Scattering Amplitudes and BCFW in $\mathcal{N} = 2^*$ Theory

Md. Abhishek[1], Subramanya Hegde, Dileep P. Jatkar, and Arnab Priya Saha

[1]Harish-Chandra Research Institute

August 3, 2022

Introduction

- The $\mathcal{N} = 2^*$ theory is a massless $\mathcal{N} = 2$ vector multiplet coupled to massive adjoint hypermultiplet. In the massless limit of this theory, we recover the $\mathcal{N} = 4$ SYM theory.
- So the $\mathcal{N} = 2^*$ theory is beautifully perched between the reasonably well-understood $\mathcal{N} = 4$ theory and the wild variety of $\mathcal{N} = 2$ theories.
- With this motivation we have studied the massive amplitudes in the Coulomb branch of $\mathcal{N} = 2^*$ theory using the massive spinor-helicity variables and massive supersymmetric BCFW formalism.

On-shell multiplets

- The $\frac{1}{2}$ BPS $\mathcal{N}=4$ SYM multiplet in terms of long $\mathcal{N}=2$ SUSY multiplet is,

$$\begin{aligned}\mathcal{W} &= \phi + \eta_I^a \psi_a^I - \frac{1}{2}\eta_I^a \eta_J^b (\epsilon^{IJ}\phi_{(ab)} + \epsilon_{ab} W^{(IJ)}) \\ &+ \frac{1}{3}\eta_I^b \eta_{Jb} \eta^{Ja} \tilde{\psi}_a^I + \eta_1^1 \eta_1^2 \eta_2^1 \eta_2^2 \tilde{\phi}. \end{aligned} \tag{1}$$

  where, $a = \{1,2\}$ is the $\mathcal{N}=2$ SUSY index and $I = \{1,2\}$ is the $SU(2)$ little group index.

- We can expand the above supermultiplet in terms of the Grassmann variables η_I^2 to obtain,

$$\mathcal{W} = \Phi + \eta_I^2 \mathcal{W}^I - \frac{1}{2}\eta_I^2 \eta_J^2 \epsilon^{IJ} \bar{\Phi}, \tag{2}$$

- The above decomposition yielded $\mathcal{N}=2$ massive hypermultiplet $(\Phi, \bar{\Phi})$ and $\mathcal{N}=2$ half-BPS SYM multiplet $(\mathcal{W}^I)$ respectively.

Amplitudes using BCFW formulation

- By taking proper projection of the amplitude in $\mathcal{N}=4$ SYM theory we can get various massive three point amplitudes, $A_3\left[\mathcal{W}_1^I, \bar{\Phi}_2, \Phi_3\right], A_3\left[\mathcal{W}_1^I, \bar{\mathcal{W}}_2^J, \mathcal{W}_3^K\right]$, in $\mathcal{N}=2^*$ theory.
- Using these three-point amplitudes we have calculated the four-point amplitudes, $A_4\left[\mathcal{W}_1^I, \bar{\mathcal{W}}_2^J, \Phi_3, \bar{\Phi}_4\right], A_4\left[\Phi_1, \bar{\Phi}_2, \Phi_3, \bar{\Phi}_4\right], A_4[\mathcal{W}_1^I, \bar{\mathcal{W}}_2^J, \mathcal{W}_3^K, \bar{\mathcal{W}}_4^L]$, by massive supersymmetric BCFW recursion.
- In BCFW formulation we got a contribution from the pole at infinity which was nicely canceled with little group non-covariant part of the integrand.

Conclusions and future directions

- It would be interesting to extend this analysis to loop amplitudes.
- Also we can extend this construction for the supersymmetric theories with higher spin particles.
- It would be also interesting to see if the amplitudes of $\mathcal{N} = 2^*$ theory can be put in a CHY-like formulation.

Thanks!

Evaluation of Feynman integrals via series expansions

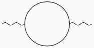
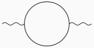
Tommaso Armadillo, Université Catholique de Louvain

3rd August 2022

Amplitudes 2022 Summer school, Prague

Feynman integrals & differential equations

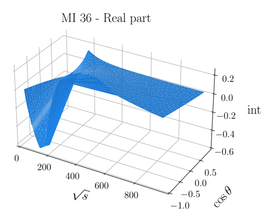
- One of the many difficulties in the evaluation of **squared amplitudes** is the computation of **Feynman integrals**
- One possible way to evaluate them is the **differential equation method**.

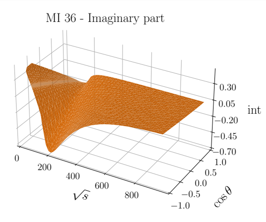
$$\frac{d}{dx}$$

$$+\frac{1}{2}\left[\frac{1}{x}-\frac{(D-3)}{(1+x)}\right]$$

$$=-\frac{(D-2)}{4m^2}\left[\frac{1}{x}-\frac{1}{(1+x)}\right]$$


- Problem: Sometimes is not possible to write the solution in terms of some known classes of functions, such as **Generalized Poli-Logarithms**.

- **Solution:** Solve the system via **series expansion techniques!**
- We implemented this technique in the publicly available Mathematica package **SeaSyde** (**S**eries **E**xpansion **A**pproach for **SY**stems of **D**ifferential **E**quations)
- For example:

$\Longrightarrow$





Thank you for the attention!

Soft limits of Cosmological EFTs

Connor Armstrong
with Humberto Gomez, Renann Lipinski Jusinskas, Arthur Lipstein, and Jiajie Mei
based on [2204.08931 and upcoming]

August 2022, Amplitudes Summer School

Durham University

de Sitter Wavefunction Coefficients

$\sim$ curved space amplitudes

- Particles propagate and interact in the bulk – we then integrate over the time coordinate so the wavefunction coefficient depends on 3d momenta with arbitrary masses
- We get a Laurent series in $E = k_1 + \cdots + k_n$. Residues in E recover flat space amplitudes

Calculating with Scalar EFTs

- Feynman diagrams generalise to Witten diagrams which have extra propagators to take states to the boundary
- Inspired by the Cosmological scattering equations, we can alternatively write as Casimir operators acting on a contact diagram

$$\mathcal{D}_a \cdot \mathcal{D}_b = \frac{1}{2}\left(P_a^i K_{bi} + K_{ai} P_b^i - M_{aij} M_b^{ij}\right) + D_a D_b$$

$$\mathcal{C}_n = \int \frac{\mathrm{d}\eta}{\eta^{d+1}} \prod_{i=1}^{n} \mathcal{K}_\nu(k_a, \eta)$$

- For example, 4pt DBI amplitude is

$$\mathcal{A}_4^{(4)} = (\mathcal{D}_1 \cdot \mathcal{D}_2 \mathcal{D}_3 \cdot \mathcal{D}_4 + \text{Cyc.}[234])\, \mathcal{C}_4$$

Soft Limits

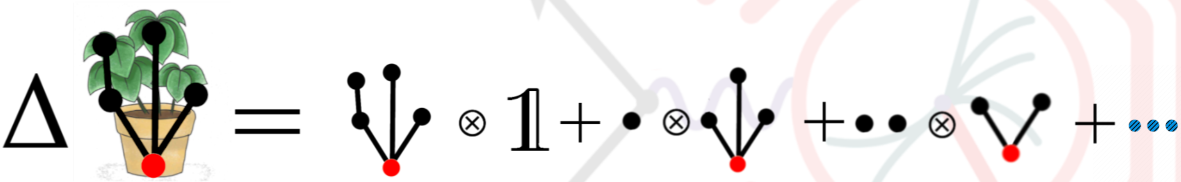
- The operator description (being analogous to momentum space) is convenient for taking soft limits
- We can then probe the conditions on d and Δ under which the dS wavefunction coefficients have Adler zero behavior
- For NLSM, DBI and sGal, imposing specific $\mathcal{O}(k^{\sigma})$ soft scaling looks to uniquely fix Δ as well as curvature corrections in the Lagrangian for arbitrary d
- Hope is to link hidden symmetries to soft theorems in dS

Numerical UV subtraction: the story of a dead basil plant

Daniele Artico,
Amplitudes 2022

Phenomenology of Elementary Particles

Hopf Algebraic structure of UV divergences

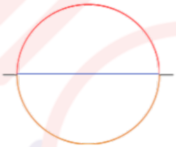


$(\phi_b \circ S) * \phi_a(X)$ is the renormalized value of X

A. Connes and D. Kreimer, arXiv:hep-th/9808042

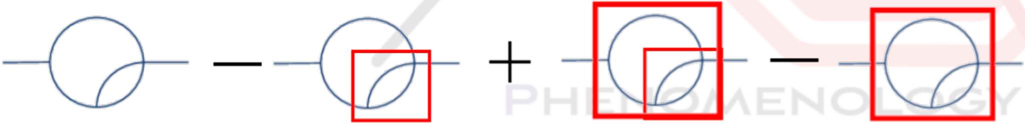
Local UV subtraction and LTD

$$\int d^4k_1 d^4k_2 \frac{1}{[(k_1^0)^2-E_1^2][(k_2^0)^2-E_2^2][(k_1^0+k_2^0+p_0)^2-E_3^2]} =$$



$=$

$$= \int d^3\vec{k}_1 d^3\vec{k}_2 \frac{1}{2E_1 2E_2} \frac{1}{(E_1+E_2+E_3+p_0)(E_1+E_2-E_3+p_0)} + \dots$$



Z. Capatti, V. Hirschi, D. Kermanschah, A. Pelloni and B. Ruijl, JHEP 04 (2020), 096 doi:10.1007/JHEP04(2020)096 [arXiv:1912.09291 [hep-ph]].

And this is how you remove all the leaves

Daniele Artico
PhD student @ HU Berlin
YouTube Project @ NonStandard Models
Activist @ Scientist Rebellion

Color-Kinematics Duality & Double Copy from the String-Inspired Formalism

Filippo Maria Balli
(filippo.m.balli@unimore.it)

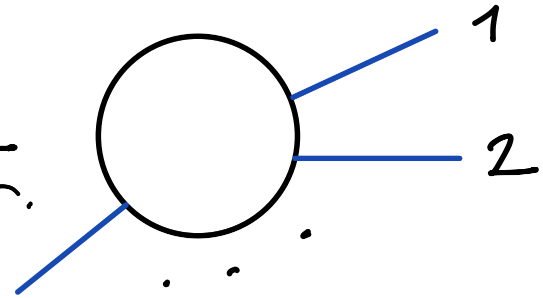
Amplitudes '22 Summer School – GONG SHOW

UNIMORE
UNIVERSITÀ DEGLI STUDI DI MODENA E REGGIO EMILIA

UPPSALA UNIVERSITET

* Worldline methods for the computation of QFT quantities in flat & curved space

* Main tool : Bern-Kosower formula for color-ordered one-loop n-gluon correlat.

$$A^n(1,2,\dots n) = \sum_{\text{diagr.}}$$ 

* Worldline methods for the computation of QFT quantities in flat & curved space

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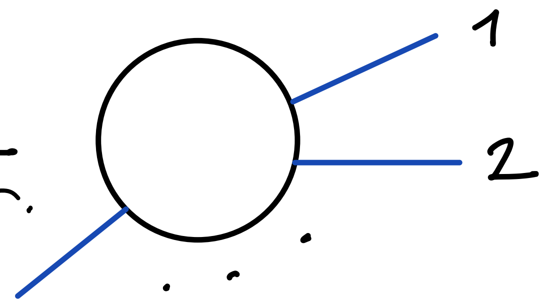
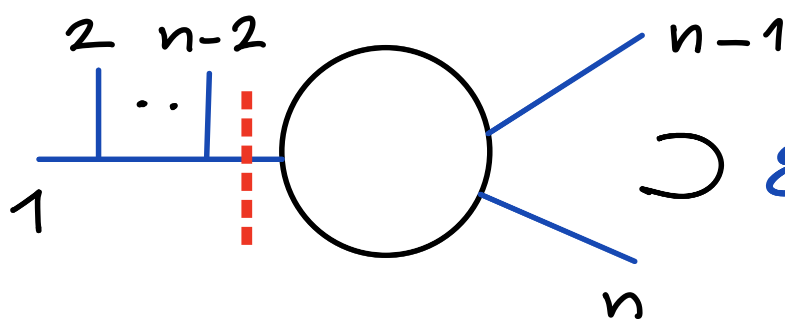


* Reducible contr. with IBP & pinching rules

⟶ (n−2) pinch



$$\supset \varepsilon_{12..n-2}\cdot R_{n-1}$$

* $\varepsilon_{12..n}$ appears naturally in BCJ gauge

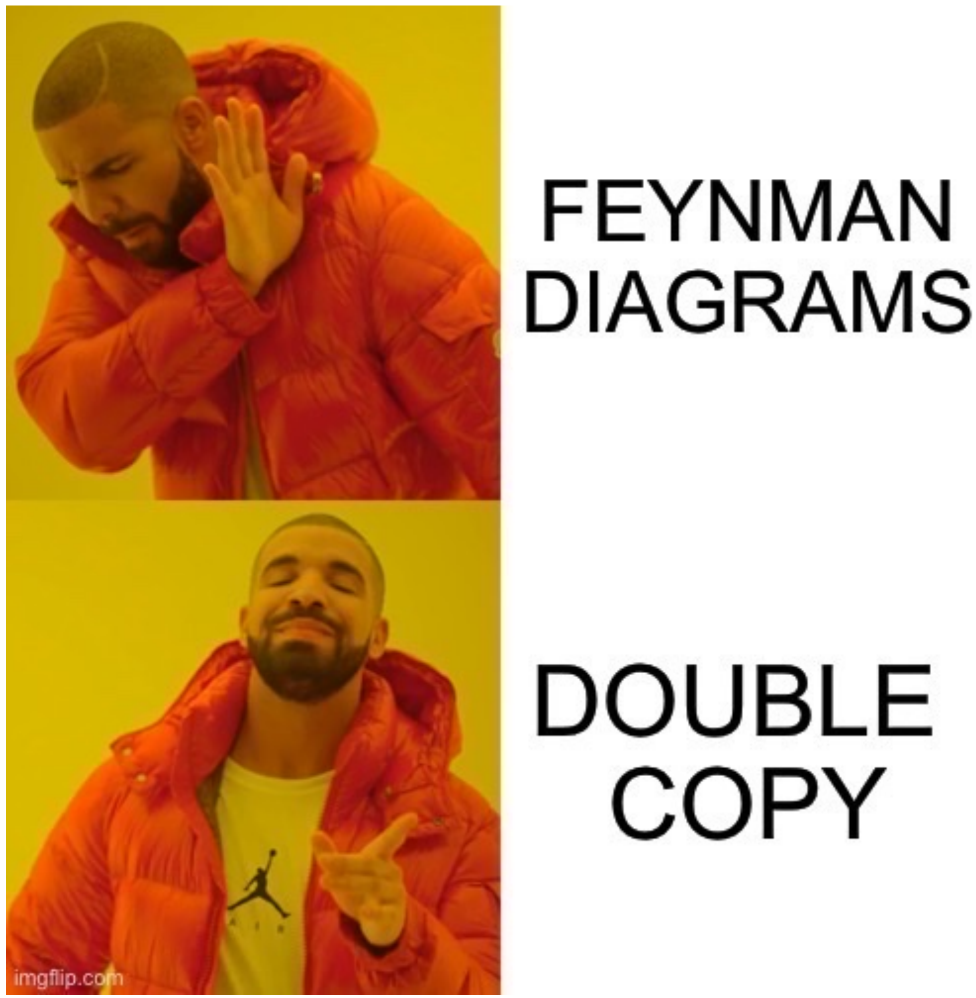
* A^n @ tree level from Berends-Giele relations in a C-R dual repres.

* why do we care?

* A^n @ tree level from Berends-Giele relations in a C-R dual repres.

* why do we care?

Perturbative quantum gravity :



* Next: what about one-loop?

* At Uppsala U. ⟶ string amplit. using chiral-splitting method

* References:

  - 2105.13709

  - 2110.04853

* POSTER by C. Schubert next week!

Multi-fold Mellin-Barnes with applications to Feynman integrals

Sumit Banik
Indian Institute of Science (IISc), Bengaluru, India

Based on: Phys. Rev. Lett. 127, 151601, (2021)
Phys. Rev. D 102, 091901(R), (2020)

In collaboration with : **B. Ananthanarayan** (IISc)
S. Friot(Paris-Saclay Univ.)

August 3, 2022

- Feynman integrals are hard to evaluate as go to higher order in perturbation expansion.

- We focus to study and evaluate Feynman integrals via its MB representation.

- Feynman integral can be converted into a MB integral using

$$\frac{1}{(A+B)^{\alpha}} = \frac{1}{\Gamma(\alpha)} \int_{-i\infty}^{+i\infty} \frac{dz}{2\pi i} \Gamma(-z)\Gamma(\alpha+z) A^{-\alpha-z} B^{z}$$

- AMBRE [J. Gluza, K. Kajda, T. Riemann 2007] automatizes the derivation of MB integrals.

- A multi-fold MB integral is of the form :

$$\int_{-i\infty}^{+i\infty} \frac{dz_1}{2\pi i} \cdots \int_{-i\infty}^{+i\infty} \frac{dz_N}{2\pi i} \; \frac{\prod_{i=1}^{k} \Gamma^{a_i}(\mathrm{e}_i \cdot \mathrm{z} + g_i)}{\prod_{j=1}^{l} \Gamma^{b_j}(\mathrm{f}_j \cdot \mathrm{z} + h_j)} x_1^{z_1} \cdots x_N^{z_N}$$

where a_i, b_j, k, l and N are positive integers.

Mellin-Barnes (MB) integrals can be solved as follows,

- Iterative Evaluation: MB.m [M. Czakon 2006], MBResolve.m [A.V. Smirnov, V.A. Smirnov 2009] etc.

- Non-Iterative Evaluation: Tsikh's method [M. Passare, A. Tsikh and O. Zhdanov, 1994] works for two-fold MB.

- We unveil a surprising connection between MB and conic hulls in [Phys. Rev. Lett. 127, 151601, (2021)].

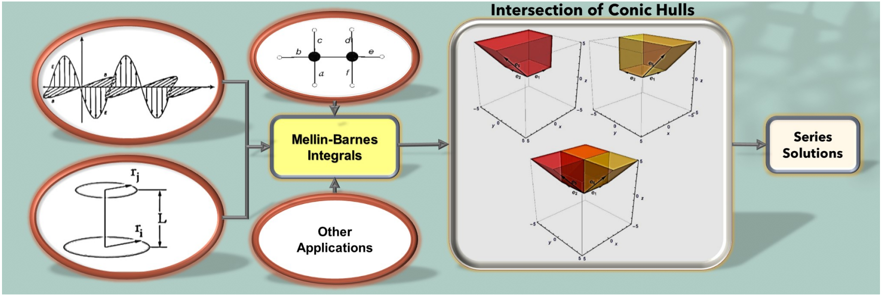
- We have automatized the method in the form of a `Mathematica` package `MBConicHulls.wl`.

- We solved the conformal hexagon and double box Feynman integrals in [Phys. Rev. D 102, 091901(R), (2020)]
- We proved that massive *n*-point one-loop conformal integrals can be written as a single hypergeometric series in [Phys. Rev. D 103, 096008, (2021)]
- We showed the limitations of the Method of Brackets in [arXiv: 2112.09679]

Future Plans

- Perform ϵ-expansion of Feynman integrals using the conic hull method.
- Study the GKZ system of MB integrals.
- Counting masters integrals from MB integrals, extending [M. Kalmykov, B. Kniehl 2017].

Schematic Outline of the method



Thank You

The N = 4 SUSY EFTHedron with a Mass Gap

Justin Berman

In collaboration with Aidan Herderschee and Henriette Elvang

LEINWEBER CENTER FOR THEORETICAL PHYSICS
UNIVERSITY OF MICHIGAN

Main Question

If we impose EFTHedron, N = 4 SUSY, and mass gap constraints, what do we get?

- If we also include monodromy, we isolate the open string [1].

<u>Four Gluon Superamplitudes:</u>

EFT Ansatz - $$\mathcal{A}[1234] = \delta^4(Q)\delta^4(\widetilde{Q})\left(\frac{a}{su} + \sum_{q\leq k} a_{k,q} s^{k-q} u^q\right)$$

Higher k ↔ higher derivative order terms in EFT expansion

Dispersive - $$\mathcal{A}[1234] = \delta^4(Q)\delta^4(\widetilde{Q})\left(\frac{a}{su} + \sum_i p_i \frac{G_{\ell_i}\left(1 + \frac{2u}{M_i^2}\right)}{s - M_i^2}\right)$$

[1] Huang, Liu, Rodina, Wang; arxiv: 2008.02293

Projections of the EFTHedron

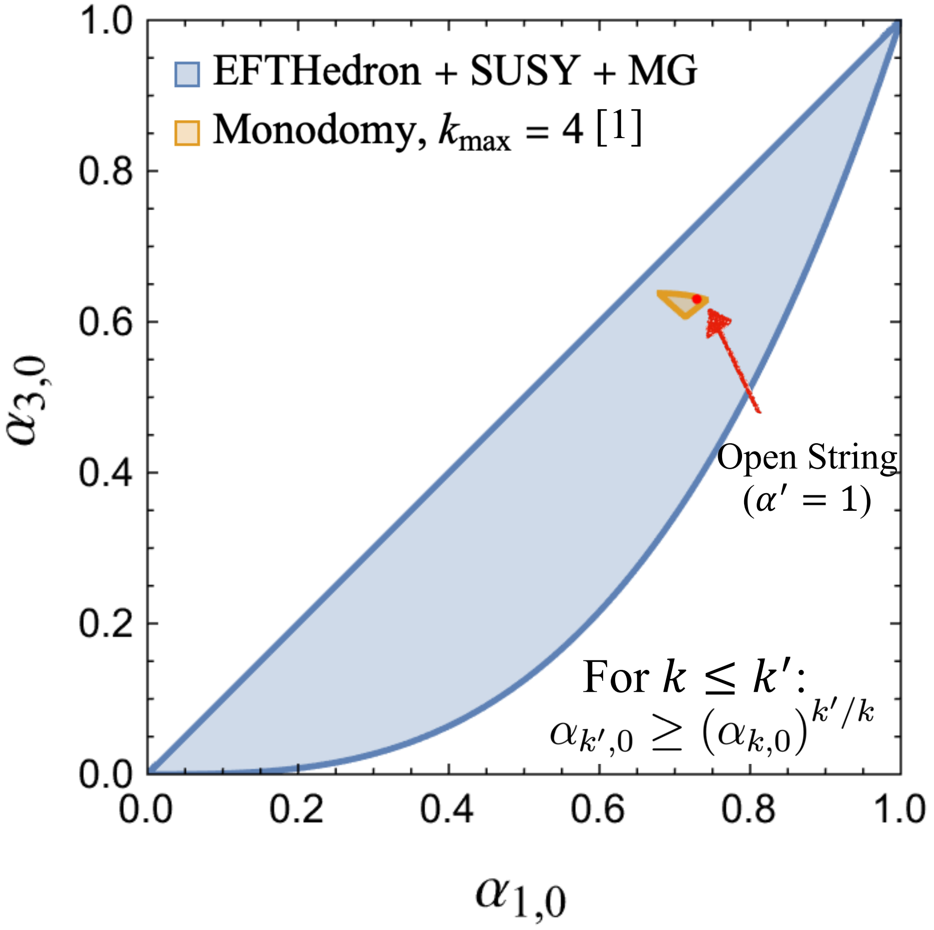


Figure 2: Analytic $(\alpha_{1,0}, \alpha_{3,0})$ region, independent of k_{max}.

$$\alpha_{k,q} = \frac{a_{k,q}}{a_{0,0}}$$

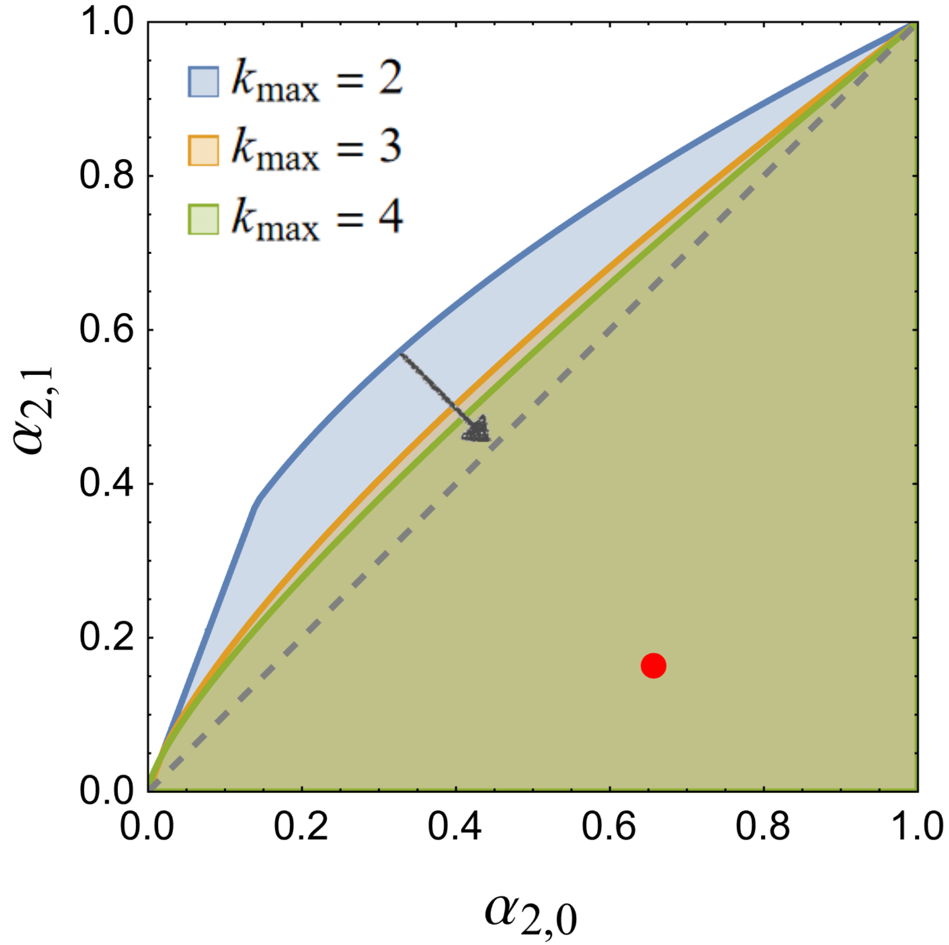


Fig 3: Analytic $(\alpha_{2,0}, \alpha_{2,1})$ region for $k_{max} \leq 4$.

[1] Huang, Liu, Rodina, Wang; arxiv: 2008.02293

Corner Amplitudes

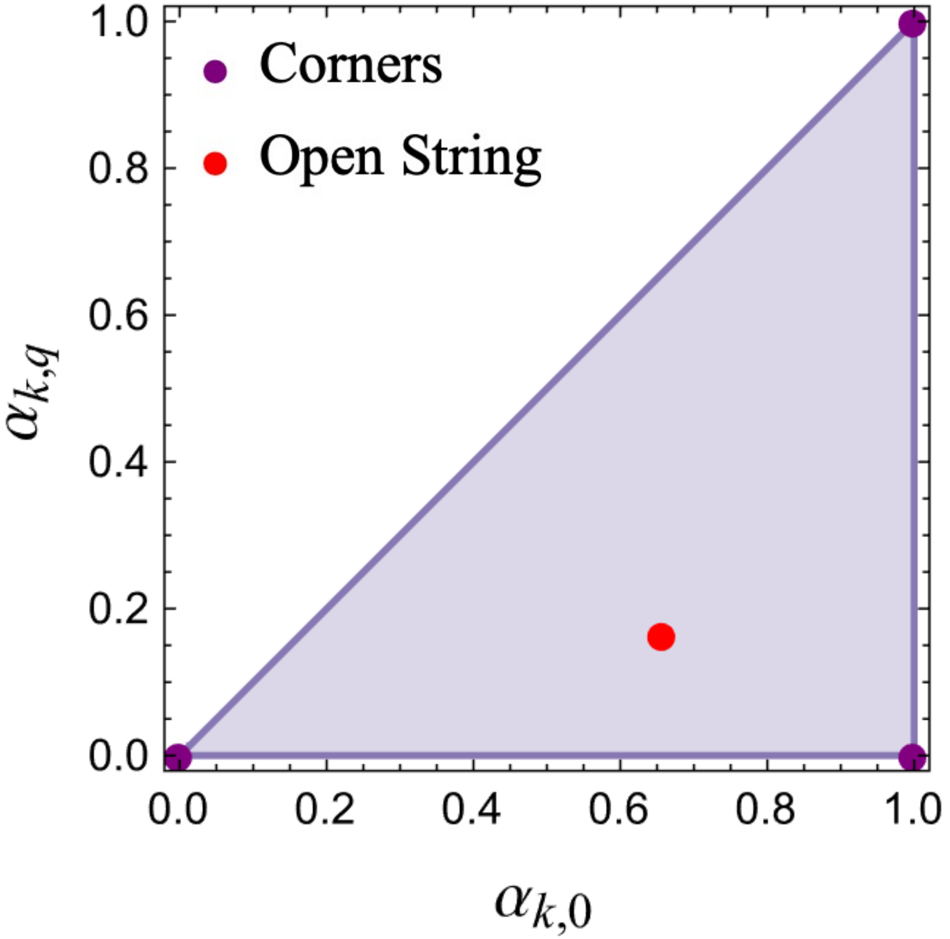


Figure 6: The conjectured $(\alpha_{2,0}, \alpha_{2,1})$ triangle with $M_{\text{gap}}^2 = 1$ and marked theories.

(0,0): $$\mathcal{A}[1234] = \delta^4(Q)\delta^4(\widetilde{Q})\left(\frac{a}{su} + a_{0,0}\right)$$

(1,0): $$\mathcal{A}[1234] = \delta^4(Q)\delta^4(\widetilde{Q}) \times \left(\frac{a}{su} + \frac{1}{M_{\text{gap}}^2 - s} + \frac{1}{M_{\text{gap}}^2 - u} - \frac{1}{M_{\text{gap}}^2}\right)$$

(1,1) $$\mathcal{A}[1234] = \delta^4(Q)\delta^4(\widetilde{Q}) \times \left(\frac{a}{su} + \frac{1}{(s - M_{\text{gap}}^2)(u - M_{\text{gap}}^2)}\right)$$

Scattering of Gravitons and Spinning Massive States from Compact Numerators

Amplitudes 2022 Gong Show

Taro Valentin Brown

Center for Quantum Mathematics and Physics (QMAP), Davis

Based on work with N.E.J. Bjerrum-Bohr and H. Gomez, 2011.10556

August 3, 2022

CHY

Recap

- Recall how Cachazo, He, and Yuan found how to construct d-dimensional scattering amplitudes from

$$\mathcal{A}(1,\ldots,n) = \int \mathrm{d}\Omega_{\mathsf{CHY}}\, \mathcal{I}_L \times \mathcal{I}_R \tag{1}$$

  with measure $\mathrm{d}\Omega_{\mathsf{CHY}} = z_{rs}^2 z_{st}^2 z_{tr}^2 \prod_{\substack{i=1 \\ i\neq r,s,t}} \mathrm{d}z_i\, \delta(S_i)$.
- S_i's are the scattering equations.
- Left and right integrands depend on specific theory, but often contain a Pfaffian.

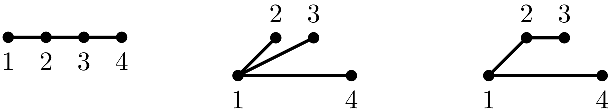
BCJ

Massive particles

- Can be extended to include massive particles
- CHY Pfaffian in DDM-basis

$$\mathrm{Pf}'(\Psi_{1n}) = \sum_{\beta \in S_{n-2}} N(1,\beta,n)\mathrm{PT}(1,\beta,n), \tag{2}$$

- Increasing tree algorithm to calculate numerators using combinatorial diagrams [A. Edison, F. Teng, S. He, O. Schlotterer]



$$(\epsilon_1 \cdot f_2 \cdot f_3 \cdot \epsilon_4), \quad (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot k_1)(\epsilon_3 \cdot k_1), \quad (\epsilon_1 \cdot \epsilon_4)(\epsilon_3 \cdot f_2 \cdot k_1) \tag{3}$$

Our main results:

- Summing over reference orders and collecting in exponentials decreases number of diagrams needed from

$$(N-1)! \quad \rightarrow \quad (N-2)! \tag{4}$$

- Exchanging field strength tensors and polarization vectors yield expression with scalars and quarks.
- Using double copy one e.g. obtains

$$\begin{aligned} \mathcal{M}(1_1, 2_2, 3_1) &= -(\epsilon_2 \cdot k_1)^2 \left(\epsilon_1 \cdot \exp\left[\frac{-f_2}{(\epsilon_2 \cdot k_1)} \right] \cdot \epsilon_3 \right) . \\ \mathcal{M}(1_{1/2}, 2_2, 3_{1/2}) &= -(\epsilon_2 \cdot k_1)^2 \left(\bar{u}_1 \exp\left[\frac{-\not{f}_2}{(\epsilon_2 \cdot k_1)} \right] \xi_3 \right) , \end{aligned} \tag{5}$$

which has similar structure to [A. Guevara, A. Ochirov, Justin Vines] minimally coupled amplitude for general spin s, but is covariant.

QCD Scattering Amplitudes in the High Energy Limit

Amplitudes 2022 Summer School Gong Show

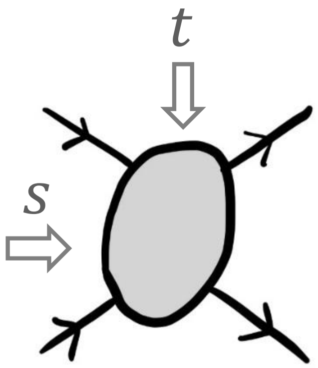
Emmet Byrne

emmet.byrne@ed.ac.uk

High-energy logarithms

At each order in perturbative QCD, large logarithms arise when the centre of mass energy is much greater than the transverse momenta of the produced partons.

Consider the dijet cross section, expanded in α_s:



$$L \equiv \log\left(\frac{s}{-t}\right) \gg 1$$

$$\alpha_s \ll 1$$

$$\alpha_s L \sim 1$$

$$\sigma^{(0)}/\sigma^{(0)} = 1$$

$$\sigma^{(1)}/\sigma^{(0)} = \alpha_s L c_0^{(1)} + \alpha_s c_1^{(1)}$$

$$\sigma^{(2)}/\sigma^{(0)} = \alpha_s^2 L^2 c_0^{(2)} + \alpha_s^2 L\, c_1^{(2)} + \alpha_s^2 c_2^{(2)}$$

$$\sigma^{(3)}/\sigma^{(0)} = \alpha_s^3 L^3 c_0^{(3)} + \alpha_s^3 L^2 c_1^{(3)} + \alpha_s^3 L c_2^{(3)} + \ldots$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

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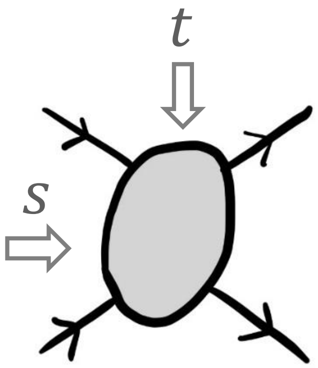

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	LL		
$\sigma^{(0)}/\sigma^{(0)} =$	1		
$\sigma^{(1)}/\sigma^{(0)} =$	$\alpha_s L c_0^{(1)}$	$+ \quad \alpha_s c_1^{(1)}$	
$\sigma^{(2)}/\sigma^{(0)} =$	$\alpha_s^2 L^2 c_0^{(2)}$	$+ \quad \alpha_s^2 L\, c_1^{(2)}$	$+ \quad \alpha_s^2 c_2^{(2)}$
$\sigma^{(3)}/\sigma^{(0)} =$	$\alpha_s^3 L^3 c_0^{(3)}$	$+ \quad \alpha_s^3 L^2 c_1^{(3)}$	$+ \quad \alpha_s^3 L c_2^{(3)} \quad + \ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

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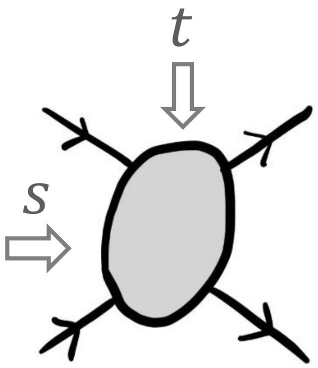

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	LL		NLL				
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$\sigma^{(1)}/\sigma^{(0)} =$	$\alpha_s L c_0^{(1)}$	$+$	$\alpha_s c_1^{(1)}$				
$\sigma^{(2)}/\sigma^{(0)} =$	$\alpha_s^2 L^2 c_0^{(2)}$	$+$	$\alpha_s^2 L\, c_1^{(2)}$	$+$	$\alpha_s^2 c_2^{(2)}$		
$\sigma^{(3)}/\sigma^{(0)} =$	$\alpha_s^3 L^3 c_0^{(3)}$	$+$	$\alpha_s^3 L^2 c_1^{(3)}$	$+$	$\alpha_s^3 L c_2^{(3)}$	$+$	...
$\vdots$	$\vdots$		$\vdots$		$\vdots$		

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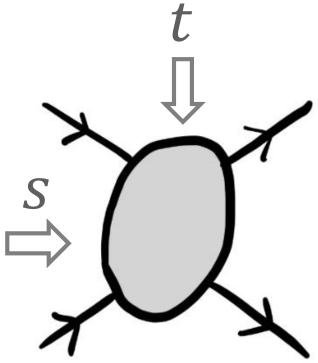

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	LL		NLL		NNLL	
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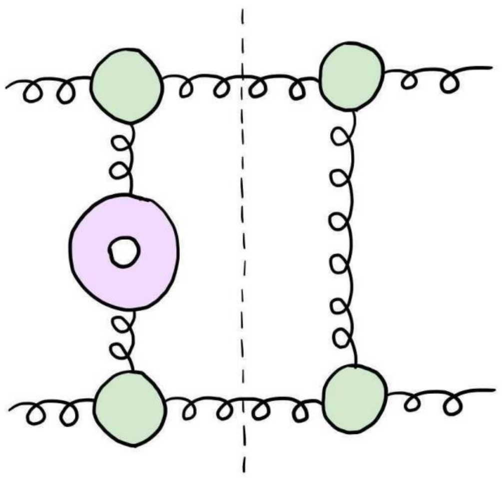
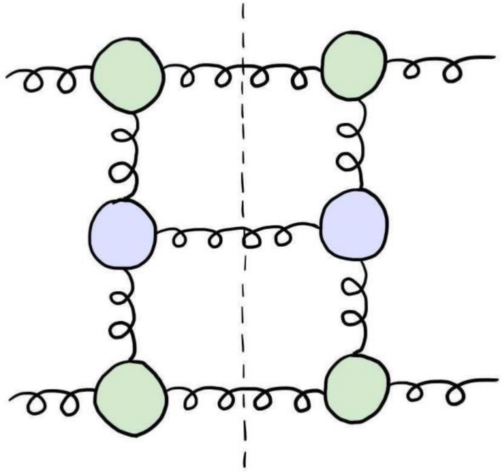
$\sigma^{(1)}/\sigma^{(0)} =$	$\alpha_s L c_0^{(1)}$	$+$	$\alpha_s c_1^{(1)}$			
$\sigma^{(2)}/\sigma^{(0)} =$	$\alpha_s^2 L^2 c_0^{(2)}$	$+$	$\alpha_s^2 L\, c_1^{(2)}$	$+$	$\alpha_s^2 c_2^{(2)}$	
$\sigma^{(3)}/\sigma^{(0)} =$	$\alpha_s^3 L^3 c_0^{(3)}$	$+$	$\alpha_s^3 L^2 c_1^{(3)}$	$+$	$\alpha_s^3 L c_2^{(3)}$	$+ \ldots$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	

The NLO dijet cross section at LL accuracy

The dependence on the initial state is captured by so called impact factors. At this accuracy they are simply helicity conserving phases.





