Weak Mixing Angle and Proton Stability in F-theory GUT

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@ Planck

Flipped SU(5) [= SU(5) x $U(1)_X$]

= A maximal subgroup of SO(10),
(if $U(1)_X$ charges properly normalized)

$$\frac{1}{2}Y = -\frac{1}{5}Z + \frac{1}{5}X \ ,$$

where $$Z = \text{diag.}\left(\frac{-1}{3}\ \frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{2}\ \frac{1}{2}\right)$$

In Flipped SU(5),

[Barr, Antoniadis etal.]

- Broken to SM gauge gp. by tensor Higgs $<10_H>$, $<10^*_H>$.
- Doublet/Triplet splitting simply works. [missing partner mech.]
- No unrealistic mass relation. [e.g. $m_e \neq m_d$]

- ***Explanation for Unification or $U(1)_X$ normalization needed .***

R parity violation in Flipped SU(5)

$10_H\, 10_i\, 10_j\, 5^*_k \;\longrightarrow\; \langle \nu^c_H \rangle \{ q_i d^c_j l_k + d^c_i d^c_j u^c_k \}$

: eff. dim. 4 R-parity viol.

$10_H 5^*_i 5^*_j 1_k \;\longrightarrow\; \langle \nu^c_H \rangle\, l_i l_j e^c_k$

: eff. dim. 4 R-parity viol.

$10_i\, 10_j\, 10_k\, 5^*_l \;\longrightarrow\; q_i q_j q_k l_l$: dim. 5 proton decay

$10_i\, 5^*_j\, 5^*_k\, 1_l \longrightarrow d^c_i u^c_j u^c_k e^c_l$: dim. 5 proton decay

Dim. 5 proton decay induced by heavy triplets

If there is an additional heavy pair of $\{5_G, 5^*_G\}$;

$$5_G = \{D_G, L_G\},\ 5^*_G = \{D^c_G, L^c_G\}$$

$$W_{unwanted} = 10_i 10_j 5_G + 10_k\, 5^*_l 5^*_G + 1_m 5^*_n 5_G + M_G 5_G 5^*_G$$

$$= (d^c_{\{k}v^c_{j\}} + q_{\{i}q_{j\}} + e^c_m u^c_n)\, D_G + (d^c_{\{i}q_{j\}} + e^c_m l_n)\, L_G$$

$$+ (d^c_k u^c_l + q_k l_l)\, D^c_G + (q_k u^c_l + v^c_k l_l)\, L^c_G$$

$\{D_G, D^c_G\}$ mediate $(1/M_G)(q_i q_j q_k l_l + d^c_k u^c_l u^c_n e^c_m)$.

To avoid d=4,5 proton decay,

In Flipped SU(5),

- 10 10 10 5* and 10 5* 5* 1 should be forbidden.
- Only one pair of { 5_h , 5^*_h } should be there.

F theory

[Vafa]

Type IIB string theory = elliptically fibred F-theory,
in which
τ (= $C_0 + ie^{-\phi}$) = complex structure of the torus

- Elliptic eqn. : $y^2 = x^3 + f(z)x + g(z)$, $\{x,y\} \in C^1 x C^1$, $\{z\} \in B$

(shape of the torus varies on B)

- Discriminant locus , $S_{GUT} = \{ 4f^3+27g^2 = 0 \}$: 7 branes
- Shape of the singularity = gauge group of the 7 branes
- E_n gauge groups are possible.

