On-Shell Diagrams, Recursion Relations, & Combinatorics

Jacob L. Bourjaily

Amplitudes 2022 Summer School
Charles University, Prague, Czech Republic

The Niels Bohr International Academy

PennState Eberly College of Science

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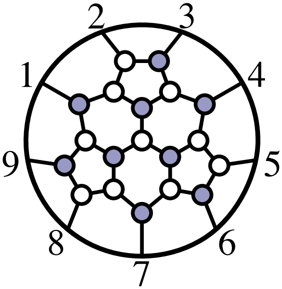
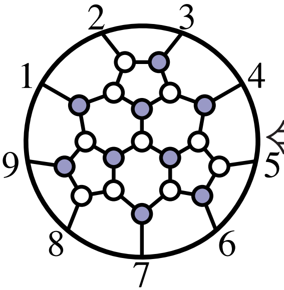
The Combinatorics and Geometry of On-Shell Physics

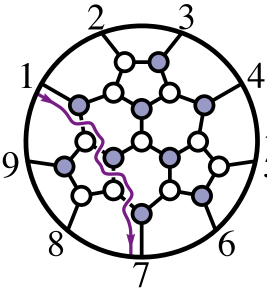


The Combinatorics and Geometry of On-Shell Physics



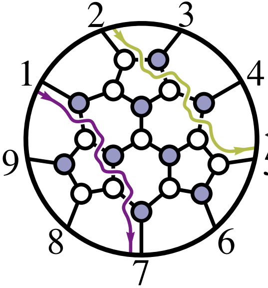
$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ & & & & & & & & \\ & & & & & & & & \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics



$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & & & & & & & & \\ 7 & & & & & & & & \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics



$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & & & & & & & \\ 7 & 5 & & & & & & & \end{pmatrix}$$

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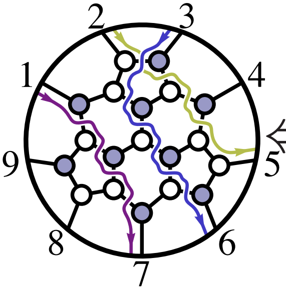

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & & & & & & \\ 7 & 5 & 6 & & & & & & \end{pmatrix}$$

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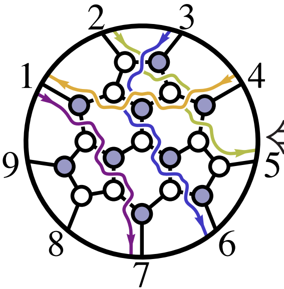
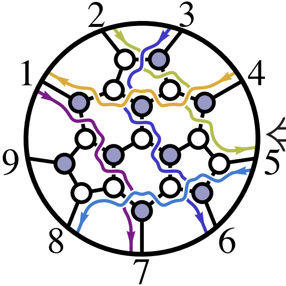


$$\Leftrightarrow\left(\begin{array}{ccccccccc}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&&&&&\\7&5&6&10&&&&&\end{array}\right)$$

The Combinatorics and Geometry of On-Shell Physics



$$\Longleftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & & \\ 7 & 5 & 6 & 10 & 8 & & & & \end{pmatrix}$$

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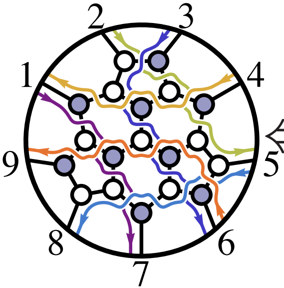

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & \\ 7 & 5 & 6 & 10 & 8 & 9 & & & \end{pmatrix}$$

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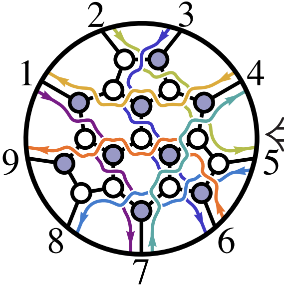
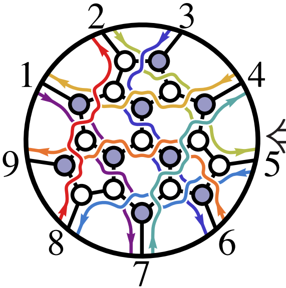


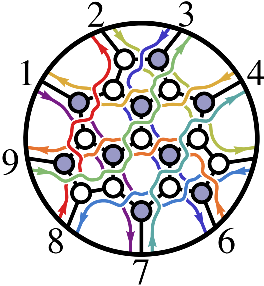
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & & \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics



$$\Leftrightarrow\left(\begin{array}{ccccccccc}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & \end{array}\right)$$

The Combinatorics and Geometry of On-Shell Physics

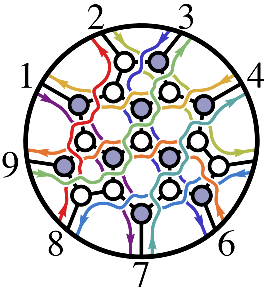


$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&1\end{pmatrix}\in G_{+}(4,9)$$

⤢ $\Updownarrow$

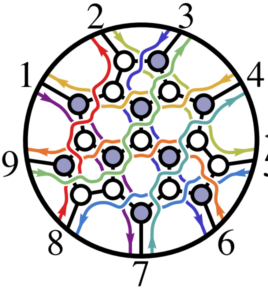


$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\left(\begin{array}{ccccccccc}1&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&\alpha_1&0&0&0&0&0&1\end{array}\right)\in G_+(4,9)$$

$\Updownarrow$

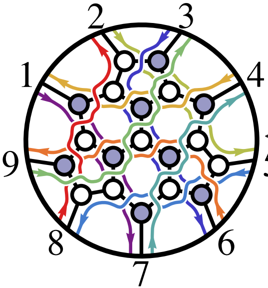


$$\Longleftrightarrow\left(\begin{array}{ccccccccc}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{array}\right)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&0&0&1\end{pmatrix}\in G_+(4,9)$$

$$\swarrow\!\!\!\nearrow \qquad \Updownarrow$$

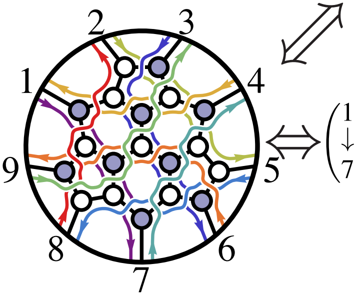


$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & 0 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$



$$\Longleftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

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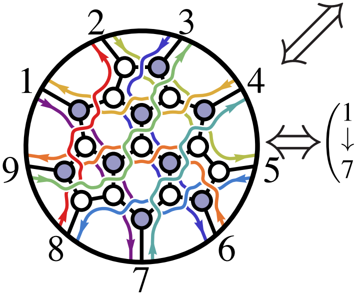
$$C(\alpha)\equiv\begin{pmatrix}1&0&0&0&0&0&0&0&0\\0&0&0&1&0&\alpha_4&0&0&0\\0&0&0&0&0&0&1&0&\alpha_3\\0&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&0&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

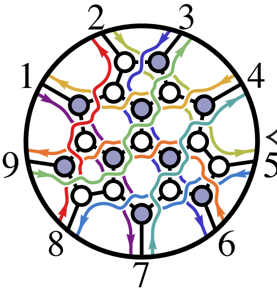


$$\Longleftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & 0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & 0 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\swarrow\!\!\!\!\nearrow \qquad \Updownarrow$$



$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

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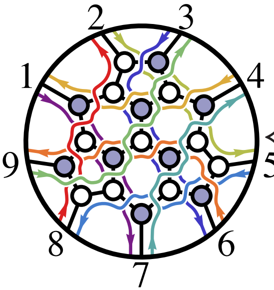
$$C(\alpha)\equiv\begin{pmatrix}1&0&\alpha_5&0&0&0&0&0&0\\0&0&0&1&0&\alpha_4&0&0&0\\0&0&0&0&0&0&1&\alpha_6&\alpha_3\\0&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&0&0&1\end{pmatrix}\in G_+(4,9)$$

⇗ ⇕



$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

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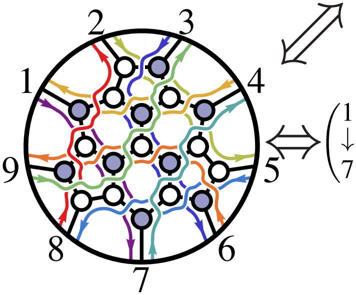
$$C(\alpha)\equiv\begin{pmatrix} 1 & 0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$



$$\Longleftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

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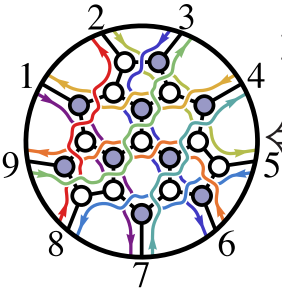
$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$



$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

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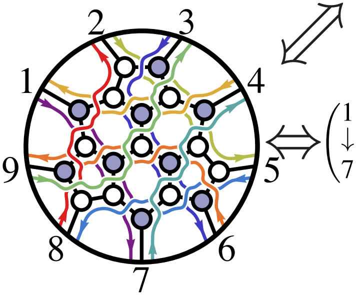
$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&\alpha_5&0&0&0&0&0&0\\0&0&0&1&0&\alpha_4&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6&\alpha_3\\-\alpha_9&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$



$$\Longleftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

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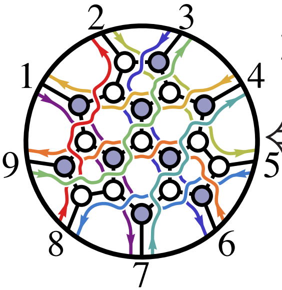
$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ -\alpha_9 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

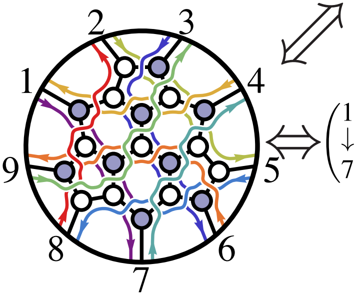
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$



The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & \alpha_5 & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3 \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$\Updownarrow$



$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

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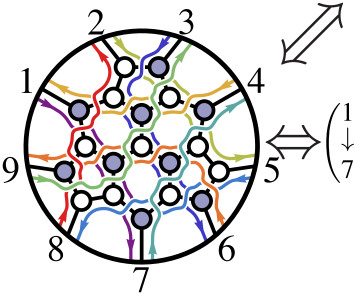
$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&\alpha_5&\alpha_{11}\,\alpha_5&0&0&0&0&0\\0&0&0&1&\alpha_{10}&\alpha_4&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6\,(\alpha_3+\alpha_{12}\,\alpha_6)\\-\alpha_9&0&\alpha_1&\alpha_{11}\,\alpha_1&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

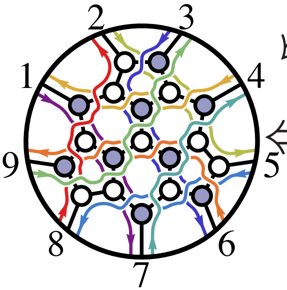


$$\Longleftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & \alpha_5 & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6\,(\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\swarrow\!\!\nearrow \qquad \Updownarrow$$



$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

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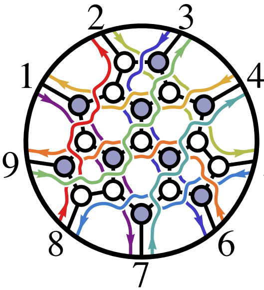
$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&(\alpha_5+\alpha_{14}\,\alpha_8)&\alpha_{11}\,\alpha_5&0&0&0&0&0\\0&0&0&1&\alpha_{10}&(\alpha_4+\alpha_{13}\,\alpha_{10})&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6&(\alpha_3+\alpha_{12}\,\alpha_6)\\-\alpha_9&0&\alpha_1&\alpha_{11}\,\alpha_1&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$

⤢ $\qquad\qquad \Updownarrow$

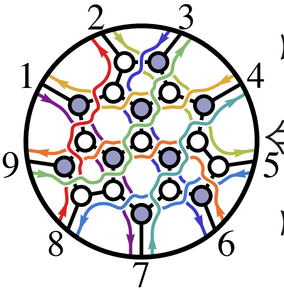


$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\left(\begin{array}{ccccccccc} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{array}\right)\in G_+(4,9)$$

$\Updownarrow$



$$\Leftrightarrow\left(\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{array}\right)$$

$\Updownarrow$

$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\;\delta^{k\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{k\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

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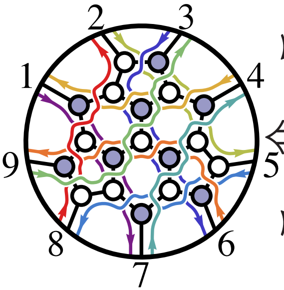
$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&(\alpha_5+\alpha_{14}\,\alpha_8)&\alpha_{11}\,\alpha_5&0&0&0&0&0\\0&0&0&1&\alpha_{10}&(\alpha_4+\alpha_{13}\,\alpha_{10})&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6&(\alpha_3+\alpha_{12}\,\alpha_6)\\-\alpha_9&0&\alpha_1&\alpha_{11}\,\alpha_1&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$



$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

$$\Updownarrow$$

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The Combinatorics and Geometry of On-Shell Physics

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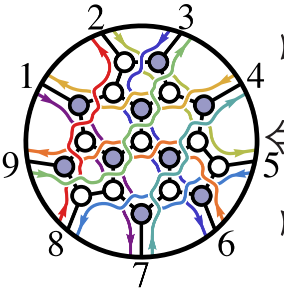

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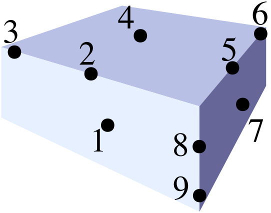

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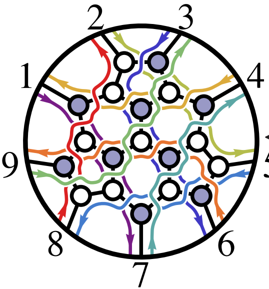

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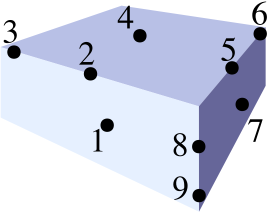

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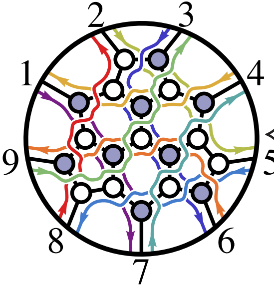

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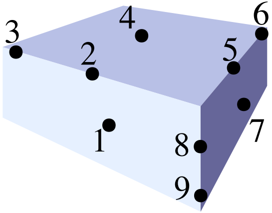

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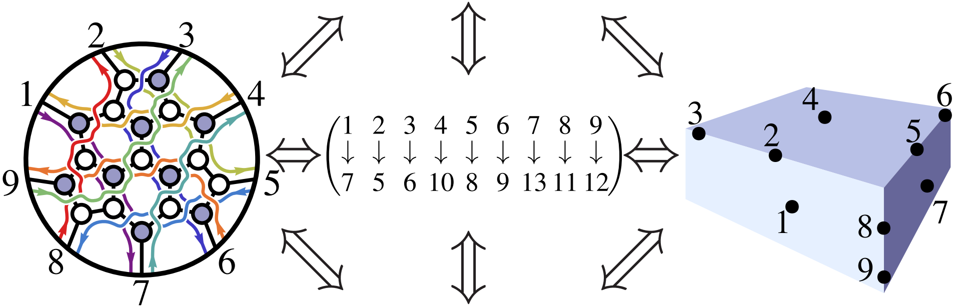

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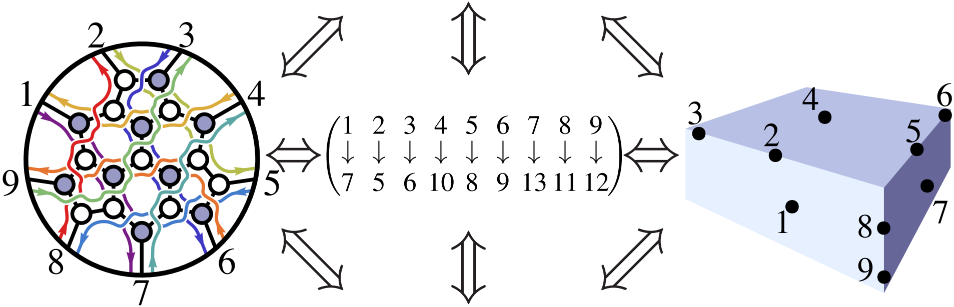

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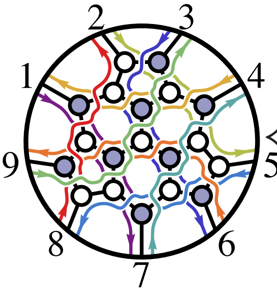

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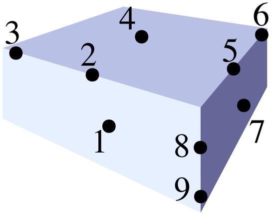

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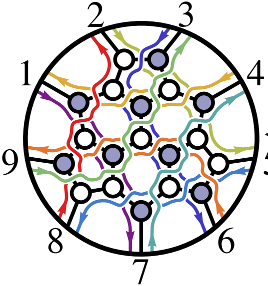

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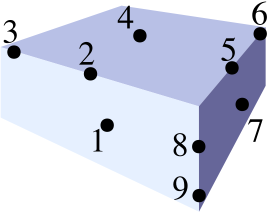

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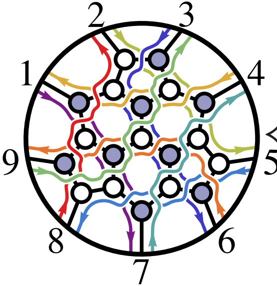

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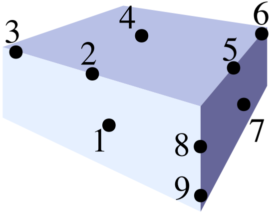

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Organization and Outline

1. On-Shell, Tree-Level Recursion Relations for Scattering Amplitudes

2. From On-Shell Physics to the (Positive) Grassmannian

   From the Bottom-Up:

   - (Combinatorially) Constructing and Computing On-Shell Functions
   - Asymptotic Symmetries of the S-Matrix: the *Yangian*

   From the Top-Down:

   - *Grassmannian* Geometry of (Generalized) Parke-Taylor ‘Amplitudes’

3. The *Positroid* Stratification of the Grassmannian
   - A Combinatorial Stratification of the Grassmannian

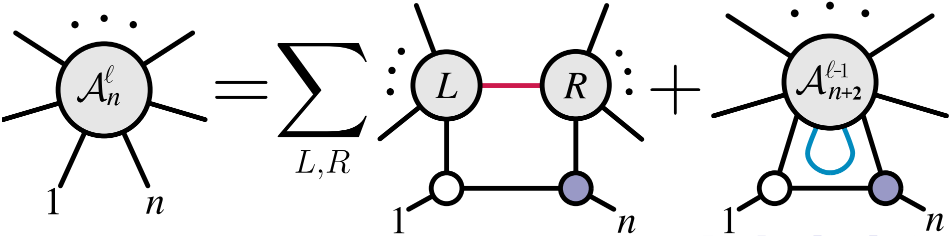
Recall from Last Lecture: the BCFW Recursion Relations

Diagrams are characterized by 'm'—the number of "minus-helicity" gluons:

$$m \equiv 2n_B + n_W - n_I.$$

For the bridge terms, we have:

$$m_L + m_R = m + 1.$$



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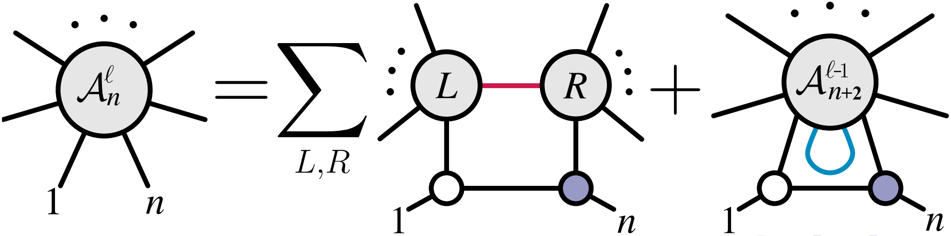

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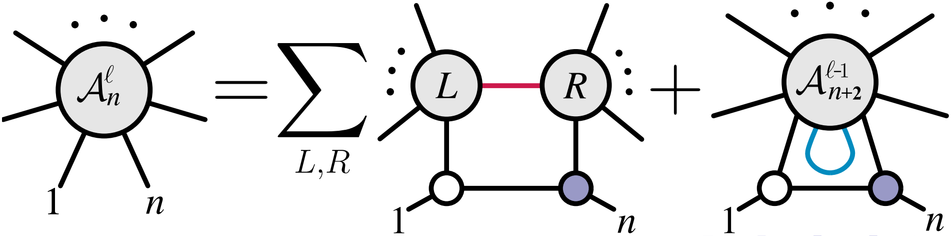

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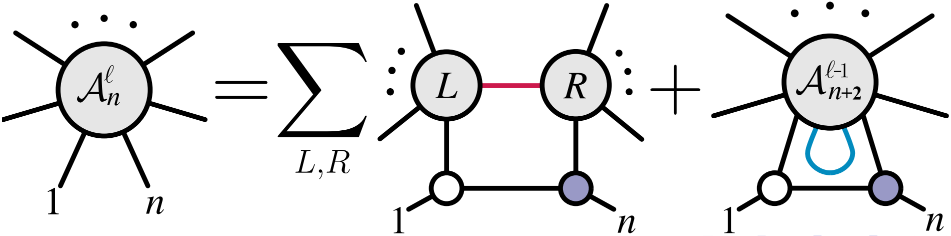

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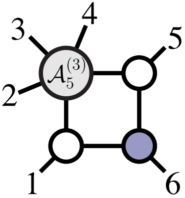

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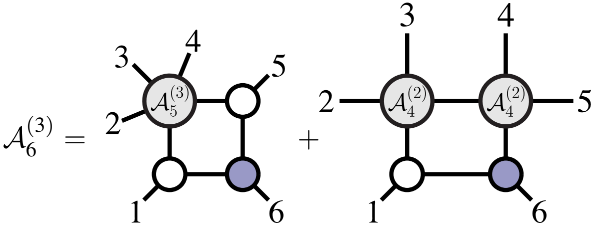

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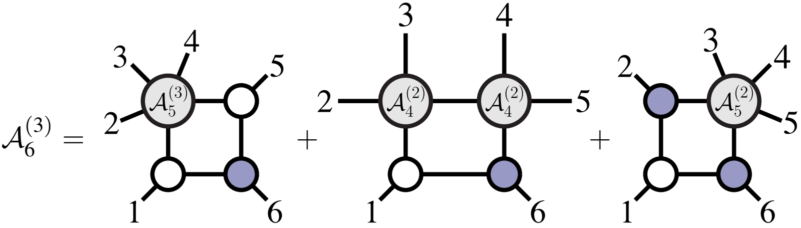

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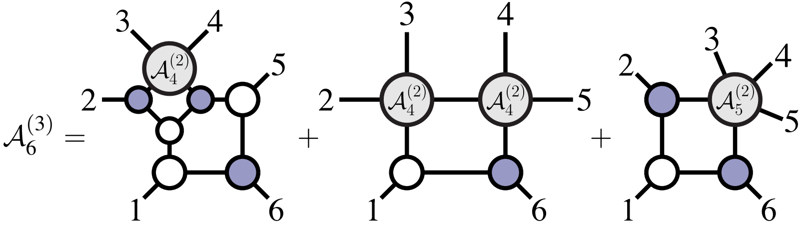

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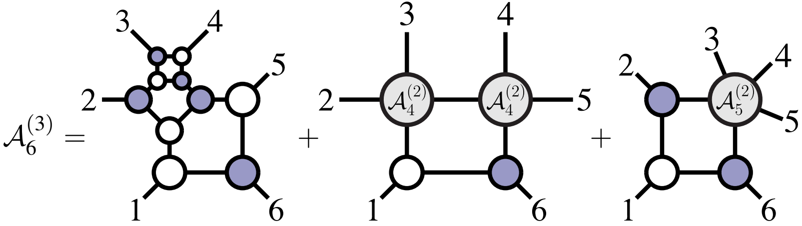

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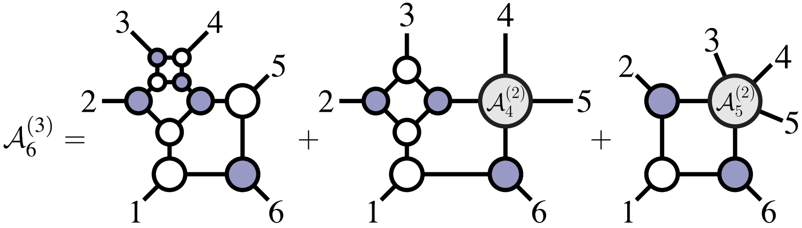

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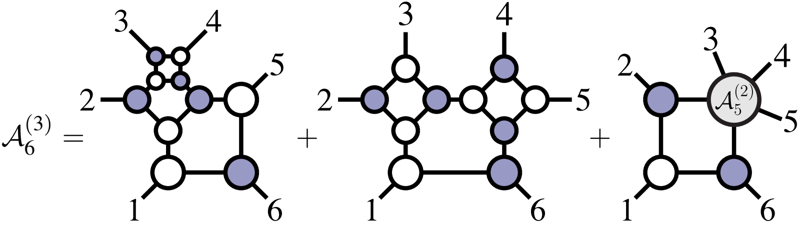

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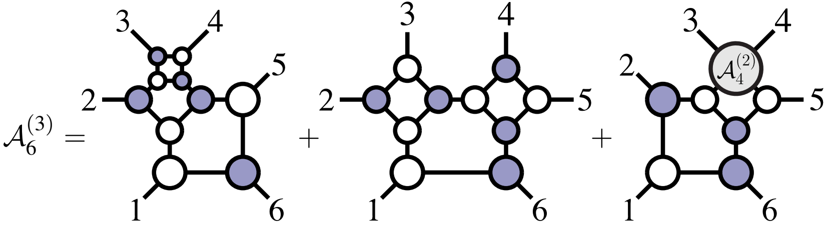

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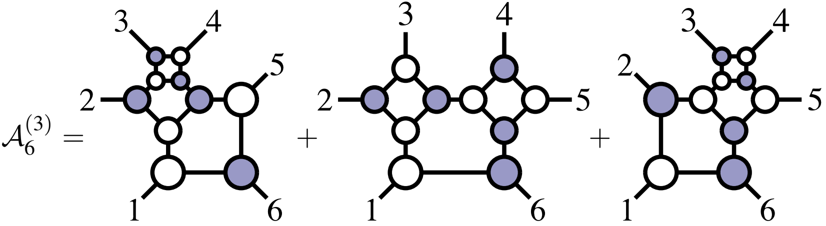

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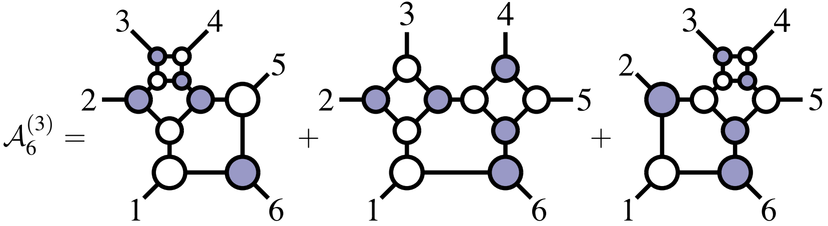

Observations regarding recursed representations of scattering amplitudes:

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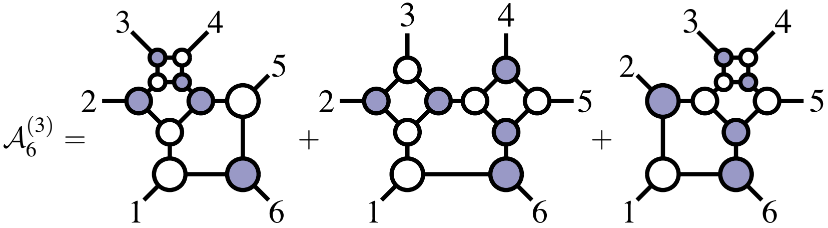

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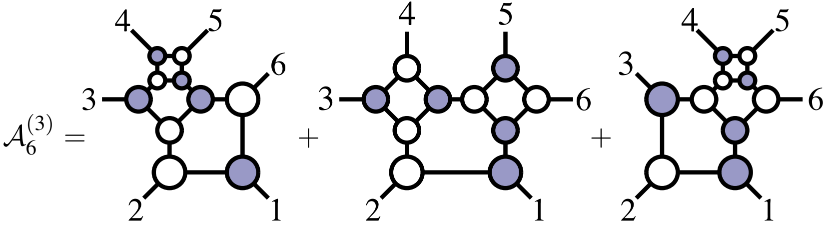

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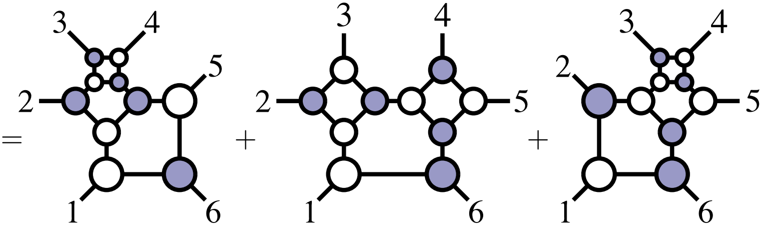

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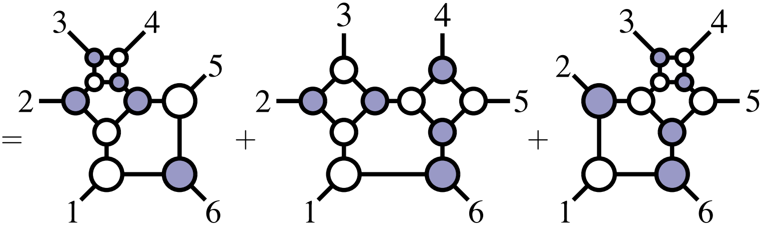

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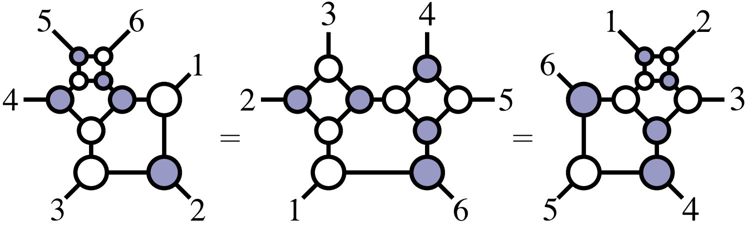

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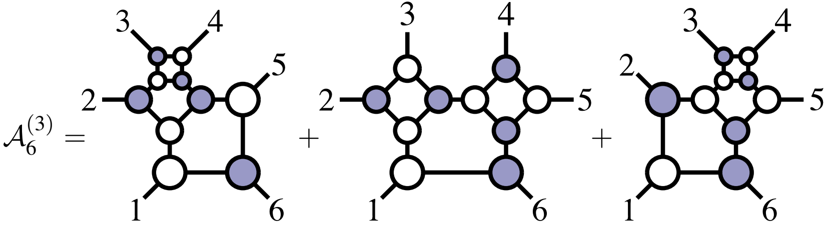

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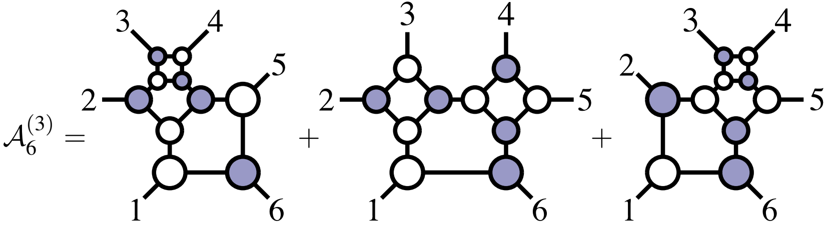

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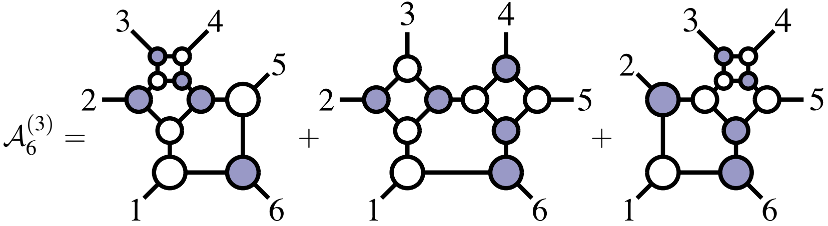

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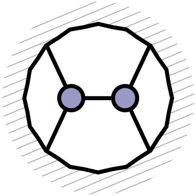

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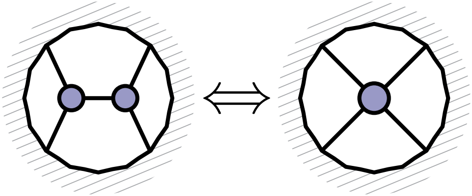

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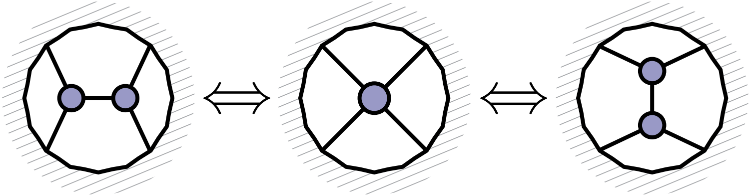

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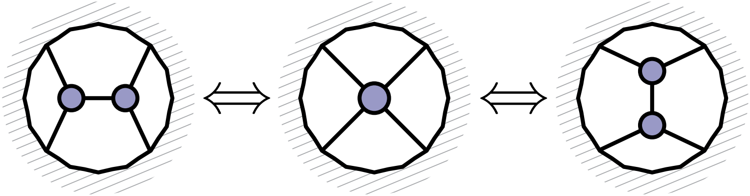

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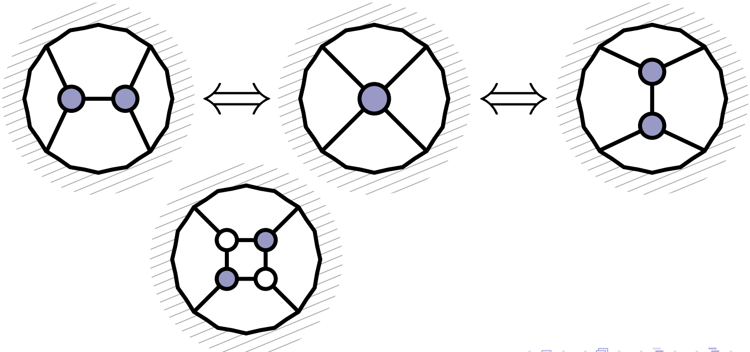

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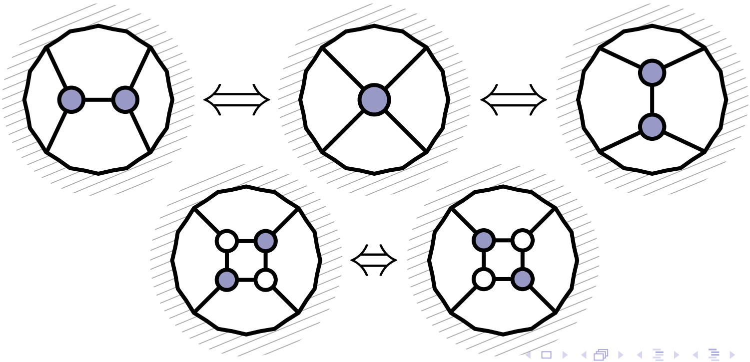

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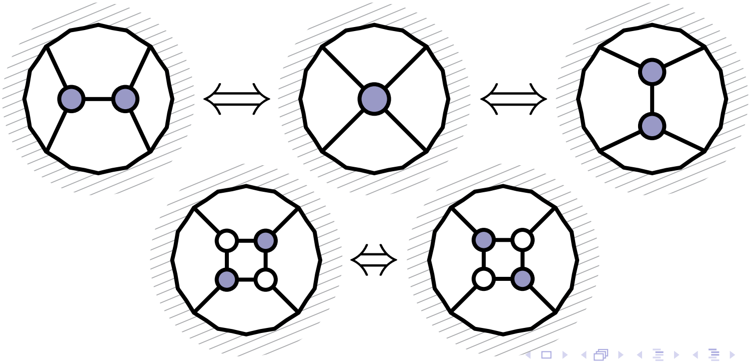

