chern-Simons real superfield, $\quad \overline{DD}\,\Omega = WW$

In this theory:

$$\mathcal{L}_{SYM} = -\frac{1}{4g^2}\, F_{\mu\nu}F^{\mu\nu} + \ldots \qquad \frac{1}{g^2} = \mathcal{F}'(C)$$

...an arbitrary function of a real field.

$$L - 2\Omega = \theta\sigma^\mu\overline{\theta}\, \epsilon_{\mu\nu\rho\sigma}(\partial^\nu B^{\rho\sigma} - \omega^{\nu\rho\sigma})$$

Gauge coupling field / 3

- Axion (vacuum angle θ) replaced by Poincaré dual $B_{\mu\nu}$:
  θ–independence in perturbation theory automatic.
- Natural relation to higher-dimensional theories (in contrast to $F \wedge F$), and to string compactifications (dilaton and string loop-counting field).
- Always dual to

$$\int d^2\theta d^2\overline{\theta}\, \mathcal{K}(S+\overline{S}) + \frac{1}{4}\int d^2\theta\, SWW + \frac{1}{4}\int d^2\overline{\theta}\, \overline{SWW}$$

S chiral, *i.e.* $1/g^2 = \mathrm{Re}\, S$, the “holomorphic coupling".

But:

All information flows from $\mathcal{F}(L - 2\Omega)$ to $\mathcal{K}(S+\overline{S})$: the “holomorphic coupling" always exists (may be hard to solve the Legendre transformation).

It does not contain any algebraic information. It is “one-loop only" since one can alvays shift $\mathrm{Re}\, S = g^{-2}$ by a S–independent constant one-loop correction.

This holomorphic coupling has a perturbative understanding in terms of a Wilsonian coupling (Shifman, Vainshtein, 1991).

Gauge coupling field / 4

Next steps are:

- Find the function $\mathcal{F}(L - 2\Omega)$ from anomaly matching.
- Two gauge-invariant superfields, to cancel two anomalies.
  - WW is chiral, identical contributions to R and scale anomalies
  - $L - 2\Omega$: contributes to scale anomalies only.

  The usual Veneziano-Yankielowicz counterterm would suffice if both R and scale anomalies would have identical coefficients.
- Use the supercurrent anomaly equation, for a theory with matter in representation R and canonical dimension.
- With this last assumption, anomalous dimensions of course disappear in the β function.

Supercurrent, anomaly matching

Typical supercurrent structure is:

$$\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha X + \chi_\alpha$$

$$D_{\dot{\alpha}} X = 0 \qquad \chi_\alpha = -\tfrac{1}{4}\overline{DD} D_\alpha \Delta \qquad \Delta = \overline{\Delta}$$

X : R and scale invariance violations

χ_α : scale invariance violation only

For theory $\mathcal{L} = \int d^2\theta d^2\overline{\theta}\, \mathcal{F}(L - 2\Omega)$ the appropriate supercurrent is

$$J_{\alpha\dot{\alpha}} = -2\mathcal{F}_{LL}(D_\alpha \hat{L})(\overline{D}_{\dot{\alpha}} \hat{L}) - 4\mathcal{F}_L W_\alpha \overline{W}_{\dot{\alpha}}$$

$$\chi_\alpha = \overline{DD} D_\alpha(\mathcal{F} - \hat{L}\mathcal{F}_L) \qquad X = 0$$

Supercurrent, anomalies / 2

Warning: with this supercurrent scale invariance does not correspond to $T^\mu{}_\mu = 0$, an improvement would be needed.

The scale anomaly counterterm is then of the form

$$\mathcal{F} = B\,\hat{L}(\ln \hat{L} - 1) \qquad\qquad \chi_\alpha = -B\,\overline{DD}D_\alpha \hat{L}$$

Similar for the real $\hat{L}$ to the VY anomaly counterm

$$\frac{1}{4}\,A\,WW(\ln WW - 1)$$

for the chiral WW.

