The Vernacular of the S-Matrix

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Amplitudes 2022 Summer School
Charles University, Prague, Czech Republic

The Niels Bohr International Academy

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Enormous Advances in Understanding Scattering Amplitudes

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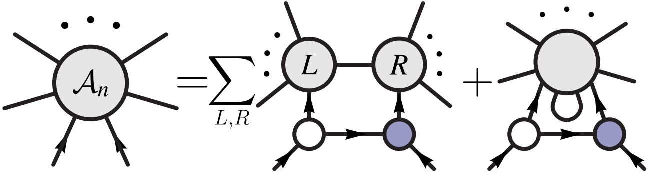

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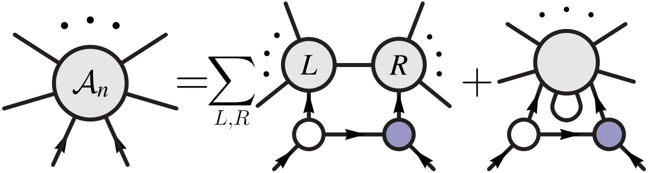

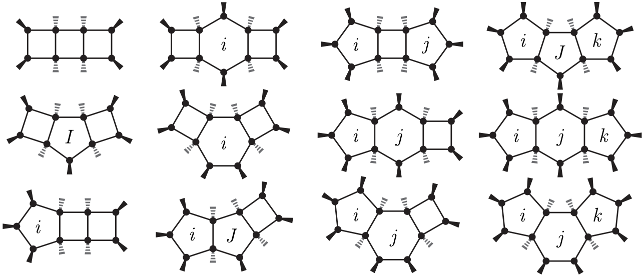

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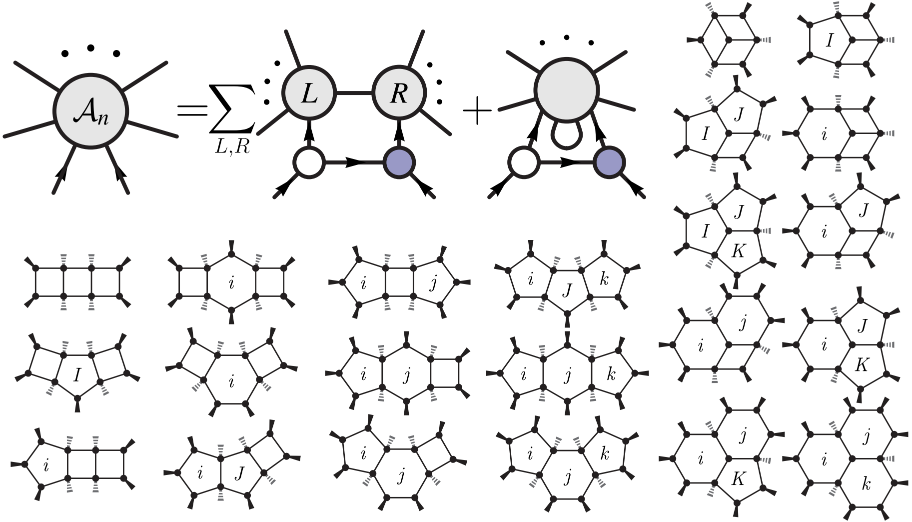

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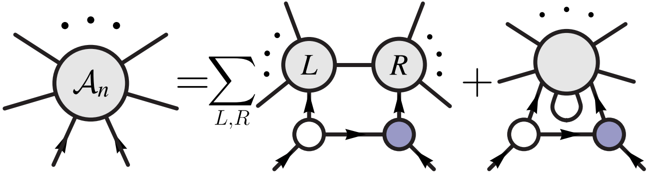

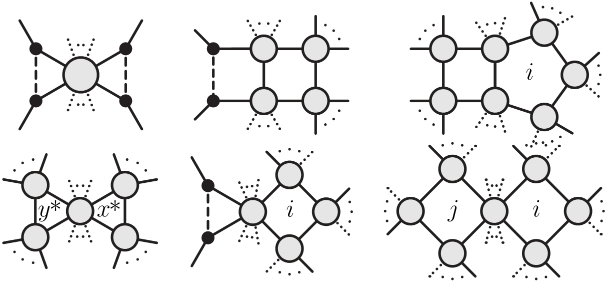

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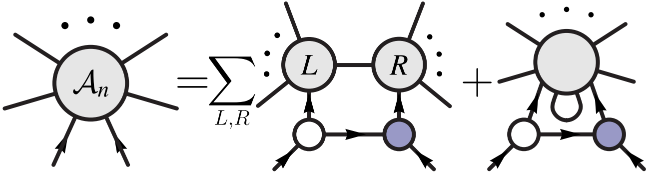

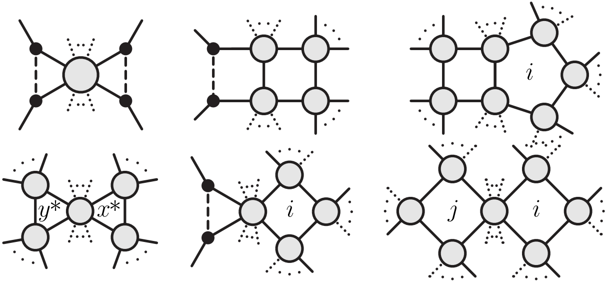

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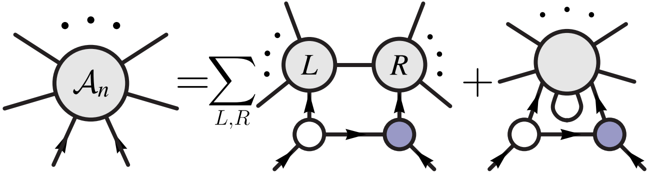

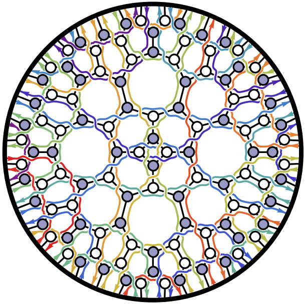

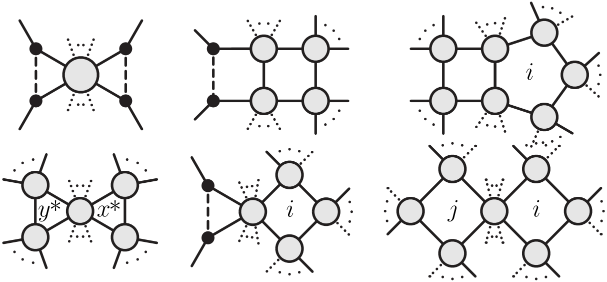

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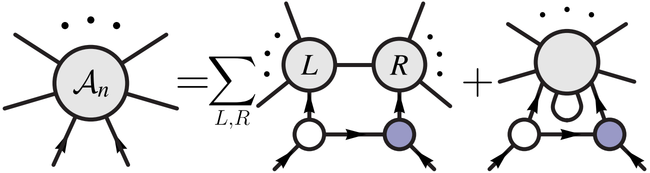

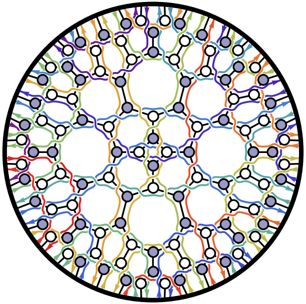

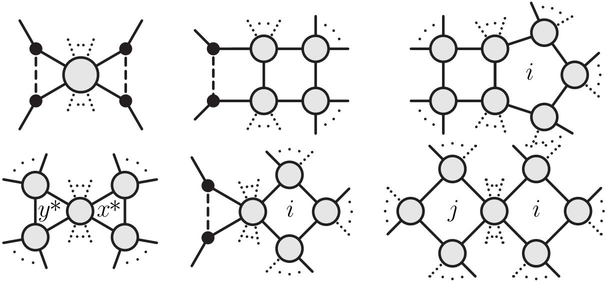

On-Shell Physics/Grassmannian Geometry Correspondence

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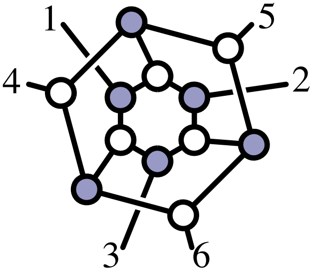
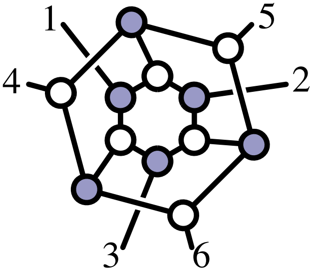


On-Shell Physics/Grassmannian Geometry Correspondence

$$f_\Gamma \equiv \prod_i \Big(\sum_{h_i, q_i} \int d^3\mathrm{LIPS}_i \Big) \prod_v \mathcal{A}_v$$



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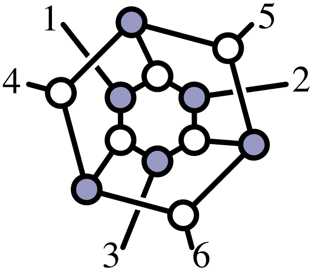

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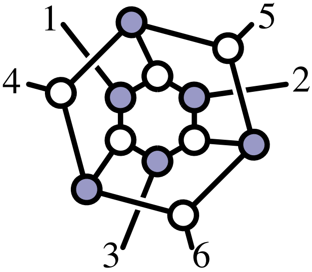

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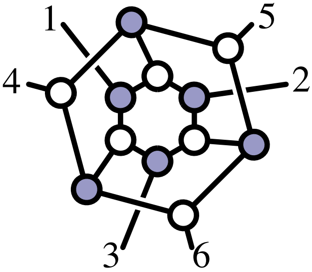

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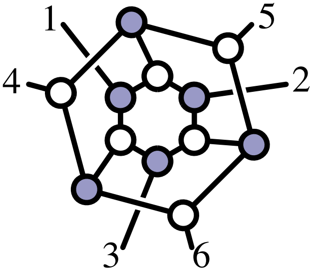

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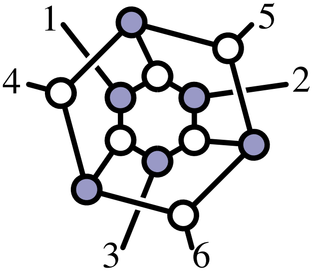

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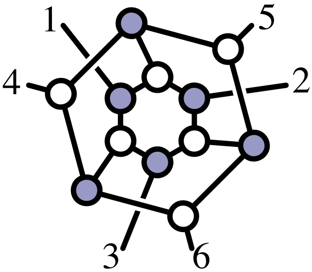

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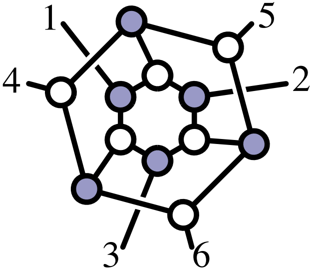

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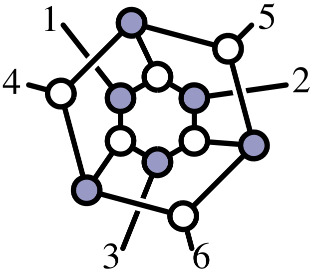

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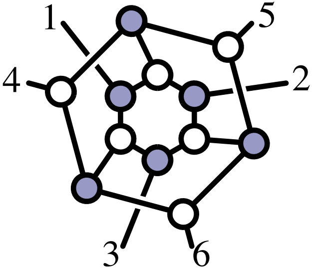

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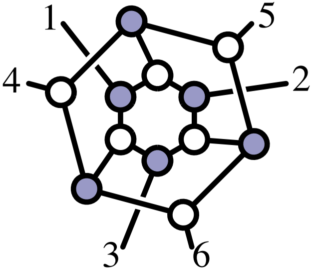

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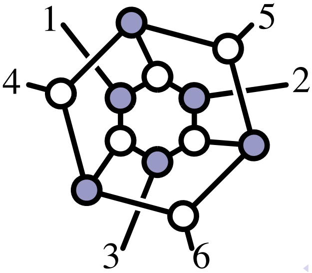

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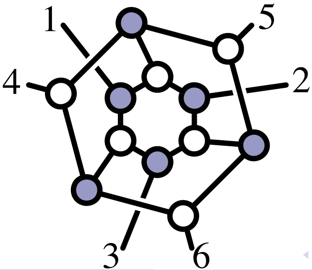

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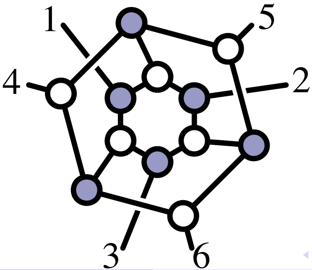

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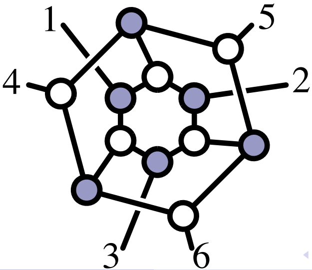

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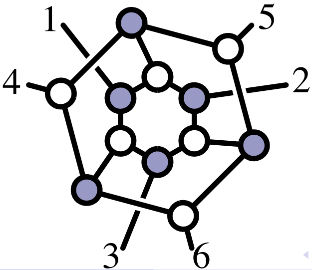

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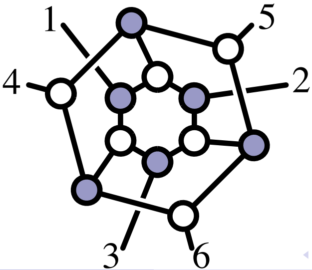

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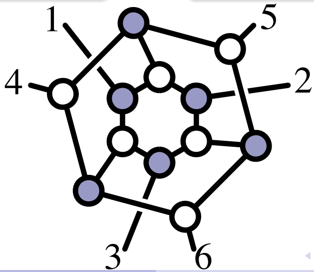
On-Shell Physics

$\Leftrightarrow$

Grassmannian Geometry



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On-Shell Physics

- on-shell diagrams

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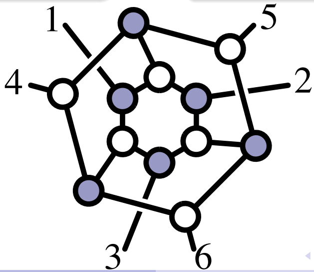

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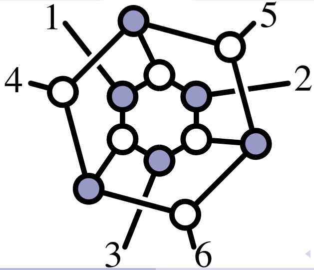
On-Shell Physics

- on-shell diagrams

$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$



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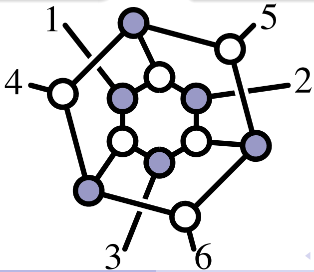

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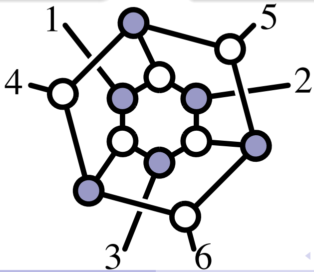
On-Shell Physics

- on-shell diagrams

$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
- volume-preserving diffeomorphisms



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On-Shell Physics

- on-shell diagrams
- physical symmetries

$\Longleftrightarrow$

Grassmannian Geometry

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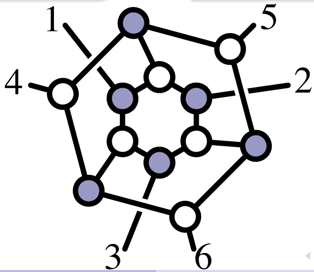

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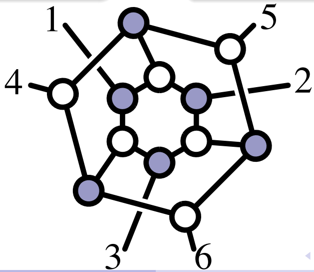
On-Shell Physics

- on-shell diagrams
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

Grassmannian Geometry

- {strata $C \in G(k,n)$, volume-form Ω_C}
- volume-preserving diffeomorphisms



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On-Shell Physics

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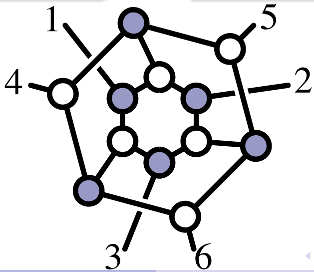
$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
- volume-preserving diffeomorphisms
  - cluster coordinate mutations



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On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

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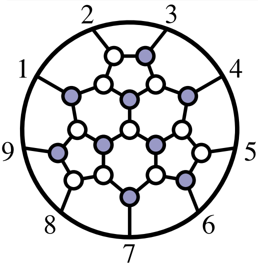

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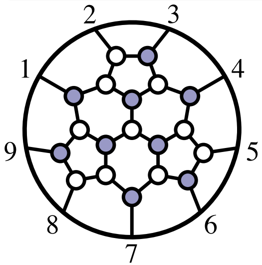
On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
  - bi-colored
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

Grassmannian Geometry

- {strata $C \in G(k,n)$, volume-form Ω_C}
- volume-preserving diffeomorphisms
  - cluster coordinate mutations



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On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
  - bi-colored, **un**directed
- physical symmetries
  - trivial symmetries (identities)

$\Longleftrightarrow$

Grassmannian Geometry

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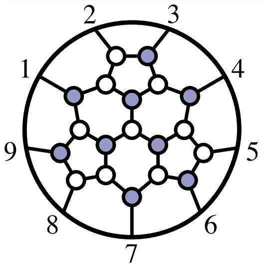
- volume-preserving diffeomorphisms
  - cluster coordinate mutations



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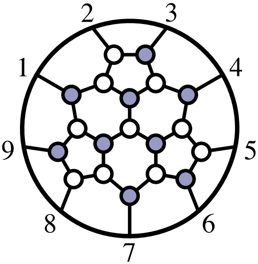
On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
  - bi-colored, **un**directed, **planar**
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C\in G(k,n)$, volume-form $\Omega_C\}$
- volume-preserving diffeomorphisms
  - cluster coordinate mutations



On-Shell Physics/Grassmannian Geometry Correspondence

$$f_\Gamma \equiv \prod_i \Big(\sum_{h_i, q_i} \int d^3\mathrm{LIPS}_i \Big) \prod_v \mathcal{A}_v \equiv \int \Omega_C \; \delta(C, p, h)$$

On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
  - bi-colored, **un**directed, **planar**
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
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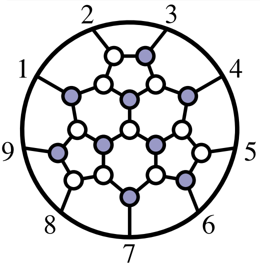

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Grassmannian Geometry

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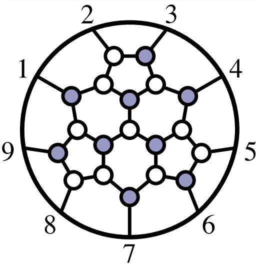

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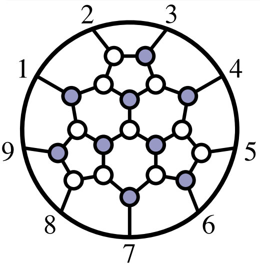

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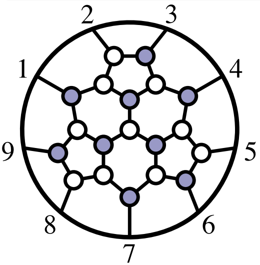

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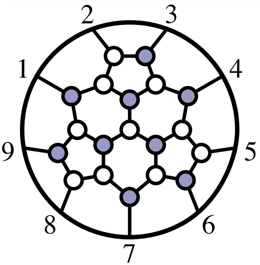

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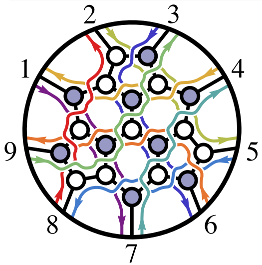

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$\Leftrightarrow$

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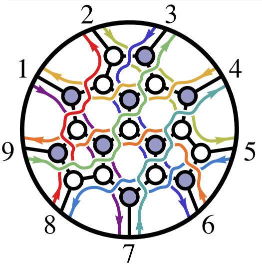

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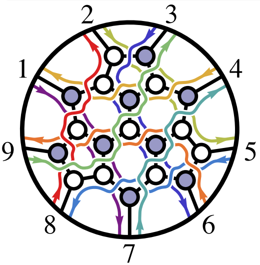

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On-Shell Physics: planar $\mathcal{N} < 4$

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  - bi-colored, **directed** , **planar**
- physical symmetries: **?**
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$\Longleftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
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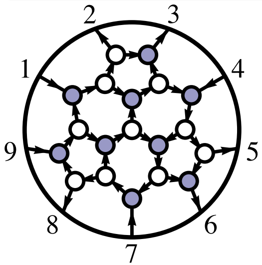

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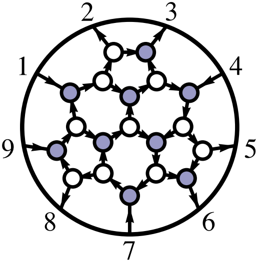

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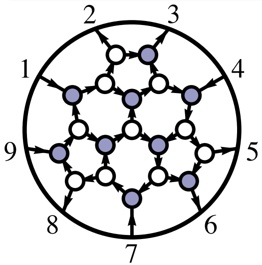

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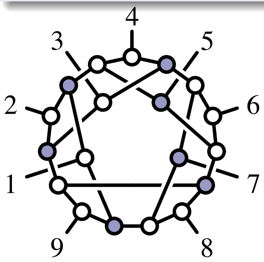

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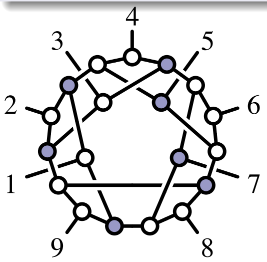

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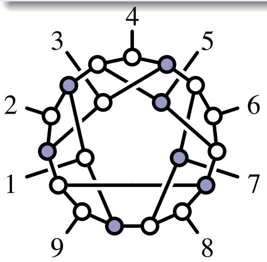

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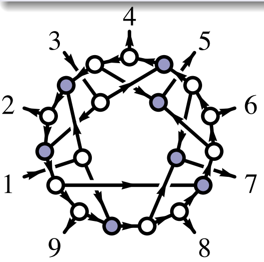


$$C^\perp \equiv \begin{pmatrix} \alpha_1 & 1 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_3 & 1 & 0 & \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_5 & 1 & \alpha_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \alpha_7 & \alpha_8 \\ 0 & 0 & 0 & 0 & 0 & \alpha_9 & 1 & \alpha_{10} & 0 \\ \alpha_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_{12} \\ 0 & \alpha_{13} & \alpha_{14} & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Omega_C \equiv \left(\frac{d\alpha_1}{\alpha_1} \wedge \cdots \wedge \frac{d\alpha_{14}}{\alpha_{14}} \right) \times \big(1 + \alpha_2 \alpha_4 \alpha_{13} (\alpha_8 + \alpha_7 \alpha_{12}) \big)^{\mathcal{N}-4}$$

On-Shell Physics/Grassmannian Geometry Correspondence

$$f_\Gamma \equiv \prod_i \Big(\sum_{h_i,q_i} \int d^3\mathrm{LIPS}_i\Big) \prod_v \mathcal{A}_v \equiv \int \Omega_C \;\delta(C,p,h)$$

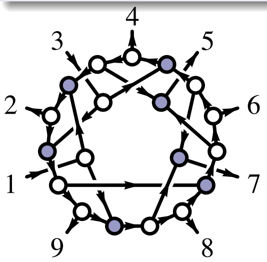
On-Shell Physics: non-planar $\mathcal{N}<4$

- on-shell diagrams
  - bi-colored, **directed** , **non**-planar
- physical symmetries: **?**
  - trivial symmetries (identities)

$\Longleftrightarrow$

Grassmannian Geometry

- {strata $C \in G(k,n)$, volume-form Ω_C}
  - cluster variety , $\big(\prod_i \frac{d\alpha_i}{\alpha_i}\big) \times \mathcal{J}^{\mathcal{N}-4}$
- volume-preserving diffeomorphisms
  - cluster coordinate mutations



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Organization and Outline

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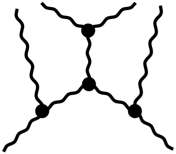

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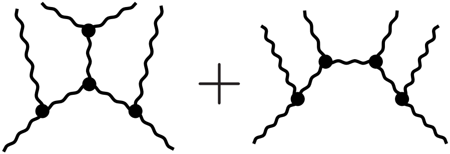

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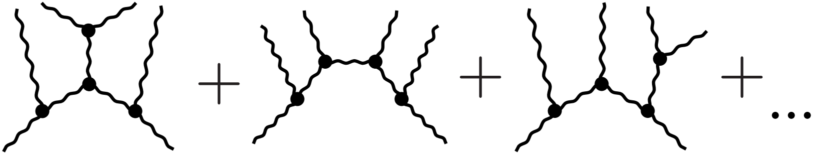

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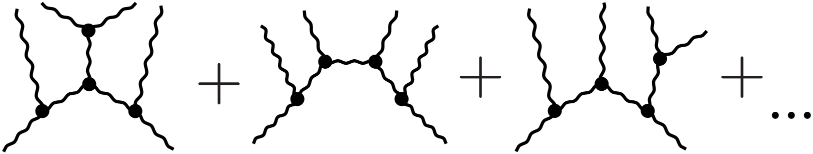

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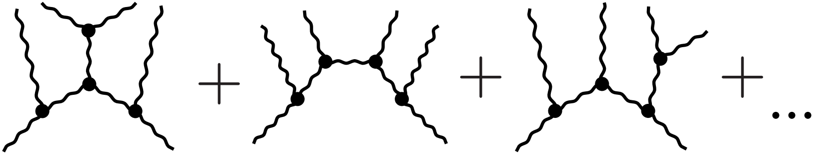

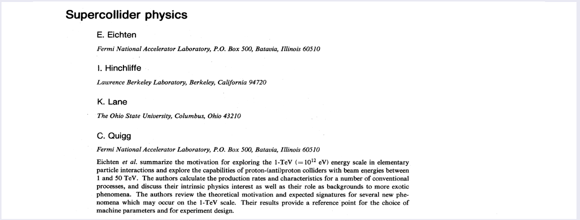

Supercollider physics

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We draw two conclusions from this very casual analysis:

At least at small-to-moderate values of E_T, two-jet events should account for most of the cross section.

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It is apparent that these questions are amenable to detailed investigation with the aid of realistic Monte Carlo simulations. Given the elementary two→three cross sections and reasonable parametrizations of the fragmentation functions, this exercise can be carried out with some degree of confidence.

