Quantum-Origin of the Matter-Antimatter Asymmetry

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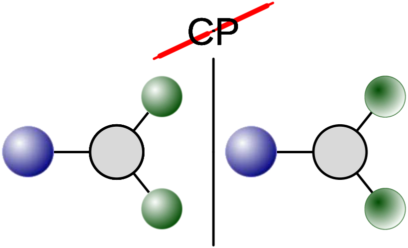
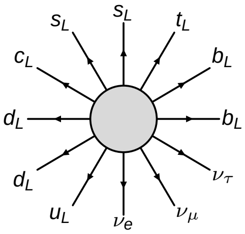
Outline

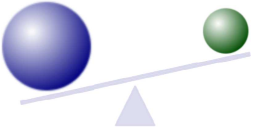
1. Baryogenesis via Leptogenesis
2. Leptogenesis and Kinetic Theory
3. Systematic Kadanoff-Baym Approach
4. Application to Phenomenological Theories
5. Thermal Masses and Modified Dispersion Relations

Baryogenesis via Leptogenesis

$$\eta_B \simeq 6 \cdot 10^{-10}$$







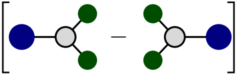
$$\mathcal{L} = \mathcal{L}_{SM} + \tfrac{1}{2}\bar{N}_i\left(i\partial\!\!\!/ - M_i\right)N_i$$
$$-h_{\alpha i}\bar{\ell}_\alpha\tilde{\phi}P_R N_i - h^\dagger_{i\alpha}\bar{N}_i\tilde{\phi}^\dagger P_L \ell_\alpha$$

- kinetic equations to describe non-equilibrium process usually derived from generalized Boltzmann equations (BEs)

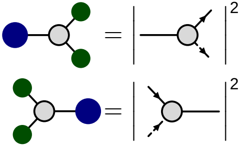
$$k^{\mu}\mathcal{D}_{\mu}f^{\ell}(X,k) = \sum_{\text{interactions of } \ell} C_k^{\ell+\cdot\leftrightarrow i+\cdot}[.f^{\ell}.](X,k)$$

- $2 \leftrightarrow 1$ collision term

$$C^{\ell\phi\leftrightarrow N_i}(k) = \frac{1}{2}\int d\Pi_p^{\phi}\, d\Pi_q^{N_i}(2\pi)^4\delta(k+p-q)\,\Big[\ \cdots\ \Big]$$



where

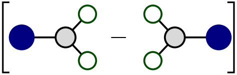


$$(1 \pm f_k^{\ell})(1 \pm f_p^{\phi})f_q^{N_i}$$

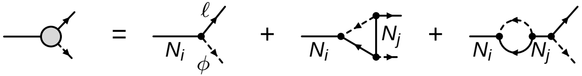
$$f_k^{\ell}f_p^{\phi}(1 \pm f_q^{N_i})$$

- $2 \leftrightarrow 1$ collision term antiparticles

$$C^{\bar\ell\bar\phi\leftrightarrow N_i}(k) = \frac{1}{2}\int d\Pi_p^{\bar\phi}\, d\Pi_q^{N_i}(2\pi)^4\delta(k+p-q)\,\Big[\ \cdots\ \Big]$$

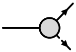


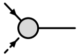
- one-loop vertex and self-energy contribution



- parametrize matrix elements by CP-violating parameter

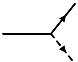
$$\left| \; \right|^2 = \frac{1}{2}(1+\epsilon_i)\,|\mathcal{M}|^2_{N_i}\,, \quad \left| \; \right|^2 = \frac{1}{2}(1-\epsilon_i)\,|\mathcal{M}|^2_{N_i}$$

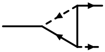


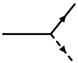


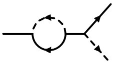
$$\epsilon_i = \frac{|\mathcal{M}|^2_{N_i\to\ell\phi} - |\mathcal{M}|^2_{N_i\to\bar\ell\bar\phi}}{|\mathcal{M}|^2_{N_i\to\ell\phi} + |\mathcal{M}|^2_{N_i\to\bar\ell\bar\phi}}$$

$$\epsilon_i = 2\frac{\Im\{(h^\dagger h)^2_{ij}\}}{(h^\dagger h)_{ii} M_i}\left[\Im\left\{ \;^* \times \; \right\} + 2\Im\left\{ \;^* \times \; \right\}\right]$$