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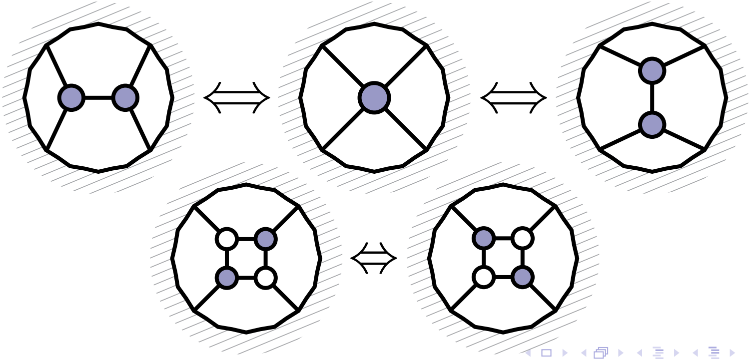

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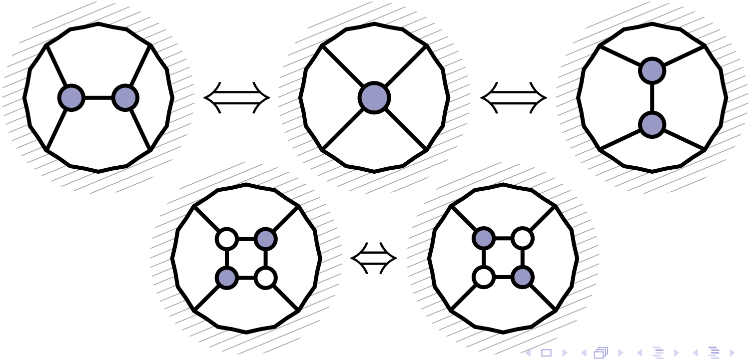

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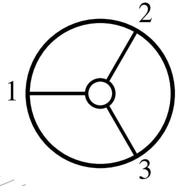

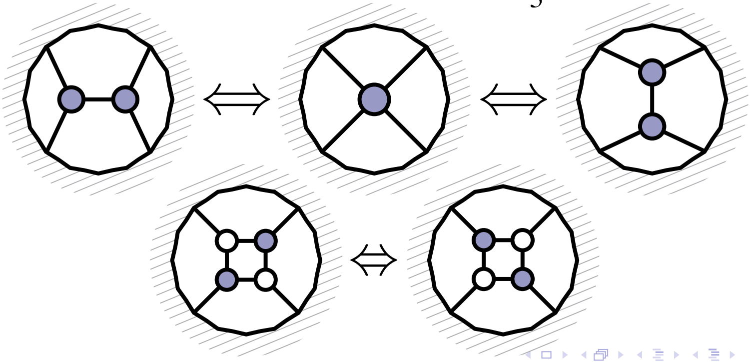

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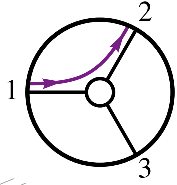

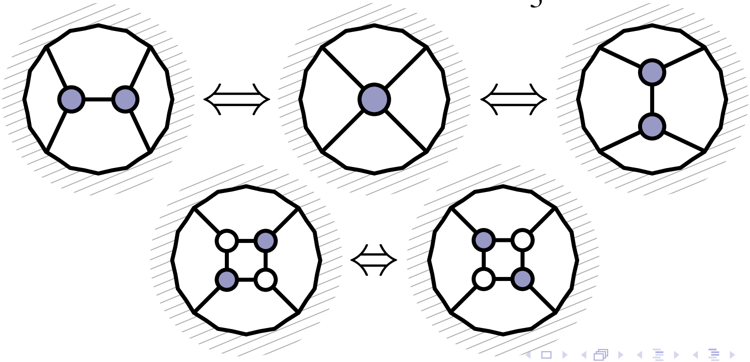

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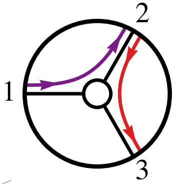

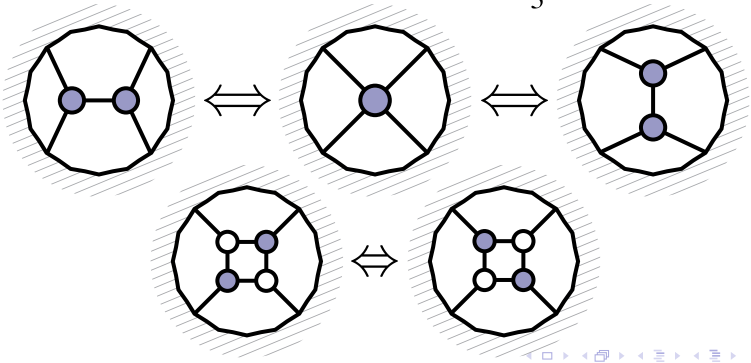

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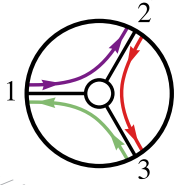

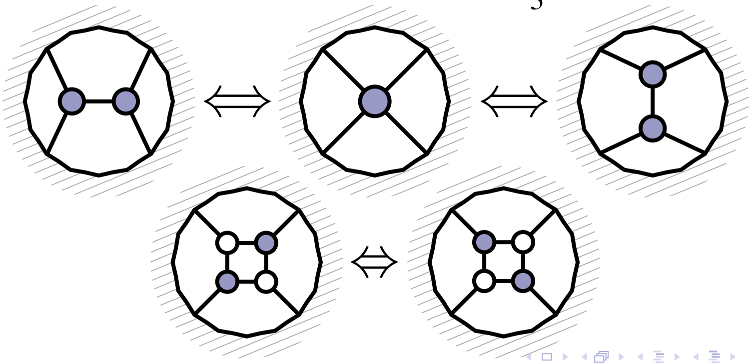

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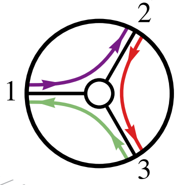

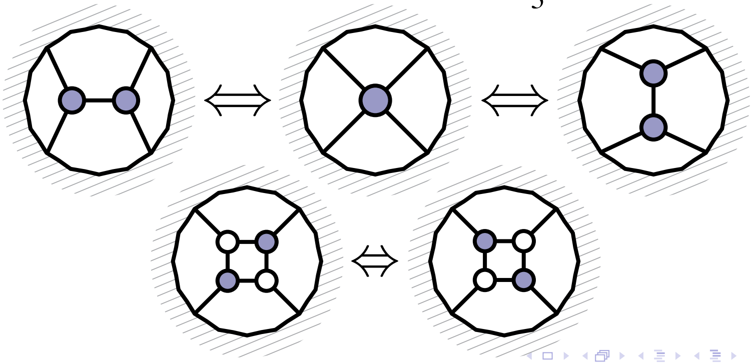

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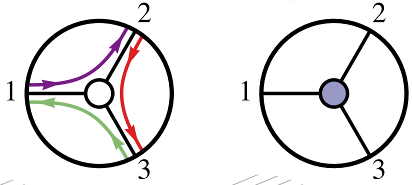

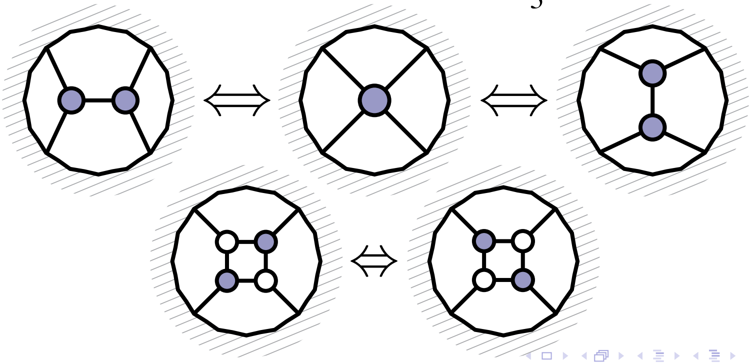

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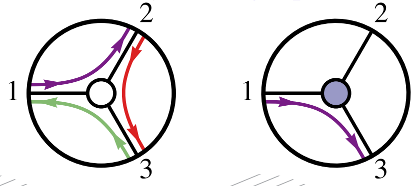

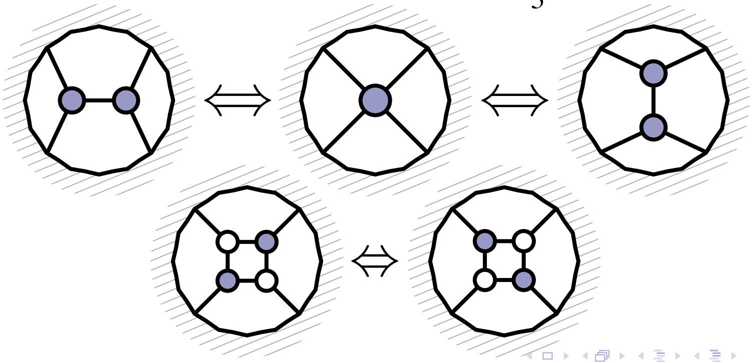

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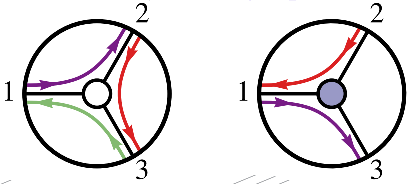

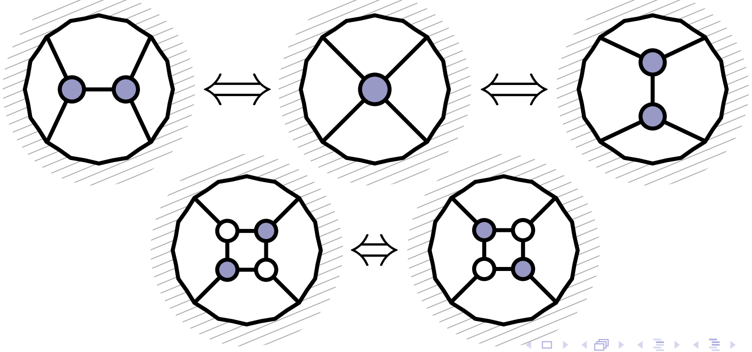

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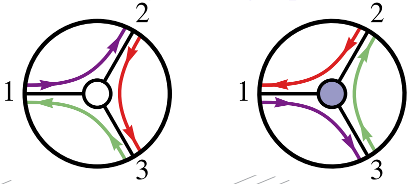

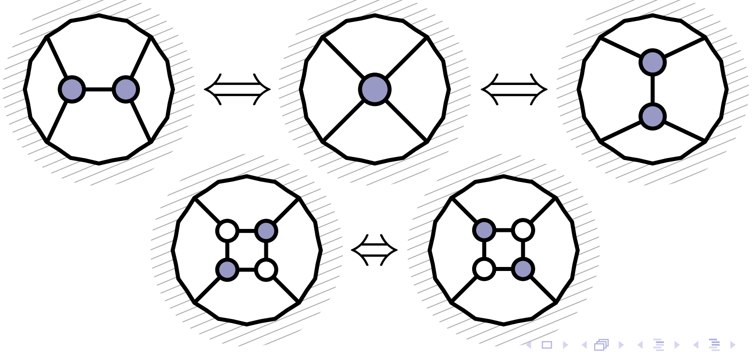

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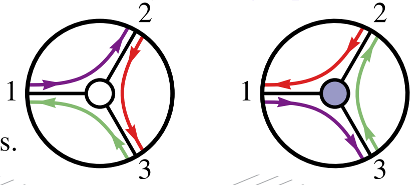

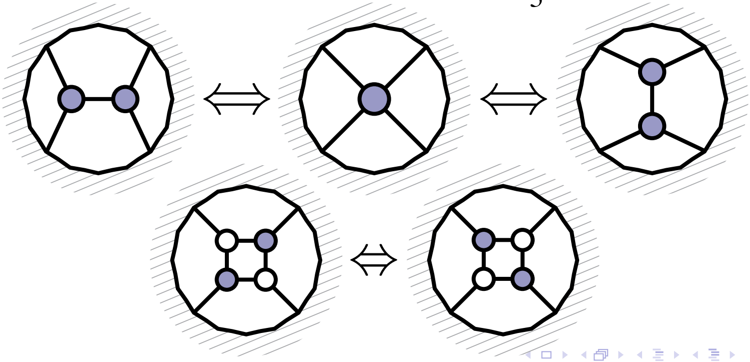

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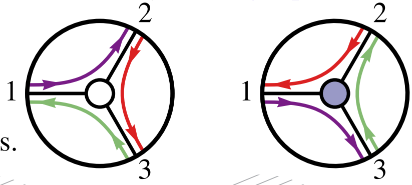

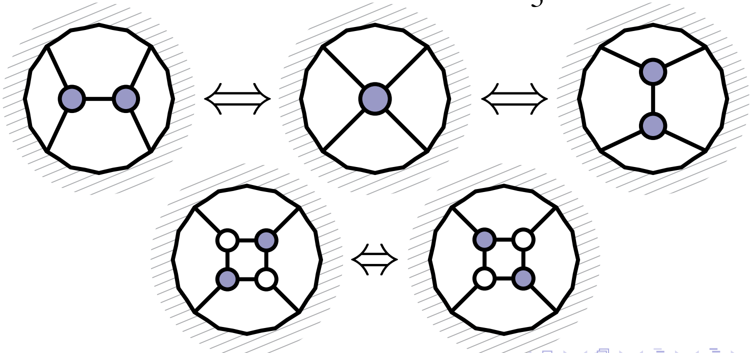

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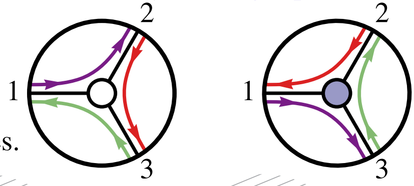

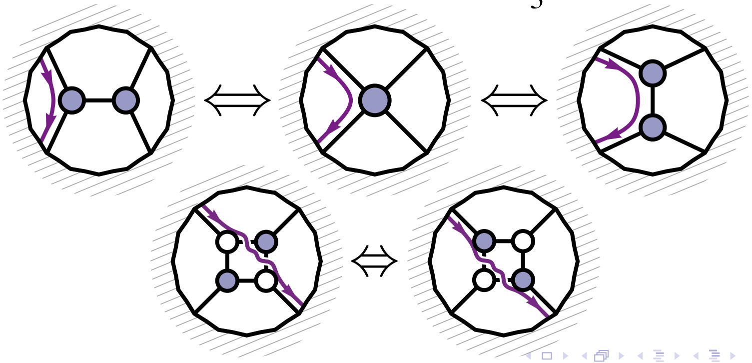

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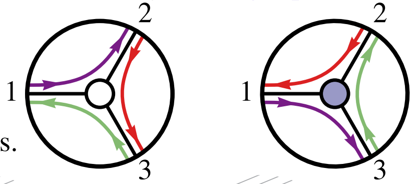

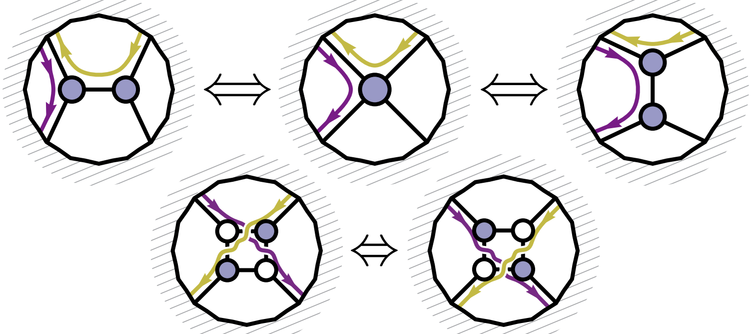

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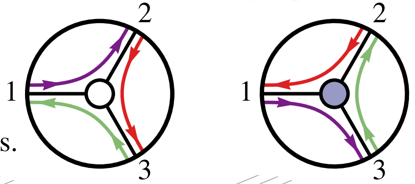

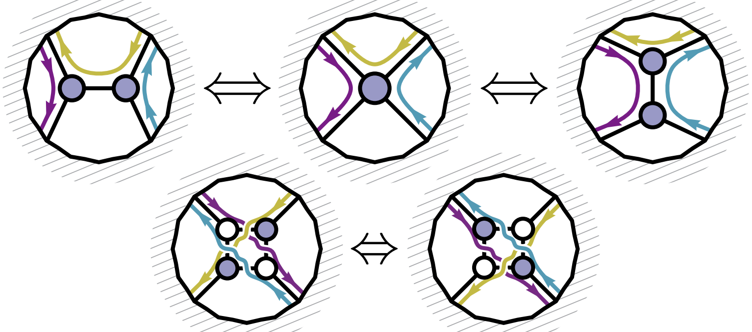

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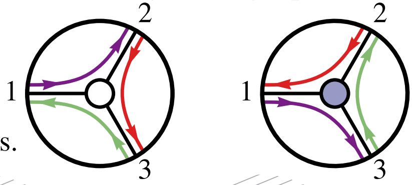

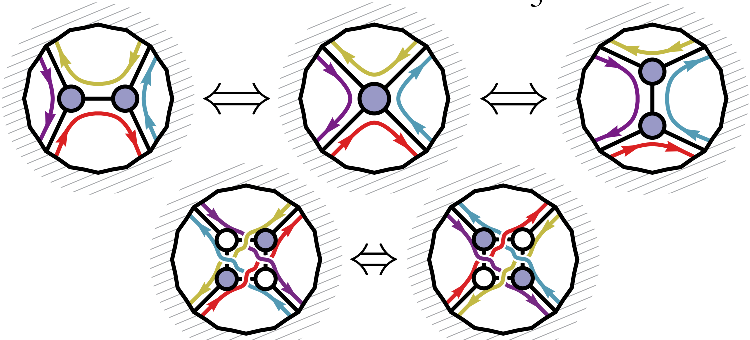

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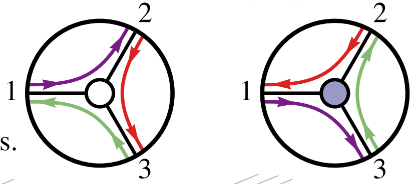

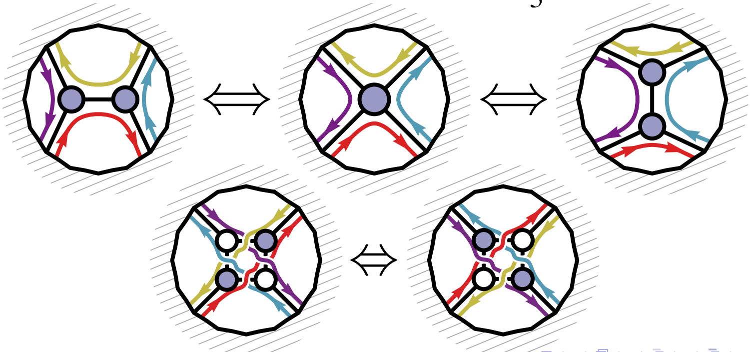

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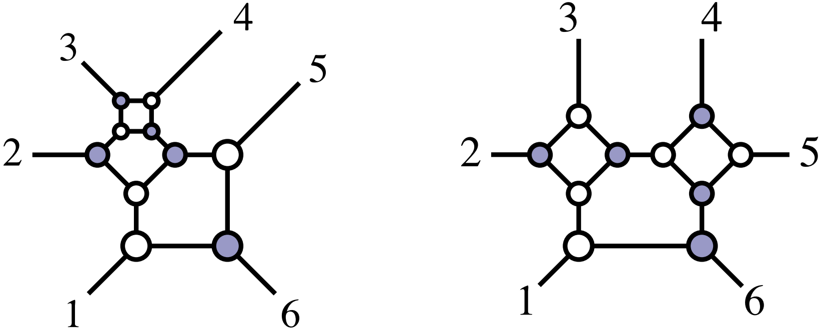

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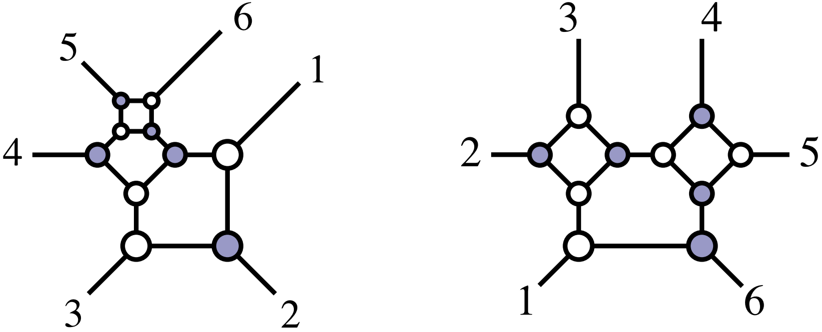

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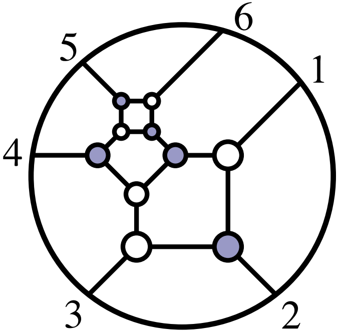

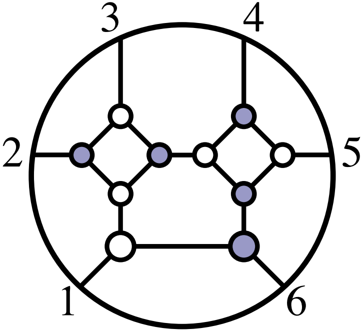

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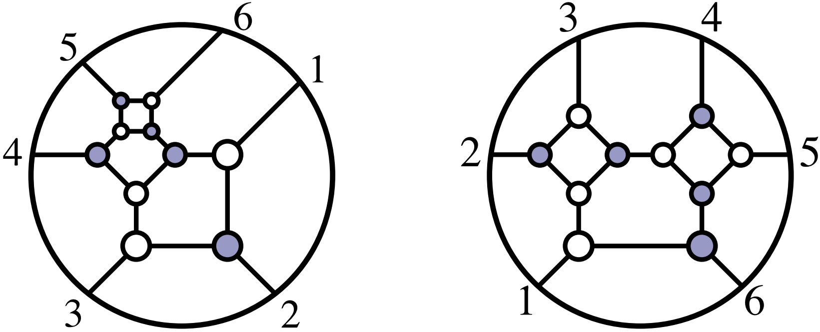

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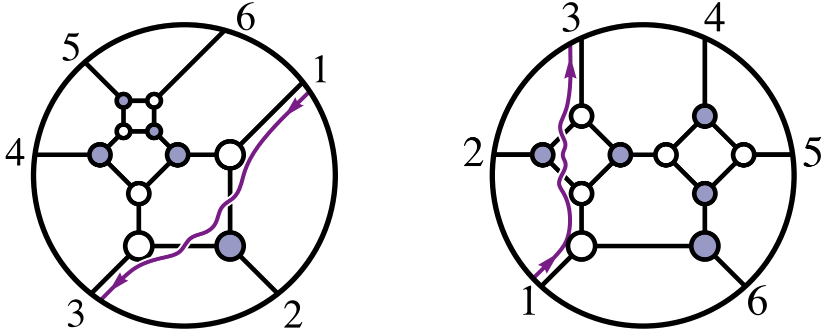

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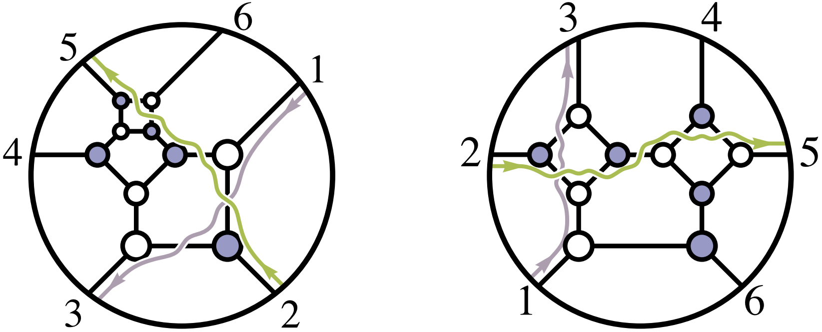

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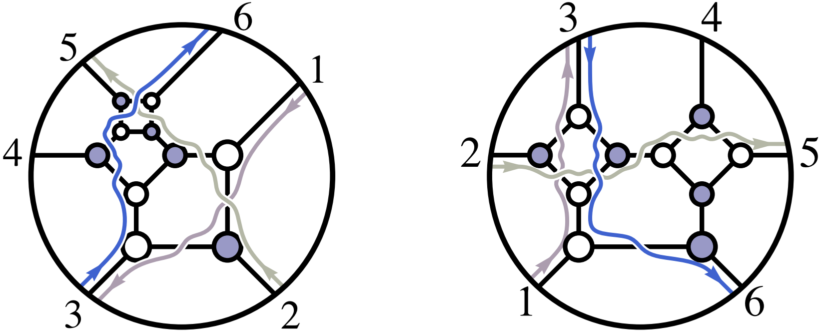

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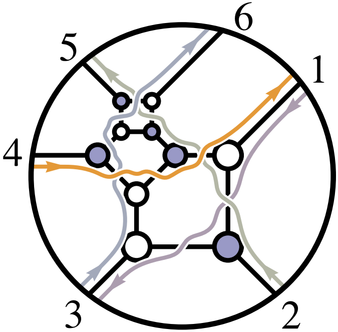

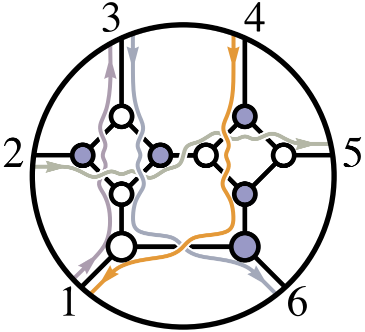

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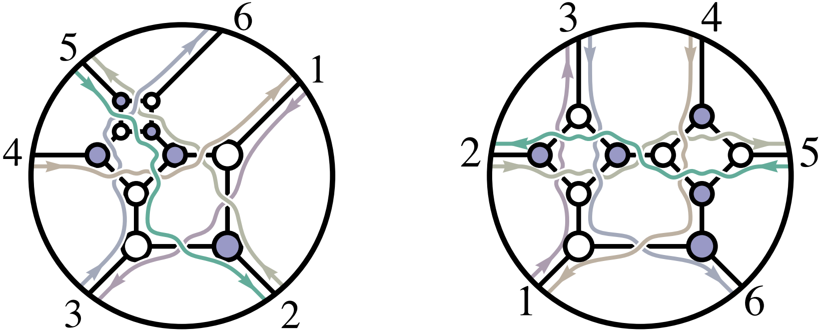

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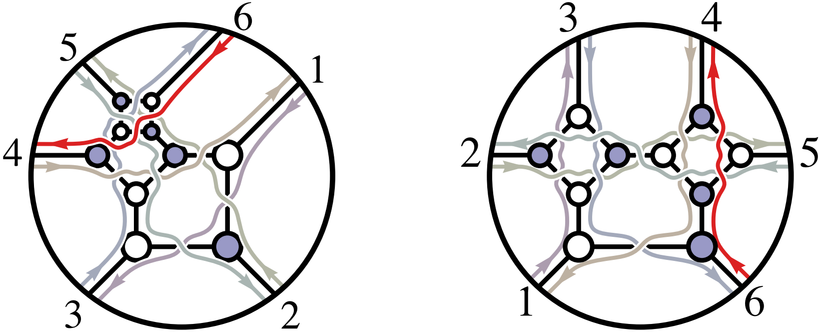

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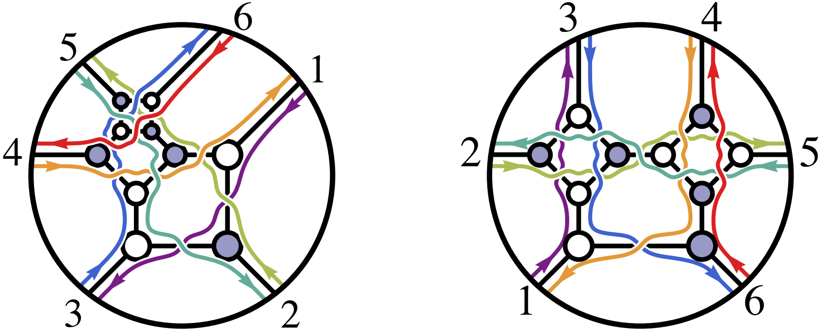

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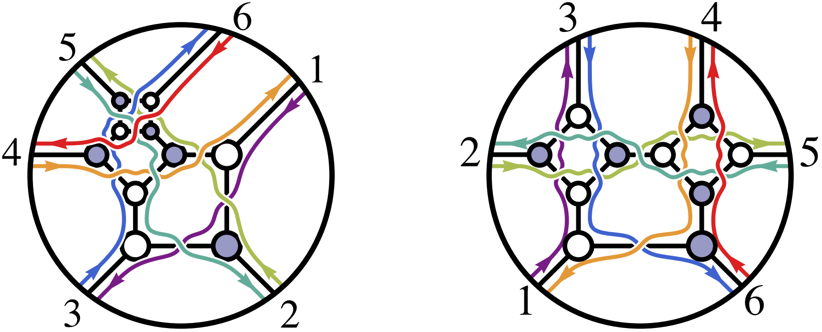

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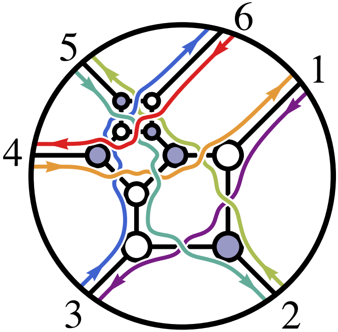

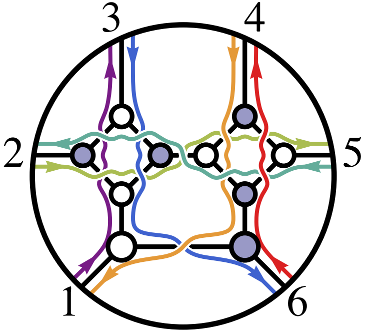

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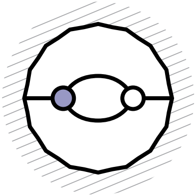

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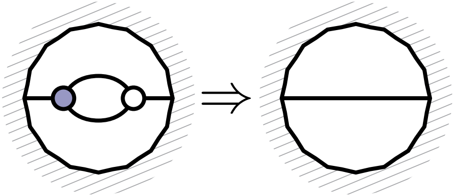

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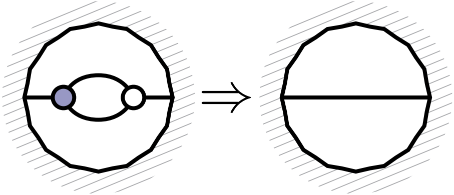

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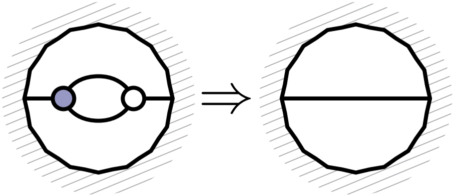

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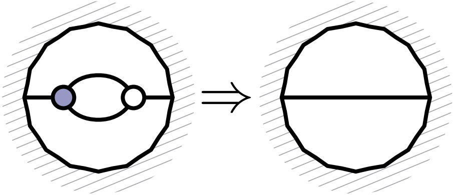

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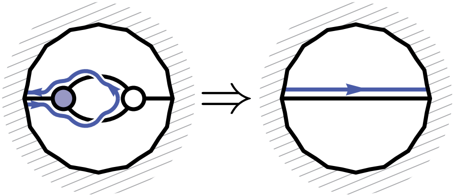

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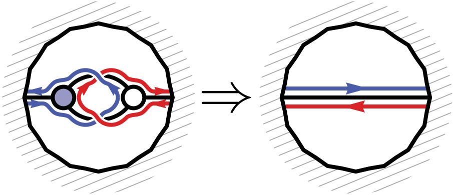

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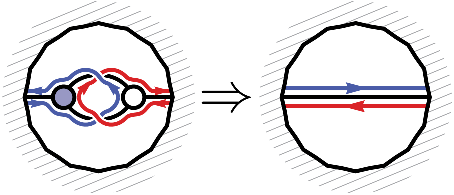
Such factors of $d\alpha/\alpha$ arising from bubble deletion encode loop integrands!