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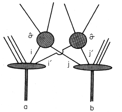
In this section we discuss the supercollider processes associated with the standard model of the weak and electromagnetic interactions (Glashow, 1961; Weinberg, 1967; Salam, 1968). By "standard model" we understand the $SU(2)_L \otimes U(1)_Y$ theory applied to three quark and lepton doublets, and with the gauge symmetry broken by a single complex Higgs doublet. The particles associated with the electroweak interactions are therefore the (left-handed) charged intermediate bosons $W^\pm$, the neutral intermedi-



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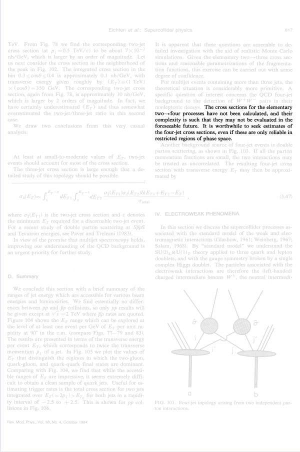

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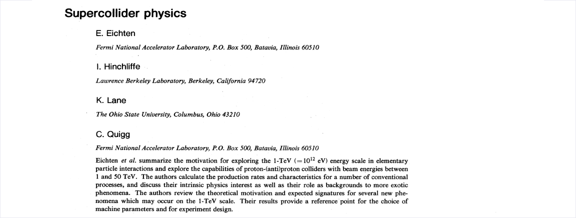

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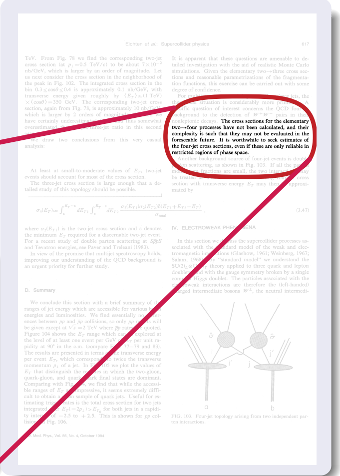
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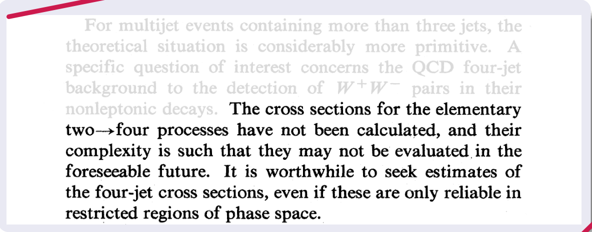
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gluons. The cross section for the scattering of two gluons with momenta p_1, p_2 into four gluons with momenta p_3, p_4, p_5, p_6 is obtained from eq. (5) by setting $I=2$ and replacing the momenta p_3, p_4, p_5, p_6 by $-p_3, -p_4, -p_5, -p_6$.

As the result of the computation of two hundred and forty Feynman diagrams, we obtain

$$A_{\binom{0}{2}}(p_1,p_2,p_3,p_4,p_5,p_6) = (\mathscr{D}^\dagger, \mathscr{D}^\dagger_\rho, \mathscr{D}^\dagger_\sigma, \mathscr{D}^\dagger_\tau)_{\binom{0}{2}} \cdot \begin{pmatrix} K & K_\rho & K_\sigma & K_\tau \\ K_\rho & K & K_\tau & K_\sigma \\ K_\sigma & K_\tau & K & K_\rho \\ K_\tau & K_\sigma & K_\rho & K \end{pmatrix} \cdot \begin{pmatrix} \mathscr{D} \\ \mathscr{D}_\rho \\ \mathscr{D}_\sigma \\ \mathscr{D}_\tau \end{pmatrix}_{\binom{0}{2}}, \qquad (6)$$

where $\mathscr{D}, \mathscr{D}_\rho, \mathscr{D}_\sigma$ and $\mathscr{D}_\tau$ are 11-component complex vector functions of the momenta p_1, p_2, p_3, p_4, p_5 and p_6, and K, K_ρ, K_σ and K_τ are constant 11×11 symmetric matrices. The vectors $\mathscr{D}_\rho, \mathscr{D}_\sigma$ and $\mathscr{D}_\tau$ are obtained from the vector $\mathscr{D}$ by the permutations $(p_2 \leftrightarrow p_3)$, $(p_5 \leftrightarrow p_6)$ and $(p_2 \leftrightarrow p_3, p_5 \leftrightarrow p_6)$, respectively, of the momentum variables in $\mathscr{D}$. The individual components of the vector $\mathscr{D}$ represent the sums of all contributions proportional to the appropriately chosen eleven basis color factors. The matrices K, which are the suitable sums over the color indices of products of the color bases, contain two independent structures, proportional to $N^4(N^2-1)$ and $N^2(N^2-1)$, respectively (N is the number of colors, $N=3$ for QCD):

$$K = \tfrac{1}{8} g^8 N^4 (N^2-1) K^{(4)} + \tfrac{1}{2} g^8 N^2 (N^2-1) K^{(2)}. \qquad (7)$$

Here g denotes the gauge coupling constant. The matrices $K^{(4)}$ and $K^{(2)}$ are given in table 1. The vector $\mathscr{D}$ is related to the thirty-three diagrams $D^{\mathrm{G}}(I=1-33)$ for two-gluon to four-scalar scattering, eleven diagrams $D^{\mathrm{F}}(I=1-11)$ for two-fermion to four-scalar scattering and sixteen diagrams $D^{\mathrm{S}}(I=1-16)$ for two-scalar to four-scalar scattering, in the following way:

$$\mathscr{D}_0 = \frac{2s_{14}}{\sqrt{|s_{15}s_{45}s_{16}s_{46}|}\, s_{23}s_{56}} \{ t_{123}^2 C^{\mathrm{G}} \cdot D_0^{\mathrm{G}} - 4 s_{14} t_{123} E(p_5+p_6, p_6) C^{\mathrm{F}} \cdot D_0^{\mathrm{F}} - 2 s_{14} G(p_5+p_6, p_5+p_6) C^{\mathrm{S}} \cdot D_0^{\mathrm{S}} \},$$

$$\mathscr{D}_2 = \frac{s_{56}}{s_{23}} C^{\mathrm{G}} \cdot D_2^{\mathrm{G}}, \qquad (8)$$

where the constant matrices $C^{\mathrm{G}}(11\times 33)$, $C^{\mathrm{F}}(11\times 11)$ and $C^{\mathrm{S}}(11\times 16)$ are given in table 2. The Lorentz invariants s_{ij} and t_{ijk} are defined as $s_{ij} = (p_i+p_j)^2$, $t_{ijk} = (p_i+p_j+p_k)^2$ and the complex functions E and G are given by

$$E(p_i, p_j) = \tfrac{1}{2}\{(p_1 p_4)(p_i p_j) - (p_1 p_i)(p_j p_4) - (p_1 p_j)(p_i p_4) + i\varepsilon_{\mu\nu\rho\lambda} p_1^\mu p_i^\nu p_j^\rho p_4^\lambda\}/(p_1 p_4),$$

$$G(p_i, p_j) = E(p_i, p_5) E(p_j, p_6), \qquad (9)$$

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TABLE 1
Matrices $K(I, J)[I = 1\text{-}11, J = 1\text{-}11]$.

Matrix $K^{(4)}$											Matrix $K^{(2)}$										
8	4	−2	2	−1	2	0	1	0	0	−1	0	0	0	0	0	0	0	0	3	3	−3
4	8	−1	1	−1	0	2	1	0	1	−1	0	0	0	0	0	0	0	0	3	3	0
−2	−1	8	4	4	1	1	2	2	1	2	0	0	0	0	0	0	0	0	0	0	0
2	1	4	8	2	−1	−1	4	1	1	1	0	0	0	0	0	0	0	0	0	0	0
−1	−1	4	2	8	1	2	4	−2	−1	4	0	0	0	0	0	0	0	0	0	0	0
2	0	1	−1	1	8	4	−1	0	1	0	0	0	0	0	0	0	0	0	3	3	0
0	2	1	−1	2	4	8	−2	0	0	0	0	0	0	0	0	0	0	0	3	3	0
1	1	2	4	4	−1	−2	8	−1	−1	2	0	0	0	0	0	0	0	0	0	0	0
0	0	2	1	−2	0	0	−1	8	4	−2	3	3	0	0	0	3	3	0	0	0	0
0	1	1	1	−1	1	0	−1	4	8	−1	3	3	0	0	0	3	3	0	0	0	0
−1	−1	2	1	4	0	0	2	−2	−1	8	−3	0	0	0	0	0	0	0	0	0	0

Matrix $K_\rho^{(4)}$											Matrix $K_\rho^{(2)}$										
0	0	0	0	1	1	0	1	1	0	−1	3	3	0	3	0	0	0	3	0	0	0
0	0	0	0	2	0	1	1	2	1	−2	3	3	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	1	1	0	0	3	0	0	3	0	0	3	3	0
0	0	0	0	1	0	0	2	0	1	0	3	0	0	0	0	3	0	0	0	0	0
1	2	0	1	0	1	2	2	0	0	2	0	0	0	0	0	0	0	0	3	3	0
1	0	1	0	1	4	2	0	0	0	−1	0	0	3	3	0	0	0	0	3	3	0
0	1	1	0	2	2	4	0	0	0	−2	0	0	0	0	0	0	0	0	0	3	0
1	1	1	2	2	0	0	4	0	0	0	3	0	0	0	0	0	0	0	3	0	0
1	2	0	0	0	0	0	0	0	2	−1	0	0	3	0	3	3	0	3	0	0	0
0	1	1	1	0	0	0	0	2	4	0	0	0	3	0	3	3	3	0	0	0	0
−1	−2	1	0	2	−1	−2	0	−1	0	4	0	0	0	0	0	0	0	0	0	0	0

Matrix $K_\sigma^{(4)}$											Matrix $K_\sigma^{(2)}$										
4	2	0	2	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	3	0	−3
2	4	0	1	0	0	1	1	0	1	0	0	0	0	0	3	0	0	3	0	0	0
0	0	4	2	2	1	1	1	2	1	0	0	0	0	0	0	0	3	0	0	0	0
2	1	2	0	1	2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	1	0	0	0	0	4	2	2	0	3	0	0	0	0	3	0	0	0	0
1	0	1	2	0	0	0	0	1	2	0	0	0	0	0	0	3	3	0	0	0	−3
0	1	1	1	0	0	0	0	2	4	0	0	0	3	0	3	3	3	0	0	0	0
1	1	1	0	0	0	0	0	2	0	1	0	3	0	0	0	0	0	3	0	0	0
0	0	2	1	4	1	2	2	0	0	−4	3	0	0	0	0	0	0	0	0	0	0
0	1	1	0	2	2	4	0	0	0	−2	0	0	0	0	0	0	0	0	0	3	0
0	0	0	0	2	0	0	1	−4	−2	4	−3	0	0	0	0	−3	0	0	0	0	0

Matrix $K_\tau^{(4)}$											Matrix $K_\tau^{(2)}$										
0	1	−1	−1	1	1	0	1	2	0	0	3	3	0	0	0	3	3	0	0	0	0
1	0	−2	−1	2	0	1	1	4	2	0	3	3	0	0	0	3	3	0	0	0	0
−1	−2	0	0	0	1	1	1	−1	1	0	0	0	3	3	3	0	0	3	0	0	0
−1	−1	0	1	0	2	1	0	1	−1	0	0	0	3	3	3	0	0	3	0	0	3
1	2	0	0	1	−1	−1	0	−2	2	1	0	0	3	3	3	0	0	3	0	0	0
1	0	1	2	−1	0	1	−2	2	4	−1	3	3	0	0	0	3	3	0	0	0	−3
0	1	1	1	−1	1	0	−1	4	8	−1	3	3	0	0	0	3	3	0	0	0	0
1	1	1	0	0	−2	−1	0	2	−2	0	0	0	3	3	3	0	0	3	0	0	0
2	4	−1	1	−2	2	4	2	1	0	−2	0	0	0	0	0	0	0	0	3	3	0
0	2	1	−1	2	4	8	−2	0	0	0	0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	1	−1	−1	0	−2	0	2	0	0	0	3	0	−3	0	0	0	0	0

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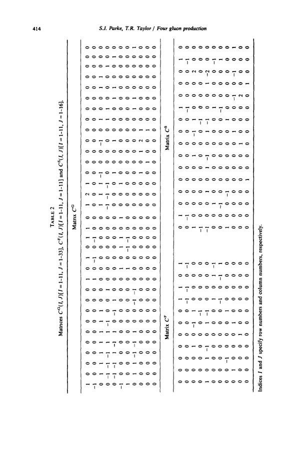

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where ε is the totally antisymmetric tensor, $\varepsilon_{xyzt} = 1$. For the future use, we define one more function,

$$F(p_i, p_j) = \{(p_1 p_4)(p_i p_j) + (p_1 p_i)(p_j p_4) - (p_1 p_j)(p_i p_4)\}/(p_1 p_4)\,. \qquad (10)$$

Note that when evaluating A_0 and A_2 at crossed configurations of the momenta, care must be taken with the implicit dependence of the functions E, F and G on the momenta p_1, p_4, p_5, p_6.

The diagrams D_2^G are listed below:

$$D_2^G(1) = \frac{\delta_2}{s_{14}s_{25}s_{36}}\{[(p_2-p_5)(p_3-p_6)][(p_1-p_4)(p_3+p_6)] - [(p_2-p_5)(p_3+p_6)] \times [(p_1-p_4)(p_3-p_6)] + [(p_2+p_5)(p_3-p_6)][(p_1-p_4)(p_2-p_5)]\}\,,$$

$$D_2^G(2) = \frac{1}{s_{25}s_{36}}\{2E(p_2-p_5, p_3-p_6) - 2E(p_3-p_6, p_2-p_5) + \delta_2[(p_2-p_5)(p_3-p_6)]\}\,,$$

$$D_2^G(3) = \frac{4}{s_{25}s_{36}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3) - [(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6) - [(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3) + [(p_1-p_2+p_5)(p_4-p_3+p_6)]E(p_5,p_6) + [p_1(p_2-p_5)]E(p_3-p_6, p_3+p_6) - [p_4(p_3-p_6)]E(p_2+p_5, p_2-p_5) + \delta_2[p_1(p_2-p_5)][p_4(p_3-p_6)]\}\,,$$

$$D_2^G(4) = \frac{-2}{s_{36}t_{125}}\{E(p_3-p_6, p_3+p_6) - \delta_2[p_4(p_3-p_6)]\}\,,$$

$$D_2^G(5) = \frac{-2}{s_{25}t_{125}}\{E(p_2+p_5, p_2-p_5) - \delta_2[p_1(p_2-p_5)]\}\,,$$

$$D_2^G(6) = \frac{\delta_2}{t_{125}}\,,$$

$$D_2^G(7) = \frac{4}{s_{12}s_{36}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3) - [(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6) - [p_4(p_3-p_6)]E(p_2, p_2-p_5)\}\,,$$

$$D_2^G(8) = \frac{4}{s_{34}s_{25}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3) - [(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3) - [p_1(p_2-p_5)]E(p_3-p_6, p_3)\}\,,$$

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$$D_2^G(9) = \frac{4}{s_{15}s_{36}t_{125}}\{[(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3)$$
$$-[(p_1-p_2+p_5)(p_4-p_3+p_6)]E(p_5,p_6)+[p_4(p_3-p_6)]E(p_5,p_2-p_5)\},$$

$$D_2^G(10) = \frac{4}{s_{25}s_{46}t_{125}}\{[(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6)$$
$$-[(p_1-p_2+p_5)(p_4-p_3+p_6)]E(p_5,p_6)+[p_1(p_2-p_5)]E(p_3-p_6,p_6)\},$$

$$D_2^G(11) = \frac{\delta_2}{s_{36}t_{124}}[s_{35}-s_{56}+s_{36}],$$

$$D_2^G(12) = \frac{-\delta_2}{s_{36}t_{145}}[s_{23}-s_{26}-s_{36}],$$

$$D_2^G(13) = \frac{\delta_2}{s_{14}s_{36}t_{124}}[s_{12}-s_{24}][s_{35}-s_{56}+s_{36}],$$

$$D_2^G(14) = \frac{\delta_2}{s_{14}s_{36}t_{145}}[s_{15}-s_{45}][s_{23}-s_{26}-s_{36}],$$

$$D_2^G(15) = \frac{\delta_2}{s_{14}s_{36}}(p_1-p_4)(p_3-p_6),$$

$$D_2^G(16) = \frac{-4}{s_{12}s_{36}t_{124}}[s_{35}-s_{56}+s_{36}]E(p_2,p_2),$$

$$D_2^G(17) = \frac{4}{s_{36}s_{45}t_{145}}[s_{23}-s_{26}-s_{36}]E(p_5,p_5),$$

$$D_2^G(18) = \frac{-4}{s_{12}s_{36}s_{45}}[2(p_1+p_2)(p_3-p_6)-s_{36}]E(p_2,p_5),$$

$$D_2^G(19) = \frac{-2}{s_{12}s_{36}}E(p_2,p_3-p_6),$$

$$D_2^G(20) = \frac{2}{s_{36}s_{45}}E(p_3-p_6,p_5),$$

$$D_2^G(21) = \frac{-4}{s_{25}s_{34}t_{134}}[s_{26}-s_{56}+s_{25}]E(p_3,p_3),$$

$$D_2^G(22) = \frac{4}{s_{16}s_{25}t_{146}}[s_{23}-s_{35}-s_{25}]E(p_6,p_6),$$

$$D_2^G(23) = \frac{4}{s_{16}s_{25}s_{34}}[2(p_1+p_6)(p_2-p_5)+s_{25}]E(p_6,p_3),$$

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$$D_2^G(24) = \frac{-2}{s_{25}s_{34}} E(p_2 - p_5, p_3)\,,$$

$$D_2^G(25) = \frac{2}{s_{16}s_{25}} E(p_6, p_2 - p_5)\,,$$

$$D_2^G(26) = \frac{-2}{s_{12}t_{125}} E(p_2, p_2 - p_5)\,,$$

$$D_2^G(27) = \frac{2}{s_{46}t_{125}} E(p_3 - p_6, p_6)\,,$$

$$D_2^G(28) = \frac{2}{s_{15}t_{125}} E(p_5, p_2 - p_5)\,,$$

$$D_2^G(29) = \frac{-2}{s_{34}t_{125}} E(p_3 - p_6, p_3)\,,$$

$$D_2^G(30) = \frac{4}{s_{12}s_{34}t_{125}} [(p_1 + p_2 - p_5)(p_4 + p_3 - p_6) - t_{125}] E(p_2, p_3)\,,$$

$$D_2^G(31) = \frac{4}{s_{12}s_{46}t_{125}} [(p_1 + p_2 - p_5)(p_4 - p_3 + p_6) + t_{125}] E(p_2, p_6)\,,$$

$$D_2^G(32) = \frac{4}{s_{15}s_{34}t_{125}} [(p_1 - p_2 + p_5)(p_4 + p_3 - p_6) + t_{125}] E(p_5, p_3)\,,$$

$$D_2^G(33) = \frac{4}{s_{15}s_{46}t_{125}} [(p_1 - p_2 + p_5)(p_4 - p_3 + p_6) - t_{125}] E(p_5, p_6)\,, \qquad (11)$$

where $\delta_2 = 1$.

The diagrams D_0^G are obtained from D_2^G by replacing δ_2 by $\delta_0 = 0$ and the functions $E(p_i, p_j)$ by $G(p_i, p_j)$.

The diagrams D_0^F are listed below:

$$D_0^F(1) = \frac{4}{s_{15}s_{34}t_{125}} \{F(p_5, p_6)E(p_3, p_5) - F(p_5, p_3)E(p_6, p_5) + [F(p_6, p_3) + s_{34}]E(p_5, p_5)\}\,,$$

$$D_0^F(2) = \frac{-4}{s_{16}s_{25}s_{34}} \{[F(p_6, p_2) + \tfrac{1}{2}s_{16}]E(p_3, p_5) + [F(p_2, p_3) + \tfrac{1}{2}s_{34}]E(p_6, p_5) - F(p_6, p_3)E(p_2, p_5)\}\,,$$

$$D_0^F(3) = \frac{4}{s_{15}s_{36}t_{125}} \{F(p_5, p_6)E(p_3, p_5) - F(p_5, p_3)E(p_6, p_5) - [F(p_3, p_6) - \tfrac{1}{2}s_{36} - \tfrac{1}{2}s_{34} + \tfrac{1}{2}s_{46}]E(p_5, p_5)\}\,,$$

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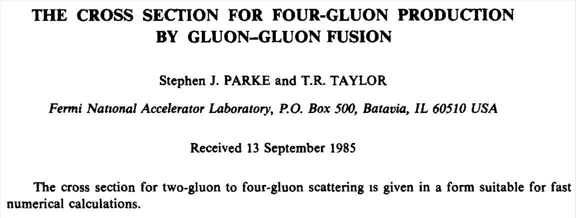

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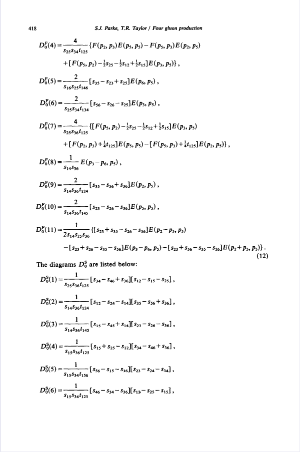

$$D_0^{\rm F}(4) = \frac{4}{s_{25}s_{34}t_{125}}\{F(p_2,p_3)E(p_5,p_5) - F(p_5,p_3)E(p_2,p_5) + [F(p_5,p_2) - \tfrac{1}{2}s_{25} - \tfrac{1}{2}s_{12} + \tfrac{1}{2}s_{15}]E(p_3,p_5)\}\,,$$

$$D_0^{\rm F}(5) = \frac{2}{s_{16}s_{25}t_{146}}[s_{35} - s_{23} + s_{25}]E(p_6,p_5)\,,$$

$$D_0^{\rm F}(6) = \frac{2}{s_{25}s_{34}t_{134}}[s_{56} - s_{26} - s_{25}]E(p_3,p_5)\,,$$

$$D_0^{\rm F}(7) = \frac{4}{s_{25}s_{36}t_{125}}\{[F(p_5,p_2) - \tfrac{1}{2}s_{25} - \tfrac{1}{2}s_{12} + \tfrac{1}{2}s_{15}]E(p_3,p_5) + [F(p_2,p_3) + \tfrac{1}{4}t_{125}]E(p_5,p_5) - [F(p_5,p_3) + \tfrac{1}{4}t_{125}]E(p_2,p_5)\}\,,$$

$$D_0^{\rm F}(8) = \frac{1}{s_{14}s_{36}}E(p_3 - p_6, p_5)\,,$$

$$D_0^{\rm F}(9) = \frac{2}{s_{14}s_{36}t_{124}}[s_{35} - s_{56} + s_{36}]E(p_2,p_5)\,,$$

$$D_0^{\rm F}(10) = \frac{2}{s_{14}s_{36}t_{145}}[s_{23} - s_{26} - s_{36}]E(p_5,p_5)\,,$$

$$D_0^{\rm F}(11) = \frac{1}{2s_{14}s_{25}s_{36}}\{[s_{23} + s_{35} - s_{26} - s_{56}]E(p_2 - p_5, p_5) - [s_{23} + s_{26} - s_{35} - s_{56}]E(p_3 - p_6, p_5) - [s_{23} + s_{56} - s_{35} - s_{26}]E(p_2 + p_5, p_5)\}\,. \qquad (12)$$

The diagrams $D_0^{\rm S}$ are listed below:

$$D_0^{\rm S}(1) = \frac{1}{s_{25}s_{36}t_{125}}[s_{34} - s_{46} + s_{36}][s_{12} - s_{15} - s_{25}]\,,$$

$$D_0^{\rm S}(2) = \frac{1}{s_{14}s_{36}t_{124}}[s_{12} - s_{24} - s_{14}][s_{35} - s_{56} + s_{36}]\,,$$

$$D_0^{\rm S}(3) = \frac{1}{s_{14}s_{36}t_{145}}[s_{15} - s_{45} + s_{14}][s_{23} - s_{26} - s_{36}]\,,$$

$$D_0^{\rm S}(4) = \frac{1}{s_{15}s_{36}t_{125}}[s_{15} + s_{25} - s_{12}][s_{34} - s_{46} + s_{36}]\,,$$

$$D_0^{\rm S}(5) = \frac{1}{s_{15}s_{34}t_{156}}[s_{56} - s_{15} - s_{16}][s_{23} - s_{24} - s_{34}]\,,$$

$$D_0^{\rm S}(6) = \frac{1}{s_{15}s_{34}t_{125}}[s_{46} - s_{34} - s_{36}][s_{12} - s_{25} - s_{15}]\,,$$

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$$D_0^S(7) = \frac{1}{s_{25}s_{34}t_{125}}[s_{36} - s_{46} + s_{34}][s_{12} - s_{15} - s_{25}]\,,$$

$$D_0^S(8) = \frac{1}{s_{16}s_{25}t_{146}}[s_{25} + s_{35} - s_{23}][s_{14} - s_{46} + s_{16}]\,,$$

$$D_0^S(9) = \frac{1}{s_{25}s_{34}t_{134}}[s_{14} + s_{34} - s_{13}][s_{26} - s_{56} + s_{25}]\,,$$

$$D_0^S(10) = \frac{1}{s_{25}s_{36}}(p_2 - p_5)(p_3 - p_6)\,,$$

$$D_0^S(11) = \frac{1}{s_{14}s_{36}}(p_1 - p_4)(p_3 - p_6)\,,$$

$$D_0^S(12) = \frac{1}{s_{16}s_{25}}(p_6 - p_1)(p_2 - p_5)\,,$$

$$D_0^S(13) = \frac{1}{s_{15}s_{34}}(p_5 - p_1)(p_3 - p_4)\,,$$

$$D_0^S(14) = \frac{1}{s_{25}s_{34}}(p_2 - p_5)(p_3 - p_4)\,,$$

$$D_0^S(15) = \frac{1}{s_{14}s_{25}s_{36}}\{[(p_2+p_5)(p_3-p_6)][(p_1-p_4)(p_2-p_5)] + [(p_2-p_5)(p_3-p_6)][(p_1-p_4)(p_3+p_6)] + [(p_1+p_4)(p_2-p_5)][(p_1-p_4)(p_3-p_6)]\}\,,$$

$$D_0^S(16) = \frac{2}{s_{16}s_{34}s_{25}}\{[(p_2-p_5)(p_3+p_4)][(p_1-p_6)(p_3-p_4)] + [(p_1+p_6)(p_3-p_4)][(p_1-p_6)(p_2-p_5)] + [(p_1-p_6)(p_2+p_5)][(p_3-p_4)(p_2-p_5)]\}\,. \quad (13)$$

The preceding list completes the result. Let us recapitulate now the numerical procedure of calculating the full cross section. First the diagrams D are calculated by using eqs. (11)–(13). The result is substituted to eq. (8) to obtain the vectors $\mathscr{D}_0$ and $\mathscr{D}_2$. After generating the vectors $\mathscr{D}_{0_\rho}$, $\mathscr{D}_{0_\sigma}$, $\mathscr{D}_{0_\tau}$, $\mathscr{D}_{2_\rho}$, $\mathscr{D}_{2_\sigma}$ and $\mathscr{D}_{2_\tau}$ by the appropriate permutations of momenta, eq. (6) is used to obtain the functions A_0 and A_2. Finally, the total cross section is calculated by using eq. (5). The FORTRAN 5 program based on such a scheme generates ten Monte Carlo points in less than a second on the heterotic CDC CYBER 175/875.