F theory model of $SU(5)_{GG}$

$y^2 + A_1 xy + A_3 y = x^3 + A_2 x^2 + A_4 x + A_6$, $\{x,y\} \in C^1 x C^1$

$E_8 \longrightarrow SU(5) \times SU(5)_\perp$

A_m = holomorphic sec. of K_B^{-m}

$b_m = \eta - m\, c_1$

$A_1 = -\, b_5 + O(z)$

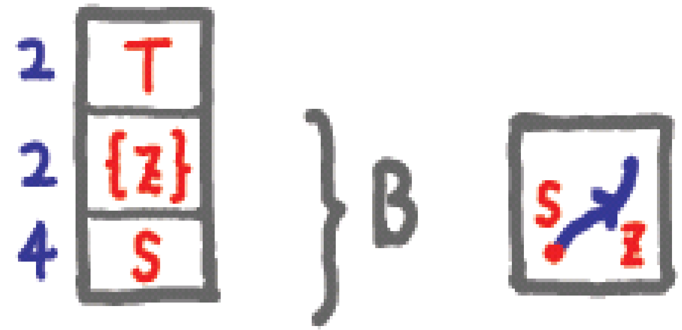
$A_2 = b_4\, z + O(z^2)$

$A_3 = -\, b_3\, z^2 + O(z^3)$

$A_4 = b_2\, z^3 + O(z^4)$

$A_6 = b_0\, z^5 + O(z^6)$

CY_4



(E_8 is unbroken if $A_1=A_2=A_3=A_4=0$.)

F theory model of $SU(5)_{GG}$

$$E_8 \longrightarrow SU(5) \times SU(5)_{\perp}$$

$$248 \longrightarrow (24,1) + (1, 24_{\perp}) + \{ (5^*, 10_{\perp}) + (10, 5_{\perp}) + c.c. \}$$

$$b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5 = 0 ,$$

$$Z = P(K_S + 0) \underset{\pi}{\longrightarrow} S_{GUT}$$

Describing the commutant group $SU(5)_{\perp}$

$$\{ U, V \} \in \{ \sigma, \sigma_\infty \} , \ \sigma_\infty = \sigma + \pi^* c_1, \ \ \sigma \cap \sigma_\infty = 0$$

$$b_m = \eta - m c_1 \qquad \eta \in H^2(S_{GUT}, \text{int.})$$

$6c_1 - t$

1st Chern class of tangent sp. of S_{GUT}

For obtaining Flipped SU(5),

$$E_8 \longrightarrow SU(5) \times SU(5)_\perp$$

$$248 \longrightarrow (24,1) + (1, 24_\perp) + \{ (5^*, 10_\perp) + (10, 5_\perp) + c.c. \}$$

$$SU(5)_\perp \longrightarrow SU(4)_\perp \times U(1)_X$$

[So $E_8 \longrightarrow \{ SU(5) \times U(1)_X \} \times SU(4)_\perp$]

$$24_\perp \longrightarrow 15_0 + 1_0 + 4_5 + 4^*_{-5}$$

$$10_\perp \longrightarrow 6_2 + 4_{-3}$$

$$5_\perp \longrightarrow 4_1 + 1_{-4}$$

$$X = (1\ 1\ 1\ 1\ ; -4) \times N_{5\perp}$$

For obtaining Flipped SU(5),

Flipped SU(5) multiplets	weights of $SU(5)_\perp$
$(1,4)_5$: e^c	$t_i - t_5$
$(5^*,6)_2$: $(d^c{}_h, H_u)$	$t_i + t_j$
$(5^*,4)_{-3}$: (u^c, L)	$t_i + t_5$
$(10,4)_1$: (d^c, Q, ν^c)	t_i
$(10,1)_{-4}$: exotic, $\in$ 45 of SO(10)	t_5

i, j = 1,2,3,4

$$(b_0U^4 + b_1U^3V + b_2U^2V^2 + b_3UV^3 + b_4V^4)(U+d_1V) = 0\,,$$

where $b_0d_1+b_1 = 0$, spectral cover (4+1) factorized !

R parity ?

Unfortunately **10 10 10 5*** is allowed.

$t_1 + t_2 + t_3 + (t_4 + t_5)$

→ More factorize the spectral cover eq.

Structure Cover

$S[U(3) \times U(1)_Z \times U(1)_X]$ is described by

$$C^{(a)} C^{(b)} C^{(d)} =$$

$$(a_0 U^3 + a_1 U^2 V + a_2 U V^2 + a_3 V^2)(b_0 U + b_1 V)(d_0 U + d_1 V) = 0 \,,$$

$b_0 = d_0 = 1$ [trivial sec.] to be consistent with 6d GS rel.