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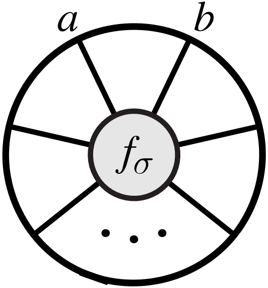

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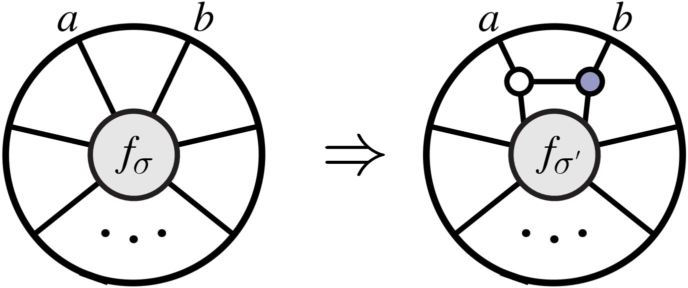

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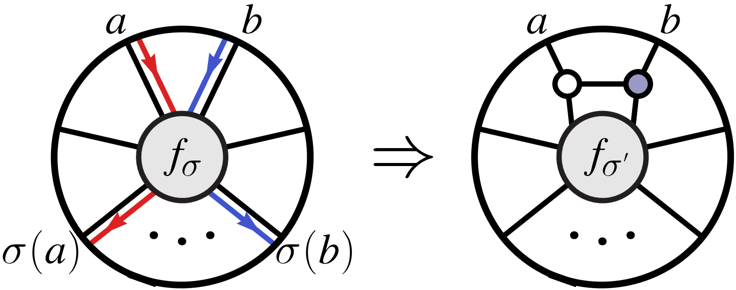

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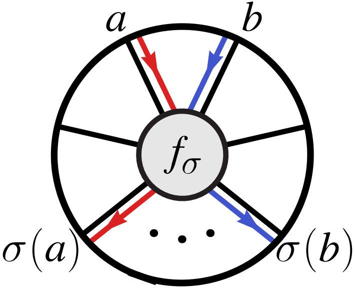

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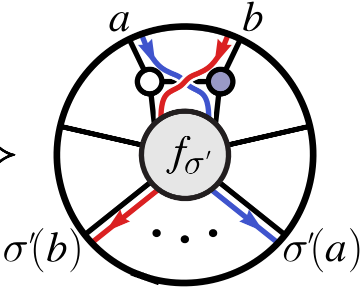

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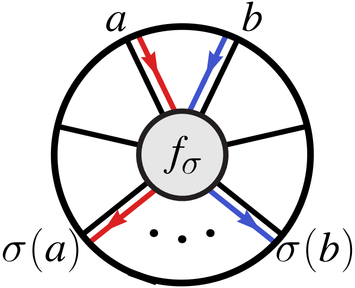

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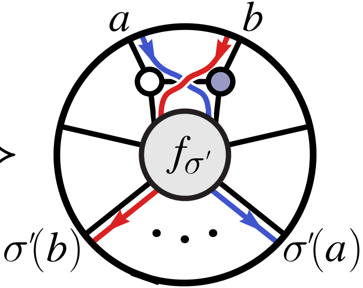

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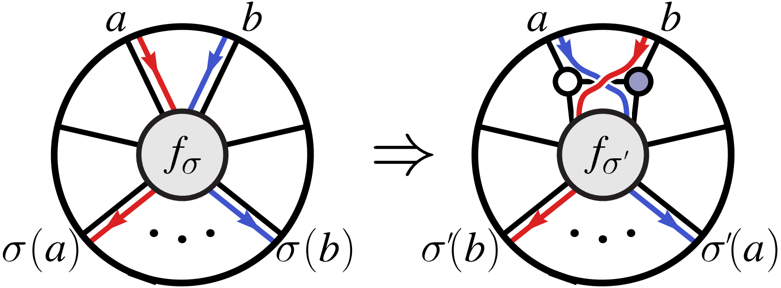

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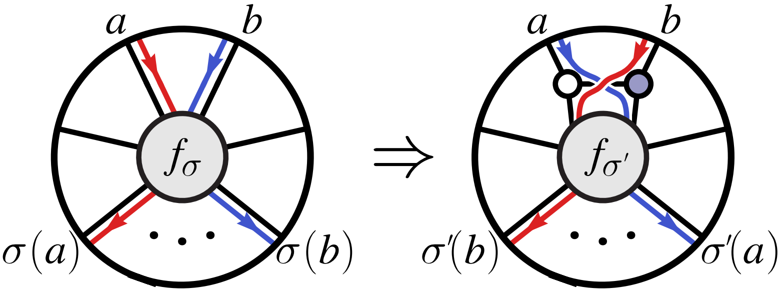

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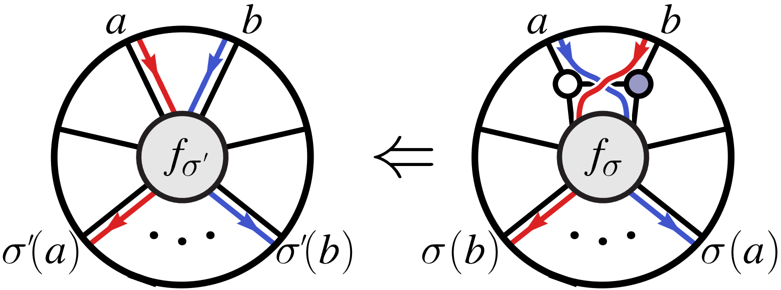

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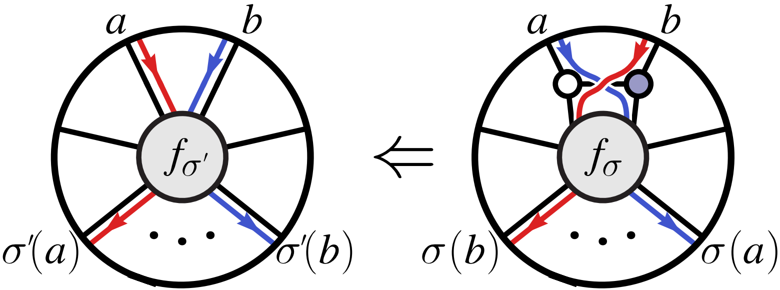
Read the other way, we can 'peel-off' bridges and thereby decompose a permutation into transpositions according to $\sigma = (a\,b) \circ \sigma'$



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There are many ways to decompose a permutation into transpositions

'Bridge' Decomposition

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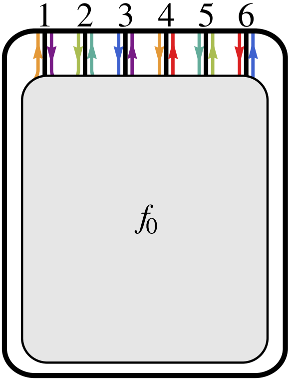

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$$\begin{array}{cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\ f_0 & \{3 & 5 & 6 & 7 & 8 & 10\} & \end{array}$$

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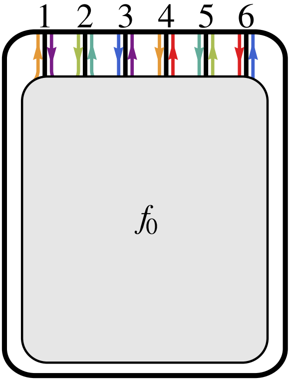

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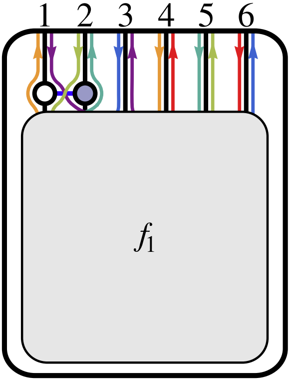

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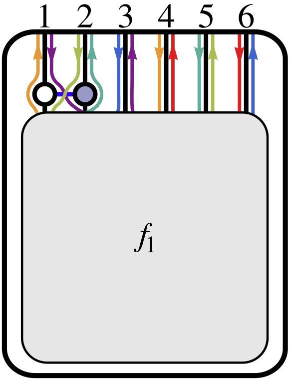

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	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$

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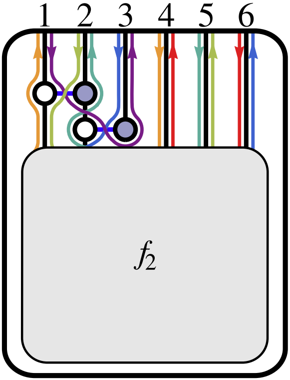

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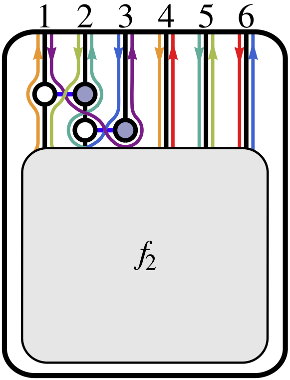

'Bridge' Decomposition

$$\begin{array}{lccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\
f_0 & \{3 & 5 & 6 & 7 & 8 & 10\} & (1\,2) \\
f_1 & \{5 & 3 & 6 & 7 & 8 & 10\} & (2\,3) \\
f_2 & \{5 & 6 & 3 & 7 & 8 & 10\} & (1\,2)
\end{array}$$

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

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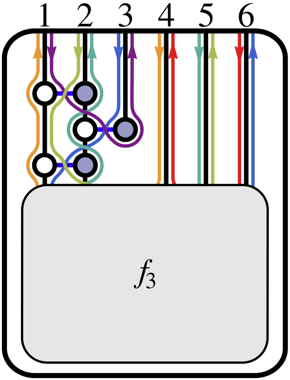

‘Bridge’ Decomposition

$$\begin{array}{lcccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & & \tau \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\
f_0 & \{3 & 5 & 6 & 7 & 8 & 10\} & & (1\,2) \\
f_1 & \{5 & 3 & 6 & 7 & 8 & 10\} & & (2\,3) \\
f_2 & \{5 & 6 & 3 & 7 & 8 & 10\} & & (1\,2) \\
f_3 & \{6 & 5 & 3 & 7 & 8 & 10\} & &
\end{array}$$

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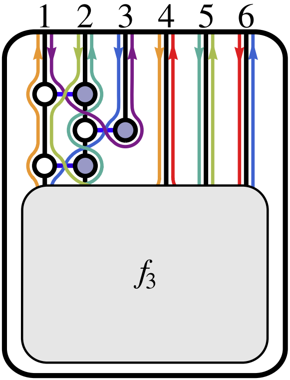

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$

Canonical Coordinates for Computing On-Shell Functions

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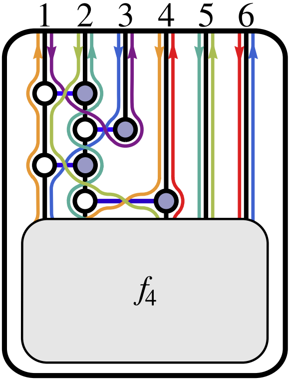

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0 {	3	5	6	7	8	10}	$(1\,2)$
f_1 {	5	3	6	7	8	10}	$(2\,3)$
f_2 {	5	6	3	7	8	10}	$(1\,2)$
f_3 {	6	5	3	7	8	10}	$(2\,4)$
f_4 {	6	7	3	5	8	10}	

Canonical Coordinates for Computing On-Shell Functions

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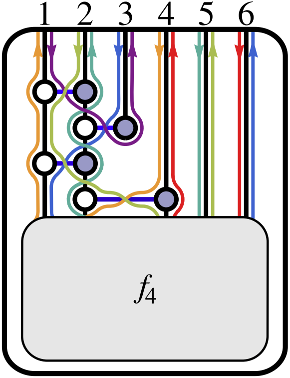

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	$\{3$	5	6	7	8	$10\}$	$(1\,2)$
f_1	$\{5$	3	6	7	8	$10\}$	$(2\,3)$
f_2	$\{5$	6	3	7	8	$10\}$	$(1\,2)$
f_3	$\{6$	5	3	7	8	$10\}$	$(2\,4)$
f_4	$\{6$	7	3	5	8	$10\}$	$(1\,2)$

Canonical Coordinates for Computing On-Shell Functions

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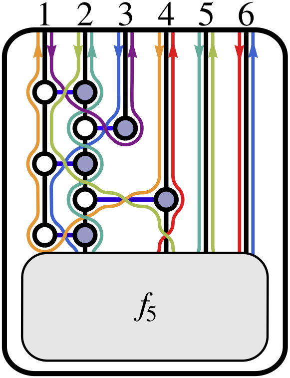

'Bridge' Decomposition

	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

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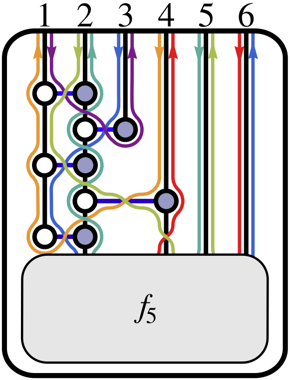

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0 {	3	5	6	7	8	10}	(1 2)
f_1 {	5	3	6	7	8	10}	(2 3)
f_2 {	5	6	3	7	8	10}	(1 2)
f_3 {	6	5	3	7	8	10}	(2 4)
f_4 {	6	7	3	5	8	10}	(1 2)
f_5 {	7	6	3	5	8	10}	(4 5)

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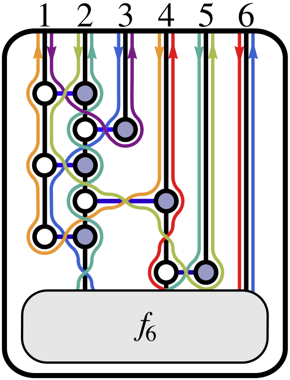
$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}f_6$$



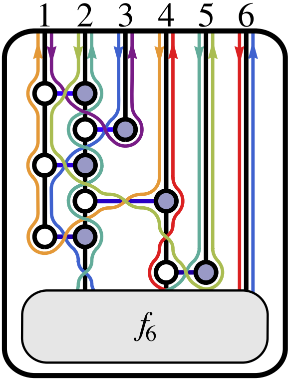
'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	$\{3$	5	6	7	8	$10\}$	$(1\,2)$
f_1	$\{5$	3	6	7	8	$10\}$	$(2\,3)$
f_2	$\{5$	6	3	7	8	$10\}$	$(1\,2)$
f_3	$\{6$	5	3	7	8	$10\}$	$(2\,4)$
f_4	$\{6$	7	3	5	8	$10\}$	$(1\,2)$
f_5	$\{7$	6	3	5	8	$10\}$	$(4\,5)$
f_6	$\{7$	6	3	8	5	$10\}$	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}f_6$$



'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)

Canonical Coordinates for Computing On-Shell Functions

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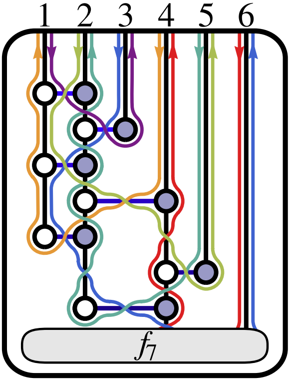

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	

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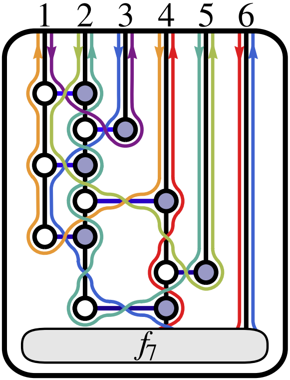

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)

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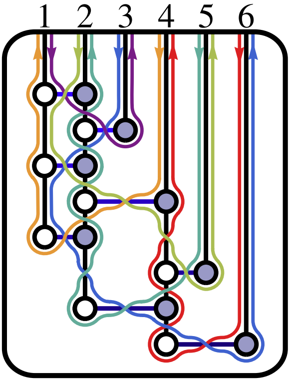

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
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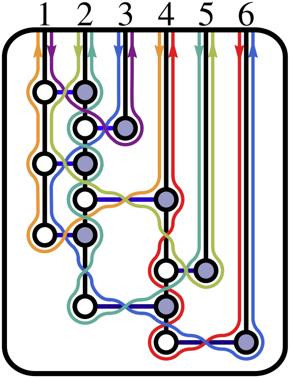

'Bridge' Decomposition

	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

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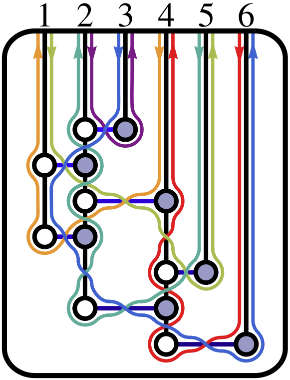

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
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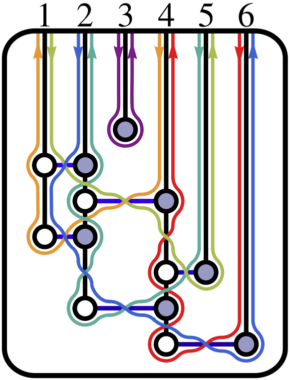

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
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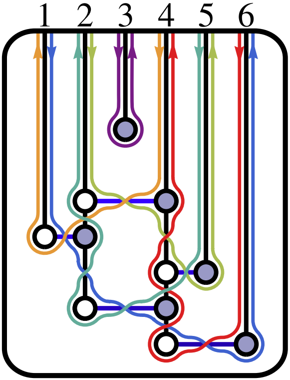

‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6 }	

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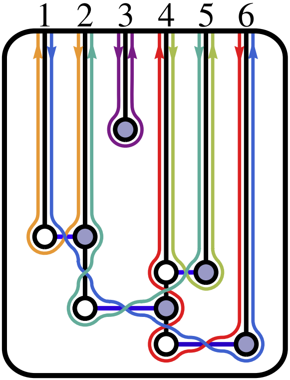

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

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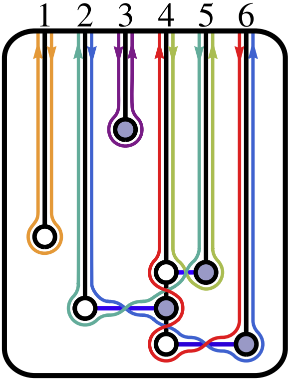

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6 }	

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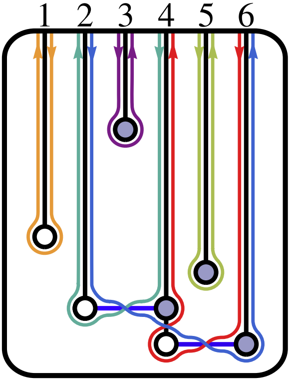

'Bridge' Decomposition

$$\begin{array}{lcccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & & \tau \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ f_6 & \{7 & 6 & 3 & 8 & 5 & 10\} & & \\ & & & & & & & & (2\,4) \\ f_7 & \{7 & 8 & 3 & 6 & 5 & 10\} & & \\ & & & & & & & & (4\,6) \\ f_8 & \{7 & 8 & 3 & 10 & 5 & 6\,\} & & \end{array}$$

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau\equiv(a\,b)$ such that $\sigma(a)<\sigma(b)$:

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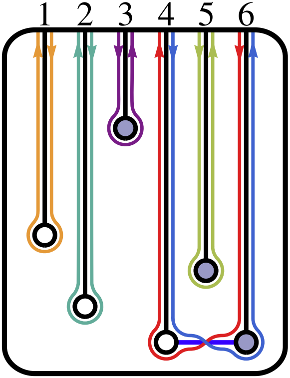

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_7	{7	8	3	6	5	10}	(4 6)
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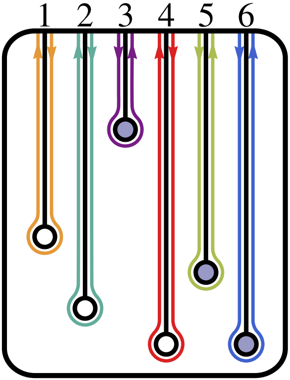

'Bridge' Decomposition

$$\begin{array}{cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \end{array}$$

$f_8\,\{7\;\;8\;\;3\;\;10\;\;5\;\;6\}$

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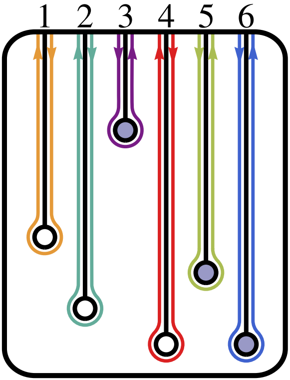

‘Bridge’ Decomposition

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}\qquad \tau$$

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$$f_8=\prod_{a=\sigma(a)+n}\Big(\delta^4(\widetilde{\eta}_a)\delta^2(\widetilde{\lambda}_a)\Big)\prod_{b=\sigma(b)}\Big(\delta^2(\lambda_b)\Big)$$

'Bridge' Decomposition

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \tau \end{array}$$

$f_8\,\{7\;\;8\;\;3\;\;10\;\;5\;\;6\}$

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

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$$C \equiv \begin{array}{c}\\ \\ \\ \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right)$$

'Bridge' Decomposition

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \end{array}$$

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$$f_8 = \delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \\ \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right)$$

'Bridge' Decomposition

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‘Bridge’ Decomposition

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \end{array}$$

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$$f_7 = \frac{d\alpha_8}{\alpha_8}\delta^{3\times4}\big(C\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}\big)$$

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

$$(4\,6): \quad c_6 \mapsto c_6 + \alpha_8\, c_4$$

'Bridge' Decomposition

$$\begin{array}{cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\ f_7 & \{7 & 8 & 3 & 6 & 5 & 10\} & (4\,6) \\ f_8 & \{7 & 8 & 3 & 10 & 5 & 6\} & \end{array}$$

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$$f_6 = \frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}\\ \left(\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)\end{array}$$

$(2\,4)$: $c_4 \mapsto c_4 + \alpha_7\, c_2$

'Bridge' Decomposition

$$\begin{array}{lcccccccl} & & 1 & 2 & 3 & 4 & 5 & 6 & \\ & & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\ f_6 & \{ & 7 & 6 & 3 & 8 & 5 & 10 & \} \\ & & & & & & & & (2\,4) \\ f_7 & \{ & 7 & 8 & 3 & 6 & 5 & 10 & \} \\ & & & & & & & & (4\,6) \\ f_8 & \{ & 7 & 8 & 3 & 10 & 5 & 6 & \} \end{array}$$

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$$f_5 = \frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

$$(4\,5): \quad c_5 \mapsto c_5 + \alpha_6\, c_4$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6}	

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$$f_4 = \frac{d\alpha_5}{\alpha_5} \cdots \frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}\\ \left(\begin{array}{cccccc} 1 & \alpha_5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)\end{array}$$

$$(1\,2):\quad c_2 \mapsto c_2 + \alpha_5\,c_1$$

'Bridge' Decomposition

$$\begin{array}{cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\ f_4 & \{6 & 7 & 3 & 5 & 8 & 10\} & (1\,2) \\ f_5 & \{7 & 6 & 3 & 5 & 8 & 10\} & (4\,5) \\ f_6 & \{7 & 6 & 3 & 8 & 5 & 10\} & (2\,4) \\ f_7 & \{7 & 8 & 3 & 6 & 5 & 10\} & (4\,6) \\ f_8 & \{7 & 8 & 3 & 10 & 5 & 6\,\} & \end{array}$$

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$$f_3 = \frac{d\alpha_4}{\alpha_4}\cdots\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}\\ \left(\begin{array}{cccccc} 1 & \alpha_5 & 0 & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & 0 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)\end{array}$$

$(2\,4)$: $c_4 \mapsto c_4 + \alpha_4\,c_2$

'Bridge' Decomposition

	$1\downarrow$	$2\downarrow$	$3\downarrow$	$4\downarrow$	$5\downarrow$	$6\downarrow$	τ
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6}	

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$$f_2 = \frac{d\alpha_3}{\alpha_3} \cdots \frac{d\alpha_8}{\alpha_8}\, \delta^{3\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{3\times2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times3}\big(\lambda\cdot C^{\perp}\big)$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_3+\alpha_5) & 0 & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & 0 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

$$(1\,2):\quad c_2 \mapsto c_2 + \alpha_3\, c_1$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_2	$\{5$	6	3	7	8	$10\}$	$(1\,2)$
f_3	$\{6$	5	3	7	8	$10\}$	$(2\,4)$
f_4	$\{6$	7	3	5	8	$10\}$	$(1\,2)$
f_5	$\{7$	6	3	5	8	$10\}$	$(4\,5)$
f_6	$\{7$	6	3	8	5	$10\}$	$(2\,4)$
f_7	$\{7$	8	3	6	5	$10\}$	$(4\,6)$
f_8	$\{7$	8	3	10	5	$6\}$	

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There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

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$$f_1 = \frac{d\alpha_2}{\alpha_2}\cdots\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

$$(2\,3):\quad c_3 \mapsto c_3 + \alpha_2\,c_2$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6 }	

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$$f_0 = \frac{d\alpha_1}{\alpha_1}\cdots\frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

$$(1\,2):\quad c_2 \mapsto c_2 + \alpha_1\, c_1$$

‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

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$$f_0 = \frac{d\alpha_1}{\alpha_1} \cdots \frac{d\alpha_8}{\alpha_8} \delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
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Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:

$$f_0 = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\frac{d\alpha_7}{\alpha_7}\frac{d\alpha_8}{\alpha_8} f_8$$

$$f_0 = \frac{d\alpha_1}{\alpha_1} \cdots \frac{d\alpha_8}{\alpha_8}\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})$$

$$C \equiv \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
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‘Bridge’ Decomposition

$$\begin{array}{cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \tau \\ f_0 & \{3 & 5 & 6 & 7 & 8 & 10\} & \\ & & & & & & & (1\,2) \\ f_1 & \{5 & 3 & 6 & 7 & 8 & 10\} & \\ & & & & & & & (2\,3) \\ f_2 & \{5 & 6 & 3 & 7 & 8 & 10\} & \\ & & & & & & & (1\,2) \\ f_3 & \{6 & 5 & 3 & 7 & 8 & 10\} & \\ & & & & & & & (2\,4) \\ f_4 & \{6 & 7 & 3 & 5 & 8 & 10\} & \\ & & & & & & & (1\,2) \\ f_5 & \{7 & 6 & 3 & 5 & 8 & 10\} & \\ & & & & & & & (4\,5) \\ f_6 & \{7 & 6 & 3 & 8 & 5 & 10\} & \\ & & & & & & & (2\,4) \\ f_7 & \{7 & 8 & 3 & 6 & 5 & 10\} & \\ & & & & & & & (4\,6) \\ f_8 & \{7 & 8 & 3 & 10 & 5 & 6\} & \end{array}$$

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$$(6\,1):\quad c_1\mapsto c_1+\alpha_0\,c_6$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
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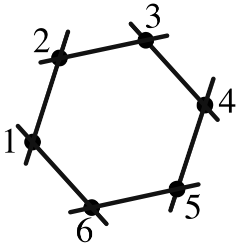

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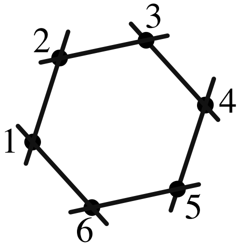

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	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
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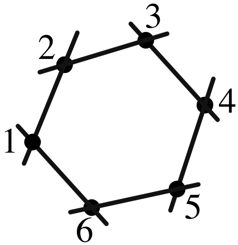

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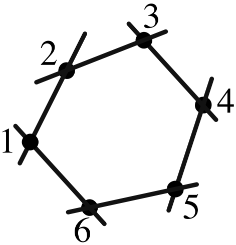

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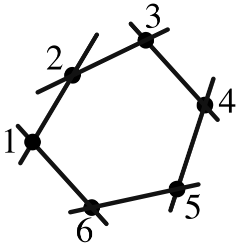

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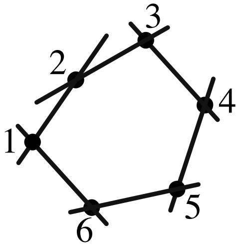

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'Bridge' Decomposition

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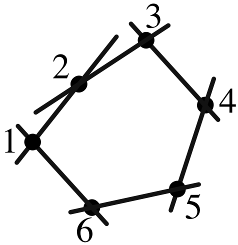

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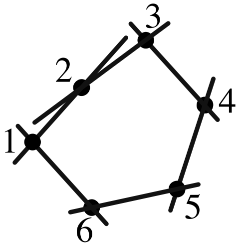

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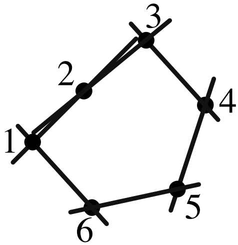

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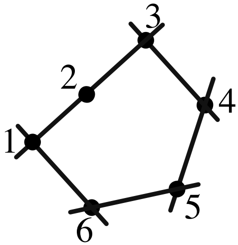

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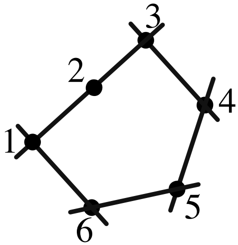

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	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	τ
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$f_6\,\{$	7	6	3	8	5	10$\}$	(2 4)
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$f_8\,\{$	7	8	3	10	5	6$\}$	

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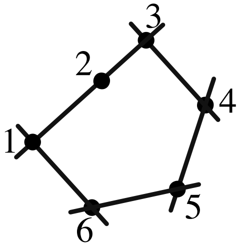

$$C \equiv \begin{array}{c} \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
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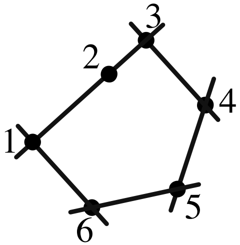

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	$(1\,2)$
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f_4	{6	7	3	5	8	10}	$(1\,2)$
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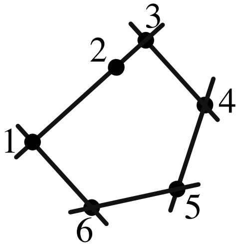

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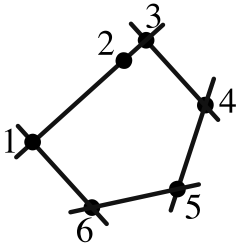

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	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
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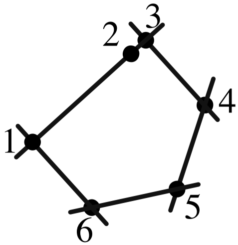

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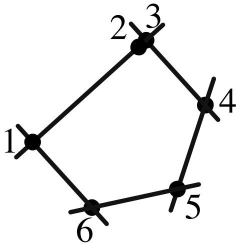

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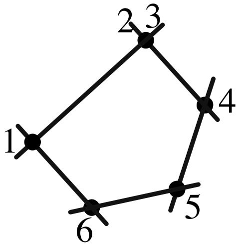

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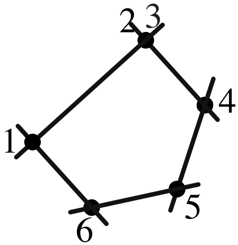

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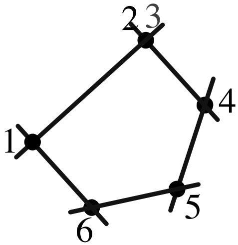

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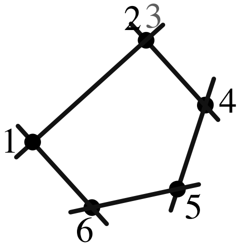

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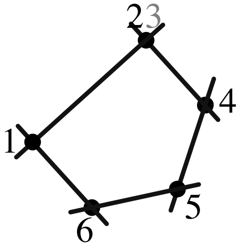

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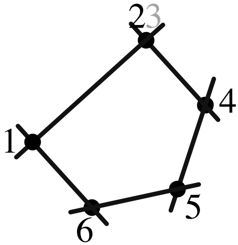

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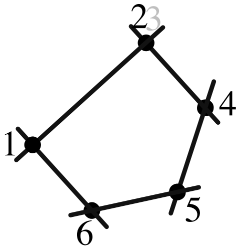

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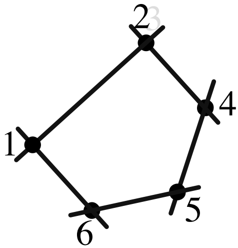

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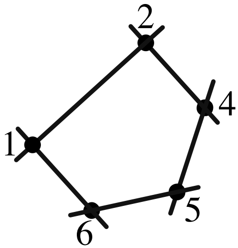

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f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
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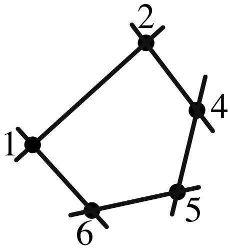

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_3+\alpha_5) & 0 & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & 0 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0 {	3	5	6	7	8	10}	(1 2)
f_1 {	5	3	6	7	8	10}	(2 3)
f_2 {	5	6	3	7	8	10}	(1 2)
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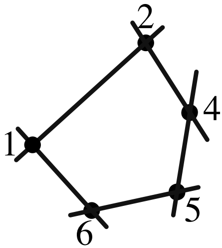

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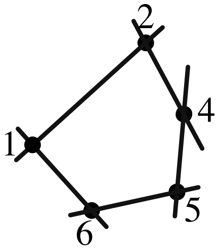

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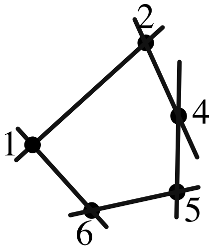

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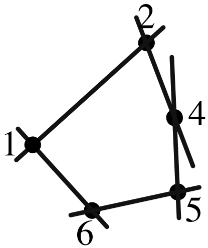

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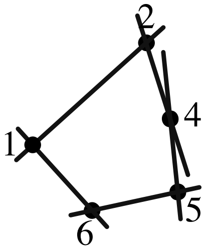

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	↓	↓	↓	↓	↓	↓	
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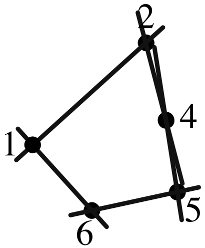

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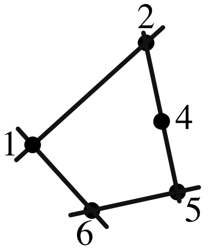

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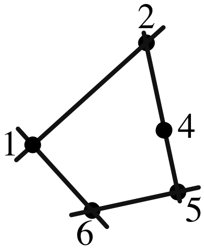

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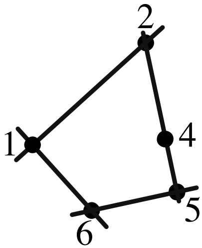

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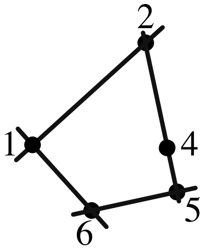

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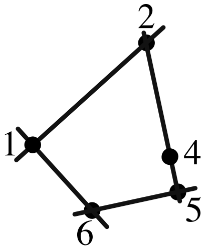

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	1 $\downarrow$	2 $\downarrow$	3 $\downarrow$	4 $\downarrow$	5 $\downarrow$	6 $\downarrow$	τ
f_0	{3	5	6	7	8	10}	(1 2)
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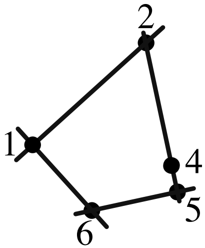

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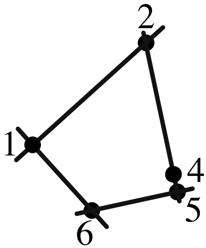

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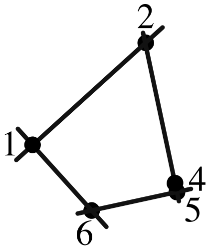

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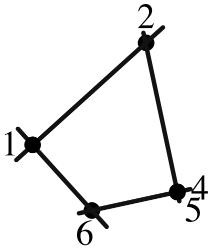

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & \alpha_5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	$\{3$	5	6	7	8	$10\}$	$(1\,2)$
f_1	$\{5$	3	6	7	8	$10\}$	$(2\,3)$
f_2	$\{5$	6	3	7	8	$10\}$	$(1\,2)$
f_3	$\{6$	5	3	7	8	$10\}$	$(2\,4)$
f_4	$\{6$	7	3	5	8	$10\}$	$(1\,2)$
f_5	$\{7$	6	3	5	8	$10\}$	$(4\,5)$
f_6	$\{7$	6	3	8	5	$10\}$	$(2\,4)$
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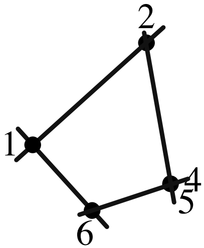

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
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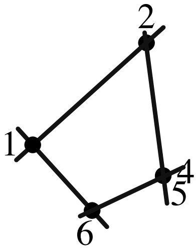

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & \alpha_5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
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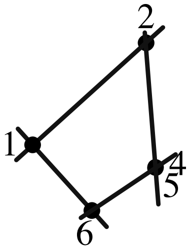

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‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	$(1\,2)$
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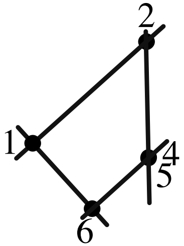

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0 {	3	5	6	7	8	10 }	(1 2)
f_1 {	5	3	6	7	8	10 }	(2 3)
f_2 {	5	6	3	7	8	10 }	(1 2)
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f_4 {	6	7	3	5	8	10 }	(1 2)
f_5 {	7	6	3	5	8	10 }	(4 5)
f_6 {	7	6	3	8	5	10 }	(2 4)
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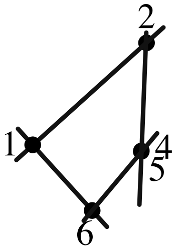

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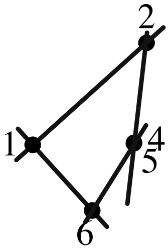

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'Bridge' Decomposition

	$1 \downarrow$	$2 \downarrow$	$3 \downarrow$	$4 \downarrow$	$5 \downarrow$	$6 \downarrow$	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
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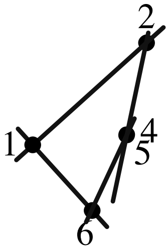

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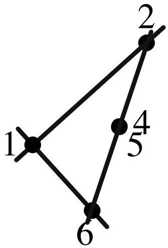

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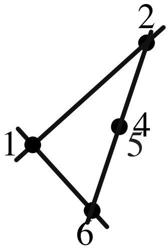

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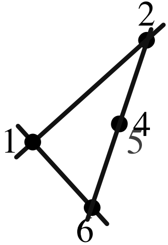

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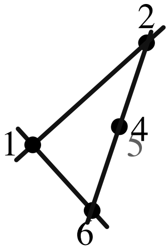

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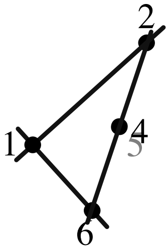

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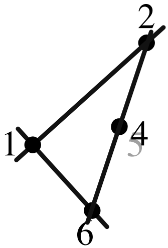

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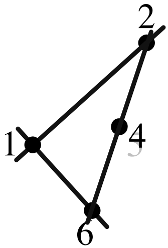

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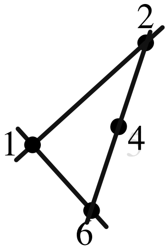

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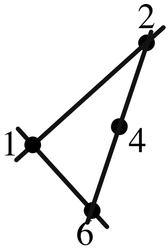
f_1 {	5	3	6	7	8	10}	(2 3)
f_2 {	5	6	3	7	8	10}	(1 2)
f_3 {	6	5	3	7	8	10}	(2 4)
f_4 {	6	7	3	5	8	10}	(1 2)
f_5 {	7	6	3	5	8	10}	(4 5)
f_6 {	7	6	3	8	5	10}	(2 4)
f_7 {	7	8	3	6	5	10}	(4 6)
f_8 {	7	8	3	10	5	6 }	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau\equiv(a\,b)$ such that $\sigma(a)<\sigma(b)$:



$$C\equiv\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6\\ \hline 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & \alpha_7 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

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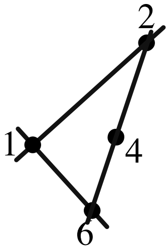