- e.g. self-energy contribution

$$\epsilon_i^S = \frac{\Im\{(h^\dagger h)^2_{ij}\}}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{M_i \Gamma_j}{M_i^2 - M_j^2}$$

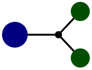
- matrix elements computed in vacuum

- self-energy, completed by finite width (Pilaftsis, Buchmüller, Plümacher)

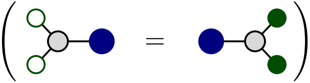
$$\epsilon_i^S \to \frac{\Im\{(h^\dagger h)_{ij}^2\}}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + (M_i\Gamma_j)^2}\,,\quad d\Pi_q^{N_i} = \int \frac{d^3\mathbf{q}\,dq_0}{(2\pi)^3}\delta(q^2 - M_i^2)$$

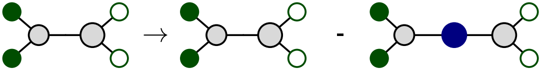
- RIS subtraction for $(n_\ell - n_{\bar\ell})$

$$\int \frac{d^3k}{(2\pi)^3 E_k^{\ell}} \left[C^{\ell\phi\leftrightarrow N_i}(k) - C^{\bar\ell\bar\phi\leftrightarrow N_i}(k)\right] = 4\epsilon \int d\Pi (2\pi)^4 \delta(k + p - q)$$



using





in $C^{\ell\phi\leftrightarrow\bar\ell\bar\phi}(k)$

- shortcomings suggest top-down derivation from Kadanoff-Baym, for toy-model (arXiv: 0807.4551, 0909.1559, 0911.4122, 1005.5385)

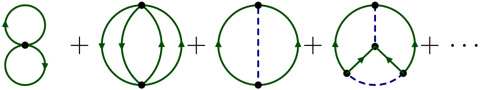
$$\mathcal{L}_{int} = -\frac{\lambda}{2!2!}(\bar{b}b)^2 - \frac{g_i}{2!}\psi_i bb - \frac{g_i^*}{2!}\psi_i \bar{b}\bar{b}\,, \quad i = 1,2$$

$$\left(N_i \to \ell\phi,\ N_i \to \bar{\ell}\bar{\phi}\right) \Longrightarrow \left(\psi_i \to bb,\ \psi_i \to \bar{b}\bar{b}\right)$$

  other approaches: (MG, AH, AK, ML; Anisimov, Buchmüller, Drewes, Mendizabal; Beneke, Garbrecht, Herranen, et.al.; De Simone, Riotto)

- Schwinger–Dyson equation (with self-energy Σ)

$$D^{-1}(x,y) = \mathscr{D}^{-1}(x,y) - \Sigma(x,y), \quad \Sigma(x,y) \equiv 2i\frac{\delta\Gamma_2[D,\Phi]}{\delta D(y,x)}$$

- 2PI functional $\Gamma_2 =$ 

Systematic Kadanoff-Baym Approach

- Kadanoff-Baym equations

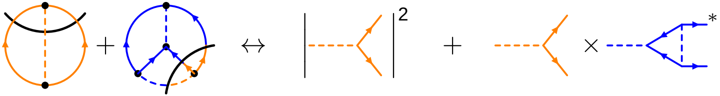
$$[\Box_x + m^2(x)]\, D_F(x,y) = \int_0^{y^0} \sqrt{-g}\, d^4z\, \Sigma_F(x,z) D_\rho(z,y) - \int_0^{x^0} \sqrt{-g}\, d^4z\, \Sigma_\rho(x,z) D_F(z,y)$$

$$[\Box_x + m^2(x)]\, D_\rho(x,y) = \int_{x^0}^{y^0} \sqrt{-g}\, d^4z\, \Sigma_\rho(x,z) D_\rho(z,y)$$

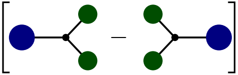
- self-energy for decay

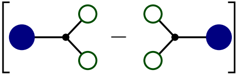
$$\Sigma^{\triangleleft}_{\gtrless}(X,k) = -|g_i|^2 \int d\Pi^4_q d\Pi^4_p (2\pi)^4 \delta(k+p-q)$$

$$\times \left[1 + \epsilon_i(X,q,p)\right] G^{ii}_{\gtrless}(X,q)\, D_{\lessgtr}(X,p)$$