Virtual corrections are given by a simple multiplicative factor, the `Regge trajectory',

$$\mathcal{M}^{(1)}_{2\to2} \xrightarrow[s\gg|t|]{} \mathcal{M}^{(0)}_{2\to2}\alpha(|q_\perp|^2)\log\left(\frac{s}{|q_\perp|^2}\right)$$

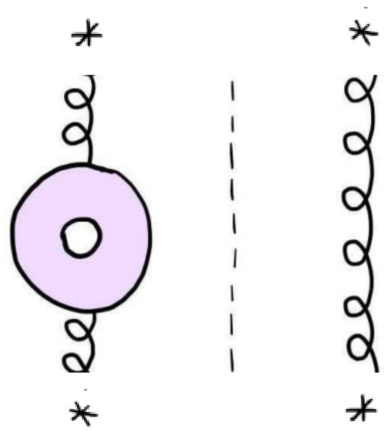
$$\alpha(|q_\perp|^2) = \frac{\alpha_s}{4\pi}N_c\kappa_\Gamma\frac{2}{\epsilon}\left(\frac{\mu^2}{|q_\perp|^2}\right)^\epsilon$$

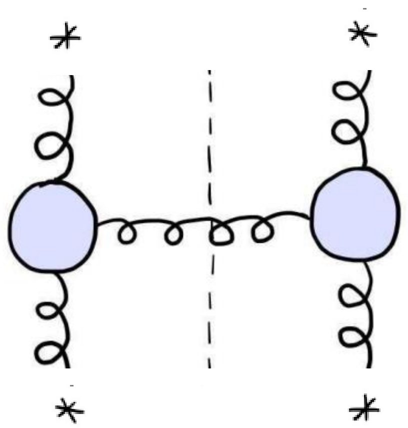
Real emissions are descrbed by a factorised `Lipatov' vertex,

$$V^{(0)g^+}(q_{1\perp},q_{2\perp}) = \frac{q^*_{1\perp}q_{2\perp}}{q_{1\perp}-q_{2\perp}}$$

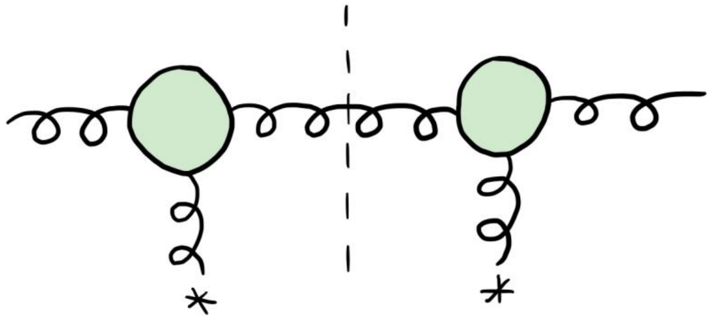
Building blocks of the LL BFKL equation

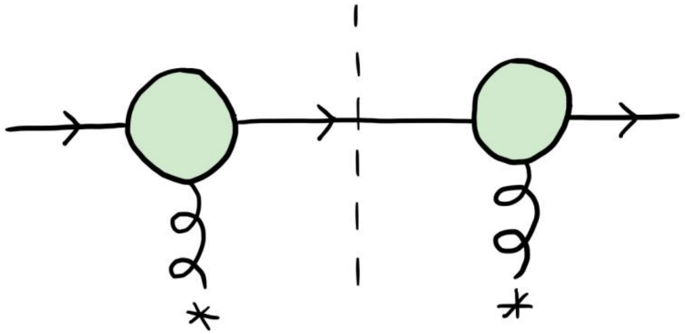
The BFKL equation describes the *t*-channel evolution of a gluon. To LL accuracy the kernel of this equation is given by the Regge trajectory and Lipatov vertex:





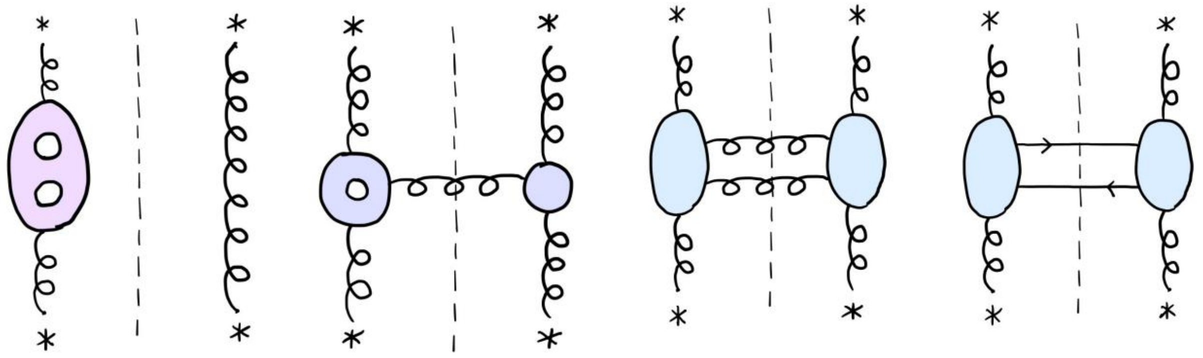
Cross sections can be computed using this framework by including the LO impact factors:



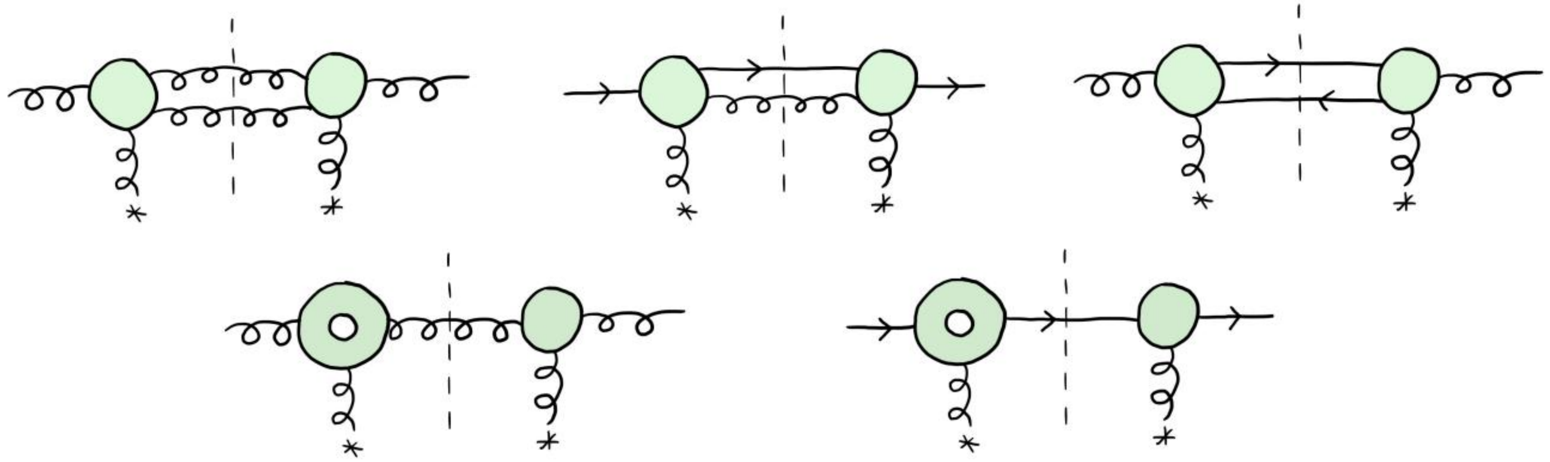


Building blocks of the NLL BFKL equation

To bring the BFKL equation to NLL accuracy, many more building blocks were required:

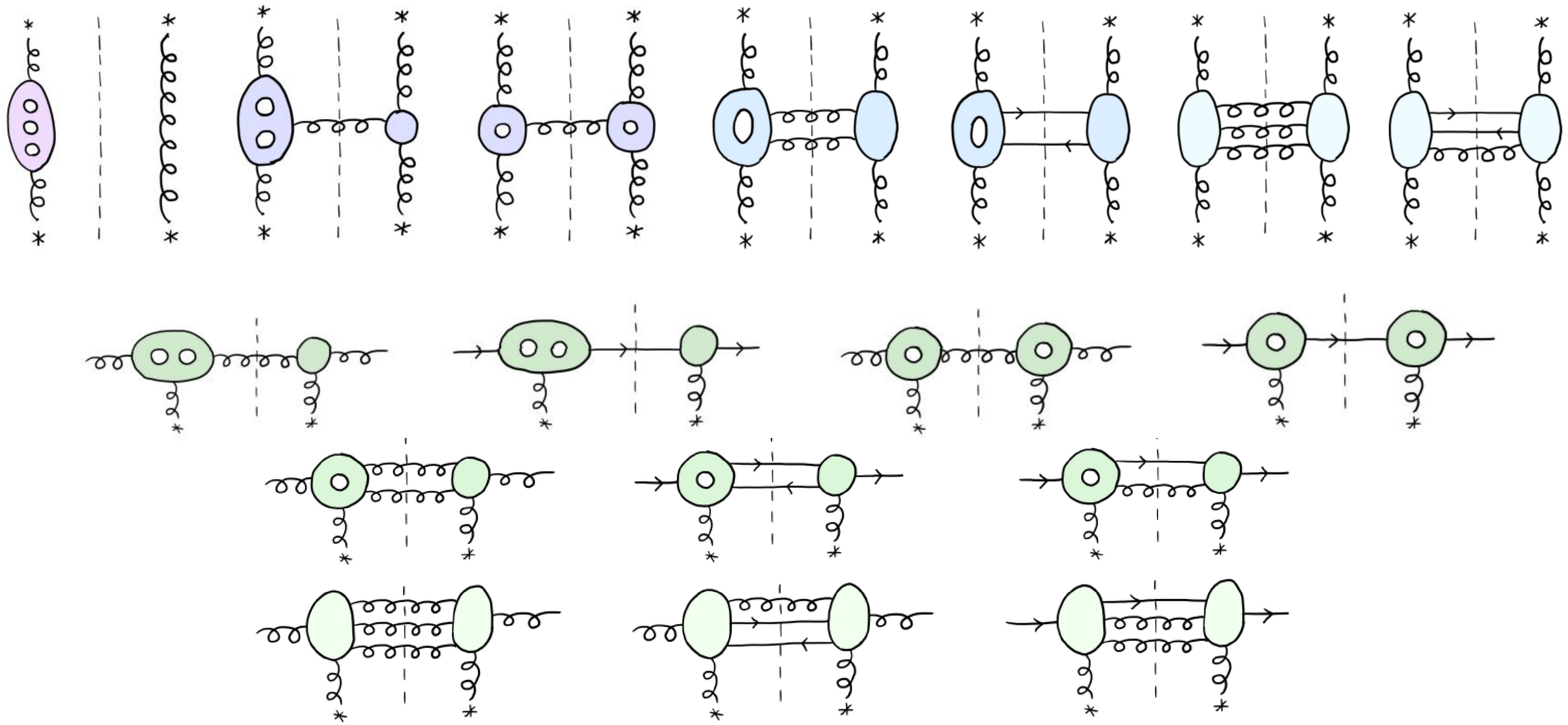


Similarly, to compute cross sections with the BFKL framework, NLO corrections to the impact factors were needed:



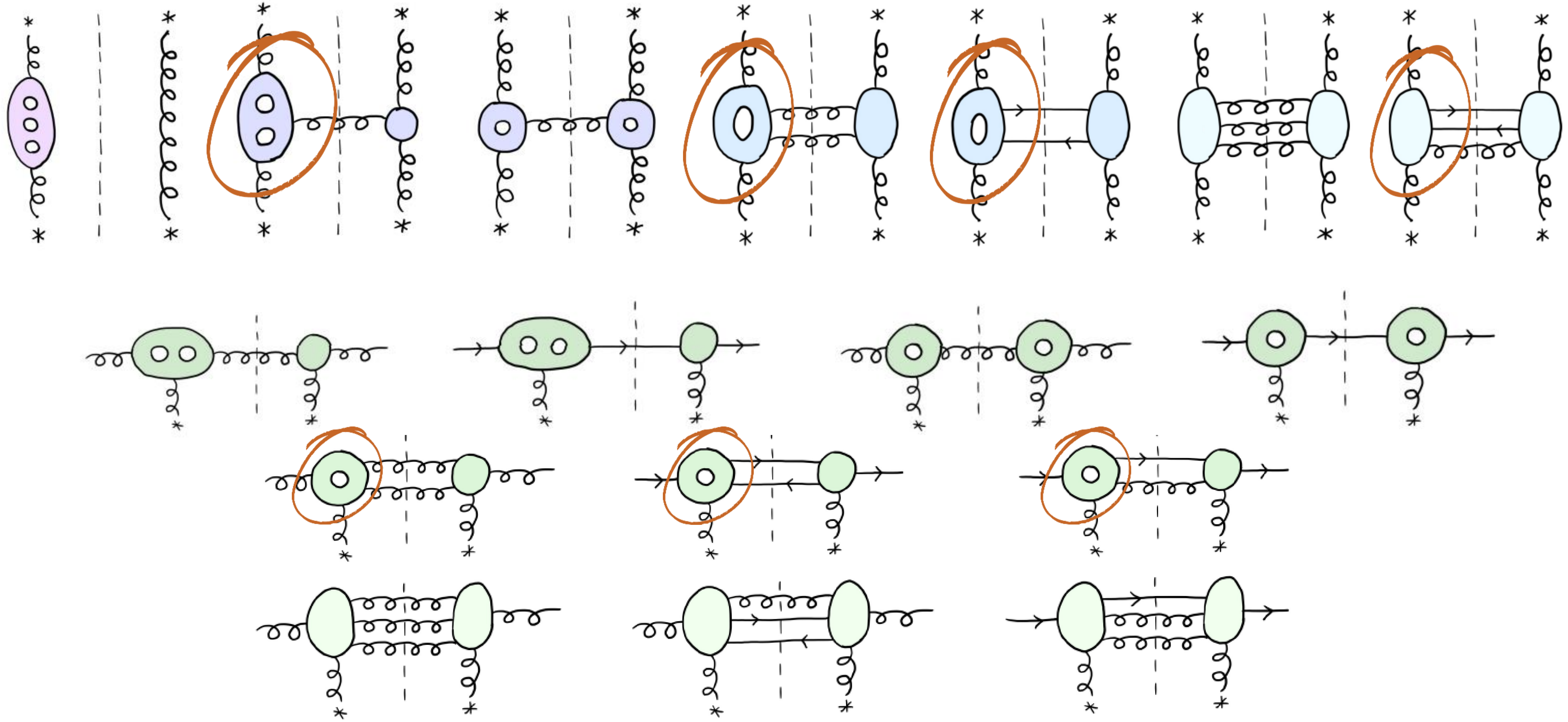
Building blocks for the NNLL BFKL equation

Some building blocks remain to bring the BFKL equation to NNLL accuracy...



Building blocks for the NNLL BFKL equation

Some building blocks remain to bring the BFKL equation to NNLL accuracy...



Ongoing Work

- At the moment, I am extracting some of the missing NNLL building blocks from QCD amplitudes.

- These high energy effects are crucial for precise and stable QCD predictions at the LHC.

- The High Energy Jets (HEJ) framework implements this resummation as a Monte Carlo event generator. I am working on bringing this framework to NLL accuracy.

Thanks for your attention!

A study of the differential equations method for Feynman loop integrals in connection with Scientific Machine Learning

Francesco Calisto

Theoretical and Mathematical Physics Master Student in Munich from October 2022

Amplitudes Summer School, Prague, 3rd August 2022

MAX PLANCK INSTITUTE FOR PHYSICS

European Research Council
Established by the European Commission

UNIVERSITÀ DEGLI STUDI DI TORINO

From Amplitudes to DE systems

A major bottleneck in scattering amplitudes calculations are Feynman loop integrals

↓

Having applied the differential equations method, our goal is to solve a canonical system of DEs:

$$d\vec{I}(\vec{x}) = \epsilon \; dA(\vec{x}) \; \vec{I}(\vec{x})$$

[Henn, 2013]

↓

New method for its numerical evaluation through Physics-Informed Neural Networks

Physics-Informed Neural Networks

Blending model-based information from physics with data to tackle complexity

- Differential equations describe dynamical systems
- Machine learning models are good at approximating (Universal Approximation Theorems), but big data are needed

$$\partial_x u = f(u, x)$$

$$\downarrow$$

$$\mathrm{L}(p) = \underbrace{\sum_{i \in \mathcal{D}} \left(\frac{\partial NN(p, x)}{\partial x} \bigg|_{x = x_i} - f(NN(p, x_i)) \right)^2}_{\text{PDE loss}} + \underbrace{\sum_{j \in \mathcal{B}} (NN(p, x_j) - u(x_j))^2}_{\text{BCs loss}}$$

Discussion and outlook

PROS

- Time cost concentrated during training
- Reduced computational cost

CONS

- Bigger networks don't guarantee higher accuracy

OUTLOOK

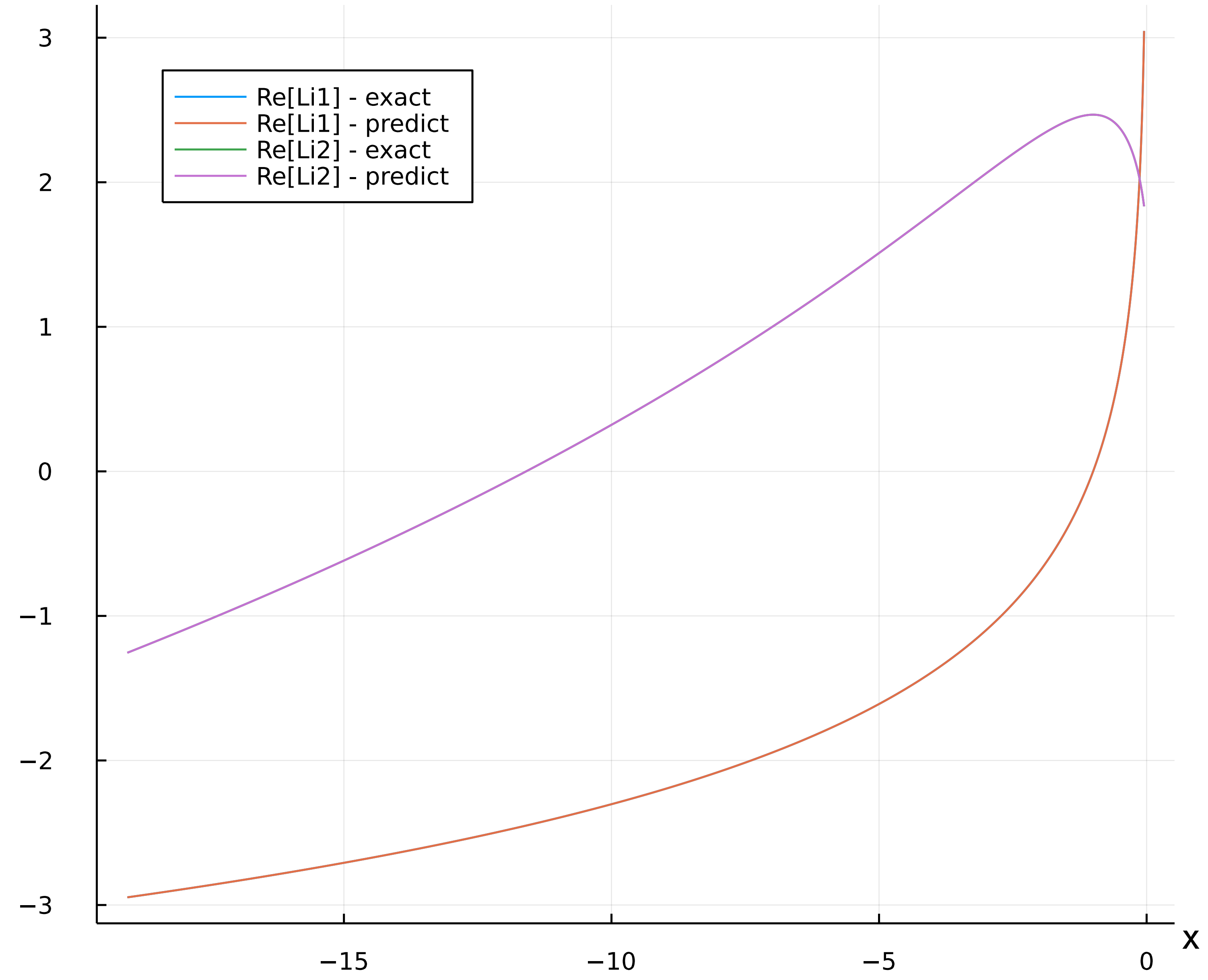
- Extension to multi-dimensional and multi-loop problems
- Separate solution for the various orders in ε
  $\longrightarrow$ explicit ε-dependence removed

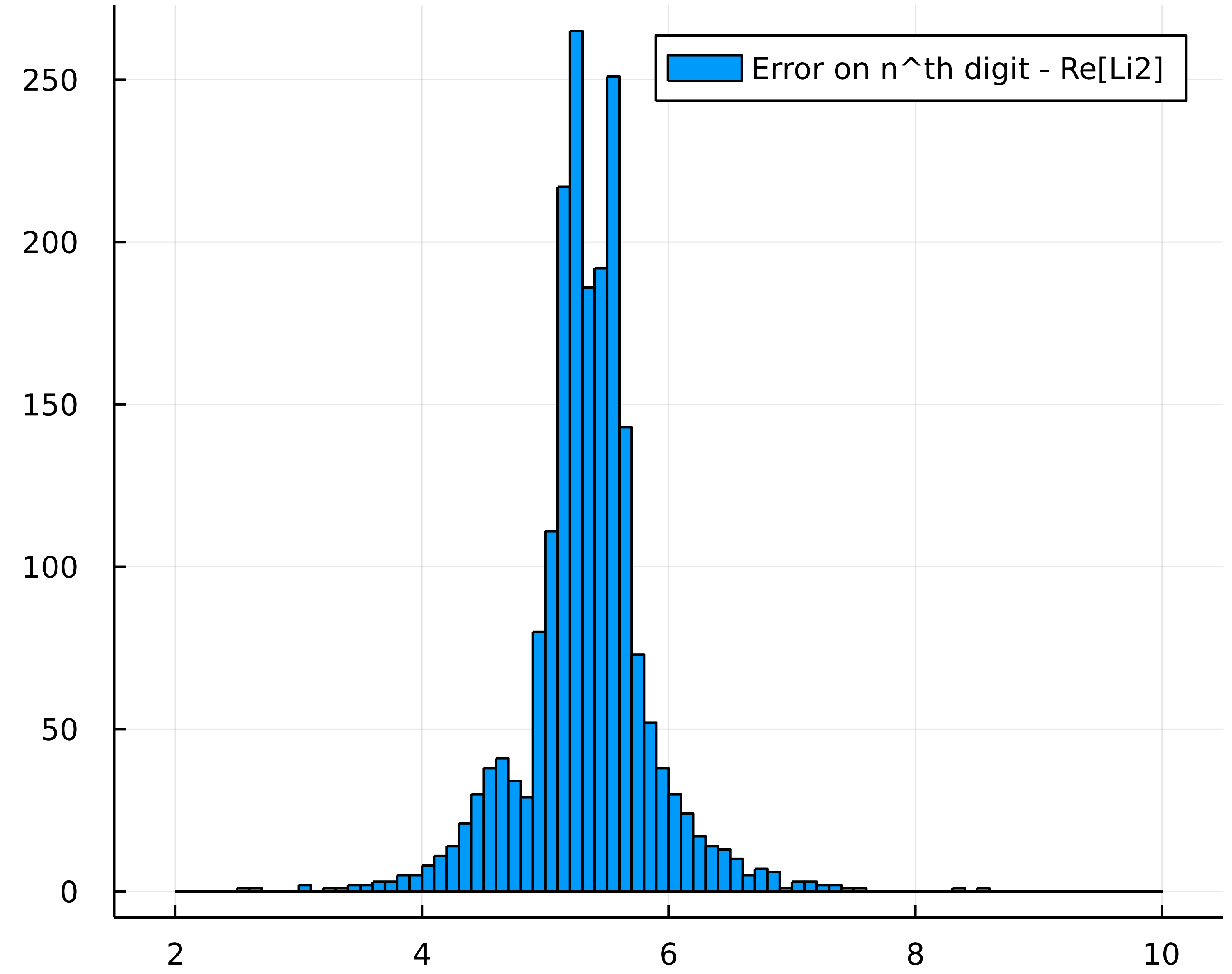
Backup

Polylogarithmic system

$$\vec{f} = (1, \mathrm{Li}_1(1-x), \mathrm{Li}_2(1-x))$$

$$\frac{d\vec{f}}{dx} = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{x-1} & 0 \end{pmatrix} \vec{f}$$

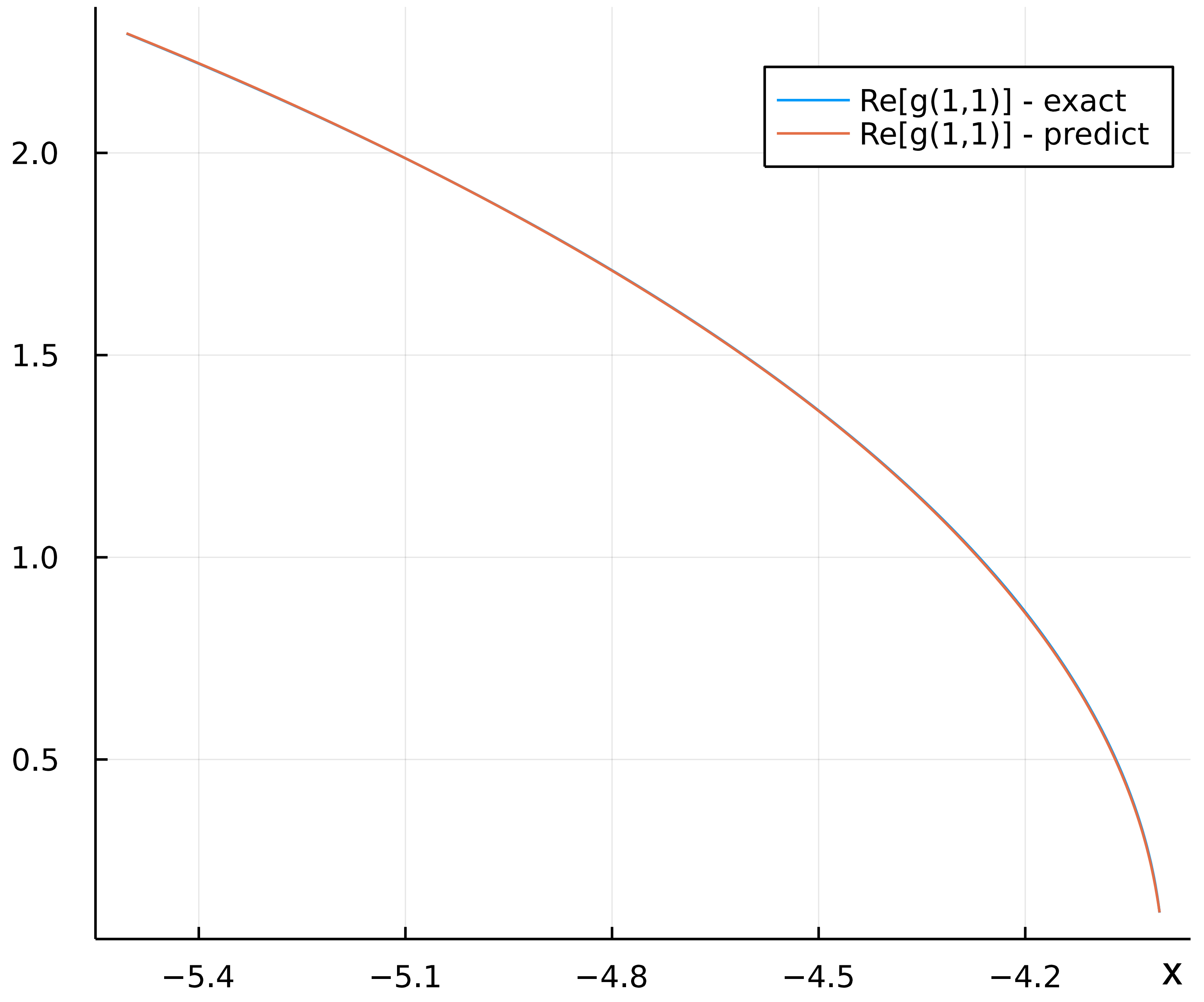


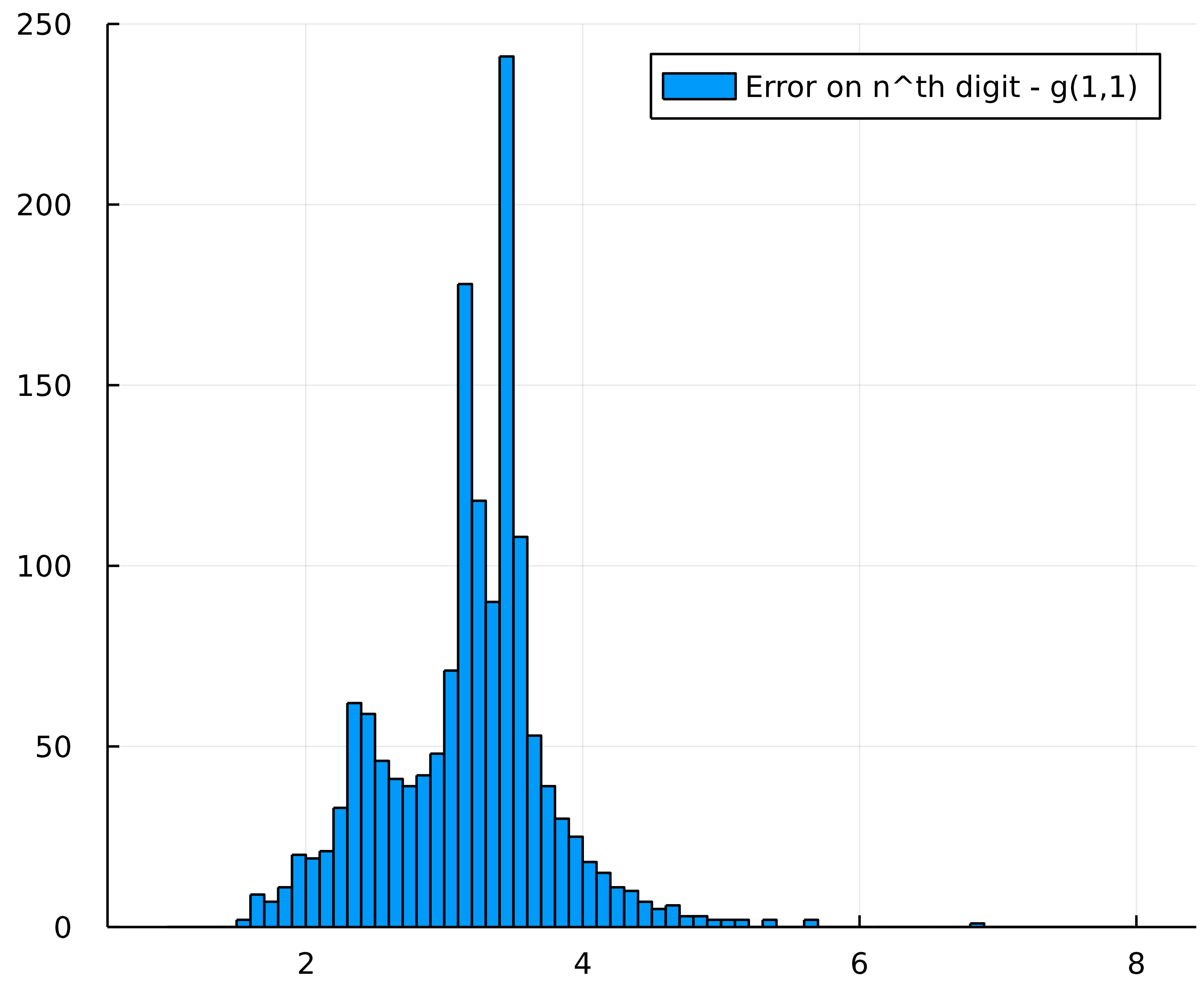


Optimization Algorithm	BFGS
Iterations	5003
Final Objective Value	2.34 e-10
Training Strategy	GridTraining
Domains	x $\in [1,3]$, dx $= 0.01$
Boundary Conditions	x $= 1, 2, 3$
Neural Network dims	$1 > 20 > 20 > 1$
Training Time	130 s

$$n_{digits} = -\log_{10}\left|\frac{u_{NN}}{u_{exact}} - 1\right|$$

One-loop bubble integral





$$\partial_x \vec{g} = \epsilon \begin{pmatrix} 0 & 0 \\ \frac{2}{\sqrt{x(x+4)}} & \frac{1}{x+4} \end{pmatrix} \vec{g}$$

$$n_{digits} = -\log_{10} \left| \frac{u_{NN}}{u_{exact}} - 1 \right|$$

Optimization Algorithm	BFGS
Iterations	3003
Final Objective Value	1.38 e-5
Training Strategy	GridTraining
Domains	$x \in [-5.5, -4]$, $dx = 0.01$
Boundary Condition	x $= -5.5, -4.75, -4$
Neural Network dims	$1 > 20 > 20 > 1$
Training Time	116 s

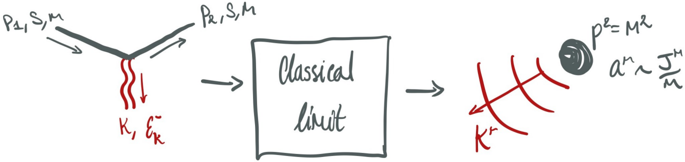
Classical Limits of Higher Spin String Amplitudes

[L.C, P. Pichini, 2207.03947]

Starting Point:

set of 3pt amplitudes $\longrightarrow$ energy-momentum tensor of Kerr BH

[GOV 1706.02314, 1812.06895]



$$\mathcal{M}_{Kerr} = (p_1 \cdot \varepsilon_k^+)^2 \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s} \qquad \varepsilon_{\mu\nu,k} T^{\mu\nu}(k) \sim (\varepsilon_k \cdot p_1)^2 \exp(k \cdot a)$$

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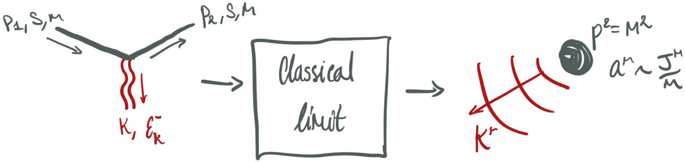


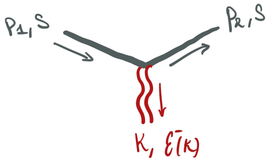
$$\mathcal{M}_{Kerr} =(p_1 \cdot \varepsilon_k^+)^2 \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s}$$

$$=(p_1 \cdot \varepsilon_k^+)^2 \left(c_0 + c_1(k\cdot a) + \ldots \ + c_{2s}(k \cdot a)^{2s}\right)$$

$$\underset{s\to\infty}{\longrightarrow}(p_1 \cdot \varepsilon_k^+)^2 \exp(k \cdot a)$$

$$\varepsilon_{\mu\nu,k} T^{\mu\nu}(k) \sim (\varepsilon_k \cdot p_1)^2 \exp(k \cdot a)$$

Comparison to interaction of leading Regge superstring states:



$$\mathcal{M}_{Kerr} = (p_1 \cdot \varepsilon_k^+)^2 \sum_{n=0}^{2s} c_n \langle (k \cdot a_{(s)})^n \rangle$$

$$\mathcal{M}_{string} = (p_1 \cdot \varepsilon_k^+)^2 \sum_{n=0}^{2s} \tilde{c}_n \langle (k \cdot a_{(s)})^n \rangle$$

Spin multipole coefficients:

$\mathcal{M}_{Kerr}$: $c_n = \frac{1}{n!} \underset{s\to\infty}{\longrightarrow} \frac{1}{n!}$ *spin universality*

$\mathcal{M}_{String}$: $\tilde{c}_n = \tilde{c}_n(s)$

$$\underset{s\to\infty}{\longrightarrow} \begin{cases} \frac{(2k+1)!}{4^k (k+1)!(k!)^3} & \text{if } n = 2k \text{ with } k \in \mathbb{N}, \\ \frac{(2k+1)!}{4^k ((k+1)!)^2 (k!)^2} & \text{if } n = 2k+1 \text{ with } k \in \mathbb{N}. \end{cases}$$

Classical Limits of Higher Spin String Amplitudes

[L.C, P. Pichini, 2207.03947]

Results:

Closed/coupling to gravity:

$$\lim_{s\to\infty} \mathcal{M}_{Kerr} = (p_1 \cdot \varepsilon_k^+)^2 \exp(k \cdot a) \quad \text{[GOV]}$$

$$\lim_{s\to\infty} \mathcal{M}_{String} = (p_1 \cdot \varepsilon_k^+)^2 \left[I_0(k \cdot a) + I_1(k \cdot a)\right]^2 \quad \text{[LC, Pichini]}$$

Open/coupling to electromagnetism:

$$\lim_{s\to\infty} \mathcal{A}_{Kerr} = (p_1 \cdot \varepsilon_k^+) \exp(k \cdot a) \quad \text{[GOV]}$$

$$\lim_{s\to\infty} \mathcal{A}_{String} = (p_1 \cdot \varepsilon_k^+) \left[I_0(2k \cdot a) + I_1(2k \cdot a)\right] \quad \text{[LC, Pichini]}$$

Theoretical Physics of the Early Universe
Department of Physics
Technical University of Munich

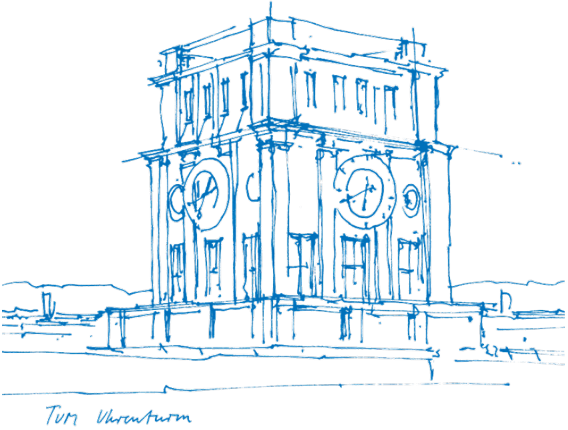
False Vacuum Decay and Functional Determinants

The Fubini Lipatov instanton

Matthias Carosi

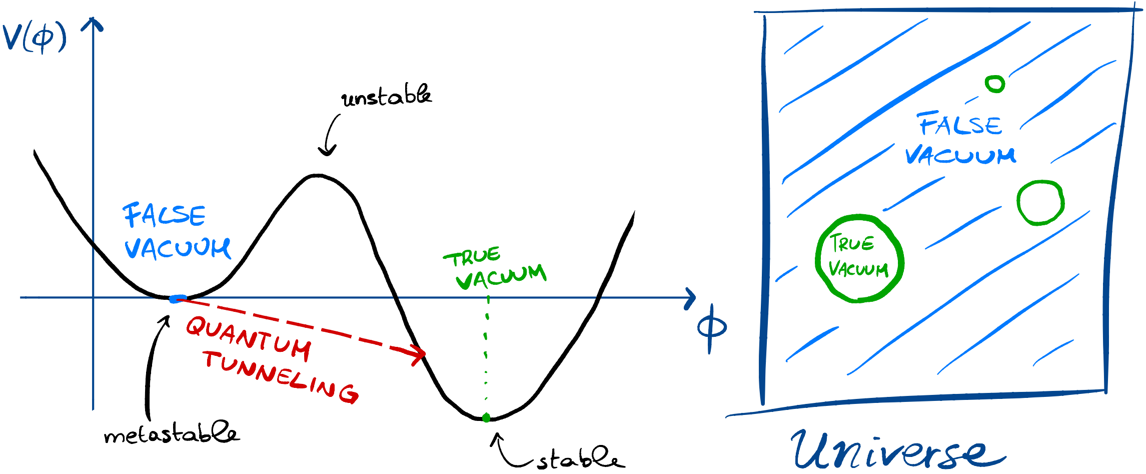
Theoretical Physics of the Early Universe
Department of Physics
Technical University of Munich

Amplitudes 2022 Summer School



Metastability in the quantum world

False vacua and quantum tunneling



A saddle point approximation

The Coleman bounce

$$\Gamma = \operatorname{Im} E_{FV}$$

A saddle point approximation
The Coleman bounce

$$\Gamma = \mathrm{Im}\, E_{FV}$$

$$= \mathcal{N} e^{-S[\phi_b]} \left(\frac{\det(S''[\phi_b])}{\det(S''[\phi_{FV}])} \right)$$

↑ normalization constant

ϕ_b = Coleman bounce

ϕ_{FV} = False vacuum

$$S''[\bar{\Phi}] = -\Delta_D + V''(\bar{\Phi})$$

A saddle point approximation

The Coleman bounce

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$$= \mathcal{N} e^{-S[\phi_b]} \left(\frac{\det(S''[\phi_b])}{\det(S''[\phi_{FV}])} \right)$$

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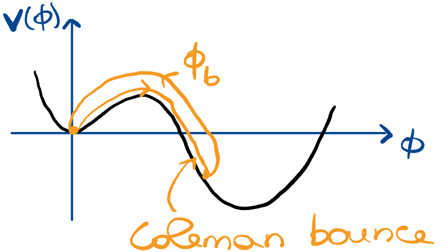
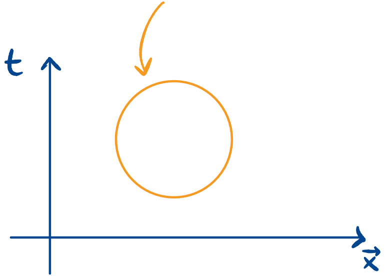
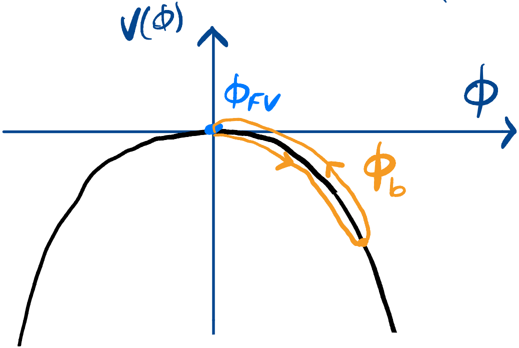




The Fubini-Lipatov instanton

The issue with scale invariance

$$V(\phi) = -\frac{\lambda}{n}\phi^n \qquad \begin{pmatrix} \lambda > 0 \\ n = \frac{2D}{D-2} \end{pmatrix}$$



The Fubini-Lipatov instanton

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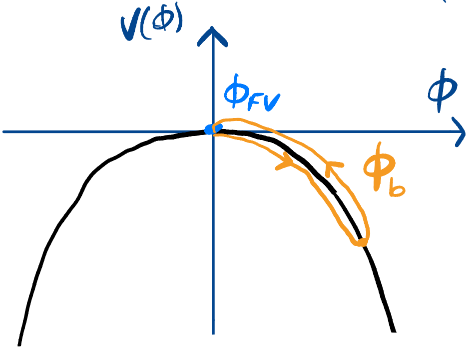
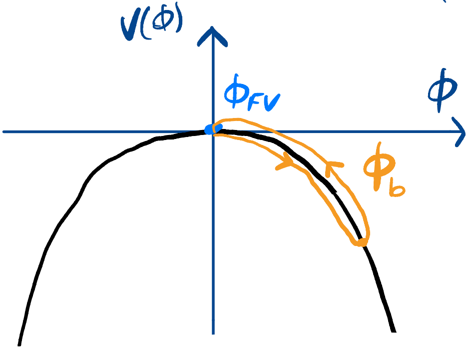


$$\phi_b(r) = \left[\frac{D(D-2)}{\lambda}\left(\frac{R}{R^2+(r-r_0)^2}\right)^2\right]^{\frac{D-2}{4}}$$

The Fubini-Lipatov instanton

The issue with scale invariance

$$V(\phi) = -\frac{\lambda}{n}\phi^n \qquad \left(\begin{array}{c} \lambda > 0 \\ n = \frac{2D}{D-2} \end{array} \right)$$



$$r^2 = \sum_{i=1}^{D} x_i^2$$

$$\phi_b(r) = \left[\frac{D(D-2)}{\lambda} \left(\frac{R}{R^2 + (r - r_0)^2} \right)^2 \right]^{\frac{D-2}{4}}$$

SCALE INVARIANCE → IR ∞ !