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0 {	3	5	6	7	8	10 }	$(1\,2)$
f_1 {	5	3	6	7	8	10 }	$(2\,3)$
f_2 {	5	6	3	7	8	10 }	$(1\,2)$
f_3 {	6	5	3	7	8	10 }	$(2\,4)$
f_4 {	6	7	3	5	8	10 }	$(1\,2)$
f_5 {	7	6	3	5	8	10 }	$(4\,5)$
f_6 {	7	6	3	8	5	10 }	$(2\,4)$
f_7 {	7	8	3	6	5	10 }	$(4\,6)$
f_8 {	7	8	3	10	5	6 }	

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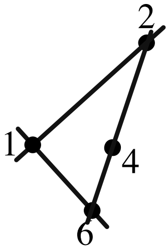

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

'Bridge' Decomposition

	1 $\downarrow$	2 $\downarrow$	3 $\downarrow$	4 $\downarrow$	5 $\downarrow$	6 $\downarrow$	τ
f_0 {	3	5	6	7	8	10}	$(1\,2)$
f_1 {	5	3	6	7	8	10}	$(2\,3)$
f_2 {	5	6	3	7	8	10}	$(1\,2)$
f_3 {	6	5	3	7	8	10}	$(2\,4)$
f_4 {	6	7	3	5	8	10}	$(1\,2)$
f_5 {	7	6	3	5	8	10}	$(4\,5)$
f_6 {	7	6	3	8	5	10}	$(2\,4)$
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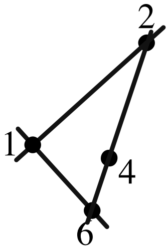

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
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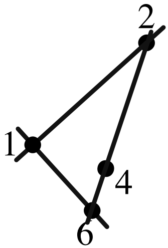

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
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f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
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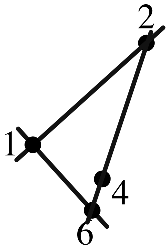

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
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f_3	{6	5	3	7	8	10}	$(2\,4)$
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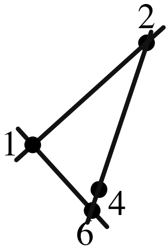

$$C \equiv \begin{array}{c} \\ \\ \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
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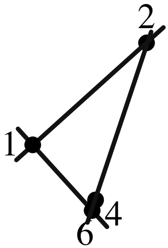

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
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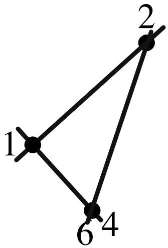

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
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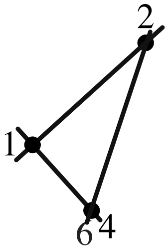

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‘Bridge’ Decomposition

	1 $\downarrow$	2 $\downarrow$	3 $\downarrow$	4 $\downarrow$	5 $\downarrow$	6 $\downarrow$	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
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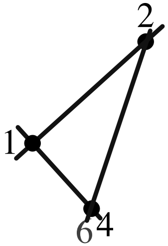

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	1	2	3	4	5	6	
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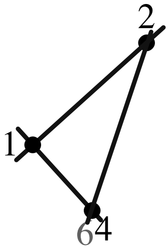

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	1	2	3	4	5	6	τ
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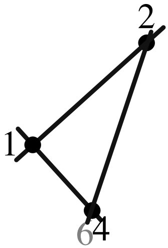

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‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
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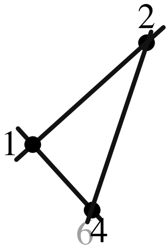

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‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
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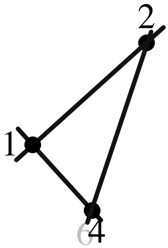

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	↓	↓	↓	↓	↓	↓	
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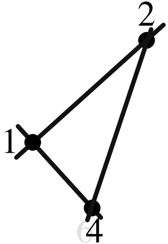

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
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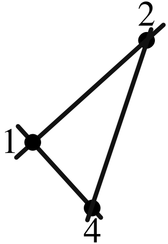
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau\equiv(a\,b)$ such that $\sigma(a)<\sigma(b)$:



$$C\equiv\left(\begin{array}{cccccc}1&2&3&4&5&6\\ \hline 1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&0&1&0&0\end{array}\right)$$

‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
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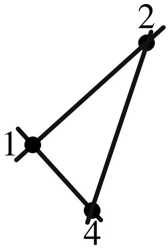

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right)$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
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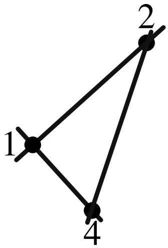

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right)$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
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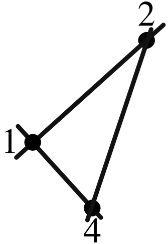

$$C\equiv\left(\begin{array}{cccccc}1&2&3&4&5&6\\ \hline 1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&0&1&0&0\end{array}\right)$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

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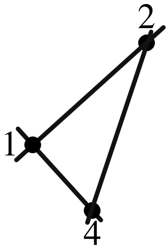

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right)$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

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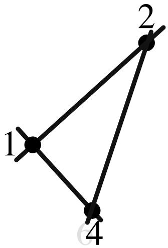

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right) \end{array}$$

$$(4\,6): \quad c_6 \mapsto c_6 + \alpha_8\, c_4$$

'Bridge' Decomposition

	1 $\downarrow$	2 $\downarrow$	3 $\downarrow$	4 $\downarrow$	5 $\downarrow$	6 $\downarrow$	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
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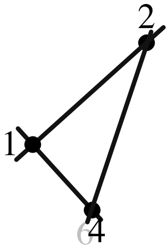

$$C\equiv\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	$(1\,2)$
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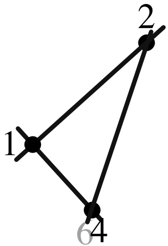

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
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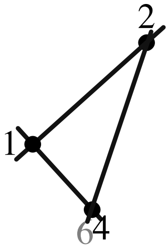

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
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f_1	{5	3	6	7	8	10}	(2 3)
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f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
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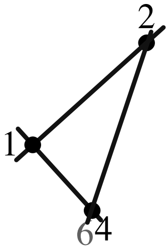

$$C \equiv \begin{array}{c} \\ \\ \\ \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
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f_5	{7	6	3	5	8	10}	$(4\,5)$
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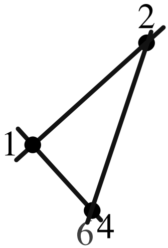

$$C \equiv \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline & 1 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}$$

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
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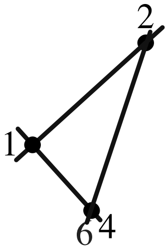

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

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‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0 {	3	5	6	7	8	10}	$(1\,2)$
f_1 {	5	3	6	7	8	10}	$(2\,3)$
f_2 {	5	6	3	7	8	10}	$(1\,2)$
f_3 {	6	5	3	7	8	10}	$(2\,4)$
f_4 {	6	7	3	5	8	10}	$(1\,2)$
f_5 {	7	6	3	5	8	10}	$(4\,5)$
f_6 {	7	6	3	8	5	10}	$(2\,4)$
f_7 {	7	8	3	6	5	10}	$(4\,6)$
f_8 {	7	8	3	10	5	6}	

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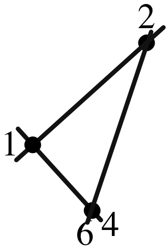

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

$$(4\,6): \quad c_6 \mapsto c_6 + \alpha_8\, c_4$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
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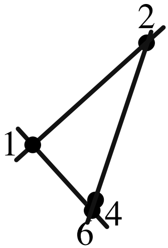

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right)$$

$$(2\,4):\quad c_4 \mapsto c_4 + \alpha_7\, c_2$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6}	

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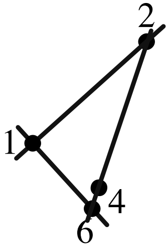

$$C \equiv \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline & 1 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 1 & 0 & \alpha_7 & 0 & 0 \\ & 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}$$

$$(2\,4): \quad c_4 \mapsto c_4 + \alpha_7\, c_2$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
f_8	{7	8	3	10	5	6}	

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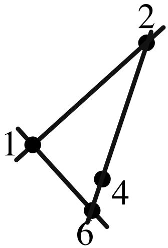

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left(\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_8 \end{array}\right) \end{array}$$

$$(2\,4): \quad c_4 \mapsto c_4 + \alpha_7\, c_2$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
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f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
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Canonical Coordinates for Computing On-Shell Functions

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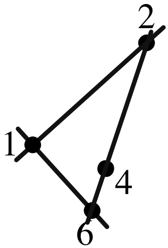

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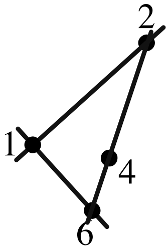

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
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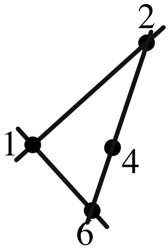

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	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
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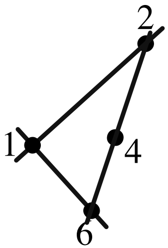

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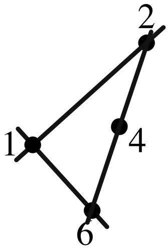

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	↓	↓	↓	↓	↓	↓	
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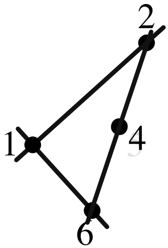

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$(4\,5): \quad c_5 \mapsto c_5 + \alpha_6\, c_4$

'Bridge' Decomposition

	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
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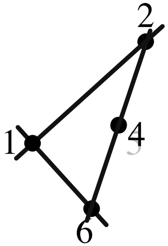

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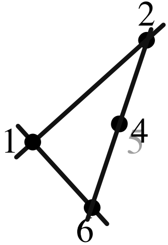

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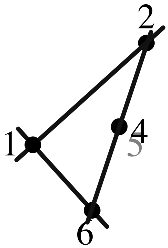

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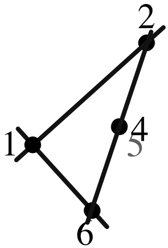

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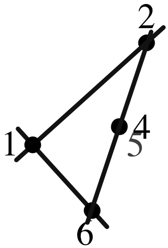

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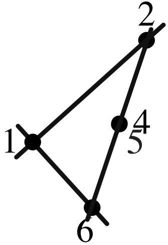

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'Bridge' Decomposition

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f_4 {	6	7	3	5	8	10}	(1 2)
f_5 {	7	6	3	5	8	10}	(4 5)
f_6 {	7	6	3	8	5	10}	(2 4)
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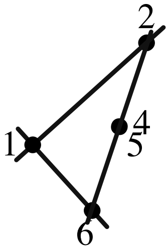

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'Bridge' Decomposition

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f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
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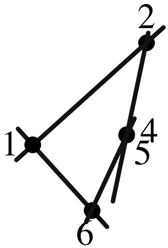

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & \alpha_5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

$$(1\,2): \quad c_2 \mapsto c_2 + \alpha_5\, c_1$$

'Bridge' Decomposition

	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
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f_6	{7	6	3	8	5	10}	$(2\,4)$
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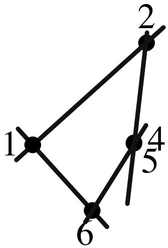

$$C \equiv \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & \alpha_5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_7 & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}$$

$$(1\,2): \quad c_2 \mapsto c_2 + \alpha_5\, c_1$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
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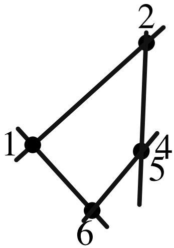

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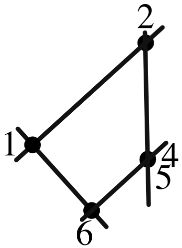

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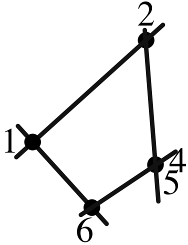

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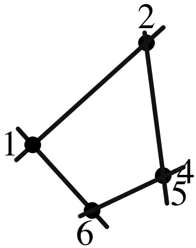

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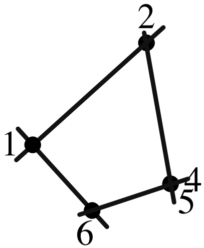

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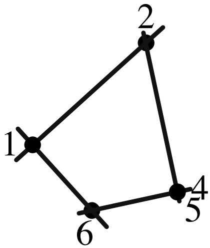

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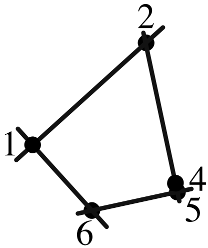

$$C\equiv\left(\begin{array}{cccccc} 1&2&3&4&5&6\\ \hline 1&\alpha_5&0&\alpha_4\alpha_5&0&0\\ 0&1&0&(\alpha_4+\alpha_7)&\alpha_6\alpha_7&0\\ 0&0&0&1&\alpha_6&\alpha_8\end{array}\right)$$

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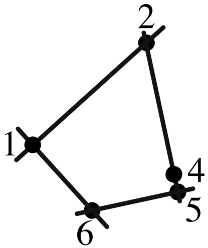

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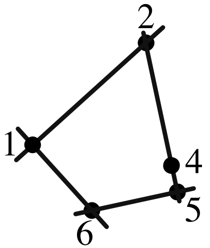

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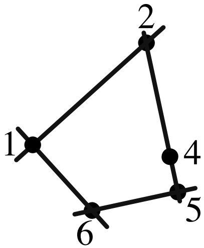

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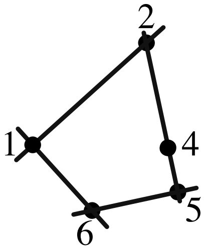

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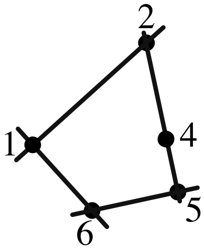

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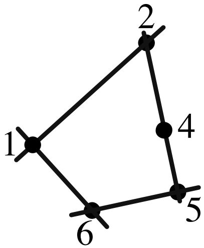

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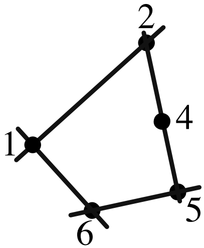

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	↓	↓	↓	↓	↓	↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
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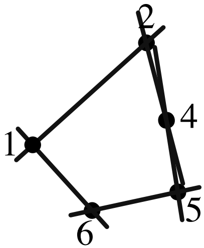

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_3+\alpha_5) & 0 & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & 0 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

$$(1\,2):\quad c_2 \mapsto c_2 + \alpha_3\, c_1$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
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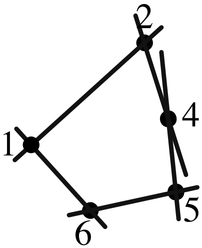

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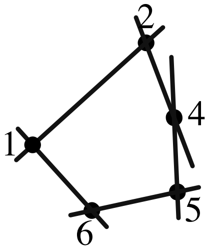

$$C\equiv\left(\begin{array}{cccccc}1&2&3&4&5&6\\\hline 1&(\alpha_3+\alpha_5)&0&\alpha_4\alpha_5&0&0\\0&1&0&(\alpha_4+\alpha_7)&\alpha_6\alpha_7&0\\0&0&0&1&\alpha_6&\alpha_8\end{array}\right)$$

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	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
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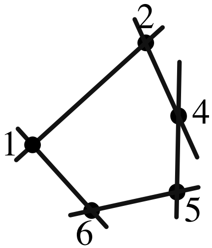

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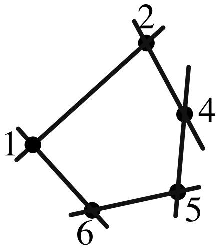

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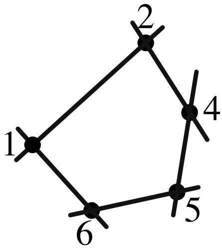

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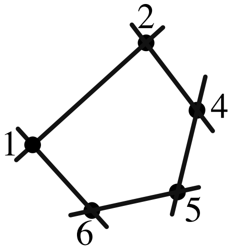

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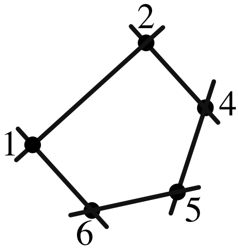

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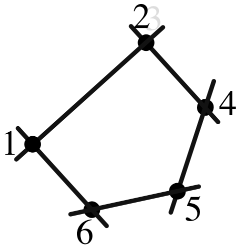

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‘Bridge’ Decomposition

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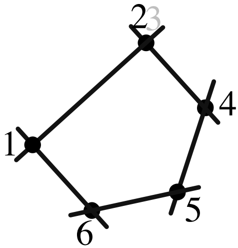

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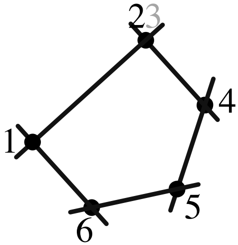

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	1	2	3	4	5	6	
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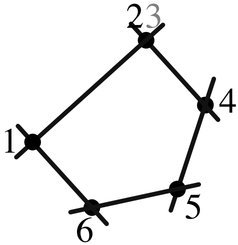

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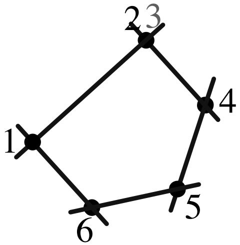

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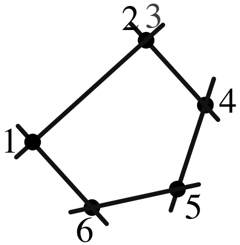

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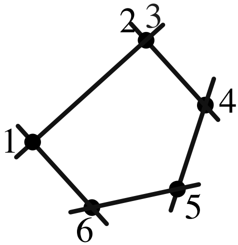
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:



$$C \equiv \begin{array}{c}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}\\ \left(\begin{array}{cccccc} 1 & (\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)\end{array}$$

$$(2\,3): \quad c_3 \mapsto c_3 + \alpha_2\, c_2$$

'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
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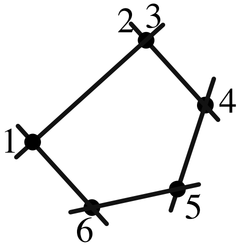

$$C\equiv\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6\\ \hline 1 & (\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0\\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0\\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
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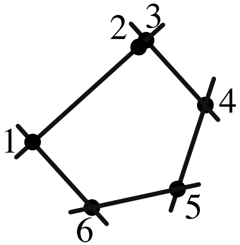

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

$$(1\,2): \quad c_2 \mapsto c_2 + \alpha_1\, c_1$$

'Bridge' Decomposition

	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
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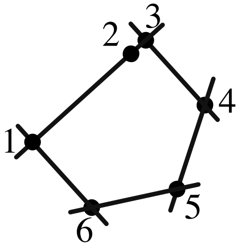

$$C \equiv \begin{array}{c} \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
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f_3	{6	5	3	7	8	10}	(2 4)
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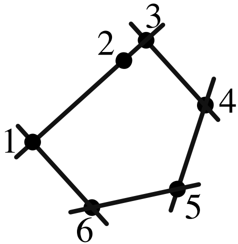

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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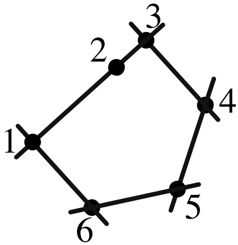

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
f_0	$\{3$	5	6	7	8	$10\}$	$(1\,2)$
f_1	$\{5$	3	6	7	8	$10\}$	$(2\,3)$
f_2	$\{5$	6	3	7	8	$10\}$	$(1\,2)$
f_3	$\{6$	5	3	7	8	$10\}$	$(2\,4)$
f_4	$\{6$	7	3	5	8	$10\}$	$(1\,2)$
f_5	$\{7$	6	3	5	8	$10\}$	$(4\,5)$
f_6	$\{7$	6	3	8	5	$10\}$	$(2\,4)$
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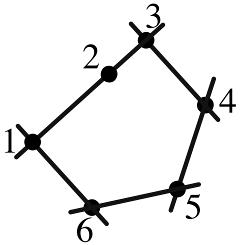

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	$1\downarrow$	$2\downarrow$	$3\downarrow$	$4\downarrow$	$5\downarrow$	$6\downarrow$	τ
f_0	$\{3$	5	6	7	8	$10\}$	$(1\,2)$
f_1	$\{5$	3	6	7	8	$10\}$	$(2\,3)$
f_2	$\{5$	6	3	7	8	$10\}$	$(1\,2)$
f_3	$\{6$	5	3	7	8	$10\}$	$(2\,4)$
f_4	$\{6$	7	3	5	8	$10\}$	$(1\,2)$
f_5	$\{7$	6	3	5	8	$10\}$	$(4\,5)$
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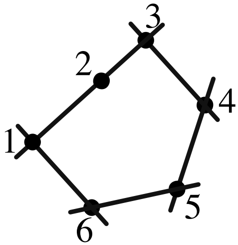

$$C \equiv \begin{array}{c} \\ \\ \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
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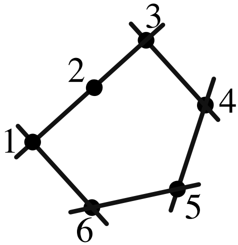

$$C \equiv \begin{array}{c} \\ \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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‘Bridge’ Decomposition

	1 $\downarrow$	2 $\downarrow$	3 $\downarrow$	4 $\downarrow$	5 $\downarrow$	6 $\downarrow$	τ
f_0	{3	5	6	7	8	10}	(1 2)
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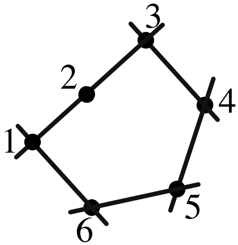

$$C \equiv \begin{array}{c} \\ \\ \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ 0 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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'Bridge' Decomposition

	1 $\downarrow$	2 $\downarrow$	3 $\downarrow$	4 $\downarrow$	5 $\downarrow$	6 $\downarrow$	τ
f_0	{3	5	6	7	8	10}	$(1\,2)$
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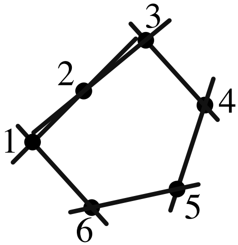

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$$(6\,1): \quad c_1 \mapsto c_1 + \alpha_0\, c_6$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
f_2	{5	6	3	7	8	10}	$(1\,2)$
f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
f_5	{7	6	3	5	8	10}	$(4\,5)$
f_6	{7	6	3	8	5	10}	$(2\,4)$
f_7	{7	8	3	6	5	10}	$(4\,6)$
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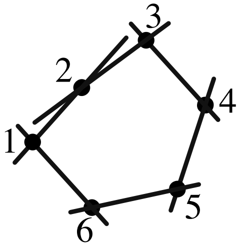

$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ \alpha_0\alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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‘Bridge’ Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	$(1\,2)$
f_1	{5	3	6	7	8	10}	$(2\,3)$
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f_3	{6	5	3	7	8	10}	$(2\,4)$
f_4	{6	7	3	5	8	10}	$(1\,2)$
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f_6	{7	6	3	8	5	10}	$(2\,4)$
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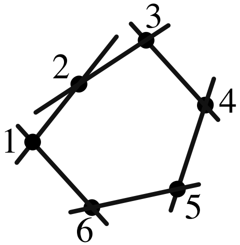

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
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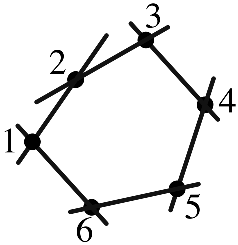

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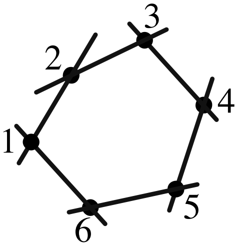
$$(6\,1):\quad c_1 \mapsto c_1 + \alpha_0\, c_6$$

'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

There are many ways to decompose a permutation into transpositions—*e.g.*, always choose the first transposition $\tau \equiv (a\,b)$ such that $\sigma(a) < \sigma(b)$:



$$C \equiv \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ \alpha_0\alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

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	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
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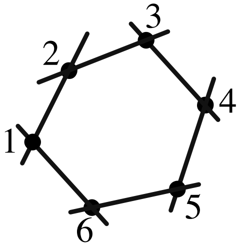

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f_4	{6	7	3	5	8	10}	(1 2)
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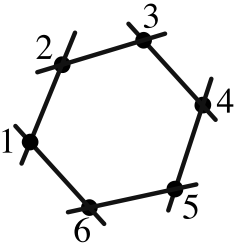

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	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
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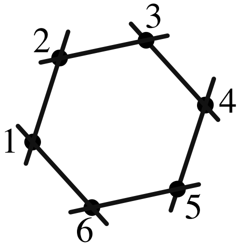

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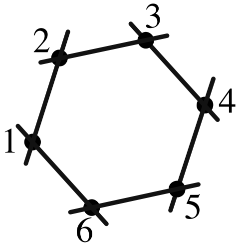

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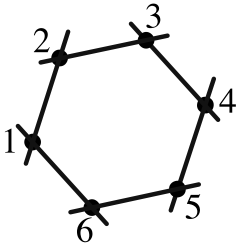

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \end{array} \\ \left(\begin{array}{cccccc} 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ \alpha_0\alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right) \end{array}$$

'Bridge' Decomposition

	1 $\downarrow$	2 $\downarrow$	3 $\downarrow$	4 $\downarrow$	5 $\downarrow$	6 $\downarrow$	τ
f_0	{3	5	6	7	8	10}	(1 2)
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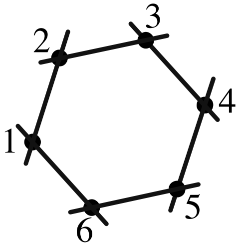

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'Bridge' Decomposition

	1	2	3	4	5	6	τ
	↓	↓	↓	↓	↓	↓	
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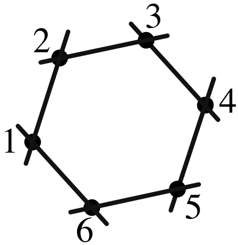
Canonical Coordinates for Computing On-Shell Functions



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'Bridge' Decomposition

	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
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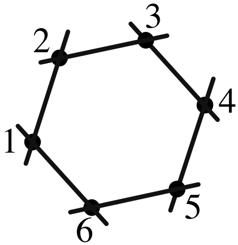
Canonical Coordinates for Computing On-Shell Functions

$$\mathcal{L}_{6,3} \equiv \frac{d\alpha_0}{\alpha_0} \cdots \frac{d\alpha_8}{\alpha_8}$$



$$C \equiv \begin{array}{c} \\ \\ \\ \end{array}\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (\alpha_1+\alpha_3+\alpha_5) & \alpha_2(\alpha_3+\alpha_5) & \alpha_4\alpha_5 & 0 & 0 \\ 0 & 1 & \alpha_2 & (\alpha_4+\alpha_7) & \alpha_6\alpha_7 & 0 \\ \alpha_0\alpha_8 & 0 & 0 & 1 & \alpha_6 & \alpha_8 \end{array}\right)$$

‘Bridge’ Decomposition

	$1 \downarrow$	$2 \downarrow$	$3 \downarrow$	$4 \downarrow$	$5 \downarrow$	$6 \downarrow$	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
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f_4	{6	7	3	5	8	10}	(1 2)
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Canonical Coordinates for Computing On-Shell Functions

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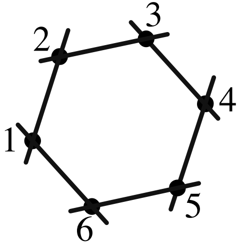

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‘Bridge’ Decomposition

	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
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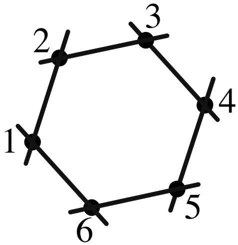
Canonical Coordinates for Computing On-Shell Functions

$$\mathcal{L}_{6,3} \equiv \frac{d\alpha_0}{\alpha_0} \cdots \frac{d\alpha_8}{\alpha_8} = \frac{d^{3\times 6}C}{\mathrm{vol}(GL(3))} \frac{1}{(123)(234)(345)(456)(561)(612)}$$



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‘Bridge’ Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
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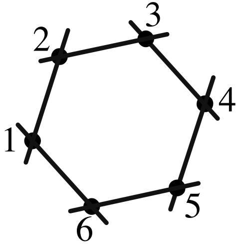
Canonical Coordinates for Computing On-Shell Functions

$$\mathcal{L}_{n,k} \equiv \frac{d\alpha_1}{\alpha_1}\cdots\frac{d\alpha_{k(n-k)}}{\alpha_{k(n-k)}} = \frac{d^{k\times n}C}{\text{vol}(GL(k))}\,\frac{1}{(1\cdots k)\,(2\cdots k{+}1)\cdots(n\cdots k{-}1)}$$



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Canonical Coordinates for Computing On-Shell Functions

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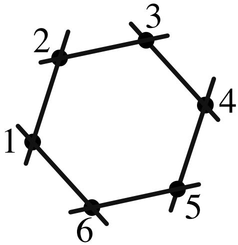

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'Bridge' Decomposition

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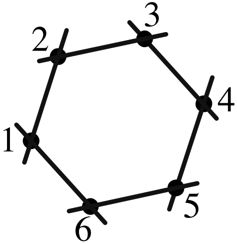

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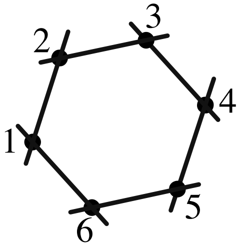
‘Bridge’ Decomposition

	1	2	3	4	5	6	
	↓	↓	↓	↓	↓	↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
f_8	{7	8	3	10	5	6}	

Canonical Coordinates for Computing On-Shell Functions

$$\mathcal{L}_{n,k}\equiv\frac{d\alpha_1}{\alpha_1}\cdots\frac{d\alpha_{k(n-k)}}{\alpha_{k(n-k)}}=\frac{d^{k\times n}C}{\mathrm{vol}(GL(k))}\,\frac{1}{(1\cdots k)\,(2\cdots k{+}1)\cdots(n\cdots k{-}1)}$$



$$C\equiv\left(\begin{array}{cccccc}1&2&3&4&5&6\\\hline 1&(\alpha_1+\alpha_3+\alpha_5)&\alpha_2(\alpha_3+\alpha_5)&\alpha_4\alpha_5&0&0\\0&1&\alpha_2&(\alpha_4+\alpha_7)&\alpha_6\alpha_7&0\\\alpha_0\alpha_8&0&0&1&\alpha_6&\alpha_8\end{array}\right)$$

‘Bridge’ Decomposition

	1 ↓	2 ↓	3 ↓	4 ↓	5 ↓	6 ↓	τ
f_0	{3	5	6	7	8	10}	(1 2)
f_1	{5	3	6	7	8	10}	(2 3)
f_2	{5	6	3	7	8	10}	(1 2)
f_3	{6	5	3	7	8	10}	(2 4)
f_4	{6	7	3	5	8	10}	(1 2)
f_5	{7	6	3	5	8	10}	(4 5)
f_6	{7	6	3	8	5	10}	(2 4)
f_7	{7	8	3	6	5	10}	(4 6)
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Consider all the *iterated factorizations* of the Parke-Taylor Amplitude:

[illegible]

$$\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

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$\lambda \in G(2,4)$

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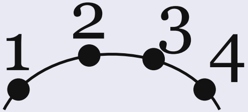

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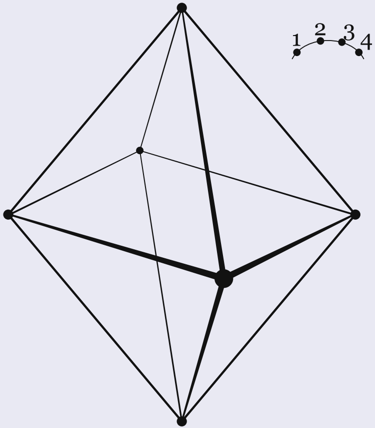

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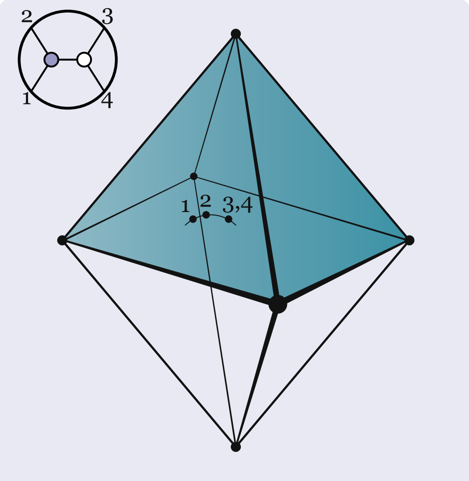
$$\begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 \\ \lambda_1^2 & \lambda_2^2 & 0 & 0 \end{pmatrix}$$

$$\lambda_3 \longrightarrow \alpha_{3\,4}\lambda_4$$



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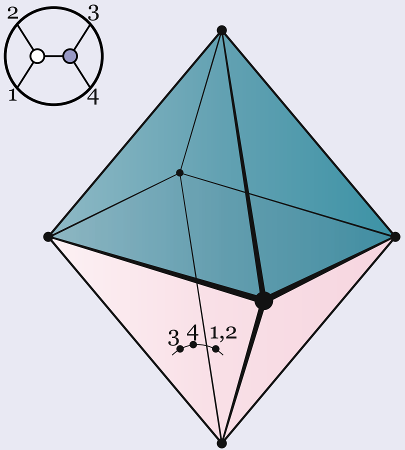

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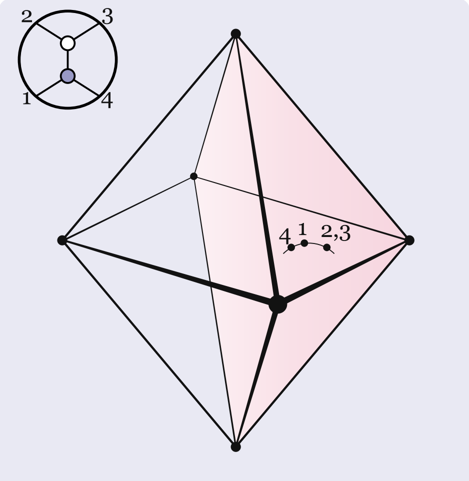

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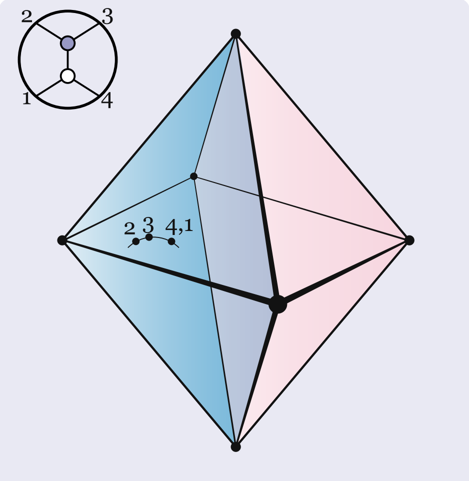

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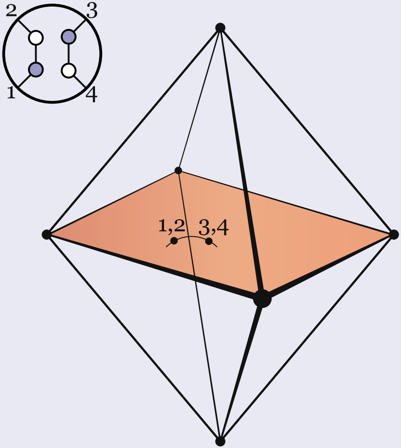

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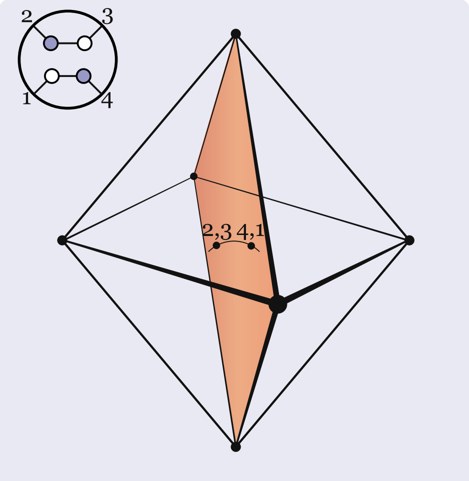