- quantum-corrected BEs

$$C^{b\,b\leftrightarrow\psi_i}(k)=\frac{1}{2}\int d\Pi_p^{b}\, d\Pi_q^{\psi_i}(2\pi)^4\delta(k+p-q)\left(1+\epsilon_i\right)$$ 

$$C^{\textcircled{b}\textcircled{b}\leftrightarrow\psi_i}(k)=\frac{1}{2}\int d\Pi_p^{\textcircled{b}}\, d\Pi_q^{\psi_i}(2\pi)^4\delta(k+p-q)\left(1-\epsilon_i\right)$$ 

- CP-violating parameter

$$\epsilon_i=\epsilon_i^V+\epsilon_i^S,\quad \epsilon_i^V=-\frac{1}{8\pi}\frac{|g_j|^2}{M_i^2}\Im\left(\frac{g_i g_j^*}{g_i^* g_j}\right)\int\frac{d\Omega_l}{4\pi}\frac{1+f_{E_1}^{b}+f_{E_2}^{b}}{M_j^2/M_i^2+\frac{1}{2}(1+\cos\theta_l)}$$

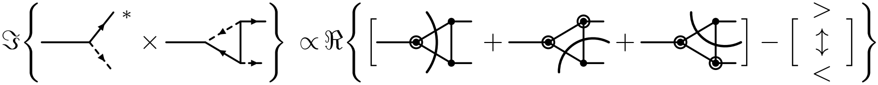
$$\text{with } E_{1,2}=\frac{1}{2}\left[E_q^{\psi_1}\mp|\mathbf{q}|\cos\theta_{ql'}\right]$$

- thermal CP-violating parameter

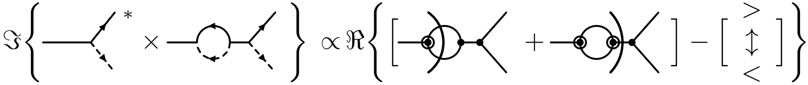
$$\epsilon_i^{V,th}=-\frac{1}{8\pi}\frac{|g_j|^2}{M_i^2}\Im\left(\frac{g_i g_j^*}{g_i^* g_j}\right)\int\frac{d\Omega_l}{4\pi}\frac{1+f_{E_1}^{b}+f_{E_2}^{b}+2f_{E_1}^{b}f_{E_2}^{b}}{M_j^2/M_i^2+\frac{1}{2}(1+\cos\theta_l)}$$

Application to Phenomenological Theories

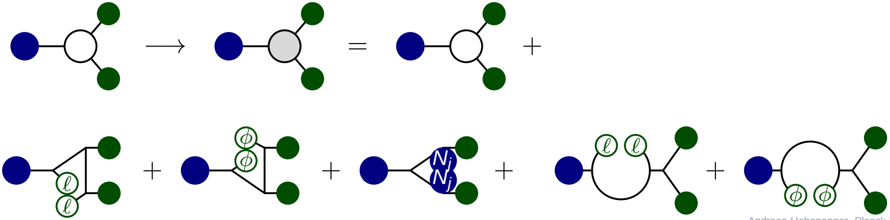
- toy-model results applied to phenomenological scenarios (arXiv: 1002.0331) using causal products in real-time formalism (Kobes 1991)



- self-energy contribution

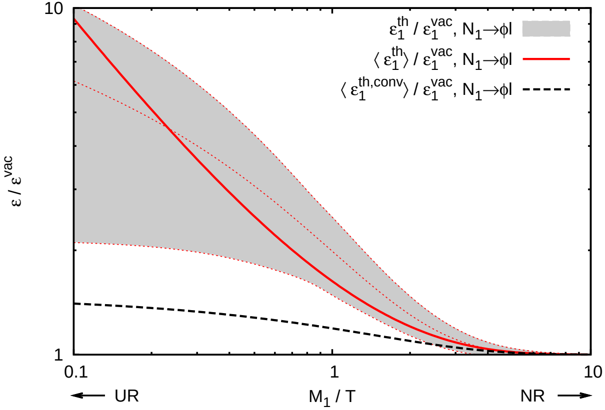


- vertex and self-energy contribution depend on $1 + f^{\phi} - f^{\ell}$ (MG, AH, AK, ML; Beneke, Garbrecht, Herranen, et.al.)