⇕

instanton size (R)

position of the instanton (r_0)

⇕

TRANSLATIONAL SYMMETRY

Rescaling functional determinants

The effect of the zero modes

Strategy:

1) Subtract zero-modes

$$\rightarrow \int dx_0 \int dR \, J_{tr} \, J_{dil} \left(\frac{\det'(-\Delta_D + V''(\phi_b))}{\det(-\Delta_D)} \right)^{-1/2}$$

Rescaling functional determinants

The effect of the zero modes

Strategy:

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$$\rightarrow \int dx_0 \int dR \, J_{tr} \, J_{dil} \left(\frac{\det'(-\Delta_D + V''(\phi_b))}{\det(-\Delta_D)} \right)^{-1/2}$$

2) Rescale (1707.08124 Andreassen, Frost, Schwartz)

$$\frac{\det'(-\Delta_D + V''(\phi_b))}{\det(-\Delta_D)} \longrightarrow \frac{\det'\left(\frac{1}{V''(\phi_b)}\Delta_D - 1\right)}{\det\left(\frac{1}{V''(\phi_b)}\Delta_D\right)}$$

Rescaling functional determinants

The effect of the zero modes

FINDINGS

$K \neq 1$ induced by zero modes

$$\frac{\det'(-\Delta_D + V''(\phi_b))}{\det(-\Delta_D)} = K \, \frac{\det'\left(\frac{1}{V''(\phi_b)}\Delta_D - 1\right)}{\det\left(\frac{1}{V''(\phi_b)}\Delta_D\right)}$$

$\Rightarrow$ subtracting zero-modes <u>does not</u> commute with rescaling K

$$K \sim \frac{1}{R^D} \Rightarrow \int_0^\infty \frac{dR}{R^D}$$

UV divergence

Probing the Double Copy Landscape: the KLT Bootstrap

Alan (Shih-Kuan) Chen

Leinweber Center for Theoretical Physics, The University of Michigan

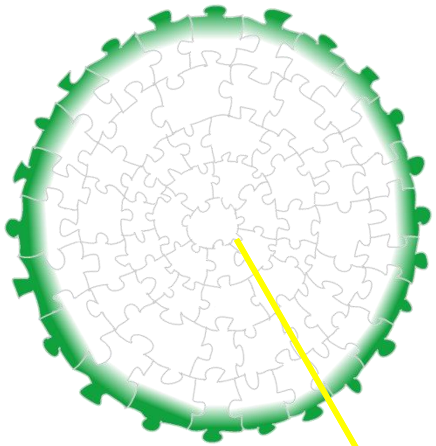
In collaboration with Henriette Elvang

Standard Double-Copy

$\times \qquad \times \qquad =$

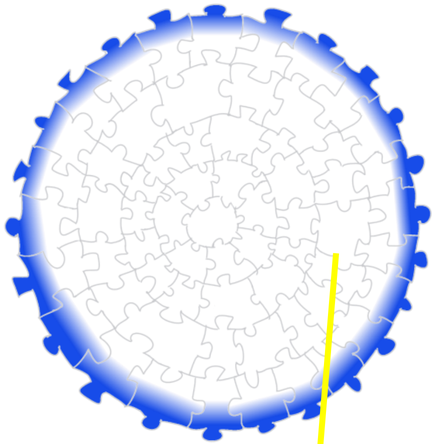
$$\sum_{(n-3)!\,\text{basis}} (\text{Gauge}) \cdot \left(m^{\text{BAS}}\right)^{-1} \cdot \left(\widetilde{\text{Gauge}}\right) = (\text{Gravity})$$

Standard Double-Copy



$\mathcal{A}_n^{\mathrm{L}}$

$\times \qquad \times$

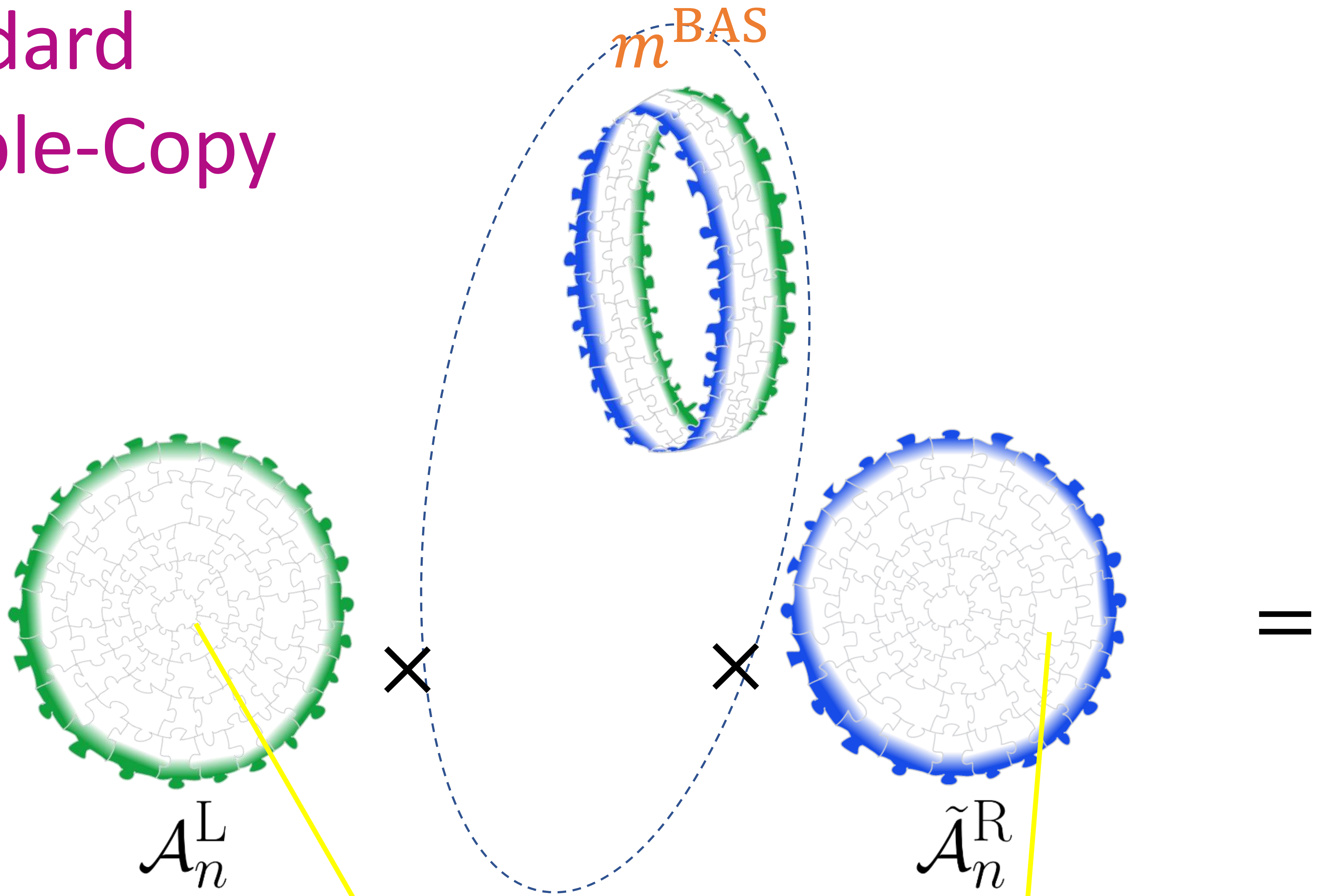


$\tilde{\mathcal{A}}_n^{\mathrm{R}}$

$=$

$$\sum_{(n-3)!\text{basis}} (\text{Gauge}) \cdot \left(m^{\text{BAS}}\right)^{-1} \cdot \left(\widetilde{\text{Gauge}}\right) = (\text{Gravity})$$

Standard Double-Copy



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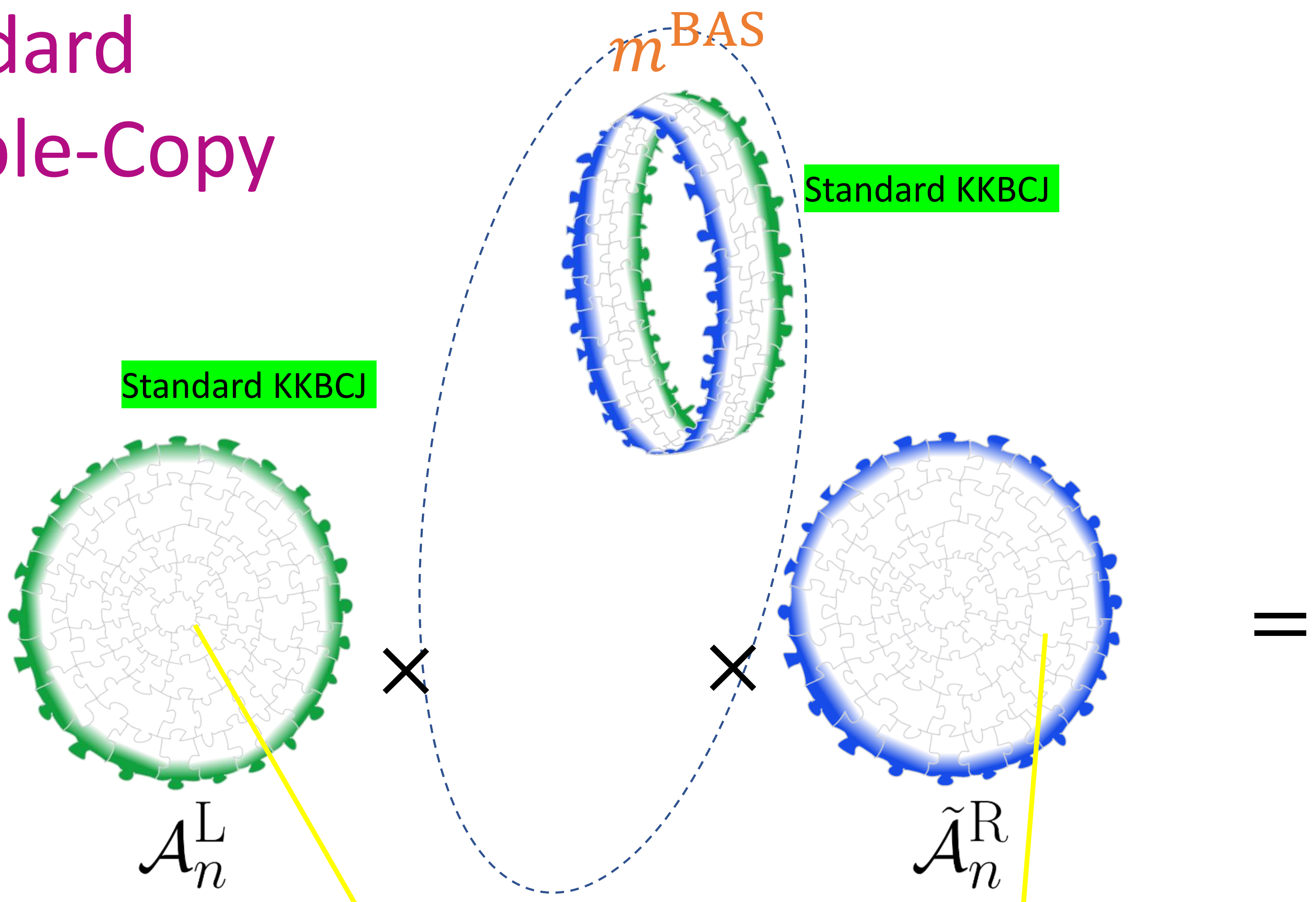

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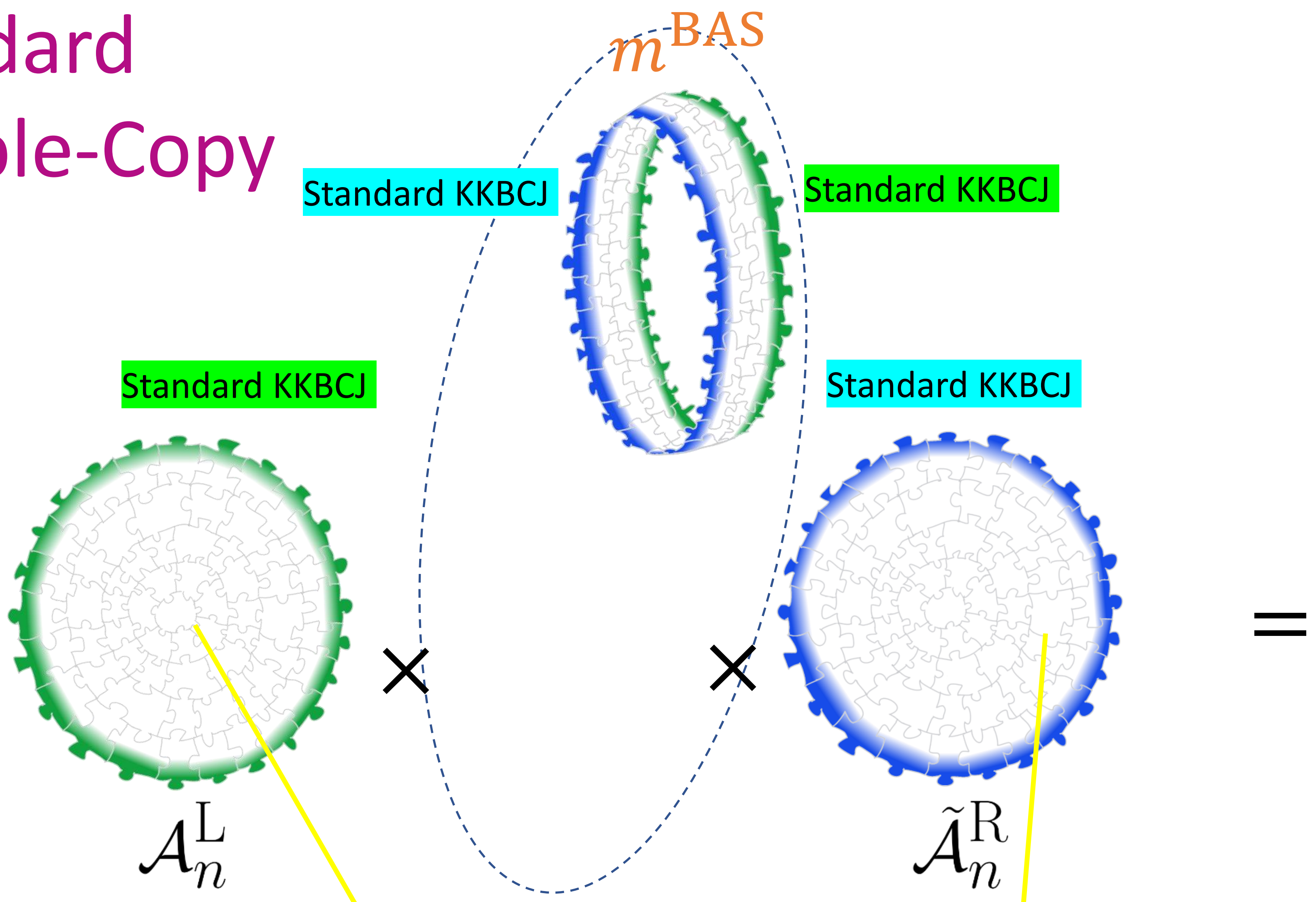

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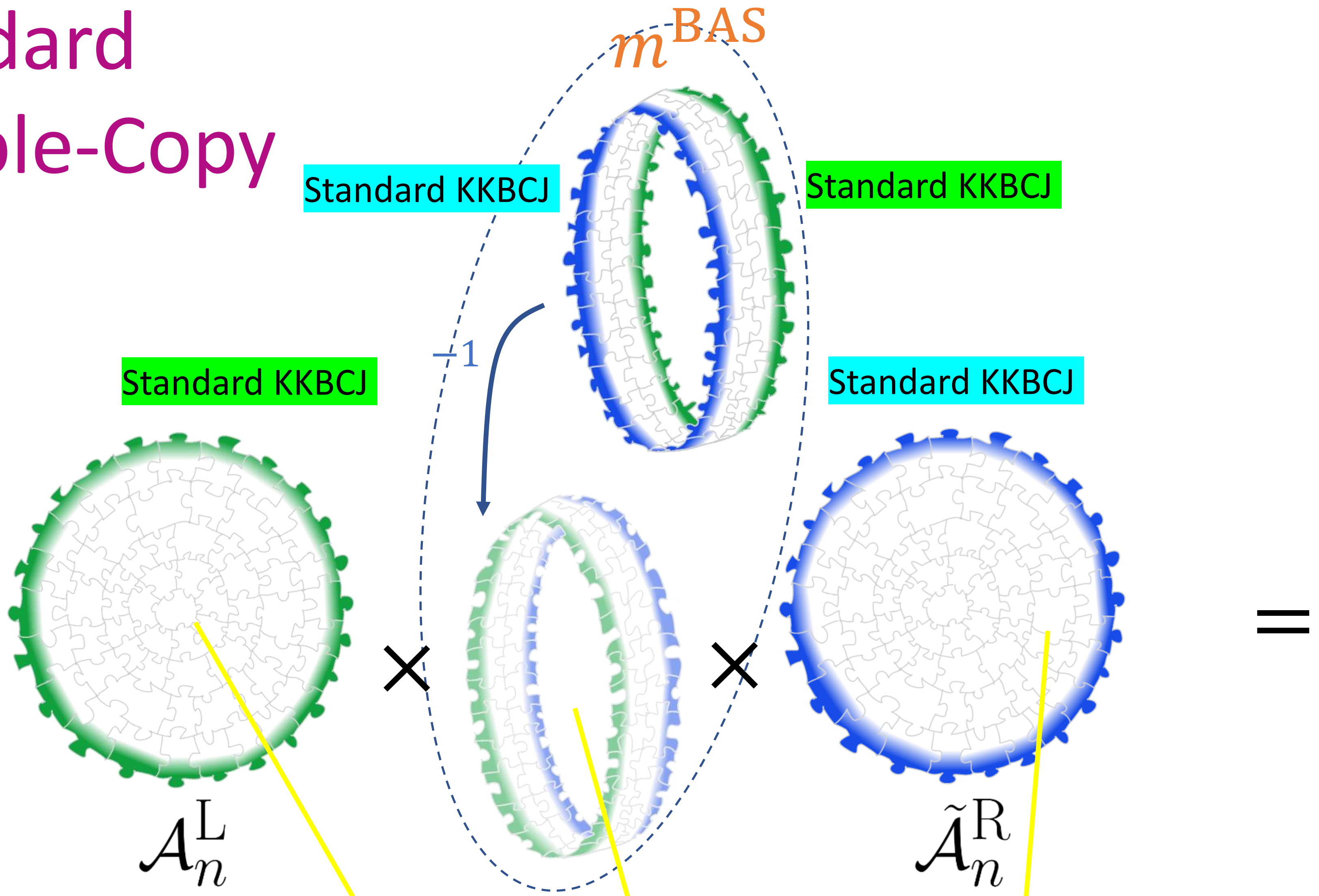

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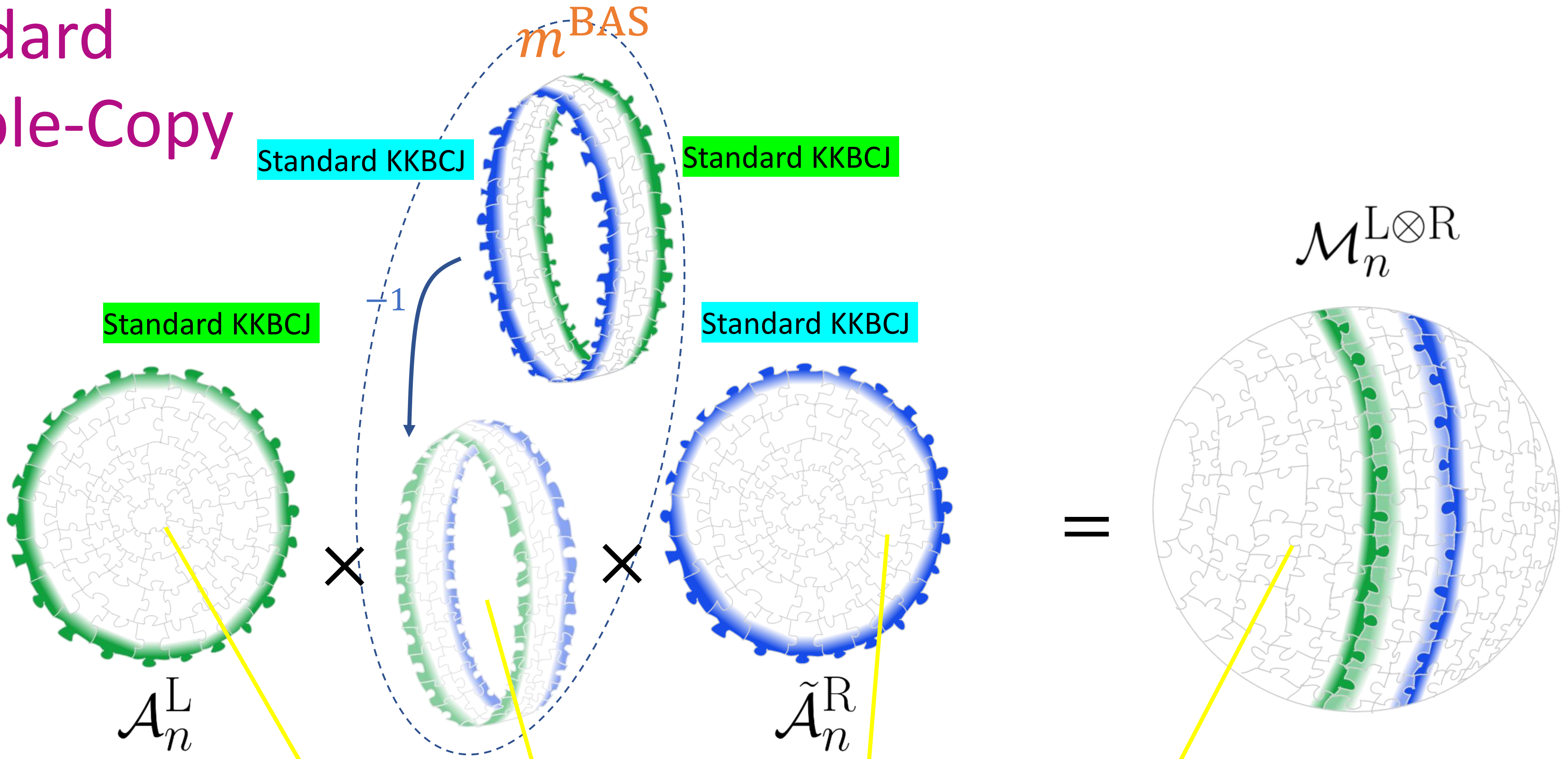
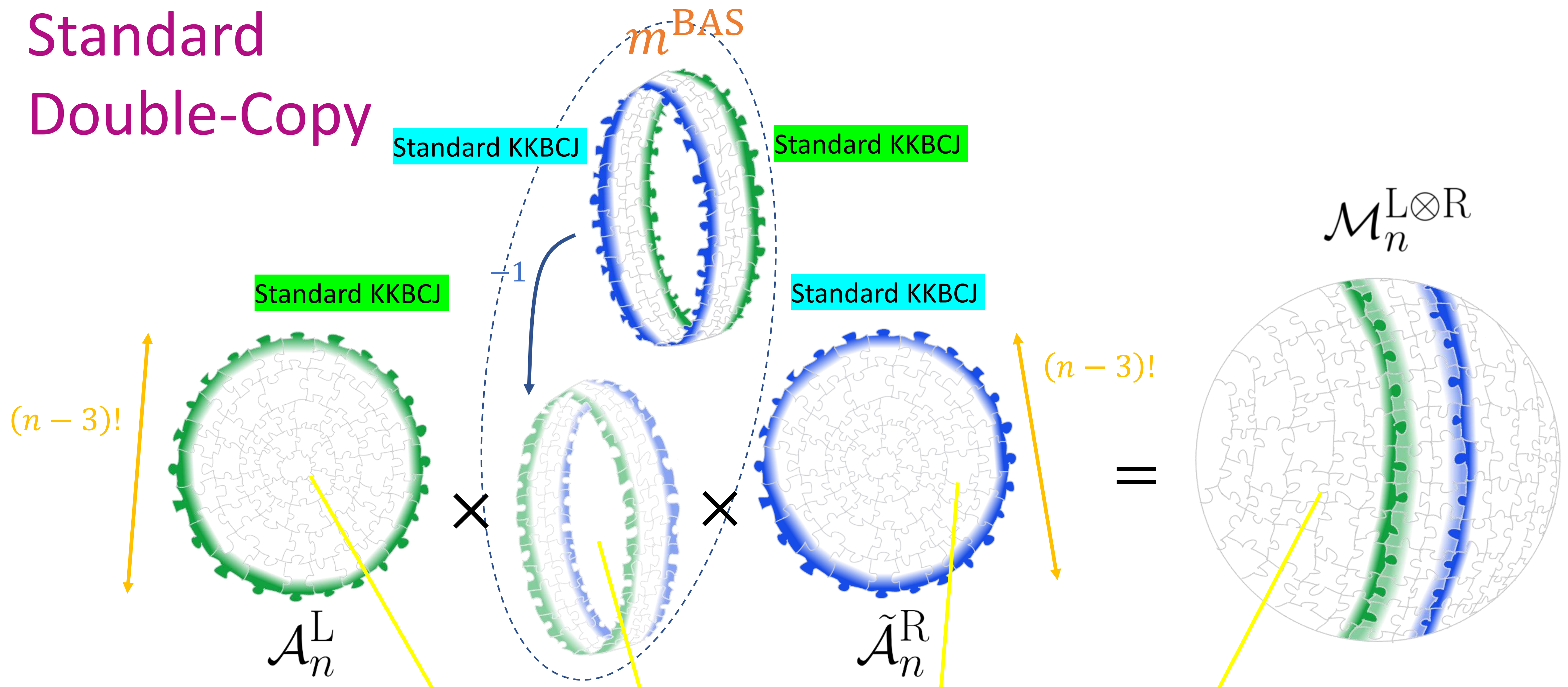


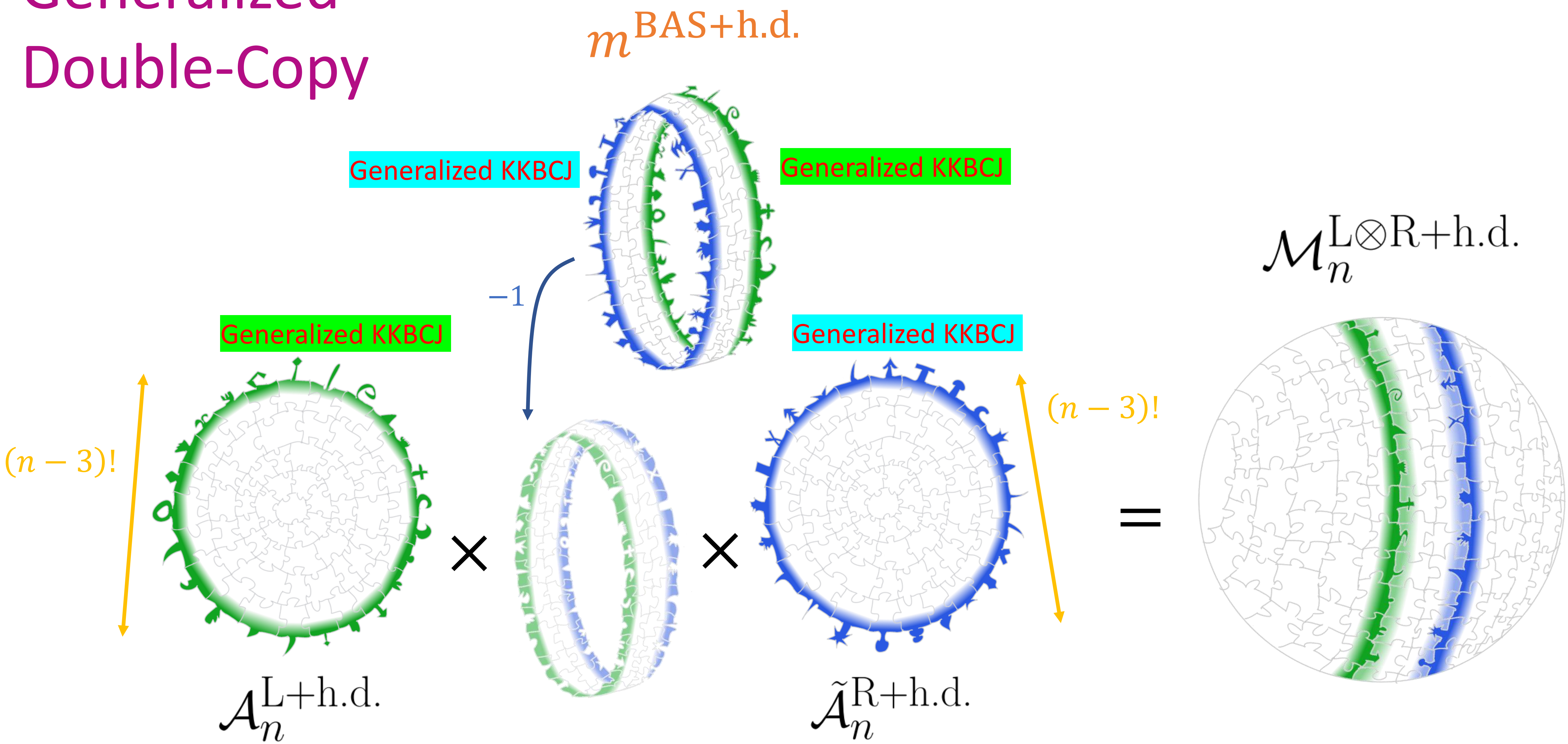
$$\sum_{(n-3)!\text{basis}} (\text{Gauge}) \cdot \left(m^{\text{BAS}}\right)^{-1} \cdot \left(\widetilde{\text{Gauge}}\right) = (\text{Gravity})$$

Standard Double-Copy

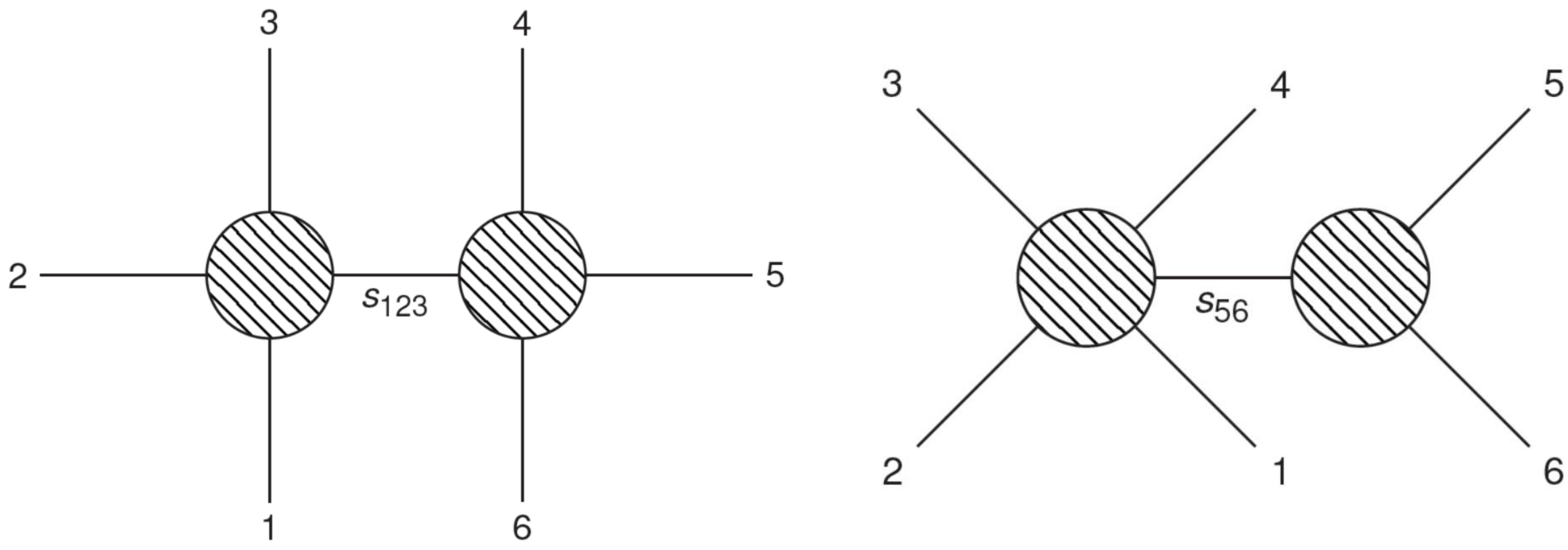


$$\sum_{(n-3)!\text{basis}} (\text{Gauge}) \cdot \left(m^{\mathrm{BAS}}\right)^{-1} \cdot \left(\widetilde{\text{Gauge}}\right) = (\text{Gravity})$$

Generalized Double-Copy



New Constraints from 6pts Factorization



- Higher order terms depend on the lower order terms in a <u>non-linear way</u>

Emergence of Stringy Kernels!

- 4pt

$$m[\cdot|\cdot] = m^{\text{stringy}}[\cdot|\cdot] \times U(s,t,u) + O(s^8)$$

$$U(s,t,u) = g^2 \text{Exp}\left[-U_1 \cdot \frac{s^3+t^3+u^3}{3} - U_2 \cdot \frac{s^5+t^5+u^5}{5} - U_3 \cdot \frac{s^7+t^7+u^7}{7}\right]$$

Emergence of Stringy Kernels!

- 4pt

$$m[\cdot|\cdot] = m^{\text{stringy}}[\cdot|\cdot] \times U(s,t,u) + O(s^8)$$

$$U(s,t,u) = g^2 \text{Exp}\left[-U_1 \cdot \frac{s^3+t^3+u^3}{3} - U_2 \cdot \frac{s^5+t^5+u^5}{5} - U_3 \cdot \frac{s^7+t^7+u^7}{7}\right]$$

- String monodromy seems to be the only possible UV-completion of the field theory KKBCJ

N-EXTENDED SUPERSYMMETRY AND POLYTOPIC REPRESENTATION THEORY

Aleksander Cianciara
Brown University

GOAL

Recursively generate N-extended supermultiplets given minimal representations of off-shell, N=1 supermultiplets. This corresponds to recursive construction of higher order permutahedra (permutation polytopes) from lower order permutahedra.

Watson and Crick (1956)

STRUCTURE OF SMALL VIRUSES

IT is a striking fact that almost all small viruses are either rods or spheres. The purpose of this article is to explain this observation by means of the following simple hypothesis: a small virus contains identical sub-units, packed together in a regular manner. It has been suggested before[1] that viruses are constructed from sub-units; but the idea has not previously been described in precise terms or put forward as a general feature of all small viruses.

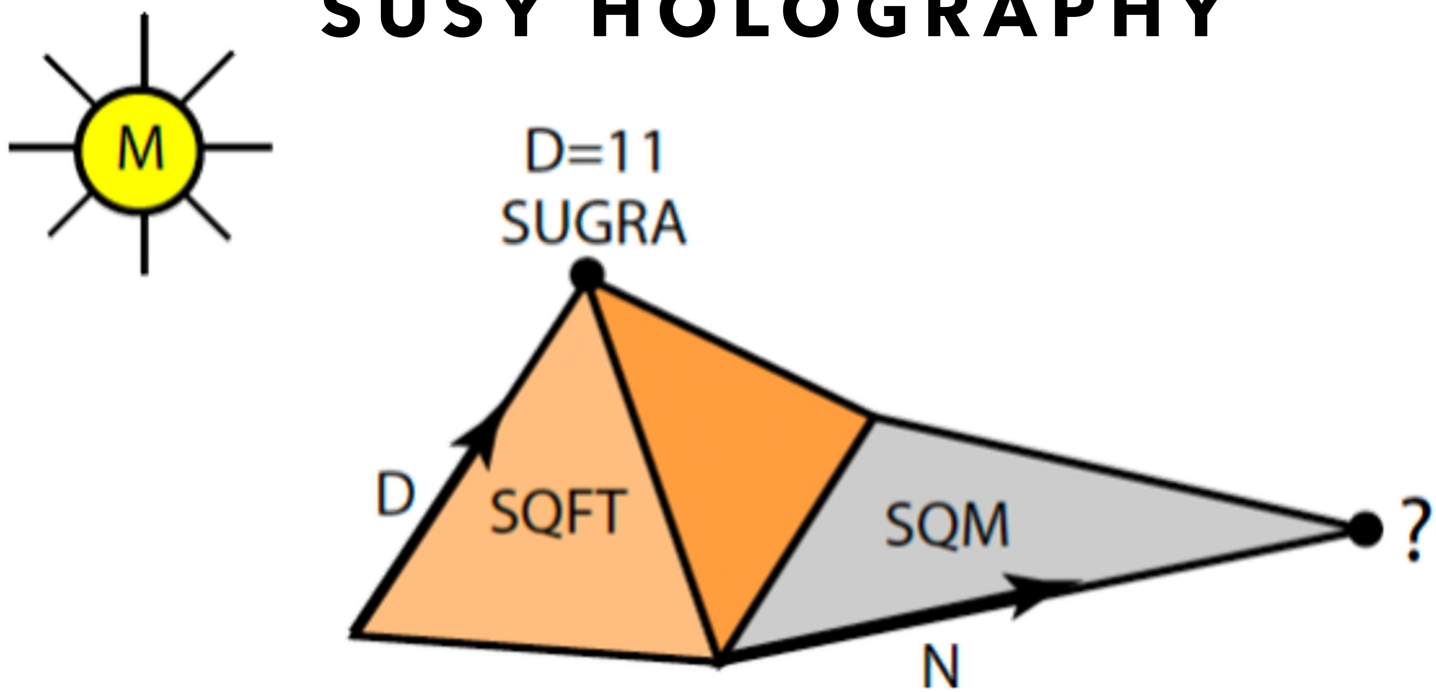
CONCEPTUAL FOUNDATIONS

- The N-extended supersymmetry algebra (Super-Poincare Algebra) without central charges in 1D is generated by the time-derivative ∂_τ and the N Hermitian operators (supercharges) $Q_1,\ldots,Q_N$ which satisfy the relations

$$\{ Q_I \, , \, Q_J \} = 2i \, \delta_{IJ} \, \partial_\tau, \quad [\partial_\tau \, , \, Q_I] = 0, \quad I, J = 1, \cdots , N.$$

- Through the process of dimensional reduction (SUSY holography), we can reduce a higher dimensional N=1 theory to a theory in 1D with higher N.

SUSY HOLOGRAPHY



CONCEPTUAL FOUNDATIONS

- These 1D theories are described by Garden Algebras GR(d, N), which are real Clifford algebras. Dimensionally reducing the 4D, N=1 Chiral multiplet to the 1D, N=4 chiral multiplet leads to the supersymmetric transformation equations

$$\begin{aligned} \mathrm{D_I}\,\Phi_i &= i\,(\mathrm{L_I})_i{}^{\hat{k}}\,\Psi_{\hat{k}}, \\ \mathrm{D_I}\,\Psi_{\hat{k}} &= (\mathrm{R_I})_{\hat{k}}{}^{i}\,\frac{\mathrm{d}}{\mathrm{d}t}\,\Phi_i. \end{aligned} \quad \Rightarrow \quad \begin{aligned} (\mathrm{L_I})_i{}^{\hat{j}}\,(\mathrm{R_J})_{\hat{j}}{}^{k} + (\mathrm{L_J})_i{}^{\hat{j}}\,(\mathrm{R_I})_{\hat{j}}{}^{k} &= 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^{k} \\ (\mathrm{R_J})_{\hat{i}}{}^{j}\,(\mathrm{L_I})_j{}^{\hat{k}} + (\mathrm{R_I})_{\hat{i}}{}^{j}\,(\mathrm{L_J})_j{}^{\hat{k}} &= 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{i}}{}^{\hat{k}} \end{aligned}$$

**Where $\mathbf{\Phi}_i$ are the bosonic fields, $\mathbf{\Psi}_k$ are the fermionic fields, and L_I and R_I encode the action of the supercharges among the component fields.

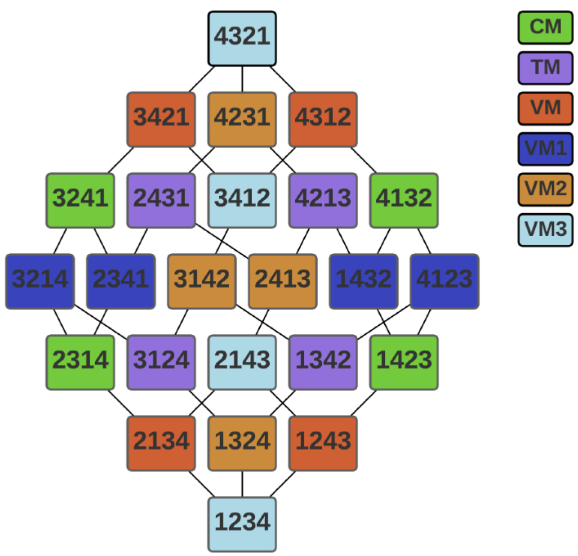
1D, N=4 SUPERMULTIPLETS

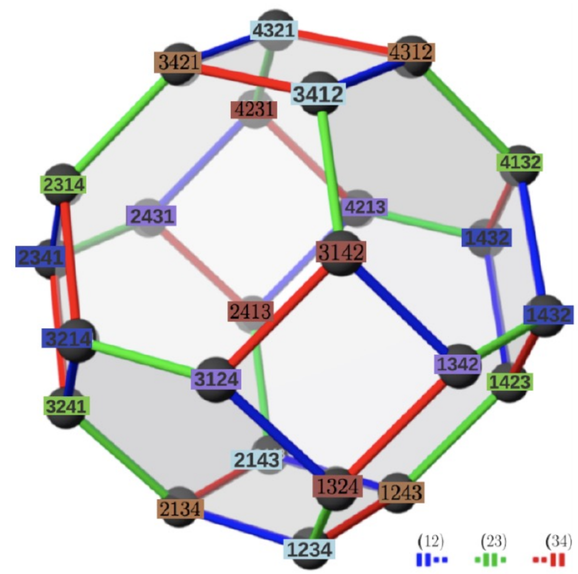
$$
\begin{array}{rl}
 & \quad\ \mathrm{L}_1 \qquad\ \ \mathrm{L}_2 \qquad\ \ \mathrm{L}_3 \qquad\ \ \mathrm{L}_4 \\
\{CM\} \equiv & \{\, \langle 1423 \rangle,\ \langle 2314 \rangle,\ \langle 3241 \rangle,\ \langle 4132 \rangle \,\} \\
\{VM\} \equiv & \{\, \langle 2413 \rangle,\ \langle 1324 \rangle,\ \langle 4231 \rangle,\ \langle 3142 \rangle \,\} \\
\{TM\} \equiv & \{\, \langle 1342 \rangle,\ \langle 2431 \rangle,\ \langle 3124 \rangle,\ \langle 4213 \rangle \,\} \\
\{VM_1\} \equiv & \{\, \langle 4123 \rangle,\ \langle 1432 \rangle,\ \langle 2341 \rangle,\ \langle 3214 \rangle \,\} \\
\{VM_2\} \equiv & \{\, \langle 3421 \rangle,\ \langle 4312 \rangle,\ \langle 2134 \rangle,\ \langle 1243 \rangle \,\} \\
\{VM_3\} \equiv & \{\, \langle 3412 \rangle,\ \langle 4321 \rangle,\ \langle 1234 \rangle,\ \langle 2143 \rangle \,\}
\end{array}
$$

E.g.

$$
\langle 2143 \rangle = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
$$

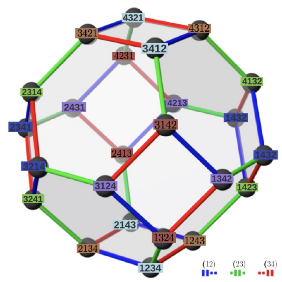
S_4 Permutahedron

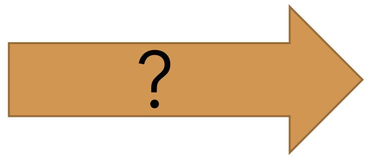


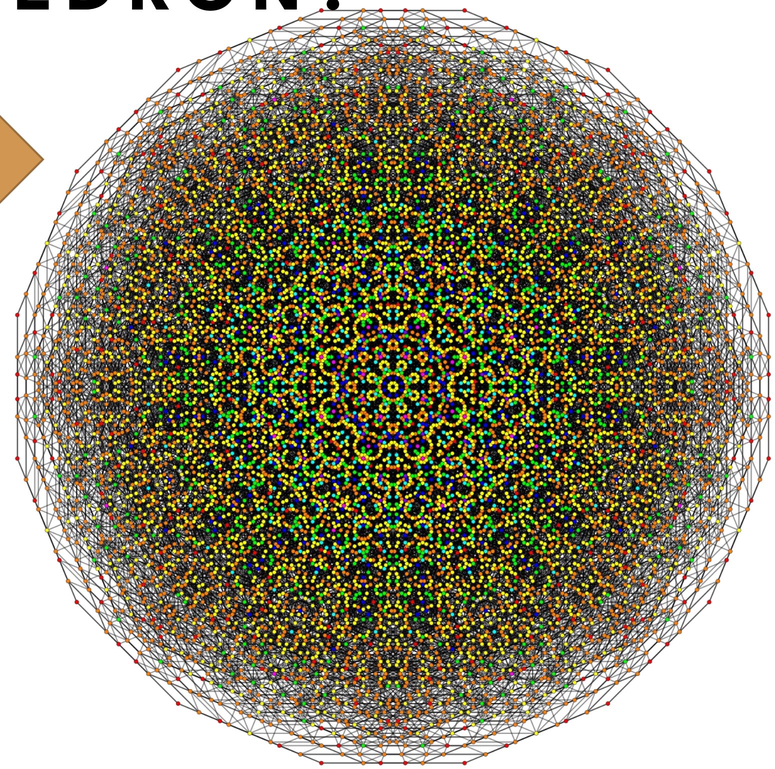


** 'Magic number' invariant exists that constrains locations of supercharges on permutahedron

EMBEDDING N=4 SUPERMULTIPLETS INTO N=8 PERMUTAHEDRON?





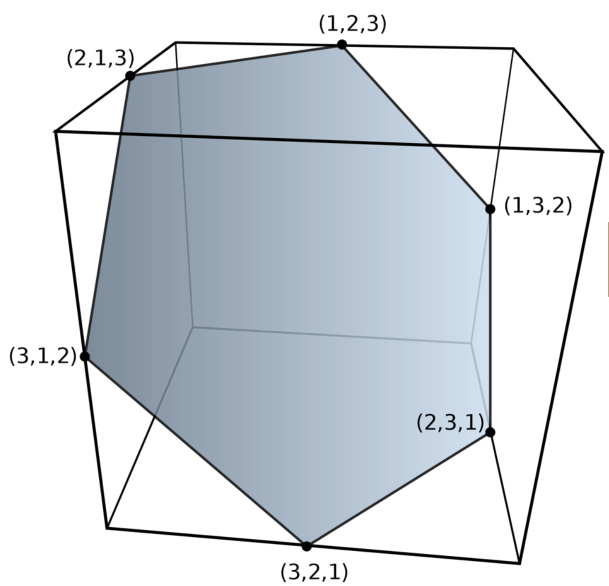


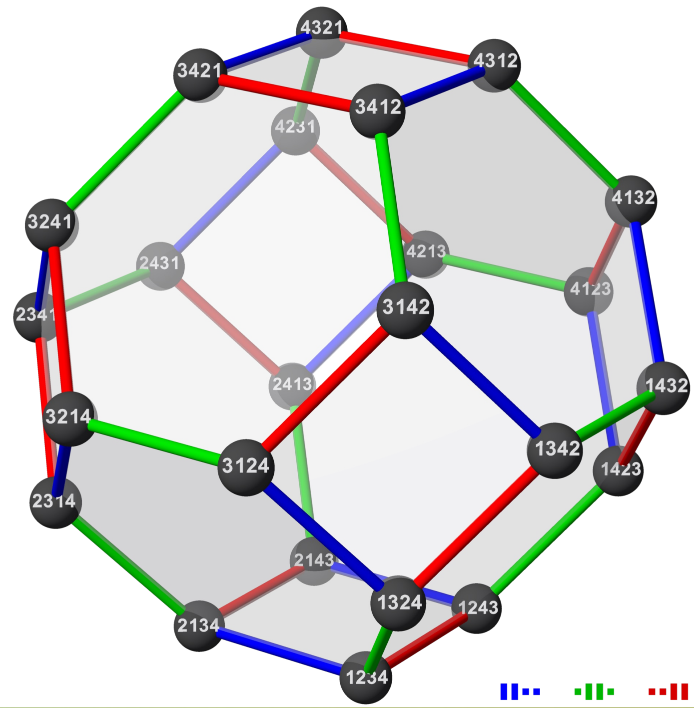
For N = 8, a convenient set for our required matrices is given by,

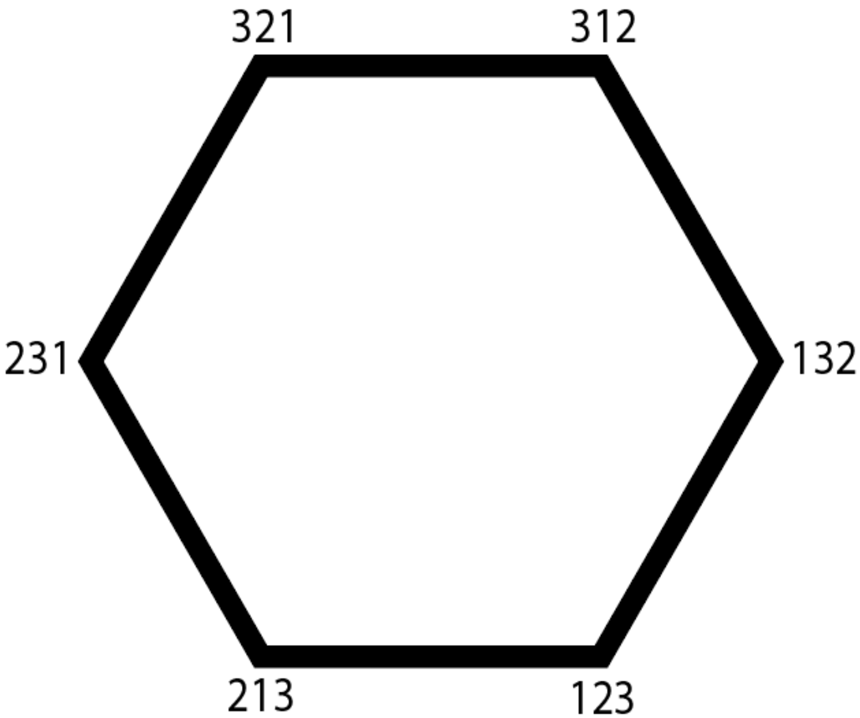
$$
\begin{array}{lllllllllllllllllll}
L_1 & = & i\mathrm{I} & \otimes & \sigma^3 & \otimes & \sigma^2 & = & R_1 & ; & L_5 & = & i\sigma^2 & \otimes & \mathrm{I} & \otimes & \sigma^1 & = & R_5 \ ; \\
L_2 & = & i\sigma^3 & \otimes & \sigma^2 & \otimes & \mathrm{I} & = & R_2 & ; & L_6 & = & i\sigma^2 & \otimes & \mathrm{I} & \otimes & \sigma^3 & = & R_6 \ ; \\
L_3 & = & i\mathrm{I} & \otimes & \sigma^1 & \otimes & \sigma^2 & = & R_3 & ; & L_7 & = & i\sigma^2 & \otimes & \sigma^2 & \otimes & \sigma^2 & = & R_7 \ ; \\
L_4 & = & i\sigma^1 & \otimes & \sigma^2 & \otimes & \mathrm{I} & = & R_4 & ; & L_8 & = & \mathrm{I} & \otimes & \mathrm{I} & \otimes & \mathrm{I} & = & -R_8 \ .
\end{array}
\tag{A.3}
$$

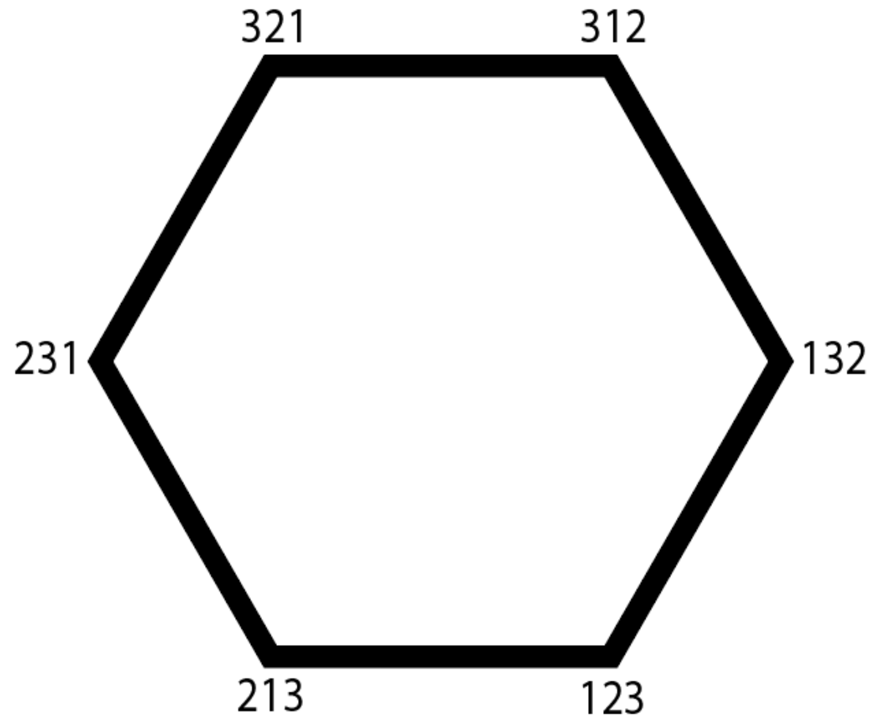
N = 5, 6, and 7 can be formed by taking any set of 5, 6 or 7 of the N = 8 matrices, respectively.