[K. Choi]

$\therefore$ $a_1 = -a_0(b_1 + d_1)$ [$SU(5)_\perp$ traceless condi.]

$$a_1/a_0 \sim t_1 + t_2 + t_3$$
$$a_2/a_0 \sim t_1 t_2 + t_2 t_3 + t_3 t_1$$
$$a_3/a_0 \sim t_1 t_2 t_3$$
$$b_1/b_0 \sim t_4$$
$$d_1/d_0 \sim t_5$$

Field Spectrum (matt. curve)

$\mathbf{10}_1$: $\prod_i t_i \to 0$;
$\mathbf{10}_1'$: $t_4 \to 0$;

$\mathbf{5}^*_{-3}$: $\prod_i (t_i + t_5) \to 0$;
$\mathbf{5}^*_{-3}{}'$: $t_4 + t_5 \to 0$;

$\mathbf{1}_5$: $\prod_i (t_i - t_5) \to 0$;
$\mathbf{1}_5'$: $t_4 - t_5 \to 0$;

$\mathbf{5}_{-2}$: $\prod_{i<j} (-t_i - t_j) \to 0$;
$\mathbf{5}^*_2{}'$: $\prod_i (t_i + t_4) \to 0$;

$\mathbf{1}_0$: $\prod_i (t_i - t_4) \to 0$;
$\mathbf{10}_{-4}'$: $t_5 \to 0$;

$$E_8 \longrightarrow SU(5) \times U(1)_X \times SU(4)_\perp$$

$$\begin{aligned} 248 \longrightarrow\ & (24,1)_0 + (1,15)_0 + (1,1)_0 \\ & + \{(1,4)_5 + (10,4)_1 + (10,1)_{-4} \\ & + (5,6^*)_{-2} + (5^*,4)_{-3} + \text{c.c.}\} \end{aligned}$$

$$X = (1,\ 1,\ 1;\ 1;\ -4)$$

$$Z = (1,\ 1,\ 1; -3;\ 0)$$

where $t_i, t_j = \{t_1, t_2, t_3\}$

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$+ \{(1,4)_5 + (10, 4)_1 + (10,1)_{-4}$
$+ (5, 6^*)_{-2} + (5^*, 4)_{-3} + c.c. \}$

$X = (1, 1, 1; 1; -4)$

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where $t_i, t_j = \{ t_1, t_2, t_3 \}$

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$\mathbf{5^*_{-3}{}'}$: $t_4 + t_5 \to 0$;

$\mathbf{1_5}$: $\prod_i (t_i - t_5) \to 0$;
$\mathbf{1_5'}$: $t_4 - t_5 \to 0$;

$\mathbf{5_{-2}}$: $\prod_{i<j} (-t_i - t_j) \to 0$;
$\mathbf{5^*_2{}'}$: $\prod_i (t_i + t_4) \to 0$;

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1_5 : $\prod_i (t_i - t_5) \to 0$;

$1_5'$: $t_4 - t_5 \to 0$;

5_{-2} : $\prod_{i<j} (-t_i - t_j) \to 0$;

$5^{*\prime}_{2}$: $\prod_i (t_i + t_4) \to 0$;

1_0 : $\prod_i (t_i - t_4) \to 0$;

$10'_{-4}$: $t_5 \to 0$;

$$E_8 \to SU(5) \times U(1)_X \times SU(4)_\perp$$

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$\mathbf{1}_5'$: $t_4 - t_5 \to 0$;

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$$X = (1,\ 1,\ 1;\ 1;\ -4)$$

$$Z = (1,\ 1,\ 1; -3;\ 0)$$

where $t_i, t_j = \{t_1, t_2, t_3\}$

Flux for 4d Chiral Spectrum

Turn on fluxes only on $C^{(a)}$ and $C^{(b)}$ to preserve **SO(10) structure**:

$$\Gamma_a = \lambda\{3\sigma - \pi_a{*}(\eta - 3c_1)\} + \pi_a{*}\,\zeta/3 \quad \text{with } \pi_a : C^{(a)} \to S_{GUT}$$

$$\Gamma_b = -\pi_b{*}\,\zeta \quad \text{with } \pi_b : C^{(b)} \to S_{GUT}$$

(traceless)

$\therefore$ $\mathbf{10}_{-4}$ remains vector-like.