Application to Phenomenological Theories

- enhancement of ϵ_i in SM+3ν_R

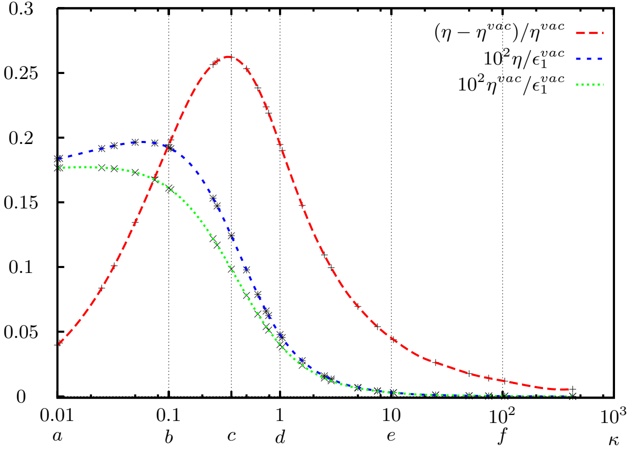


$$\epsilon_i^{S,th} = -\frac{1}{8\pi}\frac{\Im\left\{(h^\dagger h)_{ij}^2\right\}}{(h^\dagger h)_{ii}}\frac{M_i M_j}{M_i^2 - M_j^2}\int\frac{d\Omega_l}{4\pi}(1-\cos\theta_l)\left\{1 - f_{E_1}^{\ell} + f_{E_2}^{\phi}\right\}$$

$$\epsilon_i^{V,th} = \frac{1}{16\pi}\frac{\Im\left\{(h^\dagger h)_{ij}^2\right\}}{(h^\dagger h)_{ii}}\frac{M_j}{M_i}\int\frac{d\Omega_l}{4\pi}\frac{1-\cos\theta_l}{M_j^2/M_i^2 + \frac{1}{2}(1+\cos\theta_l)}\left\{1 - f_{E_1}^{\ell} + f_{E_2}^{\phi}\right\}$$

Application to Phenomenological Theories

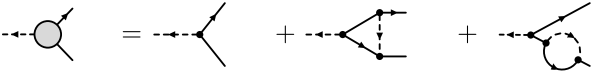
- relative increase of final baryon asymmetry from solution of full BEs (toy-model)



- sizable effect for intermediate washout-factor

Thermal Masses and Modified Dispersion Relations

- thermal masses of leptons and Higgs take large values $m_{\ell} \sim 0.2T$, $m_{\phi} \sim 0.4T$, $\phi \to \ell + N_i$ for $m_{\phi} > m_{\ell} + M_i$



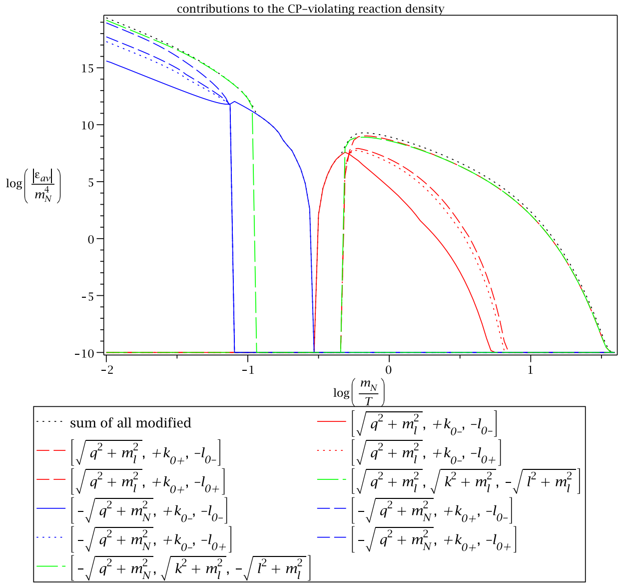
ϵ_i for Higgs decay depends on $f^{\phi} + f^{\ell}$

