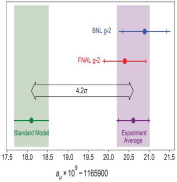
Muon $g-2$ — physics beyond the SM at an e^+e^- collider

Dominik Stöckinger, TU Dresden

Seminar series on physics potential of e+e- Higgs/Top/EW factories, 4th March 2022

Collaborators: Peter Athron, Csaba Balasz, Douglas Jacob, Wojciech Kotlarski, Hyejung Stöckinger-Kim

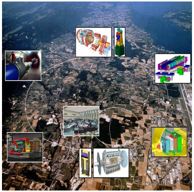
$g-2$



. . .

e^+e^--collider

LHC



1 Overview and SM theory

2 $g-2$ and BSM — important general remarks

3 Examples part 1

4 Examples of chirality-flip enhanced models

5 General lessons and conclusions

Outline

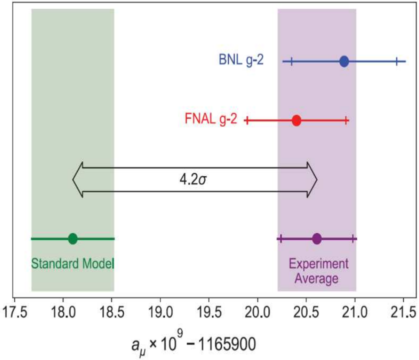
1. Overview and SM theory
2. $g-2$ and BSM — important general remarks
3. Examples part 1
4. Examples of chirality-flip enhanced models
5. General lessons and conclusions

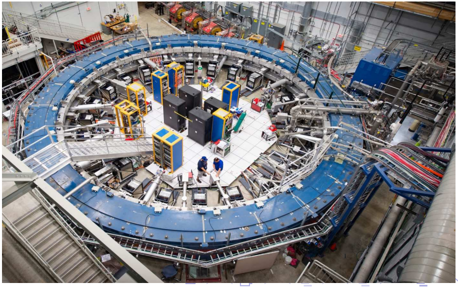
Finally: Fermilab Run 1 versus Theory Initiative SM value

2006: BNL experiment

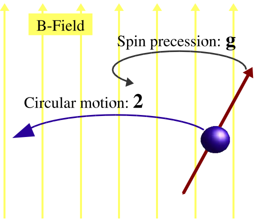
2013: moved ring to Fermilab

2021: first results

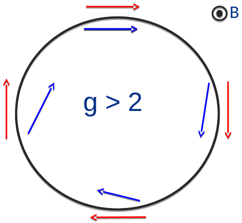




Muon magnetic moment: definition of $g = 2(1 + a_\mu)$



$$H_{\mathrm{magnetic}} = -g \frac{e}{2m_\mu} \vec{B} \cdot \vec{S}$$



circular motion: $\omega_c = -\frac{e}{m_\mu} B$

spin precession: $\omega_s = -\frac{g}{2} \frac{e}{m_\mu} B$

$\rightarrow$ measure $\omega_a = \omega_s - \omega_c = -a_\mu \frac{e}{m_\mu} B$

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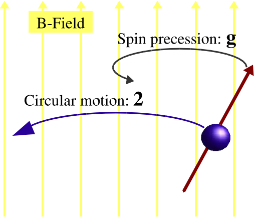

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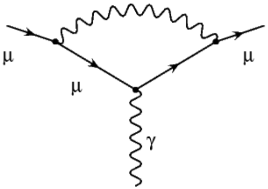
$g = 2$ is special for measurement.
$g = 1$ is the result for classical charge/mass distributions.
What is g for a relativistic quantum particle???

Computation in QFT, Chirality flips and muon mass



$$\mathcal{L}_m = -m\, \overline{\psi_L}\psi_R + h.c.$$

$$\mathcal{L}_{\text{eff}} = \frac{Qe}{2} c^* \bar{\psi}_L \sigma_{\mu\nu} \psi_R F^{\mu\nu} + h.c.$$

$$a_\mu = -2m_\mu \text{Re}(c) \quad d_\mu = Qe\, \text{Im}(c)$$

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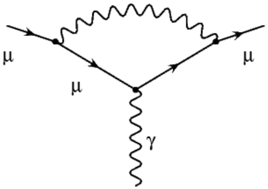

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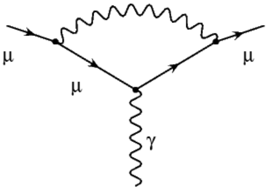

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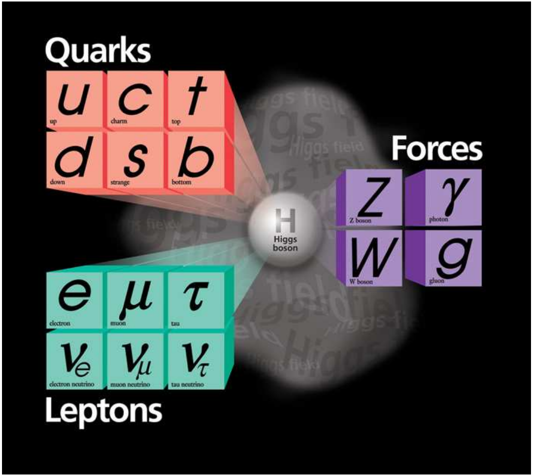
$$a_\mu \sim m_\mu \times (\text{some VEV}) \times (\psi_{L\leftrightarrow R}\text{-flipping param.}) \times \frac{(\text{other couplings})}{M^2_{\text{typical}}}$$

$$m_\mu(\text{SM}) \sim (\text{SM Higgs-VEV}) \times (\text{muon Yukawa coupling})$$

Standard Model of particle physics (est. 1967...1973))

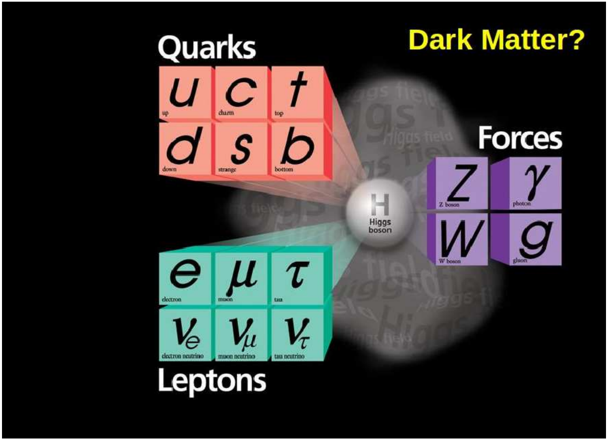


SM very well confirmed!

- relativistic QFT
- gauge invariance
- spontaneous EWSB

a_μ sensitive to all particles and forces via quantum fluctuations!

Open questions require Beyond the Standard Model (BSM) physics



Open questions!