Flux quantization condi. : $3(1/2 + \lambda) \in \text{int.}$

$$-(\lambda - 1/2)\eta + (3\lambda - 1/2)c_1 + \zeta/3 \in H^2(S_{GUT}, \text{int.})$$

[Blumenhagen etal.]

Flux for 4d Chiral Spectrum

Riemann-Roch-Hirzebruch index theorem:

$$n(R) = n_R - n_{R*} = \text{index } i\not\nabla = \int_\Sigma \text{tr } F$$

$$= \boldsymbol{\Sigma}_R \cap \boldsymbol{\Gamma}|_{\boldsymbol{\sigma}}$$

Field Spectrum (# of Families)

10_1 : $\prod_i t_i \to 0$; $-(\lambda\eta - \zeta/3)\cdot(\eta - 3c_1) + c_1 \cdot \zeta = ?$

$10_1'$: $t_4 \to 0$; $c_1 \cdot \zeta = ?$

5^*_{-3} : $\prod_i (t_i + t_5) \to 0$; $-(\lambda\eta - \zeta/3)\cdot(\eta - 3c_1) + c_1 \cdot \zeta = ?$

$5^*_{-3}{}'$: $t_4 + t_5 \to 0$; $c_1 \cdot \zeta = ?$

1_5 : $\prod_i (t_i - t_5) \to 0$; $-(\lambda\eta - \zeta/3)\cdot(\eta - 3c_1) + c_1 \cdot \zeta = ?$

$1_5'$: $t_4 - t_5 \to 0$; $c_1 \cdot \zeta = ?$

5_{-2} : $\prod_{i,j} (-t_i - t_j) \to 0$; $-(\lambda\eta + 2\,\zeta/3)\cdot(\eta - 3c_1) = ?$

$5^*_2{}'$: $\prod_i (t_i + t_4) \to 0$; $-(\lambda\eta + 2\,\zeta/3)\cdot(\eta - 3c_1) = ?$

1_0 : $\prod_i (t_i - t_4) \to 0$; $-(\lambda\eta - 4\,\zeta/3)\cdot(\eta - 3c_1) = ?$

$10_{-4}'$: $t_5 \to 0$; 0

Flux for 4d Chiral Spectrum

For $\{10_1, 5^*_{-3}, 1_5\} = 3$, $\{5_{-2}, 5^*_2\} = 1$,

and $\{10_1', 5^*_{-3}', 1_5'\} = 0$,

Require $\underline{\lambda\eta\cdot(\eta - 3c_1) = -7/3,\ \ \eta\cdot\zeta = 2,\ \ c_1\cdot\zeta = 0}$,

Flux quantization condi. satisfied by taking

$\lambda = 1/6$ and $(\eta + \zeta)/3 \in H^2(S_{GUT}, Z)$

Field Spectrum (# of Families)

10_1 : $\prod_i t_i \to 0$; $-(\lambda\eta - \zeta/3)\cdot(\eta - 3c_1) + c_1\cdot\zeta = 3$

$10_1'$: $t_4 \to 0$; $c_1\cdot\zeta = 0$

5^*_{-3} : $\prod_i (t_i + t_5) \to 0$; $-(\lambda\eta - \zeta/3)\cdot(\eta - 3c_1) + c_1\cdot\zeta = 3$

$5^*_{-3}{}'$: $t_4 + t_5 \to 0$; $c_1\cdot\zeta = 0$

1_5 : $\prod_i (t_i - t_5) \to 0$; $-(\lambda\eta - \zeta/3)\cdot(\eta - 3c_1) + c_1\cdot\zeta = 3$

$1_5'$: $t_4 - t_5 \to 0$; $c_1\cdot\zeta = 0$

5_{-2} : $\prod_{i,j} (-t_i - t_j) \to 0$; $-(\lambda\eta + 2\zeta/3)\cdot(\eta - 3c_1) = 1$