- leptons have modified dispersion relations (due to partially full Fermi-sea)
(Weldon; Giudice et.al.; Plümacher, Kießig)

$$d\Pi_q^{\ell} = \int \frac{d^3\mathbf{k}\, dk_0}{(2\pi)^3} \delta(k^2 - m_{\ell}^2) \longrightarrow \int \frac{d^3\mathbf{k}\, dk_0}{(2\pi)^3} \delta(\delta_{\ell}(k))$$

$\delta_{\ell}(k) = 0$ has particle and hole solutions, may be derived using HTL-resummation

Thermal Masses and Modified Dispersion Relations



Conclusions

- quantum-corrected BEs derived in hierarchical case
  - consistent structure (no double-counting problem)
  - include medium effects (thermal QFT in RTF reconciled)
  - quantitative corrections can be significant (but further thermal effects must be included)
  - improved equations for quasi-degenerate case
  - gradient corrections contribute even in LTE (can be neglected in standard scenarios)
- ongoing and future work
  - numeric analysis of quasi-degenerate case in SM
  - analysis of 2-2 scattering contributions
  - off-shell equations needed for resonant scenarios
  - leptogenesis through neutrino oscillations (νMSM)

Derivation from Kadanoff-Baym

- Kadanoff-Baym equations

$$[\Box_x + m^2(x)]D_F(x,y) = \int_0^{y^0} \sqrt{-g}d^4z\,\Sigma_F(x,z)D_\rho(z,y) - \int_0^{x^0} \sqrt{-g}d^4z\,\Sigma_\rho(x,z)D_F(z,y)$$

$$[\Box_x + m^2(x)]D_\rho(x,y) = \int_{x^0}^{y^0} \sqrt{-g}d^4z\,\Sigma_\rho(x,z)D_\rho(z,y)$$

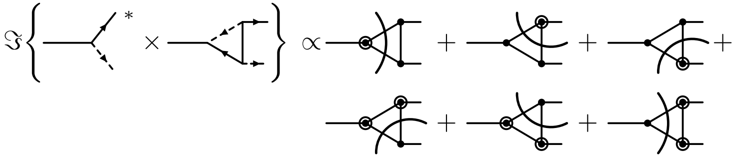
- "quantum-corrected BEs" result from systematic computation
  - introduce center and relative coordinates (covariantly), $t_{init} \to -\infty$
  - Wigner transformation wrt. relative coordinate
  - gradient expansion wrt. center coordinate
  - neglect Poisson brackets (memory effects)
  - Kadanoff-Baym ansatz and quasi-particle approximation

$$[k^\mu \mathcal{D}_\mu f^{b}(X,k)]D_\rho(X,k) = -\frac{1}{2}[D_>(X,k)\Sigma_<(X,k) - \Sigma_>(X,k)D_<(X,k)]$$

where $$D_{\gtrless}(X,p) \equiv D_F(X,p) \pm \frac{1}{2}D_\rho(X,p) = \begin{cases} (1+f^{b}(X,k))D_\rho(X,p) \\ f^{b}(X,k)D_\rho(X,p) \end{cases}$$

Time Ordered Cutting

- loops computed with thermal cutting rules (Kobes&Semenoff 1986)



$$D_a(p) = \frac{i}{p^2 - m_a^2 + i\epsilon} + 2\pi f^{a,eq}(p)\delta(p^2 - m_a^2)\,,$$

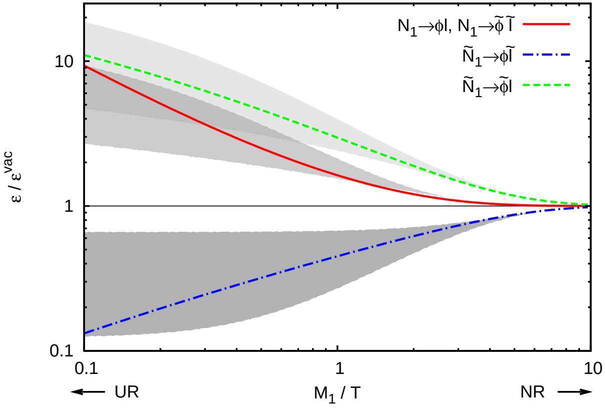
$$D_a^{\pm}(p) = 2\pi\left[\Theta(\pm p_0) + f^{a,eq}(p)\right]\delta(p^2 - m_a^2)$$