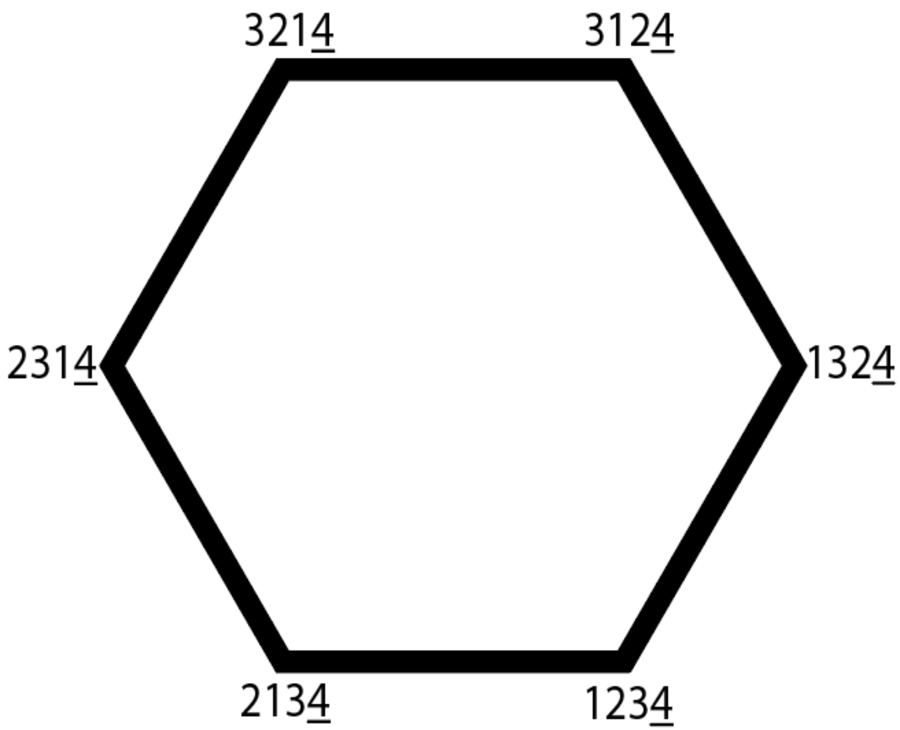
SIMPLER TO ASK IF N=3 PERMUTAHEDRA EMBEDS INTO N=4 PERMUTAHEDRA

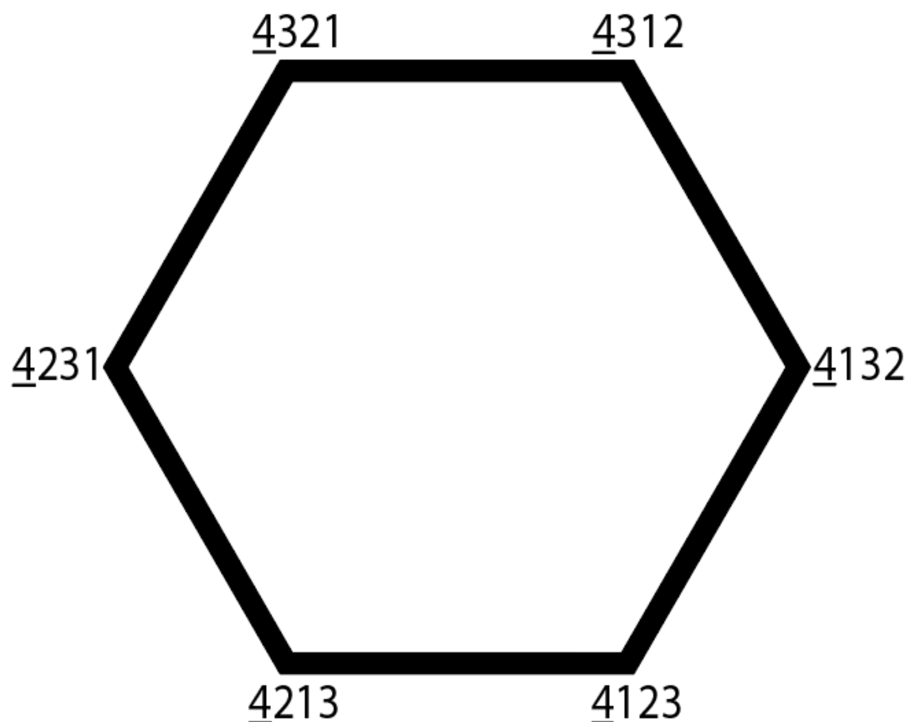


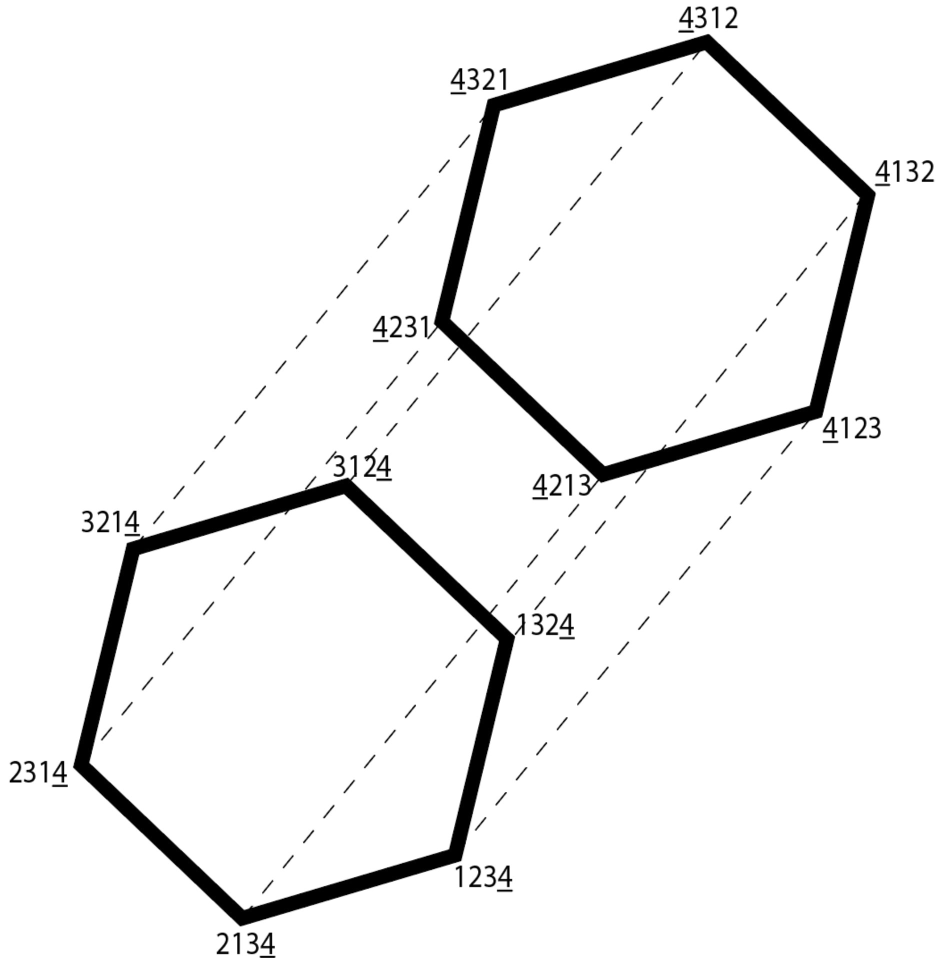



321
312
231
132
213
123


321
312
231
132
213
123


3214
3124
2314
1324
2134
1234


4321
4312
4231
4132
4213
4123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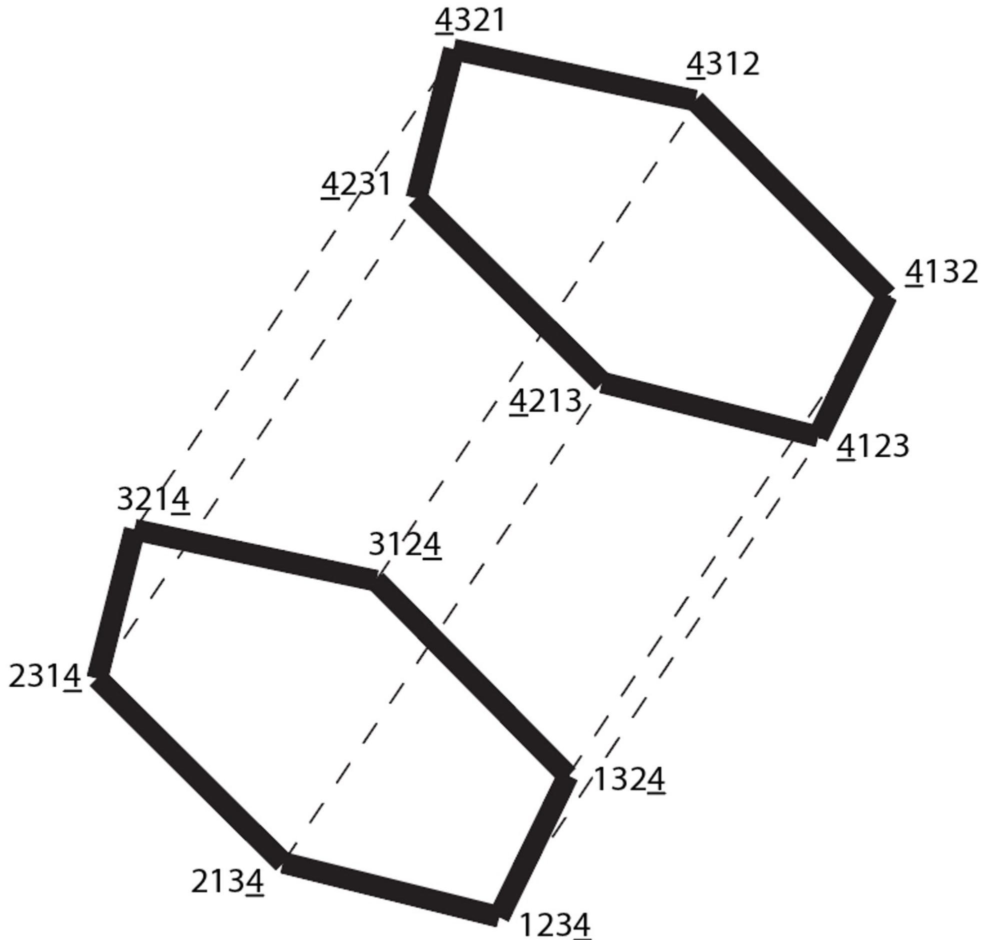

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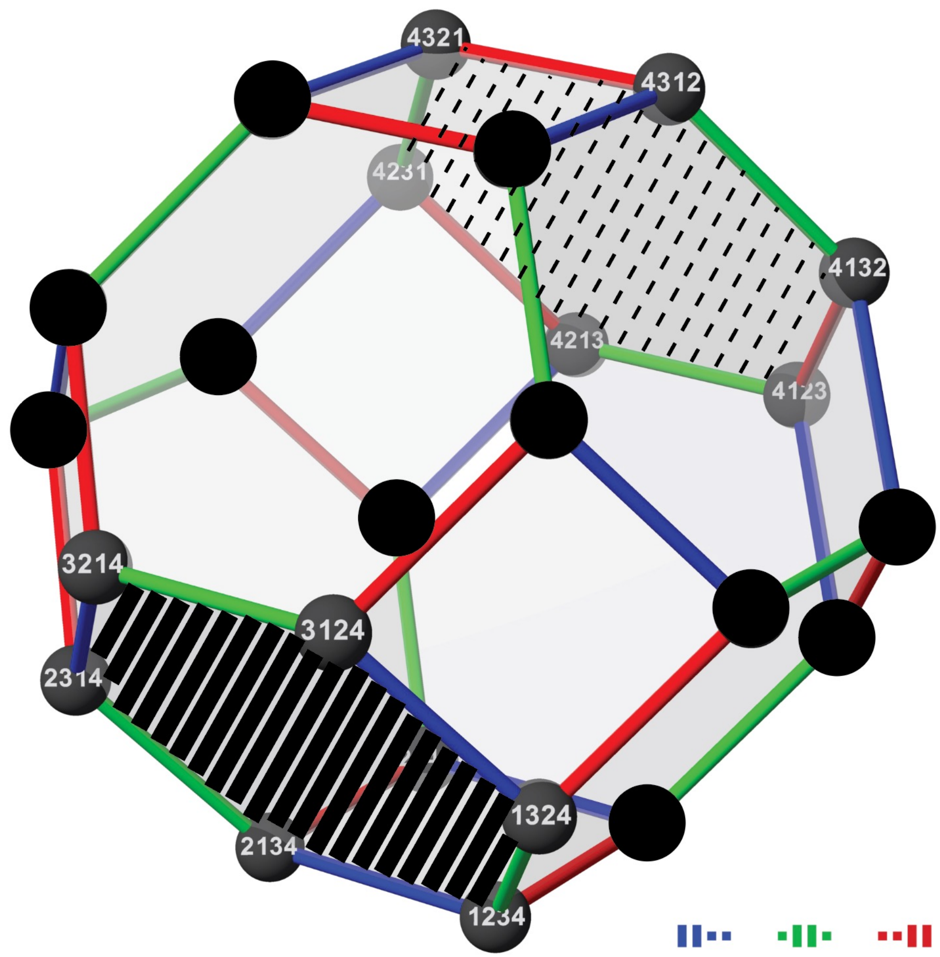

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3124
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1324
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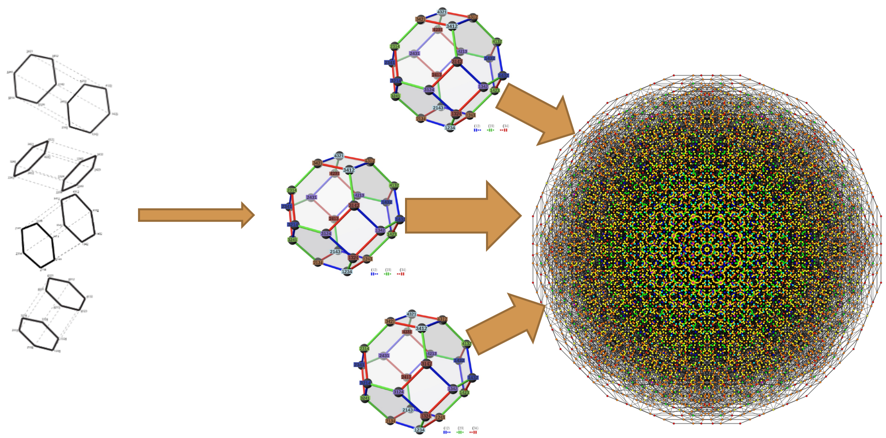
1234

POLYTOPIC REPRESENTATION THEORY

- Recursively construct N-extended supermultiplets (represented by higher dimensional permutahedra) using lower dimensional permutahedra as building blocks.



Types of 2-faces		
$\mathcal{N}$	{4}	{6}
3	0	1
4	6	8
5	90	60
6	1080	480
7	12600	4200
8	151200	40320

CITATIONS

- *The 300 "Correlators" Suggests 4D, N = 1 SUSY Is a Solution to a Set of Sudoku Puzzles, Cianciara et al.*
- *A Note on Exemplary off-shell Constructions of 4D, N=2 Supersymmetry, Bristow et al.*
- *Adinkra (In)Equivalence From Coxeter Group Representations: A Case Study by Chappell II et al.*
- *4D, N = 1 Supersymmetry Genomics (I), Gates Jr. et al.*
- *Adinkras: A Graphical Technology for Supersymmetric Representation Theory, Faux et al.*
- *One-dimensional σ-models with N = 5, 6, 7, 8 off-shell supersymmetries by Gonzales et al*

Scattering Amplitudes & Form Factors in Effective Field Theories

Stefano De Angelis

Queen Mary University of London

soon CEA Saclay

MY RESEARCH

I year	II year	III year	IV year
• form factors in (s)YM theories • d-dimensional gen. unitarity • 6d formalism			

MY RESEARCH

I year

- form factors in (s)YM theories
- d-dimensional gen. unitarity
- 6d formalism

II year

- binary problem in classical GR, from Scattering Amplitudes
- higher-derivative interactions:

$$\mathcal{L} = \mathcal{L}_{EH} + R^3 + R^4 + \ldots$$

III year

IV year

MY RESEARCH

I year

- form factors in (s)YM theories
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III year

- application to SMEFT
- classification of EFT interactions
- tree-level amplitudes from (non-BCFW-like) on-shell recursion relations
- one-loop anomalous dimension in the SMEFT

IV year

- BLACK HOLE INFORMATION PARADOX
- DOUBLE HOLOGRAPHY (AdS/CFT)

MY RESEARCH

I year

- form factors in (s)YM theories
- d-dimensional gen. unitarity
- 6d formalism

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- binary problem in classical GR, from Scattering Amplitudes
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III year

- application to SMEFT
- classification of EFT interactions
- tree-level amplitudes from (non-BCFW-like) on-shell recursion relations
- one-loop anomalous dimension in the SMEFT

IV year

- back to classical GR
- still broad interest in the SMEFT
- intership @ Wolfram Research
- writing the thesis

Double Field Theory as the Double Copy of Yang-Mills

- Background: BCJ double copy [Bern, Carrasco and Johansson 2008]

$$\text{Colour} \rightarrow \text{Kinematics}$$
$$(\text{Yang-Mills (YM)} \rightarrow \text{Gravity})$$

- What?: Extension of BCJ to off-shell, gauge invariant action (including non-linear gauge transformations).
- Why?: First principle understanding/constructive proof outside of amplitude realm.

Double Field Theory as the Double Copy of Yang-Mills

- How?: L_∞-algebras. For Yang-Mills [Zeitlin 2008]

$$X_{YM} = \mathfrak{g}_{\text{colour}} \otimes \mathcal{K}$$

Following BCJ philosophy $\quad \mathfrak{g}_{\text{colour}} \to \bar{\mathcal{K}} \quad$ we obtain Double Field Theory (DFT)

$$X_{DFT} = \bar{\mathcal{K}} \otimes \mathcal{K}$$

We have successfully constructed DFT from YM up to cubic order, including non-linear gauge structure!

Double Field Theory as the Double Copy of Yang-Mills

- How?: L_∞-algebras. For Yang-Mills [Zeitlin 2008]

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We have successfully constructed DFT from YM up to cubic order, including non-linear gauge structure!

That's all. Thank you very much for your attention!

Properties of the Conformal Yangian in Scalar and Gauge Field Theories

[arXiv : 2207.14806]

Nikos Dokmetzoglou

University of North Carolina at Chapel Hill

Amplitudes 2022 Summer School

August 3rd, 2022

SO(2,n)
Yangian

Conformal Yangian
Algebra and Generators

SO(2,n)
Yangian

$$\left[J^{AB}, J^{CD}\right] = f^{ABCD}{}_{EF} J^{EF}, \qquad \left[J^{AB}, \widehat{J}^{CD}\right] = f^{ABCD}{}_{EF} \widehat{J}^{EF}$$

$$\left[\widehat{J}^{AB}, \left[\widehat{J}^{CD}, J^{EF}\right]\right] + \left[\widehat{J}^{CD}, \left[\widehat{J}^{EF}, J^{AB}\right]\right] + \left[\widehat{J}^{EF}, \left[\widehat{J}^{AB}, J^{CD}\right]\right]$$
$$= \frac{1}{24} f^{AB}{}_{GHMN} f^{CD}{}_{IJOP} f^{EF}{}_{KLQR} f^{MNOPQR} \left\{J^{GH}, J^{IJ}, J^{KL}\right\}$$

$$J^{AB} = \sum_{i=1}^{N} J_i^{AB}, \qquad \widehat{J}^{AB} = \frac{1}{2} f^{AB}{}_{CDEF} \sum_{1 \le i < j \le N} J_i^{CD} J_j^{EF}$$

Conformal Yangian
Algebra and Generators

SO(2,n)
Yangian

Conformal Yangian Algebra and Generators

$$[J^{AB}, J^{CD}] = -\eta^{AC} J^{BD} - \eta^{BD} J^{AC} + \eta^{AD} J^{BC} + \eta^{BC} J^{AD}$$

$$J^{AB} = -J^{BA}, \quad 0 \leq A, B \leq n+1,$$
$$\eta^{AB} = \text{diagonal}\,(1, -1, -1, -1, \ldots, -1, 1)$$

SO(2,n) Yangian

$$\left[J^{AB}, J^{CD}\right] = f^{ABCD}{}_{EF} J^{EF}, \qquad \left[J^{AB}, \widehat{J}^{CD}\right] = f^{ABCD}{}_{EF} \widehat{J}^{EF}$$

$$\left[\widehat{J}^{AB}, \left[\widehat{J}^{CD}, J^{EF}\right]\right] + \left[\widehat{J}^{CD}, \left[\widehat{J}^{EF}, J^{AB}\right]\right] + \left[\widehat{J}^{EF}, \left[\widehat{J}^{AB}, J^{CD}\right]\right]$$
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$$\left[J^{AB},J^{CD}\right]=f^{ABCD}{}_{EF}J^{EF},\qquad \left[J^{AB},\widehat{J}^{CD}\right]=f^{ABCD}{}_{EF}\widehat{J}^{EF}$$

$$\left[\widehat{J}^{AB},\left[\widehat{J}^{CD},J^{EF}\right]\right]+\left[\widehat{J}^{CD},\left[\widehat{J}^{EF},J^{AB}\right]\right]+\left[\widehat{J}^{EF},\left[\widehat{J}^{AB},J^{CD}\right]\right]$$
$$=\frac{1}{24}\,f^{AB}{}_{GHMN}f^{CD}{}_{IJOP}f^{EF}{}_{KLQR}f^{MNOPQR}\left\{J^{GH},J^{IJ},J^{KL}\right\}$$

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$$[J^{AB},J^{CD}]=-\eta^{AC}J^{BD}-\eta^{BD}J^{AC}+\eta^{AD}J^{BC}+\eta^{BC}J^{AD}$$

$$J^{AB}=-J^{BA},\quad 0\le A,B\le n+1,$$
$$\eta^{AB}=\text{diagonal}\,(1,-1,-1,-1,\dots,-1,1)$$

Conformal Yangian Algebra and Generators

SO(2,n)
Yangian

$$\left[J^{AB}, J^{CD}\right] = f^{ABCD}{}_{EF} J^{EF}, \qquad \left[J^{AB}, \widehat{J}^{CD}\right] = f^{ABCD}{}_{EF} \widehat{J}^{EF}$$

$$\left[\widehat{J}^{AB}, \left[\widehat{J}^{CD}, J^{EF}\right]\right] + \left[\widehat{J}^{CD}, \left[\widehat{J}^{EF}, J^{AB}\right]\right] + \left[\widehat{J}^{EF}, \left[\widehat{J}^{AB}, J^{CD}\right]\right] = \frac{1}{24} f^{AB}{}_{GHMN} f^{CD}{}_{IJOP} f^{EF}{}_{KLQR} f^{MNOPQR} \left\{J^{GH}, J^{IJ}, J^{KL}\right\}$$

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Differential Operator Representation in Momentum-Space:

$$P_i^\mu = k_i^\mu, \quad L_i^{\mu\nu} = k_i^\mu \partial_i^\nu - k_i^\nu \partial_i^\mu + \Sigma_i^{\mu\nu}, \quad D_i = d + k_i^\nu \partial_{i\nu},$$
$$K_i^\mu = 2\, d\, \partial_i^\mu + 2\, k_i^\nu \partial_{i\nu} \partial_i^\mu - k_i^\mu \partial_i^\nu \partial_{i\nu} - 2\, \Sigma_i^{\mu\nu} \partial_{i\nu}$$

$$J_i^{\mu\nu} = L_i^{\mu\nu}, \quad J_i^{n\mu} = \frac{1}{2}(P_i^\mu - K_i^\mu),$$
$$J_i^{n+1,\mu} = \frac{1}{2}(P_i^\mu + K_i^\mu), \quad J_i^{n,n+1} = D_i$$

Conformal Yangian Algebra and Generators

SO(2,n) Yangian

$$\left[J^{AB},J^{CD}\right]=f^{ABCD}{}_{EF}J^{EF},\qquad \left[J^{AB},\widehat{J}^{CD}\right]=f^{ABCD}{}_{EF}\widehat{J}^{EF}$$

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$$=\frac{1}{24}\,f^{AB}{}_{GHMN}f^{CD}{}_{IJOP}f^{EF}{}_{KLQR}f^{MNOPQR}\left\{J^{GH},J^{IJ},J^{KL}\right\}$$

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$$K_i^\mu=2\,d\,\partial_i^\mu+2\,k_i^\nu\partial_{i\nu}\partial_i^\mu-k_i^\mu\partial_i^\nu\partial_{i\nu}-2\,\Sigma_i^{\mu\nu}\partial_{i\nu}$$

$$J_i^{\mu\nu}=L_i^{\mu\nu},\quad J_i^{n\mu}=\frac{1}{2}(P_i^\mu-K_i^\mu),$$
$$J_i^{n+1,\mu}=\frac{1}{2}(P_i^\mu+K_i^\mu),\quad J_i^{n,n+1}=D_i$$

Conformal Yangian Algebra and Generators

SO(2,n) Yangian

Proof of the Serre relation

- For N =1, $J^{AB}=J_1^{AB}$, $\widehat{J}^{AB}=0$
- Single site Serre condition:

$$0=f^{AB}{}_{GHMN}f^{CD}{}_{IJOP}f^{EF}{}_{KLQR}f^{MNOPQR}\left\{J_1^{GH},J_1^{IJ},J_1^{KL}\right\}$$

- **Scalar** field representation: consistent **off-shell**, for **arbitrary** space-time dimensions **n** and conformal spin **d**
- **Gauge** field representation: consistent only **on-shell**, for **n = 4** and **d = 1**

$$\left[J^{AB}, J^{CD}\right] = f^{ABCD}{}_{EF} J^{EF}, \qquad \left[J^{AB}, \widehat{J}^{CD}\right] = f^{ABCD}{}_{EF} \widehat{J}^{EF}$$

$$\left[\widehat{J}^{AB}, \left[\widehat{J}^{CD}, J^{EF}\right]\right] + \left[\widehat{J}^{CD}, \left[\widehat{J}^{EF}, J^{AB}\right]\right] + \left[\widehat{J}^{EF}, \left[\widehat{J}^{AB}, J^{CD}\right]\right] = \frac{1}{24} f^{AB}{}_{GHMN} f^{CD}{}_{IJOP} f^{EF}{}_{KLQR} f^{MNOPQR} \{J^{GH}, J^{IJ}, J^{KL}\}$$

$$J^{AB} = \sum_{i=1}^{N} J_i^{AB}, \qquad \widehat{J}^{AB} = \frac{1}{2} f^{AB}{}_{CDEF} \sum_{1 \leq i < j \leq N} J_i^{CD} J_j^{EF}$$

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$$P_i^\mu = k_i^\mu, \quad L_i^{\mu\nu} = k_i^\mu \partial_i^\nu - k_i^\nu \partial_i^\mu + \Sigma_i^{\mu\nu}, \quad D_i = d + k_i^\nu \partial_{i\nu},$$

$$K_i^\mu = 2\, d\, \partial_i^\mu + 2\, k_i^\nu \partial_{i\nu} \partial_i^\mu - k_i^\mu \partial_i^\nu \partial_{i\nu} - 2\, \Sigma_i^{\mu\nu} \partial_{i\nu}$$

$$J_i^{\mu\nu} = L_i^{\mu\nu}, \quad J_i^{n\mu} = \frac{1}{2}(P_i^\mu - K_i^\mu),$$

$$J_i^{n+1,\mu} = \frac{1}{2}(P_i^\mu + K_i^\mu), \quad J_i^{n,n+1} = D_i$$

Level-0 Generators : $P^\mu,\ L^{\mu\nu},\ D,\ K^\mu$

Level-1 Generators : $\widehat{P}^\mu,\ \widehat{L}^{\mu\nu},\ \widehat{D},\ \widehat{K}^\mu$

Conformal Yangian Algebra and Generators

<u>Proof of the Serre relation</u>

- For N =1, $J^{AB} = J_1^{AB}, \quad \widehat{J}^{AB} = 0$
- Single site Serre condition:

$$0 = f^{AB}{}_{GHMN} f^{CD}{}_{IJOP} f^{EF}{}_{KLQR} f^{MNOPQR} \{J_1^{GH}, J_1^{IJ}, J_1^{KL}\}$$

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- **Gauge** field representation: consistent only **on-shell**, for **n = 4** and **d = 1**

Scalar $\lambda\, \phi^3$ Theory Off-Shell Tree Graphs

SO(2,n) Yangian

Pure Yang-Mills Theory On-Shell Tree Amplitudes

$$\left[J^{AB}, J^{CD}\right] = f^{ABCD}{}_{EF} J^{EF}, \qquad \left[J^{AB}, \widehat{J}^{CD}\right] = f^{ABCD}{}_{EF} \widehat{J}^{EF}$$

$$\left[\widehat{J}^{AB}, \left[\widehat{J}^{CD}, J^{EF}\right]\right] + \left[\widehat{J}^{CD}, \left[\widehat{J}^{EF}, J^{AB}\right]\right] + \left[\widehat{J}^{EF}, \left[\widehat{J}^{AB}, J^{CD}\right]\right] = \frac{1}{24} f^{AB}{}_{GHMN} f^{CD}{}_{IJOP} f^{EF}{}_{KLQR} f^{MNOPQR} \left\{J^{GH}, J^{IJ}, J^{KL}\right\}$$

$$J^{AB} = \sum_{i=1}^{N} J_i^{AB}, \qquad \widehat{J}^{AB} = \frac{1}{2} f^{AB}{}_{CDEF} \sum_{1 \le i < j \le N} J_i^{CD} J_j^{EF}$$

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$$P_i^\mu = k_i^\mu, \quad L_i^{\mu\nu} = k_i^\mu \partial_i^\nu - k_i^\nu \partial_i^\mu + \Sigma_i^{\mu\nu}, \quad D_i = d + k_i^\nu \partial_{i\nu},$$
$$K_i^\mu = 2\, d\, \partial_i^\mu + 2\, k_i^\nu \partial_{i\nu} \partial_i^\mu - k_i^\mu \partial_i^\nu \partial_{i\nu} - 2\, \Sigma_i^{\mu\nu} \partial_{i\nu}$$

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Scalar $\lambda\, \phi^3$ Theory Off-Shell Tree Graphs

SO(2,n) Yangian

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$$\widehat{P}^\mu\ k_{[I,J]}^2 = \left[\left(2d \sum_{j=1}^{N} j k_j^\mu\right) - 2\left(k_{[1,I-1]}^\mu - k_{[J+1,N]}^\mu\right)\right] k_{[I,J]}^2$$

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$$\widehat{P}^\mu \ \ \delta^n(k_1 + \ldots + k_N)\ A_N^\Delta(k_1, \ldots, k_N)$$
$$= \delta^n(k_1 + \ldots + k_N)$$
$$\cdot \left[\left(2d \sum_{j=1}^{N} j\, k_j^\mu \right) + 2 \sum_{\substack{[I,J] \\ k^2_{[I,J]} \in \Delta}} \left(k^\mu_{[1,I-1]} - k^\mu_{[J+1,N]} \right) \right] A_N^\Delta(k_1, \ldots, k_N)$$

Nikos Dokmetzoglou [https://tarheels.live/nikosdokmetzoglou/] UNC Chapel Hill

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$$\widehat{P}'^\mu \equiv \widehat{P}^\mu + \sum_{j=1}^{N} c^\Delta_{N,j} P_j^\mu, \quad \widehat{P}'^\mu \ \delta^n(k_1 + \ldots + k_N)\, A_N^\Delta(k_1, \ldots, k_N) = 0$$

$$c^\Delta_{N,j} = -2d\, j - 2 \sum_{\substack{[I,J] \\ k^2_{[I,J]} \in \Delta}} \begin{cases} 1 \text{ if } j \in [1, I-1] \\ -1 \text{ if } j \in [J+1, N] \end{cases}$$

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$$\widehat{J}'^{AB}\ \delta^n(k_1+\dots+k_N)\ A_N^\Delta(k_1,\dots,k_N) = 0, \quad \text{for } n=6 \text{ and } d=2$$

since $\widehat{P}'^\mu\ \delta^n(k_1+\dots+k_N)\ A_N^\Delta(k_1,\dots,k_N) = 0$ and

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For n = 4 and d = 1, on-shell:

$$\widehat{P}^{\mu\alpha_1\alpha_2\alpha_3}{}_{\gamma_1\gamma_2\gamma_3} \ A^{a\gamma_1,\, b\gamma_2,\, c\gamma_3}$$
$$= g\, f_{abc}\, (2\pi)^4 \delta^4(k_1 + k_2 + k_3)$$
$$\cdot \Big(4k_1^\mu \left(\eta^{\alpha_2\alpha_3} k_2^{\alpha_1} + \eta^{\alpha_3\alpha_1} k_3^{\alpha_2} + 2\eta^{\alpha_1\alpha_2} k_1^{\alpha_3}\right) - 4k_3^\mu \left(2\eta^{\alpha_2\alpha_3} k_2^{\alpha_1} + \eta^{\alpha_3\alpha_1} k_3^{\alpha_2} + \eta^{\alpha_1\alpha_2} k_1^{\alpha_3}\right)$$
$$+ \eta^{\mu\alpha_1} \left(-4k_1^{\alpha_2} k_1^{\alpha_3}\right) + \eta^{\mu\alpha_2} \left(-4k_2^{\alpha_1} k_2^{\alpha_3}\right) + \eta^{\mu\alpha_3} \left(-4k_3^{\alpha_1} k_3^{\alpha_2}\right) \Big)$$
$$= g\, f_{abc}\, (2\pi)^4 \delta^4(k_1 + k_2 + k_3) \big(tr(\gamma^{\alpha_2}\gamma^\zeta\gamma^{\alpha_3}\gamma^\omega\gamma^{\alpha_1}\gamma^\mu) k_{1\omega} k_{2\zeta}$$
$$- tr(\gamma^{\alpha_3}\gamma^\zeta\gamma^{\alpha_2}\gamma^\omega\gamma^{\alpha_1}\gamma^\mu) k_{1\omega} k_{3\zeta} + tr(\gamma^{\alpha_3}\gamma^\zeta\gamma^{\alpha_1}\gamma^\omega\gamma^{\alpha_2}\gamma^\mu) k_{2\omega} k_{3\zeta} \big)$$

Similarly for the 4-point gluon partial amplitude

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$$\widehat{J}'^{AB} \ \delta^n(k_1 + \ldots + k_N)\, A_N^\Delta(k_1, \ldots, k_N) = 0, \quad \text{for } n = 6 \text{ and } d = 2$$

since $\widehat{P}'^{\mu} \ \delta^n(k_1 + \ldots + k_N)\, A_N^\Delta(k_1, \ldots, k_N) = 0$ and

$$\widehat{J}^{AB} \ \delta^n(k_1 + \ldots + k_N)\, A_N^\Delta(k_1, \ldots, k_N) = 0, \quad \text{for } n = 6 \text{ and } d = 2$$

CHY Scattering Equations Formalism

For n = 4 and d = 1, on-shell:

$$\widehat{P}^{\mu\alpha_1\alpha_2\alpha_3}_{\gamma_1\gamma_2\gamma_3} \ A^{a\gamma_1,\ b\gamma_2,\ c\gamma_3}$$
$$= g\, f_{abc}\, (2\pi)^4 \delta^4(k_1 + k_2 + k_3)$$
$$\cdot \Big(4k_1^\mu \left(\eta^{\alpha_2\alpha_3} k_2^{\alpha_1} + \eta^{\alpha_3\alpha_1} k_3^{\alpha_2} + 2\eta^{\alpha_1\alpha_2} k_1^{\alpha_3}\right) - 4k_3^\mu \left(2\eta^{\alpha_2\alpha_3} k_2^{\alpha_1} + \eta^{\alpha_3\alpha_1} k_3^{\alpha_2} + \eta^{\alpha_1\alpha_2} k_1^{\alpha_3}\right)$$
$$+ \eta^{\mu\alpha_1} \left(-4k_1^{\alpha_2} k_1^{\alpha_3}\right) + \eta^{\mu\alpha_2} \left(-4k_2^{\alpha_1} k_2^{\alpha_3}\right) + \eta^{\mu\alpha_3} \left(-4k_3^{\alpha_1} k_3^{\alpha_2}\right) \Big)$$
$$= g\, f_{abc}\, (2\pi)^4 \delta^4(k_1 + k_2 + k_3)\, \big(tr(\gamma^{\alpha_2}\gamma^\zeta\gamma^{\alpha_3}\gamma^\omega\gamma^{\alpha_1}\gamma^\mu) k_{1\omega} k_{2\zeta}$$
$$- tr(\gamma^{\alpha_3}\gamma^\zeta\gamma^{\alpha_2}\gamma^\omega\gamma^{\alpha_1}\gamma^\mu) k_{1\omega} k_{3\zeta} + tr(\gamma^{\alpha_3}\gamma^\zeta\gamma^{\alpha_1}\gamma^\omega\gamma^{\alpha_2}\gamma^\mu) k_{2\omega} k_{3\zeta}\big)$$

Similarly for the 4-point gluon partial amplitude

Nikos Dokmetzoglou [https://tarheels.live/nikosdokmetzoglou/]
UNC Chapel Hill

$$\left[J^{AB}, J^{CD}\right] = f^{ABCD}{}_{EF} J^{EF}, \qquad \left[J^{AB}, \widehat{J}^{CD}\right] = f^{ABCD}{}_{EF} \widehat{J}^{EF}$$

$$\left[\widehat{J}^{AB}, \left[\widehat{J}^{CD}, J^{EF}\right]\right] + \left[\widehat{J}^{CD}, \left[\widehat{J}^{EF}, J^{AB}\right]\right] + \left[\widehat{J}^{EF}, \left[\widehat{J}^{AB}, J^{CD}\right]\right] = \frac{1}{24} f^{AB}{}_{GHMN} f^{CD}{}_{IJOP} f^{EF}{}_{KLQR} f^{MNOPQR} \left\{J^{GH}, J^{IJ}, J^{KL}\right\}$$

$$J^{AB} = \sum_{i=1}^{N} J_i^{AB}, \qquad \widehat{J}^{AB} = \frac{1}{2} f^{AB}{}_{CDEF} \sum_{1 \leq i < j \leq N} J_i^{CD} J_j^{EF}$$

Differential Operator Representation in Momentum-Space:

$$P_i^\mu = k_i^\mu, \quad L_i^{\mu\nu} = k_i^\mu \partial_i^\nu - k_i^\nu \partial_i^\mu + \Sigma_i^{\mu\nu}, \quad D_i = d + k_i^\nu \partial_{i\nu},$$
$$K_i^\mu = 2\, d\, \partial_i^\mu + 2\, k_i^\nu \partial_{i\nu} \partial_i^\mu - k_i^\mu \partial_i^\nu \partial_{i\nu} - 2\, \Sigma_i^{\mu\nu} \partial_{i\nu}$$

$$J_i^{\mu\nu} = L_i^{\mu\nu}, \quad J_i^{n\mu} = \frac{1}{2}(P_i^\mu - K_i^\mu),$$
$$J_i^{n+1,\mu} = \frac{1}{2}(P_i^\mu + K_i^\mu), \quad J_i^{n,n+1} = D_i$$

Level-0 Generators : $P^\mu,\ L^{\mu\nu},\ D,\ K^\mu$

Level-1 Generators : $\widehat{P}^\mu,\ \widehat{L}^{\mu\nu},\ \widehat{D},\ \widehat{K}^\mu$

Proof of the Serre relation

- For N =1, $J^{AB} = J_1^{AB}, \quad \widehat{J}^{AB} = 0$
- Single site Serre condition:

$$0 = f^{AB}{}_{GHMN} f^{CD}{}_{IJOP} f^{EF}{}_{KLQR} f^{MNOPQR} \left\{J_1^{GH}, J_1^{IJ}, J_1^{KL}\right\}$$

- **Scalar** field representation: consistent **off-shell**, for **arbitrary** space-time dimensions **n** and conformal spin **d**
- **Gauge** field representation: consistent only **on-shell**, for **n = 4** and **d = 1**

Conformal Yangian Algebra and Generators

Scalar $\lambda\, \phi^3$ Theory Off-Shell Tree Graphs

SO(2,n) Yangian

Pure Yang-Mills Theory On-Shell Tree Amplitudes

$$A_N^\Delta(k_1, \ldots, k_N) = (-1)^{N+1} \prod_{k^2_{[I,J]} \in \Delta} \frac{1}{k^2_{[I,J]}}$$

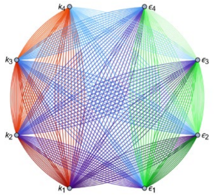
$$c^\Delta_{N,j} = -2d\, j - 2 \sum_{k^2_{[I,J]} \in \Delta} \begin{cases} 1 \text{ if } j \in [1, I-1] \\ -1 \text{ if } j \in [J+1, N] \end{cases}$$

$$\widehat{J}'^{AB} \ \delta^n(k_1 + \ldots + k_N)\ A_N^\Delta(k_1, \ldots, k_N) = 0, \quad \text{for } n = 6 \text{ and } d = 2$$

since $\widehat{P}'^\mu \ \delta^n(k_1 + \ldots + k_N)\ A_N^\Delta(k_1, \ldots, k_N) = 0$ and

$$\widehat{J}^{AB} \ \delta^n(k_1 + \ldots + k_N)\ A_N^\Delta(k_1, \ldots, k_N) = 0, \quad \text{for } n = 6 \text{ and } d = 2$$

CHY Scattering Equations Formalism



For n = 4 and d = 1, on-shell:

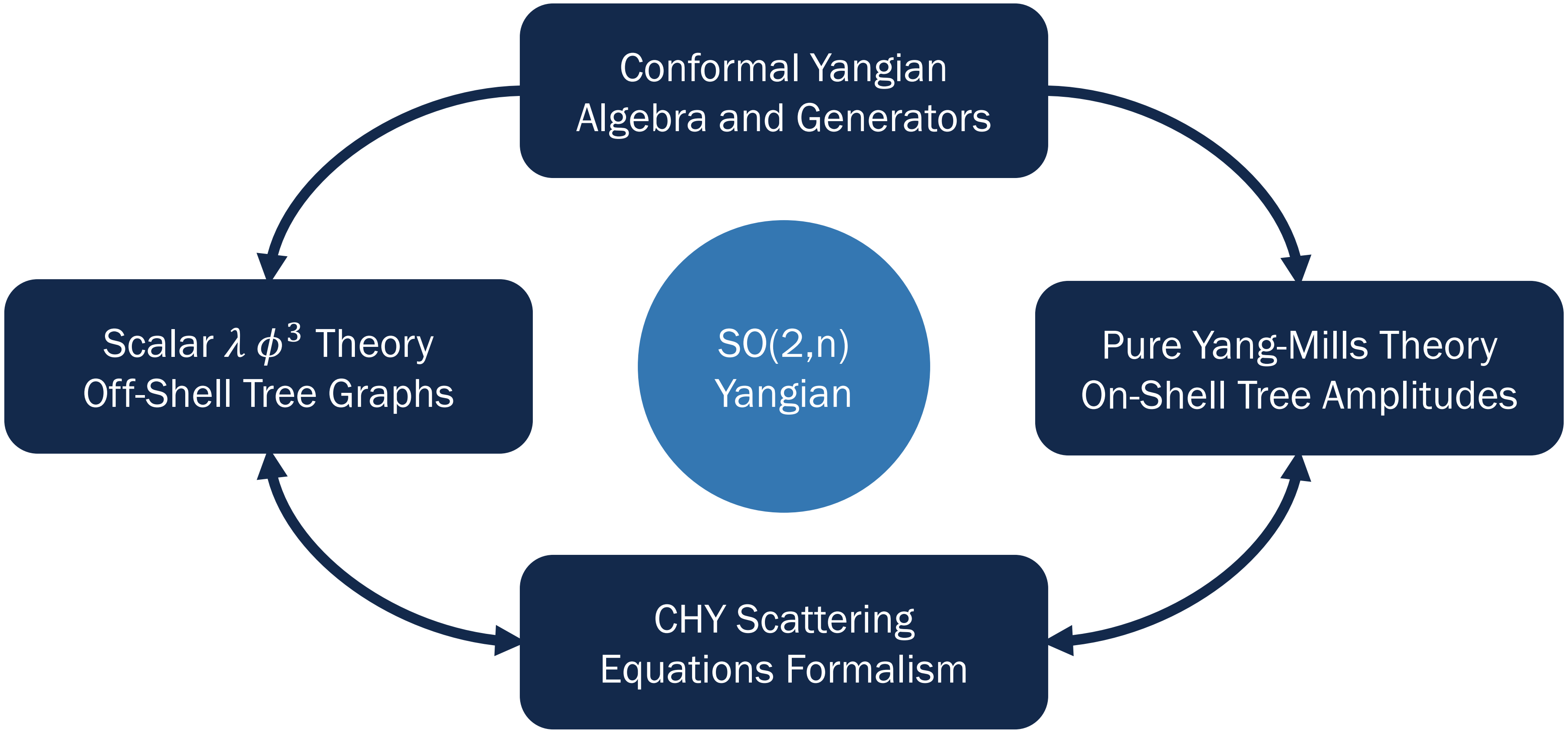
$$\widehat{P}^{\mu\alpha_1\alpha_2\alpha_3}_{\gamma_1\gamma_2\gamma_3} \ A^{a\gamma_1,\ b\gamma_2,\ c\gamma_3}$$
$$= g\, f_{abc}\, (2\pi)^4 \delta^4(k_1 + k_2 + k_3)$$
$$\cdot \Big(4k_1^\mu \left(\eta^{\alpha_2\alpha_3} k_2^{\alpha_1} + \eta^{\alpha_3\alpha_1} k_3^{\alpha_2} + 2\eta^{\alpha_1\alpha_2} k_1^{\alpha_3}\right) - 4k_3^\mu \left(2\eta^{\alpha_2\alpha_3} k_2^{\alpha_1} + \eta^{\alpha_3\alpha_1} k_3^{\alpha_2} + \eta^{\alpha_1\alpha_2} k_1^{\alpha_3}\right)$$
$$+ \eta^{\mu\alpha_1} \left(-4k_1^{\alpha_2} k_1^{\alpha_3}\right) + \eta^{\mu\alpha_2} \left(-4k_2^{\alpha_1} k_2^{\alpha_3}\right) + \eta^{\mu\alpha_3} \left(-4k_3^{\alpha_1} k_3^{\alpha_2}\right) \Big)$$
$$= g\, f_{abc}\, (2\pi)^4 \delta^4(k_1 + k_2 + k_3) \left(tr(\gamma^{\alpha_2}\gamma^\zeta\gamma^{\alpha_3}\gamma^\omega\gamma^{\alpha_1}\gamma^\mu) k_{1\omega} k_{2\zeta}\right.$$
$$\left. - tr(\gamma^{\alpha_3}\gamma^\zeta\gamma^{\alpha_2}\gamma^\omega\gamma^{\alpha_1}\gamma^\mu) k_{1\omega} k_{3\zeta} + tr(\gamma^{\alpha_3}\gamma^\zeta\gamma^{\alpha_1}\gamma^\omega\gamma^{\alpha_2}\gamma^\mu) k_{2\omega} k_{3\zeta}\right)$$

Similarly for the 4-point gluon partial amplitude

Nikos Dokmetzoglou [https://tarheels.live/nikosdokmetzoglou/]
UNC Chapel Hill

Properties of the Conformal Yangian in Scalar and Gauge Field Theories

[arXiv: 2207.14806]


Conformal Yangian
Algebra and Generators
Scalar $\lambda\,\phi^3$ Theory
Off-Shell Tree Graphs
SO(2,n)
Yangian
Pure Yang-Mills Theory
On-Shell Tree Amplitudes
CHY Scattering
Equations Formalism

Smooth Points on Positroid Varieties

Joseph Fluegemann

Cornell University

Amplitudes 2022 Summer School Gong Show

Positroid Variety

A positroid variety is a subset of the Grassmannian $Gr(k,n)$ defined by cyclic rank conditions.