Given the complexity of the final result, it is very important to have some reliable testing procedures available for numerical calculations. Usually in QCD, the multi-gluon amplitudes are tested by checking the gauge invariance. Due to the specifics

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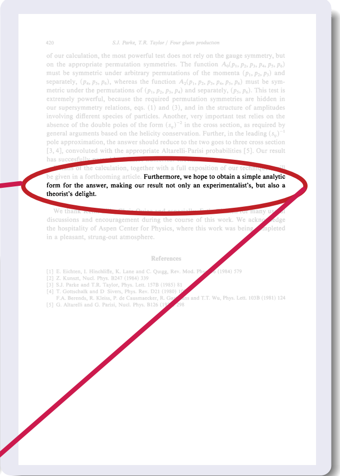

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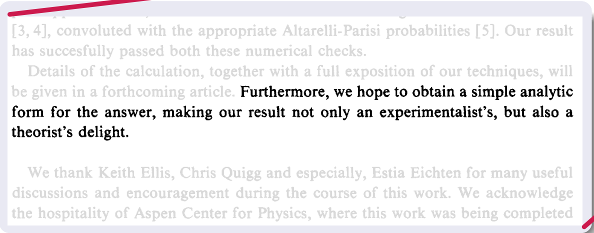

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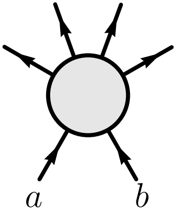

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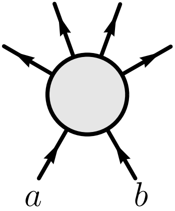

$$= \frac{\langle a\,b\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle\langle 6\,1\rangle}\,\delta^{2\times 2}\big(\lambda\cdot\widetilde{\lambda}\big)$$

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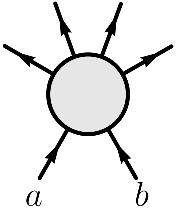

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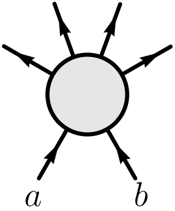

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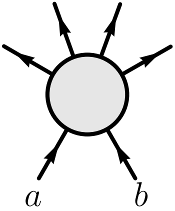

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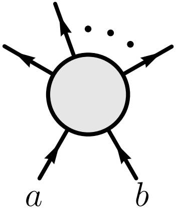

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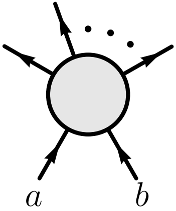

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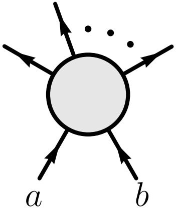

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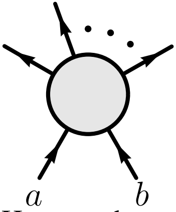

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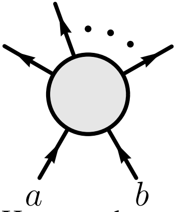

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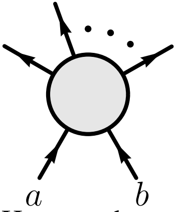

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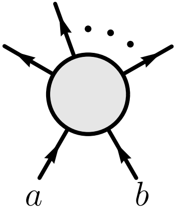

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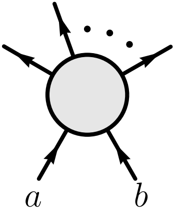

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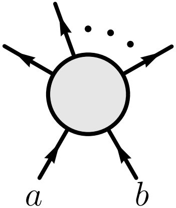

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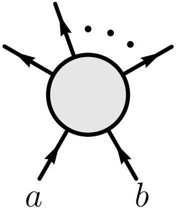

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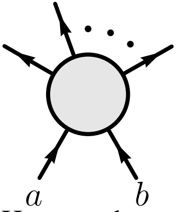

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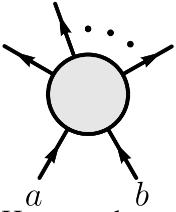

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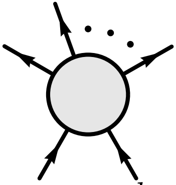

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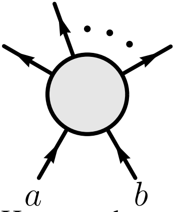

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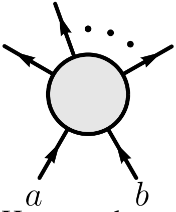

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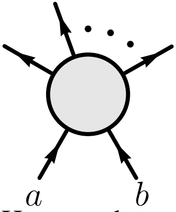

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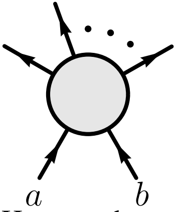

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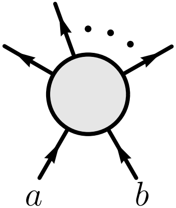

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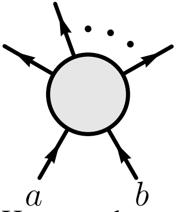

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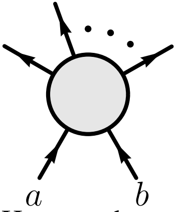

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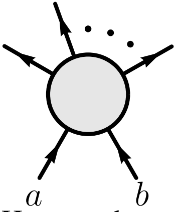

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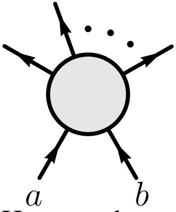

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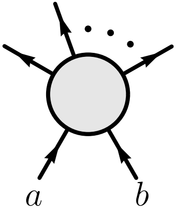

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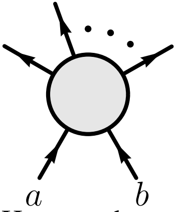

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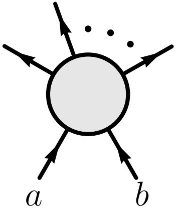

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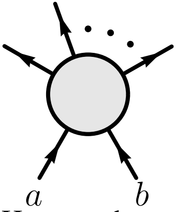

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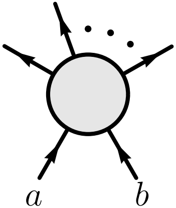

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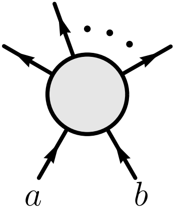

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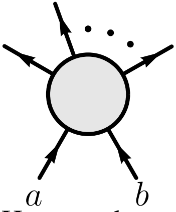

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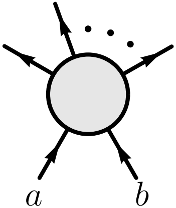

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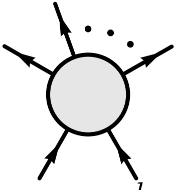

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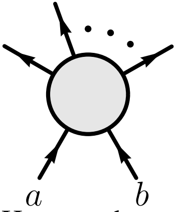

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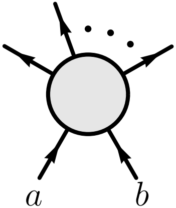

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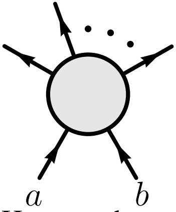

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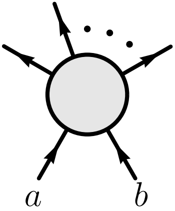

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The **Grassmannian** $G(k,n)$: the linear span of k vectors in $\mathbb{C}^n$.

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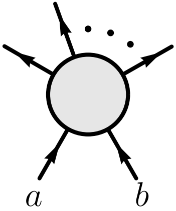

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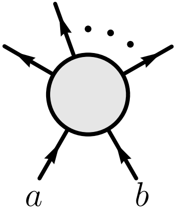

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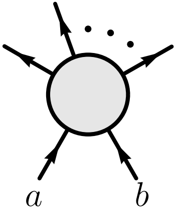

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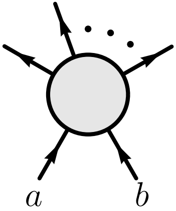

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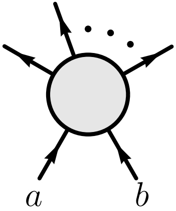

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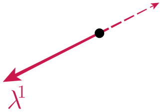

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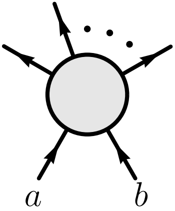

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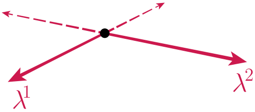

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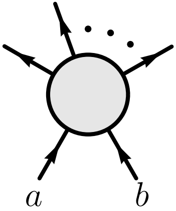

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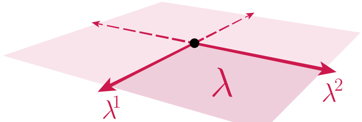

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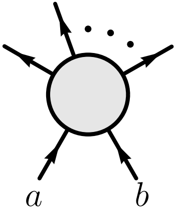

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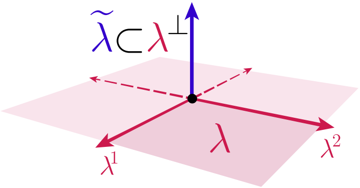

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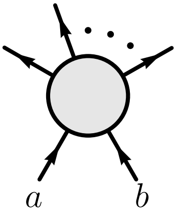

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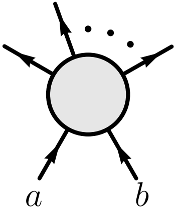

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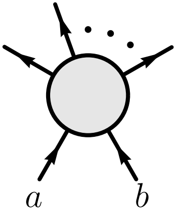

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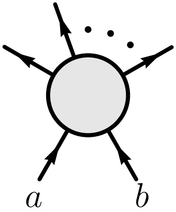

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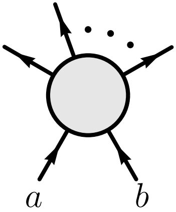

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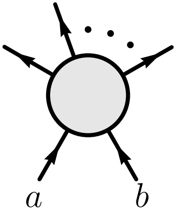

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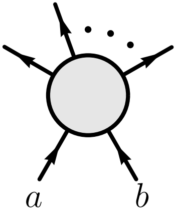

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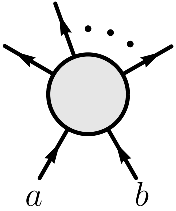

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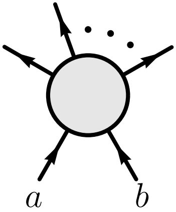

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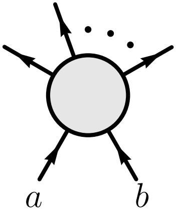

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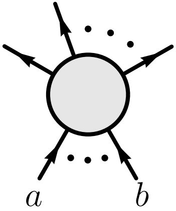

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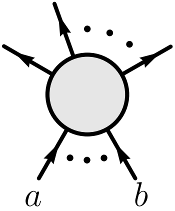

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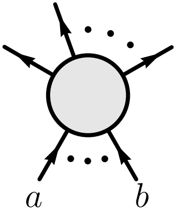

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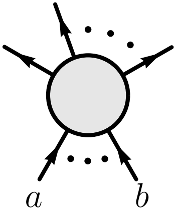

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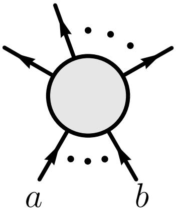

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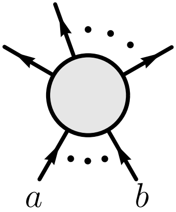

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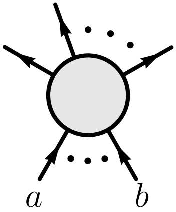

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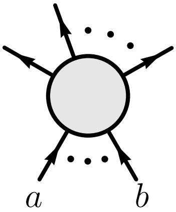

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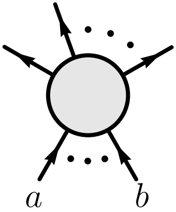

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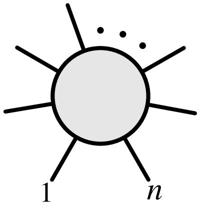
$\mathcal{A}_n \equiv$ 

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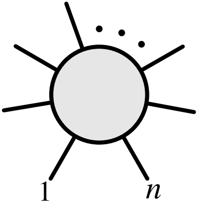

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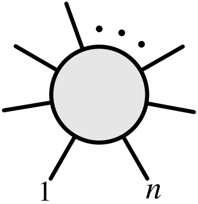
$\mathcal{A}_n \equiv$ 

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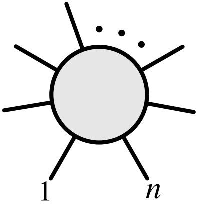
$\mathcal{A}_n \equiv$ 

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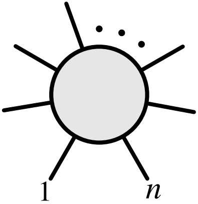
$\mathcal{A}_n \equiv$ 

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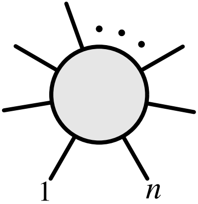
$\mathcal{A}_n \equiv$ 

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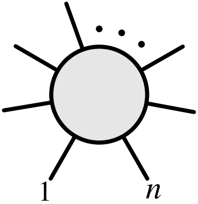
$\mathcal{A}_n \equiv$ 

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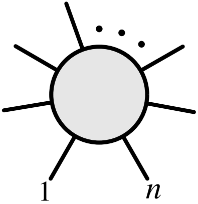
$\mathcal{A}_n \equiv$ 

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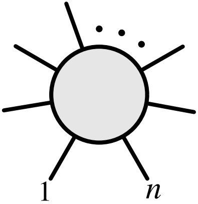
$\mathcal{A}_n \equiv$ 

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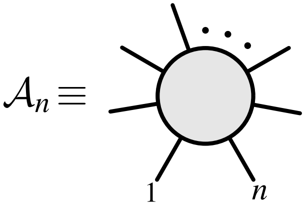

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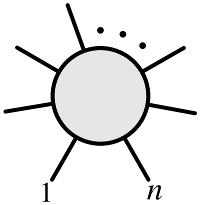
$\mathcal{A}_n \equiv$ 

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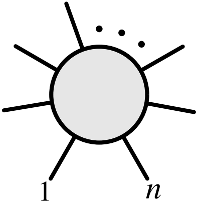
$\mathcal{A}_n \equiv$ 

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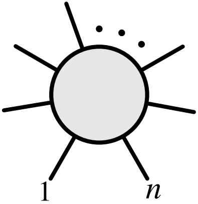
$\mathcal{A}_n \equiv$ 

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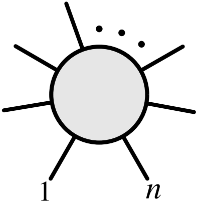
$\mathcal{A}_n \equiv$ 

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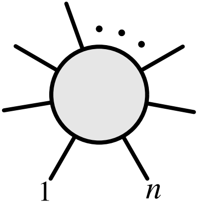
$\mathcal{A}_n \equiv$ 

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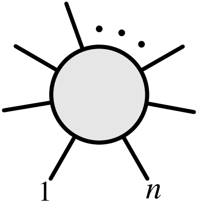
$\mathcal{A}_n \equiv$ 

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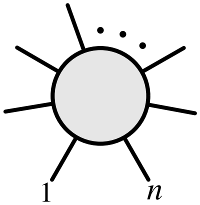
$\mathcal{A}_n \equiv$ 

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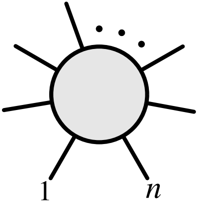
$\mathcal{A}_n \equiv$ 

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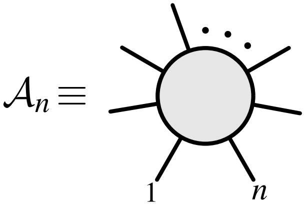

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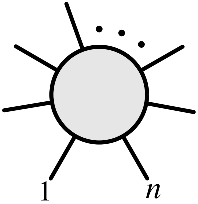
$\mathcal{A}_n \equiv$ 

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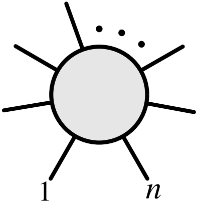
$\mathcal{A}_n \equiv$ 

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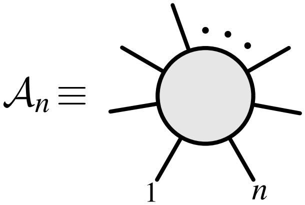

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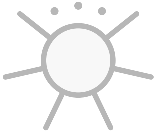

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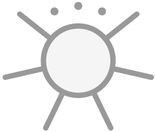

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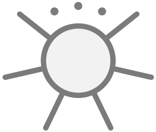

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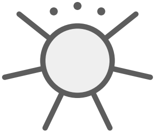

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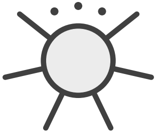

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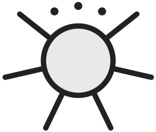

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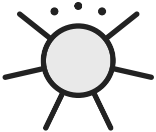

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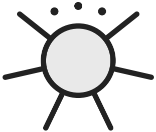

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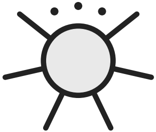

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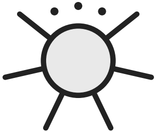

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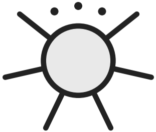

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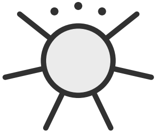

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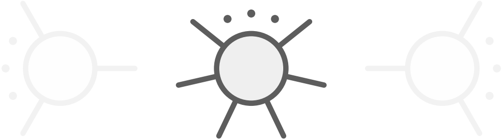

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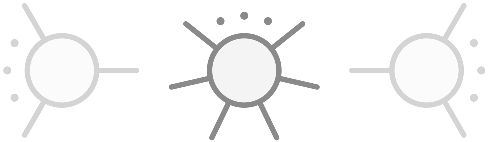

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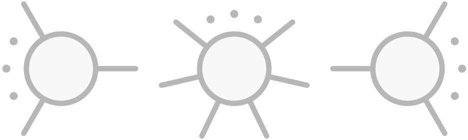

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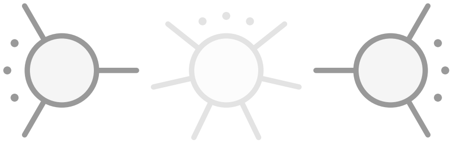

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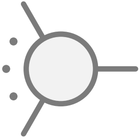
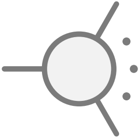

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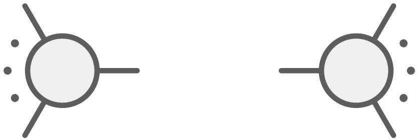

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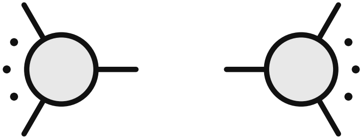

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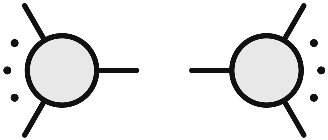

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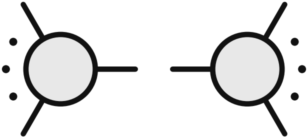

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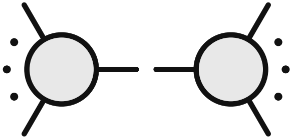

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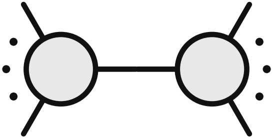

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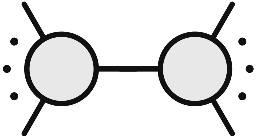

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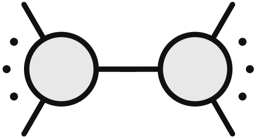

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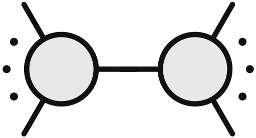

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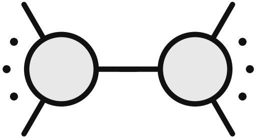

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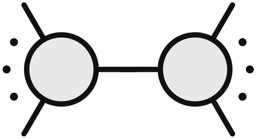

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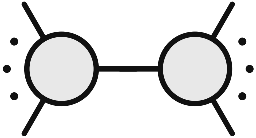

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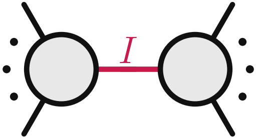

Internal Particles:

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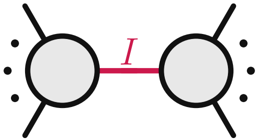

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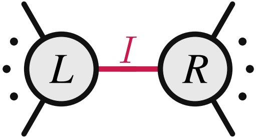

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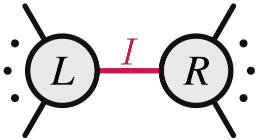

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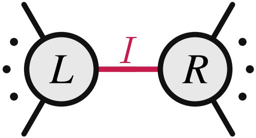

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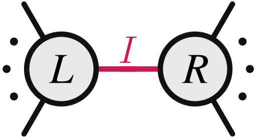

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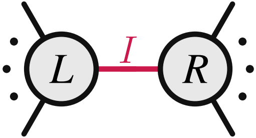

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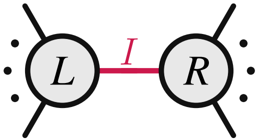

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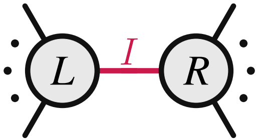

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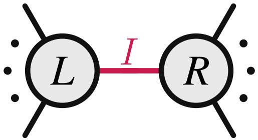

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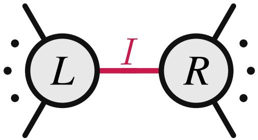

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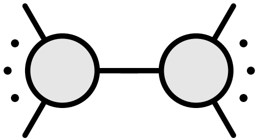

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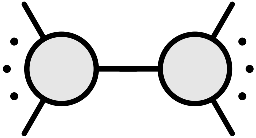

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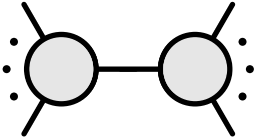

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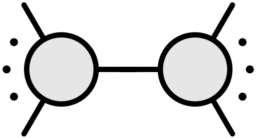

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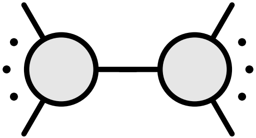

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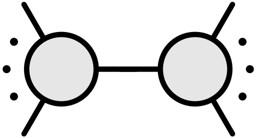

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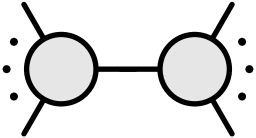

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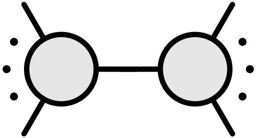

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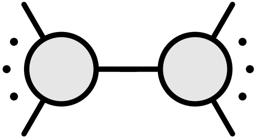

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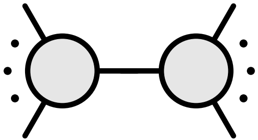

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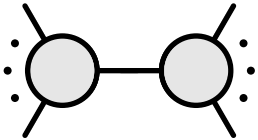

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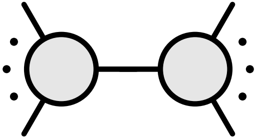

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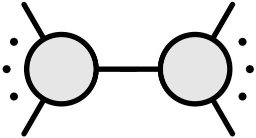

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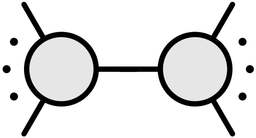

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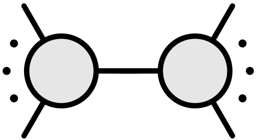

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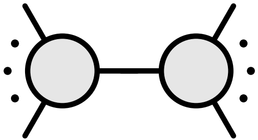

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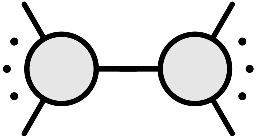

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Momentum conservation and Poincaré-invariance **uniquely** fix the kinematical dependence of the amplitude for three massless particles (to all loop orders!).