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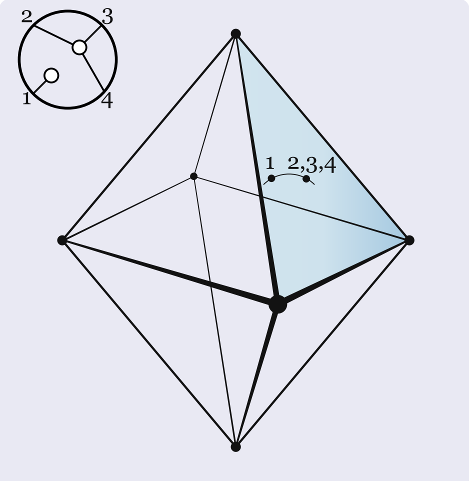

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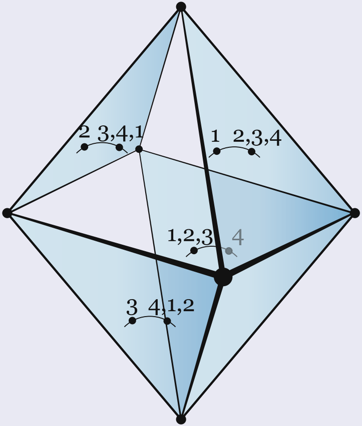

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$$\lambda_4 \longrightarrow \alpha_{4\,1}\lambda_1$$
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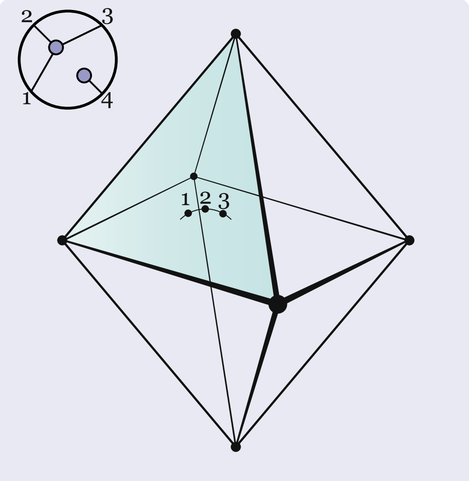
$$(\therefore\, \lambda_4 = 0)$$



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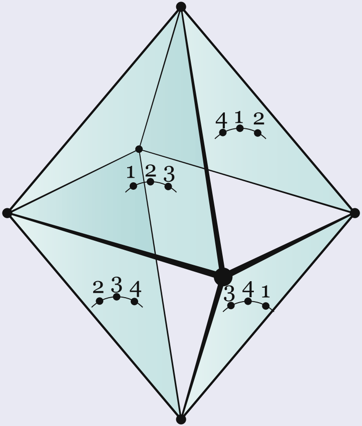

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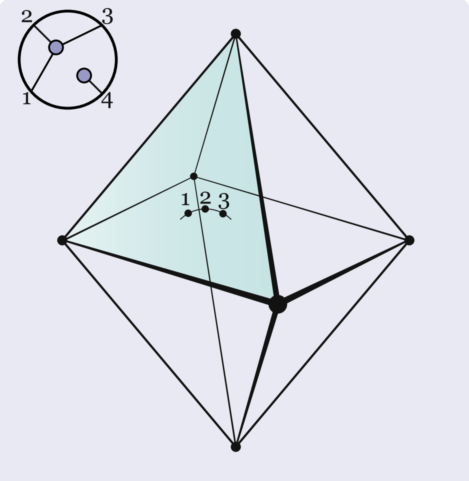

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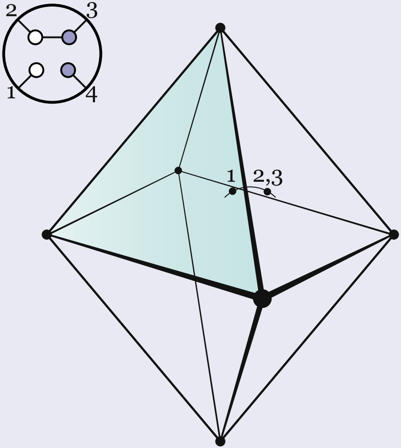

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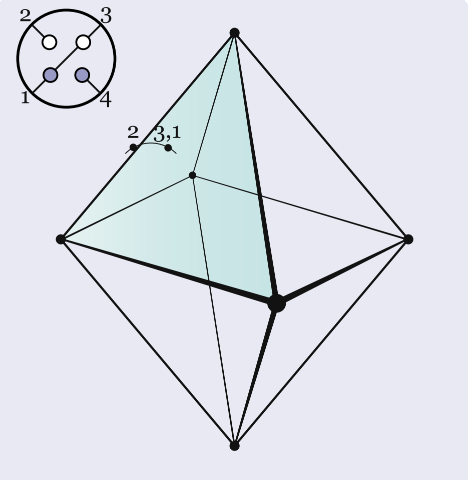

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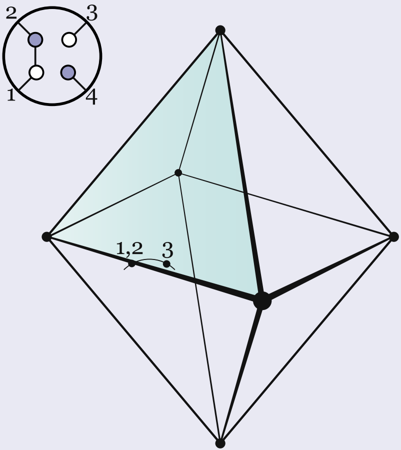

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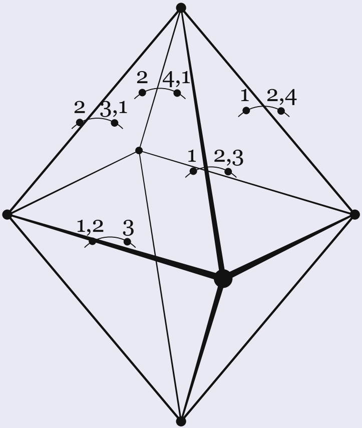
$$\begin{aligned}\lambda_4 &\longrightarrow \alpha_{4\,1}\lambda_1\\ \lambda_4 &\longrightarrow \alpha_{4\,3}\lambda_3\\ \lambda_3 &\longrightarrow \alpha_{3\,2}\lambda_2\end{aligned}$$



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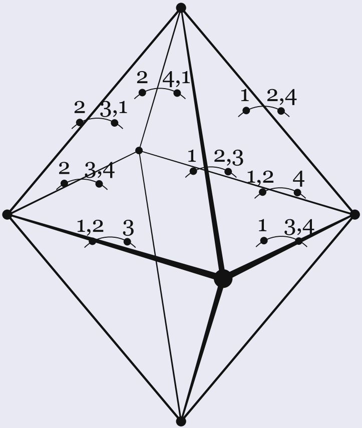
$$\begin{aligned} \lambda_4 &\longrightarrow \alpha_{4\,1}\lambda_1 \\ \lambda_4 &\longrightarrow \alpha_{4\,3}\lambda_3 \\ \lambda_3 &\longrightarrow \alpha_{3\,2}\lambda_2 \end{aligned}$$



The Grassmannian Geometry of Scattering Amplitudes

Consider all the *iterated factorizations* of the Parke-Taylor Amplitude:

$\mathcal{A}_4^{(2)}(1,2,3,4):$

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$\lambda\in G(2,4)\qquad\lambda_a\in\mathbb{P}^1$

$$\begin{pmatrix}\lambda_1^1 & \lambda_2^1 & \lambda_3^1 & 0\\ \lambda_1^2 & 0 & 0 & 0\end{pmatrix}$$

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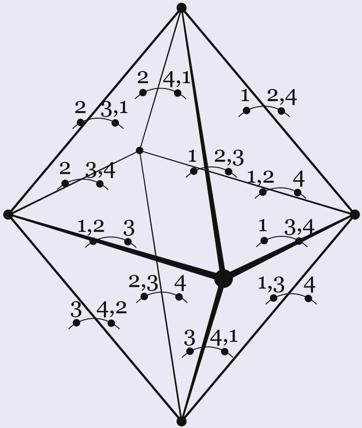

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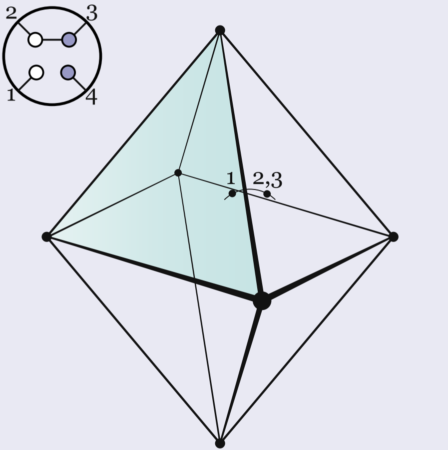

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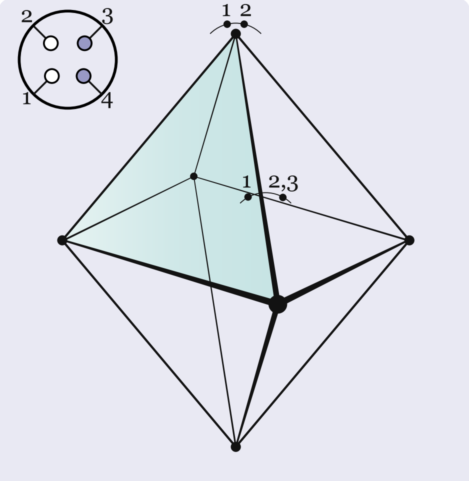
$$\begin{aligned}\lambda_4 &\longrightarrow \alpha_{4\,1}\lambda_1\\ \lambda_4 &\longrightarrow \alpha_{4\,3}\lambda_3\\ \lambda_3 &\longrightarrow \alpha_{3\,2}\lambda_2\\ \lambda_3 &\longrightarrow \alpha_{3\,1}\lambda_1\end{aligned}$$



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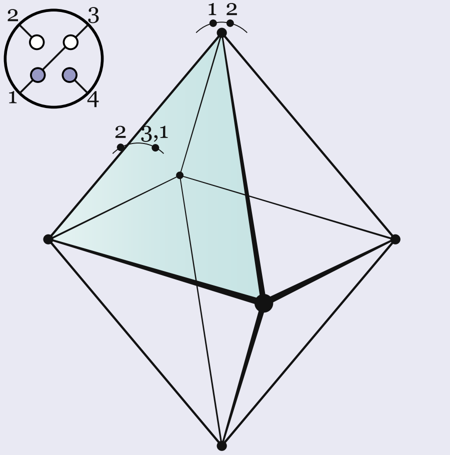

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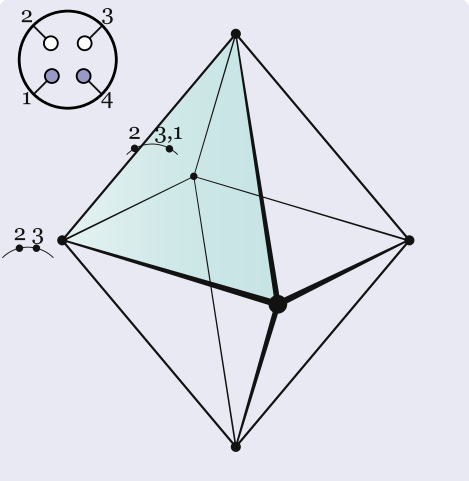

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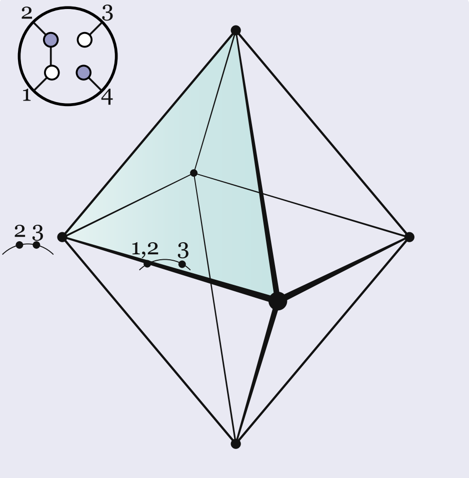

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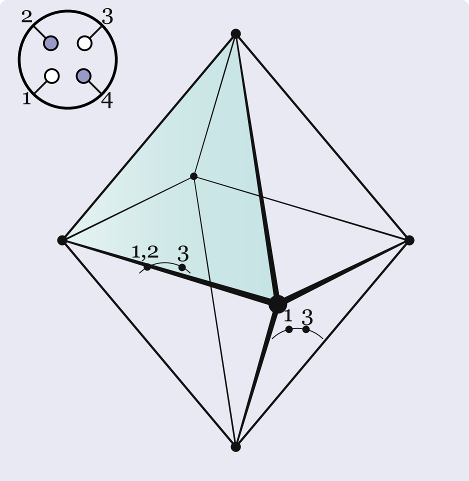

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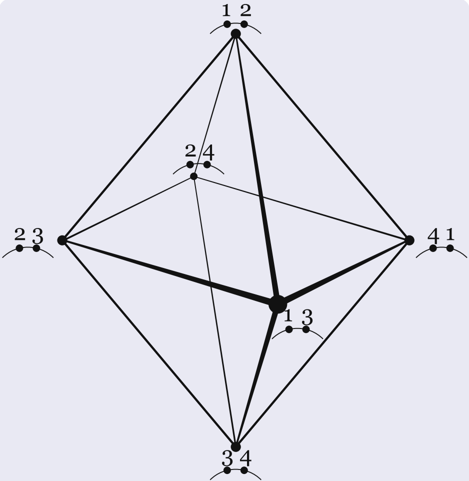

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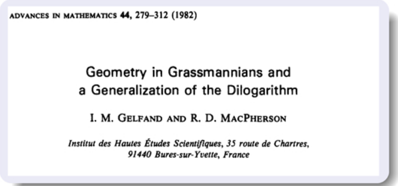
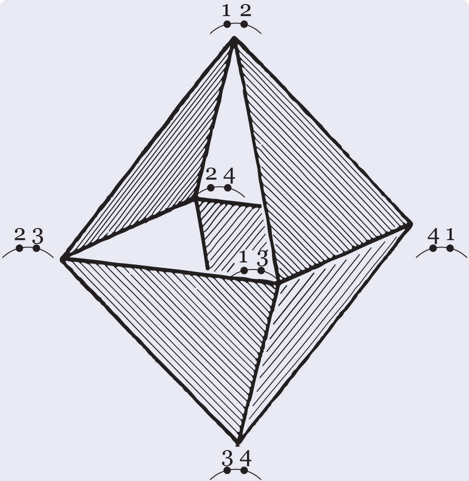


ADVANCES IN MATHEMATICS **44**, 279–312 (1982)

Geometry in Grassmannians and
a Generalization of the Dilogarithm

I. M. GELFAND AND R. D. MACPHERSON

Institut des Hautes Études Scientifiques, 35 route de Chartres,
91440 Bures-sur-Yvette, France



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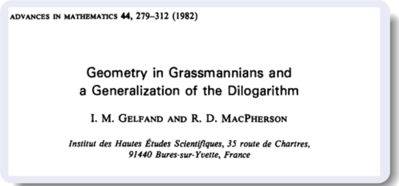

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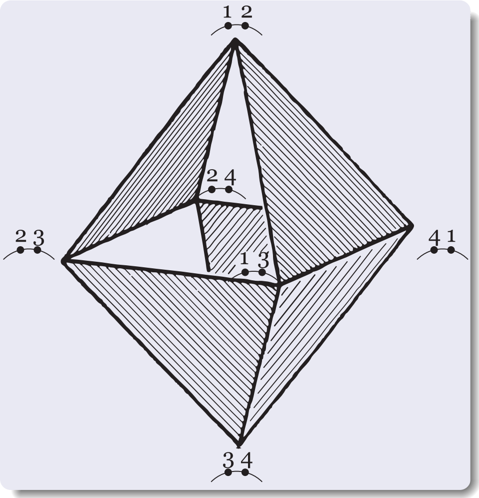

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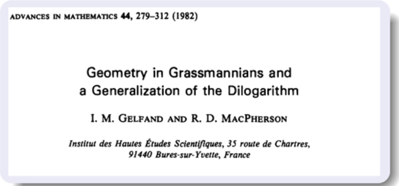

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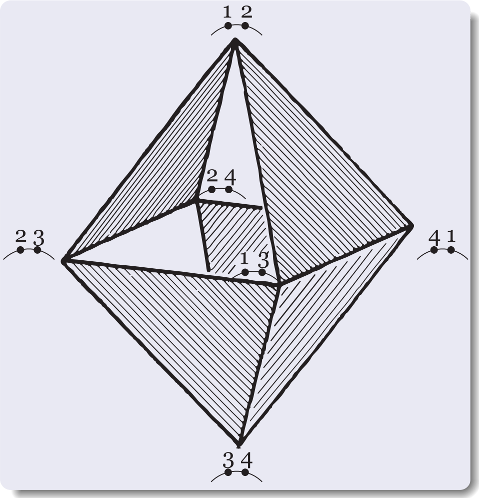

Parke-Taylor 'Amplitudes' and Grassmannian Residues

Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 2 negative-helicity gluons

$$\mathcal{A}_n^{(2)} = \frac{\delta^{2\times4}\big(\lambda\cdot\widetilde{\eta}\big)\,\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\cdots\langle n\,1\rangle}$$

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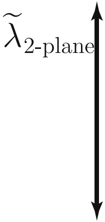

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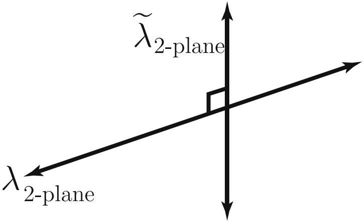

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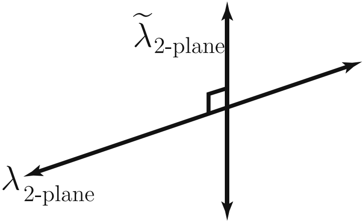
Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with m negative-helicity gluons:

$$\mathcal{A}_n^{(m)} \overset{?}{=} \frac{\delta^{m\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{m\times2}\big(C\cdot\widetilde{\lambda}\big)}{\langle1\,\cdots\,m\rangle\langle2\,\cdots\,m{+}1\rangle\cdots\langle n\,\cdots\,m{-}1\rangle}$$

$$C\equiv\begin{pmatrix} c_1^1 & c_2^1 & c_3^1 & \cdots & c_n^1\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ c_1^m & c_2^m & c_3^m & \cdots & c_n^m\end{pmatrix}$$



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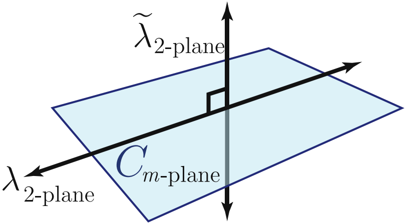

In order for momentum conservation, $\delta^{2\times 2}\big(\lambda\cdot\widetilde{\lambda}\big)$, to be part of the constraints, we must have that $C \supset \lambda$

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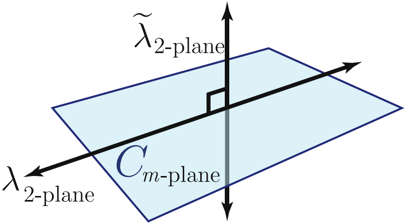

In order for momentum conservation, $\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)$, to be part of the constraints, we must have that $C\supset\lambda$, imposed via $\delta^{2\times(n-m)}\big(\lambda\cdot C^{\perp}\big)$

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$$\mathcal{A}_6^{(3)} \overset{?}{=} \frac{\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})}{\langle 1\,2\,3\rangle\langle 2\,3\,4\rangle\langle 3\,4\,5\rangle\langle 4\,5\,6\rangle\langle 5\,6\,1\rangle\langle 6\,1\,2\rangle}$$

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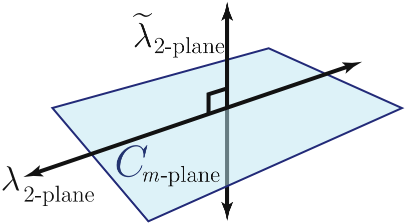

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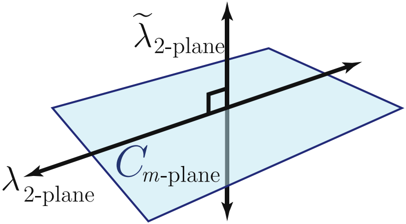

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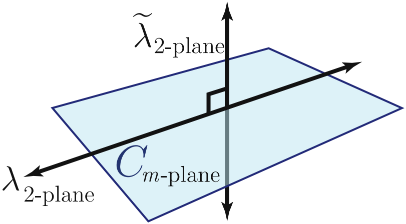
$$\dim(C) = 3\times6 - 3\times3 = 9$$



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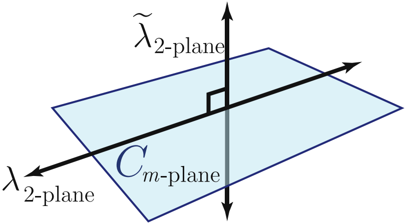
$$C \equiv \begin{pmatrix} c_1^1 & c_2^1 & c_3^1 & c_4^1 & c_5^1 & c_6^1 \\ c_1^2 & c_2^2 & c_3^2 & c_4^2 & c_5^2 & c_6^2 \\ c_1^3 & c_2^3 & c_3^3 & c_4^3 & c_5^3 & c_6^3 \end{pmatrix}$$

$$\dim(C) = 3\times 6 - 3\times 3 = 9 = 8 + 1$$



Parke-Taylor 'Amplitudes' and Grassmannian Residues

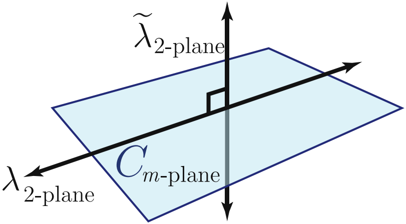
Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 3 negative-helicity gluons—e.g.,

$$\mathcal{A}_6^{(3)} \overset{?}{=} \frac{\delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{3\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times3}(\lambda\cdot C^{\perp})}{\langle 1\,2\,3\rangle\langle 2\,3\,4\rangle\langle 3\,4\,5\rangle\langle 4\,5\,6\rangle\langle 5\,6\,1\rangle\langle 6\,1\,2\rangle}$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ c_1^3 & c_2^3 & c_3^3 & c_4^3 & c_5^3 & c_6^3 \end{pmatrix}$$

$$\dim(C) = 3\times6 - 3\times3 = 9 = 8 + 1$$



Parke-Taylor ‘Amplitudes’ and Grassmannian Residues

Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint \frac{d\tau}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)} \delta^{3\times4}(C\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ c_1^3 & c_2^3 & c_3^3 & c_4^3 & c_5^3 & c_6^3 \end{pmatrix}$$

$$\dim(C) = 3\times 6 - 3\times 3 = 9 = 8 + 1$$

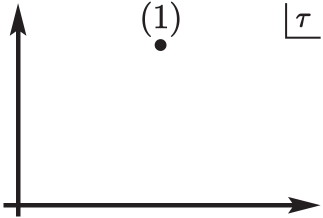
Parke-Taylor 'Amplitudes' and Grassmannian Residues

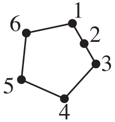
Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint_{\langle 123\rangle = 0} \frac{d\tau \qquad \delta^{3\times 4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & c_4^3 & c_5^3 & c_6^3 \end{pmatrix}$$





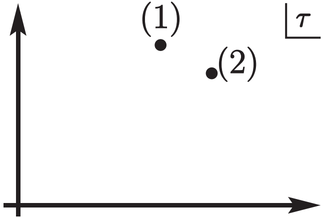
Parke-Taylor 'Amplitudes' and Grassmannian Residues

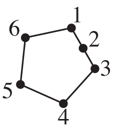
Recall the natural desire to generalize the Parke-Taylor formula according to:

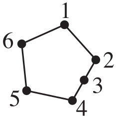
Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint_{\langle 234\rangle=0}\frac{d\tau\qquad\delta^{3\times4}(C\cdot\widetilde{\eta})\qquad\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

$$C\equiv\begin{pmatrix}\lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2\\ c_1^3 & 0 & 0 & 0 & c_5^3 & c_6^3\end{pmatrix}$$







Parke-Taylor 'Amplitudes' and Grassmannian Residues

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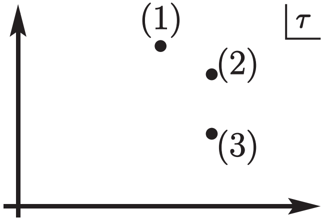
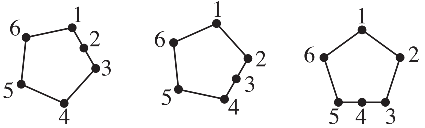
Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint\limits_{\langle 345\rangle=0} \frac{d\tau \qquad \delta^{3\times4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ c_1^3 & c_2^3 & 0 & 0 & 0 & c_6^3 \end{pmatrix}$$





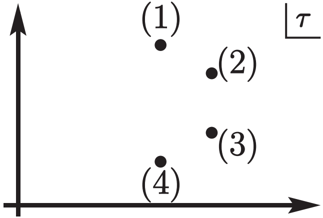
Parke-Taylor 'Amplitudes' and Grassmannian Residues

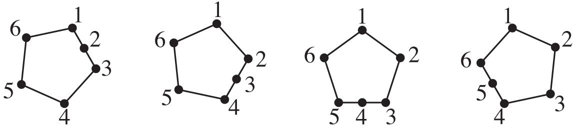
Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint_{\langle 456\rangle=0} \frac{d\tau \qquad \delta^{3\times 4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ c_1^3 & c_2^3 & c_3^3 & 0 & 0 & 0 \end{pmatrix}$$





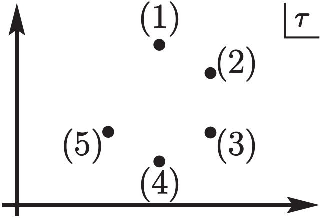
Parke-Taylor 'Amplitudes' and Grassmannian Residues

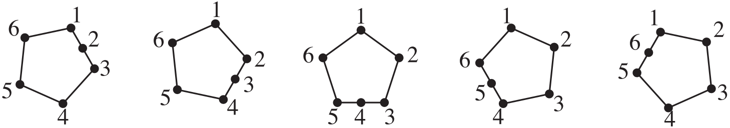
Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint_{\langle 561\rangle=0}\frac{d\tau\qquad\delta^{3\times4}(C\cdot\widetilde{\eta})\qquad\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle123\rangle(\tau)\cdot\langle234\rangle(\tau)\cdot\langle345\rangle(\tau)\cdot\langle456\rangle(\tau)\cdot\langle561\rangle(\tau)\cdot\langle612\rangle(\tau)}$$

$$C\equiv\begin{pmatrix}\lambda_1^1&\lambda_2^1&\lambda_3^1&\lambda_4^1&\lambda_5^1&\lambda_6^1\\\lambda_1^2&\lambda_2^2&\lambda_3^2&\lambda_4^2&\lambda_5^2&\lambda_6^2\\0&c_2^3&c_3^3&c_4^3&0&0\end{pmatrix}$$





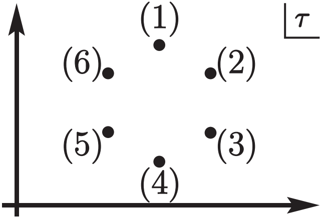
Parke-Taylor 'Amplitudes' and Grassmannian Residues

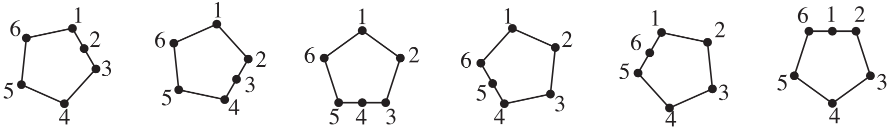
Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint\limits_{\langle 612\rangle=0}\frac{d\tau\qquad\delta^{3\times4}(C\cdot\widetilde{\eta})\qquad\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

$$C\equiv\begin{pmatrix}\lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2\\ 0 & 0 & c_3^3 & c_4^3 & c_5^3 & 0\end{pmatrix}$$





Parke-Taylor 'Amplitudes' and Grassmannian Residues

Recall the natural desire to generalize the Parke-Taylor formula according to:

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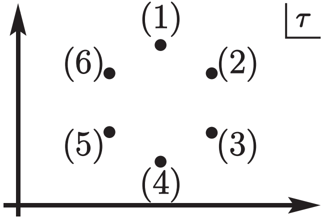
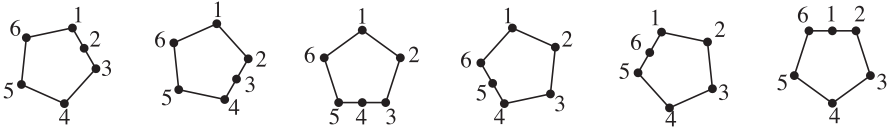
$$\oint \frac{d\tau \qquad \delta^{3\times4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle123\rangle(\tau)\cdot\langle234\rangle(\tau)\cdot\langle345\rangle(\tau)\cdot\langle456\rangle(\tau)\cdot\langle561\rangle(\tau)\cdot\langle612\rangle(\tau)}$$

$$C \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ c_1^3 & c_2^3 & c_3^3 & c_4^3 & c_5^3 & c_6^3 \end{pmatrix}$$





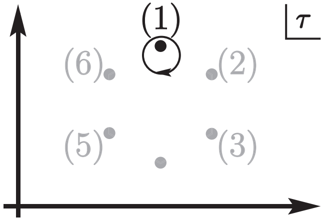
Parke-Taylor 'Amplitudes' and Grassmannian Residues

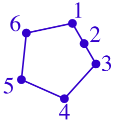
Recall the natural desire to generalize the Parke-Taylor formula according to:

Amplitudes with 3 negative-helicity gluons—e.g.,

$$\oint_{\langle 123\rangle=0}\frac{d\tau\qquad\delta^{3\times4}(C\cdot\widetilde{\eta})\qquad\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

$$C\equiv\begin{pmatrix}\lambda_1^1&\lambda_2^1&\lambda_3^1&\lambda_4^1&\lambda_5^1&\lambda_6^1\\\lambda_1^2&\lambda_2^2&\lambda_3^2&\lambda_4^2&\lambda_5^2&\lambda_6^2\\0&0&0&c_4^3&c_5^3&c_6^3\end{pmatrix}$$





Parke-Taylor 'Amplitudes' and Grassmannian Residues

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Amplitudes with 3 negative-helicity gluons—e.g.,

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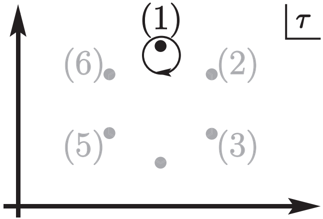
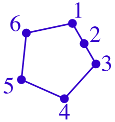
$$(1)\Leftrightarrow\begin{pmatrix}\lambda_1^1&\lambda_2^1&\lambda_3^1&\lambda_4^1&\lambda_5^1&\lambda_6^1\\\lambda_1^2&\lambda_2^2&\lambda_3^2&\lambda_4^2&\lambda_5^2&\lambda_6^2\\0&0&0&[5\,6]&[6\,4]&[4\,5]\end{pmatrix}$$





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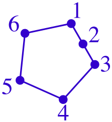
$$\frac{1}{}$$



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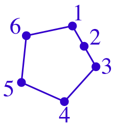
$$(1)\Leftrightarrow\left(\begin{array}{c|ccc|cc}\lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2\\ 0 & 0 & 0 & [5\,6] & [6\,4] & [4\,5]\end{array}\right)$$

$$\frac{1}{\langle2\,3\rangle[5\,6]}$$



Parke-Taylor 'Amplitudes' and Grassmannian Residues

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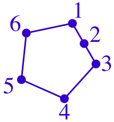
$$(1)\Leftrightarrow\left(\begin{array}{cc|ccc|c}\lambda_1^1&\lambda_2^1&\lambda_3^1&\lambda_4^1&\lambda_5^1&\lambda_6^1\\\lambda_1^2&\lambda_2^2&\lambda_3^2&\lambda_4^2&\lambda_5^2&\lambda_6^2\\0&0&0&[5\,6]&[6\,4]&[4\,5]\end{array}\right)$$

$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle}$$



Parke-Taylor 'Amplitudes' and Grassmannian Residues

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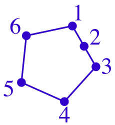
$$(1)\Leftrightarrow\left(\begin{array}{ccc|ccc} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [5\,6] & [6\,4] & [4\,5] \end{array}\right)$$

$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}}$$



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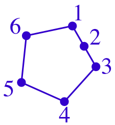
$$(1)\Leftrightarrow\left(\begin{array}{c|ccc|cc}\lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2\\ 0 & 0 & 0 & [5\,6] & [6\,4] & [4\,5]\end{array}\right)$$

$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}\langle 1|(6+5)|4]}$$



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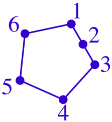
$$\oint_{\langle 123\rangle=0} \frac{d\tau \qquad \delta^{3\times4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

$$(1)\Leftrightarrow\left(\begin{array}{cc|ccc|c}\lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2\\ 0 & 0 & 0 & [5\,6] & [6\,4] & [4\,5]\end{array}\right)$$



$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}\langle 1|(6+5)|4][4\,5]\langle 1\,2\rangle}$$

Parke-Taylor 'Amplitudes' and Grassmannian Residues

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$$\oint_{\langle 123\rangle = 0} \frac{d\tau \qquad \delta^{3\times4}(C\cdot\widetilde{\eta}) \qquad \delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 123\rangle(\tau)\cdot\langle 234\rangle(\tau)\cdot\langle 345\rangle(\tau)\cdot\langle 456\rangle(\tau)\cdot\langle 561\rangle(\tau)\cdot\langle 612\rangle(\tau)}$$

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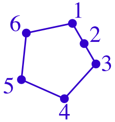

$$\frac{1}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}\langle 1|(6+5)|4][4\,5]\langle 1\,2\rangle}$$

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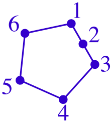

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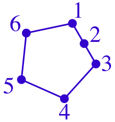
$$\mathcal{A}_6^{(3)}(+,-,+,-,+,-)=(1+r^2+r^4)\frac{\langle 2|(4+6)|5]^4}{\langle 2\,3\rangle[5\,6][6|(5+4)|3\rangle s_{4\,5\,6}\langle 1|(6+5)|4][4\,5]\langle 1\,2\rangle}$$



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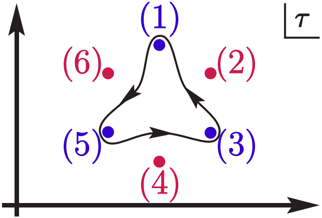
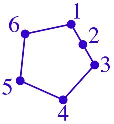
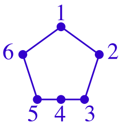
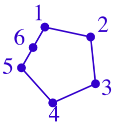


$$\mathcal{A}_6^{(3)}(+,-,+,-,+,-) = (1)+(3)+(5)$$







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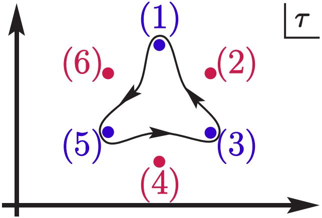

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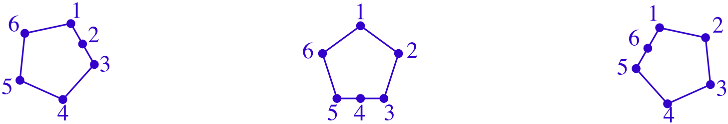

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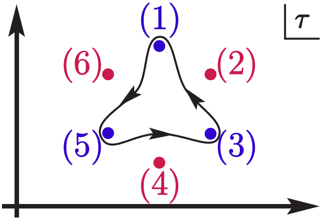

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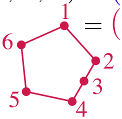

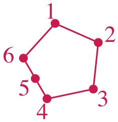

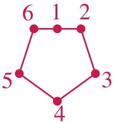

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A complete, $GL(k)$-invariant description of any contour of $\mathcal{L}_{n,k}$ would be a list of all the ranks of spaces spanned by all consecutive chains of columns.