For example, take $Gr(2,4)$. An element of this Grassmannian is represented by a 2-by-4 matrix:

$$\begin{matrix} 1 & 2 & 2 & 3 \end{matrix}$$
$$\begin{pmatrix} a & a & c & e \\ b & b & d & f \end{pmatrix}$$

E.g. $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \in \Pi_{1223}$.

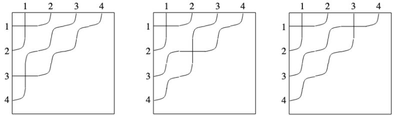
Affine Pipe Dream

-A pipe dream is a diagram with 2 types of tiles filling a grid: elbows and  and crosses +.

-The permutation (e.g. 1223 in the previous slide) can be read from the pipe dream.

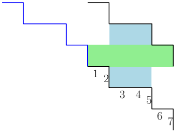
-The shape of the pipe dream represents an open neighborhood around a point in the Grassmannian.

Example:

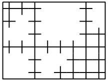


All the (reduced) pipe dreams for the permutation $\pi = 2143$.

I proved that a positroid variety is smooth (has no singular points) in a given neighborhood if you draw its pipe dreams, look at all the maximal rectangles:



and find that within these maximal rectangles, all crosses occur in partitions in the northwest and southeast, or along entire rows and columns of crosses, for example:



A Loop-by-loop Approach to Dual Elliptic Feynman Integrals

Mathieu Giroux
McGill University
Based on work to appear with A. Pokraka

3rd of August 2022

Abundance of elliptic integrals at the early stages of pQFT

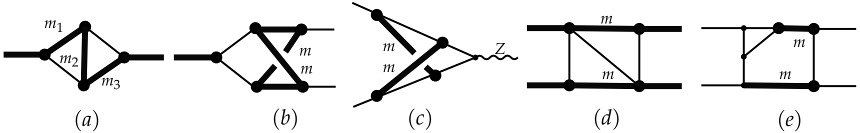


Figure: *Examples of 2-loop integrals with elliptic sectors*. *(a)* self-energy kite integral, *(b)* $t\bar{t}$ production in gluon fusion through a massive top-quark loop, *(c)* EW/QCD form factors, *(d)* Bhabha scattering in QED, *(e)* QCD/EW Drell-Yan

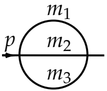
⇓

Developing efficient methods to deal with elliptic Feynman integrals is crucial to reach higher precision physics

Statement of the problem

Abstract:
We develop a **loop-by-loop*** method for the calculation of differential equations for Feynman integrals.

Focusing on the **dual** vector space to the family of fully massive sunrise integrals



$$= (\mu^2)^{(\nu_1+\nu_2+\nu_3)-D} \int \frac{\mathrm{d}^D\ell_1}{\pi^{D/2}} \frac{\mathrm{d}^D\ell_2}{\pi^{D/2}} \frac{\mathrm{d}^D\ell_3}{\pi^{D/2}} \frac{\delta^D(p-\ell_1-\ell_2-\ell_3)}{\mathrm{D}_1^{\nu_1}\mathrm{D}_2^{\nu_2}\mathrm{D}_3^{\nu_3}}$$

we obtain an **ε-form differential equation** in terms of **modular-** and **Kronecker-forms**.

*Loop-by-loop = Derive a multi-loop differential equation one loop at a time.

Why consider dual loop-by-loop?

- Dual forms

$\downarrow$

Freedom (no need to look like a Feynman integral)

- Loop-by-loop fibration

$\downarrow$

Algebraic complexity reduced (less variables)

$+$

Very explicit elliptic geometry (dual forms forced to $\sim 1/Y$)

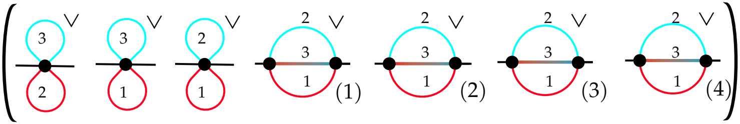
$\downarrow$

Natural/simple ansatz for *working* dual basis

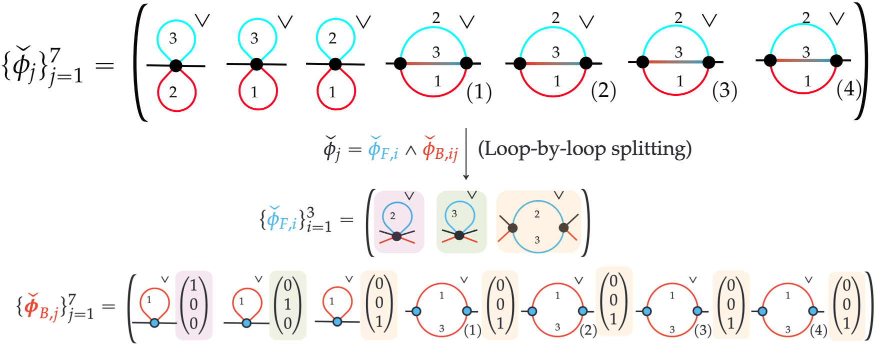
Example: The 3-scale sunrise

The dual sunrise basis is schematically given by

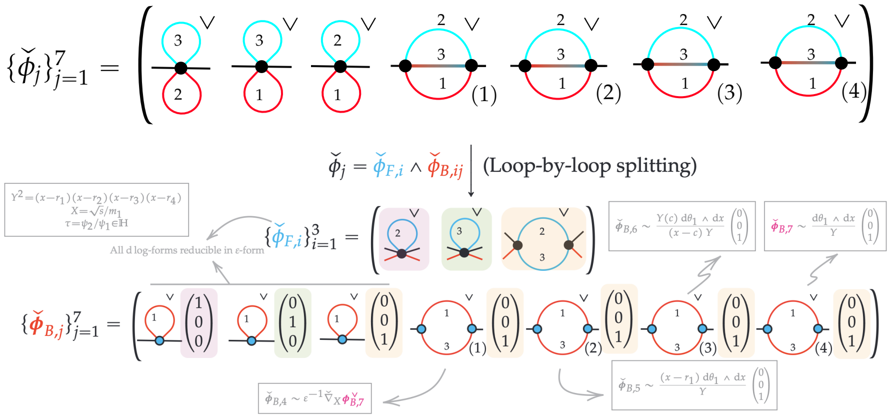
$\{\check{\phi}_j\}_{j=1}^{7} =$



Example: The 3-scale sunrise



Example: The 3-scale sunrise



- *Simple* period matrix (worth comparing with [Frellesvig, 21])

$$P = \begin{pmatrix} \pi & \pi\,\tau & 0 & 0 \\ 0 & 1/\varepsilon & 0 & 0 \\ g^{(1)}(z_c - z_\infty|\tau) + g^{(1)}(z_c + z_\infty|\tau) & \tau\left(g^{(1)}(z_c - z_\infty|\tau) + g^{(1)}(z_c + z_\infty|\tau)\right) + 4i\pi\, z_c - 2i\pi & 2i\pi & 0 \\ -2g^{(1)}(z_\infty|\tau) & -2\tau\, g^{(1)}(z_\infty|\tau) - 4i\pi\, z_\infty + 2i\pi & 0 & -2i\pi \end{pmatrix} + \mathcal{O}(\varepsilon)$$

↓

Lower triangular DE at $\mathcal{O}(\varepsilon^0)$ – i.e., linear-form

- Lower-triangular $\mathcal{O}(\varepsilon^0)$ contributions integrated out

↓

ε-form DE spanned by *modular* and *Kronnecker forms*

- Surprisingly, $\check{\boldsymbol{\Omega}}$ is **minus the transpose** of the connection in [Bogner, Weinzierl, Müller-Stach, 19]

↓

Dual bases!

Example: The 3-scale sunrise

Examples of non-trivial element in the ε-form DE

$$\check{\boldsymbol{\Omega}}_{74} = \varepsilon\, \omega_0^{\mathrm{Kr}}(\tau) \sim \varepsilon\, \mathrm{d}\tau \qquad \text{(Simplest)}$$

$$\vdots$$

$$\check{\boldsymbol{\Omega}}_{44} = 2\varepsilon \left[\omega_2^{\mathrm{Kr}}(Z_1, 2\tau) + \omega_2^{\mathrm{Kr}}(Z_2, 2\tau) + 2\omega_2^{\mathrm{Kr}}(Z_3, \tau) - 3\omega_2^{\mathrm{Kr}}(Z_3, 2\tau) - 3\eta_2(\tau)\right]$$

$$\vdots$$

$$\check{\boldsymbol{\Omega}}_{47} = -12\varepsilon \left[\omega_4^{\mathrm{Kr}}(Z_1, \tau) + \omega_4^{\mathrm{Kr}}(Z_2, \tau) + \omega_4^{\mathrm{Kr}}(Z_3, \tau) - 6\eta_4(\tau)\right]$$

(Most complicated)

Come see my poster next week for more!

Backup: Function space

Modular form $\omega_k^{\text{modular}}$ of modular weight k:

$$\omega_k^{\text{modular}}(\tau) \xrightarrow{\text{SL}(2,\mathbb{Z})} (c\tau - d)^{k-2}\omega_k^{\text{modular}}(\tau)$$

$$\eta_2(\tau) = [\underbrace{e_2(\tau) - 2e_2(2\tau)}_{\in\mathcal{M}_2(\Gamma_0(2))}]\frac{\mathrm{d}\tau}{2\pi\, i} \quad \text{and} \quad \eta_4(\tau) = \underbrace{e_4(\tau)}_{\in\mathcal{M}_4(\text{SL}(2,\mathbb{Z}))}\frac{\mathrm{d}\tau}{(2\pi\, i)^3}$$

Eisenstein-Kronecker series:

$$F(z,\alpha,\tau) = \vartheta_1'(0,\bar{q})\frac{\vartheta_1(z+\alpha,\bar{q})}{\vartheta_1(z,\bar{q})\vartheta_1(\alpha,\bar{q})} = \sum_{k=0}^{\infty} g^{(k)}(z|\tau)\alpha^{k-1}$$

Kronecker forms:

$$\omega_k^{\text{Kr}}(z,\tau) = (2\pi\, i)^{2-k}\left(g^{(k-1)}(z,\tau)\mathrm{d}z + (k-1)g^{(k)}(z,\tau)\frac{\mathrm{d}\tau}{2\pi\, i}\right)$$

Double copy construction of N=3 conformal supergravity

Subramanya Hegde
Harish-Chandra Research Institute

Based on: Ongoing work with Sanjeebani Biswal

SYM theories

- $N = 0, 1, 2, 4$ Super Yang Mills theories have been studied extensively.
- $N = 3$ SYM perturbatively is equivalent to $N = 4$ SYM.
- $N = 4$ SYM amplitude however has an $N = 3$ on-shell superspace formulation. [Elvang, Huang, Peng '11].

N=3 Conformal supergravity

- Conformal supergravity theories are theories of gravity with superconformal symmetry.
- The vielbein sits inside a Weyl multiplet which contains gauge fields for all the superconformal generators.
- They are essential to construct Poincaré supergravity theories.
- One gauge fixes additional symmetries using compensating fields to do so.
- $N = 3$ conformal supergravity was constructed only recently. [van Muiden, Van Proeyen '17; S Hegde, B Sahoo '18; S Hegde, M Mishra, B Sahoo '21]
- It is a distinct theory from $N = 4$ conformal supergravity with genuinely $N = 3$ supersymmetry.

- $N = 0, 1, 2, 4$ conformal supergravity double copy construction has been done earlier. [Johansson, Nohle '17; Johansson, Mogull, Teng '18].
- One side is $N = 0, 1, 2, 4$ SYM and the other side is the DF^2 theory.
- We have extended this to the recently constructed $N = 3$ conformal supergravity.
- We intend to extend it to six dimensions to investigate the possibility of dilaton Weyl multiplets for 6D $(1, 1)$ theory even though superconformal algebra doesn't exist here.

Thank you.

From $w_{1+\infty}$ to $\mathcal{W}_{1+\infty}$

Simon Heuveline

University of Cambridge

sph48@cam.ac.uk

August 3, 2022

Most descriptions of self-dual gravity contain a Poisson-bracket:

$$S[\bar{\phi}, \phi] = \int \bar{\phi}\Big(\partial^2\phi + \frac{\kappa}{2}\{\partial^{\alpha}_{\dot{+}}\phi, \partial_{\alpha\dot{+}}\phi\}\Big) d^4x,$$

$$\{f, g\} = \alpha^{\dot{\alpha}}\alpha^{\dot{\beta}}\epsilon^{\alpha\beta}\partial_{\alpha\dot{\alpha}} f \partial_{\beta\dot{\beta}} g.$$

$$\partial_{\alpha\dot{+}} f = \alpha^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}} f$$

Self-dual Gravity

Most descriptions of self-dual gravity contain a Poisson-bracket:

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$$\{f, g\} = \alpha^{\dot{\alpha}}\alpha^{\dot{\beta}}\epsilon^{\alpha\beta}\partial_{\alpha\dot{\alpha}} f \partial_{\beta\dot{\beta}} g.$$

$$\partial_{\alpha\dot{+}} f = \alpha^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}} f$$

This theory can be deformed by the Moyal-bracket

$$S_q[\bar{\phi}, \phi] = \int \bar{\phi}\left(\partial^2\phi + \frac{\kappa}{2}\{\partial^{\alpha}_{\dot{+}}\phi, \partial_{\alpha\dot{+}}\phi\}_q\right) d^4x,$$

$$\{f, g\}_q = \frac{1}{q} f \sinh(q\{\overleftarrow{-}, \overrightarrow{-}\}) g.$$

Amplitudes in q-deformed self-dual Gravity

Self-dual gravity has complicated 1-loop n-point amplitudes (Bern, Dixon et al, 1998):

$$\mathcal{M}_n^{(1)}(1^+,\dots,n^+) \sim \kappa^n \sum_{1\leq a<b\leq n} \sum_{M,N} h(a,M,b)h(b,N,a)\mathrm{tr}^3[aMbN]$$

with simple collinear limits

$$M_n^{(1)}(1^+,\dots,n^+) \to \frac{-\kappa}{2t(1-t)}\frac{[12]}{\langle 12\rangle} M_{n-1}^{(1)}(P^+,\dots,n^+).$$

Amplitudes in q-deformed self-dual Gravity

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$$\mathcal{M}_n^{(1)}(1^+, \ldots, n^+) \sim \kappa^n \sum_{1 \leq a < b \leq n} \sum_{M,N} h(a, M, b) h(b, N, a) \mathrm{tr}^3[aMbN]$$

with simple collinear limits

$$M_n^{(1)}(1^+, \ldots, n^+) \to \frac{-\kappa}{2t(1-t)} \frac{[12]}{\langle 12 \rangle} M_{n-1}^{(1)}(P^+, \ldots, n^+).$$

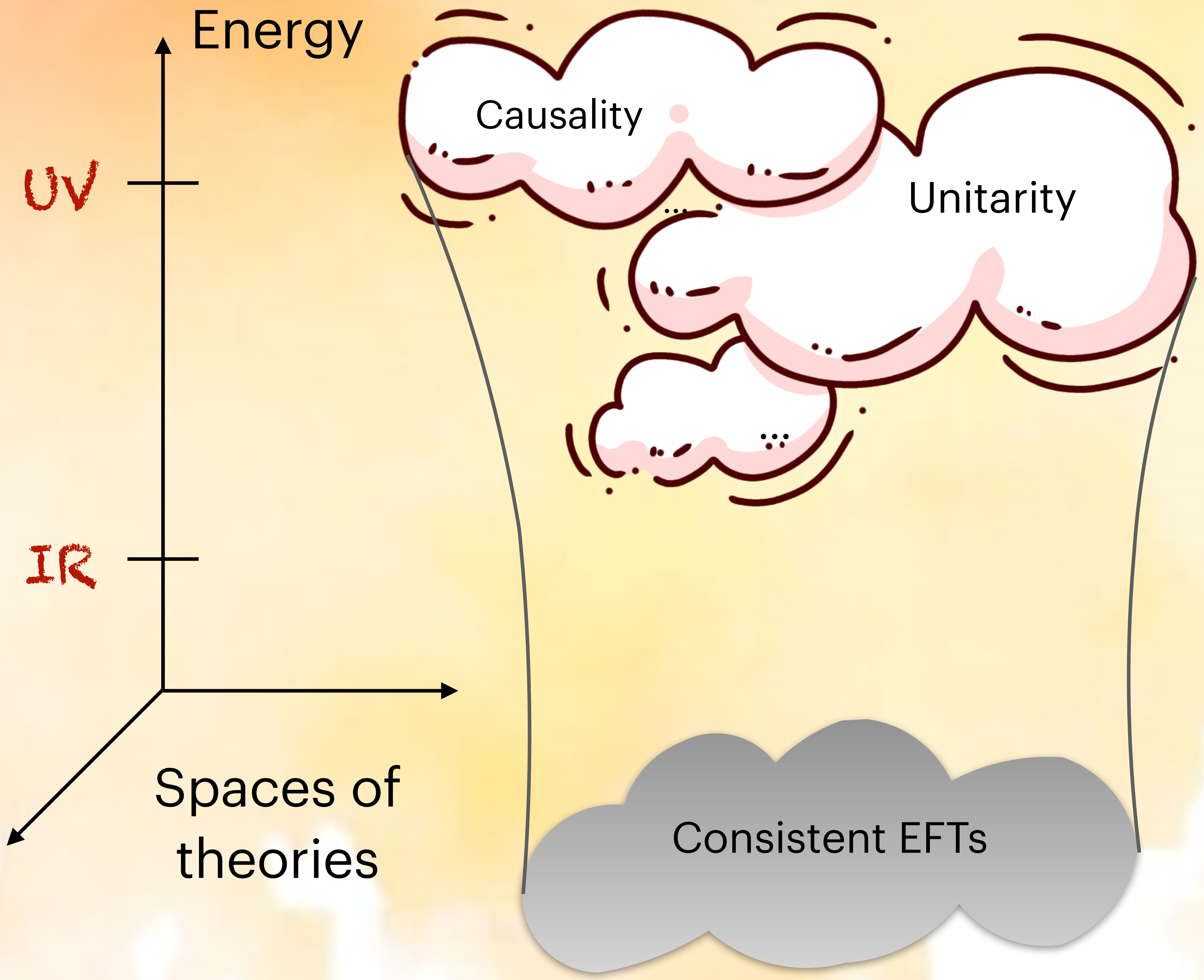
Computing the amplitudes in the Moyal-deformed theory, this gets deformed to:

$$M_{n,q}^{(1)}(1^+, \ldots, n^+) \to \frac{-\kappa}{2t(1-t)} \frac{[12]_q}{\langle 12 \rangle} M_{n-1,q}^{(1)}(P^+, \ldots, n^+) \qquad (1)$$

with $[ij]_q = \frac{1}{q\langle \alpha i \rangle \langle \alpha j \rangle} \sinh\left(q \langle \alpha i \rangle \langle \alpha j \rangle [ij]\right)$.

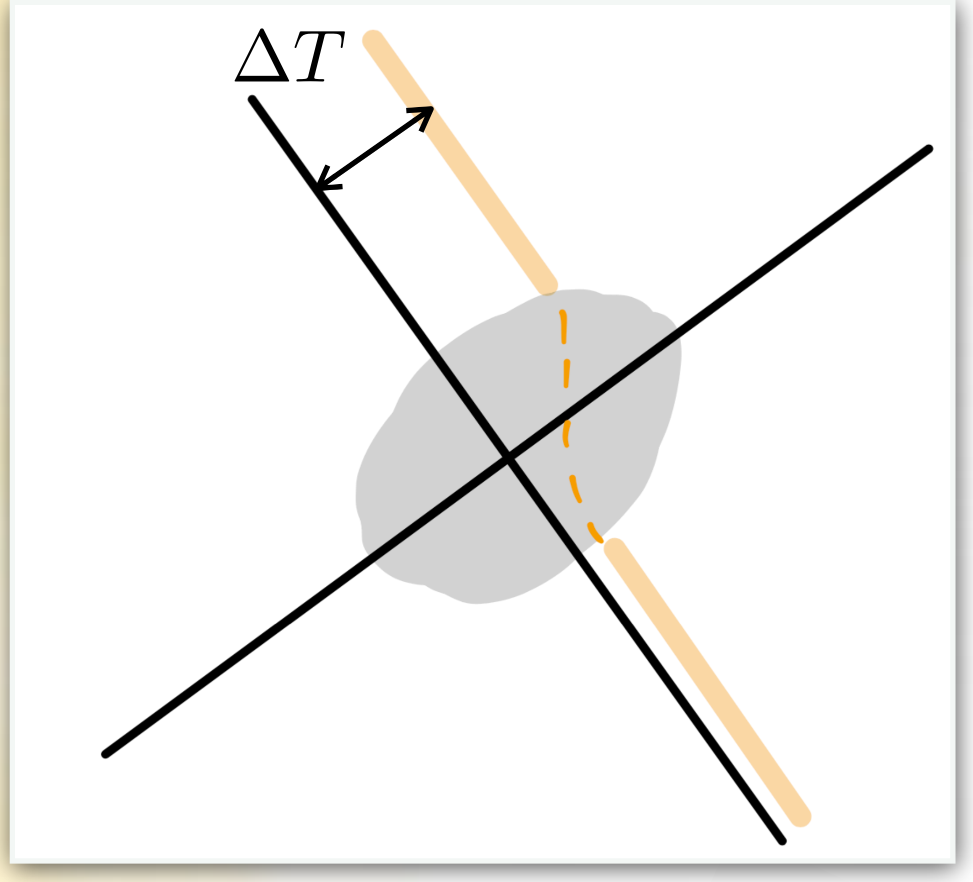
Giulia Isabella
Supervisor: Brando Bellazzini

Constrain EFTs



Why?

Transplackian scattering

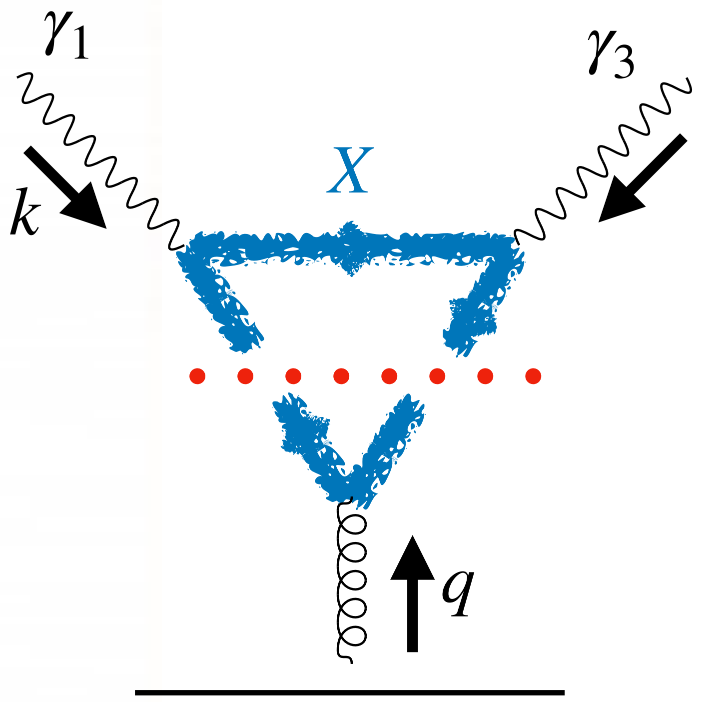


Causality VIOLATION

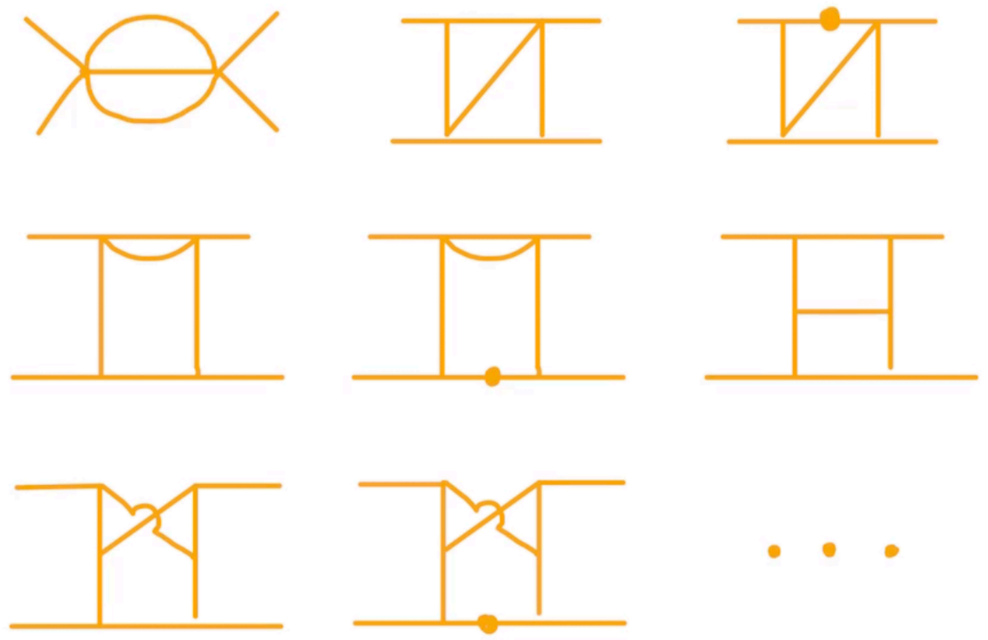
Where?

Amplitudes

Then...



Now...



How?

Wilson-line based action for gluodynamics

Based on H. Kakkad, P. Kotko, A. Stasto, arXiv:2102.11371, arXiv:2006.16188 and soon appearing.

Hiren Kakkad
Faculty of Physics, AGH UST

NCN GRANT 2021/41/N/ST2/02956
NCN GRANT DEC-2018/31/D/ST2/02731

Supervisor: Hab. Dr. Piotr Kotko
Faculty of Physics,AGH UST

Gong Show, Amplitudes 2022 Summer School

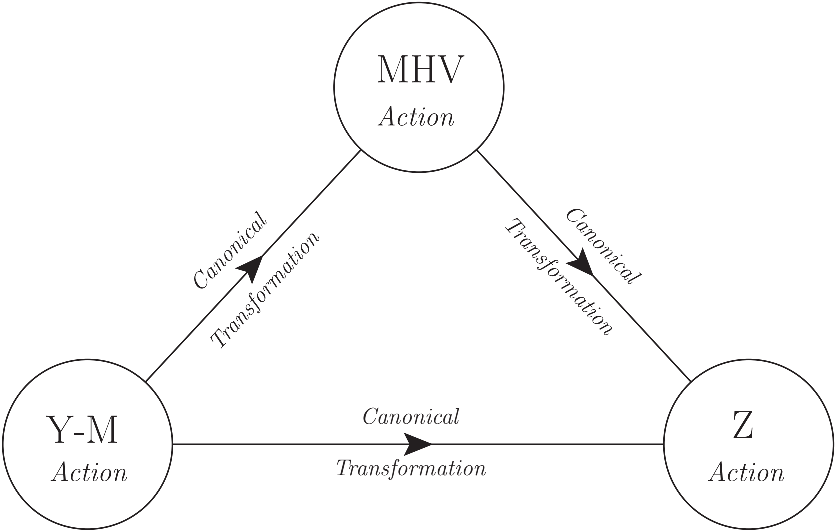
August 03, 2022

AGH University of Science and Technology

AGH

Deriving the new action

[H. Kakkad, P. Kotko, A. Stasto, 2021]- arXiv :2102.11371



Z Action

Structure of the new action

$$S_{\mathrm{Y-M}}^{(\mathrm{LC})}[Z] = \Big\{ \mathcal{L}_{-+}^{(\mathrm{LC})} + \mathcal{L}_{--++}^{(\mathrm{LC})} + \mathcal{L}_{--+++}^{(\mathrm{LC})} + \mathcal{L}_{--++++}^{(\mathrm{LC})} + \dots$$

$$+ \mathcal{L}_{---++}^{(\mathrm{LC})} + \mathcal{L}_{---+++}^{(\mathrm{LC})} + \mathcal{L}_{---++++}^{(\mathrm{LC})} + \dots$$

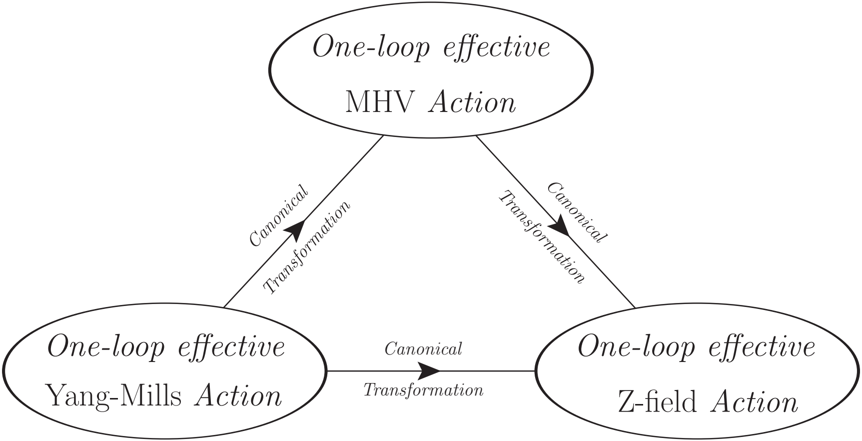
$$\vdots$$

$$+ \mathcal{L}_{---\dots-++}^{(\mathrm{LC})} + \mathcal{L}_{---\dots-+++}^{(\mathrm{LC})} + \mathcal{L}_{---\dots-++++}^{(\mathrm{LC})} + \dots \Big\}$$

Important features

- There are no three point interaction vertices.
- At the classical level there are no all-plus, all-minus, as well as $(-+\cdots+)$, $(-\cdots-+)$ vertices.
- There are MHV vertices, $(--+\cdots+)$, corresponding to MHV amplitudes in the on-shell limit.
- There are $\overline{\mathrm{MHV}}$ vertices, $(-\cdots-++)$, corresponding to $\overline{\mathrm{MHV}}$ amplitudes in the on-shell limit.
- All vertices have the form which can be easily calculated.

[H. Kakkad, P. Kotko, A. Stasto]- appearing soon



Thank You for your Time!

I invite you to my poster next week at the conference.

From Hopf to braided L_∞-algebras

Toni Kodžoman

Division of Theoretical Physics
Ruđer Bošković Institute, Zagreb

based on: 2204.01352

Clay J. Grewcoe, Larisa Jonke, Toni Kodžoman, Georgios Manolakos

Gong Show Talk, 3.8.2022.

HRZZ

How are Hopf and L_∞-algebras related?

- What are L_∞-algebras?
  - generalizations of Lie algebras with infinitely many higher brackets
  - underlying geometry useful in understanding quantization issues in field theory and gravity

- Introducing noncommutative deformations can be achieved via the Drinfel'd twist approach on a symmetry Hopf algebra

- We first show that an L_∞-algebra can be extended to a graded Hopf algebra with a differential D
  - after twisting the extended algebra, one further twists its modules
  - brings forth the construction of the braided L_∞-algebra
  - we identify the Hopf algebra morphisms φ with strict L_∞-morphisms

$$\varphi \circ D_* = D^{\mathcal{F}} \circ \varphi$$

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THANK YOU FOR YOUR ATTENTION!

Amplitudes in Jacob-Wick's helicity basis

Hojin Lee

Seoul National University

Amplitudes Summer School 2022

With Sangmin Lee and Subhajit Mazumdar

Partial Wave Expansion in helicity basis

- We can **partial-wave** decompose S-matrix for rotation-invariant scattering

$$\langle \text{out}|S|\text{in}\rangle = \sum_j M_j(\text{energy}) B_j(\text{angle})$$

- The angular part can be written in terms of, for example, $Y_{lm}(\hat{k})$, but also

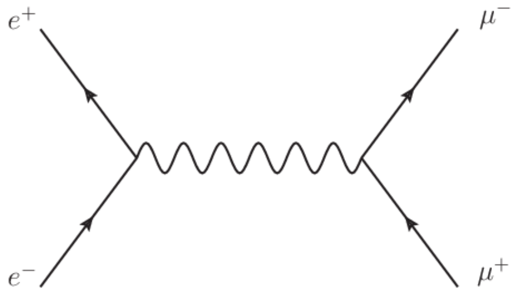
$$B^j(\hat{k}', \hat{k})_{h',h} = \left[(k'_+|^{j+h'} (k'_-|^{j-h'} \right]_{\text{ts}} \left[|k_+)^{j+h} |k_-)^{j-h} \right]_{\text{ts}}$$

- Helicity basis has many advantages (e.g. manifest **gauge** and **frame dependence**) and can be naturally extended to complicated problems (e.g. **Monopole Scattering**)

[Csaki, Hong, Shirman, Telem, Terning, Waterbury]

Amplitudes in helicity basis: from QED

- Starting from QED Feynman diagrams (e.g. electron-muon scattering)



$$\mathcal{M}^{I_3 I_4}{}_{I_1 I_2} = \frac{e^2}{s}\left[\bar{u}^{I_3}(p_3)\gamma_\mu v^{I_4}(p_4)\right]\left[\bar{v}_{I_2}(p_2)\gamma^\mu u_{I_1}(p_1)\right]$$

$$u_I(p) = \begin{pmatrix} \lambda_{aI} \\ \tilde{\lambda}^{\dot{a}}{}_I \end{pmatrix}, \quad v^I(p) = \begin{pmatrix} -\lambda_{aJ} \\ \tilde{\lambda}^{\dot{a}}{}_J \end{pmatrix} \epsilon^{JI},$$

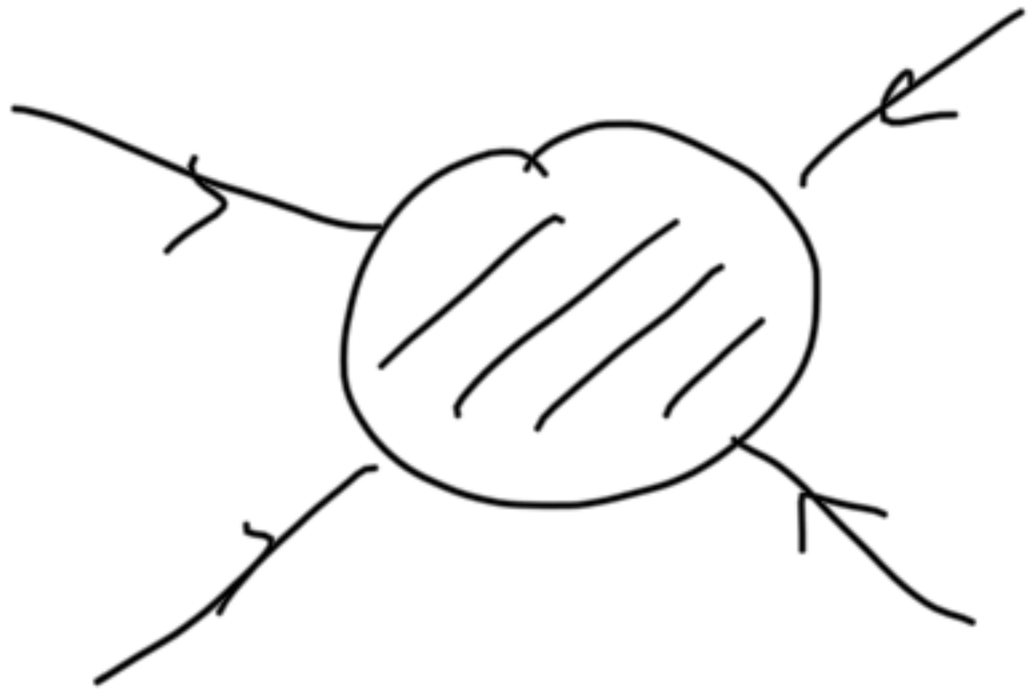
- To where? Monopole Scattering, Celestial Amplitude, Graviton, … (Work in progress)

Thank you!