$5^*_2{}'$: $\prod_i (t_i + t_4) \to 0$; $-(\lambda\eta + 2\zeta/3)\cdot(\eta - 3c_1) = 1$

1_0 : $\prod_i (t_i - t_4) \to 0$; $-(\lambda\eta - 4\zeta/3)\cdot(\eta - 3c_1) = 5$

$10_{-4}'$: $t_5 \to 0$; 0

Base Manifold

Require $\lambda\, \eta \cdot (\eta - 3c_1) = -7/3$, $\eta \cdot \zeta = 2$, $c_1 \cdot \zeta = 0$,

Flux quantization condi. satisfied by taking

$\lambda = 1/6$ and $(\eta + \zeta)/3 \in H^2(S_{GUT}, Z)$

All the condi. satisfied only if $S_{GUT} = dP_2$,

i.e. $-K_S = c_1 = 3H - E_1 - E_2$, and $\eta = 2H$, $\zeta = H - 3E_1$

[$H \cdot H = 1$, $E_i \cdot E_j = -\delta_{ij}$, $H \cdot E_i = 0$]

Global embedding is easily done !!

[Blumenhagen etal]

Dim. 5 proton decay?

10_1 $\{t_1, t_2, t_3\}$; 5^*_{-3} $\{t_1+t_5, t_2+t_5, t_3+t_5\}$;

1_5 $\{t_1-t_5, t_2-t_5, t_3-t_5\}$; 1_0 $\{t_1-t_4, t_2-t_4, t_3-t_4\}$

$W \supset 10_1 10_1 10_1 5^*_{-3} + 10_1 5^*_{-3} 5^*_{-3} 1_5$

are forbidden,

because $(+t_4)$ is missing.

Field Spectrum (Higgs fields)

To avoid dim. 5 proton decay,

Require <u>Absolute # of $\{5_{-2}, 5^*_2\} = 1$</u>.

In principle, it is possible.

$W \supset 1_0\, 5_{-2} 5^*_2$ (``μ term'') ,

where we have five 1_0s.

The bare μ-term ($M_G\, 5_{-2} 5^*_2$) is forbidden.

Field Spectrum (Higgs fields)

Net # of $10_1 = 3$

Regard $1\mathrm{x}\{10_1, 10^*_{-1}\}$ as $\{10_H, 10^*_H\}$.

$$W \supset M_G 10_H 10^*_H + (10_H 10^*_H)^2/M_G + \cdots .$$

Assume $\langle 10_H \rangle = \langle 10^*_H \rangle \sim M_G$ at a SUSY vacuum.

In the strongly coupled heterotic string theory (``heterotic M''), the fundamental scale becomes coincident with the GUT scale. F-theory is dual to the heterotic M theory.

VEV distinguishes 10_H and 10_i .

$U(1)_X$ normalization

SO(10) normalization for $U(1)_X$

gives

the MSSM gauge coupling unification

with $\sin^2\theta_W = 3/8$,

under which $X \rightarrow X\,/\,(40)^{1/2}$

i.e.

$10_{1/\sqrt{40}}$, $5^*_{-3/\sqrt{40}}$, $1_{5/\sqrt{40}}$

$5_{-2/\sqrt{40}}$, $5^*_{2/\sqrt{40}}$ etc.

$U(1)_X$ normalization

In our case, at first glance,

$U(1)_X \subset SU(5)_\perp$

$|(2,2,2,2,2)|^2 / L(5^*) = |(1,1,1,1;-4)|^2 / L(5^*_\perp)$

$= 40 = 1/N_{5^*}{}^2$

$\mathrm{tr}_R T^a T^b = L(R)\, \delta^{ab}$

$L(F) = ½$ for $SU(n)$

So $X \longrightarrow X / (40)^{1/2}$

when $U(1)_X \subset SU(5)_\perp$

∴ MSSM gauge couplings unified

giving $\sin^2\theta_w = 3/8$.