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Ranks of Consecutive Chains of Columns

$$\begin{array}{c}
|1\cdots n{-}1| \\
|1\cdots n{-}2| \\
\vdots \\
|1\ \cdots\ 2| \\
|1\ \cdots\ 1|
\end{array}$$

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Ranks of Consecutive Chains of Columns

$|1 \;\cdots\; n|$

$|1 \cdots n{-}1|$

$|1 \cdots n{-}2|$

$\vdots$

$|1 \;\cdots\; 2|$

$|1 \;\cdots\; 1|$

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Ranks of Consecutive Chains of Columns

	$\lvert 2\cdots n{+}1\rvert$
$\lvert 1\ \cdots\ n\ \rvert$	$\lvert 2\ \cdots\ n\ \rvert$
$\lvert 1\cdots n{-}1\rvert$	$\lvert 2\ \cdots n{-}1\rvert$
$\lvert 1\cdots n{-}2\rvert$	$\lvert 2\ \cdots n{-}2\rvert$
$\vdots$	$\vdots$
$\lvert 1\ \cdots\ 2\rvert$	$\lvert 2\ \cdots\ 2\rvert$
$\lvert 1\ \cdots\ 1\rvert$	

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Ranks of Consecutive Chains of Columns

$$\begin{array}{ccccc}
 & & & & |n\cdots 2n| \\
 & |2\cdots n{+}1| & & \iddots & \vdots \\
|1\cdots n| & |2\cdots n| & \iddots & |n{-}1\cdots n| & |n\cdots n| \\
|1\cdots n{-}1| & |2\cdots n{-}1| & \iddots & |n{-}1\cdots n{-}1| & \\
|1\cdots n{-}2| & |2\cdots n{-}2| & \iddots & & \\
\vdots & \vdots & \iddots & & \\
|1\cdots 2| & |2\cdots 2| & & & \\
|1\cdots 1| & & & &
\end{array}$$

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Ranks of Consecutive Chains of Columns

$$\begin{array}{ccccc}
|1\cdots 2n| & |2\cdots 2n| & \cdots & |n{-}1\cdots 2n| & |n\cdots 2n| \\
\vdots & \iddots & \iddots & \vdots & \vdots \\
|1\cdots n| & |2\cdots n| & \iddots & |n{-}1\cdots n| & |n\cdots n| \\
|1\cdots n{-}1| & |2\cdots n{-}1| & \iddots & |n{-}1\cdots n{-}1| & \\
|1\cdots n{-}2| & |2\cdots n{-}2| & \iddots & & \\
\vdots & \vdots & \iddots & & \\
|1\cdots 2| & |2\cdots 2| & & & \\
|1\cdots 1| & & & &
\end{array}$$

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Ranks of Consecutive Chains of Columns

$$\begin{array}{ccccc}
|1\cdots 2n| & |2\cdots 2n| & \cdots & |n{-}1\cdots 2n| & |n\cdots 2n| \\
\vdots & \iddots & \iddots & \vdots & \vdots \\
|1\cdots n| & |2\cdots n| & \iddots & |n{-}1\cdots n| & |n\cdots n| \\
|1\cdots n{-}1| & |2\cdots n{-}1| & \iddots & |n{-}1\cdots n{-}1| & \\
|1\cdots n{-}2| & |2\cdots n{-}2| & \iddots & & \\
\vdots & \vdots & \iddots & & \\
|1\cdots 2| & |2\cdots 2| & & & \\
|1\cdots 1| & & & &
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Ranks of Consecutive Chains of Columns

$$\begin{array}{ccccc}
k & k & \cdots & k & k \\
\vdots & \iddots & \iddots & \vdots & \vdots \\
k & |2\;\cdots\;n\,| & \iddots & |n{-}1\;\cdots\;n\,| & |\,n\;\cdots\;n\,| \\
|1\cdots n{-}1| & |2\cdots n{-}1| & \iddots & |n{-}1\cdots n{-}1| & \\
|1\cdots n{-}2| & |2\cdots n{-}2| & \iddots & & \\
\vdots & \vdots & \iddots & & \\
|1\;\cdots\;2| & |2\;\cdots\;2| & & & \\
|1\;\cdots\;1| & & & &
\end{array}$$

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Ranks of Consecutive Chains of Columns

k	k	$\cdots$	k	k
$\vdots$	$\iddots$	$\iddots$	$\vdots$	$\vdots$
k	$\lvert 2\ \cdots\ n\rvert$	$\iddots$	$\lvert n{-}1\ \cdots\ n\rvert$	$\lvert n\ \cdots\ n\rvert$
$\lvert 1\cdots n{-}1\rvert$	$\lvert 2\cdots n{-}1\rvert$	$\iddots$	$\lvert n{-}1\cdots n{-}1\rvert$	
$\lvert 1\cdots n{-}2\rvert$	$\lvert 2\cdots n{-}2\rvert$	$\iddots$		
$\vdots$	$\vdots$	$\iddots$		
$\lvert 1\ \cdots\ 2\rvert$	$\lvert 2\ \cdots\ 2\rvert$			
$\lvert 1\ \cdots\ 1\rvert$				

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Ranks of Consecutive Chains of Columns

k	k	$\cdots$	k	k
$\vdots$	$\iddots$	$\iddots$	$\vdots$	$\vdots$
k	$\lvert 2\ \cdots\ n\rvert$	$\iddots$	$\lvert n{-}1\ \cdots\ n\rvert$	$\lvert n\ \cdots\ n\rvert$
$\lvert 1\cdots n{-}1\rvert$	$\lvert 2\cdots n{-}1\rvert$	$\iddots$	$\lvert n{-}1\cdots n{-}1\rvert$	$\lvert n\ \cdots n{-}1\rvert$
$\lvert 1\cdots n{-}2\rvert$	$\lvert 2\cdots n{-}2\rvert$	$\iddots$	$\lvert n{-}1\cdots n{-}2\rvert$	
$\vdots$	$\vdots$	$\iddots$		
$\lvert 1\ \cdots\ 2\rvert$	$\lvert 2\ \cdots\ 2\rvert$	$\iddots$		
$\lvert 1\ \cdots\ 1\rvert$	$\lvert 2\ \cdots\ 1\rvert$			
$\lvert 1\ \cdots\ 0\rvert$				

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- let us conventionally declare $|a\ \cdots\ a{-}1| \to 0$

Ranks of Consecutive Chains of Columns

$$\begin{array}{ccccc}
k & k & \cdots & k & k \\
\vdots & \iddots & \iddots & \vdots & \vdots \\
k & |2\ \cdots\ n\ | & \iddots & |n{-}1\ \cdots\ n\ | & |n\ \cdots\ n\ | \\
|1\cdots n{-}1| & |2\cdots n{-}1| & \iddots & |n{-}1\cdots n{-}1| & 0 \\
|1\cdots n{-}2| & |2\cdots n{-}2| & \iddots & 0 & \\
\vdots & \vdots & \iddots & & \\
|1\ \cdots\ 2| & |2\ \cdots\ 2| & \iddots & & \\
|1\ \cdots\ 1| & 0 & & & \\
0 & & & &
\end{array}$$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$

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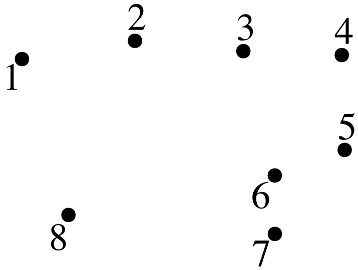

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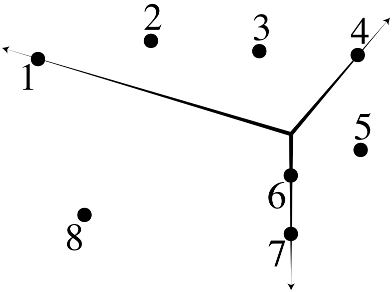

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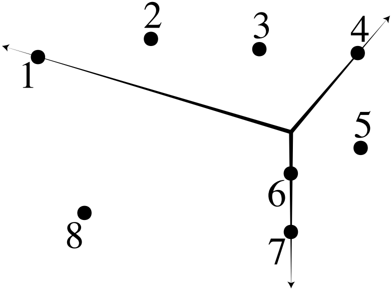

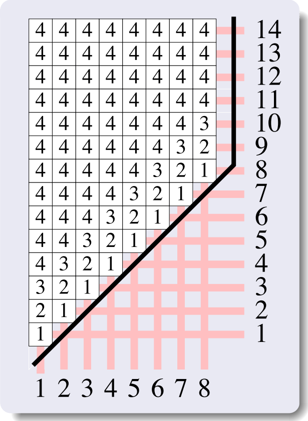

4	4	4	4	4	4	4	4	14
4	4	4	4	4	4	4	4	13
4	4	4	4	4	4	4	4	12
4	4	4	4	4	4	4	4	11
4	4	4	4	4	4	4	3	10
4	4	4	4	4	4	3	2	9
4	4	4	4	4	3	2	1	8
4	4	4	4	3	2	1		7
4	4	4	3	2	1			6
4	4	3	2	1				5
4	3	2	1					4
3	2	1						3
2	1							2
1								1
1	2	3	4	5	6	7	8	

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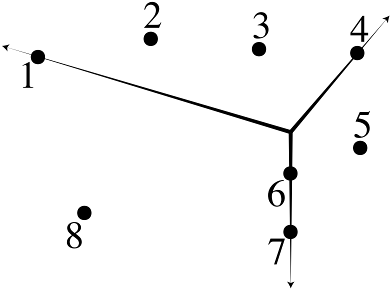

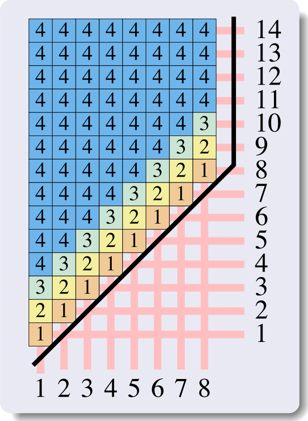

4	4	4	4	4	4	4	4	14
4	4	4	4	4	4	4	4	13
4	4	4	4	4	4	4	4	12
4	4	4	4	4	4	4	4	11
4	4	4	4	4	4	4	3	10
4	4	4	4	4	4	3	2	9
4	4	4	4	4	3	2	1	8
4	4	4	4	3	2	1		7
4	4	4	3	2	1			6
4	4	3	2	1				5
4	3	2	1					4
3	2	1						3
2	1							2
1								1
1	2	3	4	5	6	7	8	

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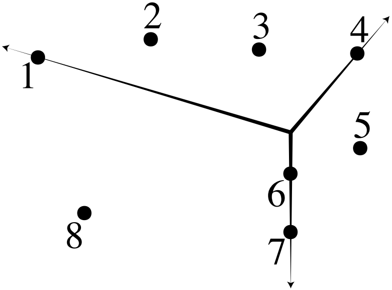
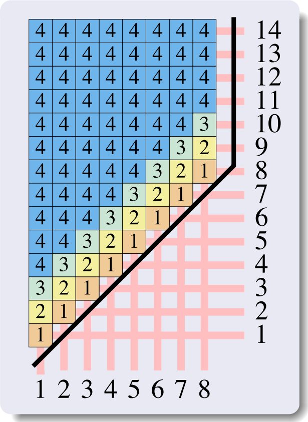
Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:





4	4	4	4	4	4	4	4	14
4	4	4	4	4	4	4	4	13
4	4	4	4	4	4	4	4	12
4	4	4	4	4	4	4	4	11
4	4	4	4	4	4	4	3	10
4	4	4	4	4	4	3	2	9
4	4	4	4	4	3	2	1	8
4	4	4	4	3	2	1		7
4	4	4	3	2	1			6
4	4	3	2	1				5
4	3	2	1					4
3	2	1						3
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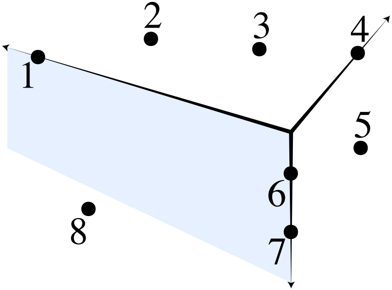

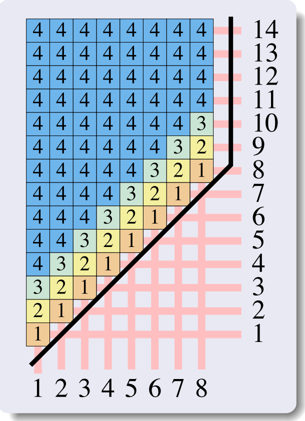

4	4	4	4	4	4	4	4	14
4	4	4	4	4	4	4	4	13
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4	4	4	4	3	2	1		7
4	4	4	3	2	1			6
4	4	3	2	1				5
4	3	2	1					4
3	2	1						3
2	1							2
1								1
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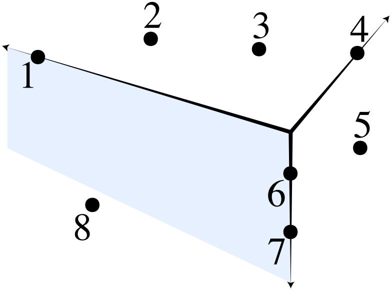

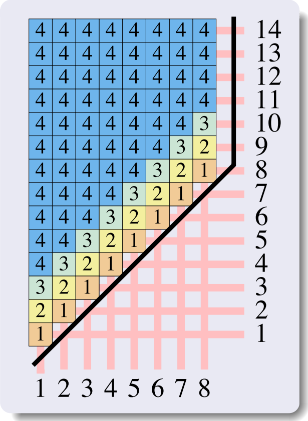

4	4	4	4	4	4	4	4	14
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4	4	4	4	3	2	1		7
4	4	4	3	2	1			6
4	4	3	2	1				5
4	3	2	1					4
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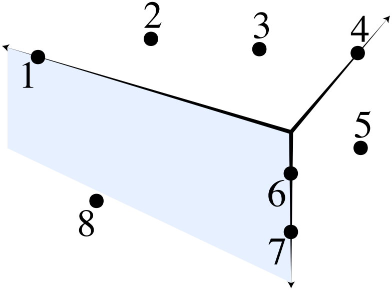

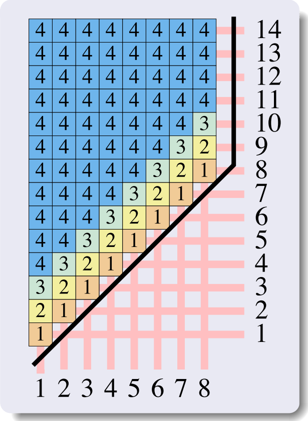

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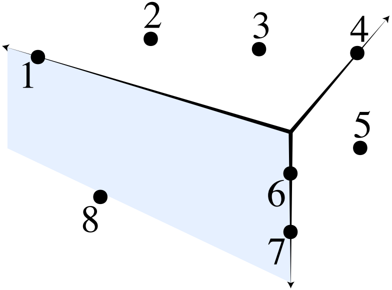

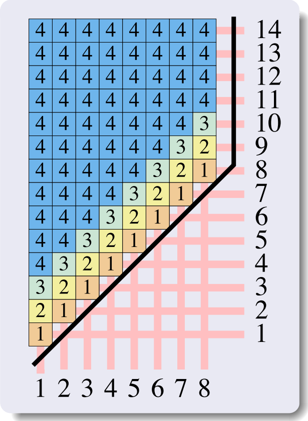

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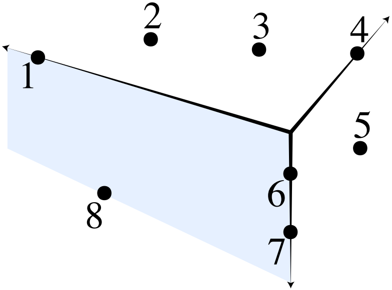

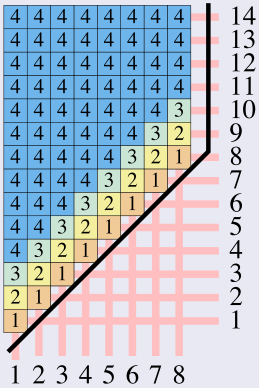

4	4	4	4	4	4	4	4	14
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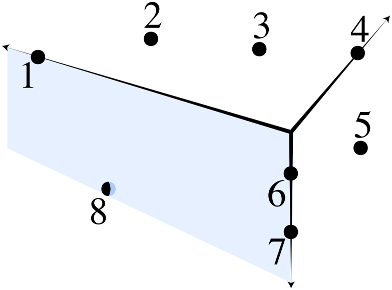

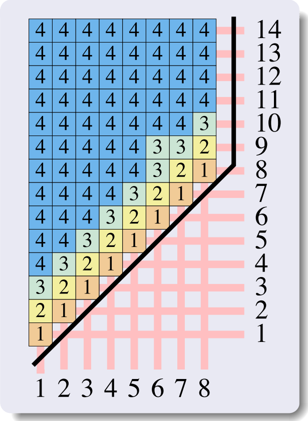

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4	4	3	2	1				5
4	3	2	1					4
3	2	1						3
2	1							2
1								1
1	2	3	4	5	6	7	8	

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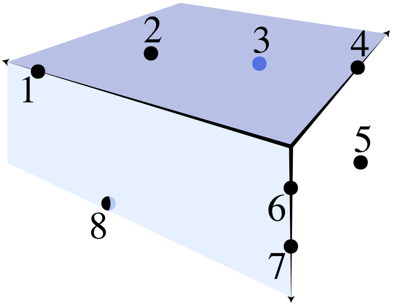

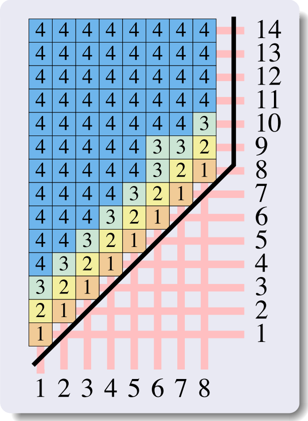

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4	3	2	1					4
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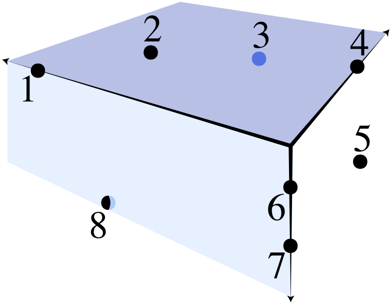

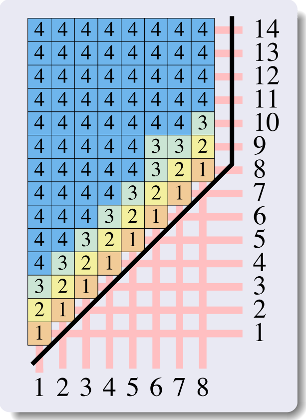

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4	3	2	1					4
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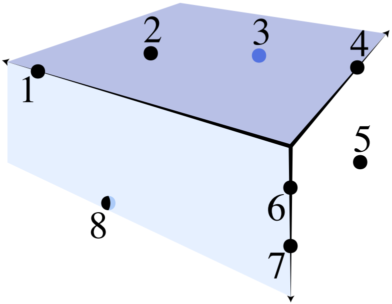

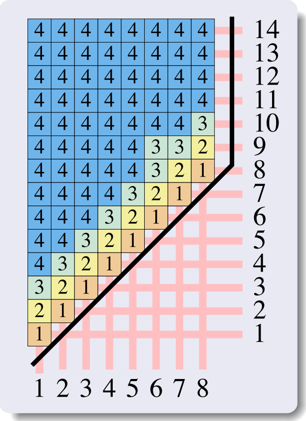

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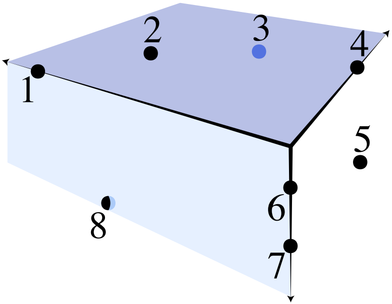

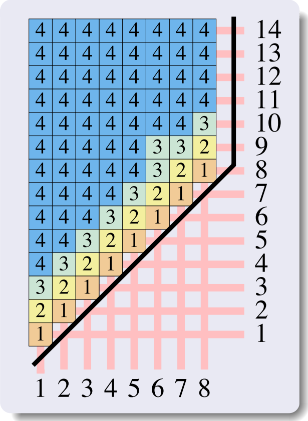

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4	4	3	2	1				5
4	3	2	1					4
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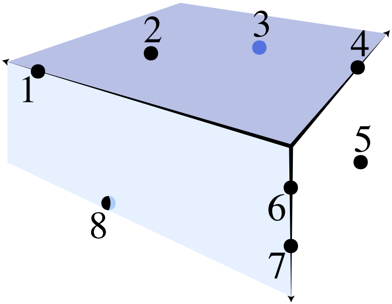

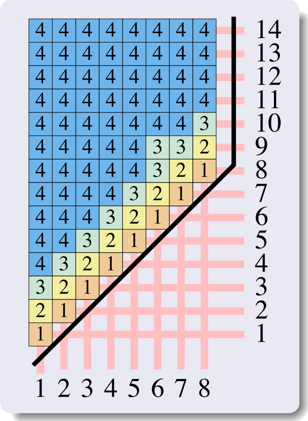

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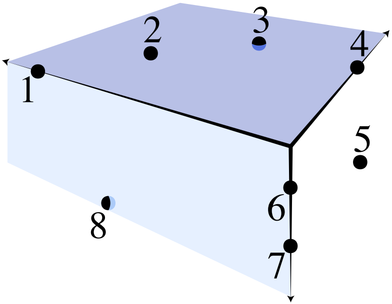

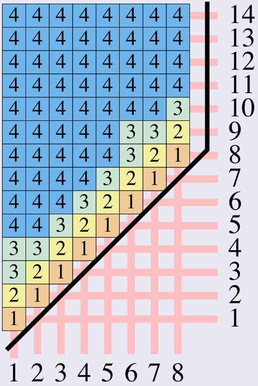

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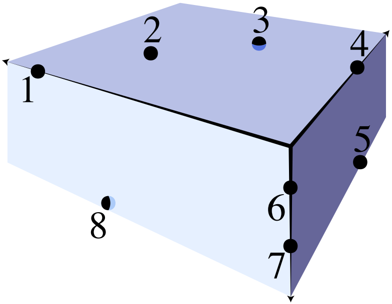

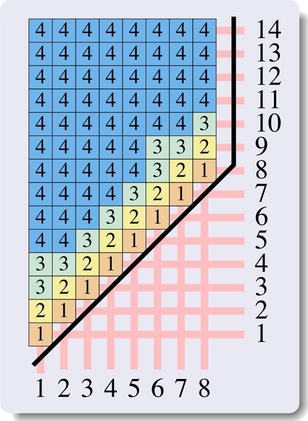

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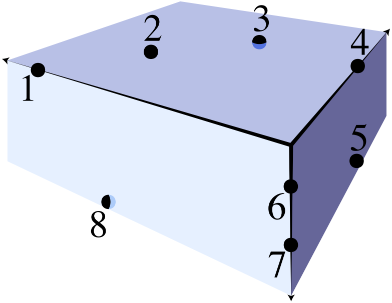

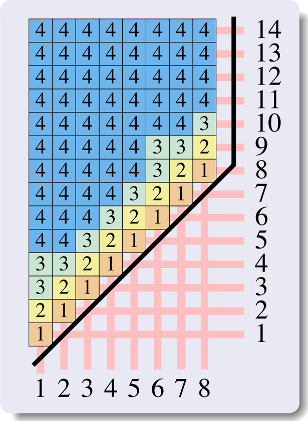

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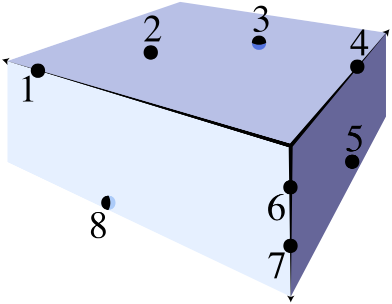

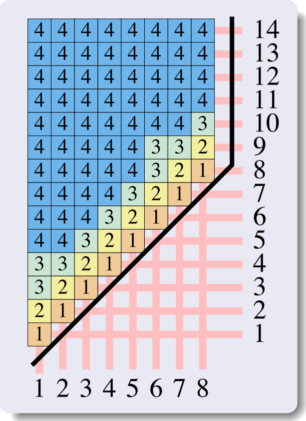

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4	4	4	3	2	1			6
4	4	3	2	1				5
3	3	2	1					4
3	2	1						3
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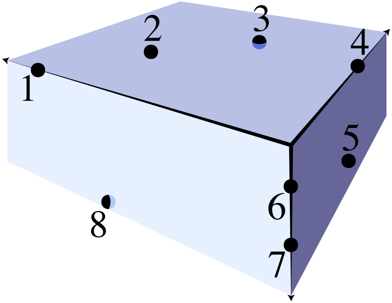

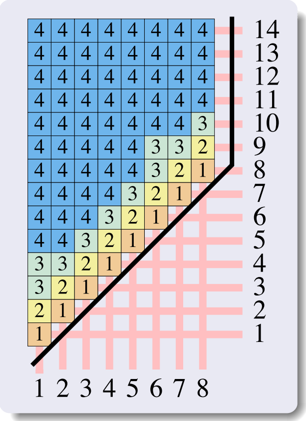

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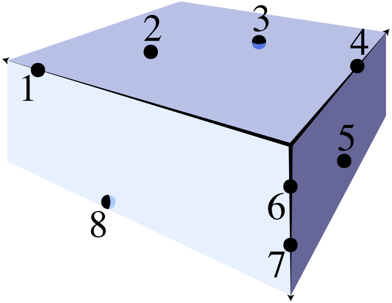

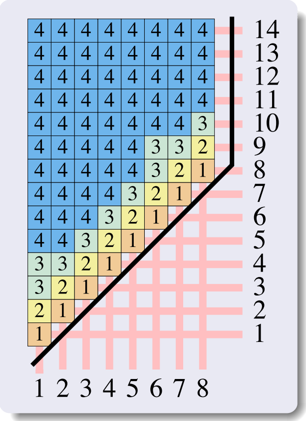

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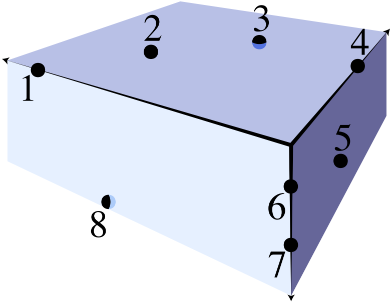

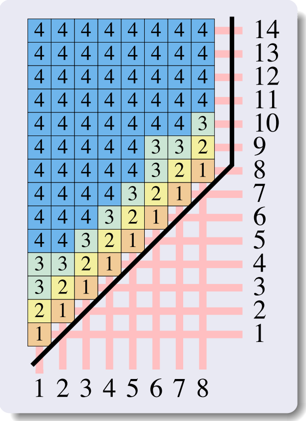

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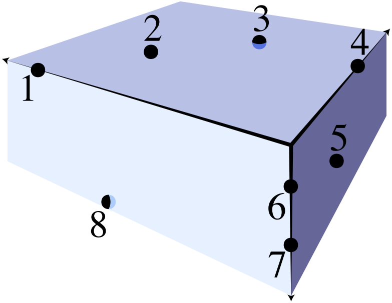

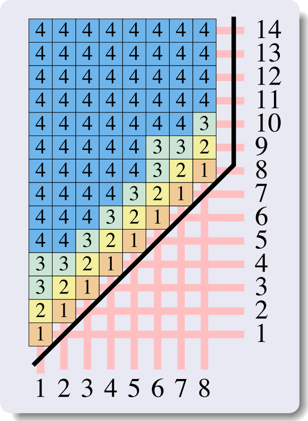

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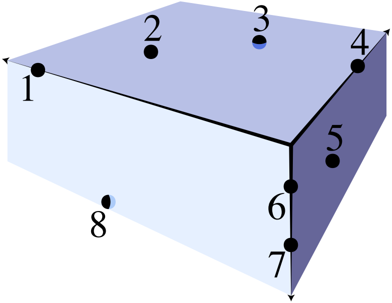

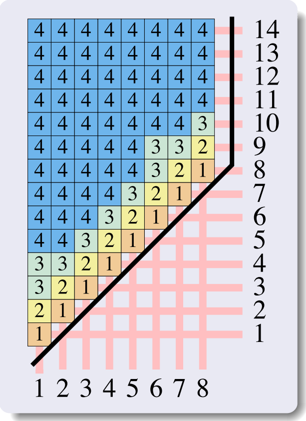

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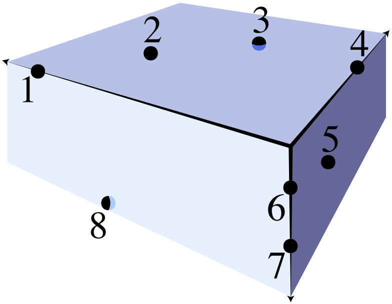

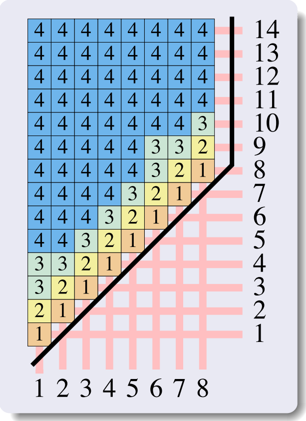

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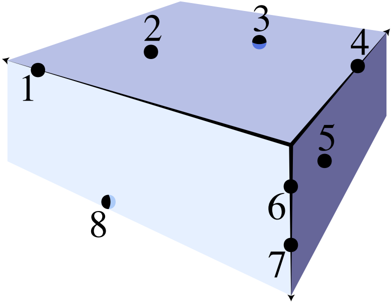
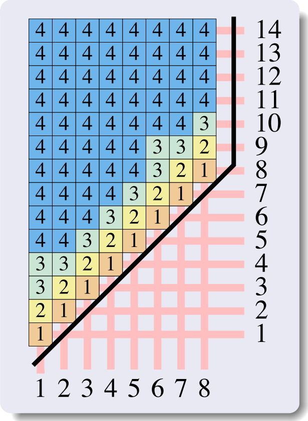
2	1							2
1								1
1	2	3	4	5	6	7	8	

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:





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4	4	4	3	2	1			6
4	4	3	2	1				5
3	3	2	1					4
3	2	1						3
2	1							2
1								1
1	2	3	4	5	6	7	8	

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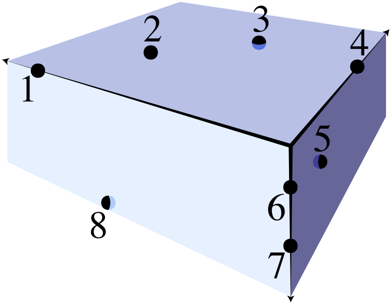

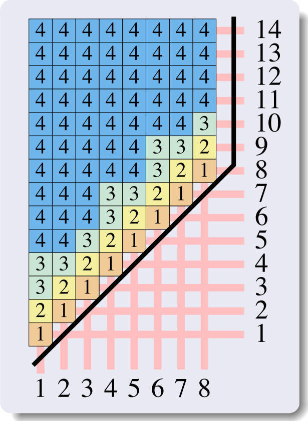

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4	4	3	2	1				5
3	3	2	1					4
3	2	1						3
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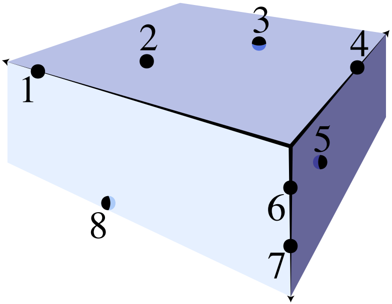

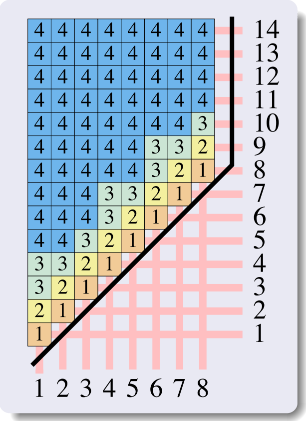

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4	4	4	3	2	1			6
4	4	3	2	1				5
3	3	2	1					4
3	2	1						3
2	1							2
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1	2	3	4	5	6	7	8	

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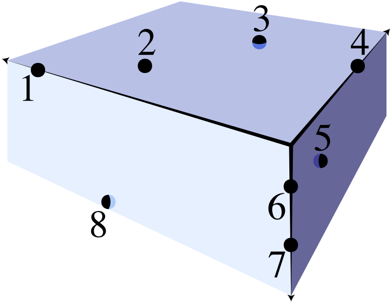

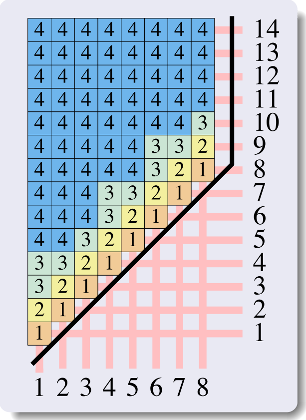

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4	4	4	3	2	1			6
4	4	3	2	1				5
3	3	2	1					4
3	2	1						3
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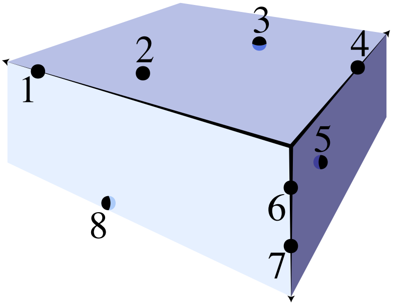

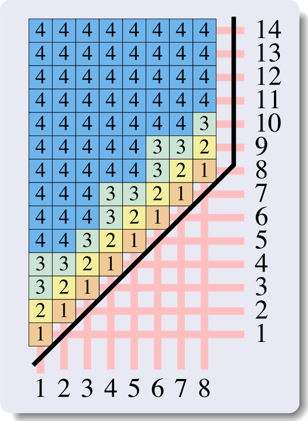

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4	4	4	3	3	2	1		7
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4	4	3	2	1				5
3	3	2	1					4
3	2	1						3
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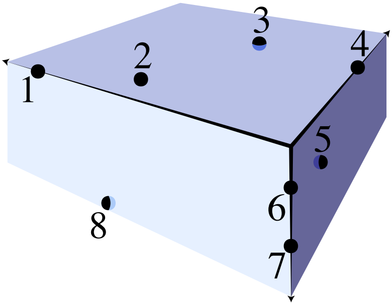

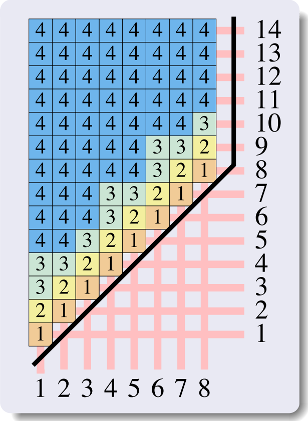

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4	4	3	2	1				5
3	3	2	1					4
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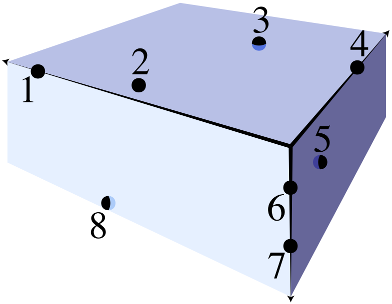

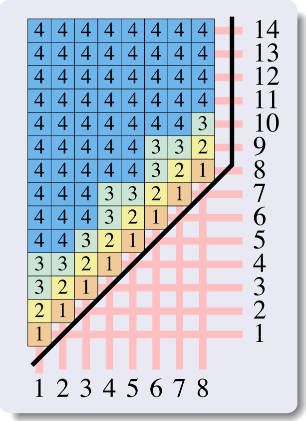

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4	4	3	2	1				5
3	3	2	1					4
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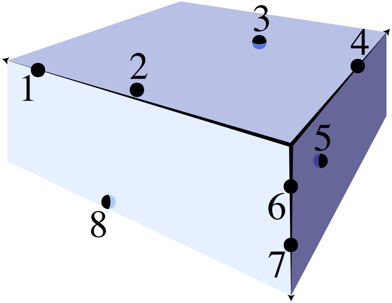

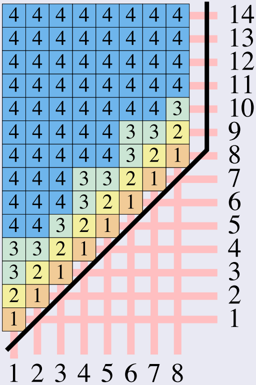

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3	3	2	1					4
3	2	1						3
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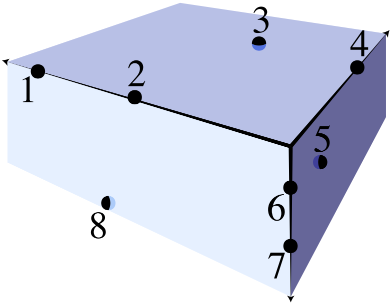