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Differential Equation for Feynman Integral

$$\partial_x \vec{g}(x;\epsilon) = \epsilon \left[\frac{a}{x} + \frac{b}{1+x} \right] \vec{g}(x,\epsilon)$$

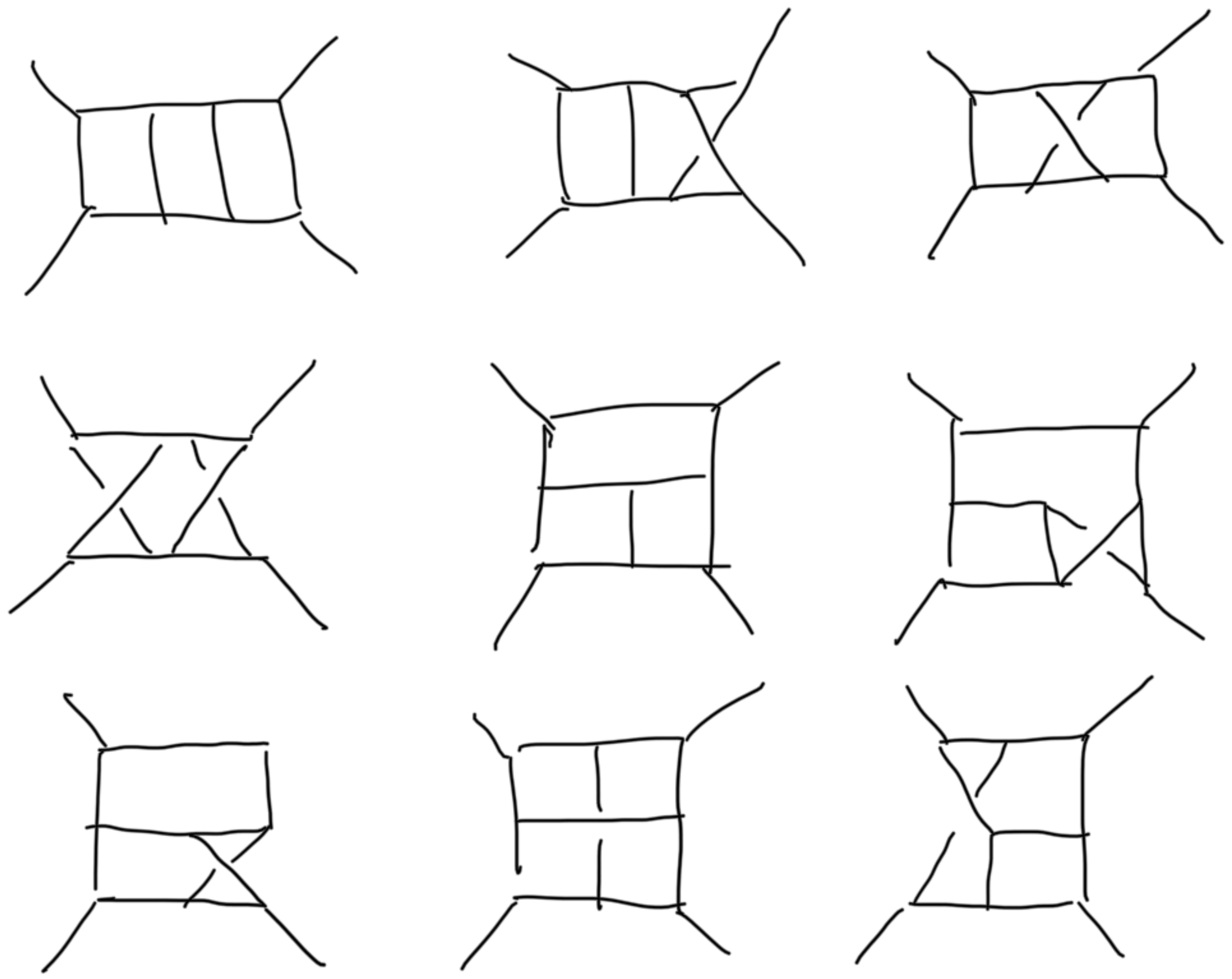


Integration-By-Part (IBP) relation

Polylogarithms

Master Integral

Canonical Form



Won Lim

With Johannes Henn, William J. Torres Bobadilla

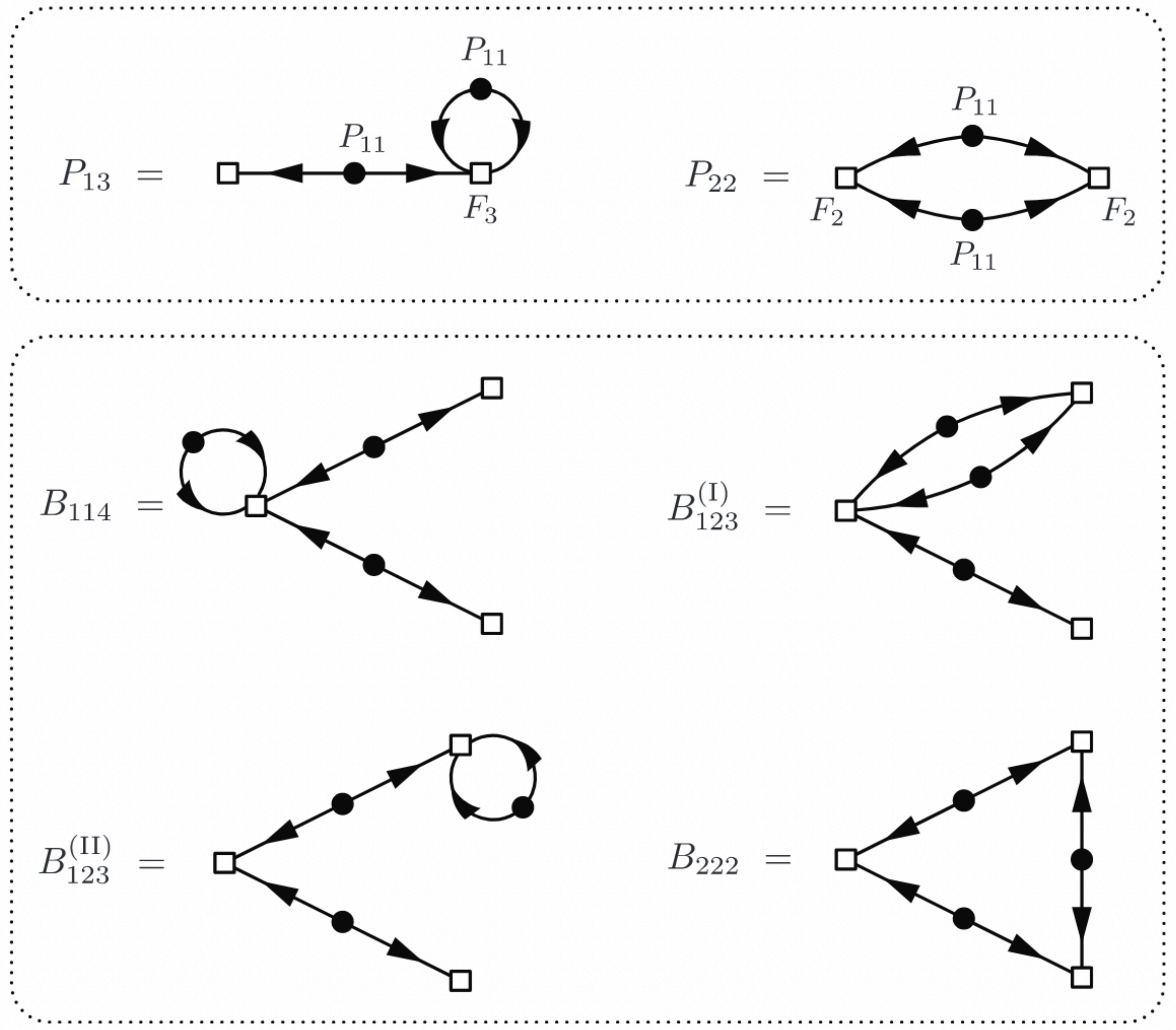
Novel Methods

Applications

Better Way for IBP

Canonical Basis

Computing Power Spectrum



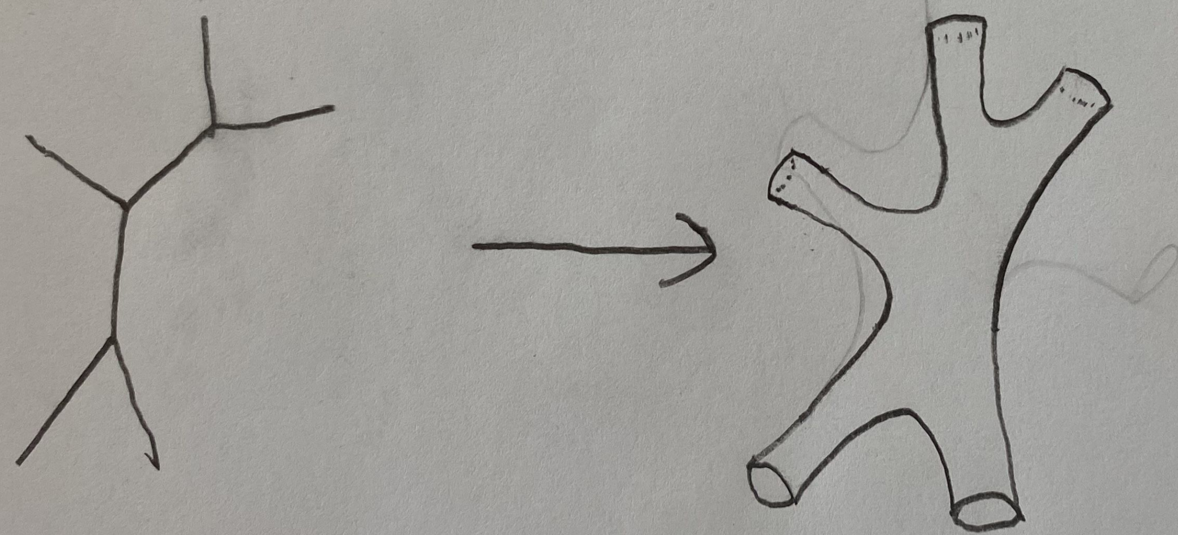
From 1505.06668

$2 \to N\,(N \gg 1)$ **graviton scattering amplitudes**

Won Lim

Main point:

Planar scalar tree amplitudes can always be fattened into a stringy integral



Stringy integrals

$$A_N = \int du \; P_N(u_{ij})$$

$du \longrightarrow$ factorizing measure $\qquad du = \alpha'^{N-3} d\Omega \left(\prod_{\substack{\text{propagators} \\ X_{ij}}} u_{ij}^{\alpha' X_{ij}} \right)$

$P_N \longrightarrow$ factorizing polynomial

Then A_N factorizes into $A_m \frac{1}{X_{ij}} A_k \qquad m+k = N+2$

"Fat Feynman" polynomial:

$$P_{FF} = \prod_{ij} u_{ij} \left(\begin{array}{l} \text{scalar amplitude with} \\ \text{propagators } X_{ij} \text{ replaced with } u_{ij} \end{array} \right)$$

P_{FF} factorizes correctly and A_N reduces to the QFT amplitude in the $\alpha' \to 0$ limit.

HOLOGRAPHYCL

ANID-SCIA-ACT210100

UNIVERSIDAD DE CHILE

Bootstrap From Unitarity: Cosmological Cutting Rules

Gabriel Marin

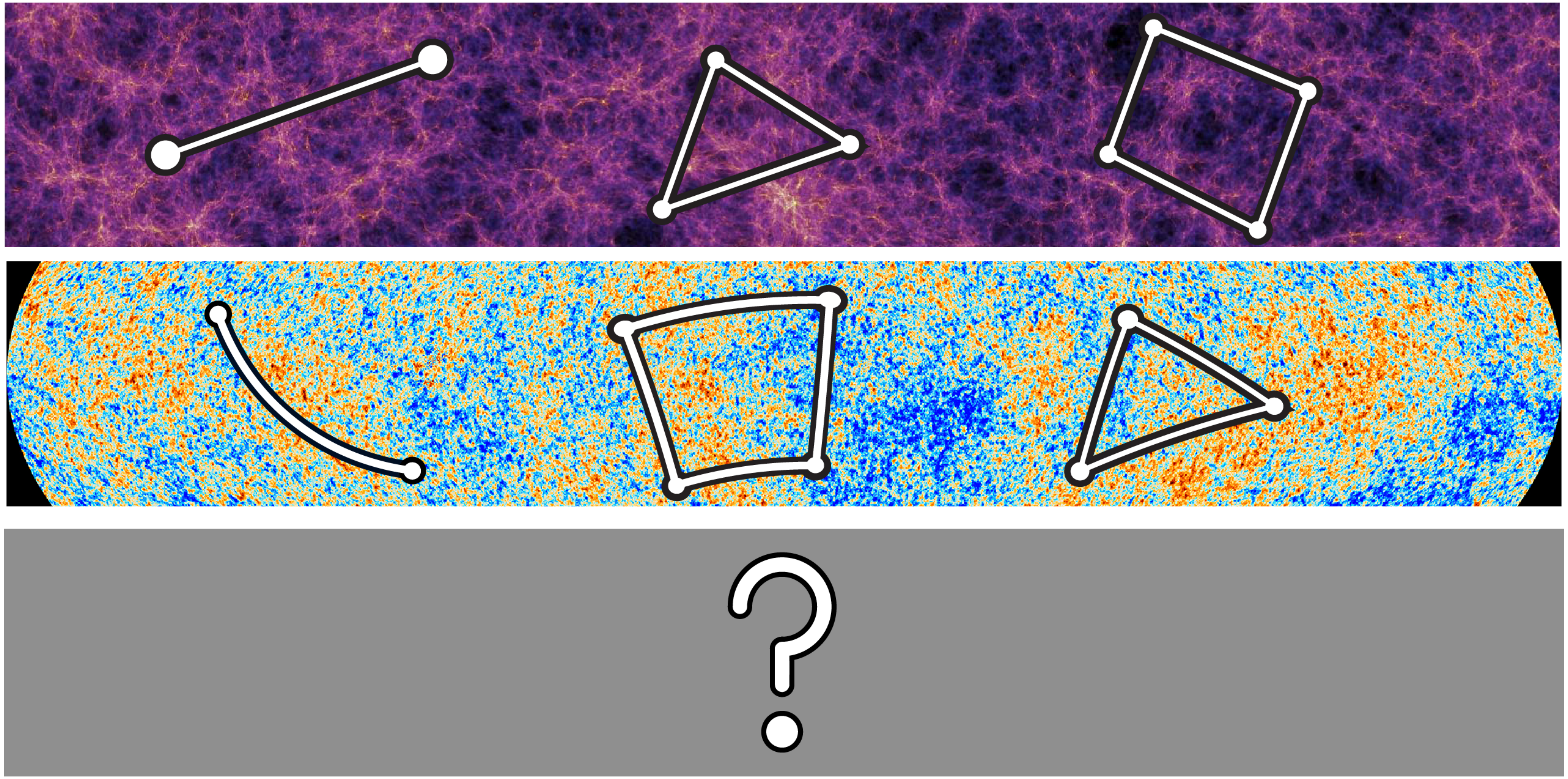
In collaboration with F. Colipí, G. Palma, and F. Rojas

Amplitudes Summer School 2022

2-6 August, 2022

- ***What was the origin of structures in the Universe?***

Correlation functions in inflationary cosmology might have a hint



Constraining CCF

- De Sitter symmetry

[1811.00024] Arkani-Hamed, Baumann, Lee, and Pimentel

- Unitarity, evolution operator in the interaction picture

$$U^\dagger U = 1, \quad \delta U + \delta U^\dagger = -\delta U \, \delta U^\dagger$$

- Cosmological Optical Theorem

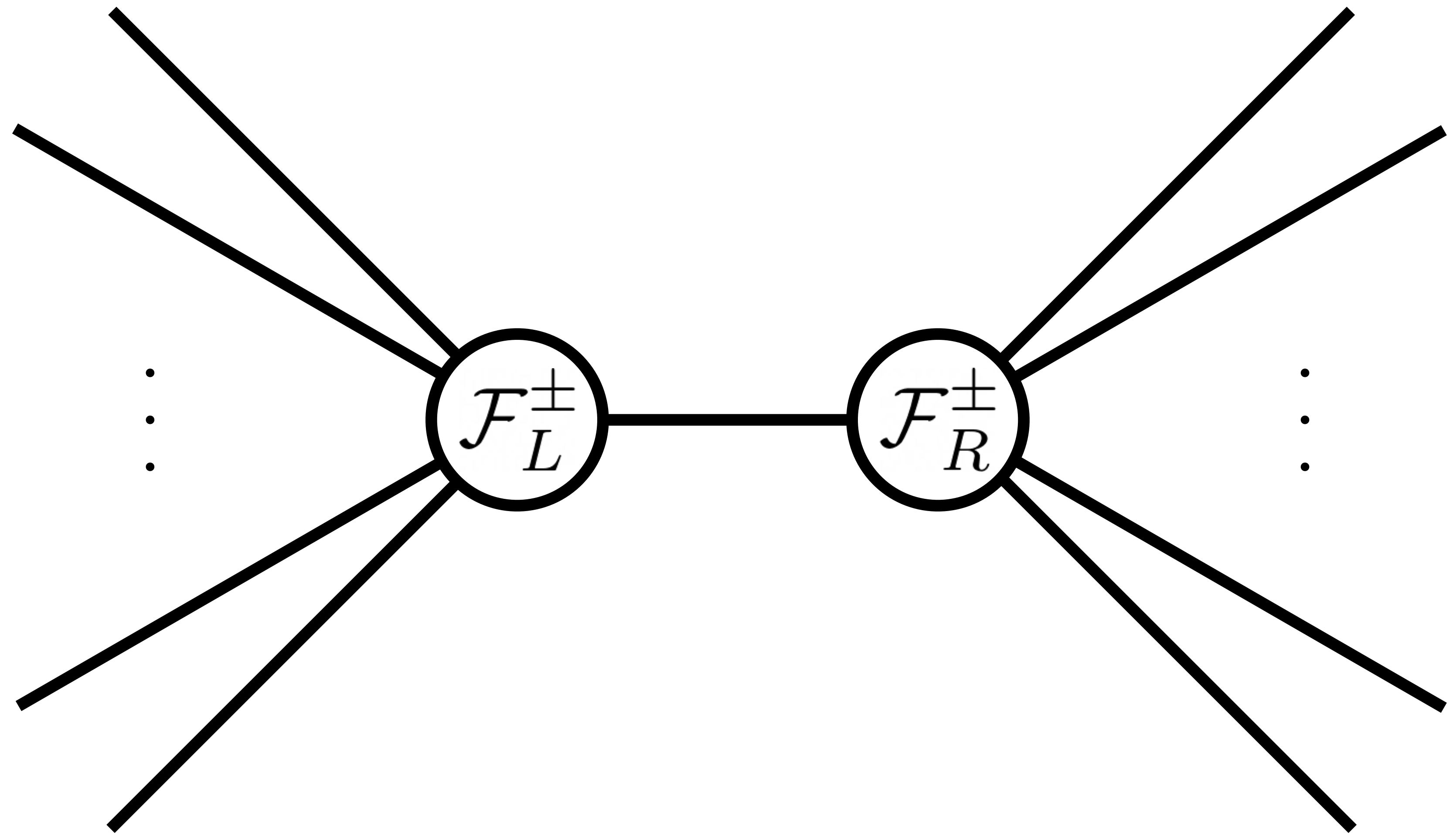
[2009.02898] Goodhew, Jazayeri, and Pajer

$$\langle \{\mathbf{k}, \alpha\}_n | \delta U(\eta_0) | 0 \rangle + \langle \{\mathbf{k}, \alpha\}_n | \delta U(\eta_0)^\dagger | 0 \rangle =$$
$$- \sum_{m=0}^{\infty} \sum_{\beta_1, \ldots, \beta_m} \int \frac{d^3 \mathbf{p}_1}{(2\pi)^3} \cdots \frac{d^3 \mathbf{p}_m}{(2\pi)^3} \langle \{\mathbf{k}, \alpha\}_n | \delta U(\eta_0) | \{\mathbf{p}, \beta\}_m \rangle \langle \{\mathbf{p}, \beta\}_m | \delta U(\eta_0)^\dagger | 0 \rangle$$

- We will use Schwinger-Keldysh
- Toy model example:

$$\mathcal{L} \supset \frac{1}{3!} g\varphi^3$$

$$\langle \varphi^n \rangle =$$



$$\underset{s}{\mathrm{Disc}}\left[\langle \varphi^n \rangle\right] \sim$$

$$\sum_{a,b=\pm} \left(\int_{-\infty}^{0} dt_L \, \underset{s}{\mathrm{Disc}} \left[\mathcal{F}_L^a(k_1, \dots, k_i, t_L) G_a(s, t_L) \right] \right) \left(\int_{-\infty}^{0} dt_R \, \underset{s}{\mathrm{Disc}} \left[G_b(s, t_R) \mathcal{F}_R^b(k_{i+1}, \dots, k_n, t_R) \right] \right)$$

$$- \sum_{a,b=\pm} ab \left(\int_{-\infty}^{0} dt_L \, i\underset{s}{\mathrm{Disc}} \left[i\mathcal{F}_L^a(k_1, \dots, k_i, t_L) G_a(s, t_L) \right] \right) \left(\int_{-\infty}^{0} dt_R \, i\underset{s}{\mathrm{Disc}} \left[iG_b(s, t_R) \mathcal{F}_R^b(k_{i+1}, \dots, k_n, t_R) \right] \right)$$


Thanks

Intersection theory: String and QFT amplitudes

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Pouria Mazloumi, Stephan Stieberger

Based on: 2201.00837

Amplitudes 2022

What do I study ??

String Theory $S = \frac{1}{2\pi\alpha'}\int d^z \ h_{\alpha\beta}g_{\mu\nu}\partial^\alpha X^\nu(z,\bar{z})\partial^\beta X^\mu(z,\bar{z});\qquad m_n = -\frac{n}{\alpha'}$

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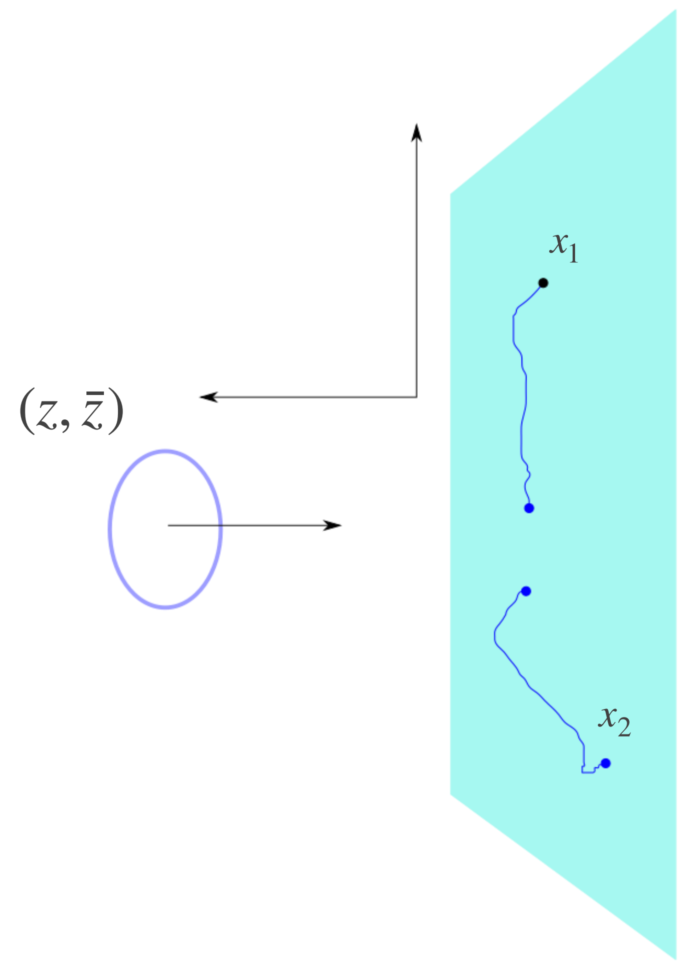

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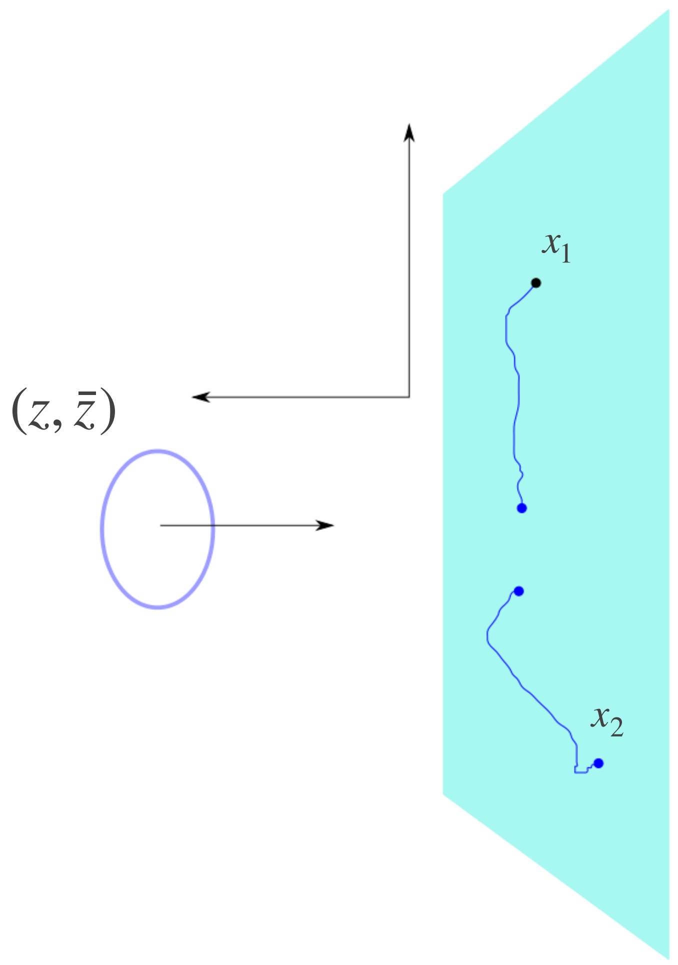

What do I study ??

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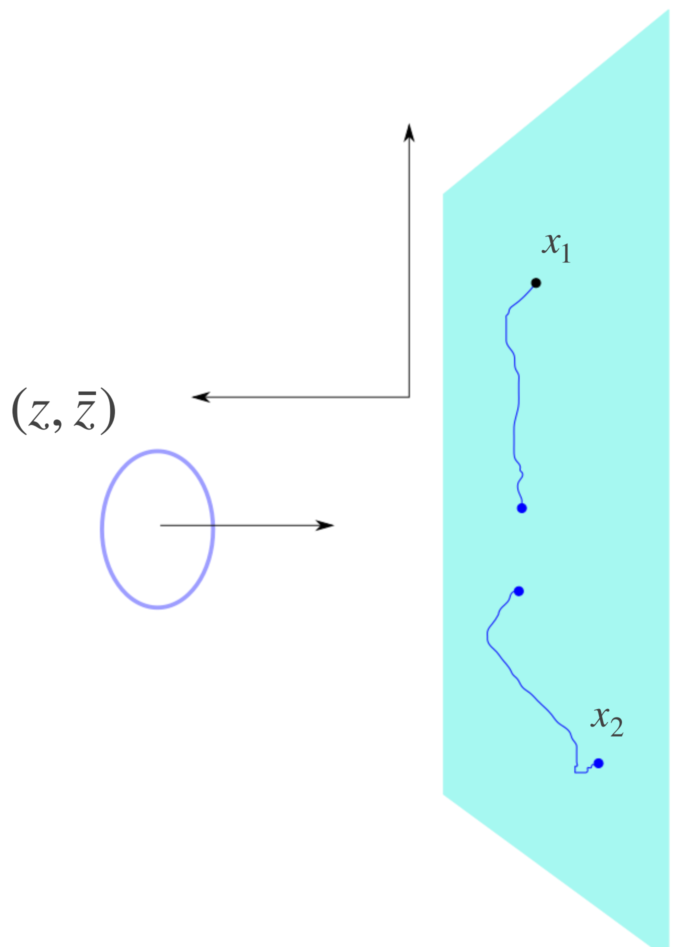

$\rightarrow$

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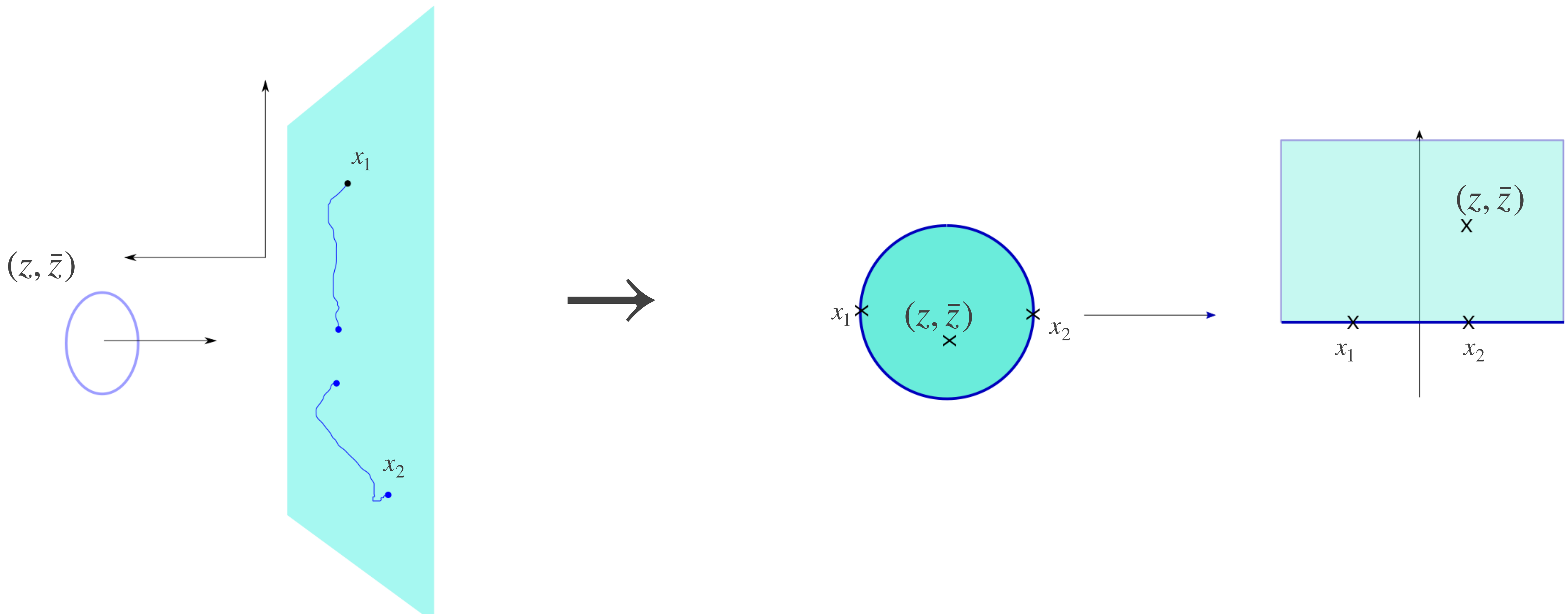

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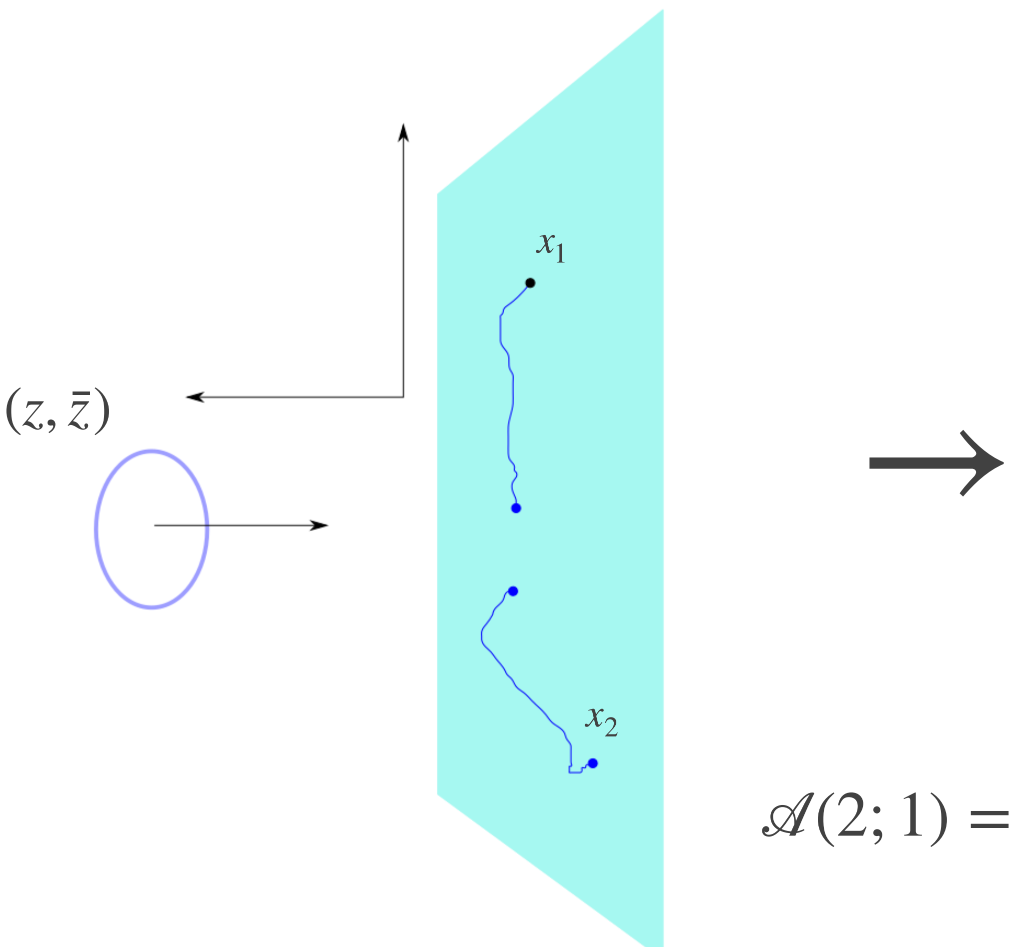
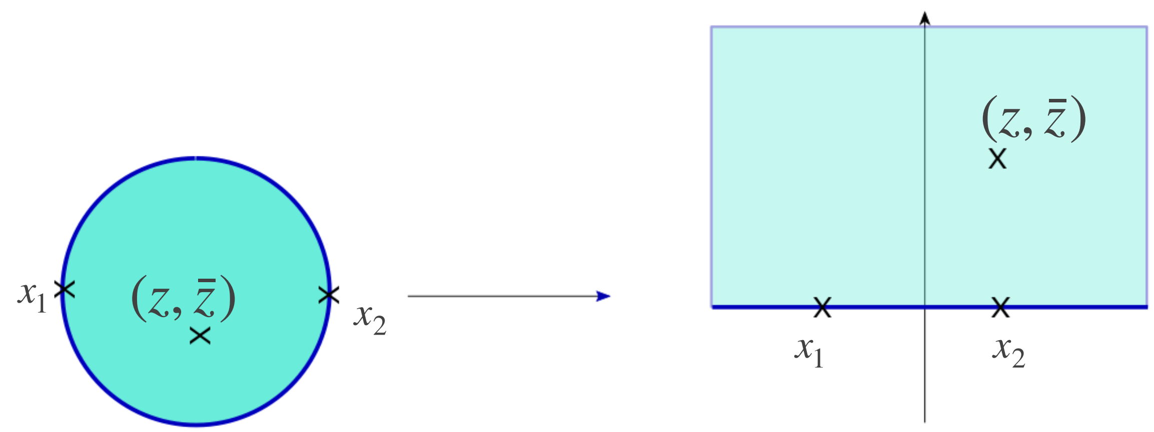


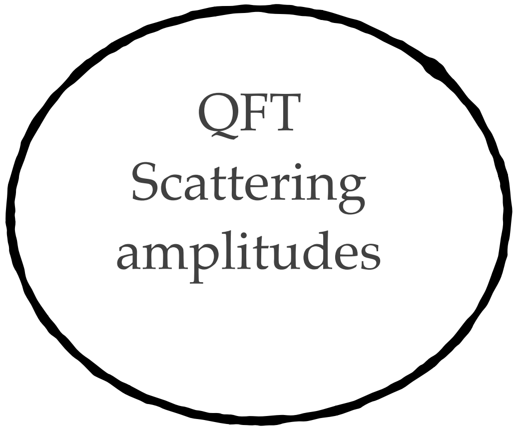


$$\mathscr{A}(2;1) = \int \frac{dx_1 dx_2 d^2z}{\text{SL(2,R)}}\ \langle V_o^{(-1)}(\varepsilon_1,k_1,x_1)V_o^{(-1)}(\varepsilon_2,k_2,x_2)V_c^{(0,0)}(\varepsilon_q,q,z,\bar{z})\rangle$$

$$= \mathcal{O}(\alpha'^{-1})\mathscr{F}\left\{(x_1-z)(x_2-z)(\bar{z}-z)\right\}$$

Web of relations

Web of relations



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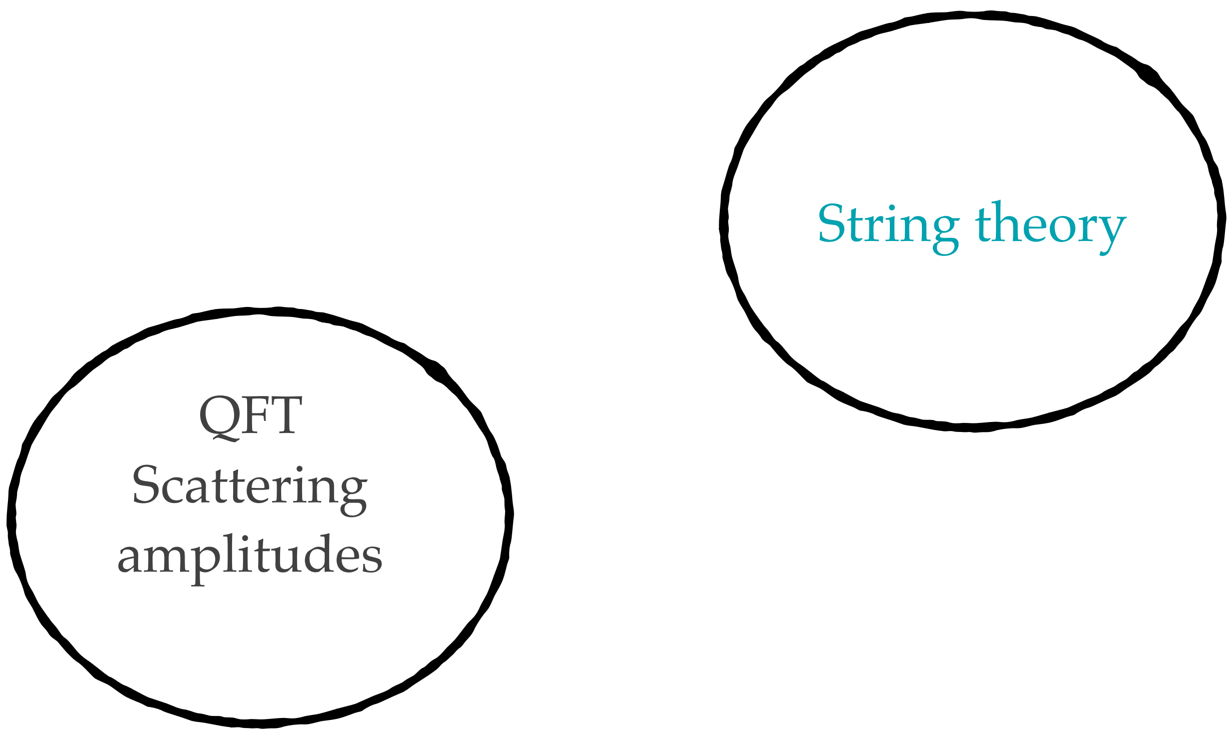

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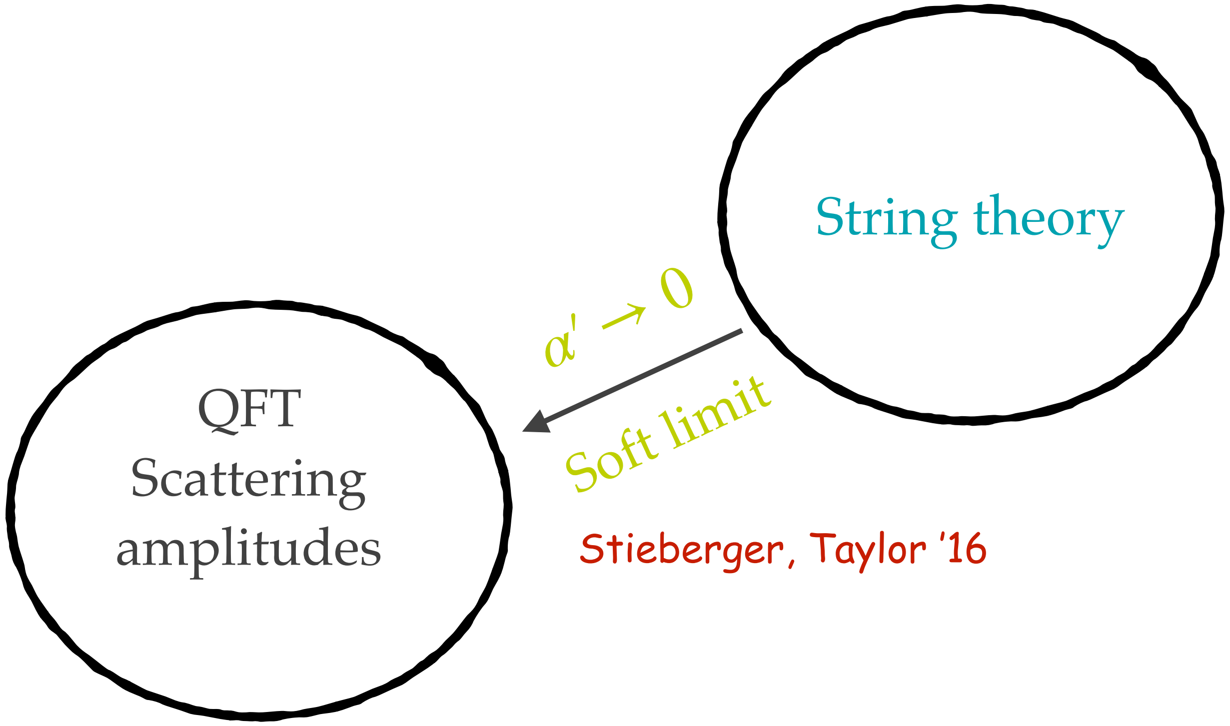

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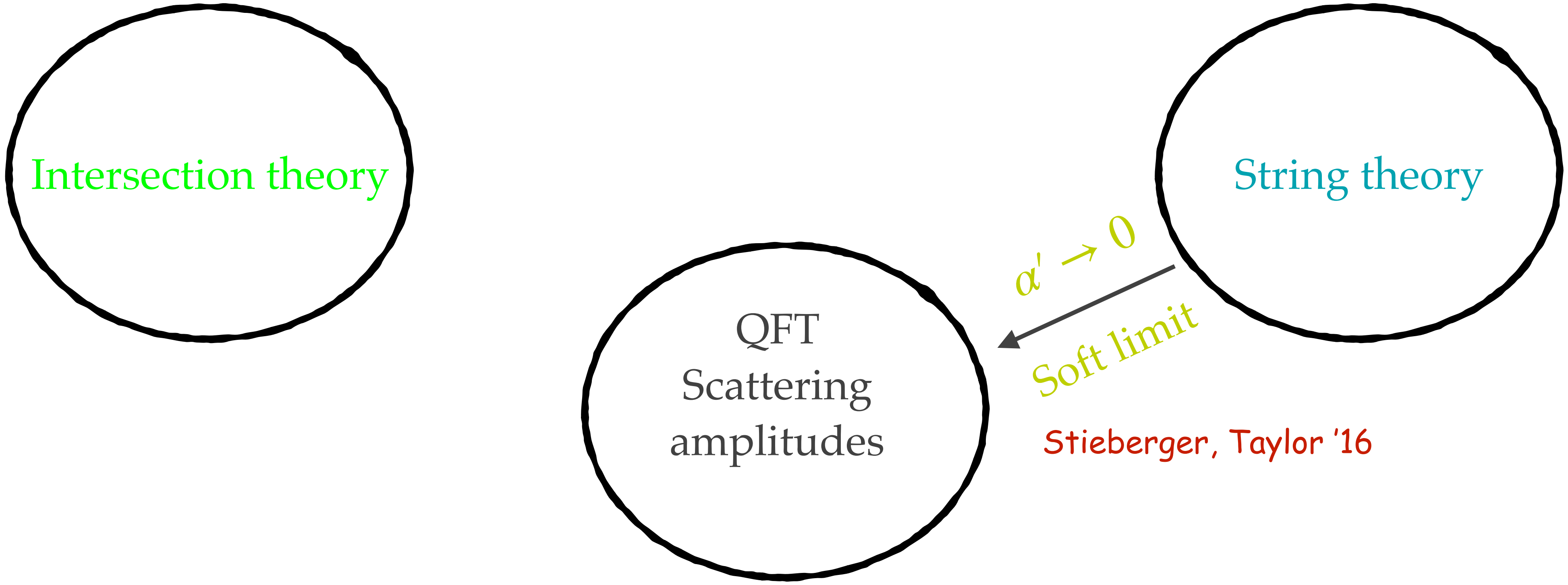

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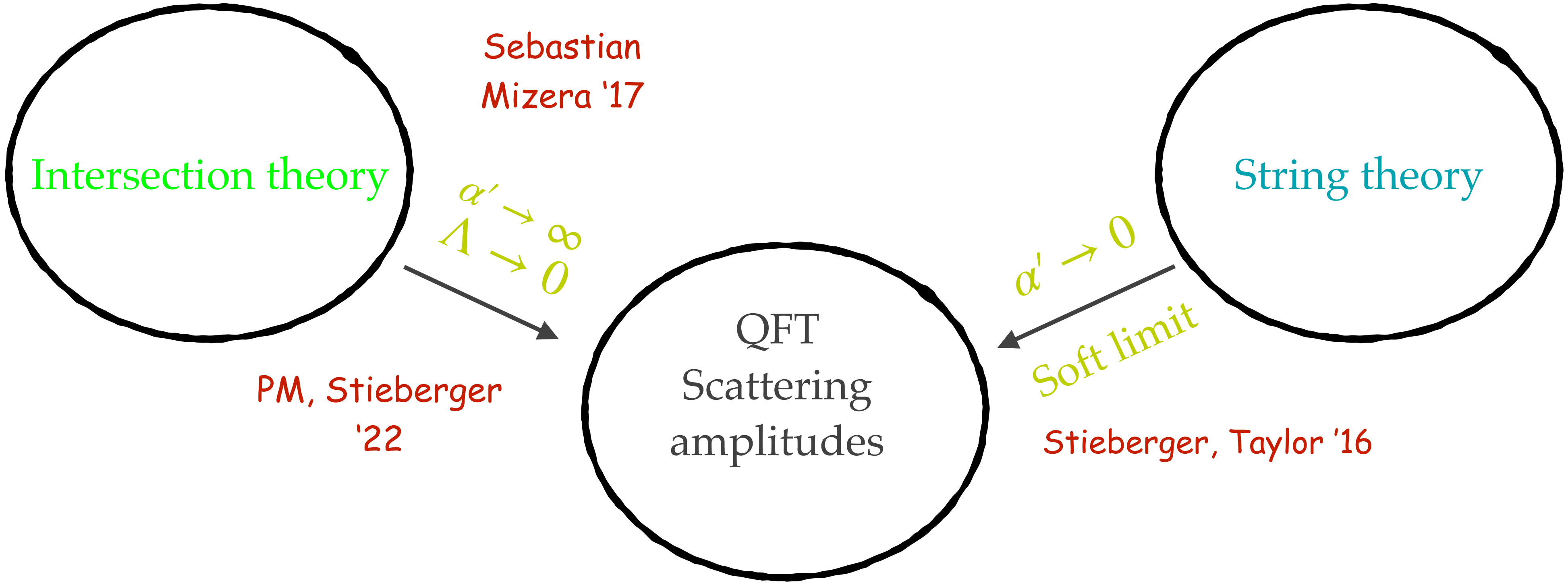