$U(1)_X$ normalization

In our case, at first glance,

$U(1)_X \subset SU(5)_\perp$

Indeed, $U(1)_X \subset SO(10)$,

∵ $U(1)_X$ is the common intersection between $SO(10)$ and $SU(5)_\perp$

$SO(10) \times SU(4)_\perp \subset E_8$

$SU(5) \times SU(5)_\perp \subset E_8$

Consistency of $U(1)_X$

From Chern-Simions interaction of Type IIB th.,

$$\int C_4 \wedge \exp(iF/2\pi)\,,$$

$$S_{induced} \sim \mathrm{tr}X^2 \int_{M4} F_X \wedge f^{(i)} \int_S c_1(L_X) \wedge \omega_i$$

In our case, $L_X = 0$, and so
the $U(1)_X$ gauge boson is masselss.

$U(1)_X$ anomalies are free.

Summary

To prohibit the dim. 4 and 5 proton decay,
the structure group in F-theory GUT should be $SU(3)_{\perp}$
or smaller, and there should be
only one pair of { 5_h , 5^*_h }.

We proposed an F-th. model of the Flip. SU(5)
considering them,
in which $\sin^2\theta_W = 3/8$ at the string scale.
We turned on fluxes only on the flavor branes.
So the gauge coupling unification shown
in the MSSM can be protected in the model.

StringVac 2011

September 5-7, 2011

Venue: The Westin Chosun Hotel, Busan

URL: http://beauty.phys.pusan.ac.kr/sv2011

Invited Speakers

Antoniadis, I. (CERN)
Cvetic, M. (U. Penn)
Faraggi, A. (Liverpool U.)
Heckman, J. (IAS)

Kawamura, Y. (Shinshu U.)
Kobayashi, T. (Kyoto U.)
Ovrut, B. (U. Penn)
Schellekens, B. (Nikhef U.)

F theory model of $SU(5)_{GG}$

$$E_8 \longrightarrow SU(5) \times SU(5)_\perp$$

$$248 \longrightarrow (24,1) + (1, 24_\perp) + \{ (5^*, 10_\perp) + (10, 5_\perp) + c.c. \}$$

$$b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5 = 0 ,$$

$$b_1/b_0 = \Sigma_i\, t_i = 0 \text{ (traceless)} ,\quad b_2/b_0 = \Sigma_{i<j}\, t_i\, t_j ,$$

$$b_3/b_0 = \Sigma_{i<j<k}\, t_i\, t_j\, t_k ,\quad b_4/b_0 = \Sigma_{i<j<k<l}\, t_i\, t_j\, t_k\, t_l ,\quad b_5/b_0 = t_1 t_2 t_3 t_4 t_5$$

$b_0 \prod_k (s + t_k) \,|_{s=0} = b_0 s^5 + b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 \,|_{s=0} = b_5 = 0$, where $s = U / V$,
$5_\perp \sim \{ t_1, t_2, t_3, t_4, t_5 \}$: weight, position of the flavor branes

$$\therefore\ \Sigma_{10} = \{ s = 0 \} \cap \{ b_5 = 0 \}$$

Elliptic equation for Flipped SU(5)

$$y^2 + A_1 xy + A_3 y = x^3 + A_2 x^2 + A_4 x + A_6$$

$$A_1 = -a_3 b_1 d_1 + O(z)$$
$$A_2 = (a_2 b_1 d_1 + a_3 b_1 + a_3 d_1) z + O(z^2)$$
$$A_3 = -(a_1 b_1 d_1 + a_2 b_1 + a_2 d_1 + a_3) z^2 + O(z^3)$$
$$A_4 = (a_0 b_1 d_1 + a_1 b_1 + a_1 d_1 + a_2) z^3 + O(z^4)$$
$$A_6 = a_0 z^5 + O(z^6)$$

In the limit of $d_1 \to 0$,
the gauge sym. is enhanced to SO(10).

[$\{10_{-4}, 10^*_4\}$ become massless.]