- need experiments!
- $g-2$ . . . LHC
- e^+e^- collider

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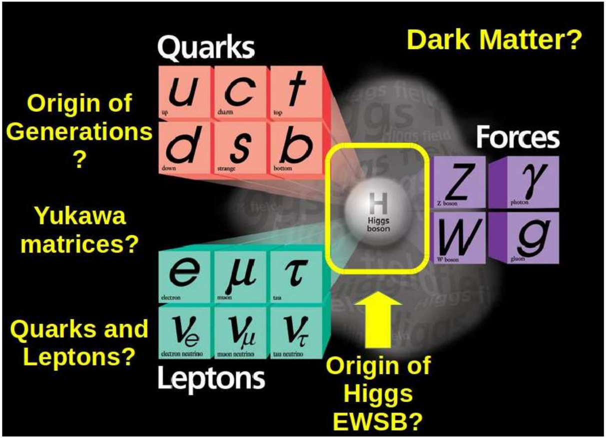

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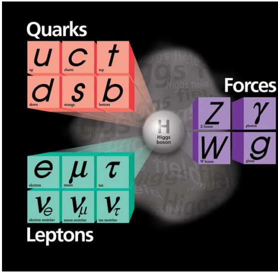
Feynman diagrams and quantum fluctuations

μ couples to B-field directly or via virtual particles



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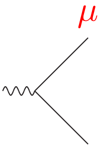
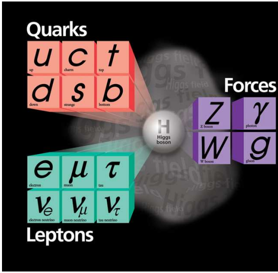


Dirac equation/direct $\rightsquigarrow$"pointlike"

$$g = 2$$



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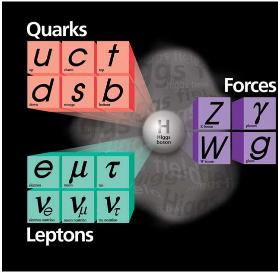

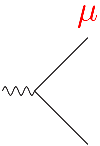

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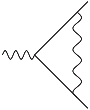
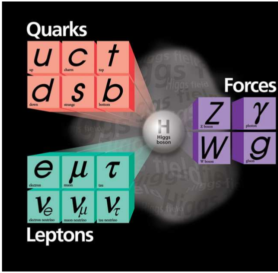


Schwinger (1948): quantum fluctuations ⇝ "non-pointlike"

$$g = 2\left(1 + \frac{\alpha}{2\pi} \qquad\qquad\qquad \right)$$

Feynman diagrams and quantum fluctuations



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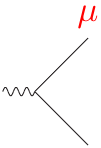

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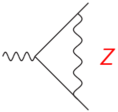
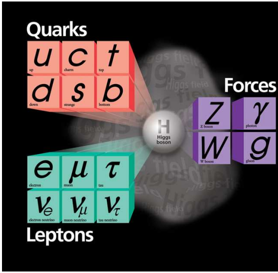


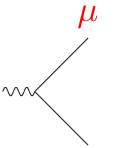
Schwinger (1948): quantum fluctuations $\rightsquigarrow$ "non-pointlike"

$$g = 2\left(1 + \frac{\alpha}{2\pi} + \sim \frac{\alpha}{2\pi}\frac{m_\mu^2}{M_Z^2}\right)$$

Feynman diagrams and quantum fluctuations

μ couples to B-field directly or via virtual particles





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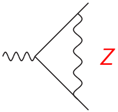

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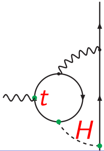
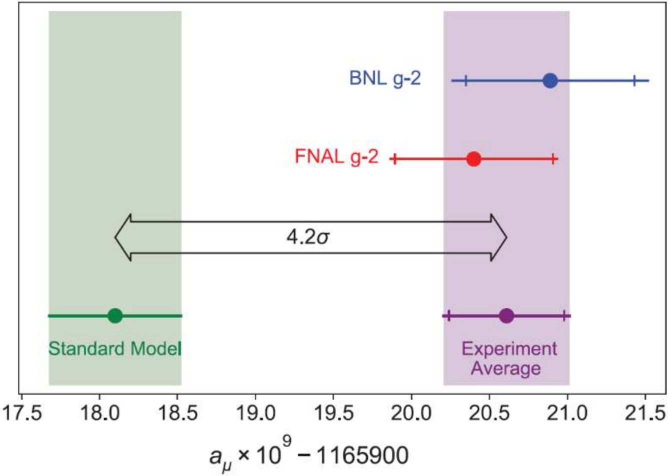


All SM particles contribute, even Higgs and top!

$$g = 2\left(1 + \ldots - 1.5 \times 10^{-11}\right)$$

Finally: Fermilab Run 1 versus Theory Initiative SM value



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Discrepancy

SM prediction too low by $\approx (25 \pm 6) \times 10^{-10}$

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Question: Which models can(not) explain it?

- Can such models be investigated at an e^+e^- collider?

Two important general points

discrepancy $\approx 2 \times a_\mu^{\mathrm{SM,weak}}$

but: expect $a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{SM,weak}} \times \left(\frac{M_W}{M_{\mathrm{NP}}}\right)^2 \times$ couplings

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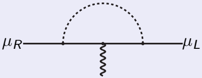
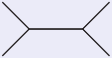
loop-induced, CP- and Flavor-conserving, chirality-flipping



compare:



EDMs, $b \to s\gamma$, $B \to \tau\nu$, $\mu \to e\gamma$

EWPO

Central formula

$$a_\mu \sim m_\mu \times \underbrace{(\text{some VEV}) \times (\mu_{L\leftrightarrow R}\text{-flipping param.})}_{\text{related to muon mass generation, potential enhancement!}} \times \frac{(\text{other couplings})}{M^2_{\text{typical}}}$$

$$m_\mu(\text{SM}) \sim (\text{SM Higgs-VEV}) \times (\text{muon Yukawa coupling})$$

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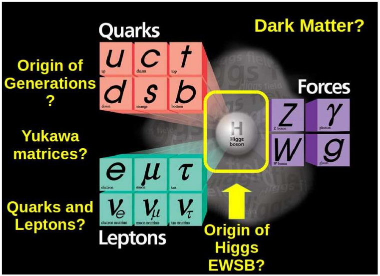
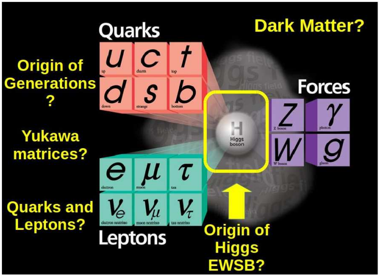
Two obvious promising directions for BSM physics

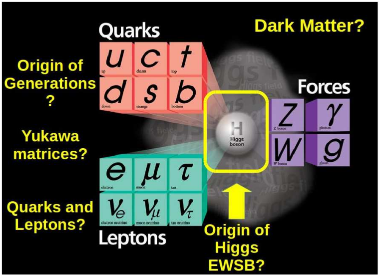


Two promising connections to big open questions



Two promising connections to big open questions

Dark Matter, dark sectors?
Hard to see in detectors but could couple to muon $\rightsquigarrow g-2$!