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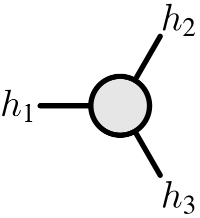

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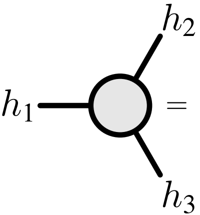
Momentum conservation and Poincaré-invariance **uniquely** fix the kinematical dependence of the amplitude for three massless particles (to all loop orders!).



$$= \; f(\lambda_1\widetilde{\lambda}_1, \lambda_2\widetilde{\lambda}_2, \lambda_3\widetilde{\lambda}_3)\,\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$

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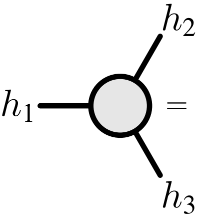

$$= f(\lambda_1\widetilde{\lambda}_1, \lambda_2\widetilde{\lambda}_2, \lambda_3\widetilde{\lambda}_3)\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$

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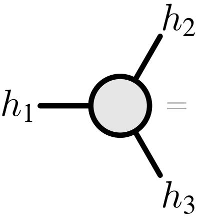

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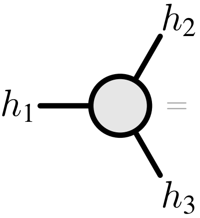

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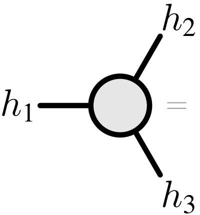

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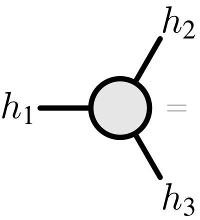

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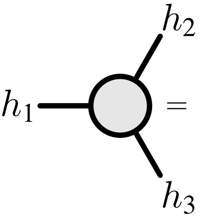

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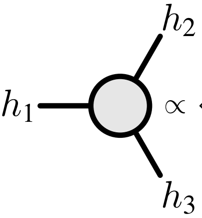

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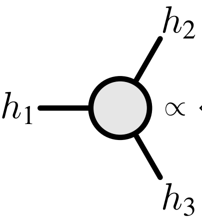

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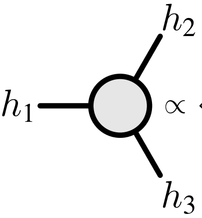

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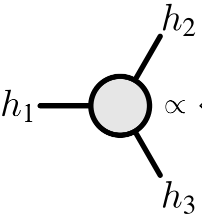

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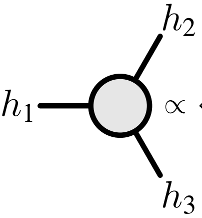

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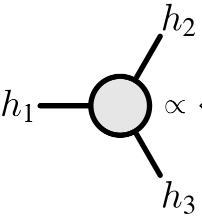

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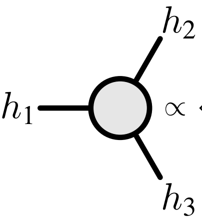

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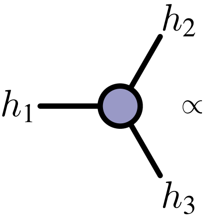

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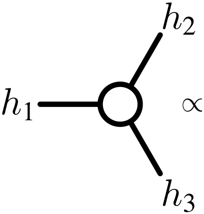

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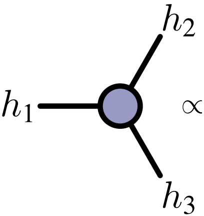

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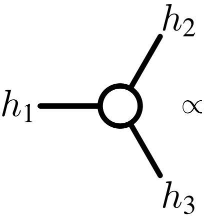

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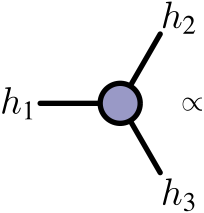

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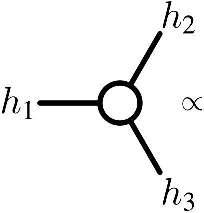

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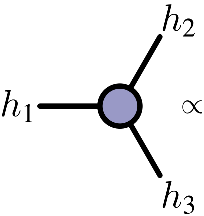

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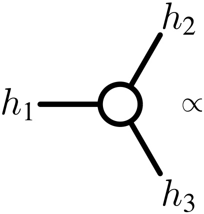

$$\propto \quad [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}$$

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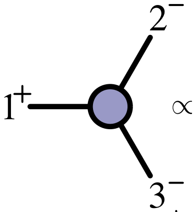
Momentum conservation and Poincaré-invariance **uniquely** fix the kinematical dependence of the amplitude for three massless particles (to all loop orders!).



$$\propto \quad \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}$$
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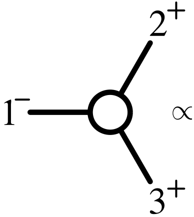

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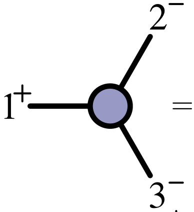
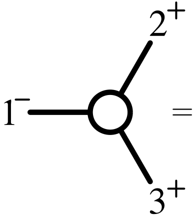


$$= \frac{\langle 2\,3\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}\,\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$



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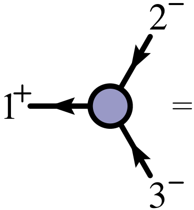

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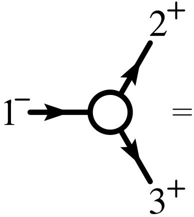

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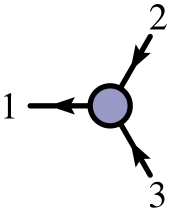

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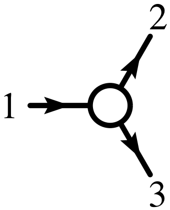

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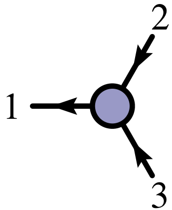

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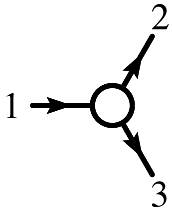

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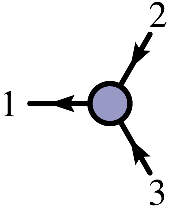

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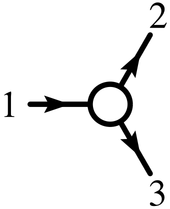

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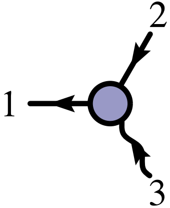

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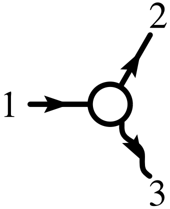

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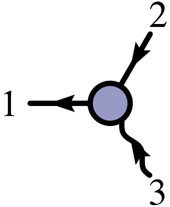

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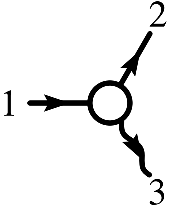

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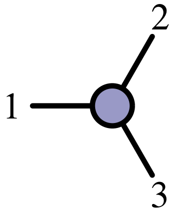
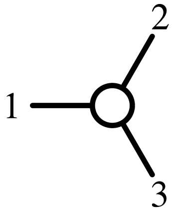


$$= \frac{\delta^{2\times4}\big(\lambda\cdot\widetilde{\eta}\big)}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}\,\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)\equiv \mathcal{A}_3^{(2)}$$



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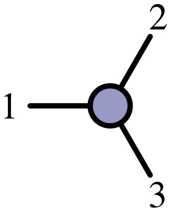

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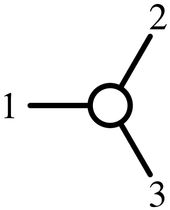

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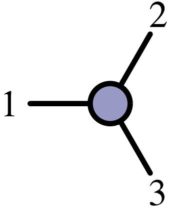

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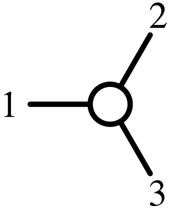

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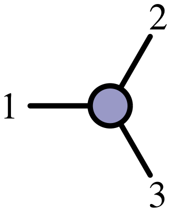

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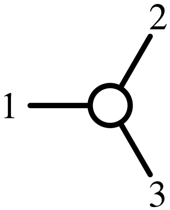

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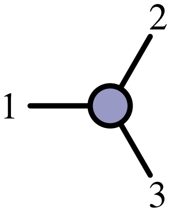

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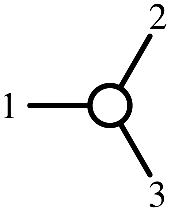

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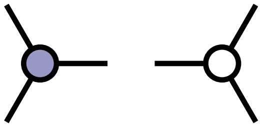

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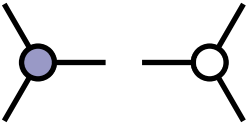

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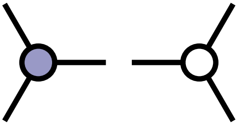

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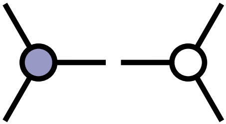

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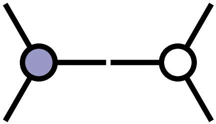

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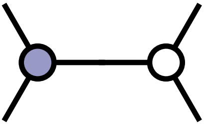

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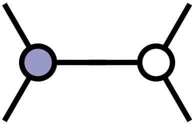

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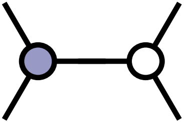

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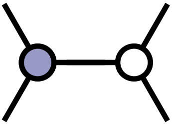

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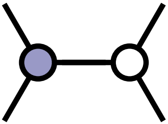

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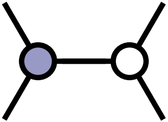

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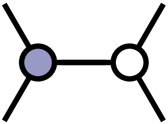

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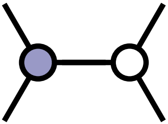

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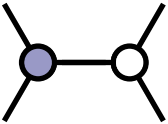

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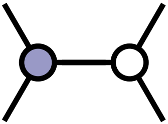

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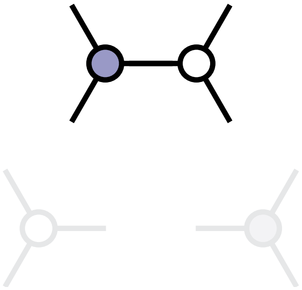

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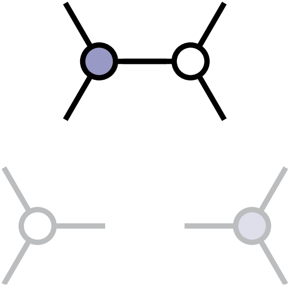

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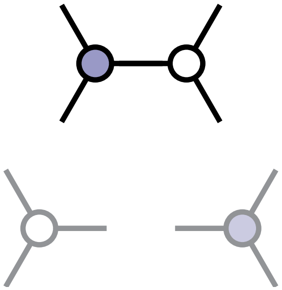

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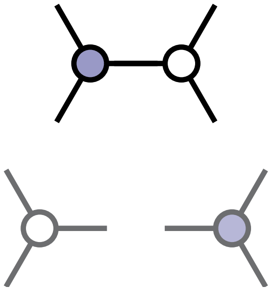

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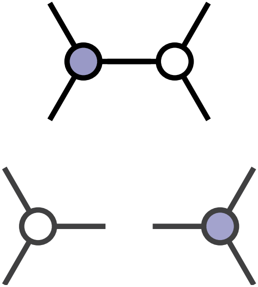

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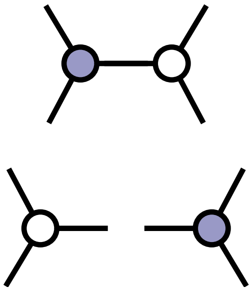

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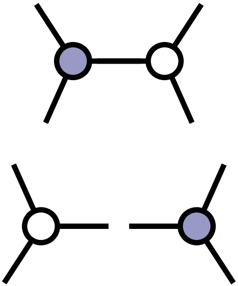

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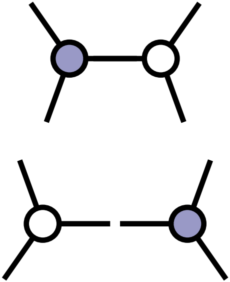

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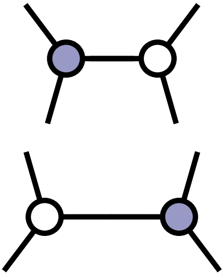

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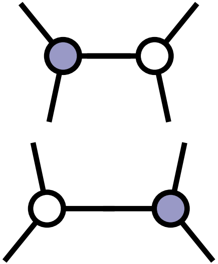

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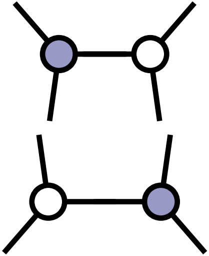

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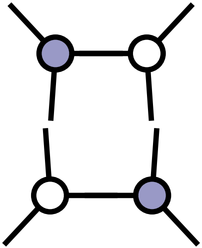

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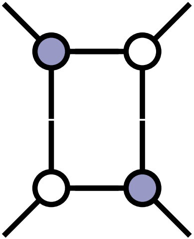

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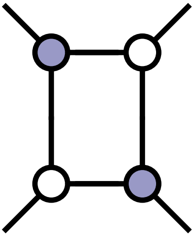

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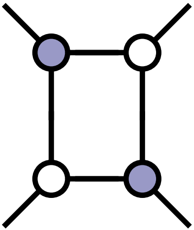

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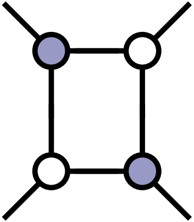

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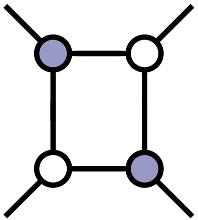

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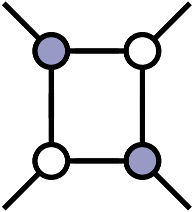

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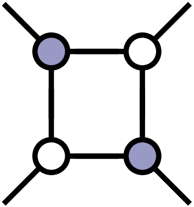

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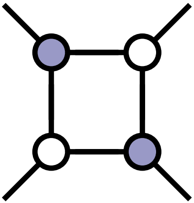

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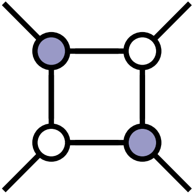

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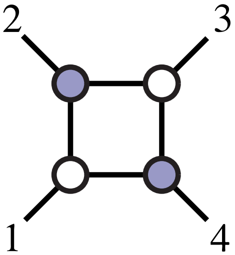

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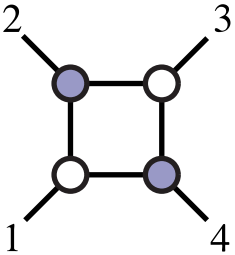

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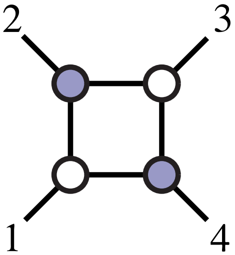

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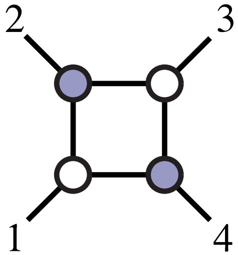

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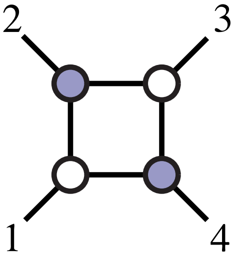

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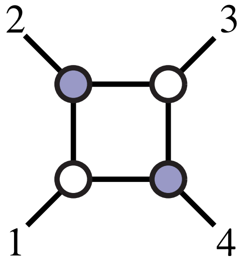

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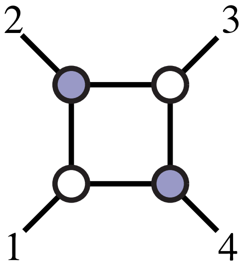

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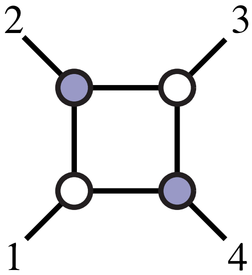

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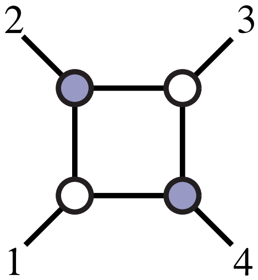

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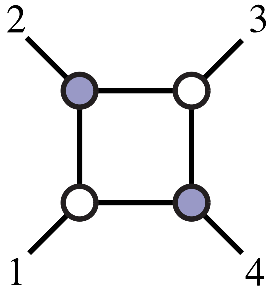

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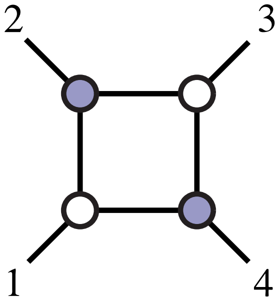

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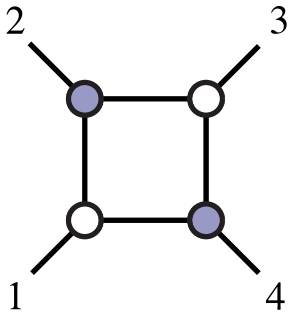

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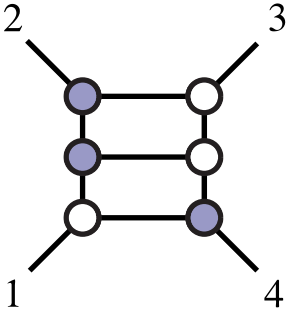

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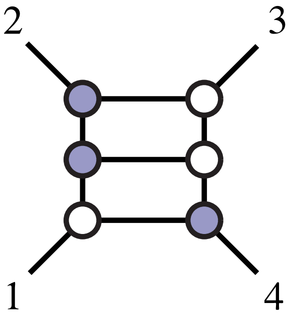

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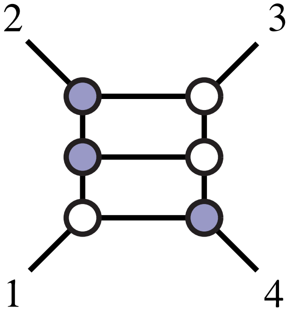

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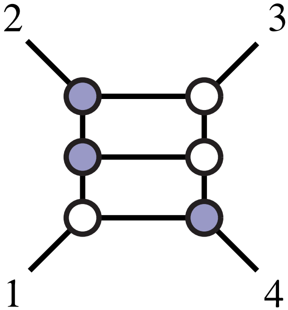

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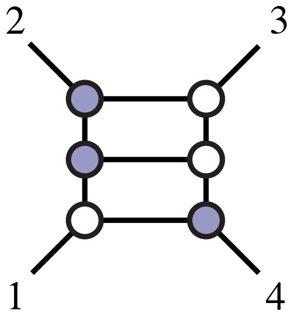

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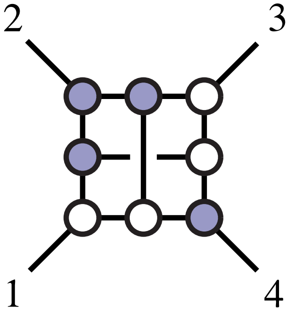

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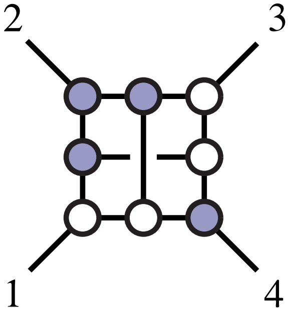

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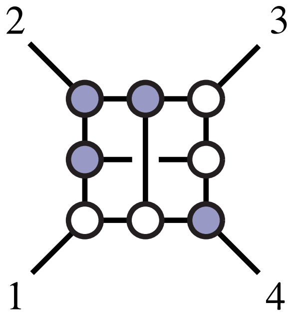

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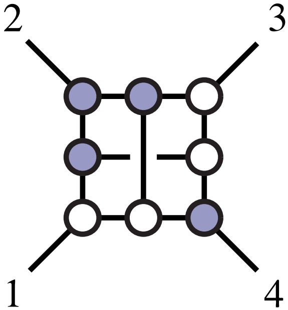

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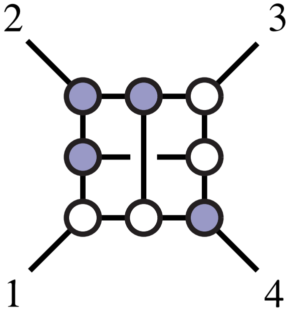

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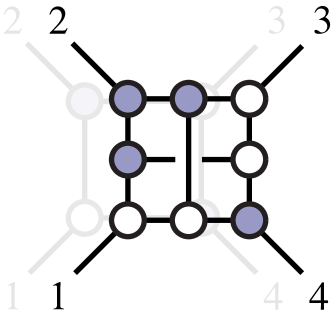

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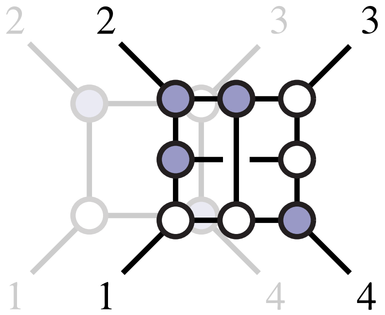

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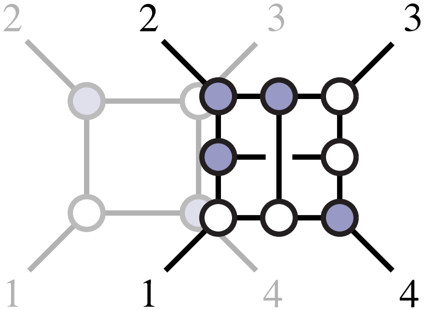

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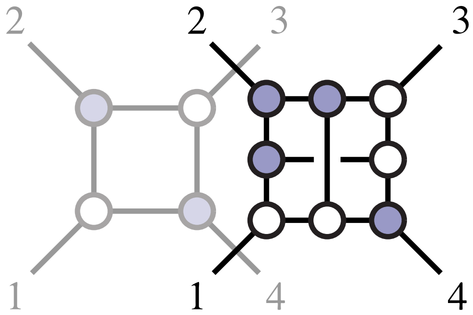

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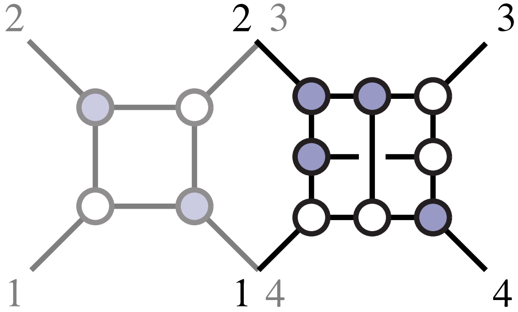

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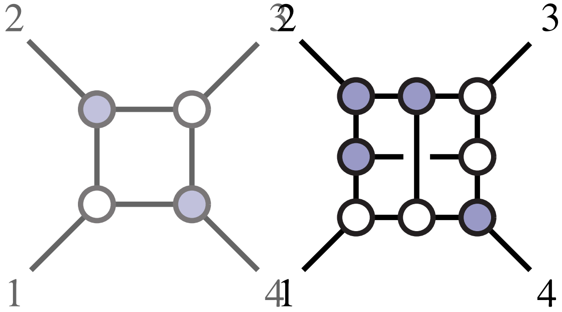

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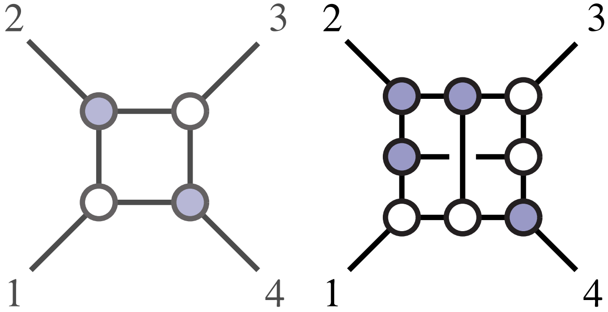

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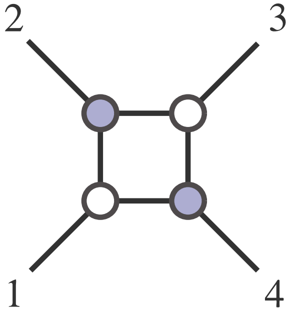

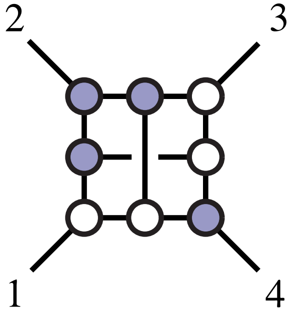

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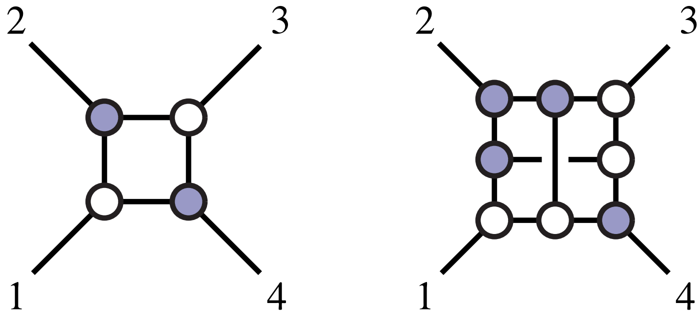

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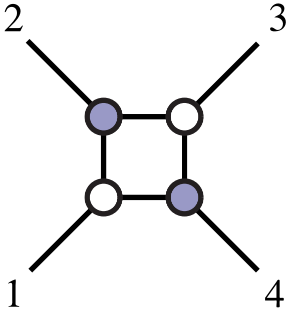

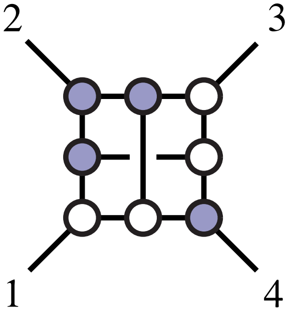

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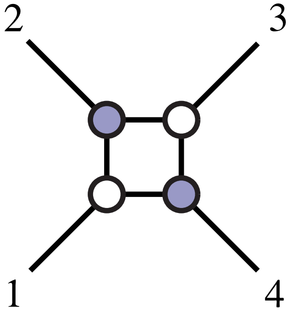

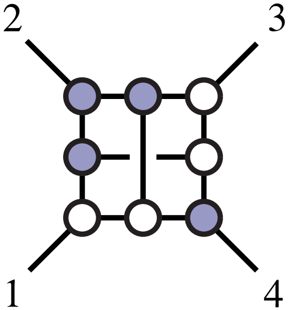

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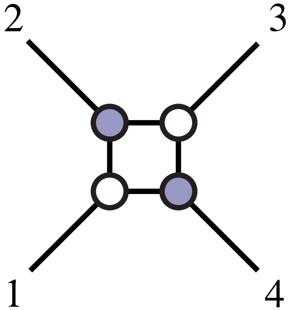

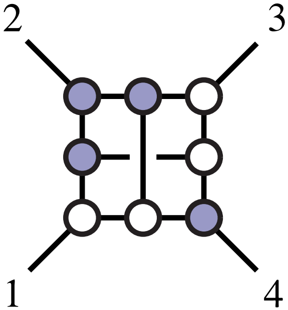

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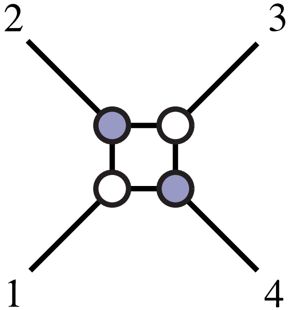

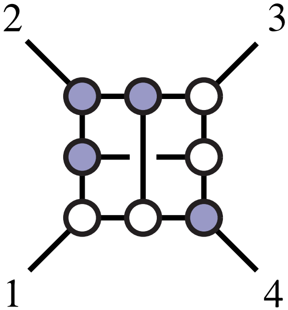

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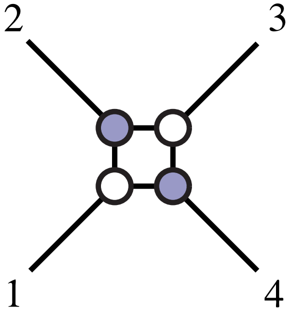

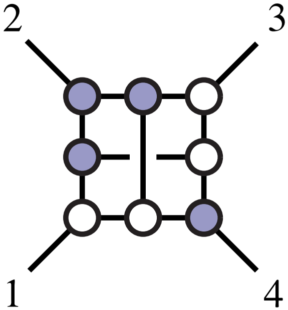

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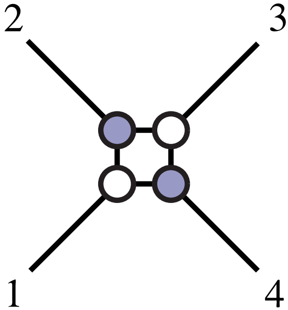

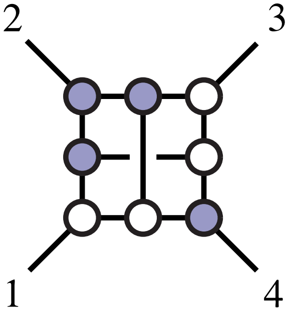

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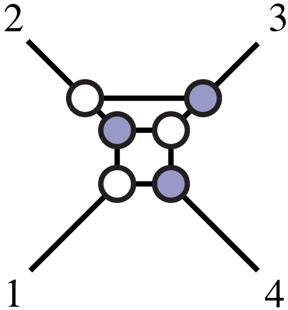

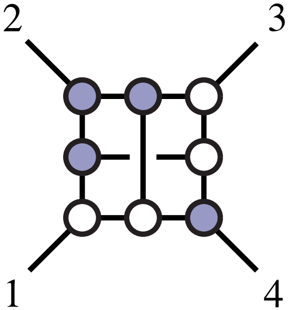

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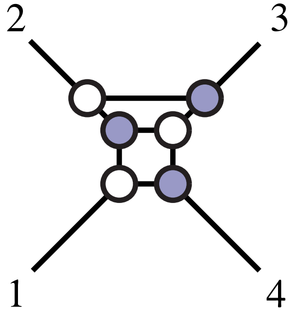