The numbers are

$$A = \frac{b_0}{24\pi^2} \qquad\qquad B = \frac{1}{8\pi^2}[C(G) - T(R)]$$

Gauge coupling field / 5

With chiral matter in representation R the appropriate Lagragian is

$$\mathcal{F} = \ln \hat{L} + \frac{1}{8\pi^2}[C(G) - T(R)]\hat{L}(\ln \hat{L} - 1)$$

Then, assuming canonical dimensions (*i.e.* anomalous dimension are missing)

$$\beta(C) = -\frac{C^2}{8\pi^2}\frac{b_0}{1 - \frac{1}{8\pi^2}[C(G) - T(R)]C} \qquad C = g^2$$

- (A variant of) NSVZ formula recovered from simple anomaly matching of scale and R symmetries.
- Anomalous dimensions and Konishi anomaly matching missing.
- One-loop only for $N = 2$ SYM (hypermultiplets have a one-loop anomalous dimension, omitted here).

Gaugino condensates

The full formulation of the effective description of gaugino condensation is:

I) Half way towards the dual theory:

$$\int d^2\theta d^2\overline{\theta}\,\mathcal{F}(L-2\Omega) = \int d^2\theta d^2\overline{\theta}\,\mathcal{F}(V) + \frac{1}{4}\int d^2\theta\, S(WW + \frac{1}{2}\overline{DD}V) + c.c.$$

The chiral S imposes $\overline{DD}V = -2WW$ on the real V; then, $V = L - 2\Omega$.

II) Use the VY prescription: $WW \longrightarrow U$ and

$$\mathcal{L}_{eff.} = \int d^2\theta d^2\overline{\theta}\,[\mathcal{F}(V) + \mathcal{K}(U,\overline{U})]$$
$$+\frac{1}{4}\int d^2\theta\,[S(U + \frac{1}{2}\overline{DD}V) + \frac{b_0}{24\pi^2}U(\ln U/\mu^3 - U) + c.c.$$

Gaugino condensates / 2

III) Rewrite as

$$\mathcal{L}_{eff.} = \int d^2\theta d^2\overline{\theta}\,[\mathcal{F}(V) + \mathcal{K}(U,\overline{U})]_{U=\frac{1}{2}\overline{DD}V}$$
$$+\frac{b_0}{96\pi^2}\int d^2\theta\, U(\ln U/\mu^3 - U)]_{U=\frac{1}{2}\overline{DD}V} + c.c.$$

- This effective theory propagates $8_B + 8_F$ (off-shell) in V,
- Minimum of the potential: the real d in $V|_{\theta\theta\overline{\theta\theta}}$ imposes:

$$|U| = |\langle\lambda\lambda\rangle| = \mu^3 \exp\left(-\frac{24\pi^2\mathcal{F}'(C)}{b_0}\right)$$

(The modulus of the gaugino condensate only).

Gaugino condensates / 3

Since

$$\mathcal{F}'(C) = \frac{1}{C} + \frac{1}{8\pi^2}[C(G) - T(R)] \ln C \qquad\qquad C = g^2(\mu)$$

the gaugino condensate is

$$|U| = |\langle\lambda\lambda\rangle| = \mu^3 C^{-1+2T(R)/b_0} \exp\left(-\frac{24\pi^2}{b_0 C}\right)$$

For super-Yang-Mills theory ($T(R) = 0$)

$$|\langle\lambda\lambda\rangle| = \frac{\mu^3}{g^2} \exp\left(-\frac{8\pi^2}{C(G)g^2}\right)$$

and again, RG independence from the scale μ implies

$$\beta(C) = -\frac{C^2}{8\pi^2} \frac{b_o}{1 - \frac{C}{8\pi^2}[C(G) - T(R)]}$$

These are the (all-order) NSVZ results.

Conclusions

- In an effective approach with a gauge coupling field, usual anomaly matching applied to superconformal symmetries (R and scale) leads to β functions similar or identical to the NSVZ formula. It then acquires an algebraic origin.
- Leads to a satisfactory effective formulation of gaugino condensates.
- Not necessarily related to perturbative schemes. For instance, the NSVZ formulation in the case of $N = 2$ SYM requires wave function renormalization $\sim 1/g^2$ to get contact with the non-renormalization theorem.
- Holomorphicity absent: $L - 2\Omega$ is real.
- Then, $N = 1$ SUSY uses essentially the same ingredients as non-supersymmetric gauge theories (QCD), with in addition the NSVZ proposal for an all-order formula for $\beta(g)$.