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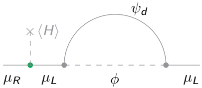
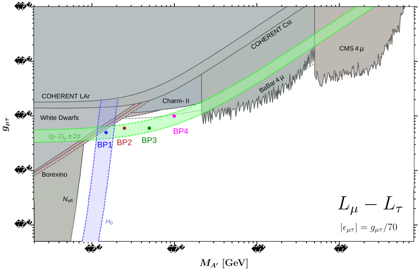


- Can do systematic model studies

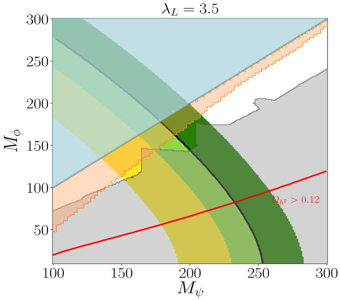
- Generic model study: certain quantum numbers viable
- But simple 2-field models cannot describe DM and a_μ simultaneously!

[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

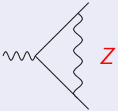
Dark Matter, dark sectors and $g-2$



Viable: 10 . . . 100 MeV [Amaral,Cerdeno,Cheek,Foldenauer'21]

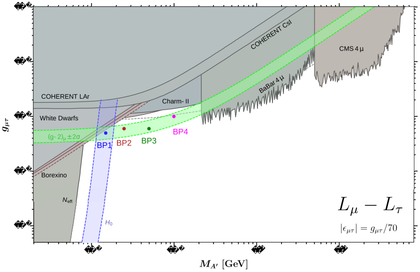


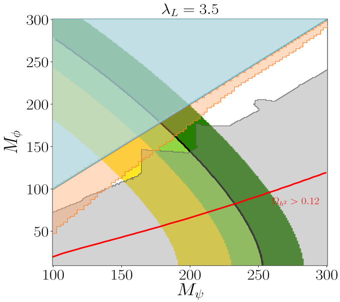
a_μ from Z' models, e.g. $L_\mu - L_\tau$ quantum number



- $Z'_{L_\mu - L_\tau}$ can explain a_μ for $M_{Z'} \sim 10$ MeV
- model with Z' plus LQ S_1, S_3 plus ν_R plus DM promising (DM$\sim$ 100 GeV, B-anomalies, ν-masses) [Heeck, Thapa '22]

Dark Matter, dark sectors and $g-2$





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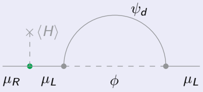
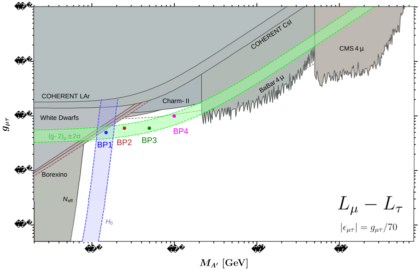
a_μ from 2-field model L

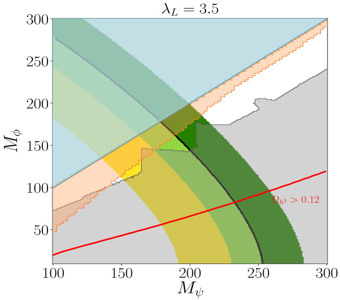


- $a_\mu \propto m_\mu{}^2 \dfrac{|\lambda_L|^2}{M^2}$
- no chiral enhancement (like SM)
- need large couplings and small masses
- Generic e^+e^- processes:
  $e^+e^- \to \mu^+\mu^- \to \mu^+\phi\psi_d, \quad e^+e^- \to \psi_d\psi_d$

  and box-contributions to $e^+e^- \to \mu^+\mu^-$ [Freitas,Lykken,Kell,Westhoff'14]

Dark Matter, dark sectors and $g-2$





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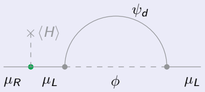
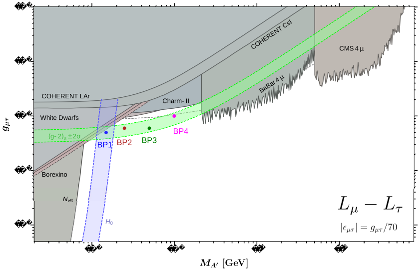
a_μ from 2-field model L



- Can do systematic analysis of models with 1, 2, . . . new fields
- General result: a_μ and DM require at least three new fields! see also: [Arcadi,Calibbi,Fedele,Mescia] on B-physics

Dark Matter, dark sectors and $g-2$



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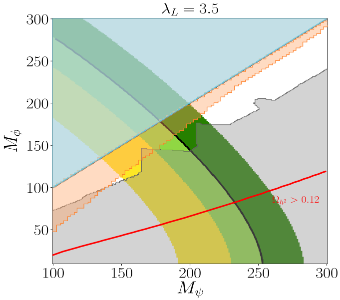
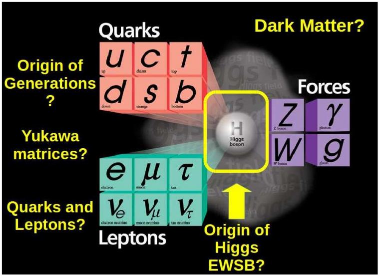


Conclusions on dark matter/dark sectors:

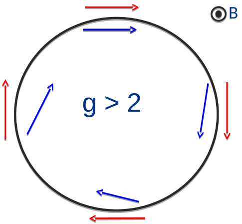
- Not trivial to accommodate a_μ and dark matter simultaneously
- But dark sector contributions generally motivated/promising
- Interesting mass range $M_{Z'} \sim 10$ MeV. . . $M_\phi \sim 200$ GeV

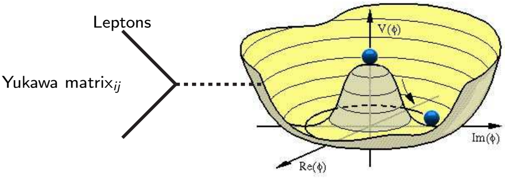


Two promising connections to big open questions

Window to the muon mass generation mechanism (Higgs/Yukawa sectors)

(continuous spin rotation requires rest mass!)





(changed by new physics?)