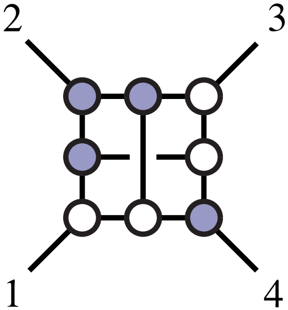

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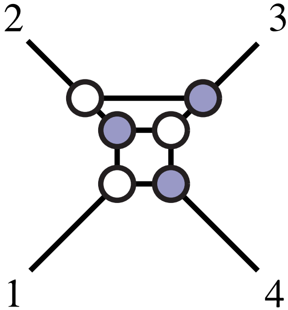

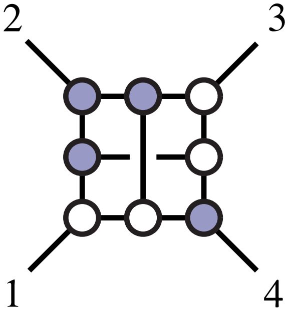

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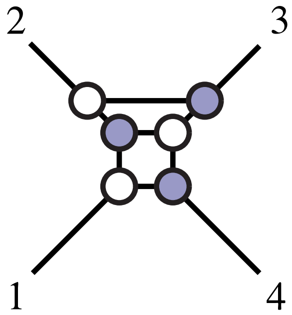

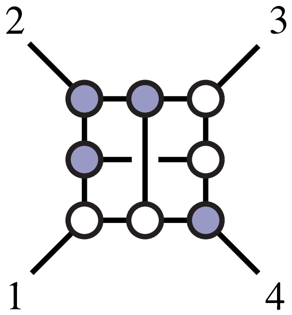

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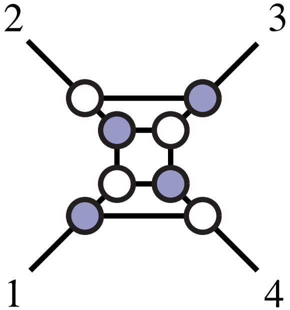

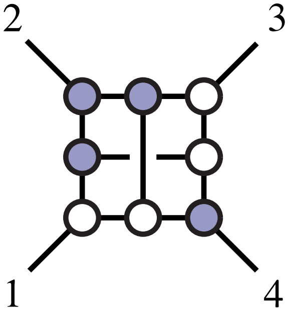

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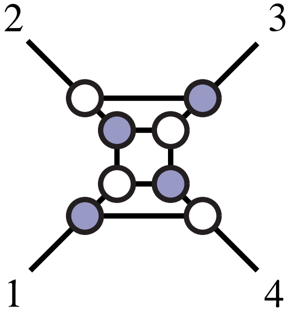

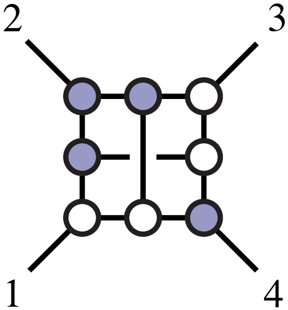

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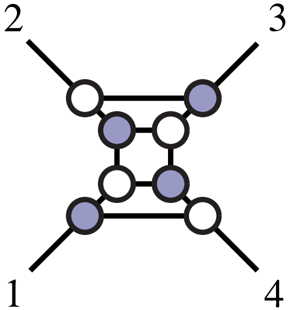

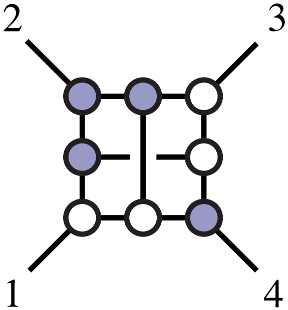

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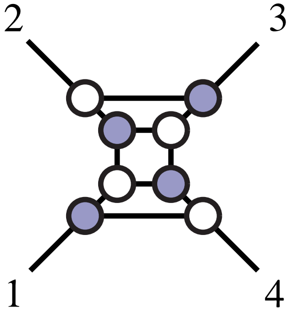

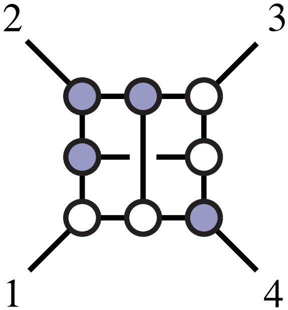

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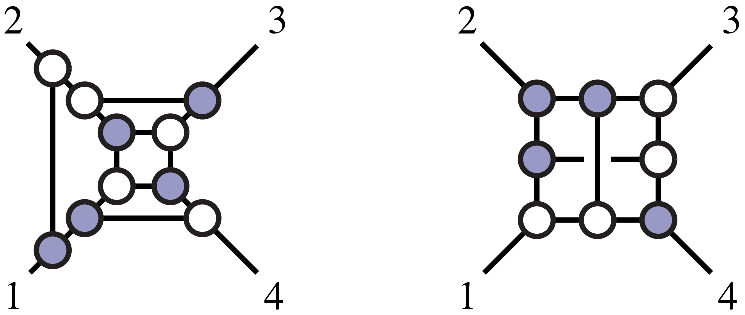

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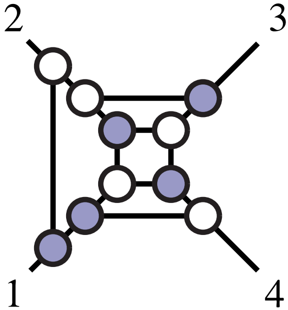

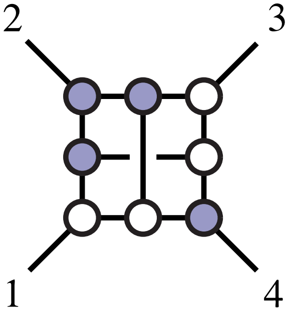

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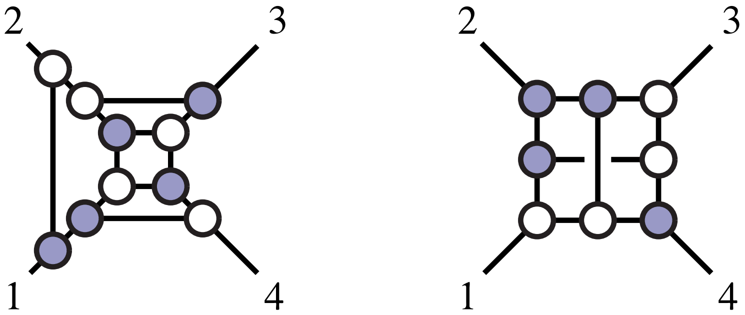

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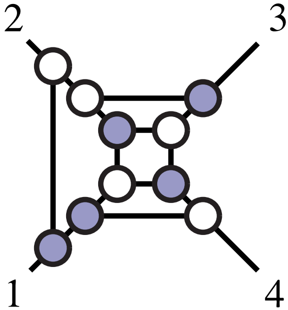

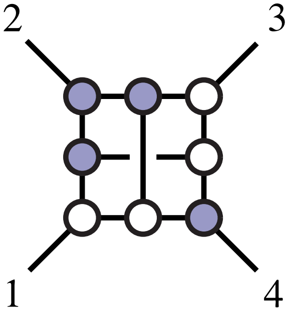

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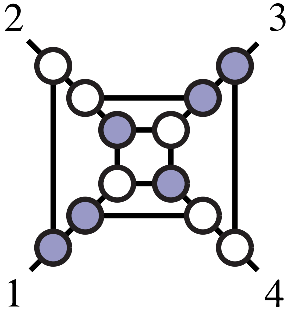

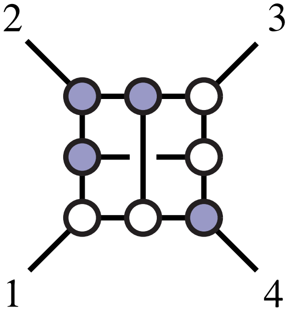

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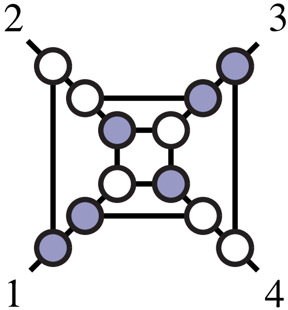

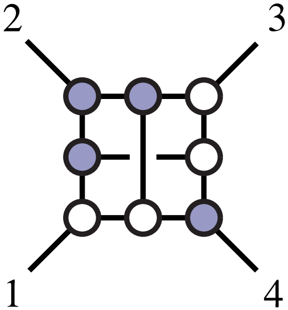

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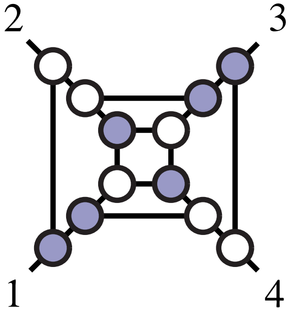

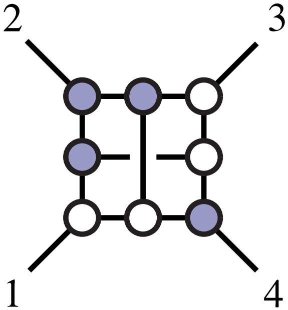

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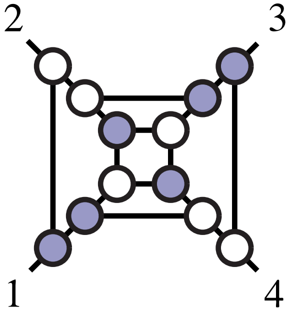

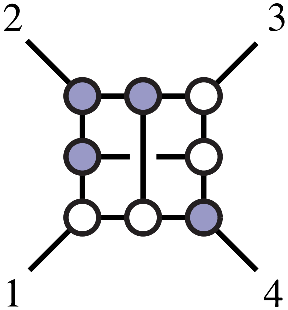

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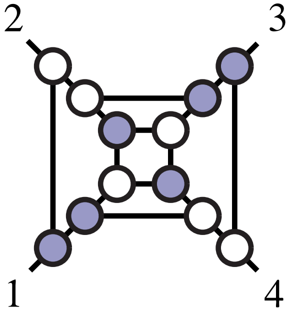

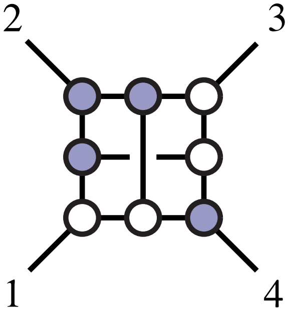

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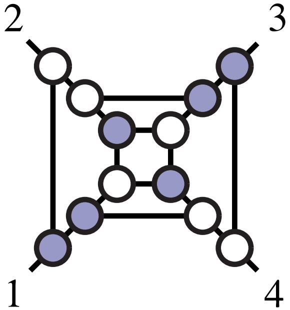

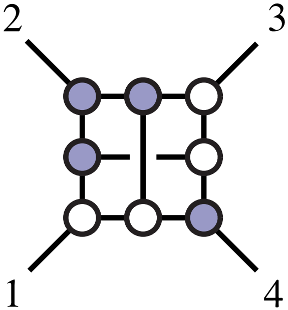

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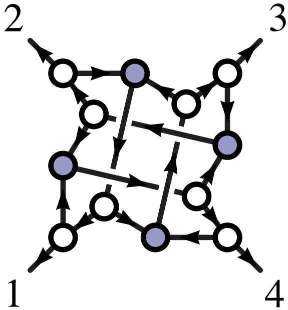

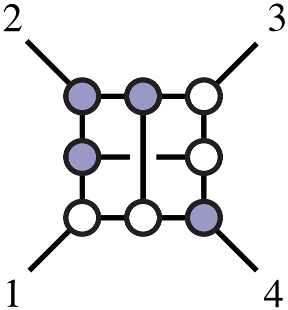

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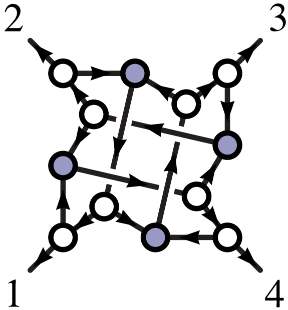

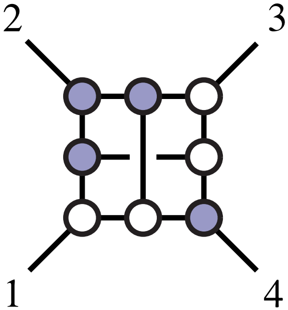

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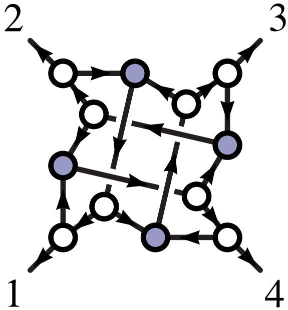

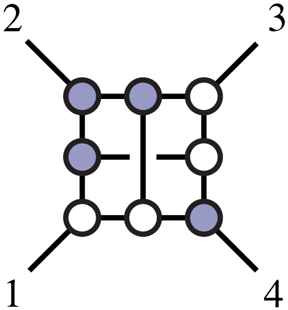

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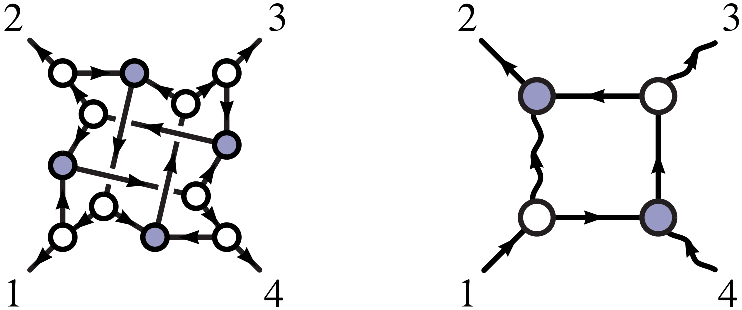

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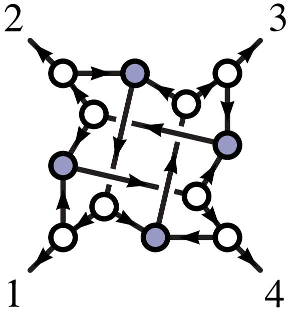

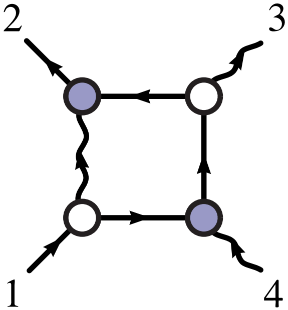

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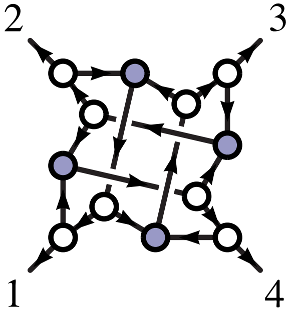

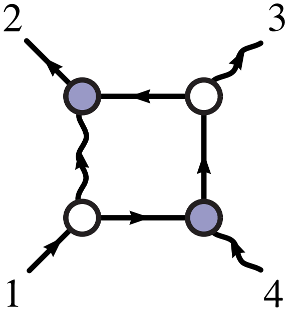

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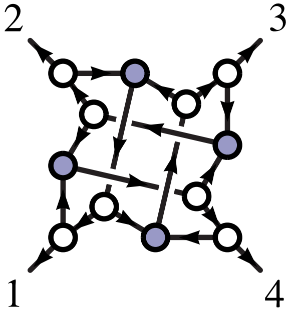

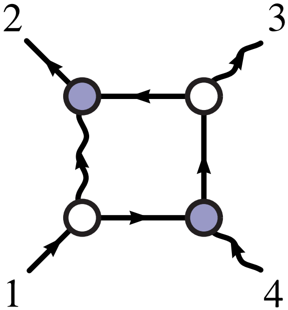

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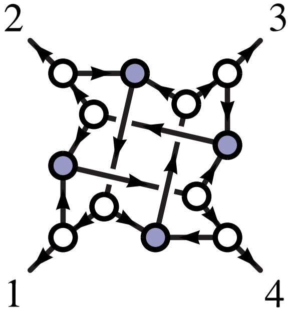

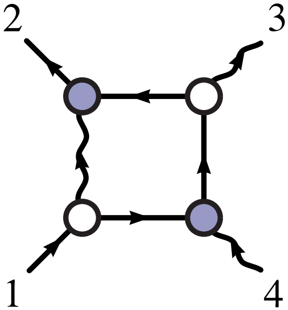

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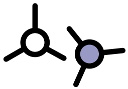

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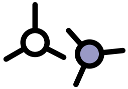

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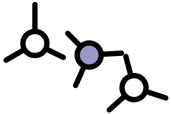

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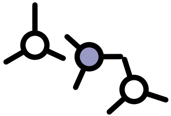

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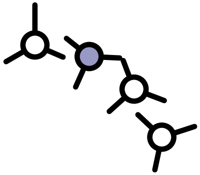

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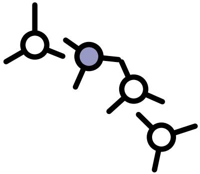

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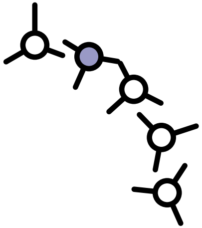

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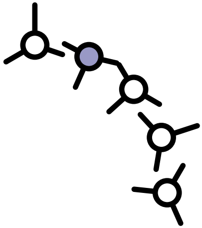

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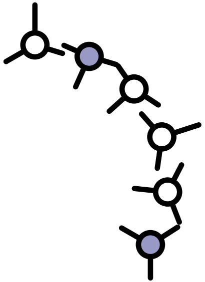

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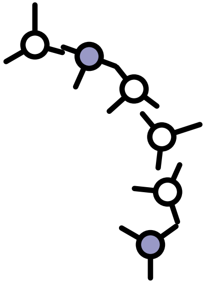

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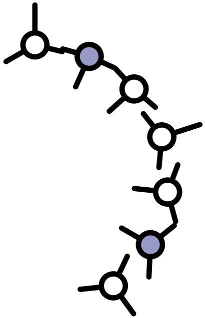

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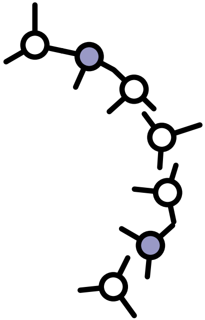

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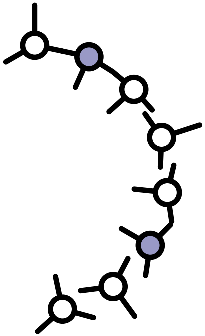

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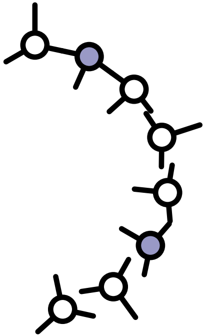

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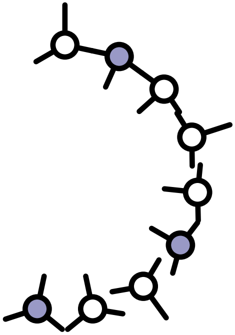

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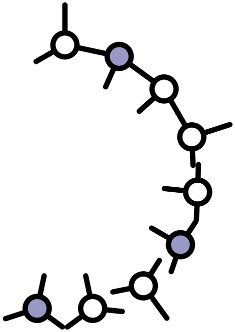

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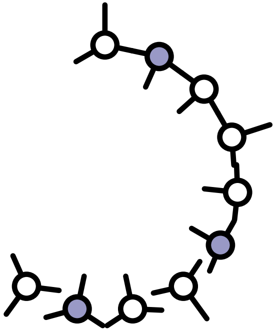

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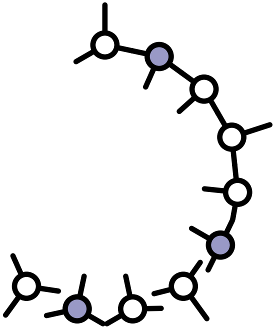

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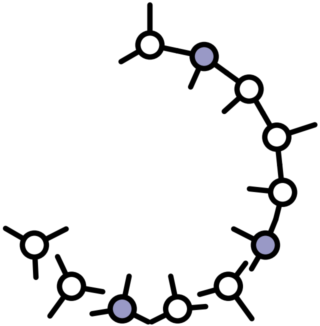

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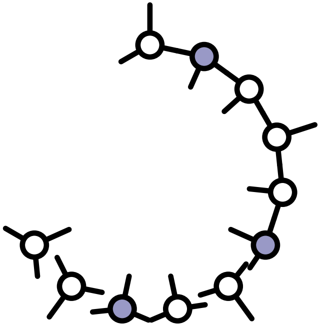

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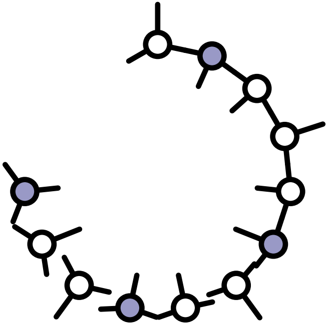

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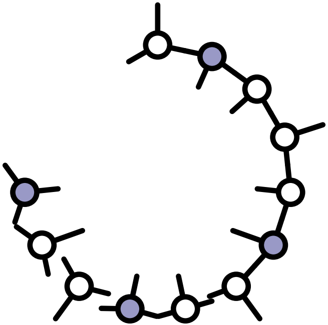

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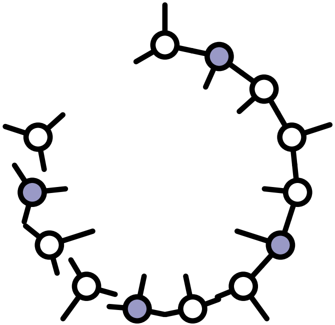

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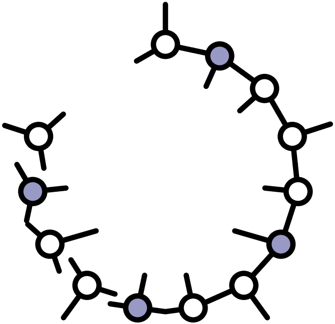

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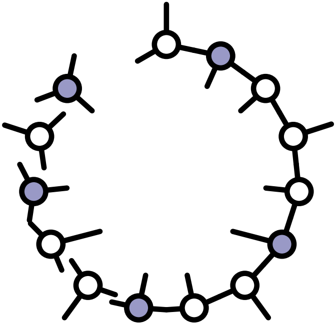

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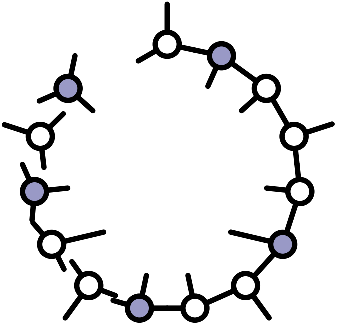

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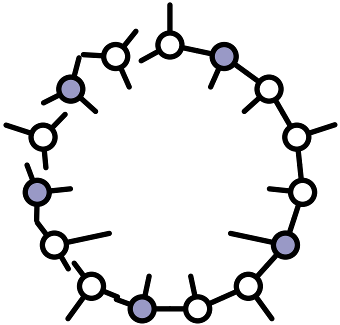

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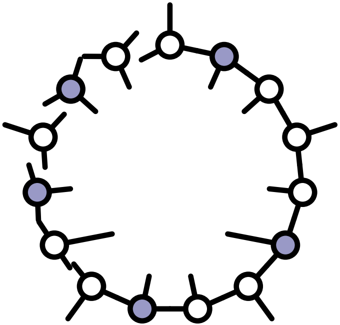

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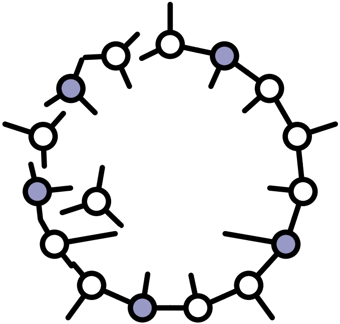

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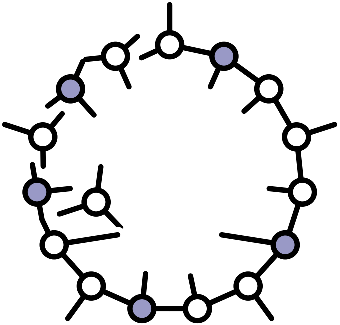

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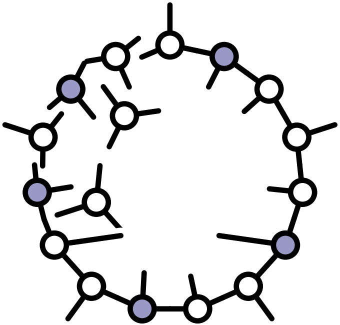

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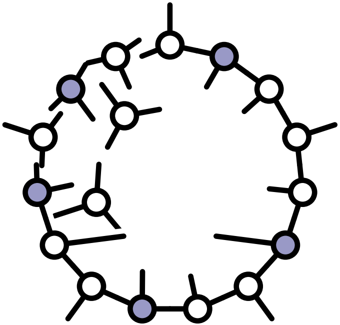

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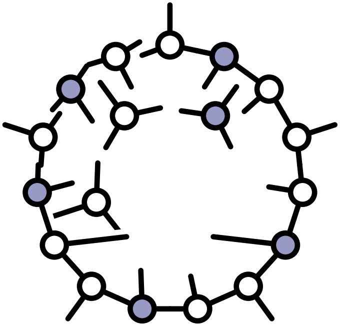

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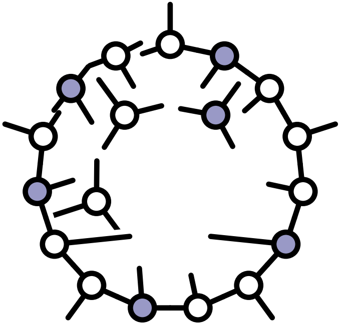

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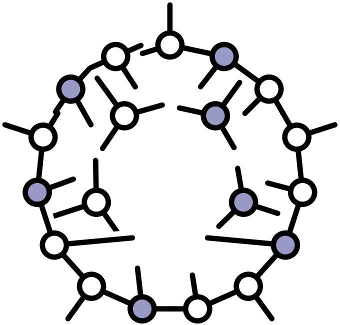

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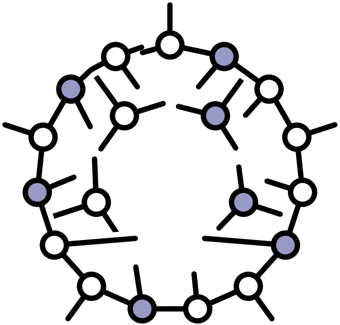

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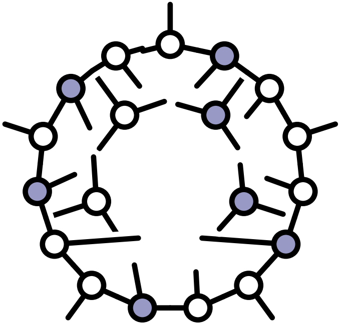