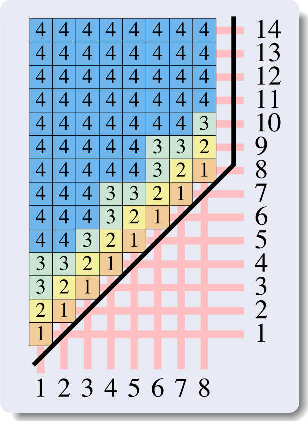

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4	4	4	4	4	3	2	1	8
4	4	4	3	3	2	1		7
4	4	4	3	2	1			6
4	4	3	2	1				5
3	3	2	1					4
3	2	1						3
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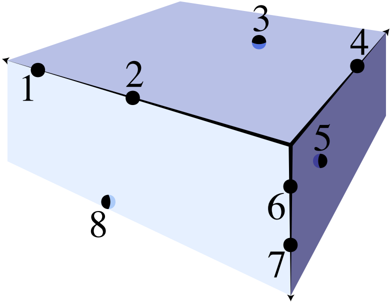

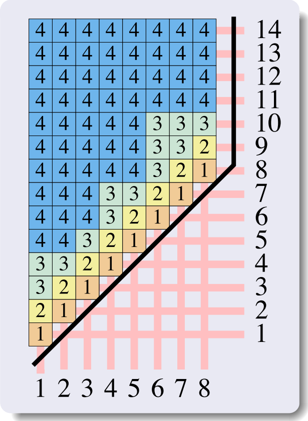

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4	4	4	4	4	3	2	1	8
4	4	4	3	3	2	1		7
4	4	4	3	2	1			6
4	4	3	2	1				5
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2	1							2
1								1
1	2	3	4	5	6	7	8	

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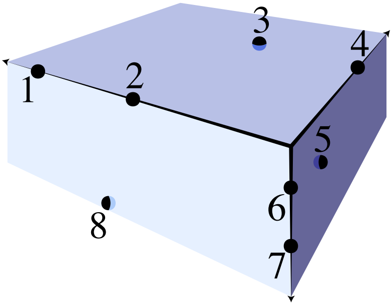

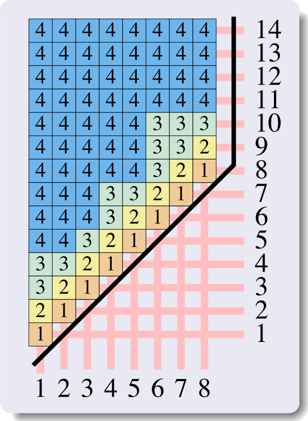

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4	4	4	3	2	1			6
4	4	3	2	1				5
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3	2	1						3
2	1							2
1								1
1	2	3	4	5	6	7	8	

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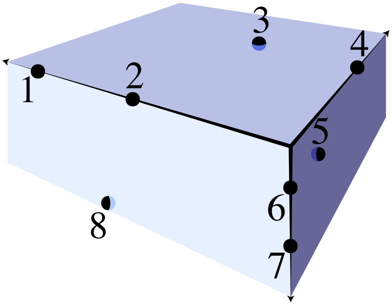

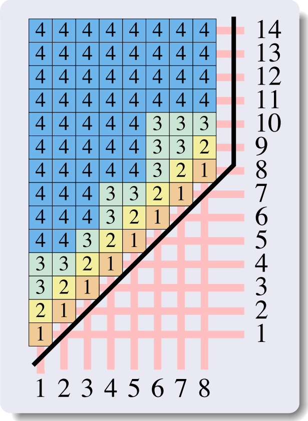

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4	4	3	2	1				5
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2	1							2
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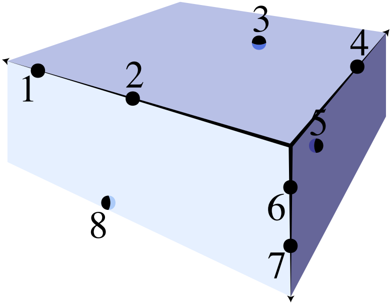

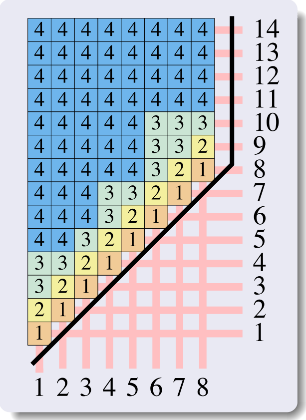

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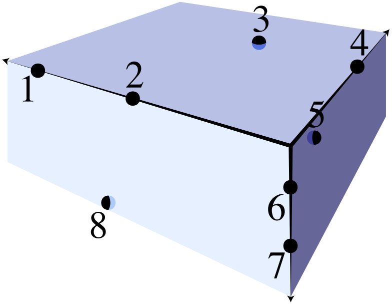

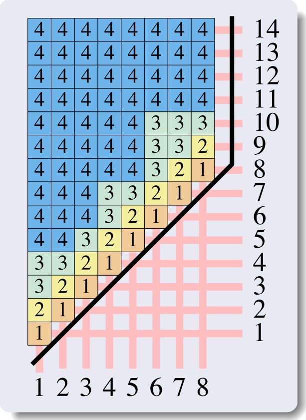

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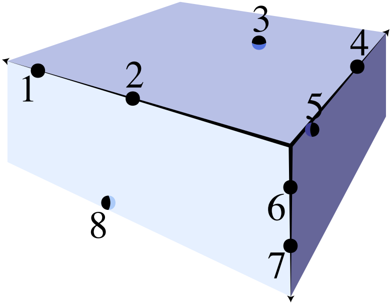

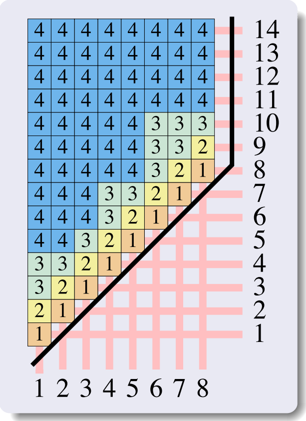

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4	4	3	2	1				5
3	3	2	1					4
3	2	1						3
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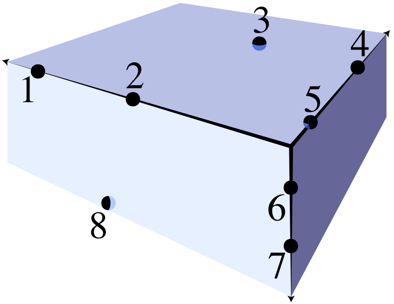

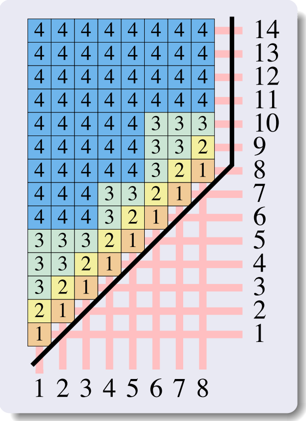

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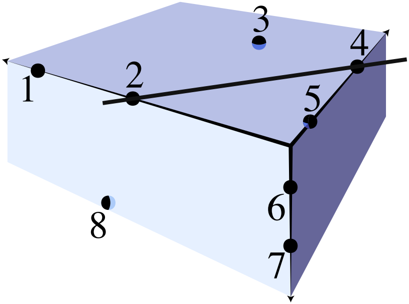
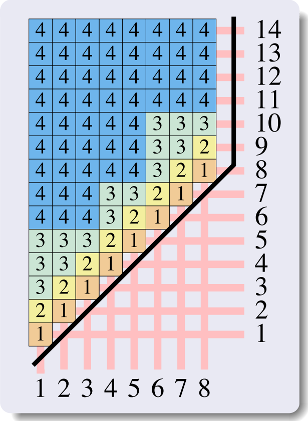
2	1							2
1								1
1	2	3	4	5	6	7	8	

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C\in G(4,8)$, $c_a\in\mathbb{P}^3$, and impose the following sequence of consecutive constraints:





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3	3	3	2	1				5
3	3	2	1					4
3	2	1						3
2	1							2
1								1
1	2	3	4	5	6	7	8	

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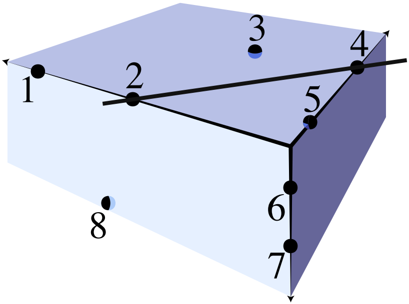

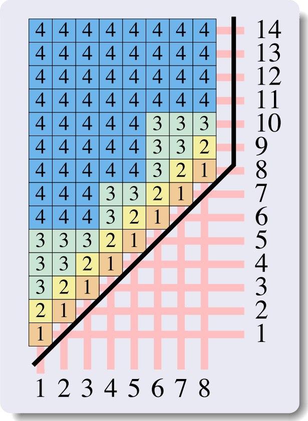

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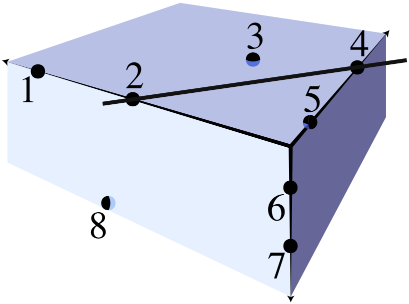

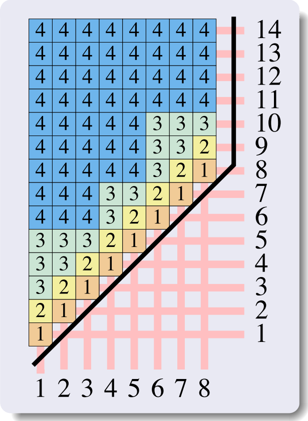

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3	3	3	2	1				5
3	3	2	1					4
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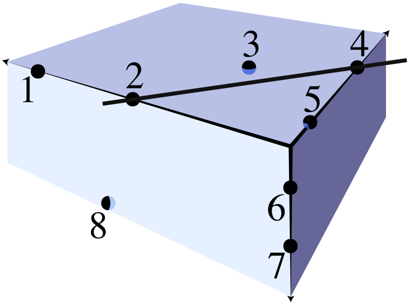

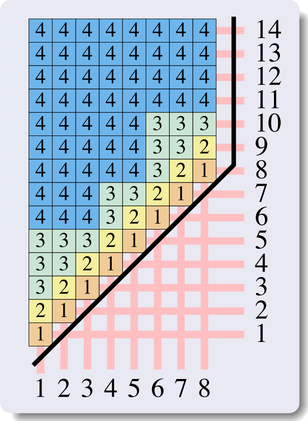

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4	4	4	3	3	2	1		7
4	4	4	3	2	1			6
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3	3	2	1					4
3	2	1						3
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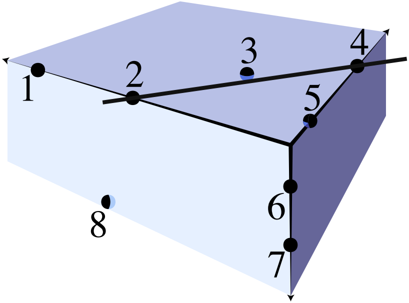

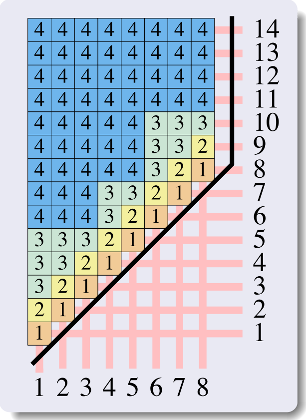

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3	3	2	1					4
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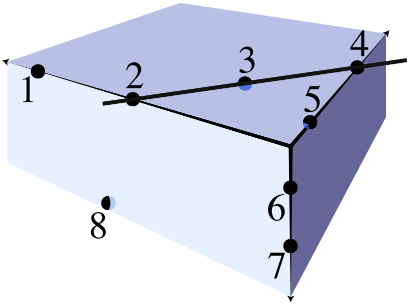

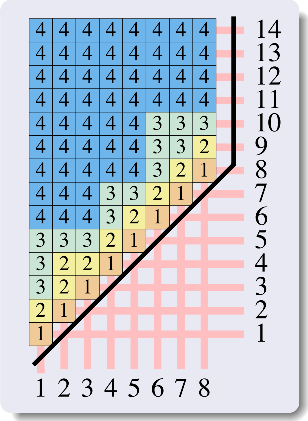

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3	3	3	2	1				5
3	2	2	1					4
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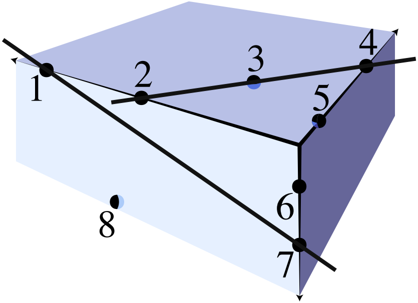

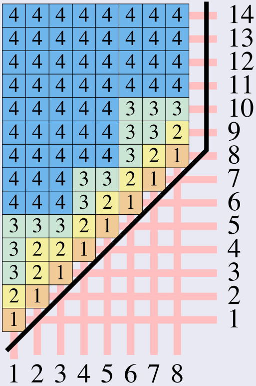

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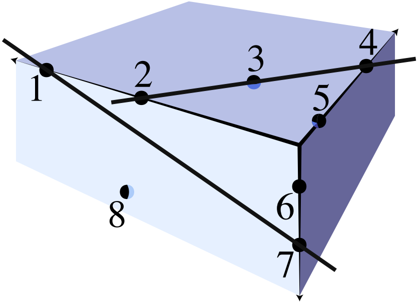

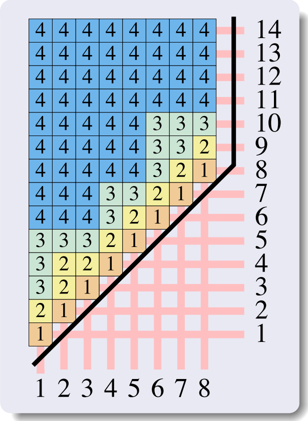

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3	3	3	2	1				5
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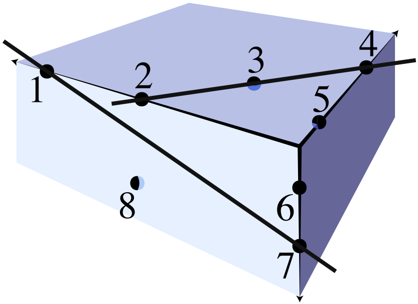

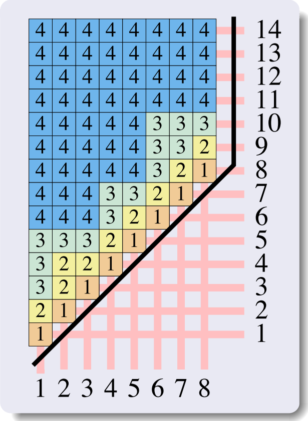

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3	3	3	2	1				5
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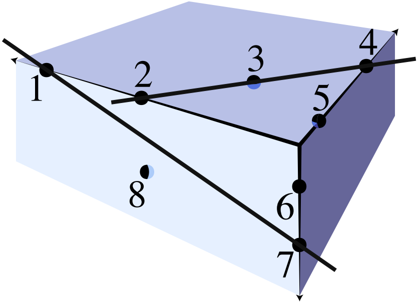

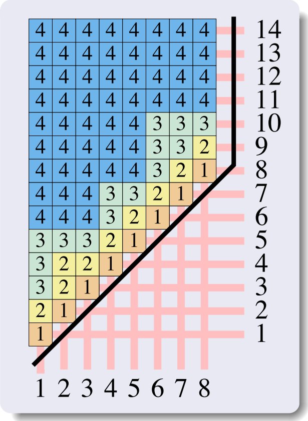

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4	4	4	4	4	3	2	1	8
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4	4	4	3	2	1			6
3	3	3	2	1				5
3	2	2	1					4
3	2	1						3
2	1							2
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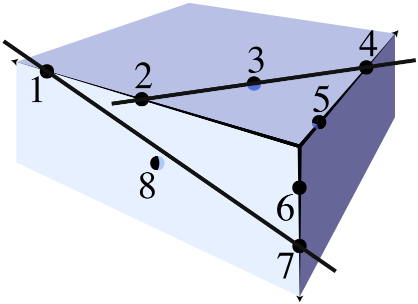

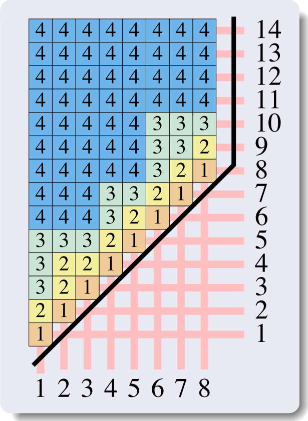

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3	2	2	1					4
3	2	1						3
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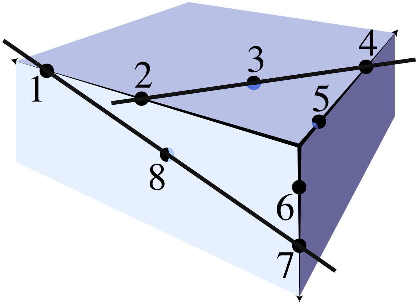

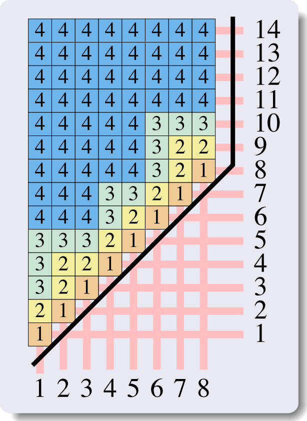

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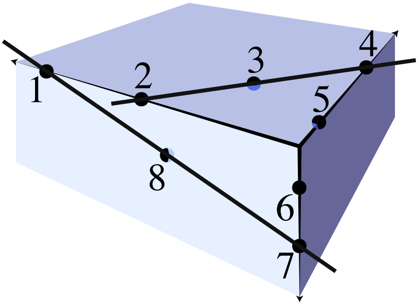

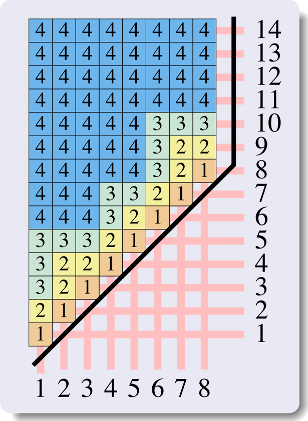

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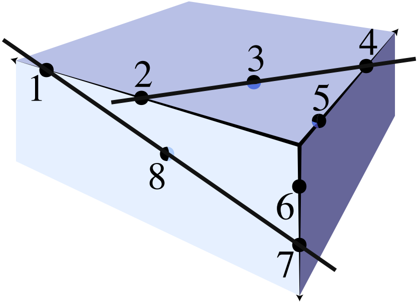

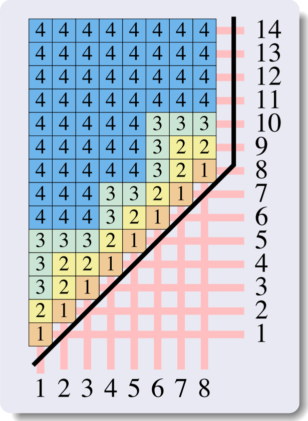

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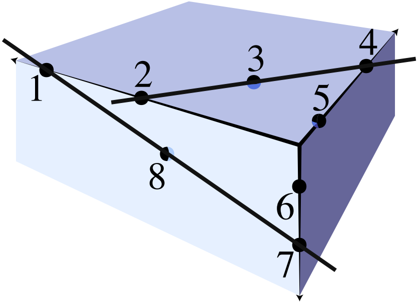

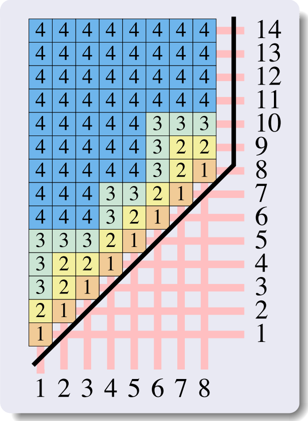

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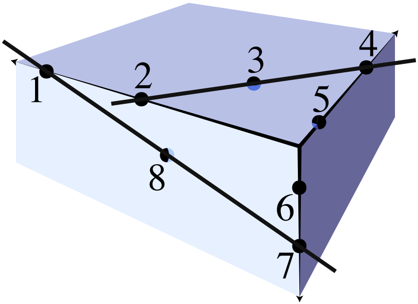

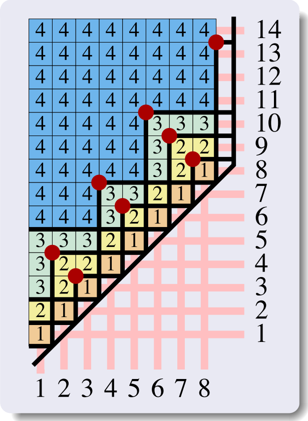

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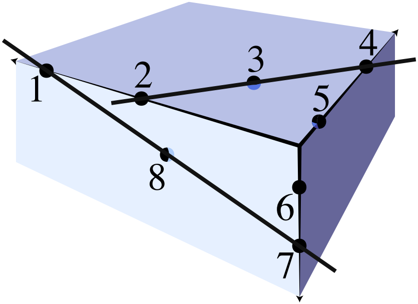
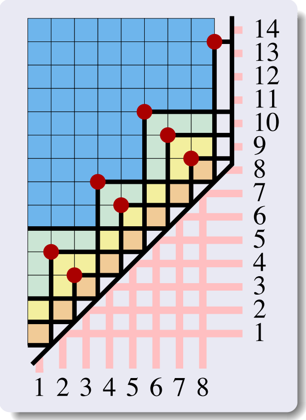
2	1							2
1								1
1	2	3	4	5	6	7	8	

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

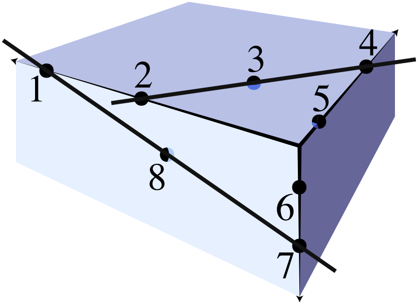
Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:

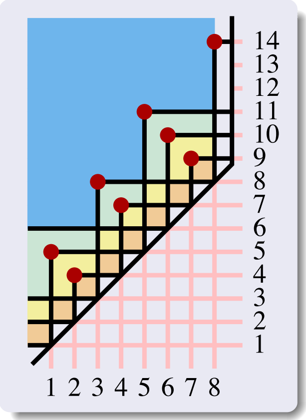




Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

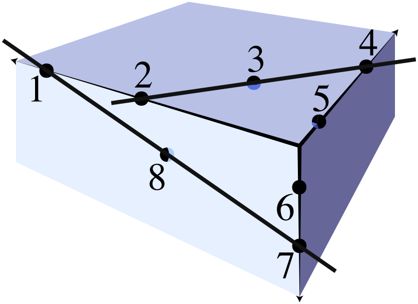
Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:

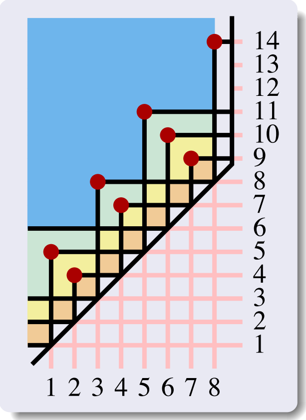




Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C\in G(4,8)$, $c_a\in\mathbb{P}^3$, and impose the following sequence of consecutive constraints:





Permutation Label

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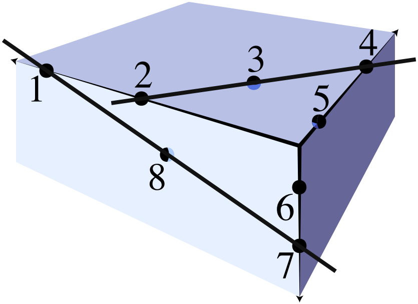
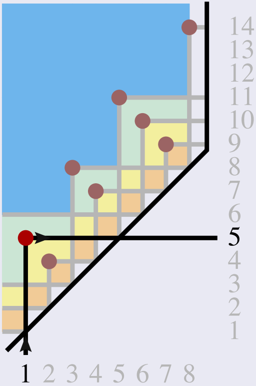
Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:



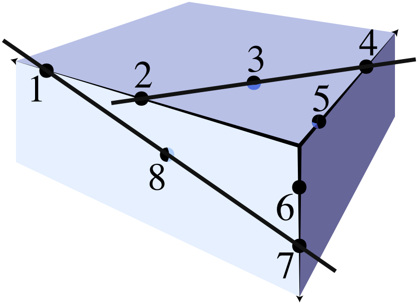


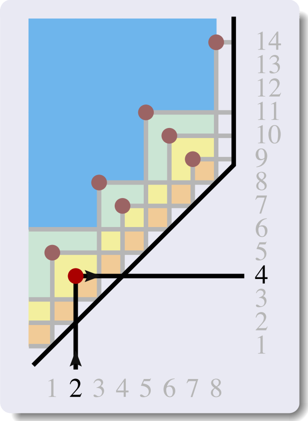
Permutation Label

$\sigma(1) = 5$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:



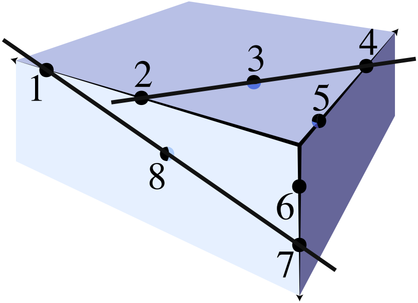


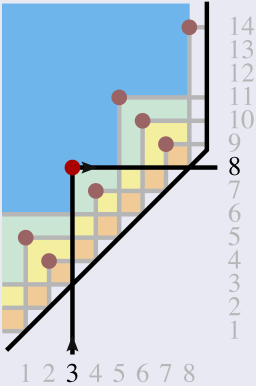
Permutation Label

$\sigma(2) = 4$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:



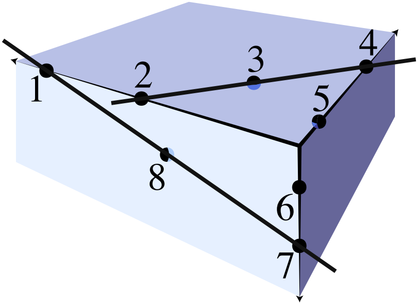


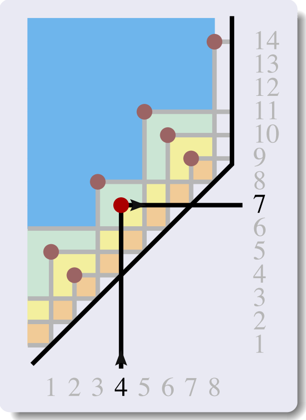
Permutation Label

$\sigma(3) = 8$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C\in G(4,8)$, $c_a\in\mathbb{P}^3$, and impose the following sequence of consecutive constraints:



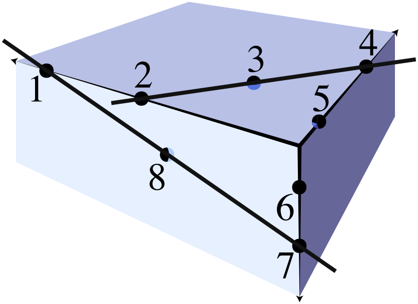


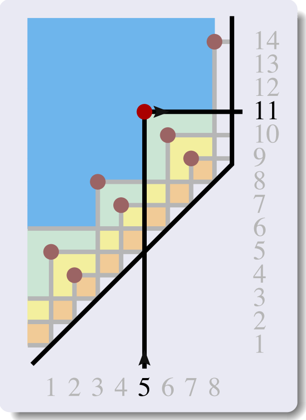
Permutation Label

$\sigma(4)=7$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C\in G(4,8)$, $c_a\in\mathbb{P}^3$, and impose the following sequence of consecutive constraints:



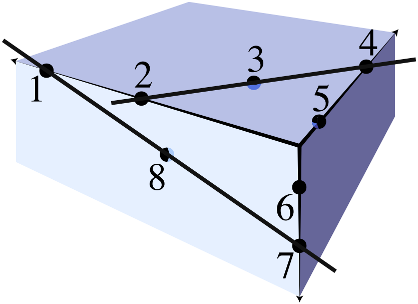


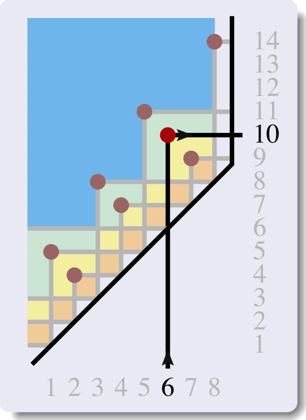
Permutation Label

$$\sigma(5) = 3+8 = 11$$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C\in G(4,8)$, $c_a\in\mathbb{P}^3$, and impose the following sequence of consecutive constraints:



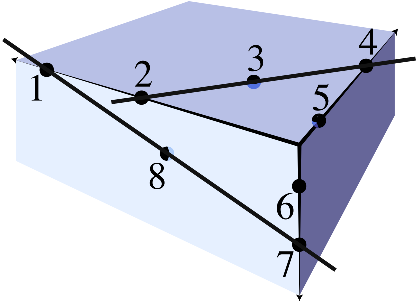


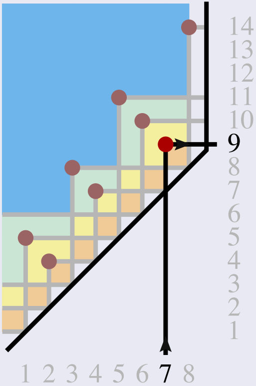
Permutation Label

$$\sigma(6)=2+8=10$$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C\in G(4,8)$, $c_a\in\mathbb{P}^3$, and impose the following sequence of consecutive constraints:



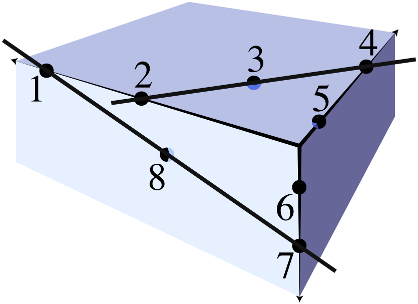


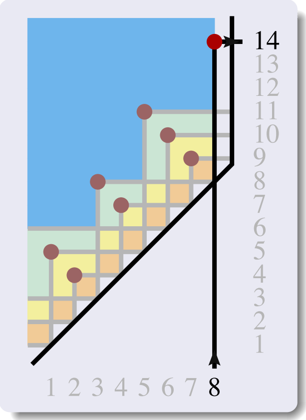
Permutation Label

$$\sigma(7)=1+8=9$$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:



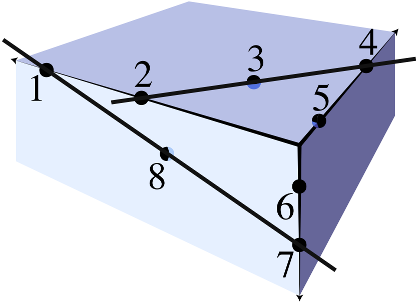


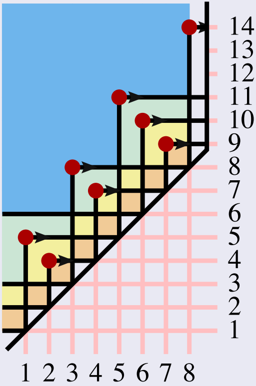
Permutation Label

$$\sigma(8) = 6 + 8 = 14$$

Exempli Gratia: Iterated Residues of $\mathcal{L}_{8,4}$

Take a *generic* $C \in G(4,8)$, $c_a \in \mathbb{P}^3$, and impose the following sequence of consecutive constraints:





Permutation Label

$\{5, 4, 8, 7, 11, 10, 9, 14\}$

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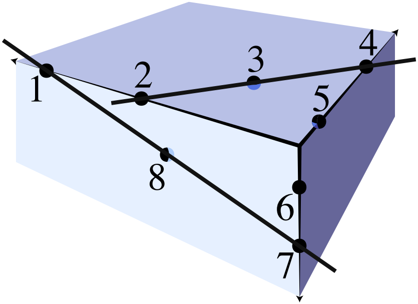

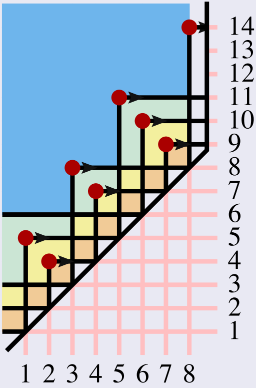

Geometric meaning of the permutation $\sigma(a)=b$

For each column a, there is a unique, **nearest** column $b\geq a$ such that $a\in\text{span}\{a+1,\ldots,b-1,b\}$

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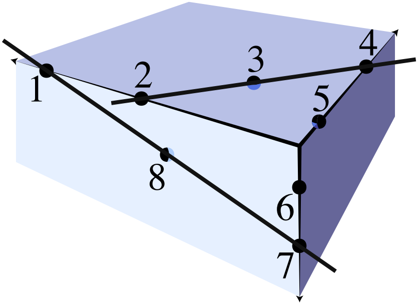

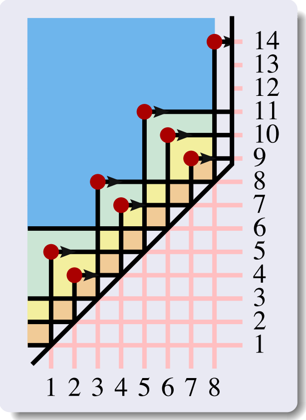

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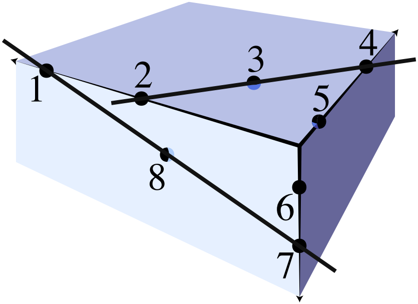
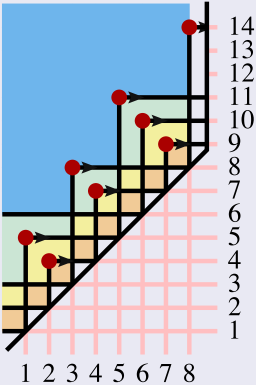
The *Positroid* Stratification of the Grassmannian

The dimensionality of the configuration is encoded by its permutation label as follows:





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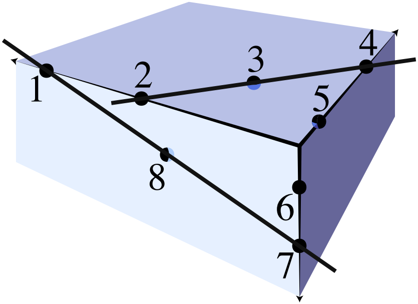

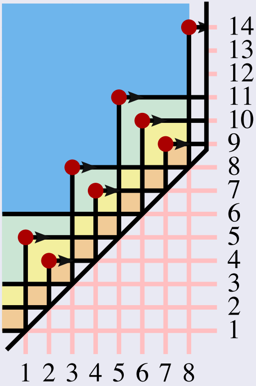

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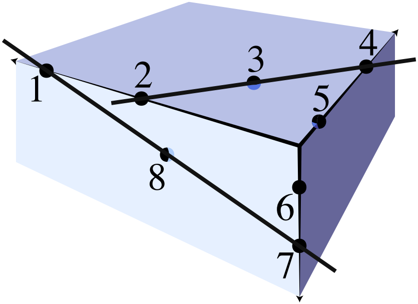

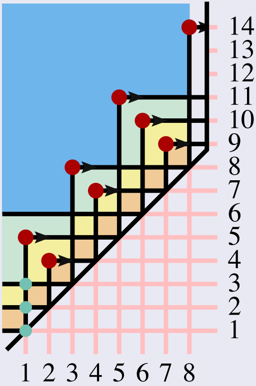

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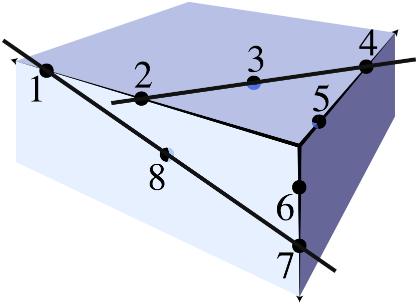