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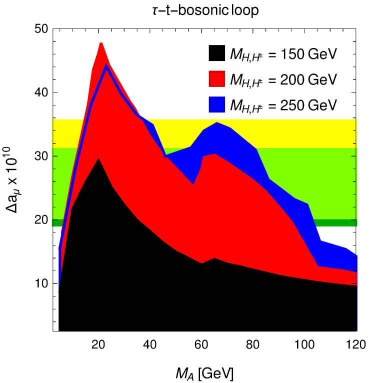
Two-Higgs doublet model: promising in specific versions

- Aligned 2-Higgs doublet model, rich new Higgs/Yukawa sectors



[2104.03691]

Details on Yukawa couplings:

Type X/lepton-specific: $Y_\ell \propto \tan\beta$ Type II: $Y_{\ell,d} \propto \tan\beta$ Aligned: $Y_\ell \propto \zeta_\ell$

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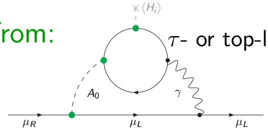
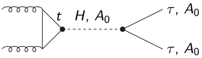
- Aligned 2-Higgs doublet model, rich new Higgs/Yukawa sectors

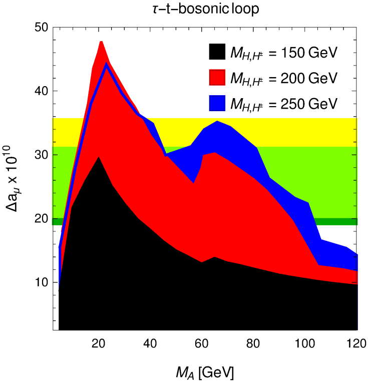
a_μ from:



τ- or top-loop

constraints (LHC, B-, τ-physics):





[2104.03691]

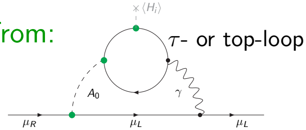
Details on Yukawa couplings:

Type X/lepton-specific: $Y_\ell \propto \tan\beta$ Type II: $Y_{\ell,d} \propto \tan\beta$ Aligned: $Y_\ell \propto \zeta_\ell$

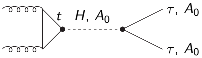
Two-Higgs doublet model: promising in specific versions

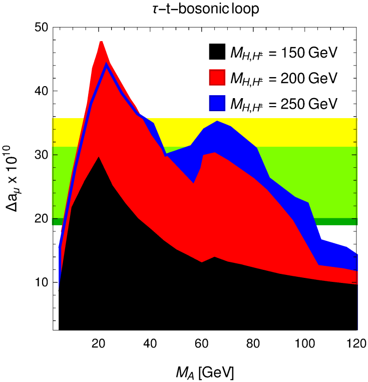
- Aligned 2-Higgs doublet model, rich new Higgs/Yukawa sectors

a_μ from:



constraints (LHC, B-, τ-physics):





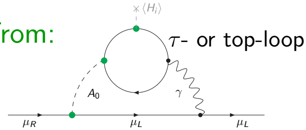
[2104.03691]

- can explain $g-2$ (but not in type I, type II)
- need large new Yukawa couplings, light pseudoscalar $M_A \sim 20 \ldots 100$ GeV
- under pressure, testable at LHC, lepton colliders, B-physics, e^+e^- collider

Two-Higgs doublet model: promising in specific versions

- Aligned 2-Higgs doublet model, rich new Higgs/Yukawa sectors

a_μ from:



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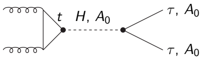
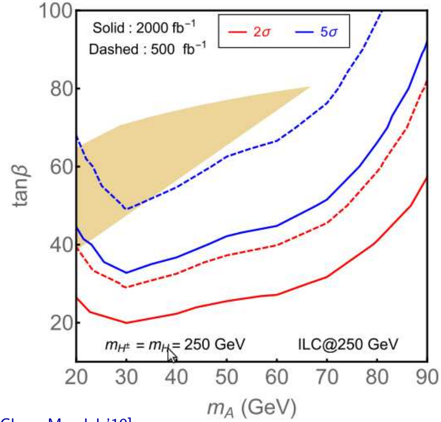
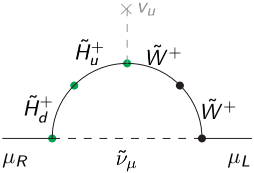




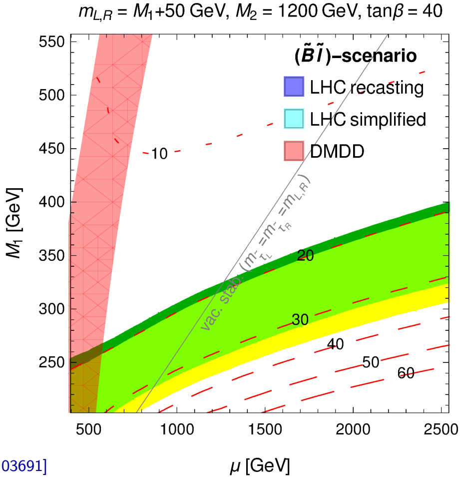
[Chun, Mondal '19]

- e^+e^- collider tests: possible at 250 GeV collider Chun, Mondal '19
- 2000 fb^{-1} can explore entire $g-2$ parameter space of type X
- "Yukawa process" $e^+e^- \to \tau\tau A \to 4\tau$

SUSY (MSSM): can explain $g-2$ and dark matter



$$a_\mu^{\rm SUSY} \approx 25 \times 10^{-10} \, \frac{\tan\beta}{50} \, \frac{\mu}{M_{\rm SUSY}} \left(\frac{500{\rm GeV}}{M_{\rm SUSY}}\right)^2$$



[2104.03691]

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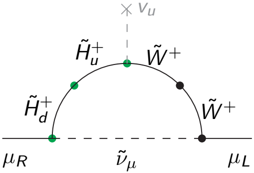

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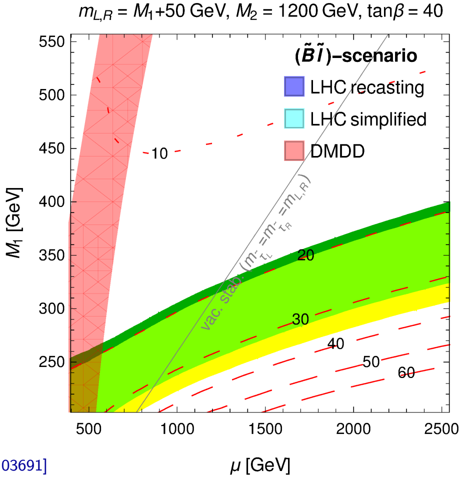
- "Dark matter mass" versus μ
- explains $g-2$ in large region (expands for $\tan\beta \neq 40$)
- DM explained by stau/slepton-coannihilation



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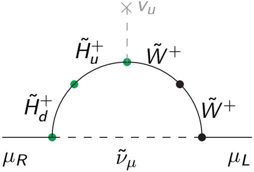