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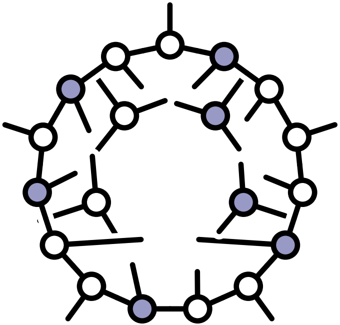

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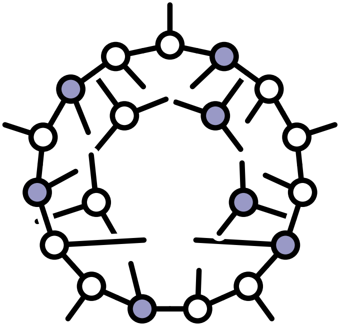

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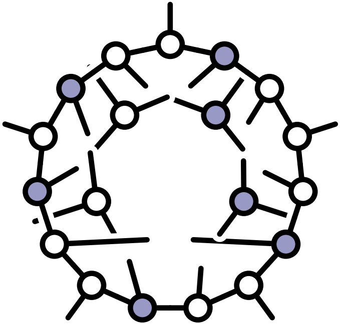

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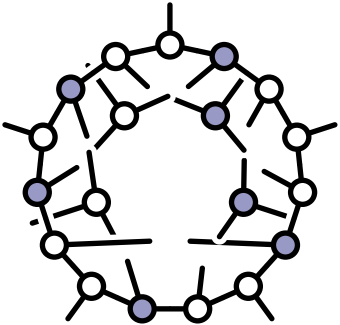

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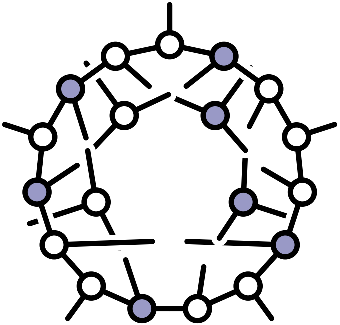

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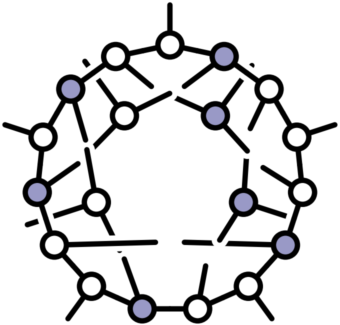

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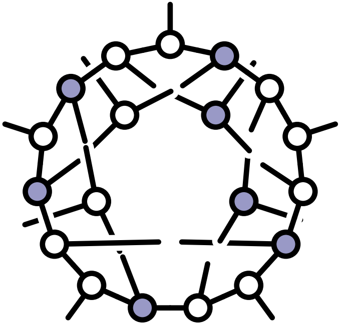

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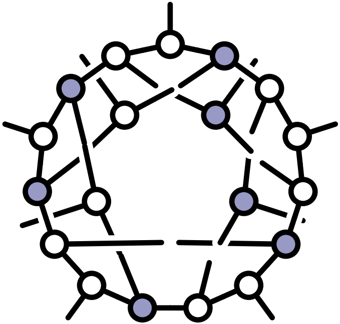

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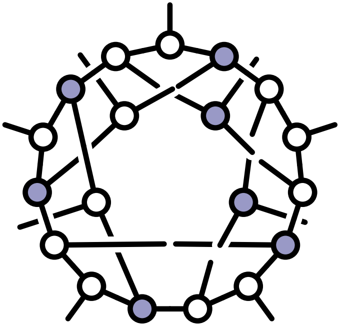

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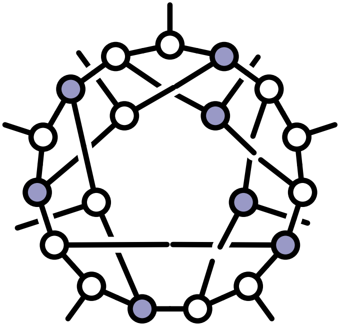

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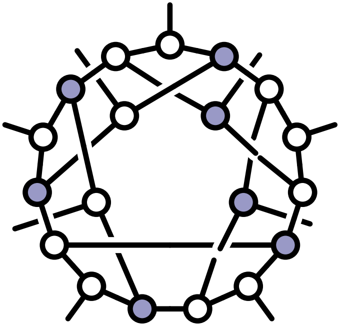

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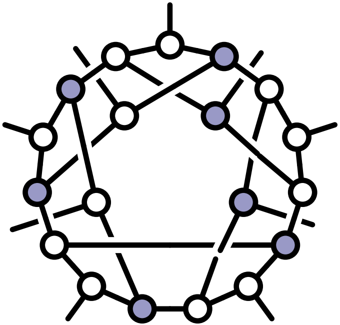

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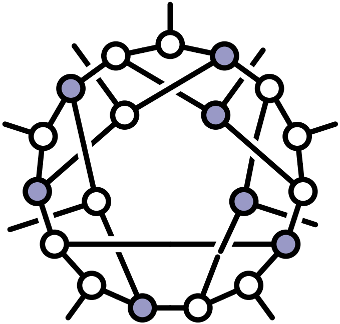

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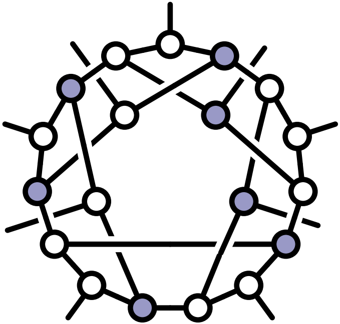

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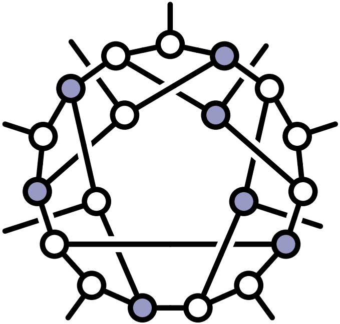

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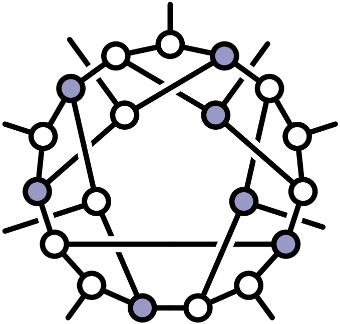

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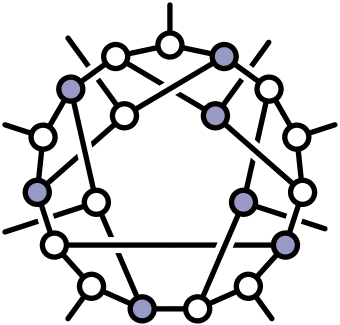

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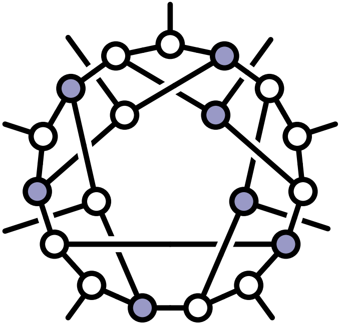

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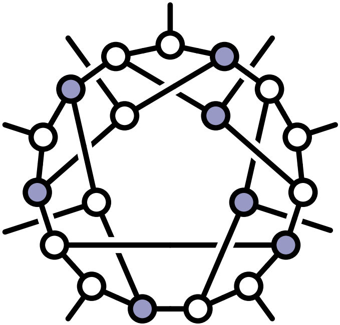

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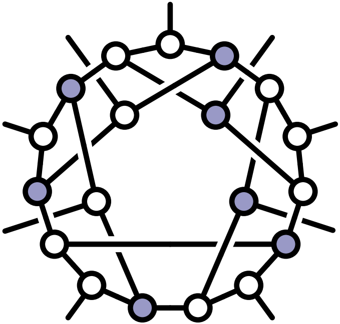

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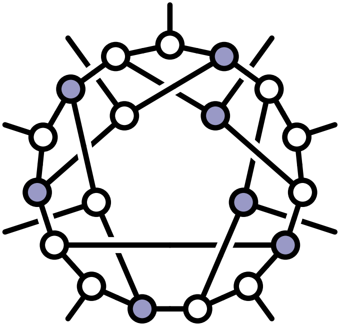

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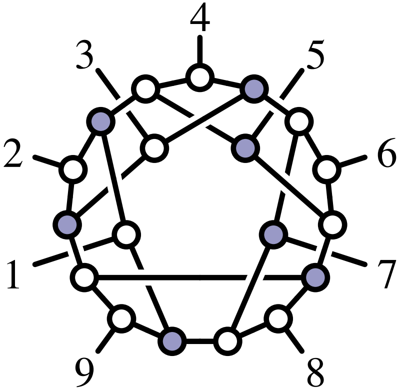

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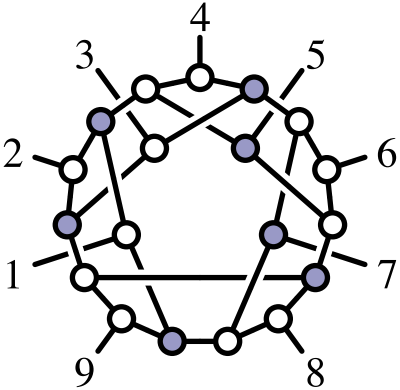

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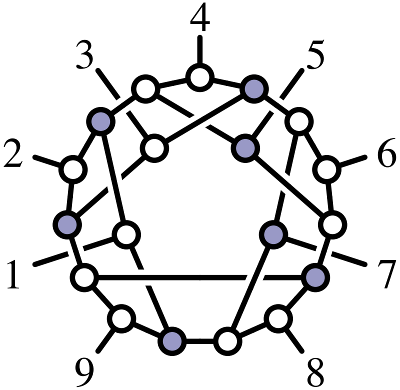

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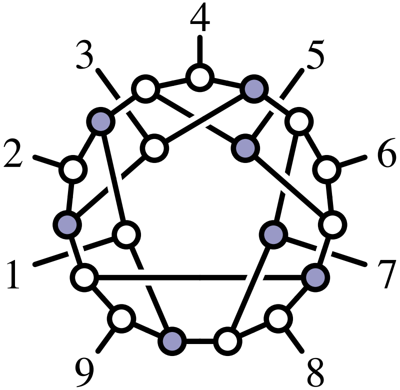

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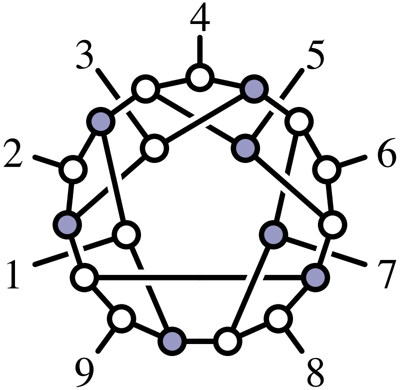

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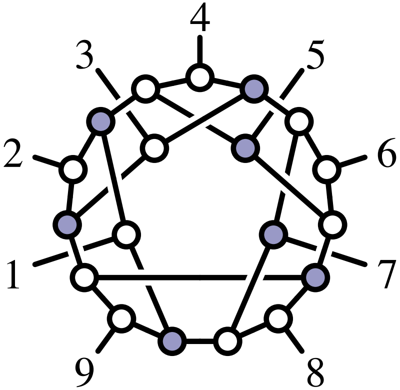

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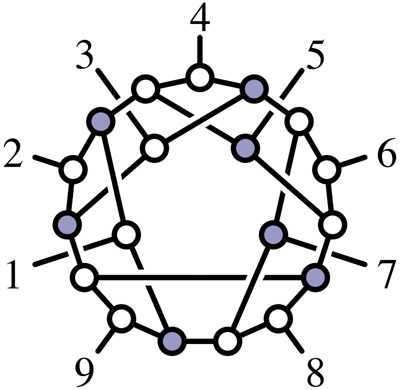

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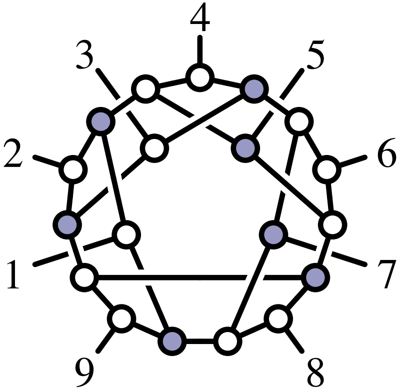

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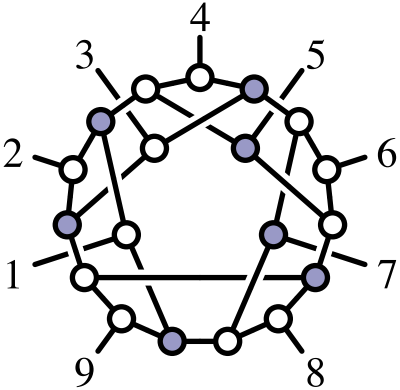

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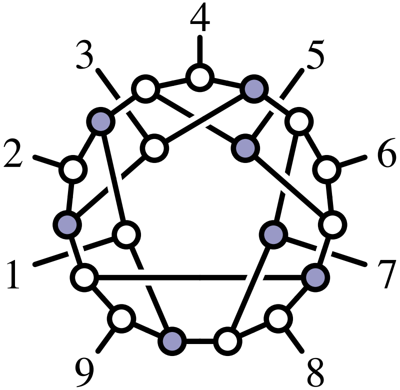

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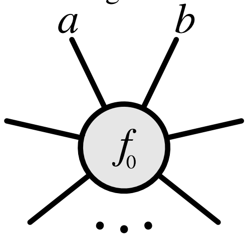

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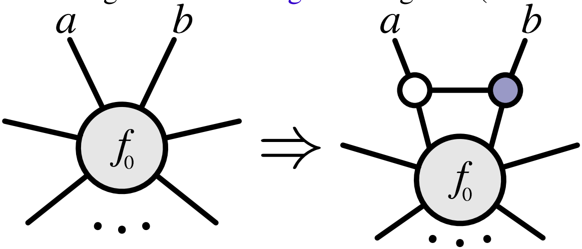

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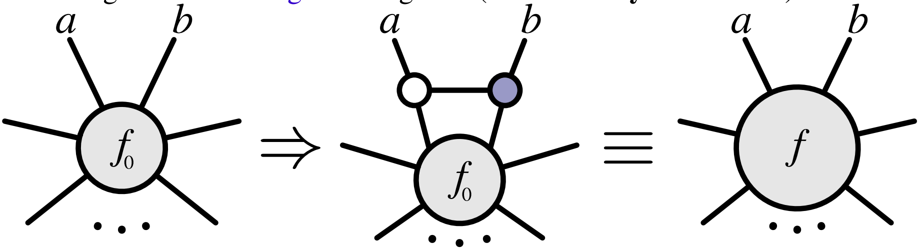

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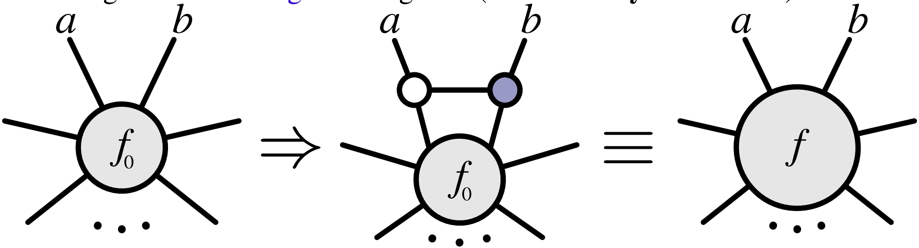

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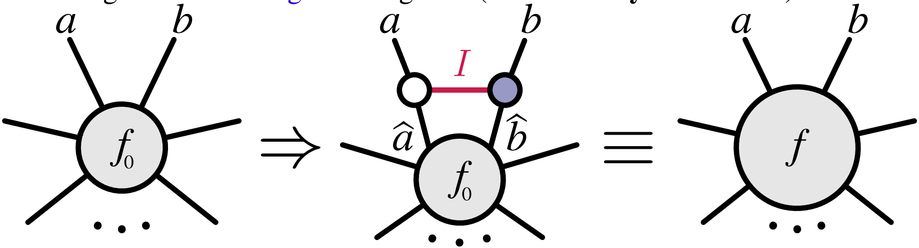

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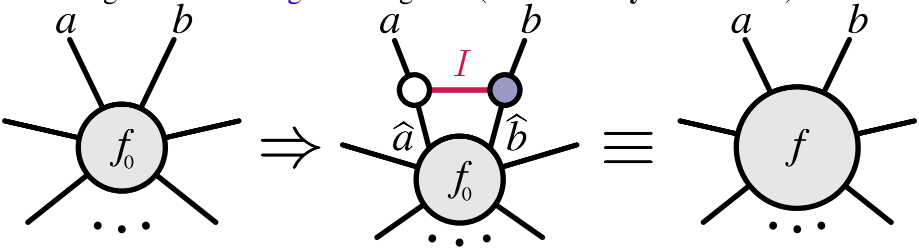

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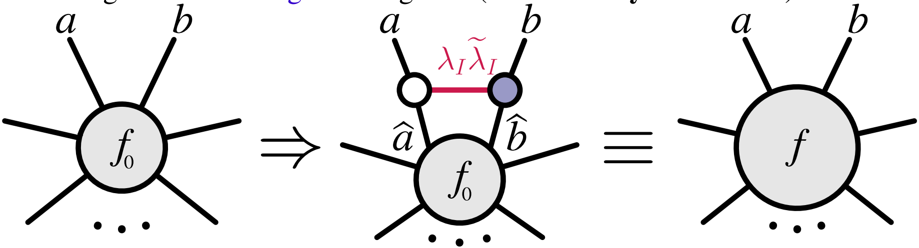

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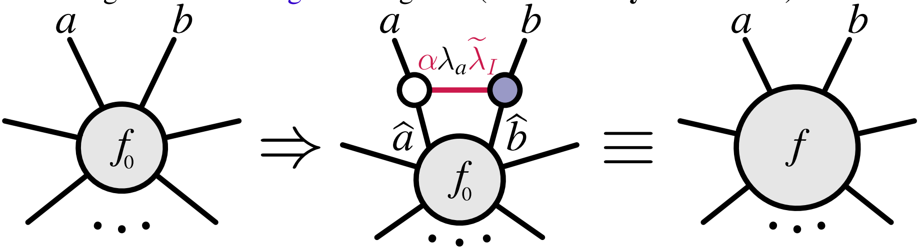

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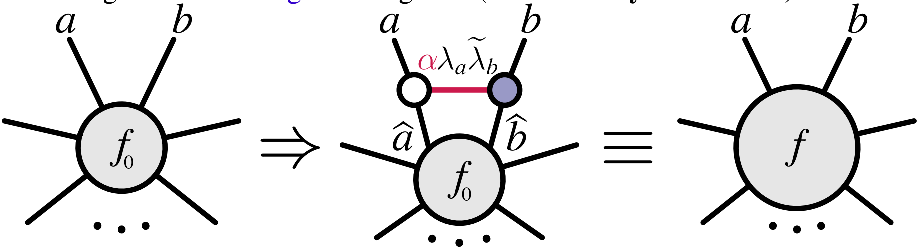

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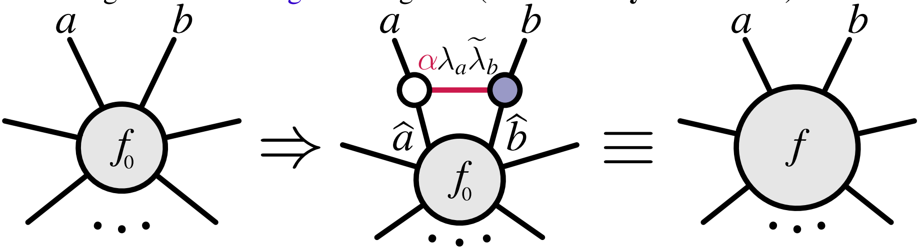

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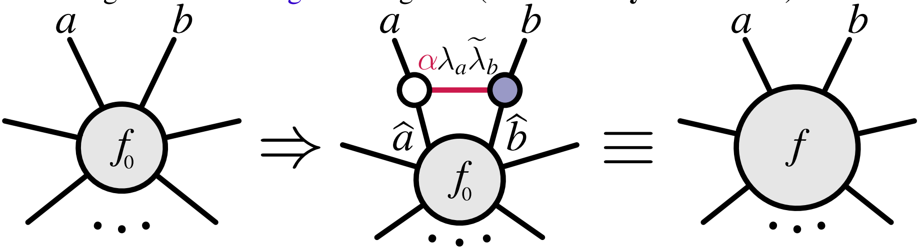

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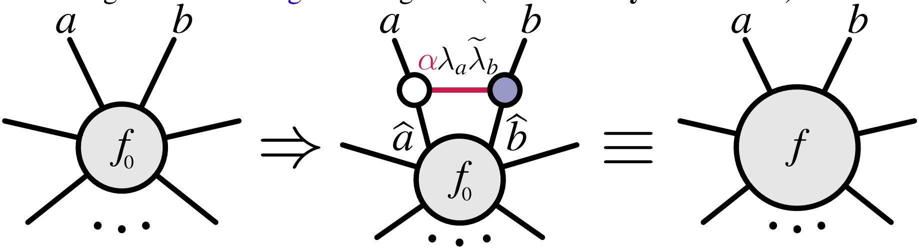

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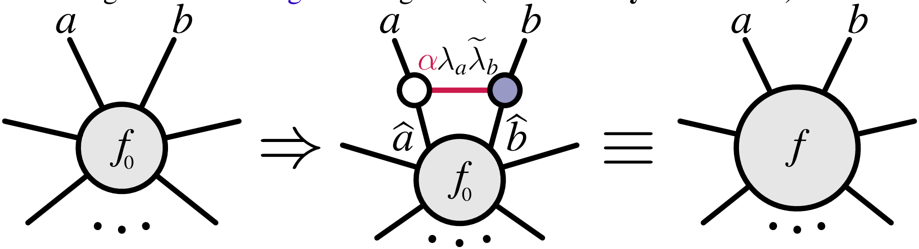

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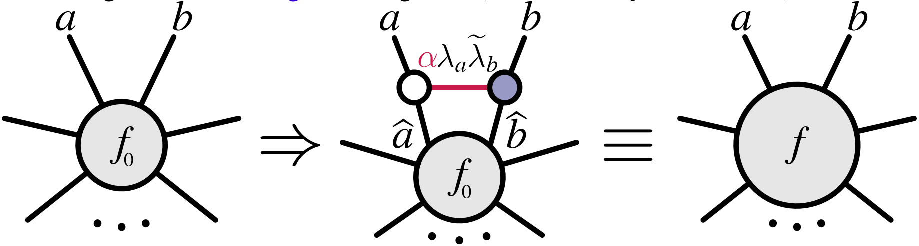

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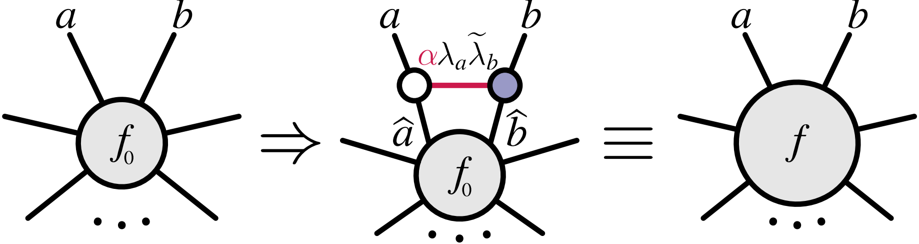

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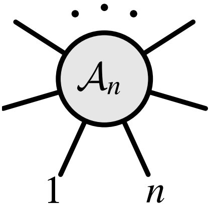

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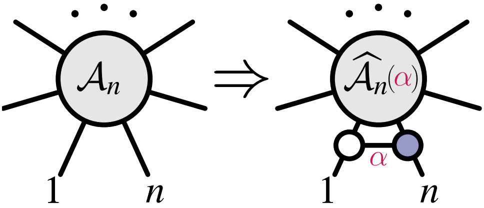

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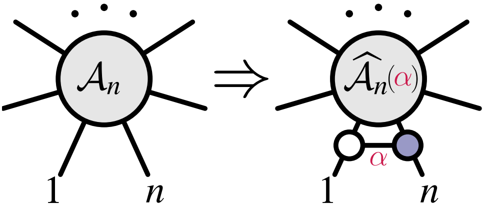

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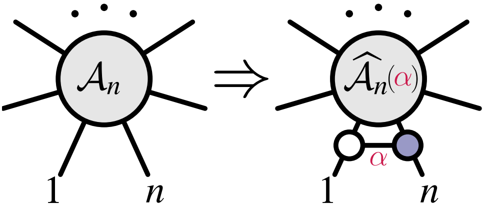

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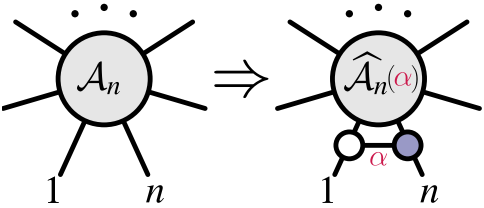

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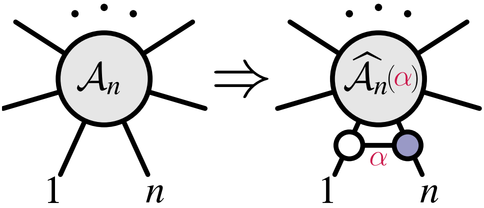

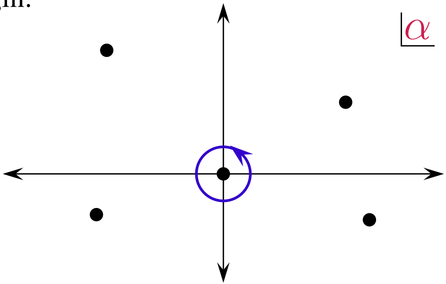

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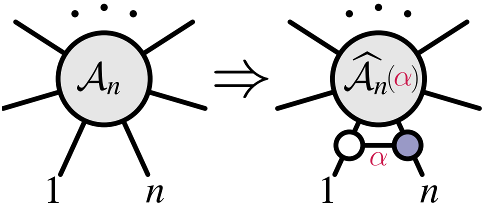

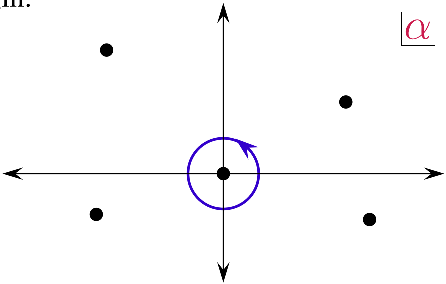

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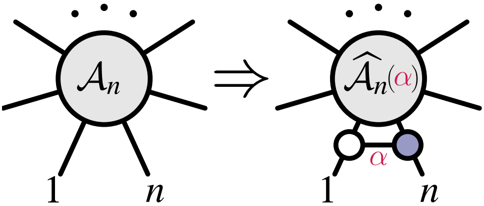

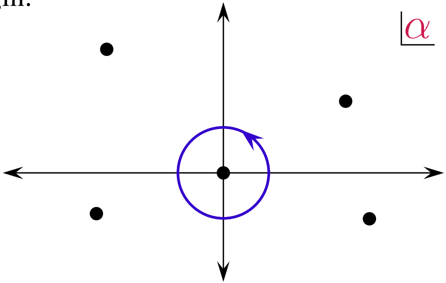

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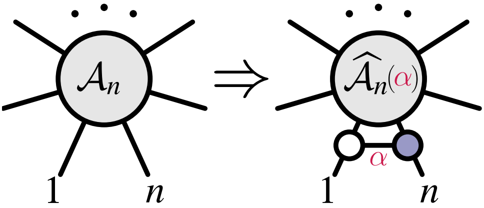