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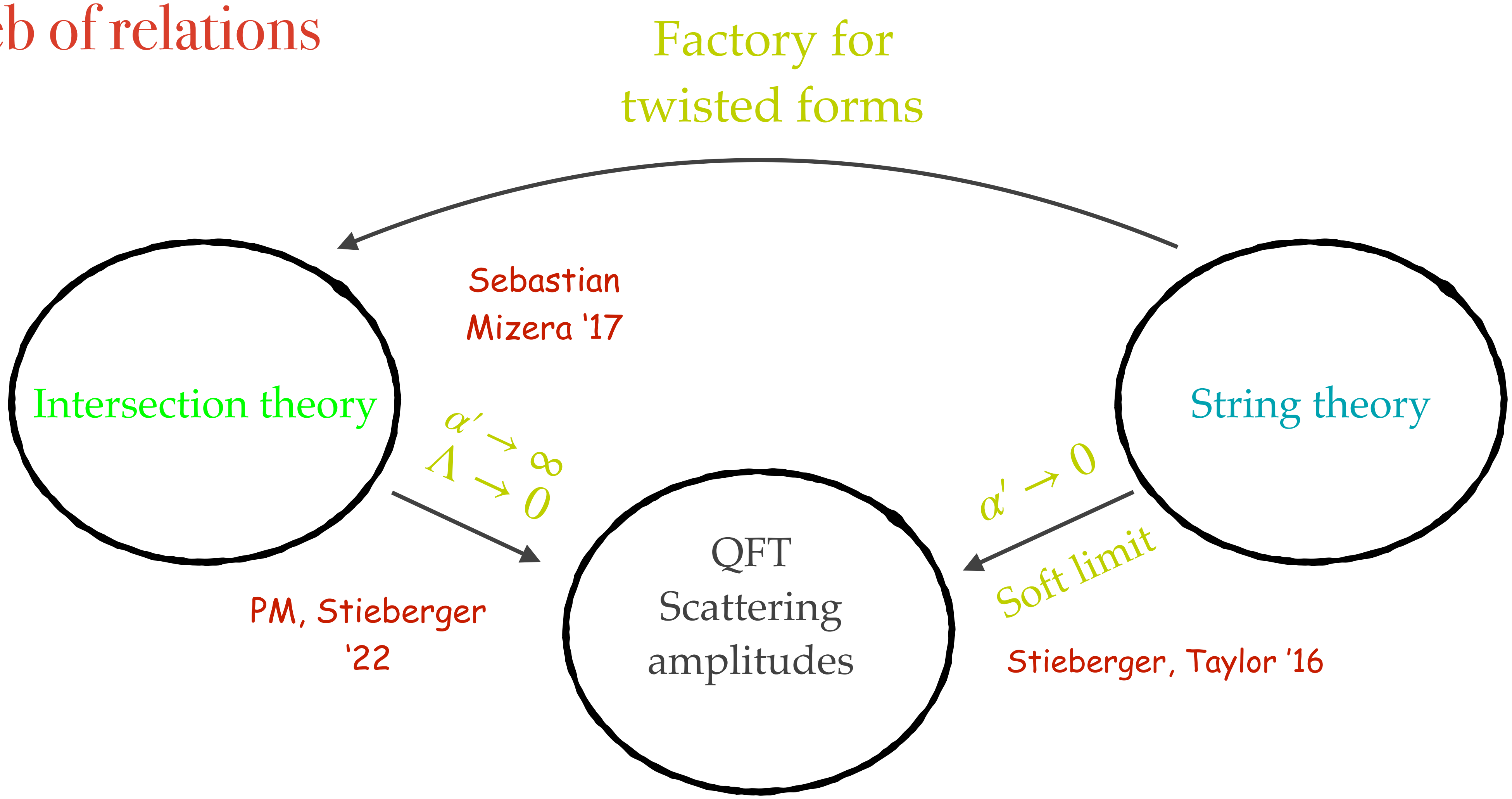

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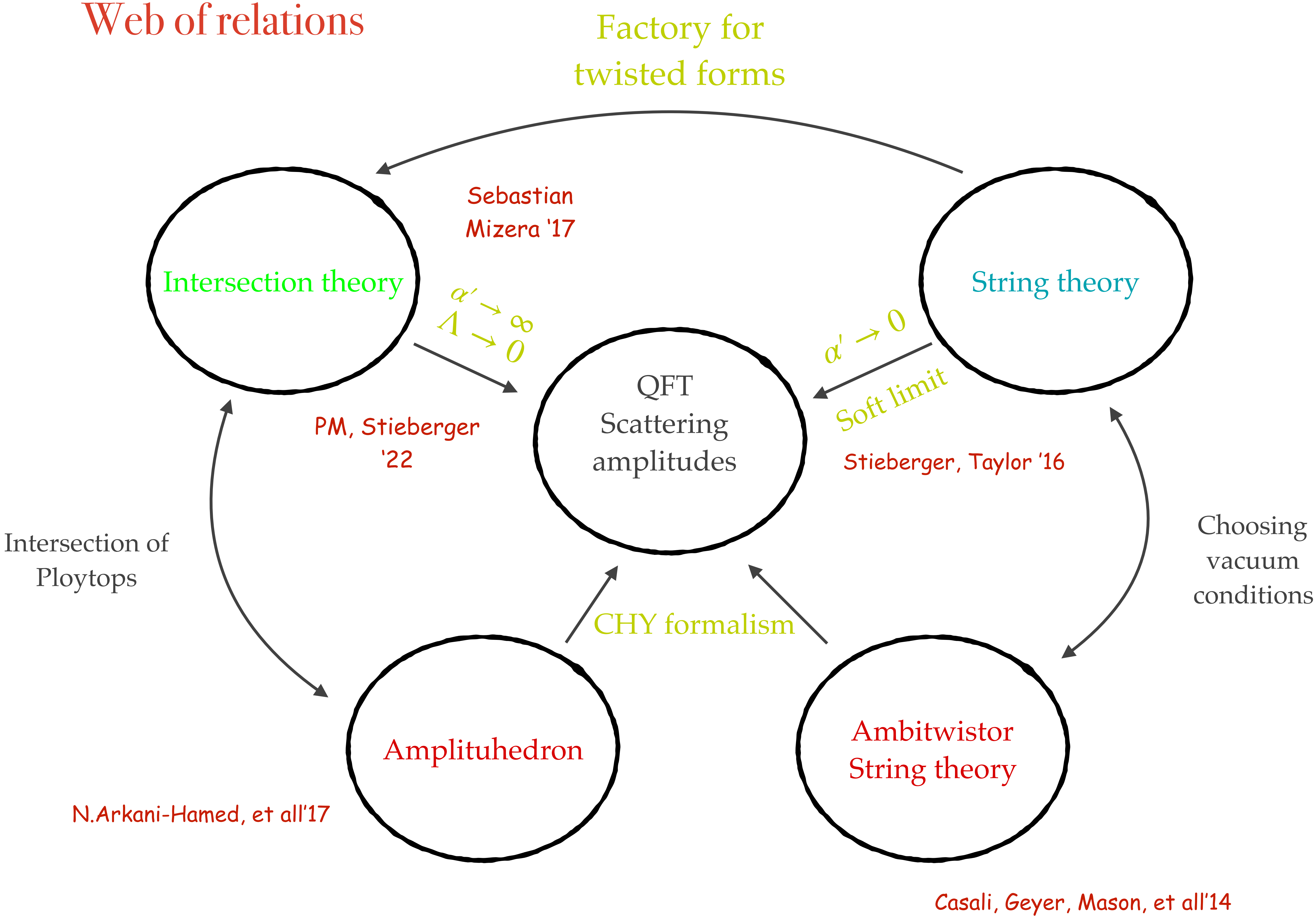
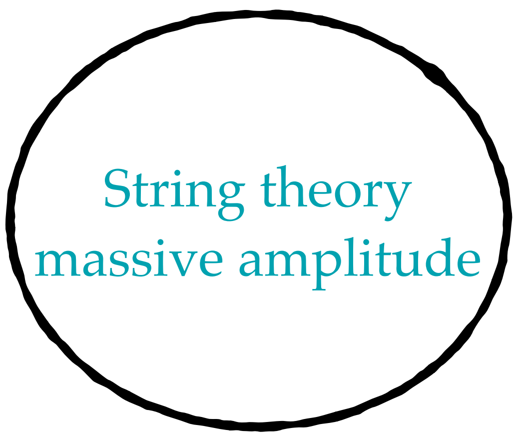


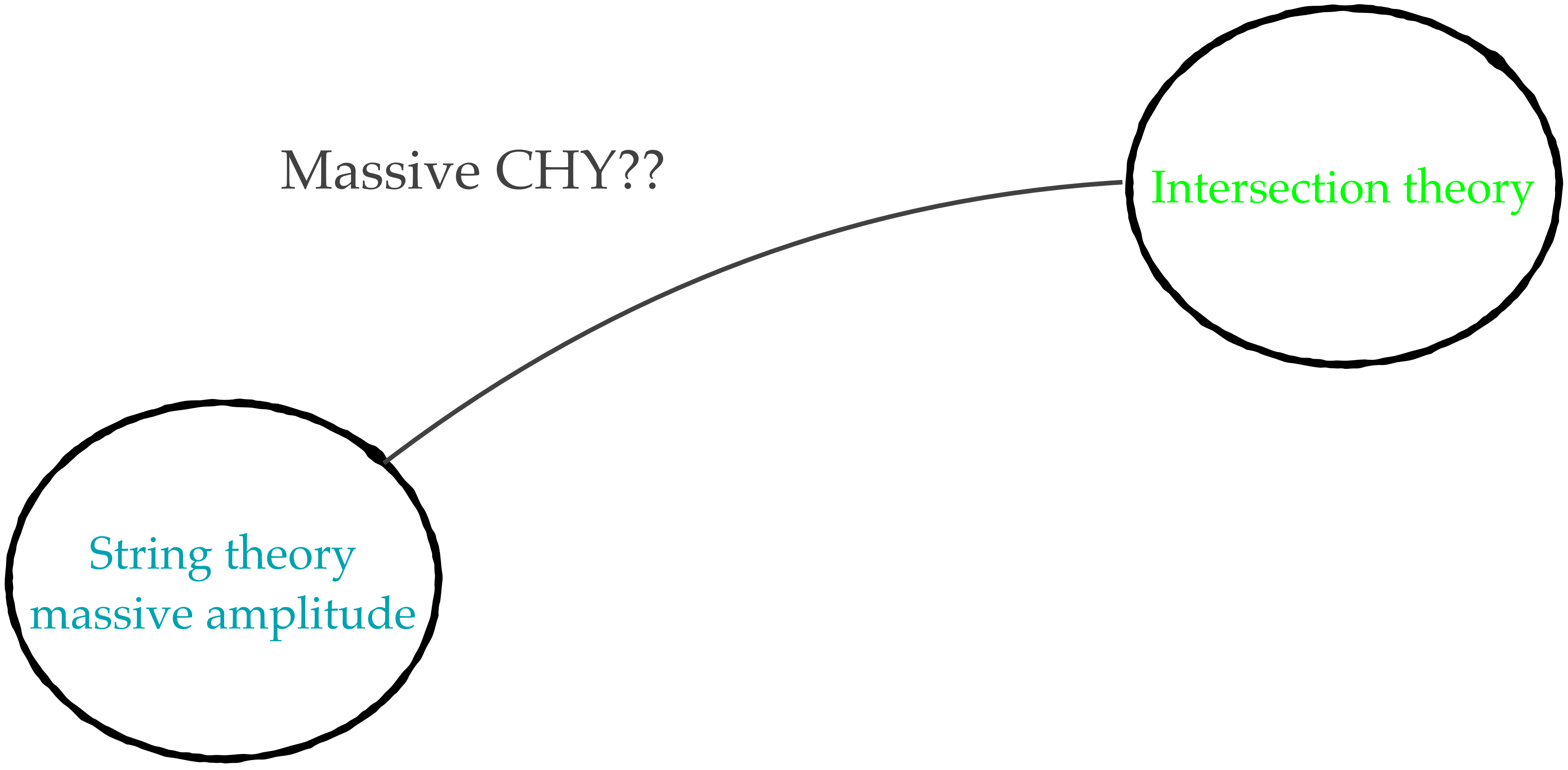
Massive amplitudes

Massive amplitudes

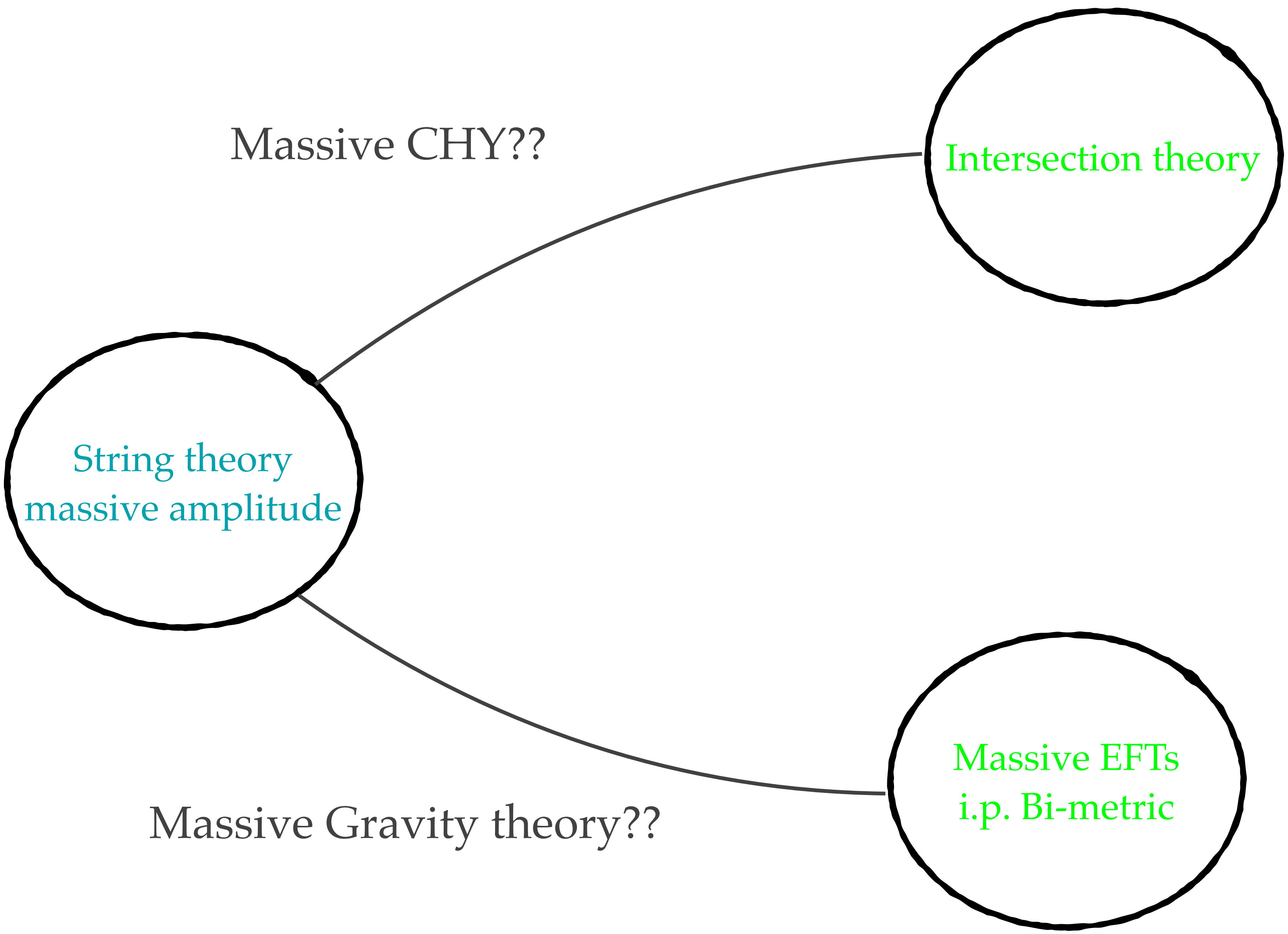


Massive amplitudes

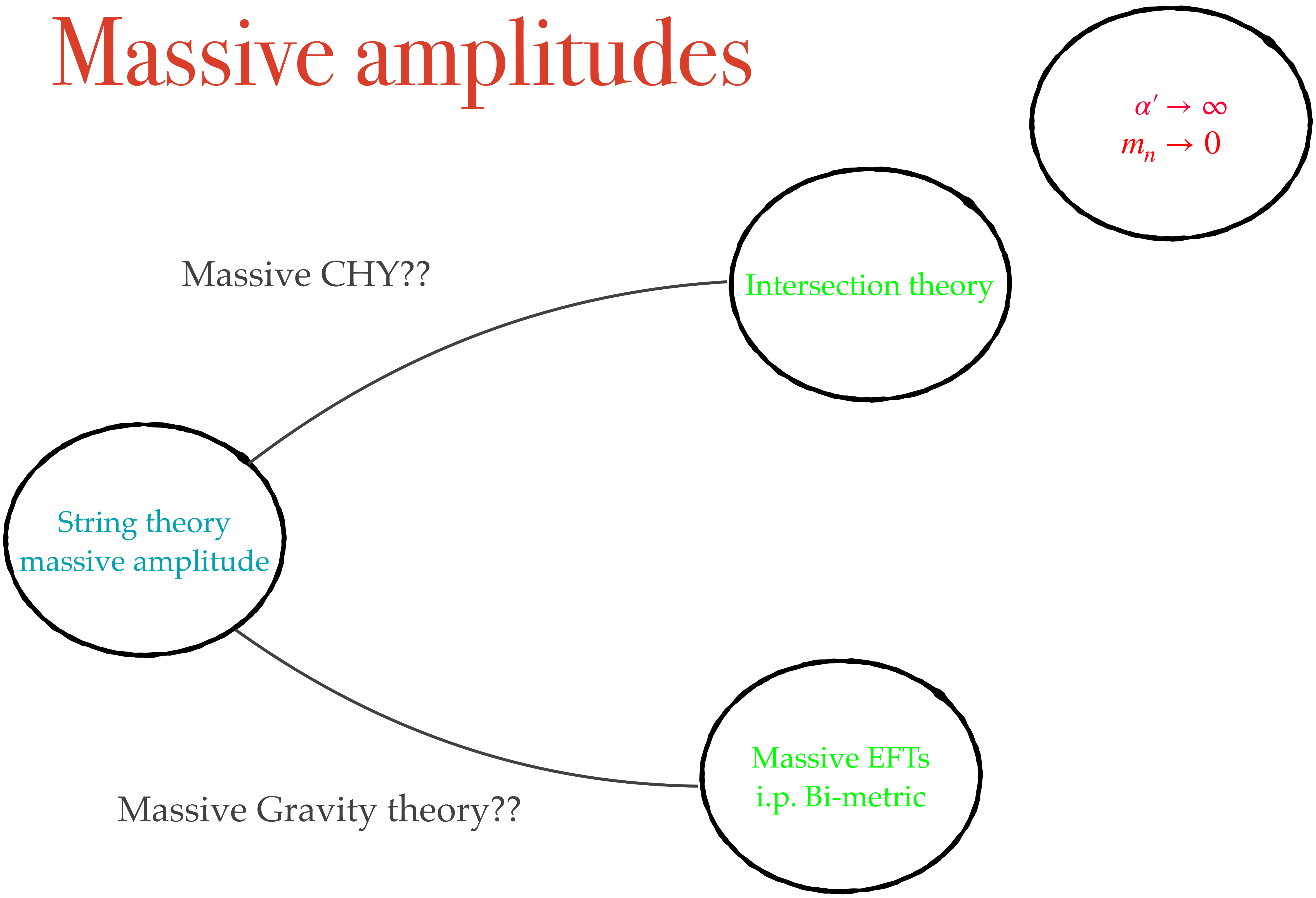
Massive CHY??



Massive amplitudes



Massive amplitudes



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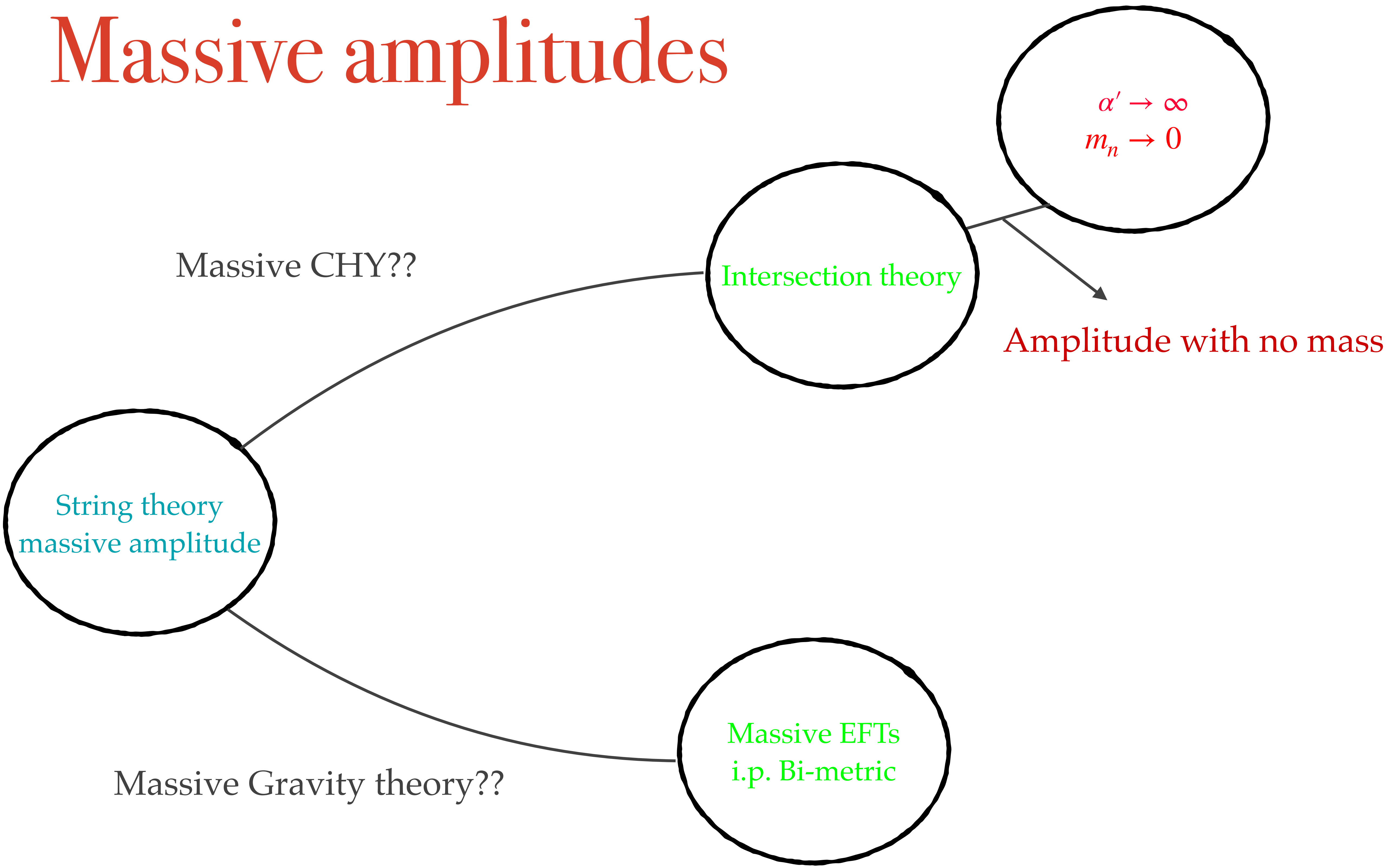
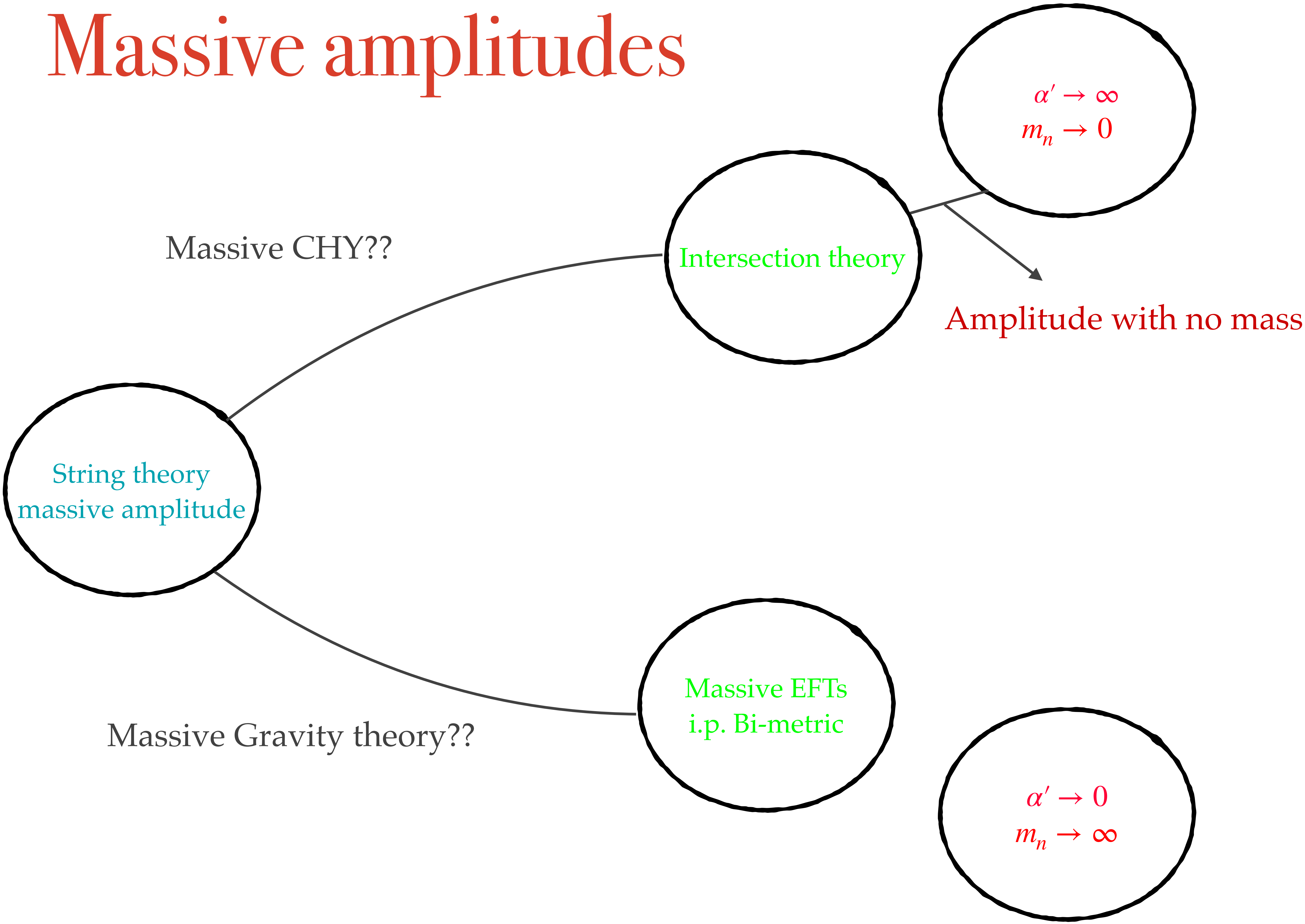
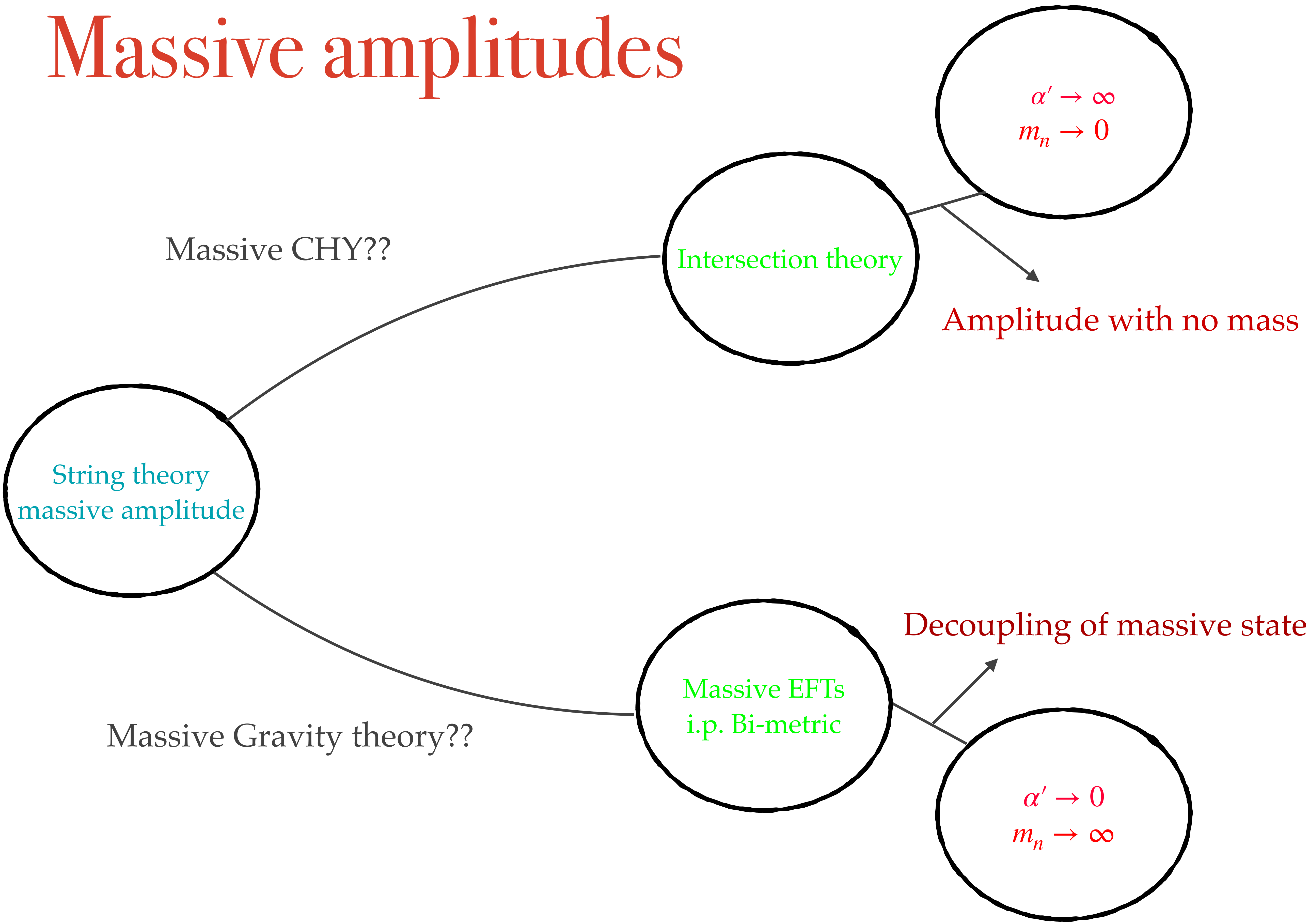


Massive amplitudes



Massive amplitudes



Thank you for your attention

Symbology for elliptic multiple polylogarithms

Roger Morales Espasa

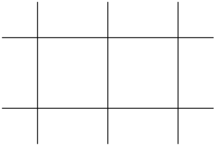
Niels Bohr Institute

Amplitudes 2022 Summer School - Gong Show

July 3, 2022

Research program

Starting at 2-loops, Feynman diagrams involve integrals over elliptic curves:



$$\Longrightarrow \qquad \int_0^\infty \frac{dx}{y}\, \mathcal{G}_3(x,y), \quad \text{where} \quad y^2 = P_4(x).$$

Research interests:

- Characterize the elliptic integrals in terms of their symbol and study its properties.
- Bootstrap the result directly from the elliptic symbol.
- Proceed to higher-loop topologies, involving Calabi-Yau varieties.

$\hookrightarrow$ In collaboration with: A. McLeod, A. Spiering, M. von Hippel, M. Wilhelm, Q. Yang, C. Zhang.

Rational Parts of Two Loop QCD Amplitudes

Siddharth Pandey

Swansea University

August 2022

Introduction

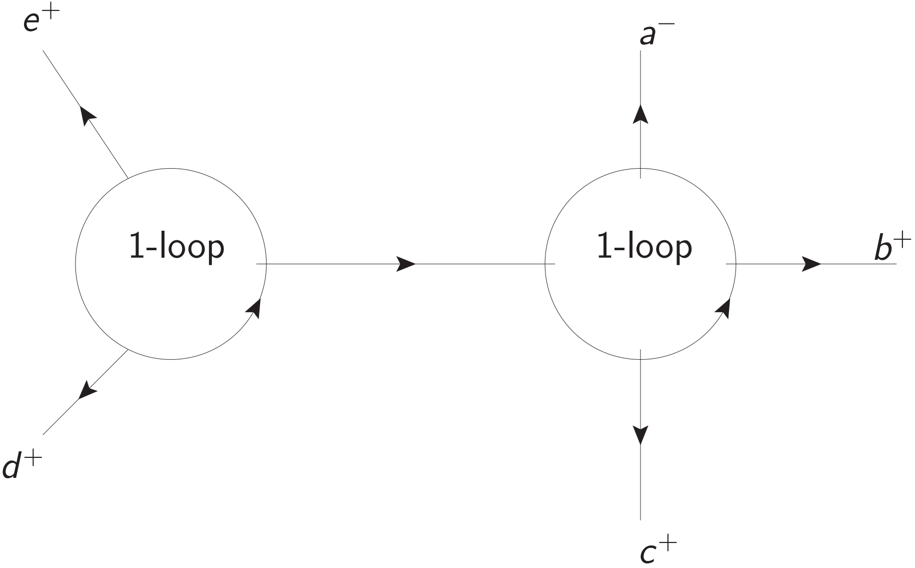
- We are currently calculating the two-loop five-point single-minus pure gluon QCD amplitude, $A_5^{(2)}(a^-, b^+, c^+, d^+, e^+)$, using analytical methods
- We use four-dimensional unitarity to calculate the cut constructible pieces of the amplitude
- This simplifies the cuts, but at the cost of missing rational terms
- We use BCFW recursion over the rational parts of lower amplitudes to reconstruct the missing rational terms

$$\frac{1}{2\pi i}\oint \frac{R(z)}{z} = R(0) + \sum_{z_j \neq 0} \mathrm{Res}\left[\frac{R(z)}{z}\right]\Big|_{z_j}$$

Double Poles

- The problem with naively applying BCFW recursion to a two-loop amplitude is that such amplitudes can have factorisations that give double poles
$$R(z) = \frac{c_2}{(z-z_i)^2} + \frac{c_1}{z-z_i} + \text{finite},$$
- This means that we may have subleading "poles under the pole" for which there are no general factorisation theorems

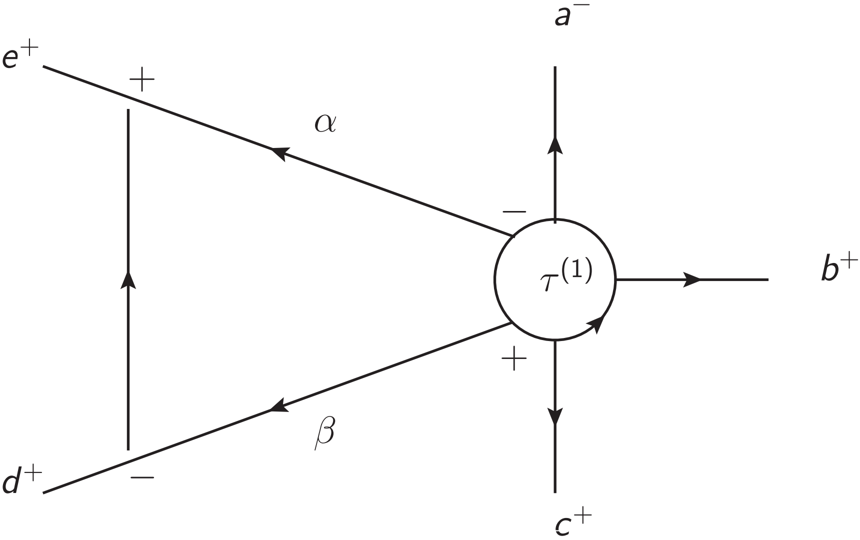
Double Poles



Augmented Recursion

- To include the subleading contributions we use "augmented Recursion"
- We "open" one of the loops and construct a 1-loop "current", $\tau^{(1)}(\beta, \alpha, a^-, b^+, c^+)$ with two off-shell legs (alpha,beta)

Current



Augmented Recursion

- We require that the current satisfies two conditions:
  1. It captures the leading singularity that we expect from the factorisation
  2. As the take the off-shell legs, on-shell, we recover the rational part of the amplitude $R^{(1)}(\beta, \alpha, a^-, b^+, c^+)$
- We must find this current and perform the loop integral, extracting the rational contributions, so that applying BCFW recursion on this expression will yield the correct rational part of the amplitude $A_5^{(2)}(a^-, b^+, c^+, d^+, e^+)$.
- We expect that the methods used to calculate the 5-point amplitude may be readily adapted to 6-point, 7-point.

Soft factors and interaction vertices in the light-cone gauge

Saurabh Pant

IISER PUNE

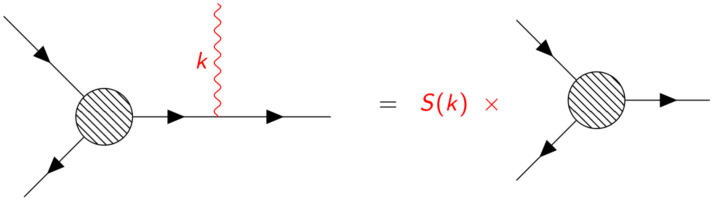
Work in progress

Amplitudes 2022

- Consider a $(n+1)$-particle amplitude $M_{n+1}(p_1, p_2, \cdots, p_n, k)$ with an external boson having momentum k.
- Weinberg's theorem states that in the soft limit $k \to 0$, the amplitude, at leading order, factorizes as follows

$$\lim_{k\to 0} M_{n+1}(p_1, p_2, \cdots, k) = S \times M_n(p_1, p_2, \cdots p_n) + \mathcal{O}(k^0)\,.$$

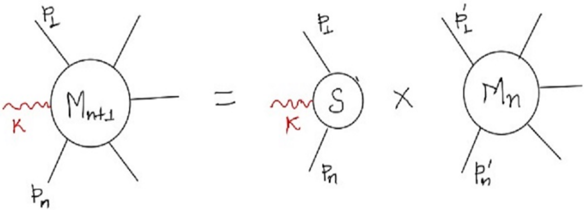
(Weinberg, 1965)



where S is the universal "soft factor" associated with the emission of the boson.

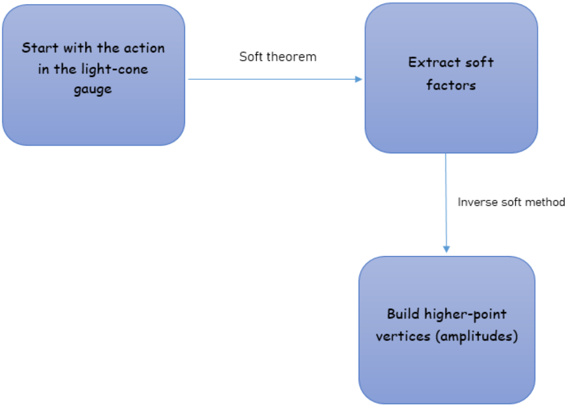
- Construct $(n+1)$-particle amplitude by multiplying a n-particle amplitude with the soft factor.

$$M_{n+1}(p_1, p_2, \cdots, p_n, k) = S(p_n, k, p_1) \times M_n(p_1', p_2, \cdots p_n') \,.$$



(Nima Arkani-Hamed, et al, 2010)

Work done



- First as an example, we constructed the higher-point MHV interaction vertices for gauge theory and gravity using the inverse soft method .
- Test the idea of the inverse soft method for $\mathcal{N} = 4$ super Yang-Mills in the light-cone superspace and then build the quartic vertex for $\mathcal{N} = 8$ supergravity.

Department of Theoretical Physics
Uppsala University, Sweden

UPPSALA
UNIVERSITET

Black Holes and Massive Higher Spins

paolo.pichini@physics.uu.se

Based on: **2107.14779, 2207.03947**

August 3, 2022

Black-Hole Amplitudes

Three-point amplitude = stress-energy tensor $T^{\mu\nu}$

$$\mathcal{L}(\phi^s, \bar{\phi}^s, h) = \mathcal{L}_{free}(\phi^s, \bar{\phi}^s) + h_{\mu\nu} T^{\mu\nu}(\phi^s, \bar{\phi}^s) + \mathcal{O}(h^2)$$

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Kerr $T^{\mu\nu}$ into on-shell graviton $\epsilon_{\mu\nu}$ [Vines; Guevara, Ochirov, Vines]

$$\varepsilon_{\mu\nu}(k) T^{\mu\nu}_{\text{BH}}(k) = (\varepsilon_k \cdot p)^2 \exp\left(\frac{k_\mu S^\mu_{\text{BH}}}{m}\right)$$

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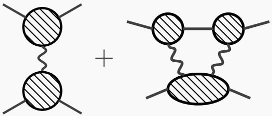
Compute binary black-hole observables:

$V_{eff} \sim$



$+ \ldots$

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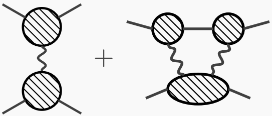
$V_{eff} \sim$  $+ \ldots$

❓ Reproduce from QFT amplitudes?

Our Results

Kerr three-point amplitudes: [Arkani-Hamed,Huang,Huang;Guevara,Ochirov,Vines;Chung,Huang,Kim,Lee;...]

$$M_s \equiv M(1\phi^s, 2\phi^s, 3h) = (\varepsilon_3 \cdot p_1)^2 [\mathbf{12}]^{2s}$$

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Covariant form: $\Rightarrow$ Lagrangian

$$\sum_{s=0}^{\infty} M_s = M_0 + M_1 + \frac{M_2 - (\varepsilon_1 \cdot \varepsilon_2)^2 M_1}{(1 + \varepsilon_1 \cdot \varepsilon_2)^2 + \frac{2}{m^2} \varepsilon_1 \cdot p_2 \, \varepsilon_2 \cdot p_1}$$

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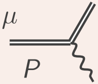
Unitarity $\Rightarrow$ Current constraint [Ferrara,Porrati,Telegdi(1992);Cucchieri,Porrati,Deser(1995)]

$$P \cdot J = \mathcal{O}(m), \;\; J^\mu \equiv$$



- Gauge invariance in the $m \to 0$ limit
- Kerr amplitudes unique up to $s = 5/2$ gravity

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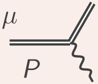

- Gauge invariance in the $m \to 0$ limit
- Kerr amplitudes unique up to $s = 5/2$ gravity

  $\Rightarrow$ Kerr $=$ elementary higher-spin particle

Our Results

Spin-3/2 Gauge Theory

$$\mathcal{L} = \mathcal{L}_{min} + \bar{\psi}_\mu \left(F^{\mu\nu} - \frac{i}{2}\gamma_5 \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \right) \psi_\nu$$

$$\mathcal{A}_4 = \frac{[4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle}{[4|p_1|3\rangle} \left(\frac{([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^2}{(s_{13}-m^2)(s_{14}-m^2)} - \frac{[\mathbf{1}4][\mathbf{2}4]\langle \mathbf{1}3\rangle\langle \mathbf{2}3\rangle}{m^4} \right)$$

Our Results

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Spin-5/2 Gravity

$$\mathcal{L} = \mathcal{L}_{min} + \bar{\psi}_{\mu\rho} \left(R^{\mu\nu\rho\sigma} - \frac{i}{2}\gamma_5 \epsilon^{\rho\sigma\alpha\beta} R^{\mu\nu}{}_{\alpha\beta} \right) \psi_{\nu\sigma}$$

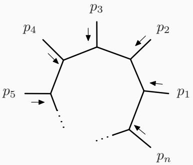
$$\mathcal{M}_4 = \frac{[\mathbf{4}1]\langle 3\mathbf{2}\rangle + [\mathbf{4}2]\langle 3\mathbf{1}\rangle}{[4|p_1|3\rangle} \left(\frac{\left([\mathbf{4}1]\langle 3\mathbf{2}\rangle + [\mathbf{4}2]\langle 3\mathbf{1}\rangle\right)^4}{s_{12}(s_{13}-m^2)(s_{14}-m^2)} - \frac{\left([\mathbf{14}][\mathbf{24}]\langle \mathbf{1}3\rangle\langle \mathbf{2}3\rangle\right)^2}{m^6} \right)$$

Thank you!