- thermal CP-violating parameter

$$\epsilon_i^{V,th} = -\frac{1}{8\pi}\frac{|g_j|^2}{M_i^2}\Im\left(\frac{g_i g_j^*}{g_i^* g_j}\right)\int\frac{d\Omega_l}{4\pi}\frac{1 + f^b_{E_1} + f^b_{E_2} + 2f^b_{E_1}f^b_{E_2}}{M_j^2/M_i^2 + \frac{1}{2}(1 + \cos\theta_l)}$$

CP-violating Parameter in MSSM

- correction to ϵ_i in MSSM



- dependence on statistics loop particles (strong enhancement for bosons)
- require solution of BEs and inclusion of further thermal effects (in particular thermal Higgs mass)

Gradient Corrections

- “quantum-corrected BEs” result from systematic computation
  - ...
  - gradient expansion wrt. center coordinate
  - ...
- formally gradient expansion

$$\int \sqrt{-g} d^4 z A(x,z) B(z,x) = \int d\Pi_p e^{-i\diamondsuit}\{A(X,p)\}\{B(X,p)\}$$

where $\diamondsuit\{.\}\{.\} = \frac{1}{2}(\partial_X^{(1)}\partial_p^{(2)} - \partial_p^{(1)}\partial_X^{(2)})\{.\}\{.\}$

- rate equations at zeroth order in the gradients

$$\frac{dY_{\psi_1}}{dz} = -\kappa z \gamma_D (Y_{\psi_1} - Y_{\psi_1}^{eq})$$
$$\frac{dY_{B-L}}{dz} = \mathcal{S}(z) - \mathcal{W}_0(z) Y_{B-L}$$

where source $\mathcal{S}(z) = \mathcal{S}_{\Delta f}(z) = \epsilon^{vac} \kappa z \gamma_D (Y_{\psi_1} - Y_{\psi_1}^{eq})$

- at higher order in gradients further source terms show up ((arXiv: 1005.5385)

$$\mathcal{S}(z) \equiv \mathcal{S}_{\Delta f}(z) + \mathcal{S}_{\dot{T}}(z) + \mathcal{S}_{\Delta f \times \dot{T}}(z)$$

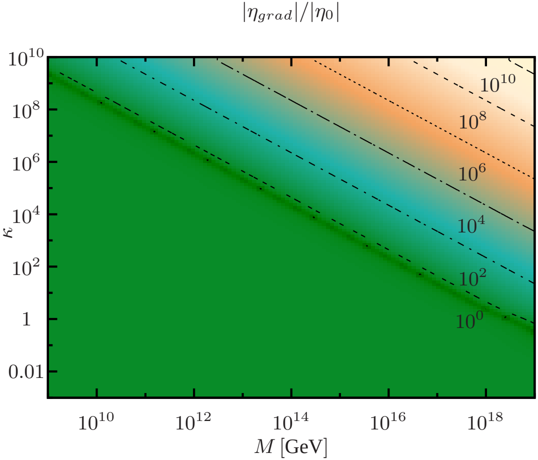
$$\mathcal{S}_{\Delta f}(z) = \epsilon^{vac}\, \kappa\, z\, \gamma_D \left(Y_{\psi_1} - Y_{\psi_1}^{eq}\right)$$

$$\mathcal{S}_{\dot{T}}(z) \simeq -\kappa \gamma_{B-L} \frac{H|_{T=M_1}}{M_1} \frac{\epsilon^{vac}}{2\pi^3}$$

- $\mathcal{S}_{\dot{T}}(z)$ contributes even in LTE
- $\mathcal{S}_{\Delta f \times \dot{T}}(z)$ may be interpreted as contribution to the CP-violating parameter

$$\epsilon_{\Delta f \times \dot{T}} = -\epsilon_i^{vac} \cdot \frac{2[k^\alpha \mathcal{D}_\alpha L_h]}{M_j^2 - M_i^2}$$

- solve rate equation with and without gradient contribution $\mathcal{S}_{\dot{T}}(z)$



- washout terms not included
- safe to neglect gradient contribution for standard leptogenesis scenarios