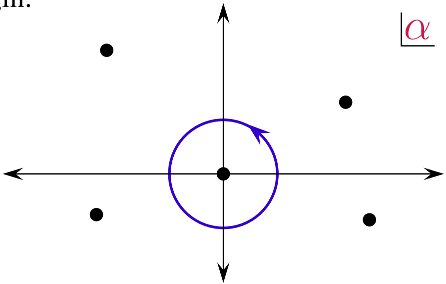

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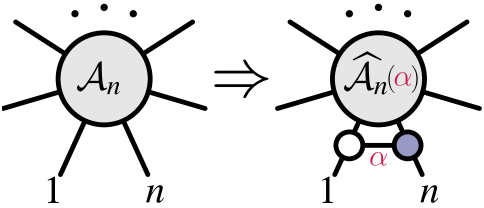

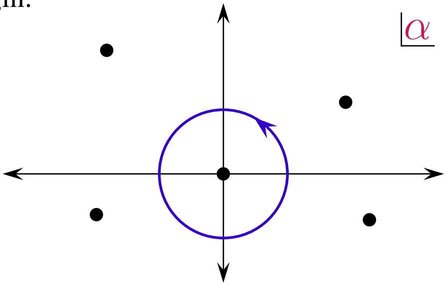

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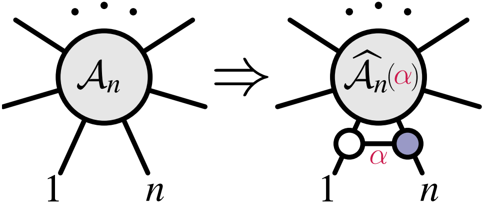

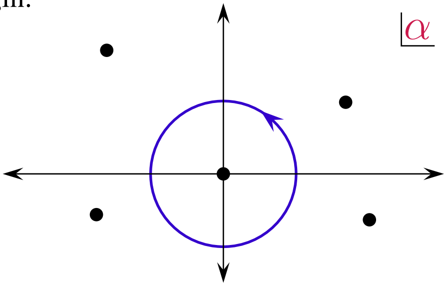

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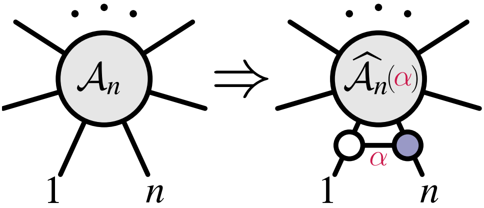

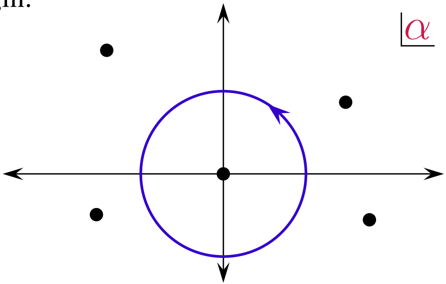

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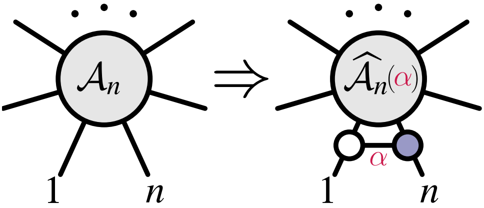

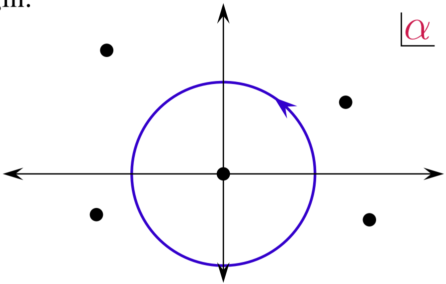

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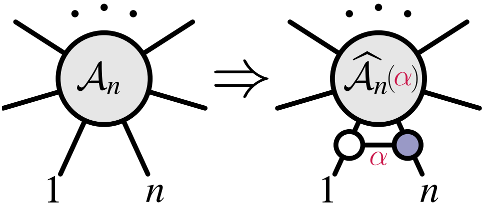

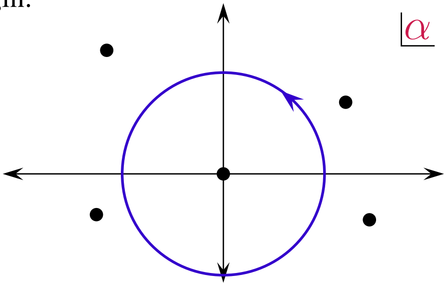

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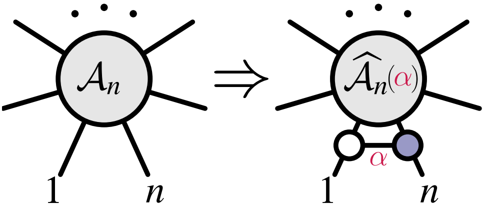

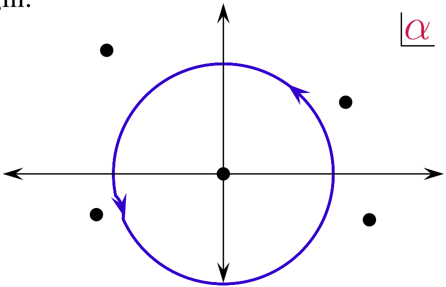

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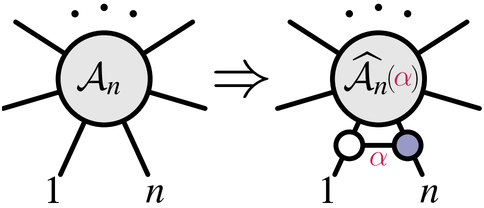

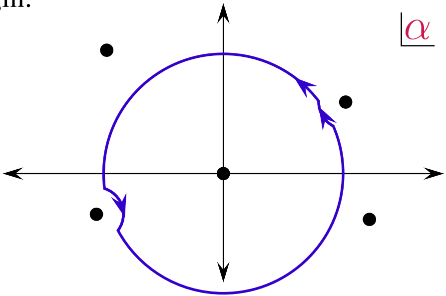

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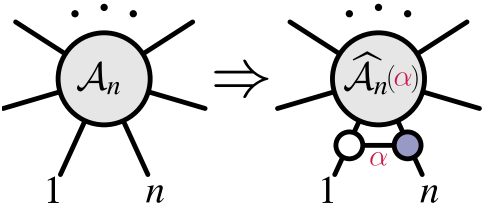

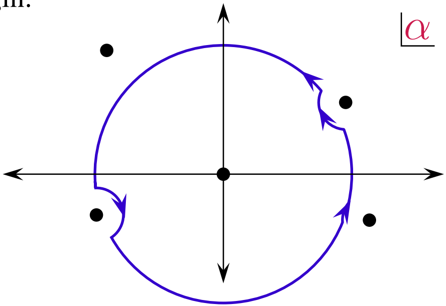

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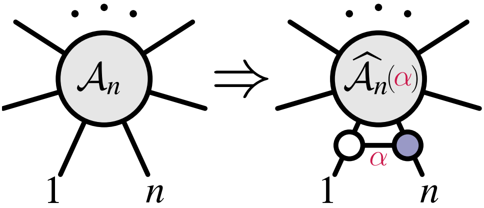

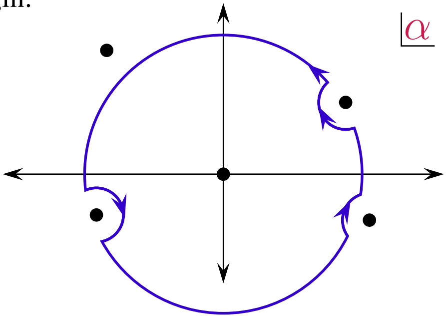

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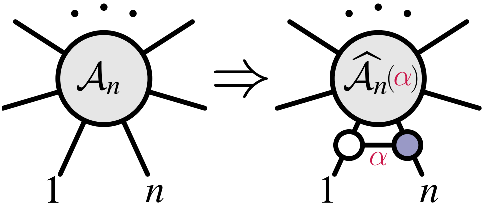

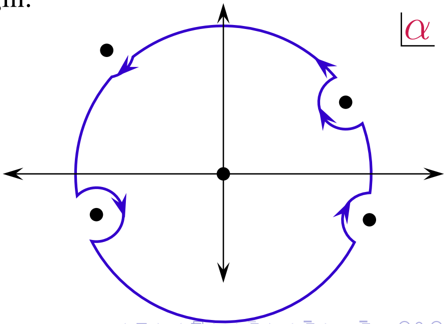

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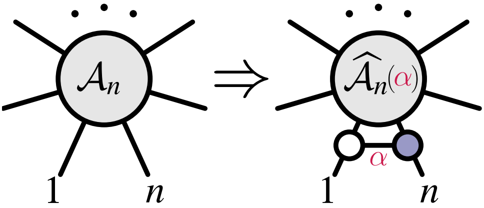

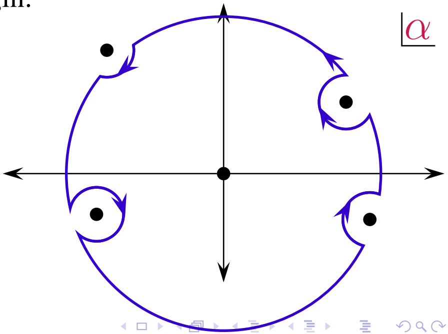

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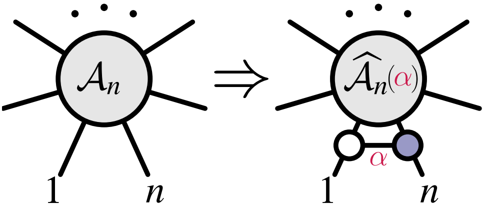

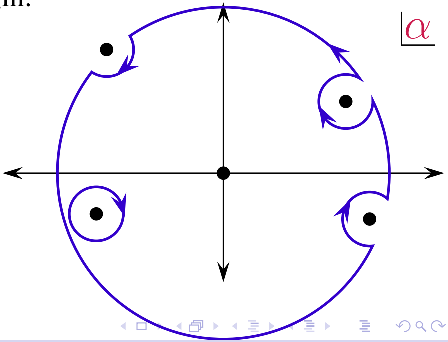

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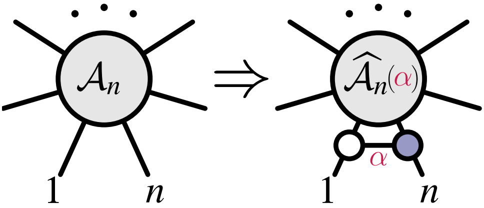

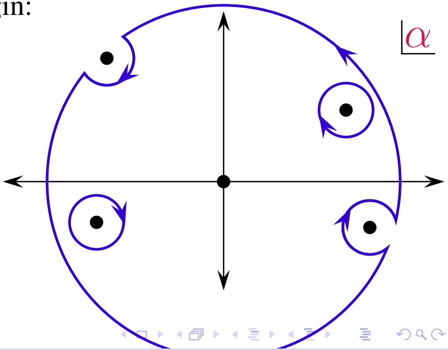

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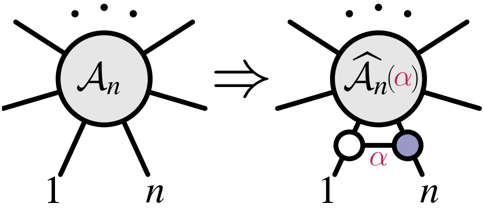

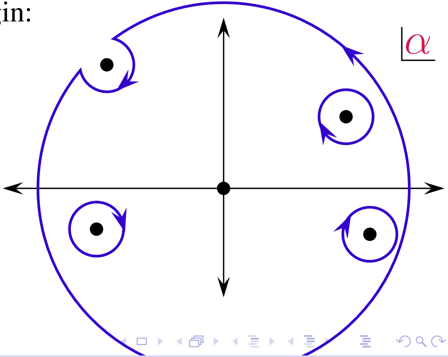

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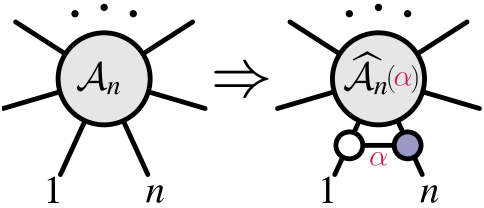

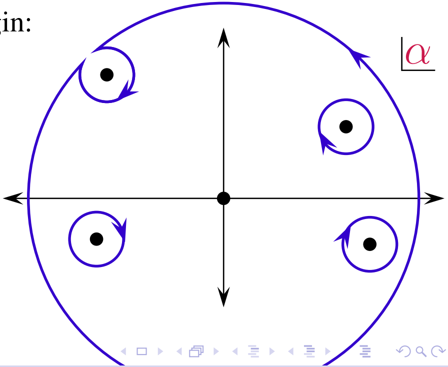

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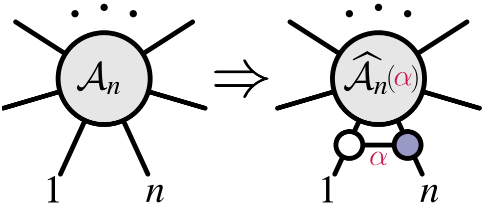

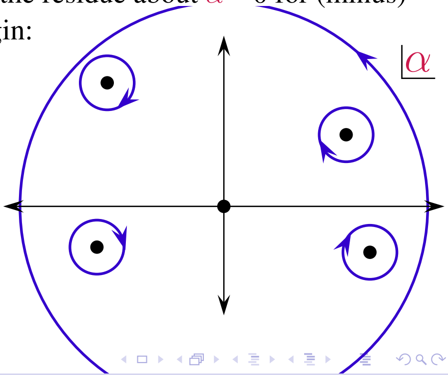

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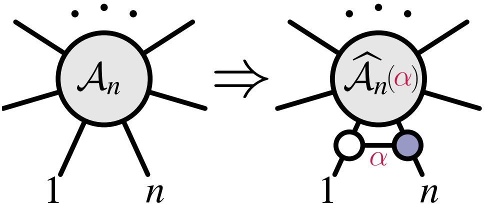

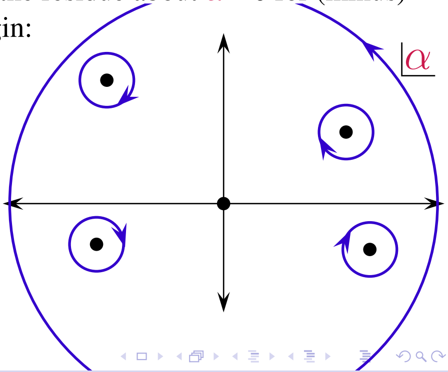

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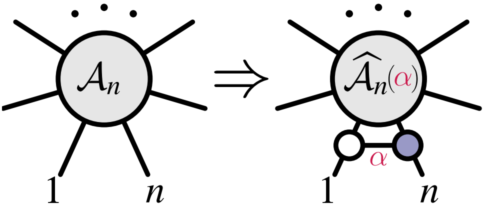

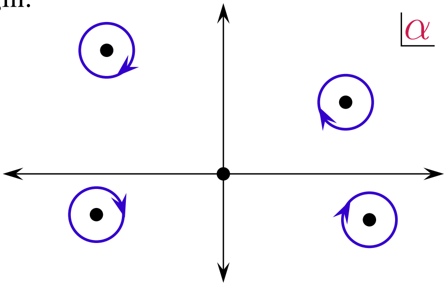

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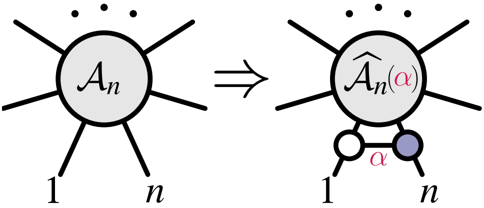

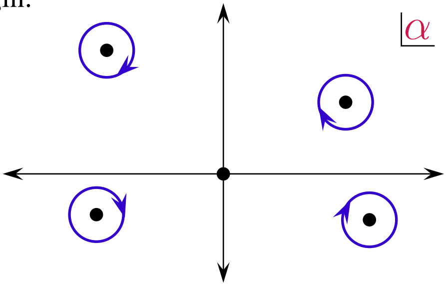

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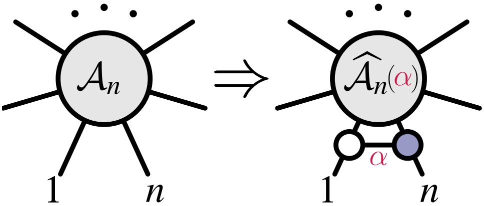

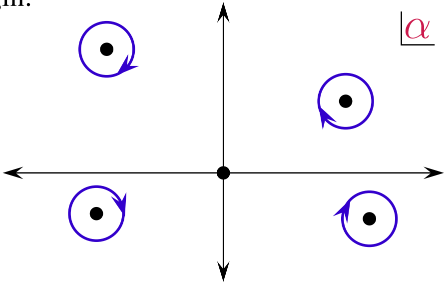

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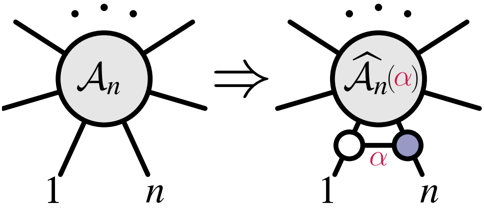

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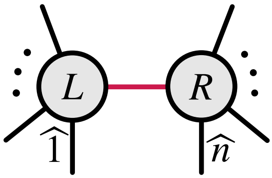

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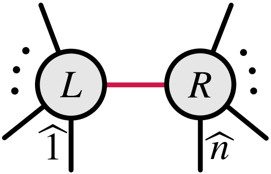

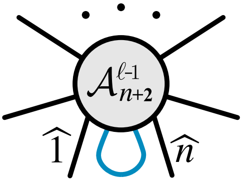

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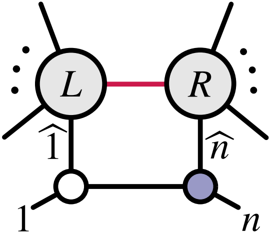

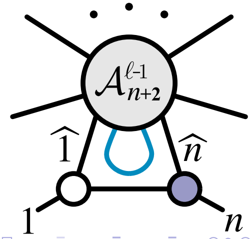

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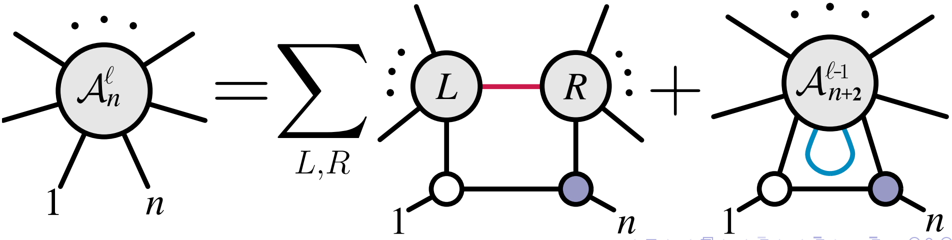

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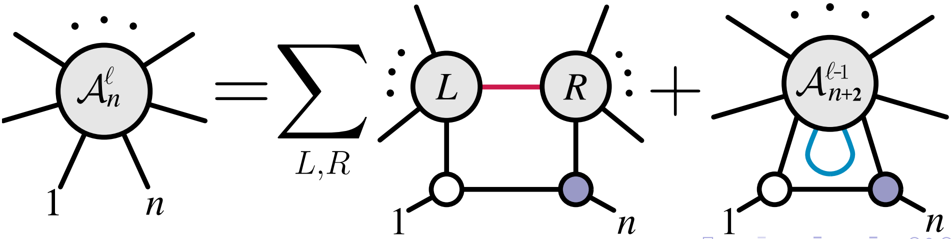

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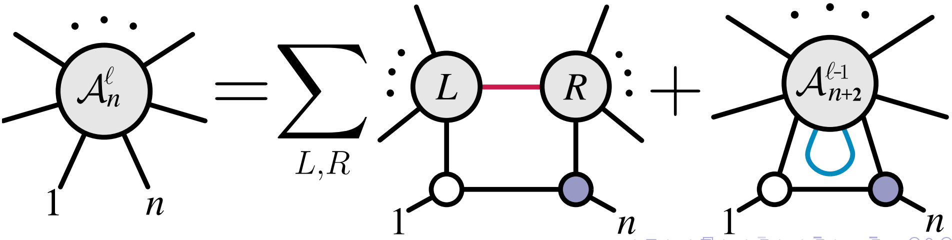

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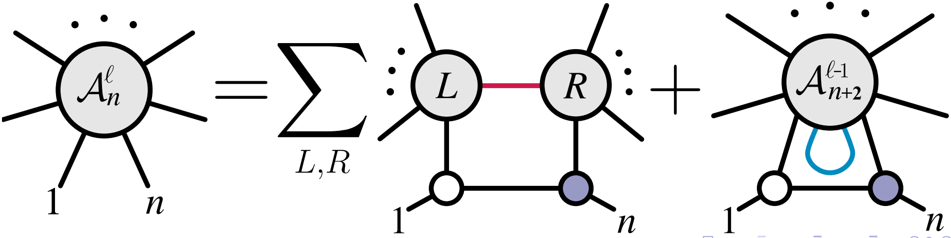

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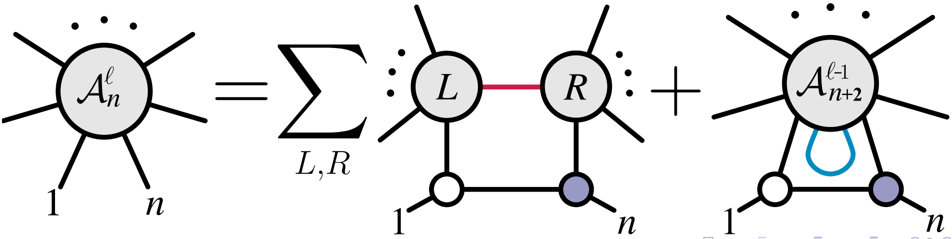

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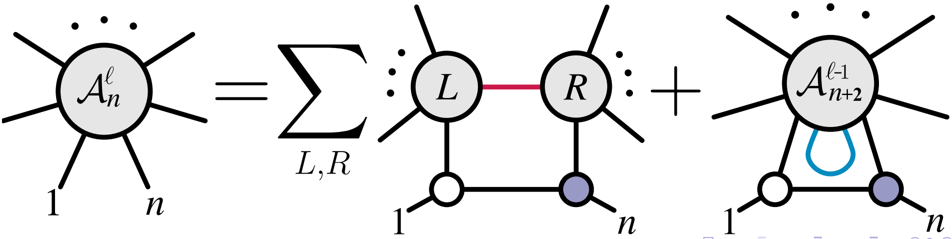

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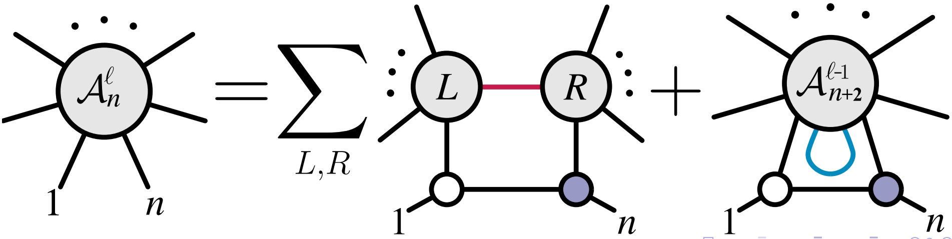

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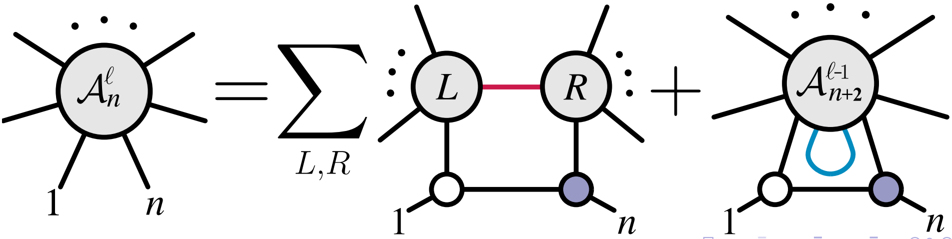

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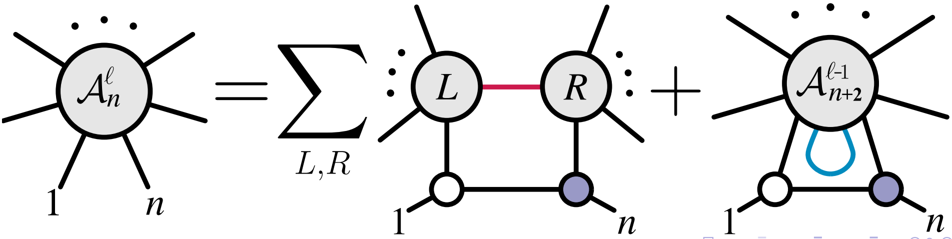

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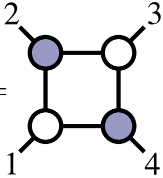
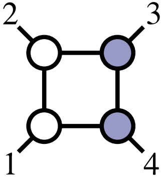
$\mathcal{A}_4^{(2)} =$  $+$ 

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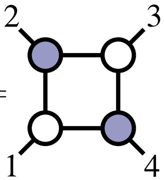
$\mathcal{A}_4^{(2)} =$ 

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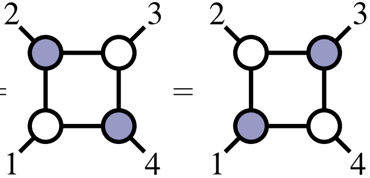

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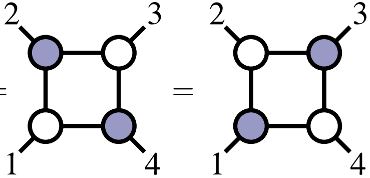
$\mathcal{A}_4^{(2)} =$ 

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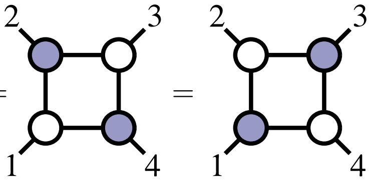

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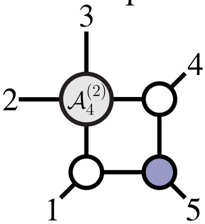

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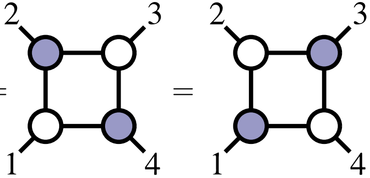
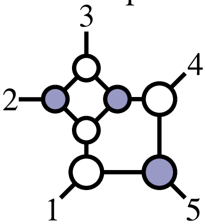
$\mathcal{A}_4^{(2)} =$ 