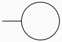
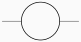
Intersection Theory and Generalized Unitarity

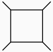
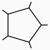
Andrzej Pokraka

Integral reduction via intersection theory



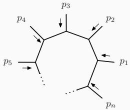
$= \mathcal{I}_{\text{one-loop } n\text{-pt}}$

$= c_{\text{tad}}(\varepsilon)$  $+ c_{\text{bub}}(\varepsilon)$  $+ c_{\text{tri}}(\varepsilon)$ 

$+ c_{\text{box}}(\varepsilon)$  $+ c_{\text{pent}}(\varepsilon)$ 

How can algebraic geometry help?

intersection number

$c_\bullet = \Big\langle \mathcal{I}^\vee_\bullet \Big|$ 

$\Big\rangle = \int_{\mathbb{C}^5} \mathcal{I}^\vee \wedge \mathcal{I}_{\text{one-loop } n\text{-pt}}$

Now known for un-deformed FI

$\underbrace{(\ell_\perp^2)^{-\varepsilon}}_{\text{m.v. fn.}} \underbrace{\frac{\mathcal{N}\, \mathrm{d}\ell_\perp^2 \wedge \mathrm{d}^4\ell}{\ell_\perp^2 D_1^{\nu_1} \cdots D_n^{\nu_n}}}_{\text{s.v. diff. form}}$

Relative twisted cohomology ← Obj. s.t. int. makes sense

↓

Localized to GU cuts → GU in generic dimension

UNIVERSITÄT BONN
Bethe Center for Theoretical Physics

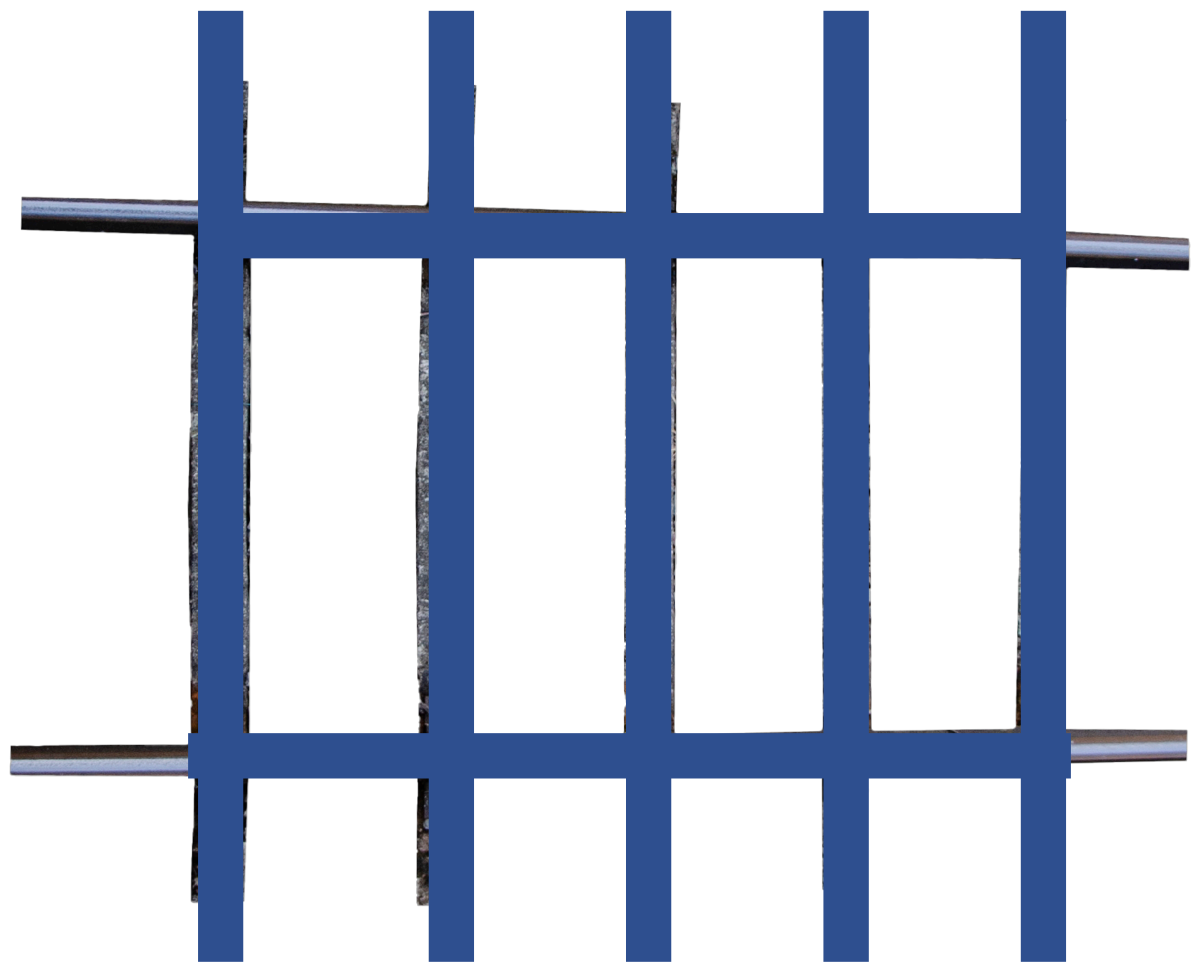
From Geometry to 2D Traintrack Integrals

Joint work with:
Claude Duhr, Albrecht Klemm, Florian Loebbert, Christoph Nega

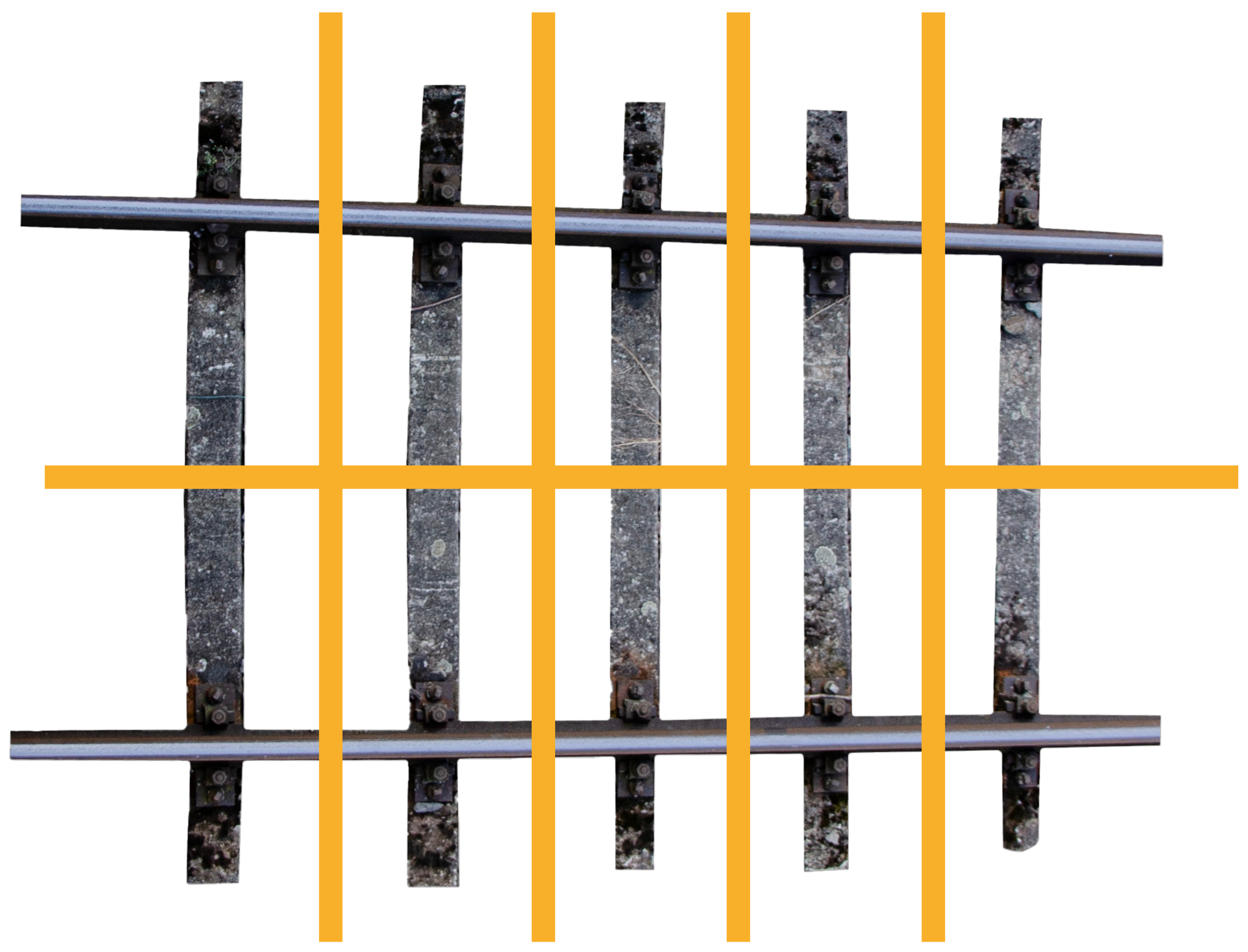
Franziska Porkert, Amplitudes 2022 Summer School, 1.8. - 7.8.2022

Traintrack integrals

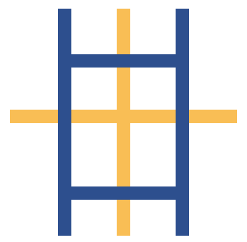
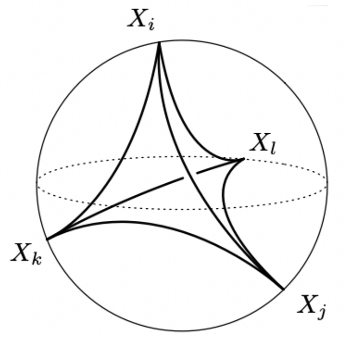
Traintrack integrals in momentum space



Traintrack integrals in dual position space



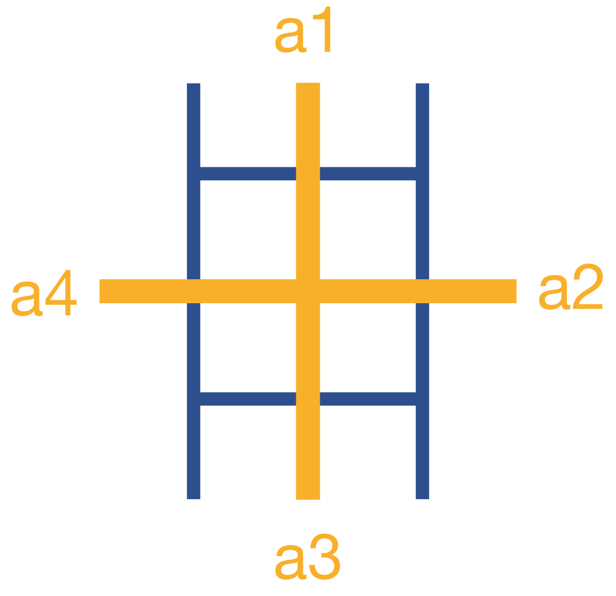
In 4 dimensions:

 $\sim \frac{1}{\sqrt{\triangle}} \mathrm{Vol}\Big($  $\Big)$ [Davydychev, Delbourgo; 1997]

Geometric interpretation for traintracks at higher loop orders?

Testcase: dual conformal traintracks in 2D

1 Loop [Corcoran, Loebbert, Miczajka; 2021]



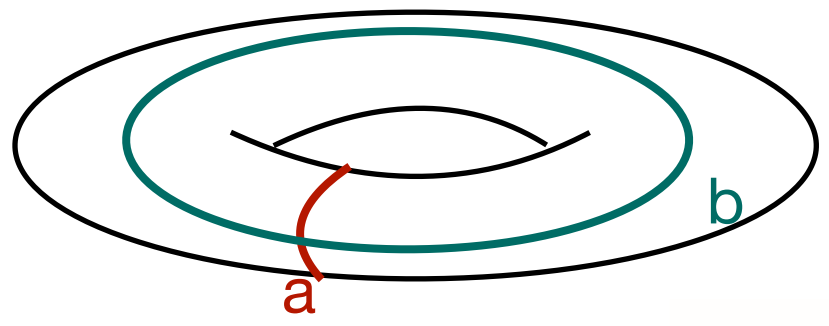
$$= \int \frac{d^2y}{|y-a_1|\,|y-a_2|\,|y-a_3|\,|y-a_4|} \quad \text{with } z = \frac{(a_1-a_2)(a_3-a_4)}{(a_1-a_3)(a_2-a_4)}$$

$$= \frac{4}{\pi} \frac{1}{|a_2-a_4||a_1-a_4|} \left[K(z)K(1-\bar{z}) - K(\bar{z})K(1-z)\right]$$

Elliptic integrals

Geometric Interpretation

$$\int \frac{d^2y}{|y-a_1||y-a_2||y-a_3||y-a_4|} \sim \frac{4}{\pi} \frac{1}{|a_2-a_4||a_1-a_4|} \mathrm{Vol}\Big(\quad\Big)$$

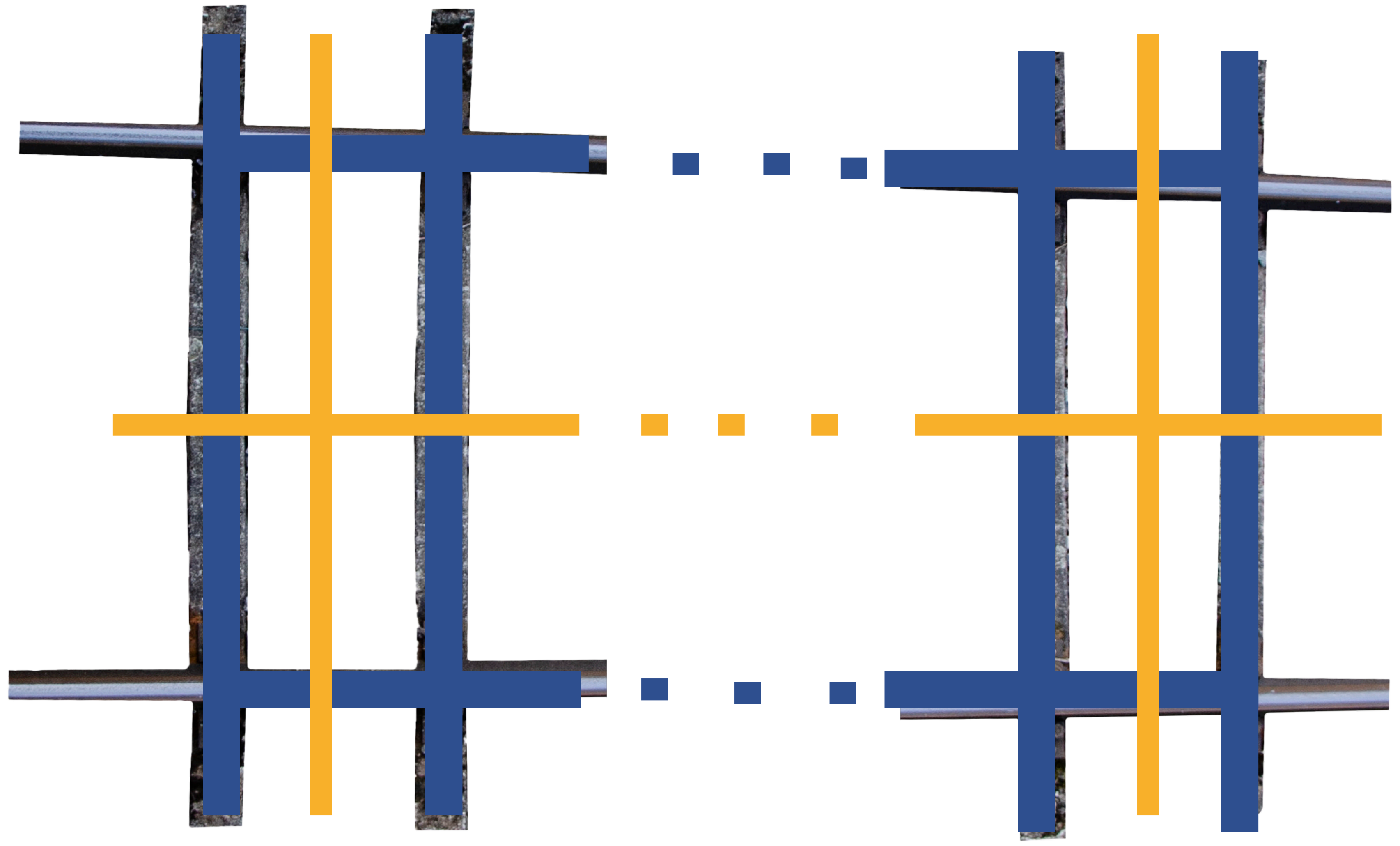


Main conjecture

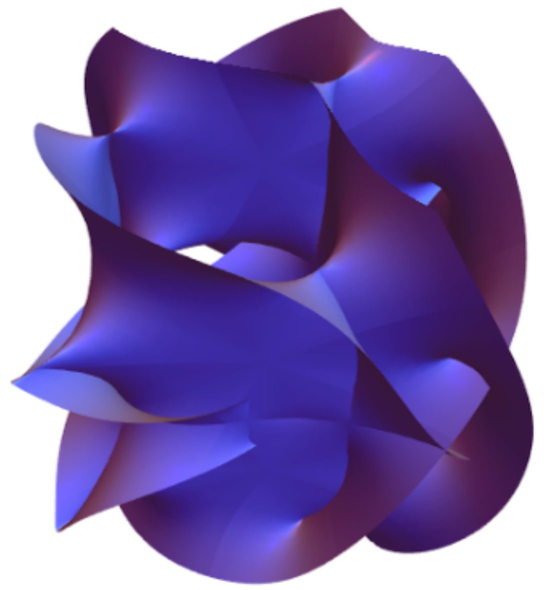
Geometric interpretation for traintracks at higher loop orders?

↓

L-Loop traintrack in 2D $\sim$ **Volume of a Calabi-Yau L-fold**

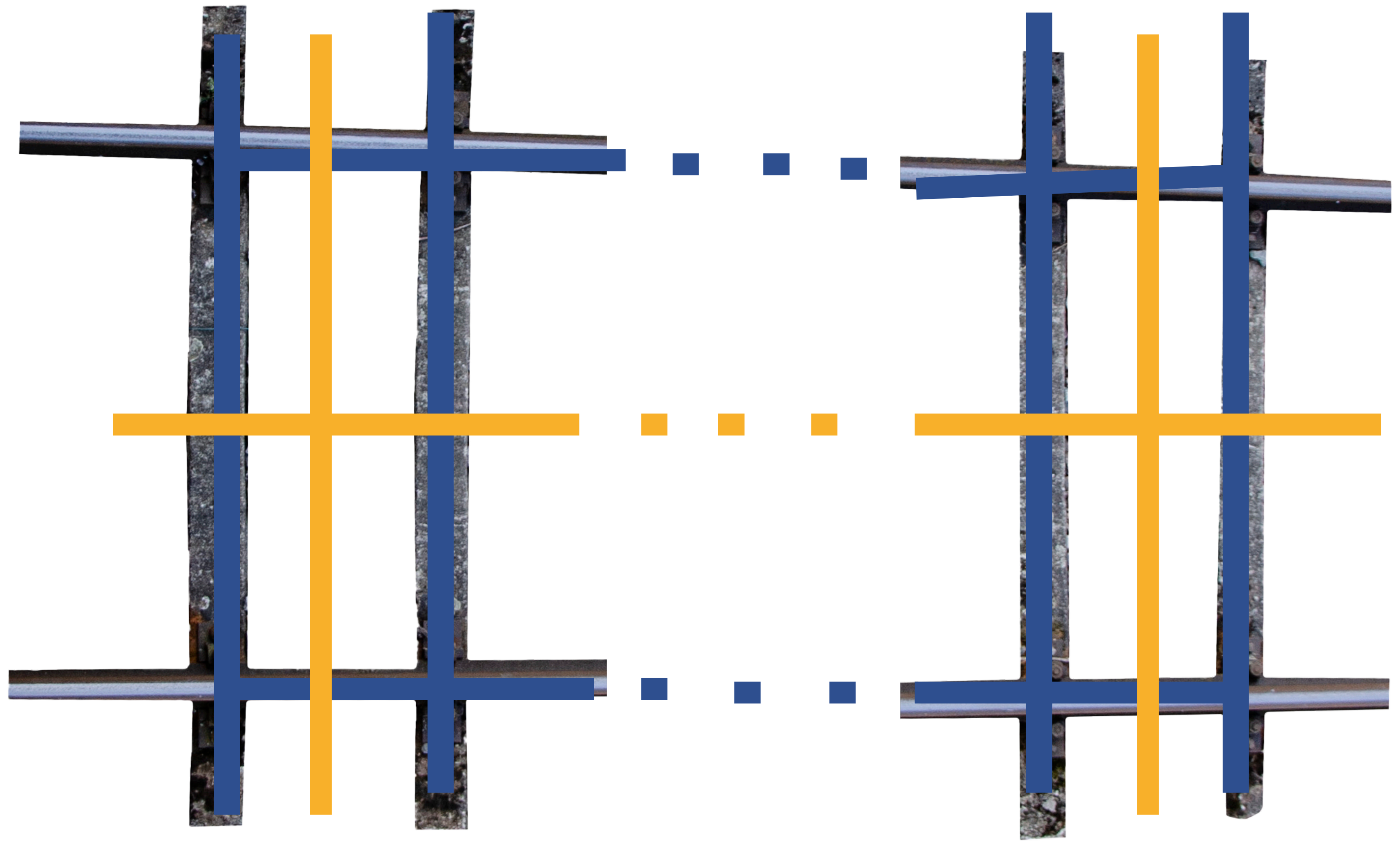


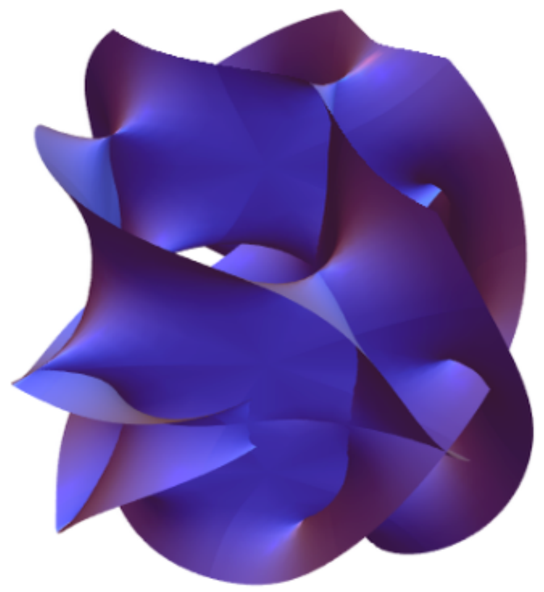
$$\sim \frac{1}{\sqrt{\Delta}} \mathrm{Vol}\Big($$



$$\Big)$$

Main conjecture + results



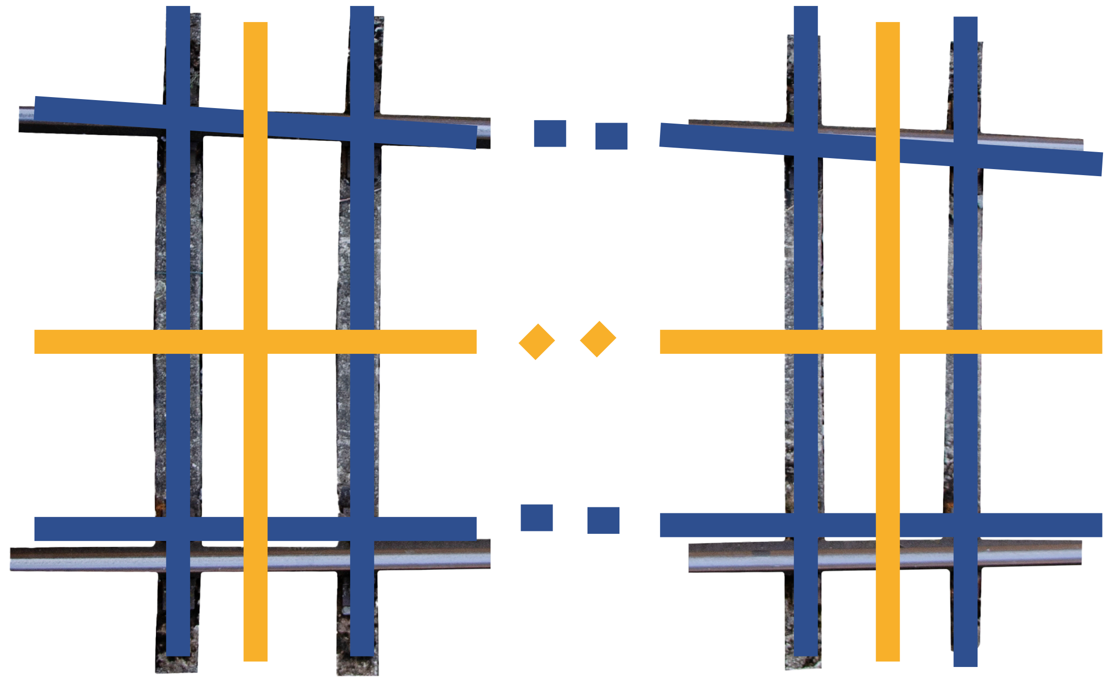
$\sim \frac{1}{\sqrt{\Delta}} \mathrm{Vol}\Big($  $\Big)$

- We reproduce known results for 2D traintrack integrals depending on one variable to high loop order.
- We obtain new results for the fully general multi-scale 2D traintrack up to three loops.

In Progress: Extension to fishnet graphs.

More details + interesting properties

→ Quick and precise calculation of 2D traintrack integrals
→ Relation between geometry and integrability



$$\sim \frac{1}{\sqrt{\Delta}} e^{-K} = \frac{1}{\sqrt{\Delta}} \bar{\Pi} \Sigma \Pi$$

Intersection Matrix

From periods via Griffiths transversality

Loops	1	2	3	...
Geometry	Elliptic Curve	K3 surface	CY 3-fold	...
Solutions	1|1	1|3|1	1|5|5|1	...

Vector of Calabi-Yau periods

Solutions of the **Picard-Fuchs differential ideal.**

- Remarkable: Picard-Fuchs differential ideal equivalent to Yangian generators (and rotations thereof)
- Solutions can be found with the Frobenius method
- Known solution pattern at MUM point

Classical physics and Quantum amplitudes

Alasdair Ross

University of Edinburgh

[2212.07556] (§2.3,§3.1)

KMOC setup

- Study classical observables like ΔP^{μ}, Δθ, ΔS^{μ} in KMOC setup

$$\Delta P^{\mu} = \langle\psi|\, S^{\dagger} P^{\mu} S\, |\psi\rangle - \langle\psi|\, P^{\mu}\, |\psi\rangle$$
$$= i\,\langle\psi|\, [P^{\mu}, T]\, |\psi\rangle + \langle\psi|\, T^{\dagger}[P^{\mu}, T]\, |\psi\rangle$$

Observables in terms of amplitudes for example,

$$\Delta P^{\mu} \sim q^{\mu} \int \mathcal{A} + \int l^{\mu} \mathcal{A}^{*} \mathcal{A}$$

Recipe:

- Make all factors of $\hbar$ explicit and rescale massless momenta $q \to \hbar\, \overline{q}$.
- Expand as series and take $\hbar \to 0$. Amplitudes are a series in $\hbar$.

- Consider some expectation value in the final state after scattering, say field strength, and/or products of operators

$$\langle F_{\mu\nu}\rangle = \langle\psi|\, S^{\dagger} F_{\mu\nu} S\, |\psi\rangle \qquad\qquad \langle F_{\mu\nu} F_{\rho\sigma}\rangle = \langle\psi|\, S^{\dagger} F_{\mu\nu} F_{\rho\sigma} S\, |\psi\rangle$$

Uncertainty

- In classical limit we should have negligible uncertainty,

$$\langle F_{\mu\nu} F_{\rho\sigma} \rangle = \langle F_{\mu\nu} \rangle \langle F_{\rho\sigma} \rangle + small$$

  - Puts constraints on the classically relevant amplitudes
  - Apply KMOC analysis

$$\langle F \rangle \sim \hbar^{7/2} \int g^3 \bar{\mathcal{A}}_5 \sim + \ldots$$

$$\langle FF \rangle \sim \hbar^5 \left(\int g^4 \bar{\mathcal{A}}_6^{(0)} + g^6 \bar{\mathcal{A}}_6^{(1)} + \ldots \right)$$

  - We learn 2 things…
    - (n-1) loops is classically relevant – here one loop
    - The scaling in hbar is $\mathcal{A}_{4+n,0} \sim \hbar^{-n/2-3}$ (non trivial checks)
  - Can be generalised with other operators too
  - Requires coherent states generally to ensure this factorisation

Thank you

BCJ RELATIONS IN $AdS_5 \times S^3$

Michele Santagata

Amplitudes 2022, Prague

Scattering of four supergluons in

$AdS_5 \times S^3$

AdS/CFT

4-pt function of half-BPS operators in

$4d\ \mathcal{N} = 2$ with flavour group G_F

The tree-level dynamics is remarkably simple, because of a hidden *8d conformal symmetry* [Alday, Behan, Ferrero, Zhou]

General structure of 4pt functions

Adjoint index of the flavour group

$$\langle \mathcal{O}_{p_1}^{I_1}(x_1, y_1)\, \mathcal{O}_{p_2}^{I_2}(x_2, y_2) \mathcal{O}_{p_3}^{I_3}(x_3, y_3) \mathcal{O}_{p_4}^{I_4}(x_4, y_4) \rangle = G_{free} + G_{int}$$

Scaling dimension

spacetime (or AdS) variable

R-symmetry+additional flavour (sphere) variables

$$G_{int} = \mathcal{P}\, \mathcal{I}\, \mathcal{A}^{I_1 I_2 I_3 I_4}(U, V, \tilde{U}, \tilde{V})$$

- The non-trivial part is the *reduced correlator*, $A(U, V, \tilde{U}, \tilde{V})$ and is a function of charges p_i, conformal and internal cross ratios:
  $U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}, \quad \tilde{U} = \frac{y_{12}^2 y_{34}^2}{y_{13}^2 y_{24}^2}, \tilde{V} = \frac{y_{14}^2 y_{23}^2}{y_{13}^2 y_{24}^2}$
- $\mathcal{A}$ is best written in terms of a generalised Mellin transform $\mathcal{M}(s, t, \tilde{s}, \tilde{t})$ which depends on AdS variables s, t, sphere variables $\tilde{s}, \tilde{t}$ as well as charges p_i [Drummond, Glew, M.S.]

Flat space amplitude

$$A(1234) = \frac{1}{s\,t}$$

$$s + t + u = 0$$

BCJ in flat space

$$s\,A(1234) = u\,A(1342)$$

$AdS_5 \times S^3$ amplitude

[Alday, Behan, Ferrero, Zhou; Drummond, Glew, M.S.]

$$\boldsymbol{s} = s + \tilde{s}, \qquad \boldsymbol{t} = t + \tilde{t}$$

$$\mathcal{M}(1234) = \frac{1}{(\boldsymbol{s}+1)\,(\boldsymbol{t}+1)}$$

$$\boldsymbol{s} + \boldsymbol{t} + \boldsymbol{u} = -3$$

BCJ in $AdS_5 \times S^3$

$$(\boldsymbol{s}+1)\,\mathcal{M}(1234) = (\boldsymbol{u}+1)\,\mathcal{M}(1342)$$

Note: they are the same for all KK modes!

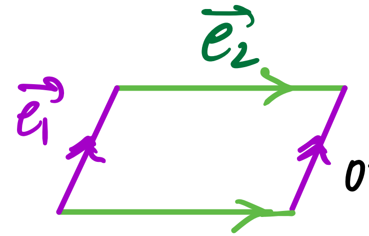
Thank you!

FLAVOR SYMMETRIES ON MAGNETIZED TORI

Start with a SUSY! theory in 6 dimensions:

T^2 $+ \mathcal{M}_4$

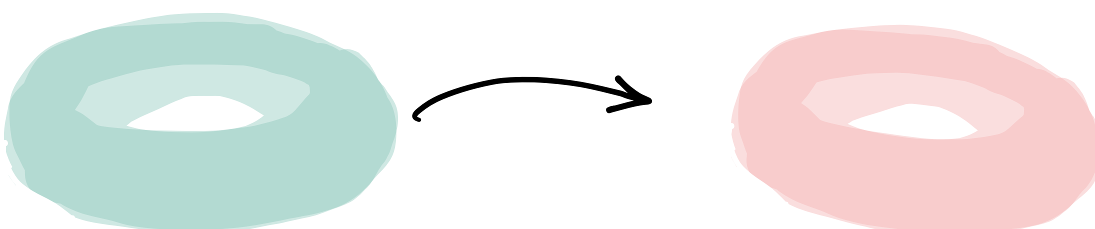
T^2 is generated by:



on $\mathbb{C}$

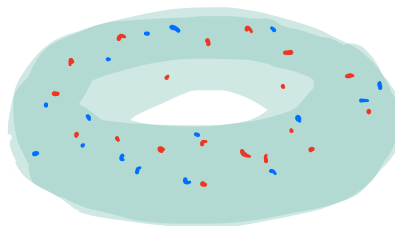
Define $\tau = \frac{e_2}{e_1}$ → governs the physical stuff, upto coordinate transformations:

$$\tau \to \frac{a\tau+b}{c\tau+d} \qquad a,b,c,d \in \mathbb{Z}, \; ad-bc=1$$



Shouldn't change physics!

Sprinkle some magnetic flux:



Solve the Dirac equation in magnetic flux background and get some zero-modes:

quantized flux!

$$\psi^{j,M} = \mathcal{N} e^{i\pi M \frac{\mathrm{Im}\, z}{\mathrm{Im}\, \tau}} \, \theta\begin{bmatrix} j/M \\ 0 \end{bmatrix}(Mz, M\tau)$$

$$\psi^{j,M} \xrightarrow[\text{transforma}^{\text{n}}]{\text{under modular}} (c\tau+d)^{k} \, (\rho(\gamma))^{jk} \, \psi^{k,M}$$

automorphy factor!

irrep under modular group $SL(2,\mathbb{Z})$

Start with U(N) gauge theory.

Add flux F:

$$F_{z\bar{z}} = \begin{pmatrix} m_a \mathbb{1}_{N_a \times N_a} & 0 & 0 \\ 0 & m_b \mathbb{1}_{N_b \times N_b} & 0 \\ 0 & 0 & m_c \mathbb{1}_{N_c \times N_c} \end{pmatrix}$$

$$U(N) \longrightarrow U(N_a) \times U(N_b) \times U(N_c)$$

coupling b/w 2 fermions & a boson ← Yukawas are an integral over our zero modes!

$$Y_{ijk} \propto \int d^2z \, \psi^{i,I_1} \psi^{j,I_2} (\psi^{k,-I_3})^*$$

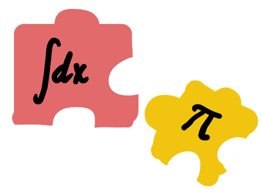
→ Looks complicated!

Use properties of theta function and Euler's theorem:

$$a^{\phi(n)} = 1 \quad \text{for } a, n \text{ coprime}$$

$\phi(n)$ → Totient function; counts number of numbers prime upto n

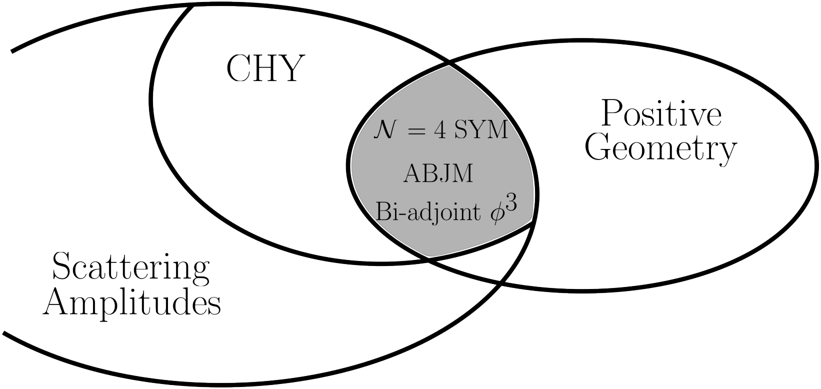
Y looks simple now! (Cute math puzzle!)

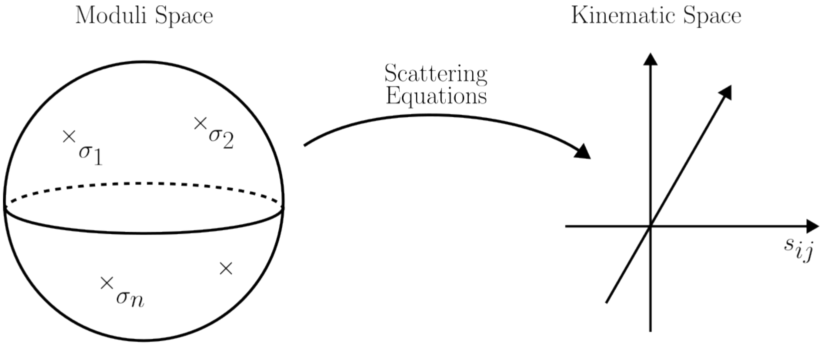


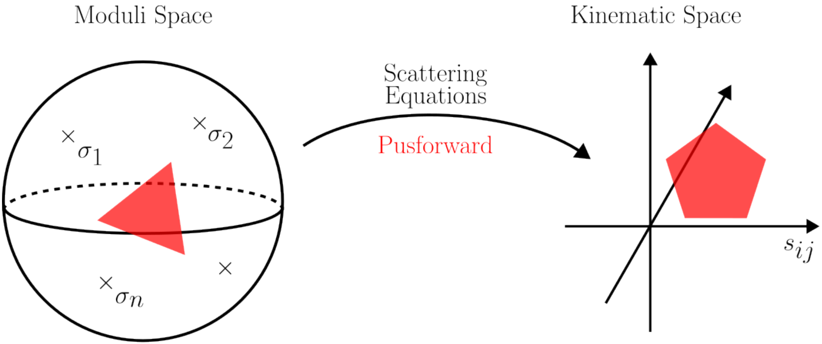
$$Y \propto \theta\begin{bmatrix} \hat{\alpha} \\ \hline \lambda \\ 0 \end{bmatrix}(\lambda\tau)$$ → Transforms same way as ψ's!

↓

Trivial to figure out symmetry group!


CHY
Positive Geometry
$\mathcal{N} = 4$ SYM
ABJM
Bi-adjoint ϕ^3
Scattering Amplitudes


Moduli Space
Kinematic Space
Scattering
Equations
σ_1
σ_2
σ_n
s_{ij}



[2206.14196] with Łukowski and Moerman

Feynman Integrals: algebra, analysis and combinatorics

Feynman integrals satisfy GKZ A-hypergeometric systems.

Systems of PDEs with a purely combinatorial description.

- Master integrals $\longleftrightarrow$ standard monomials in a Gröbner cone
- #(Master integrals) $\longleftrightarrow$ volume of polytope
- Landau equations $\longleftrightarrow$ principal A-determinant
- ϵ-expansion $\longleftrightarrow$ analytic continuation over faces of polytope
- (Canonical) differential equation $\longleftrightarrow$ Pfaffian system
- Symbol alphabet $\longleftrightarrow$ factors of prinipal A-determinant

Integrated correlators in $\mathcal{N} = 4$ SYM and periods

arXiv:2203.01890 [hep-th]

Shun-Qing Zhang

with Congkao Wen

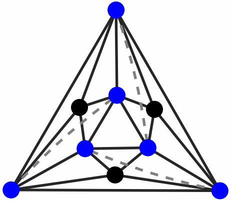
Queen Mary
University of London

Amplitudes Summer School

Prague, Aug 2022

Motivation

- Study perturbative aspects of **4-pt integrated correlators** using standard Feynman diagram.
- In perturabation, they are **periods** of conformal graphs $f_\alpha^{(L)}$



e.g. $f_4^{(5)}$

- `HyperInt` [Panzer], `HyperlogProcedures` [Schnetz] evaluate periods.
- Results: agree up to $\boldsymbol{L = 4}$ (planar) & predicts an $L = 5$ period.

Integrated Correlators in $\mathcal{N} = 4$ SYM

Consider: $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle = \text{free} + \mathcal{I}_4\, \mathcal{T}_N(U, V, \tau, \bar{\tau})$

$$\mathcal{C}_{G_N}(\tau, \bar{\tau}) = \int dU\, dV\, M(U, V)\, \mathcal{T}_N(U, V, \tau, \bar{\tau})$$

Integrated Correlators in $\mathcal{N} = 4$ *SYM*

Consider: $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle = \text{free} + \mathcal{I}_4\, \mathcal{T}_N(U, V, \tau, \bar{\tau})$

$$\mathcal{C}_{G_N}(\tau, \bar{\tau}) = \int dU\, dV\, M(U, V)\, \mathcal{T}_N(U, V, \tau, \bar{\tau})$$

- $\mathcal{C}_{G_N}(\tau, \bar{\tau})$ from *SUSY localisation* of $\mathcal{N} = 2^*$ SYM on S^4 [Pestun], [Binder, Chester, Pufu, Wang]

$$\mathcal{C}_{G_N}(\tau, \bar{\tau}) = \frac{1}{4}\Delta_\tau \partial_m^2 \log Z_{G_N}(\tau, \bar{\tau}, m)\Big|_{m=0}.$$

- Yang-Mills coupling: $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g_{YM}^2}$

Small g_{YM}, finite N

[Dorigoni, Green, Wen]

$$\mathcal{C}_{G_N N} = 4c\Big[\frac{3\zeta(3)a_{G_N}}{2} - \frac{75\zeta(5)a_{G_N}^2}{8} + \frac{735\,\zeta(7)a_{G_N}^3}{16} - \frac{6615\,\zeta(9)(1+P_{G_N})a_{G_N}^4}{32}\cdots\Big]$$

where $a_{G_N} = \lambda_{G_N}/(4\pi^2)$

- **'t Hooft coupling**:

$$\lambda_{SU(N)} = g_{_{YM}}^2\,N\,, \quad \lambda_{SO(n)} = g_{_{YM}}^2\,(n-2)\,, \quad \lambda_{USp(n)} = \frac{g_{_{YM}}^2\,(n+2)}{2}\,.$$

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- **Central charge**:

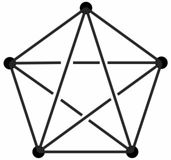
$$c_{SU(N)} = \frac{N^2-1}{4}\,, \quad c_{SO(n)} = \frac{n(n-1)}{8}\,, \quad c_{USp(n)} = \frac{n(n+1)}{8}\,.$$

- **Non-planar** factor starting at $\boldsymbol{L=4}$:

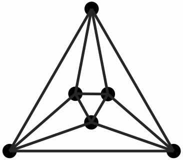
$$P_{SU(N)} = \frac{2}{7N^2}\,, \quad P_{SO(n)} = -\frac{n^2-14n+32}{14(n-2)^3}\,, \quad P_{USp(n)} = \frac{n^2+14n+32}{14(n+2)^3}\,.$$

Periods of $f^{(L)}(x_i)$

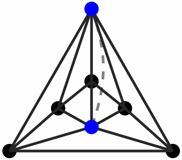
The L-loop integrand constructed by $f^{(L)}(x_i)$ [Bourjaily, Eden, Heslop, Korchemsky, Sokatchev, Tran ...]



$\mathcal{P}_{f^{(1)}} = 6\,\zeta(3)$



$\mathcal{P}_{f^{(2)}} = 6! \times \frac{1}{48} \times 20\zeta(5)$



$\mathcal{P}_{f^{(3)}} = 7! \times \frac{1}{20} \times 70\zeta(7)$

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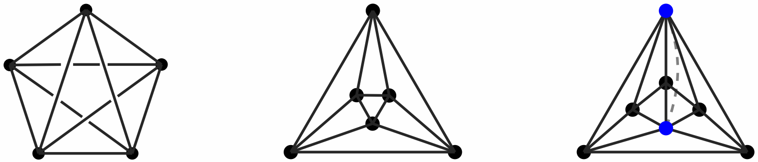


$\mathcal{P}_{f^{(1)}} = 6\,\zeta(3)$ $\quad$ $\mathcal{P}_{f^{(2)}} = 6! \times \frac{1}{48} \times 20\zeta(5)$ $\quad$ $\mathcal{P}_{f^{(3)}} = 7! \times \frac{1}{20} \times 70\zeta(7)$

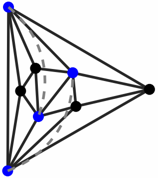
- $L = 1$: $\quad \frac{-1}{1!(-4)^1} \times \mathcal{P}_{f^{(1)}} = \boxed{\frac{3\zeta(3)}{2}}, \quad f^{(1)}(x_i) = \frac{1}{\prod_{1\le i<j\le 5} x_{ij}^2}\,.$
- $L = 2$:

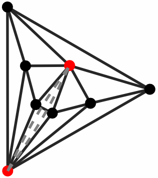
$$\frac{-1}{2!\,(-4)^2} \times \mathcal{P}_{f^{(2)}} = \boxed{\frac{-75\zeta(5)}{8}}. \quad f^{(2)}(x_i) = \frac{x_{12}^2 x_{34}^2 x_{56}^2}{48\prod_{1\le i<j\le 6} x_{ij}^2} + S_6\,.$$

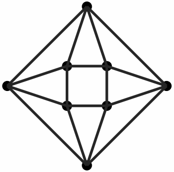
- $L = 3$:

$$\frac{-1}{3!\,(-4)^3} \times \mathcal{P}_{f^{(3)}} = \boxed{\frac{735\zeta(7)}{16}}, \quad f^{(3)}(x_i) = \frac{x_{12}^4 x_{34}^2 x_{45}^2 x_{56}^2 x_{67}^2 x_{37}^2}{20\prod_{1\le i<j\le 7} x_{ij}^2} + S_7\,.$$

Periods of $f^{(L)}(x_i)$





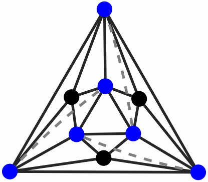


$\mathcal{P}_{f_1^{(4)}} = 8! \times \frac{1}{8} \times 252\zeta(9)$ $\mathcal{P}_{f_2^{(4)}} = 8! \times \frac{1}{24} \times 252\zeta(9)$ $\mathcal{P}_{f_3^{(4)}} = 8! \times \frac{1}{16} \times 168\zeta(9)$

- $L = 4$: 3 planar graphs,

$$\frac{-1}{4!\,(-4)^4} \times \left(\mathcal{P}_{f_1^{(4)}} + \mathcal{P}_{f_2^{(4)}} - \mathcal{P}_{f_3^{(4)}}\right) = \boxed{\frac{-6615\zeta(9)}{32}}.$$

- $L = 5$: 6 out of 7 periods done. $f_4^{(5)}$ **predicted** by localisation.



Conclusion & Outlook

- Integrated correaltors in perturbation are **periods** of conformal graphs $f_\alpha^{(L)}$.
- Explicitly verified up to $\boldsymbol{L = 4}$ (planar) for the first correlator, and $\boldsymbol{L = 3}$ for the second correaltor.
- Higher L? *Non-planar*? Higher weight $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_p \mathcal{O}_p \rangle$?

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Thank You!