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- "Dark matter mass" versus μ
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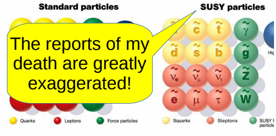
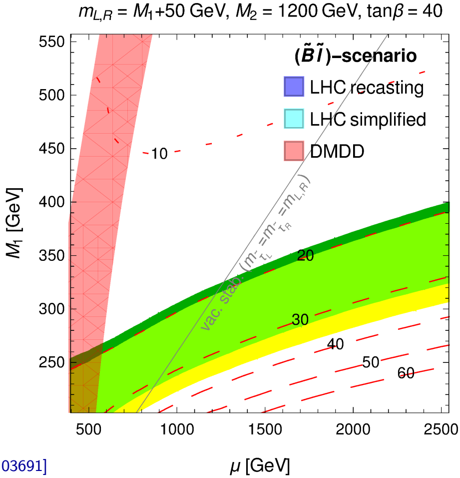
- this automatically evades (current) LHC limits





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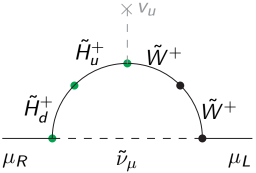

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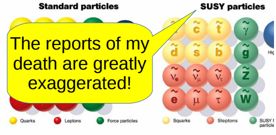
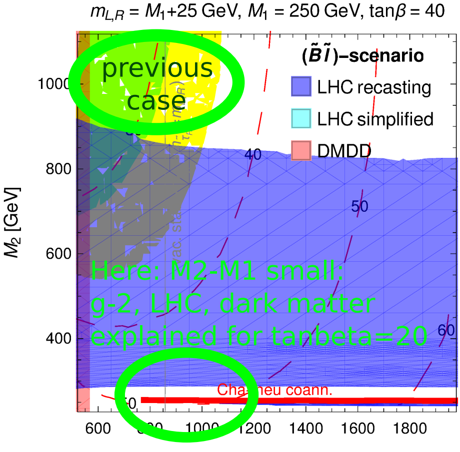
- Strong LHC limits on M_2
- DM also explained by Wino-coannihilation
- again evades (current) LHC limits





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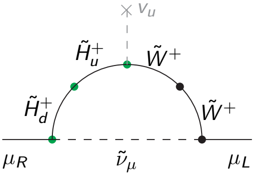
SUSY (MSSM): can explain $g-2$ and dark matter



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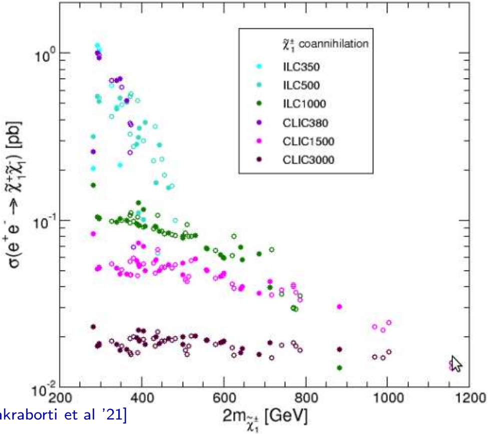
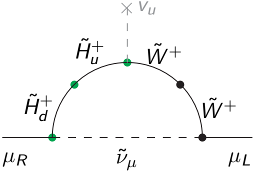
- Also Higgsino/Wino-LSP very promising
- Similar analyses:
  [Chakraborti,Heinemeyer,Saha'20/21],
  [Endo,Hamaguchi,Iwamoto,Kitahara'20/21]
- e^+e^- collider tests:
  [Chakraborti,Heinemeyer,Saha'21]
- partly accessible at 350 GeV
- $\approx$ full coverage at 1000 GeV



[Chakraborti et al '21]

SUSY (MSSM): can explain $g-2$ and dark matter



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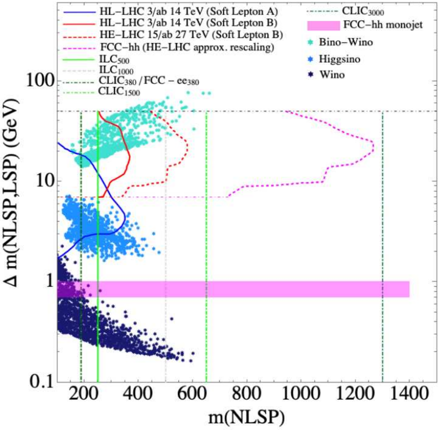
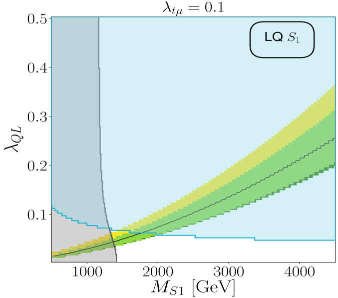
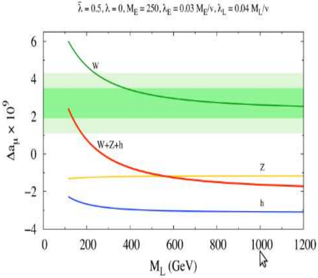


Leptoquarks: promising; vector-like leptons: similar



[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]



[Dermisek,Raval 2013, no LHC constraints here!]

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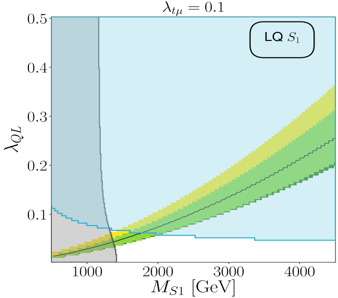

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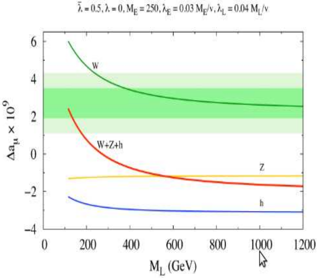
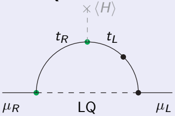


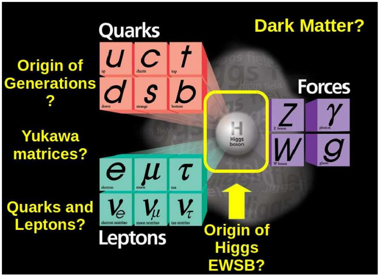
[Dermisek,Raval 2013, no LHC constraints here!]

a_μ from LQ (or VLL) $\mathcal{L}_{S_1} = -\left(\lambda_{QL} Q_3 \cdot L_2 S_1 + \lambda_{t\mu} t \mu S_1^*\right)$

Specific LQ that works:



- Chiral enhancement $\sim y_{\text{top}}, y_{\text{VLL}}$ versus y_μ
- LHC: lower mass limits
- Flavour constraints $\rightsquigarrow$ assume only couplings to muons
- Viable window above LHC (without m_μ-finetuning)



Two promising connections to big open questions

Window to the muon mass generation mechanism (Higgs/Yukawa sectors)

(continuous spin rotation requires rest mass!)