Field Spectrum (homology class)

With
$C^{(a)}C^{(b)}C^{(d)} = (a_0U^3+a_1U^2V+a_2UV^2+a_3V^2)(U+b_1V)(U+d_1V) = 0$,

- Solve the equations for

$C^{(a)} \cap \tau C^{(a)}$; $C^{(a)} \cap \tau C^{(b)}$; $C^{(a)} \cap \tau C^{(d)}$; $C^{(b)} \cap \tau C^{(b)}$; $C^{(b)} \cap \tau C^{(d)}$;
$C^{(d)} \cap \tau C^{(d)}$; etc., and
$C^{(a)} \cap C^{(b)}$; $C^{(a)} \cap C^{(d)}$; $C^{(b)} \cap C^{(d)}$, dropping $V = V^2 = 0$.

- Read off the homology class of the solutions.

Field Spectrum (homology class)

$C^{(a)} \cap \tau C^{(a)}$: $\underline{U(a_0U^2+a_3V^2) = V(a_1U^2+a_3V^2) = 0}$,

$U=a_3=0$; $\sigma \cap \pi^*(\eta-3c_1)$; $(10,3)_1$

$\underline{a_0U^2+a_2V^2 = -a_0(b_1+d_1)U^2+a_3V^2 = 0}$;

$(\pi^*\eta+2\sigma)\cap(\pi^*(\eta-c_1)+2\sigma) - 2\sigma_\infty\cap\pi^*\eta$

$=(2\sigma+\pi^*\eta) \cap \pi^*(\eta-3c_1)$; $(5^*,3^*)_2$

$C^{(a)} \cap \tau C^{(b)}$: $\underline{U-b_1V=a_0U^3+a_1U^2V+a_2UV^2+a_3V^3=0}$;

$\sigma \cap \pi^*(\eta+3\sigma)$; $(5^{*\prime},3)_2$

Similarly, from $C^{(a)}\cap\tau C^{(d)}$: $\sigma \cap \pi^*(\eta+3\sigma)$; $(5^*,3)_{-3}$

Consider also $C^{(b)}\cap\tau C^{(b)}$, $C^{(b)}\cap\tau C(d)$, $C^{(d)}\cap\tau C^{(d)}$,

$C^{(a)}\cap C^{(b)}$, $C^{(a)}\cap C^{(d)}$, and $C^{(b)}\cap C^{(d)}$.

Field Spectrum (homology class)

10_1 : $\prod_i t_i \to 0$; $\sigma \cap \pi^*(\eta - 3c_1)$

$10_1'$: $t_4 \to 0$; $\sigma \cap \pi^*(-c_1)$

5^*_{-3} : $\prod_i (t_i + t_5) \to 0$; $\sigma \cap \pi^*(\eta - 3c_1)$

$5^*_{-3}{}'$: $t_4 + t_5 \to 0$; $\sigma \cap \pi^*(-c_1)$

1_5 : $\prod_i (t_i - t_5) \to 0$; $\sigma \cap \pi^*(\eta - 3c_1)$

$1_5'$: $t_4 - t_5 \to 0$; $\sigma \cap \pi^*(-c_1)$

5_{-2} : $\prod_{i,j} (-t_i - t_j) \to 0$; $(2\sigma + \pi^*\eta) \cap \pi^*(\eta - 3c_1)$

$5^*_2{}'$: $\prod_i (t_i + t_4) \to 0$; $\sigma \cap \pi^*(\eta - 3c_1)$

1_0 : $\prod_i (t_i - t_4) \to 0$; $\sigma \cap \pi^*(\eta - 3c_1)$

$10_{-4}{}'$: $t_5 \to 0$; $\sigma \cap \pi^*(-c_1)$

Effective superpotential

$$W_{eff} = (10_H 10_H 5_h + 10_H 10_i 5_h) + 10_i 10_j 5_h$$

$$+ 10^*_H 10^*_H 5^*_h S' / M_G$$

$$+(10_H 5^*_i 5^*_h + S 5_h 5^*_h) + 10_i 5^*_j 5^*_h + 1_i 5^*_j 5_h$$

$$+ 10^*_H 10^*_H 10_i 10_j / M_G$$

- S and S' are diff. linear combi. of the five 1_0s.
- Lepton flavor violation (technical prob.)