$\mathcal{A}_5^{(2)} =$ 

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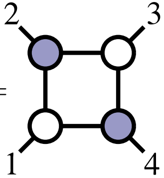
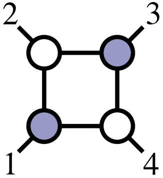
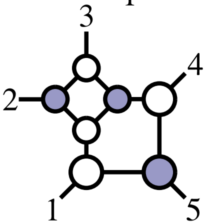
$\mathcal{A}_4^{(2)} =$ 

$=$ 

$\mathcal{A}_5^{(2)} =$ 

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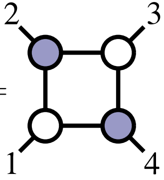

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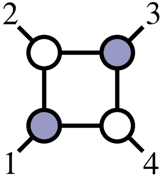

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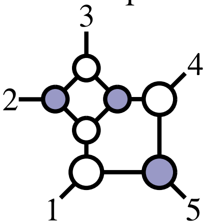

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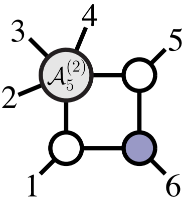

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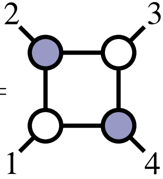

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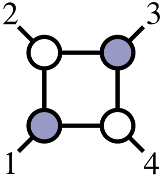

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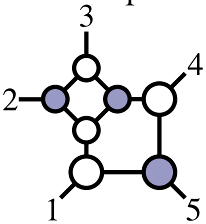

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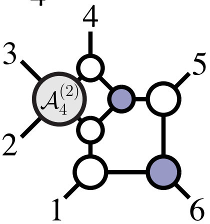

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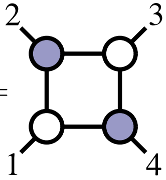

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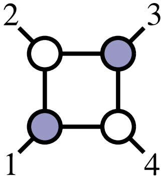

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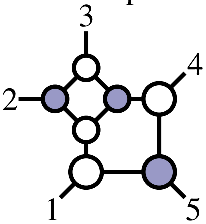

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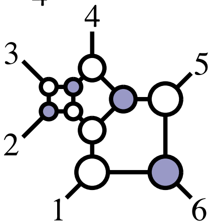

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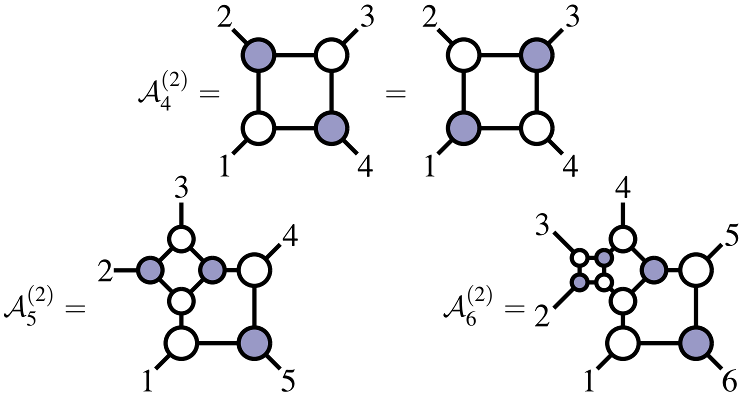

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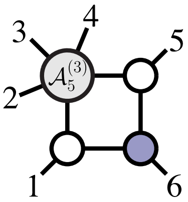
$\mathcal{A}_6^{(3)} =$ 

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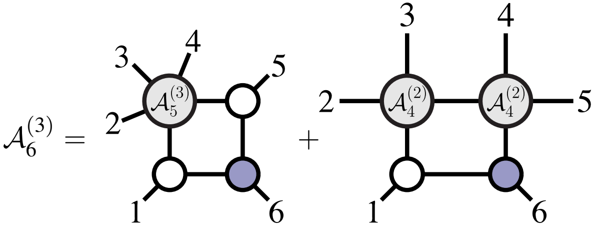

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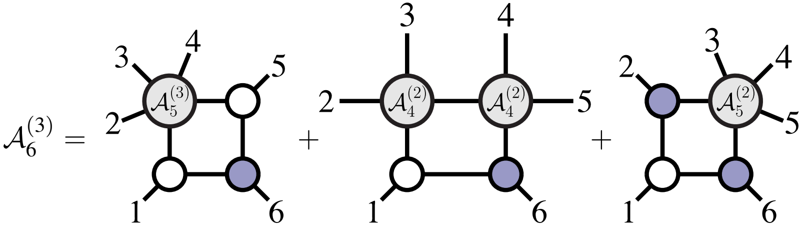

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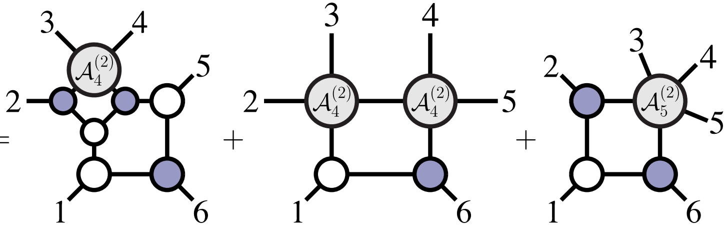
$\mathcal{A}_6^{(3)} =$ 

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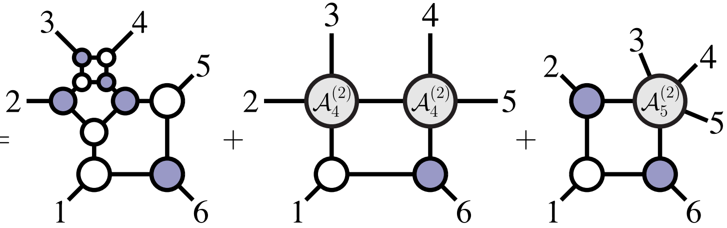

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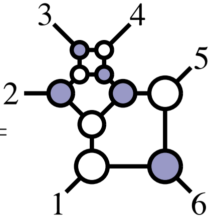
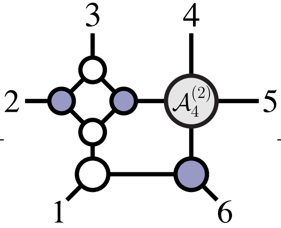
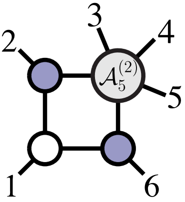
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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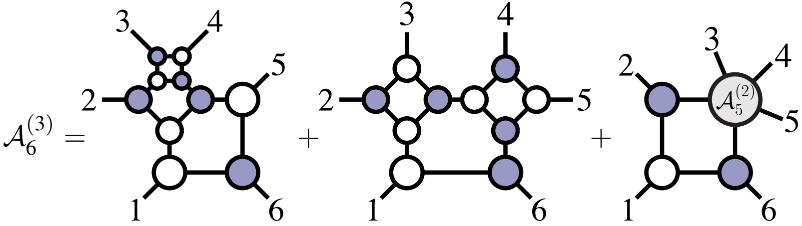

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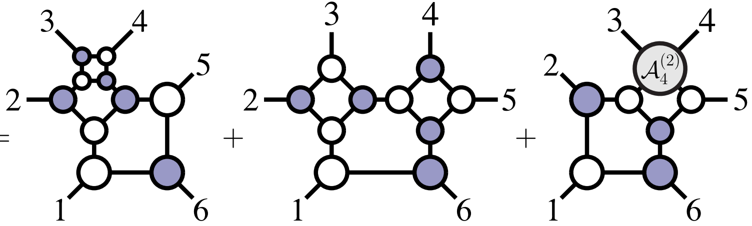

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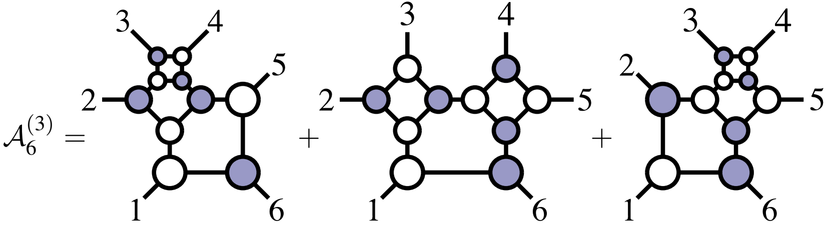

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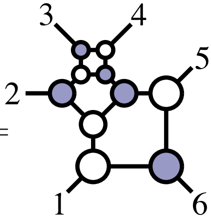
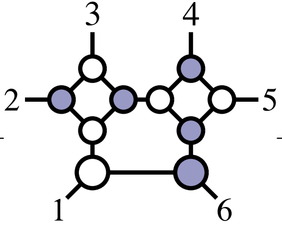
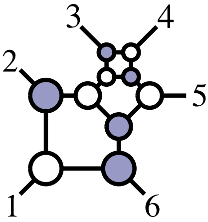
$\mathcal{A}_6^{(3)} =$  +  + 

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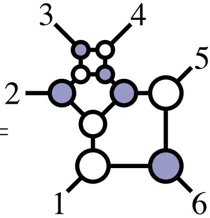
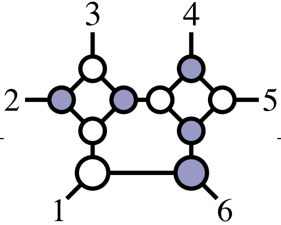
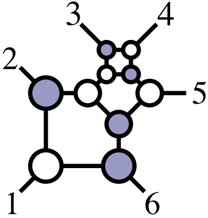
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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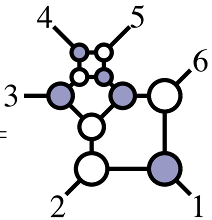
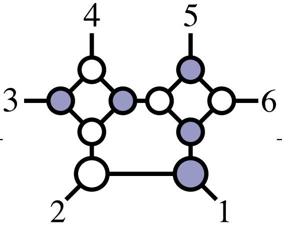
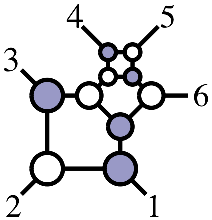
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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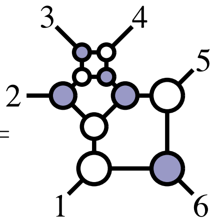
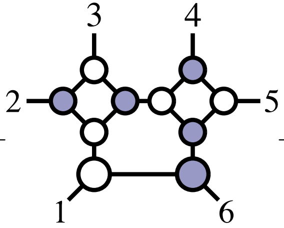
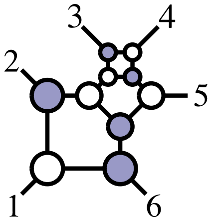
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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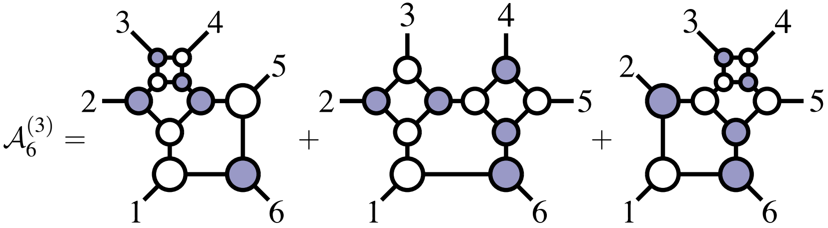

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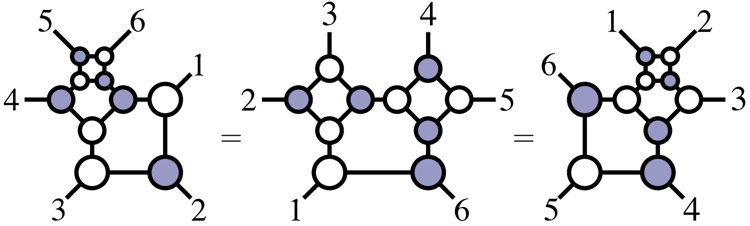

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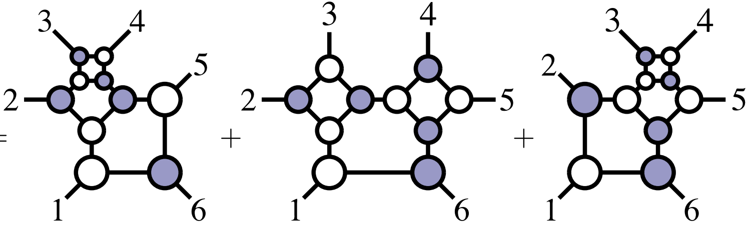

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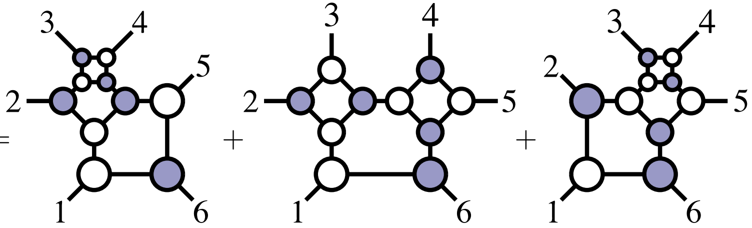

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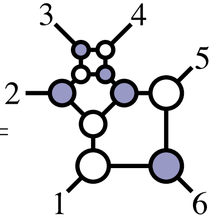
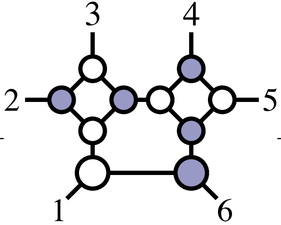
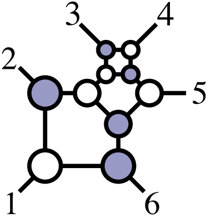
$\mathcal{A}_6^{(3)} =$  $+$  $+$ 

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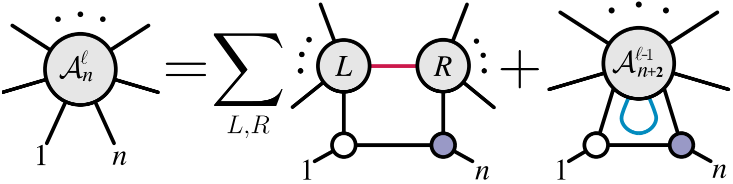

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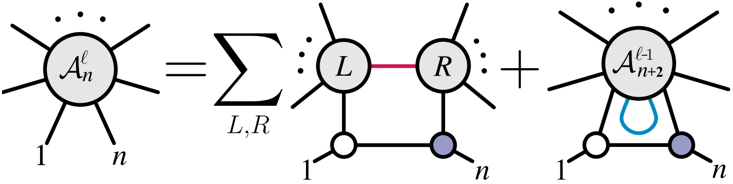

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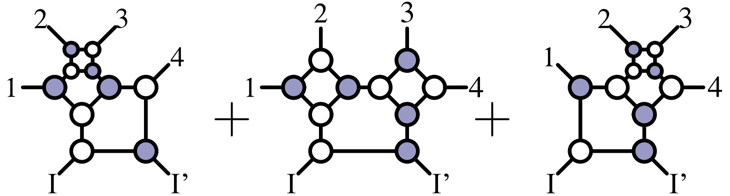

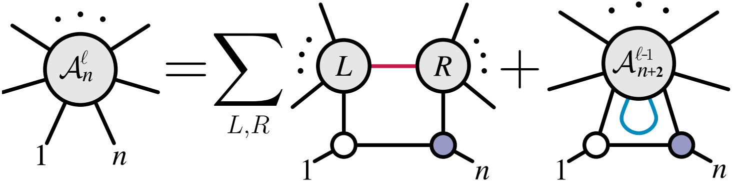

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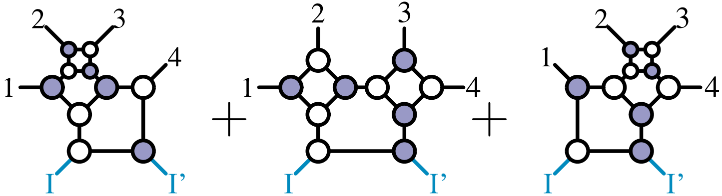

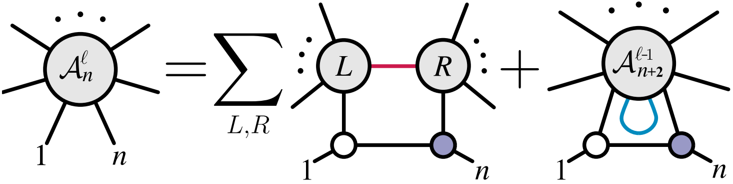

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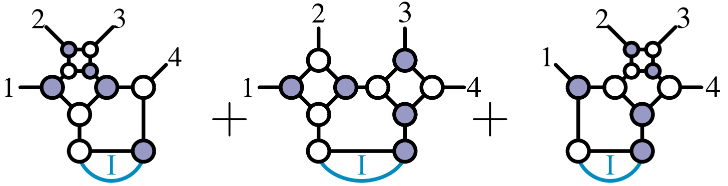

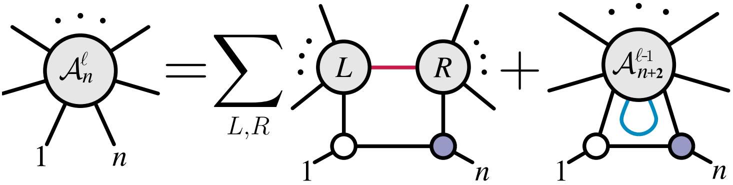

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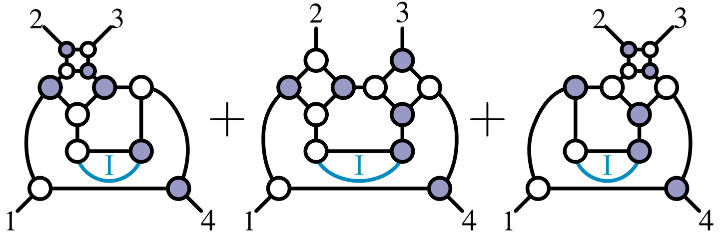

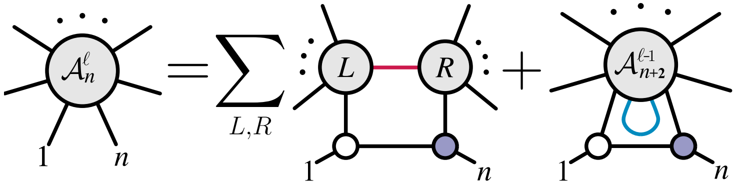

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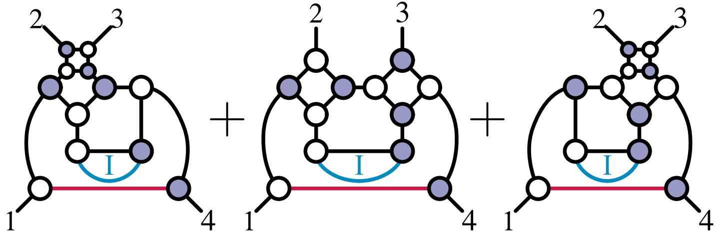

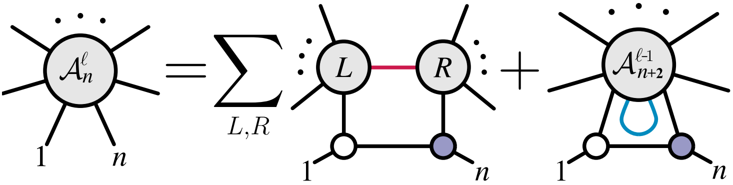

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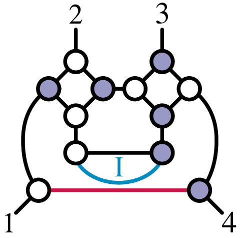

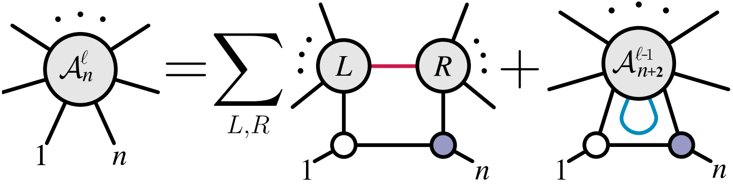

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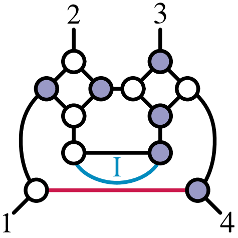

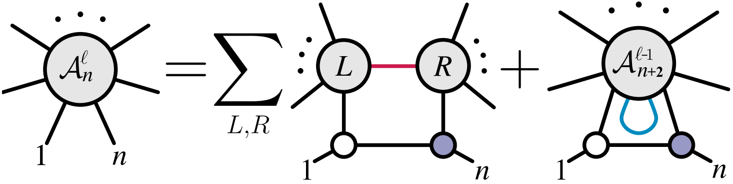

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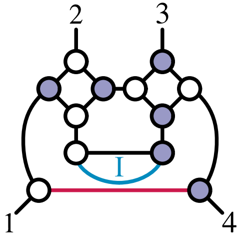

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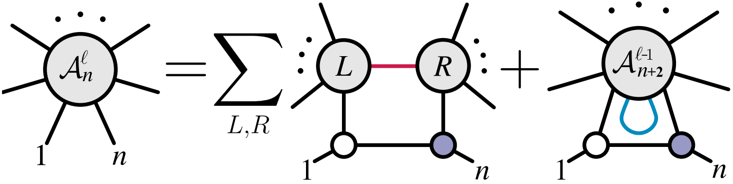

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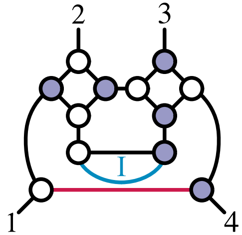

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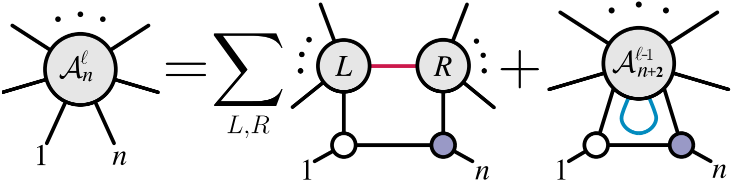

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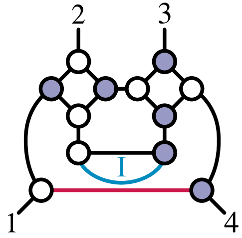

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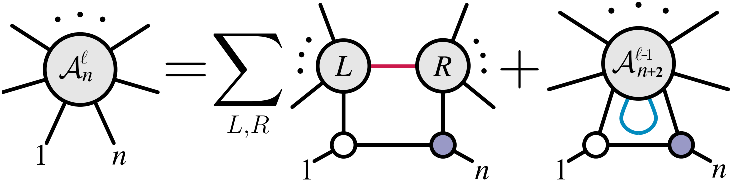

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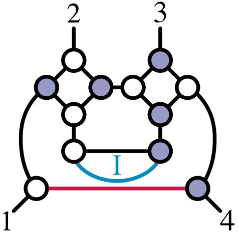

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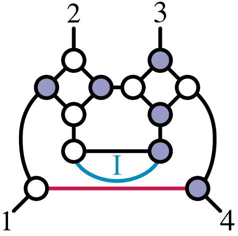

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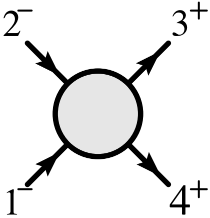

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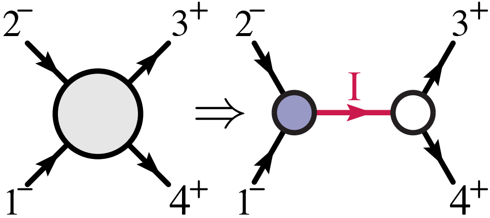

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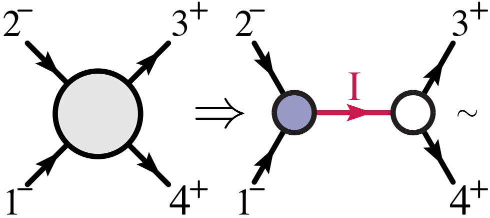

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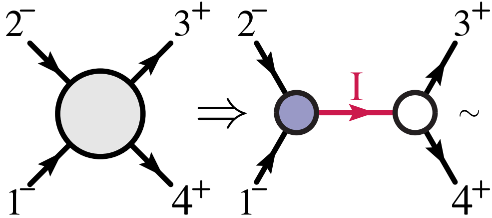

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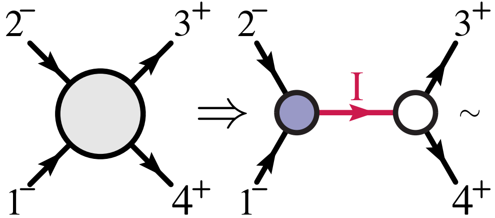

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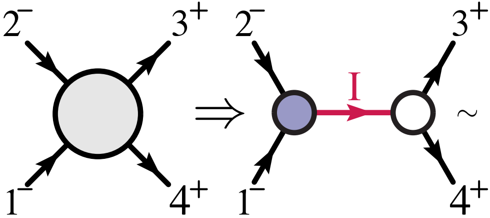

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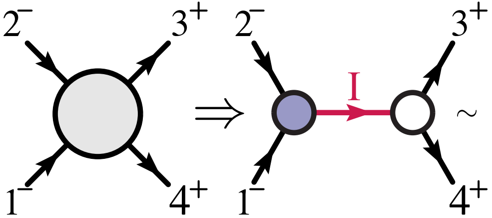

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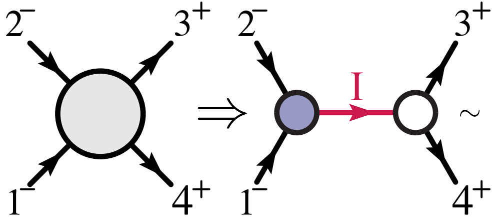

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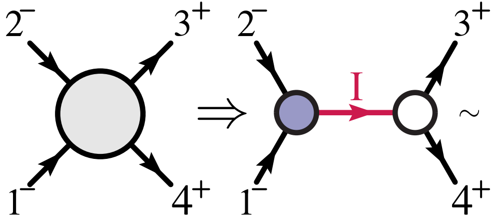

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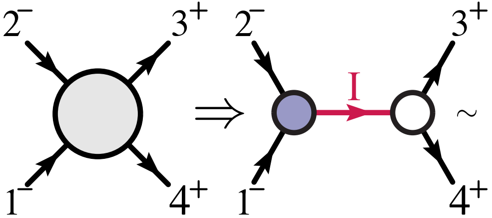

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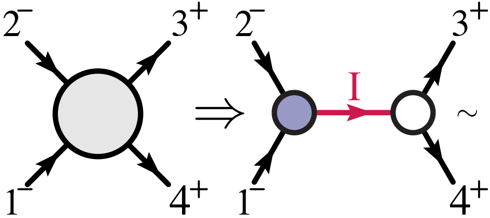

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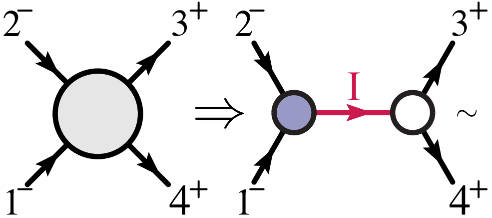

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