Conclusions for chirality-flip enhanced models

- Origin of EWSB/Higgs? of fermion masses? generations?
- Important for e^+e^- colliders!
- $g-2$ and e^+e^- colliders can constrain scenarios
- of interest: SUSY, 2HDM, vectorlike leptons, leptoquarks, ...
- mass range $M_A \sim 20$ GeV ... $M_{LQ} \sim 2$ TeV

Upper mass limit around 2 TeV

$$a_\mu \sim m_\mu \times \underbrace{(\text{some VEV}) \times (\mu_{L\leftrightarrow R}\text{-flipping param.})}_{\text{related to muon mass generation, potential enhancement!}} \times \frac{(\text{other couplings})}{M^2_{\text{typical}}}$$

$$m_\mu(\text{SM}) \sim (\text{SM Higgs-VEV}) \times (\text{muon Yukawa coupling})$$

Without chirality-flip enhancement, $\{\ldots\} = m_\mu(\text{SM})$ and a_μ can only be explained for $M_{\text{typical}} \lesssim 200$ GeV. $\rightsquigarrow e^+e^- \to \mu^+\mu^- \to \mu^+\phi\psi$

Upper mass limit around 2 TeV

$$a_\mu \sim m_\mu \times \underbrace{(\text{some VEV}) \times (\mu_{L\leftrightarrow R}\text{-flipping param.})}_{\text{related to muon mass generation, potential enhancement!}} \times \frac{(\text{other couplings})}{M^2_{\text{typical}}}$$

$$\Delta m_\mu^{\text{BSM}} \sim (\text{some VEV}) \times (\mu_{L\leftrightarrow R}\text{-flipping param.}) \times (\text{other couplings})$$

In models where

$$\Delta m_\mu^{\text{BSM}} \lesssim m_\mu$$

we have an upper limit on possible a_μ contributions and therefore, the current a_μ result can only be explained for

$$M_{\text{typical}} \lesssim 2.1\,\text{TeV}$$

This is true in all considered examples.

Outline

1. Overview and SM theory
2. $g-2$ and BSM — important general remarks
3. Examples part 1
4. Examples of chirality-flip enhanced models
5. General lessons and conclusions

e^+e^- Processes

Light, neutral: Z', A^0 with $M \sim 0.01 \ldots 100$ GeV

$$e^+e^- \to \tau\tau A \to 4\tau$$

Two new particles ϕ, ψ (at least one charged, say ψ)

$$e^+e^- \to \mu\mu \to \mu\phi\psi$$

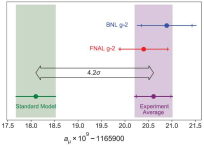
$$e^+e^- \to \psi\psi$$

If ϕ, ψ also couple to electrons: box-contributions to

$$e^+e^- \to \mu\mu$$

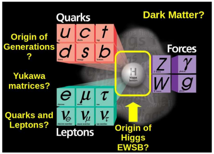
Chirality-flip enhanced models (SUSY, LQ, VLL) contain such ϕ, ψ with $M \sim 200 \ldots 2000$ GeV

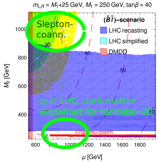
Conclusions



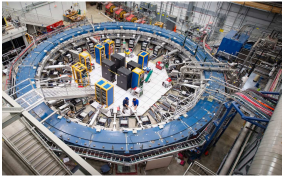
- SM prediction for $g-2$:
  - All known particles relevant (and all QFT tricks)
  - Theory Initiative: worldwide (ongoing!) effort

- BSM contributions to $g-2$:
  - large effect needed
  - Connections to deep questions
    ⇝ Connection to dark matter/dark sector?
    ⇝ Chirality flip enhancement, muon mass?
  - many models ... and constraints
  - Exp. tests:
    Higgs couplings, B-physics, CLFV, EDM, light-particle searches, e^+e^-/muon collider



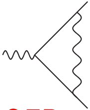


- Fermilab $g-2$ experiment
  - stat. dominated! Only 6% data used!
  - ... promising future

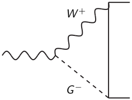


Theory Initiative prediction $a_\mu^{\rm SM} = (11\,659\,181.0 \;\; (4.3)\;) \; [10^{-10}]$

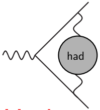
since 2017, 6 workshops, White Paper (2020), 132 authors, ongoing effort



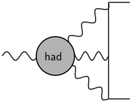
QED: 11 658 471.9 (0.0)



Weak: 15.36 (0.1)



Had vp: 684.5 (4.0)



Had lbl: 9.0 (1.7)

number = conservative combination of approaches

most precise particle theory prediction (?)

evaluation uses all conceivable

QFT methods & tricks

Theory Initiative prediction $a_\mu^{\mathrm{SM}} = (11\,659\,181.0 \;\; (4.3) \;\;)\; [10^{-10}]$

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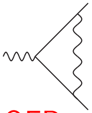

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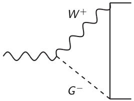

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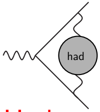

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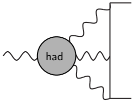

Had lbl: 9.0 (1.7)

QED:

- 4-/5-loop Feynman integrals analytical & numerical

Weak:

- first EW 2-loop calculation, also use renormalization group methods

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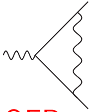

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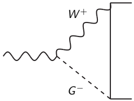

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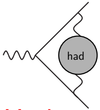
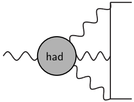


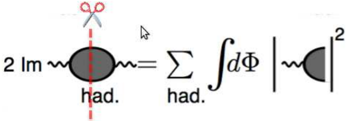
Had vp: $684.5 \;\; (4.0)$

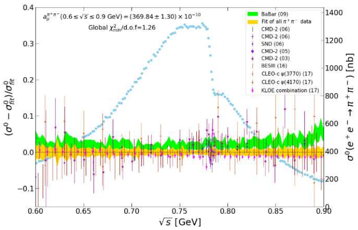


Had lbl: $9.0 \;\; (1.7)$

Hadronic vacuum polarization:

- unitarity+causality $\rightsquigarrow$ exact dispersion relation





- lattice QCD impressive progress (not yet used in TI value)

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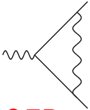

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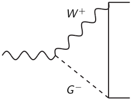
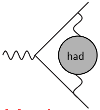
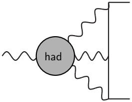


Weak: $15.36 \quad (0.1)$



Had vp: $684.5 \quad (4.0)$



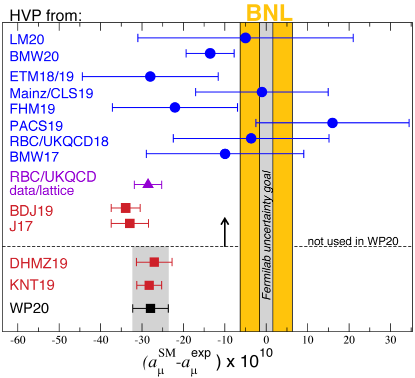
Had lbl: $9.0 \quad (1.7)$

Hadronic light-by-light:

- difficult QFT problem
- Traditionally: low-energy models
- Recently: data-driven (dispersion relations) & lattice QCD results
- consistent results
- uncertainty better under control

Details on hadronic vacuum polarization

a_μ^{HVP} : Status of Hadronic Vacuum Polarisation contributions



Lattice QCD + QED

- impressive progress, but…
- large spread between results
- tensions when looking at `Euclidean time window' comparisons
- large systematic uncertainties (e.g. from non-trivial extrapolation to continuum limit, finite size)

Dispersive/lattice hybrid (`window' method)

For WP20: Dispersive data-driven from DHMZ and KNT

TI White Paper 2020 value:

$\mathbf{a_\mu^{HVP} = 6845\ (40) \times 10^{-11}}$

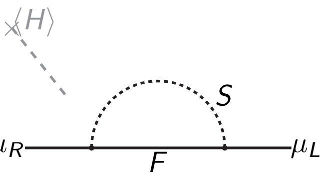
- TI WP2020 prediction uses dispersive data-driven evaluations with **minimal model dependence**
- a_μ^{HVP} **value and error** obtained **by merging** procedure ➡ accounts for tensions in input data and differences in data treatment & combination (going beyond usual χ^2_{min} inflation)

Thomas Teubner 10

BSM 1-Loop Illustration

$$\delta m_\mu = \frac{1}{16\pi^2}\left\{ \quad m_\mu \;\left[|c_L|^2 + |c_R|^2\right] B_1 + \; m_F \;\mathrm{Re}\left[c_L c_R{}^*\right] B_0 \right\}$$

$$a_\mu = \frac{m_\mu}{16\pi^2}\left\{ \frac{m_\mu}{12 m_S^2} \left[|c_L|^2 + |c_R|^2\right] F_1^C + \frac{2 m_F}{3 m_S^2}\mathrm{Re}\left[c_L c_R{}^*\right] F_2^C \right\}$$



- $|c_{L,R}|^2$-terms: $\rightsquigarrow$ simple behaviour
  chir. flip $\sim$ SM $\sim m_\mu$
- $\mathrm{Re}\left[c_L c_R{}^*\right]$-terms: $\rightsquigarrow$ tricky/deceiving:
  - $c_L c_R \neq 0$ breaks chiral sym.
  - F, S cannot be gauge eigenstates
  - often $m_F c_L c_R \propto y_\mu \langle \Phi \rangle \propto m_\mu$

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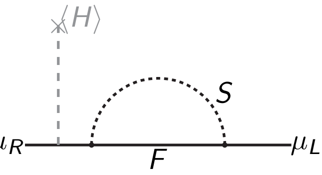

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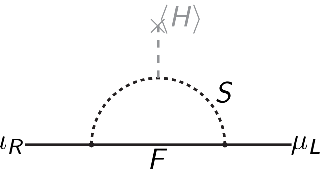

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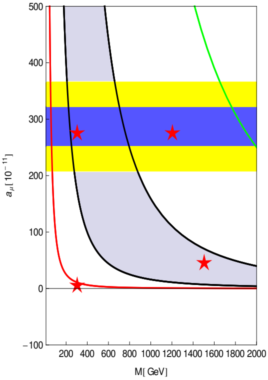
Typical behaviour: $\sim$ chirality flip ($\rightsquigarrow$ Higgs!) and masses

$$a_\mu \sim \frac{m_\mu \times (\text{some VEV}) \times (\mu_{L\leftrightarrow R}\text{-flipping parameter})}{M^2_{\text{typical}}} \left[\lesssim \frac{m_\mu^2}{M^2_{\text{typical}}} \text{ (no finetuning)} \right]$$

$$\Delta m_\mu \sim (\text{some VEV}) \times (\mu_{L\leftrightarrow R}\text{-flipping parameter})$$



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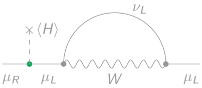
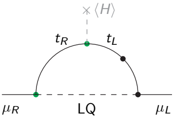
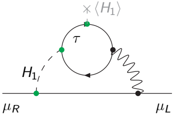
- EWSM: $\alpha \frac{m_\mu^2}{M_W^2}$

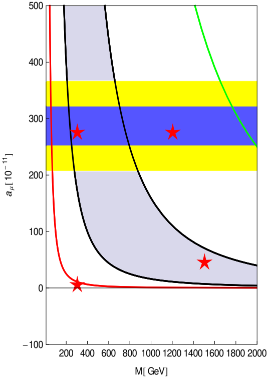


- LQ: $g_L g_R \frac{m_\mu m_t}{M^2_{\text{LQ}}}$



- 2HDM: $\alpha^2 \tan^2\beta \frac{m_\mu^2}{M_H^2}$





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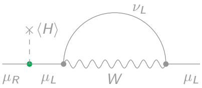

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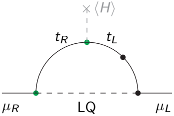
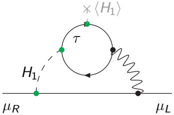


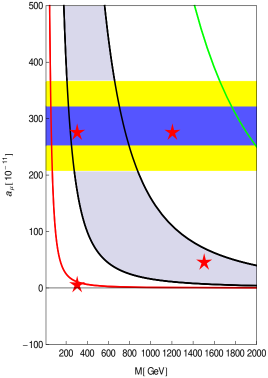
Can also involve Higgs couplings to b, c or new particles.

Beware: $\Delta m_\mu / m_\mu \sim g_L g_R m_t / m_\mu$ restricts couplings

- 2HDM: $\alpha^2 \tan^2 \beta \frac{m_\mu^2}{M_H^2}$



Well motivated; many variants; many constraints



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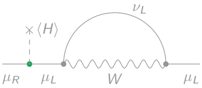
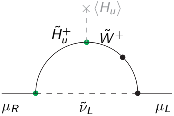
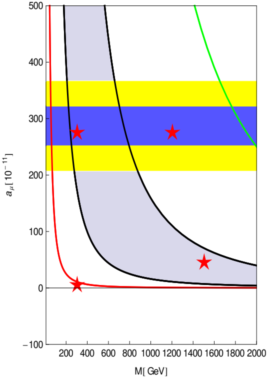


- SUSY: $\propto \frac{m_\mu^2 \tan\beta}{M^2_{\text{SUSY}}} \frac{\mu}{M_{\text{SUSY}}}$



- rad. m_μ $\sim \frac{m_\mu^2}{M^2_{\text{NP}}}$



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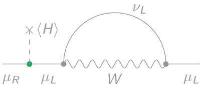
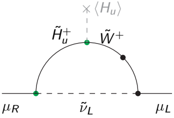


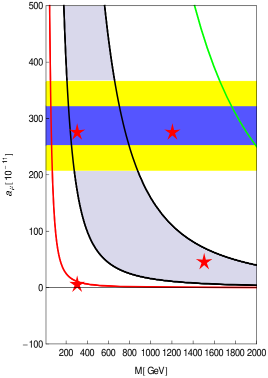
- SUSY: $\alpha \frac{m_\mu^2 \tan\beta}{M^2_{\text{SUSY}}} \frac{\mu}{M_{\text{SUSY}}}$

  Well-motivated theory. Many other advantages



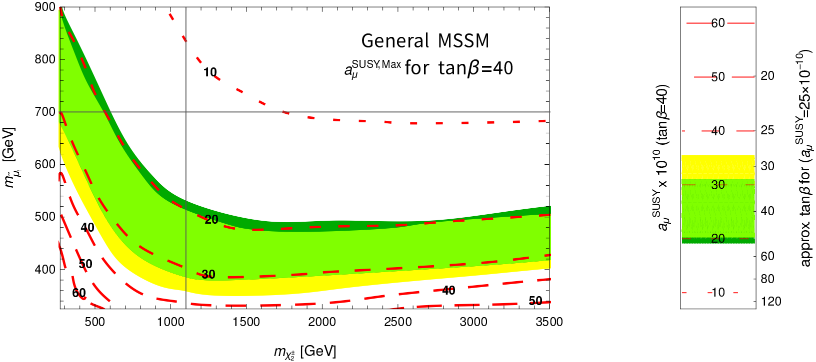
- rad. m_μ $\sim \frac{m_\mu^2}{M^2_{\text{NP}}}$

  E.g. MSSM for $\tan\beta \to \infty$ [Bach,Park,DS,Stöckinger-Kim'15]



Full MSSM overview in 7 plots

[Peter Athron, Csaba Balasz, Douglas Jacob, Wojciech Kotlarski, DS, Hyejung Stöckinger-Kim, 2104.03691]



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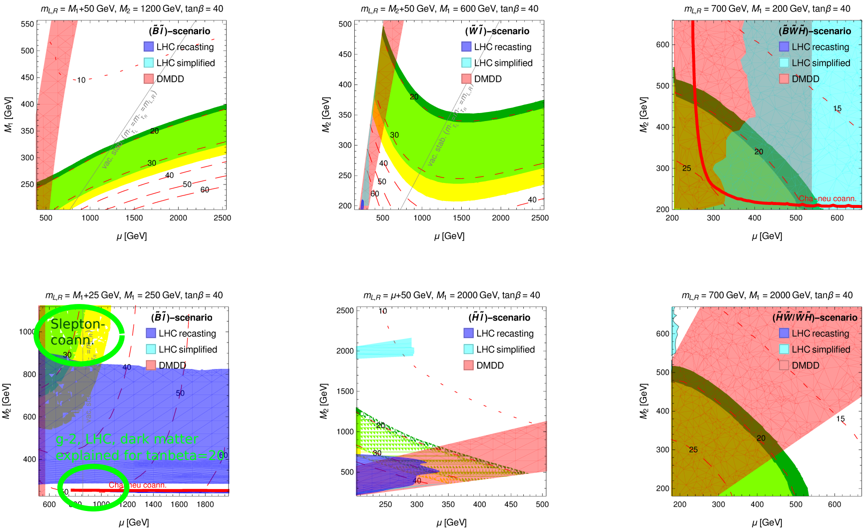
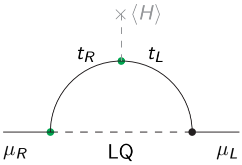
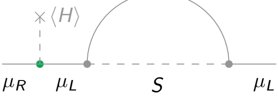


Summary: Bino-LSP: a_μ and DM. Wino-/Higgsino-LSP: a_μ. Both cha<slepton: ≈disfavoured.

DM+LHC⇒mass patterns! Coannihilation regions help! Specific cases excluded, e.g. Constrained MSSM

One-field, two-field models (renormalizable, spin $0, 1/2$)





- many models: excluded
- very special models: chiral enhancement
  specific leptoquarks, specific 2HDM versions
- however, no dark matter

Model	Spin	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Result
1	0	$(\mathbf{1},\mathbf{1},1)$	Ruled out: $\Delta a_\mu < 0$
2	0	$(\mathbf{1},\mathbf{1},2)$	Ruled out: $\Delta a_\mu < 0$
3	0	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	Special cases viable
4	0	$(\mathbf{1},\mathbf{3},-1)$	Ruled out: $\Delta a_\mu < 0$
5	0	$(\mathbf{3},\mathbf{1},1/3)$	Updated Sec. 3.2.
6	0	$(\bar{\mathbf{3}},\mathbf{1},4/3)$	Ruled out: $\Delta a_\mu < 0$
7	0	$(\bar{\mathbf{3}},\mathbf{3},1/3)$	Ruled out: $\Delta a_\mu < 0$
8	0	$(\mathbf{3},\mathbf{2},7/6)$	Updated Sec. 3.2.
9	0	$(\mathbf{3},\mathbf{2},2/3)$	Ruled out: $\Delta a_\mu < 0$
10	1/2	$(\mathbf{1},\mathbf{1},0)$	Ruled out: $\Delta a_\mu < 0$
11	1/2	$(\mathbf{1},\mathbf{1},-1)$	Ruled out: $\Delta a_\mu < 0$ or too small (disputed)
12	1/2	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	Ruled out: $\Delta a_\mu < 0$ or too small (disputed)
13	1/2	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	Ruled out: $\Delta a_\mu < 0$
14	1/2	$(\mathbf{1},\mathbf{3},0)$	Ruled out: $\Delta a_\mu < 0$
15	1/2	$(\mathbf{1},\mathbf{3},-1)$	Ruled out: $\Delta a_\mu < 0$
16	1	$(\mathbf{1},\mathbf{1},0)$	Special cases viable
17	1	$(\mathbf{1},\mathbf{2},-3/2)$	Ruled out: UV compl. $M_{Z'}$ limit

- even more models: excluded
- no chirality flip
- few models: either a_μ^{BNL} or dark matter

$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Projected LHC 14 TeV exclusion, not confirmed
	Yes	Updated Sec. 4.2
$(\mathbf{1},\mathbf{1},-1)_0 - (\mathbf{1},\mathbf{1},0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{1},0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	No	Excluded: LHC searches
	Yes	Updated Sec. 4.2
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LEP contact interactions
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{1},-1)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: LEP search
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LHC searches
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LHC searches + LEP contact interactions
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{3},0)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LHC searches
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{3},-1)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{3},-1)_0 - (\mathbf{1},\mathbf{3},0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{1},-1)_{1/2} - (\mathbf{1},\mathbf{1},0)_1$	No	Excluded: $\Delta a_\mu < 0$
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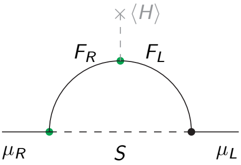
$(\mathbf{1},\mathbf{2},-1/2)_{1/2} - (\mathbf{1},\mathbf{1},-1)_1$	No	Excluded: LHC searches + LEP contact interactions

Three-field models



- many models: viable, large chirality enhancements
- can explain a_μ^{BNL} and LHC and dark matter