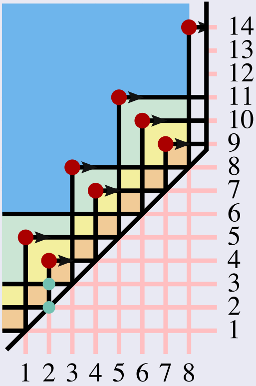

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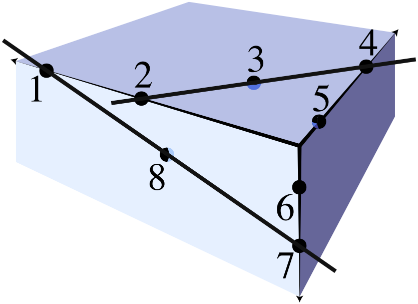

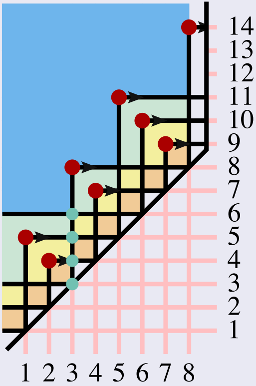

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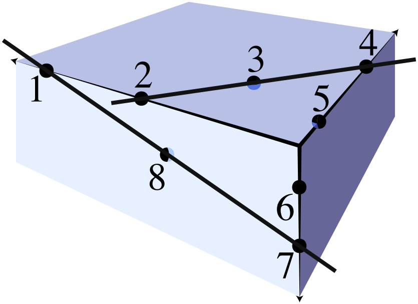

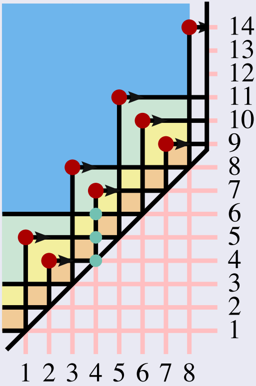

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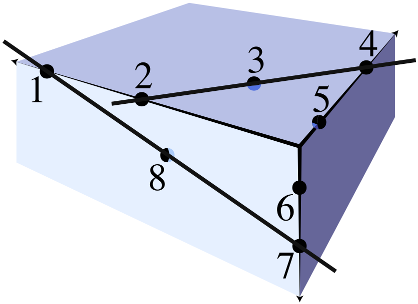

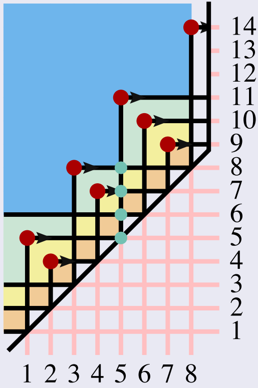

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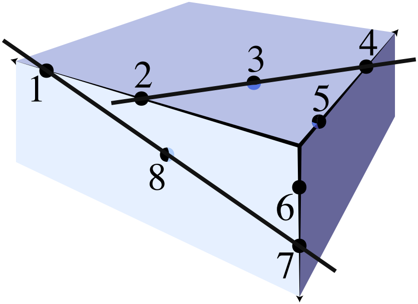

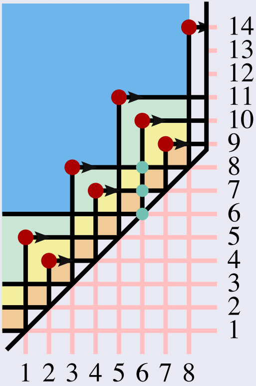

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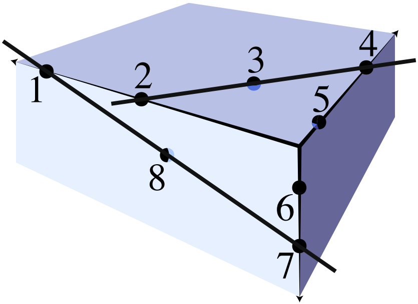

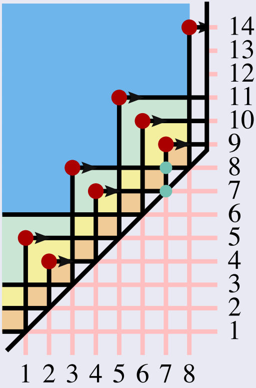

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The *Positroid* Stratification of the Grassmannian

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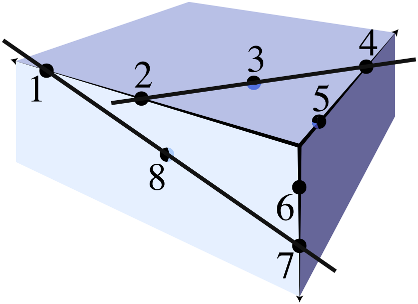

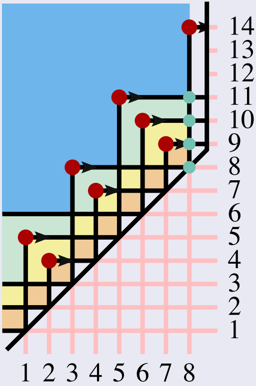

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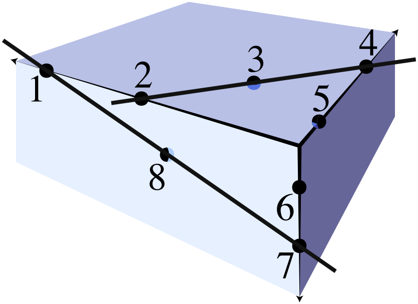

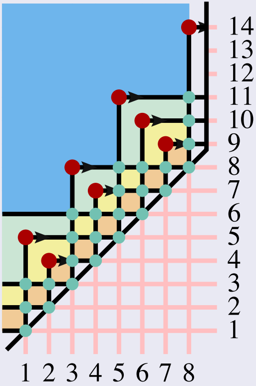

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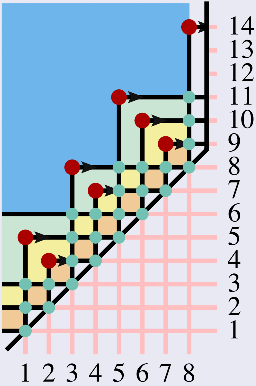
The *Positroid* Stratification of the Grassmannian

The dimensionality of the configuration is encoded by its permutation label as follows:

Dimensionality of a configuration labelled by σ:

$$\dim(\sigma) = \Big(\sum_a |a \cdots \sigma(a)|\Big) - k^2$$



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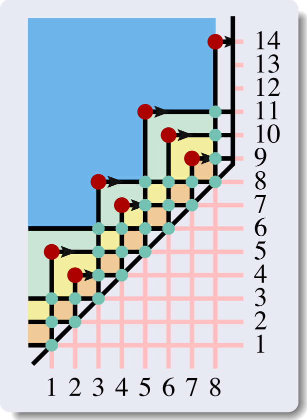

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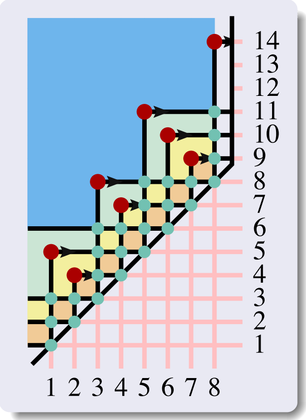

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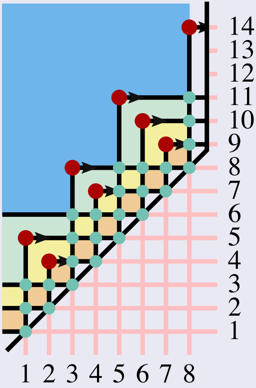

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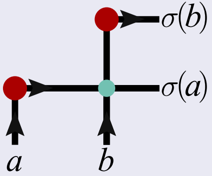

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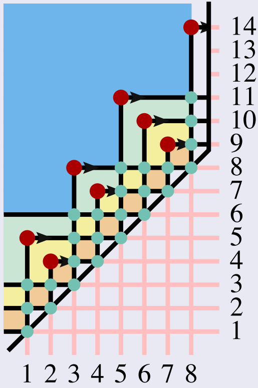

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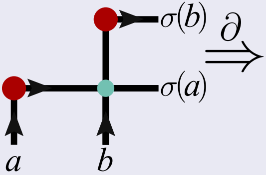

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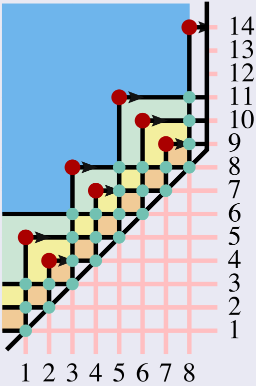

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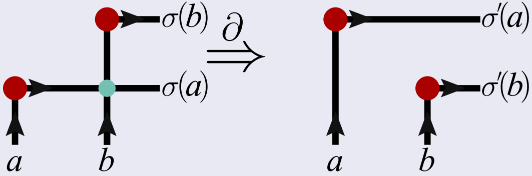

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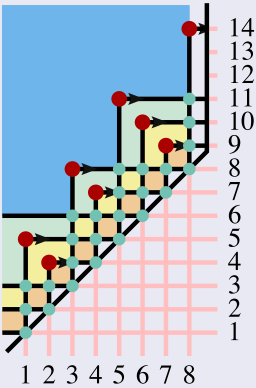

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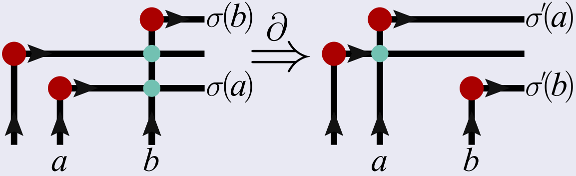

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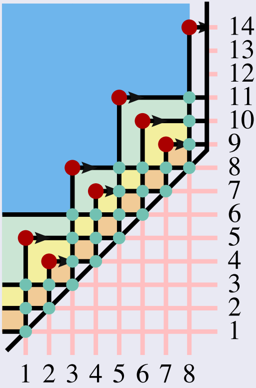

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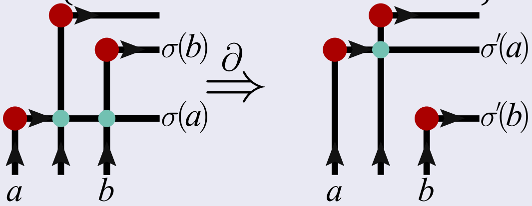

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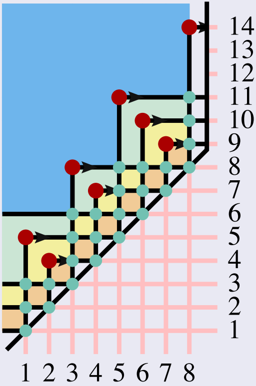

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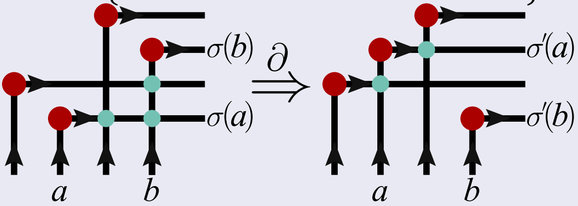

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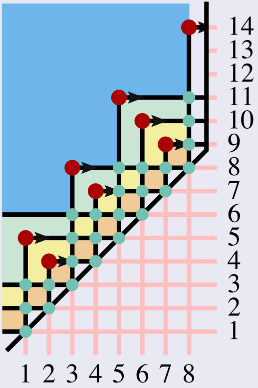

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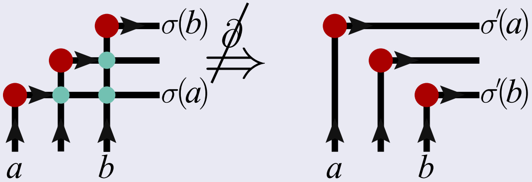

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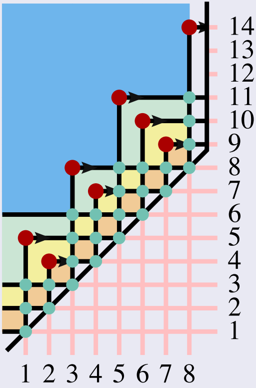

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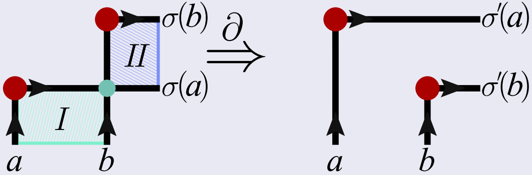

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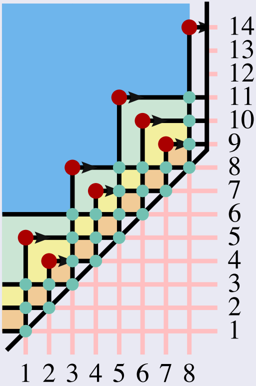

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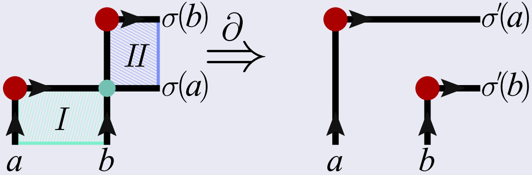

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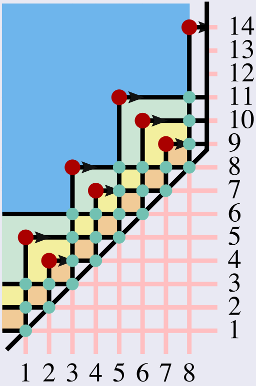

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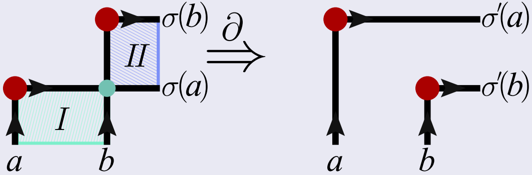

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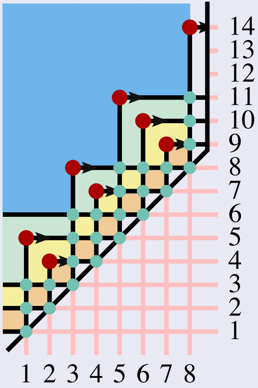

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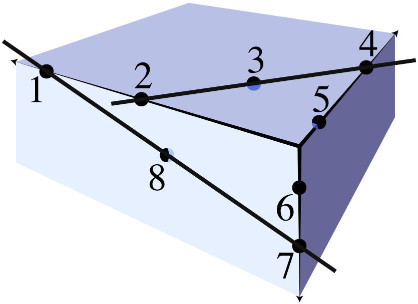

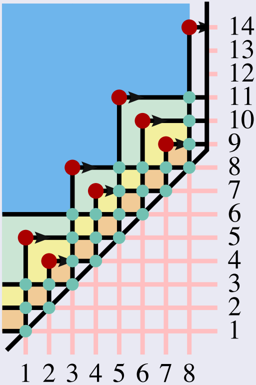

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Equivalence with On-Shell Diagram Permutation Labels

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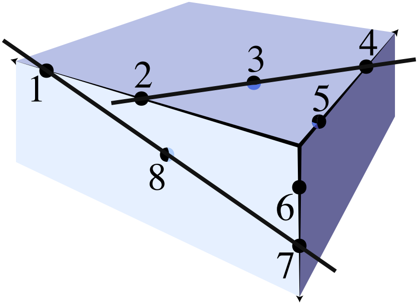

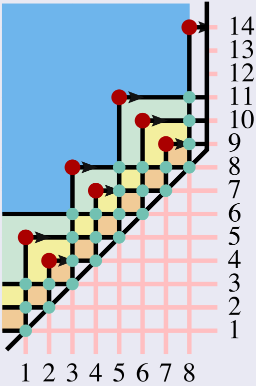

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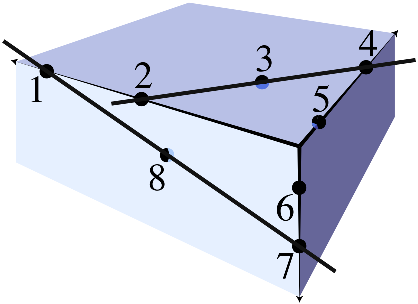

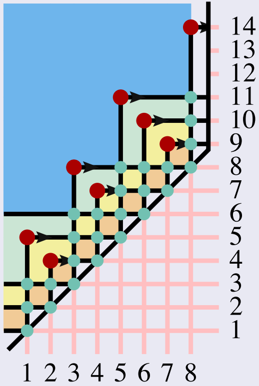

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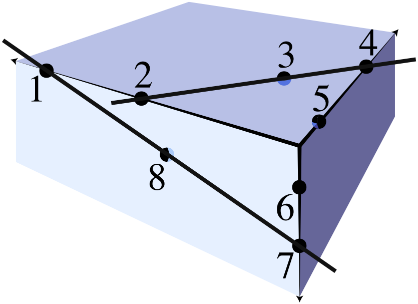

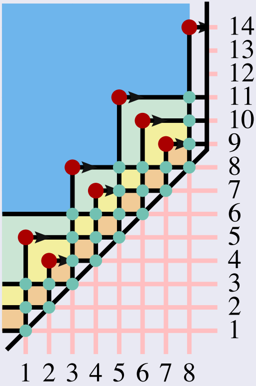

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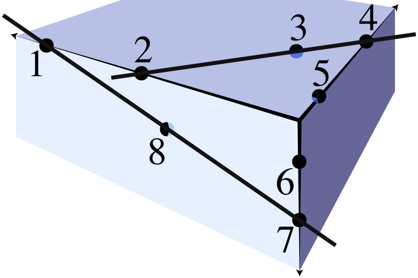

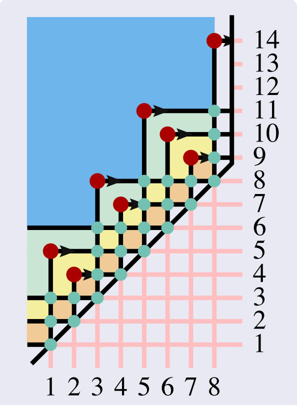

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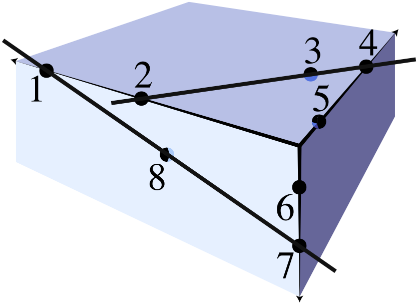

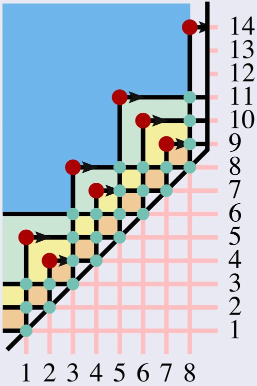

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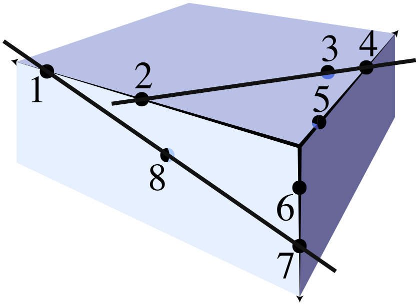

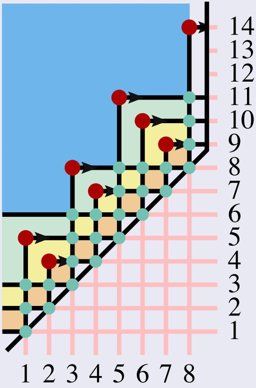

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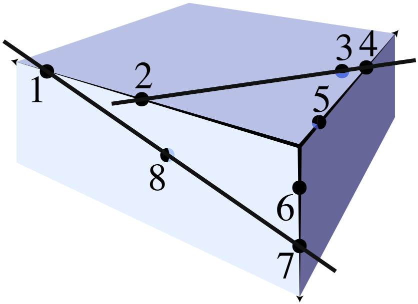

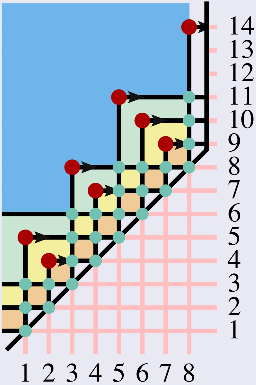

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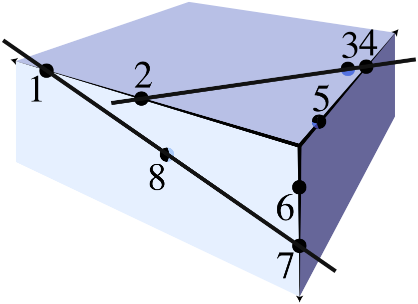

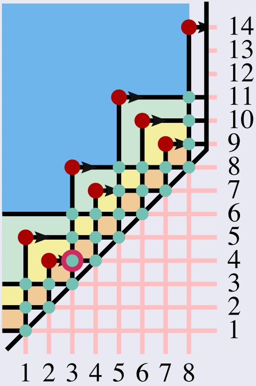

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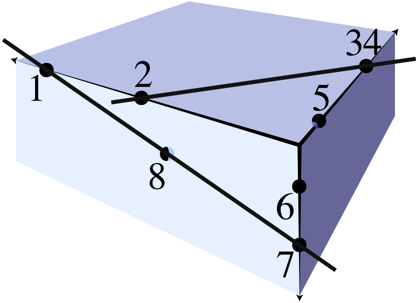

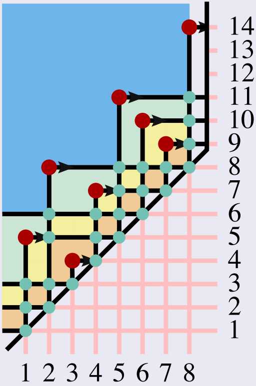

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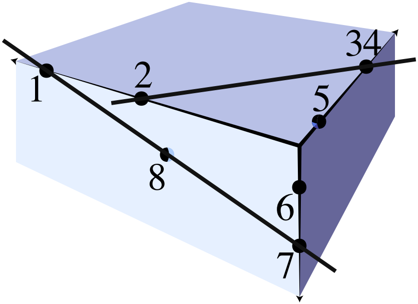

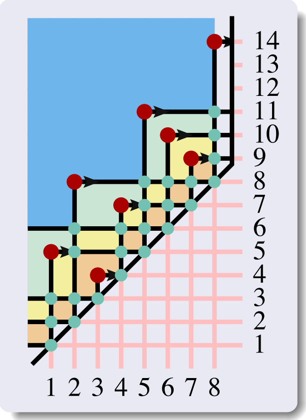

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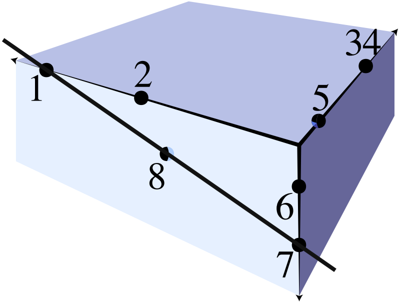

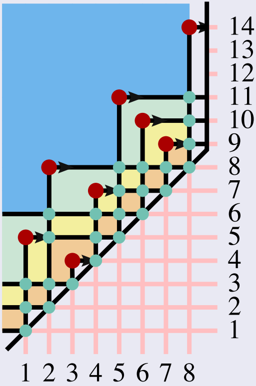

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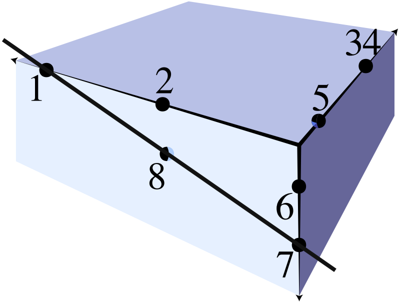

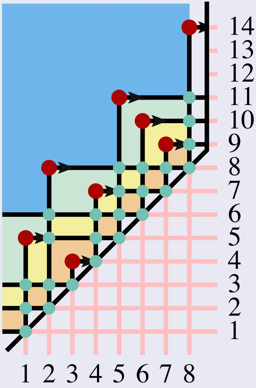

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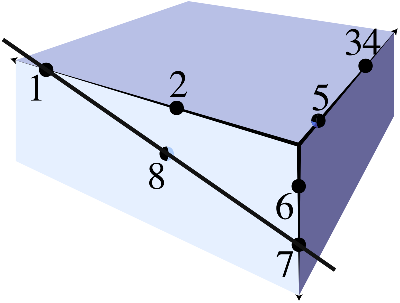

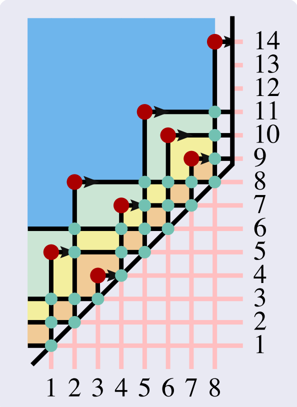

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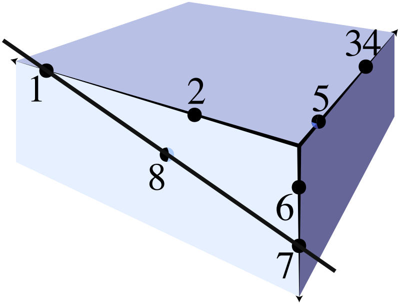

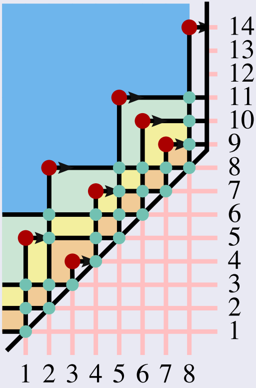

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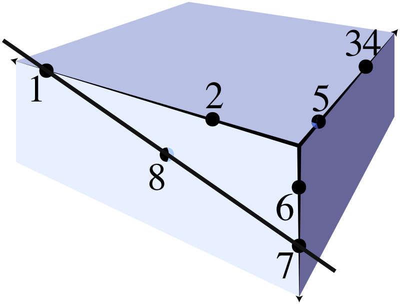

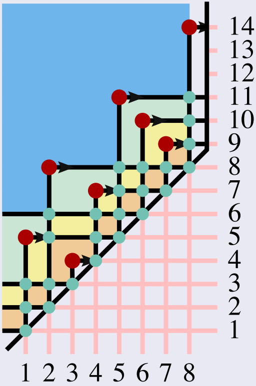

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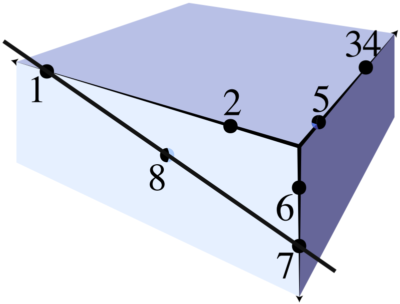

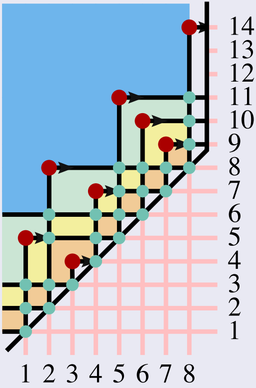

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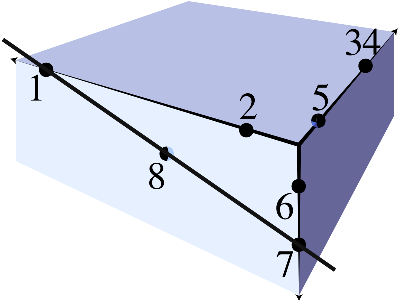

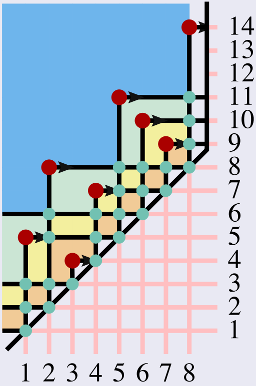

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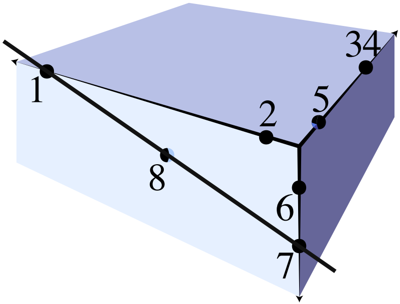

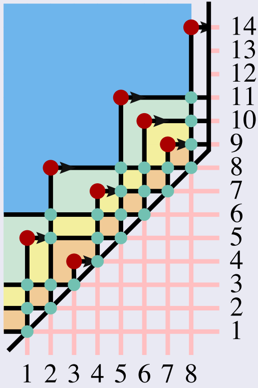

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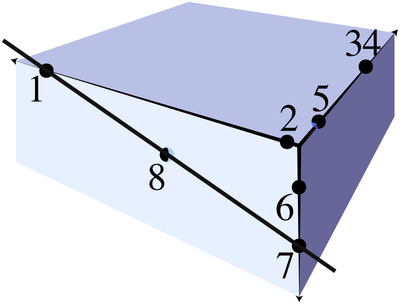

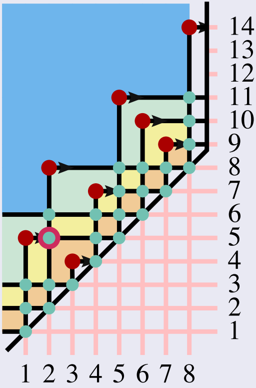

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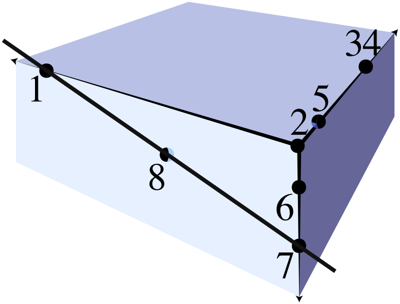

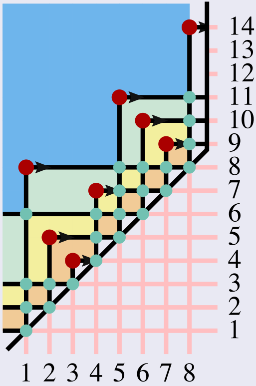

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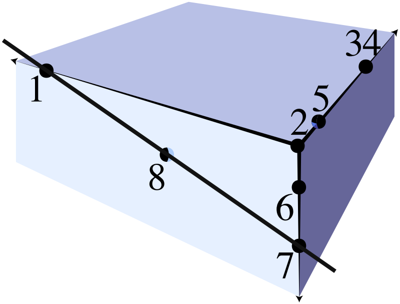

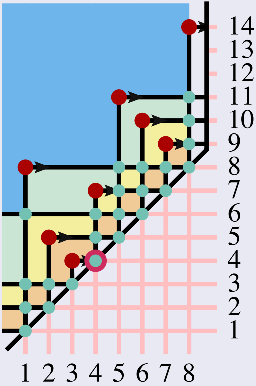

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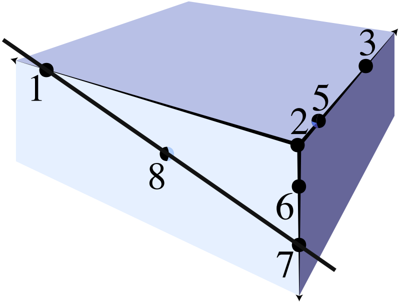

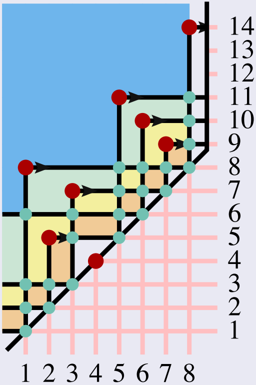

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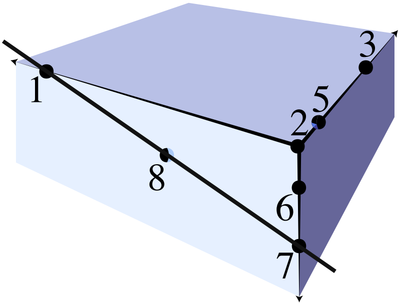

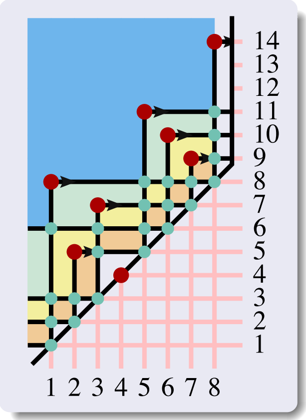

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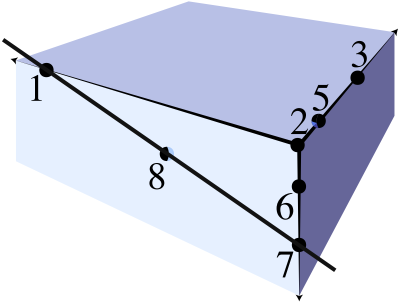

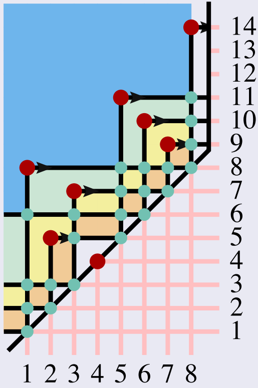

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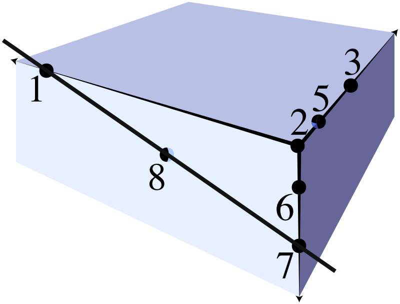

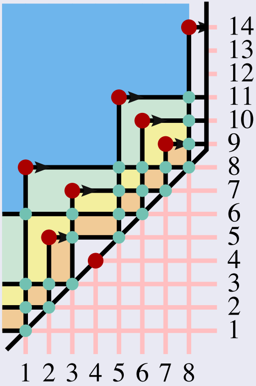

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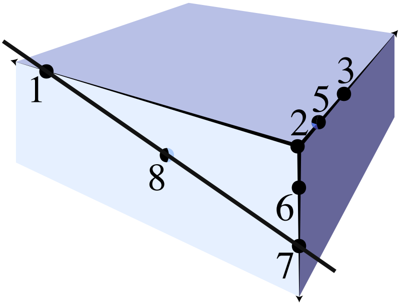

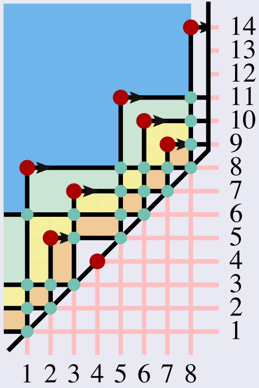

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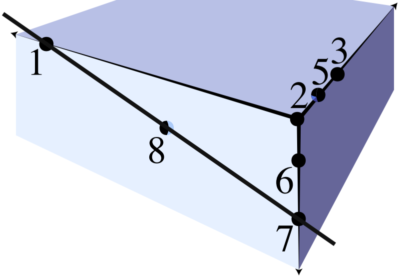

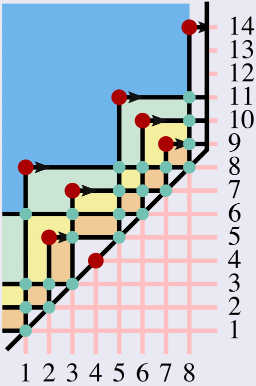

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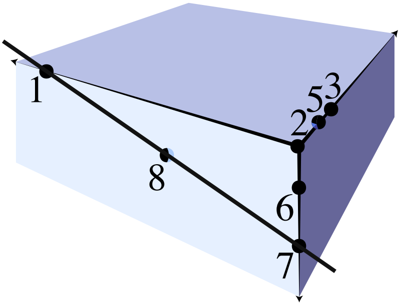

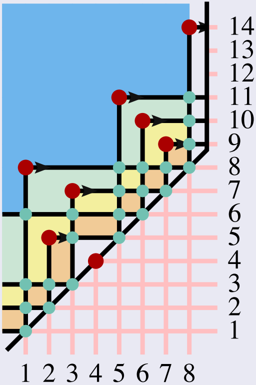

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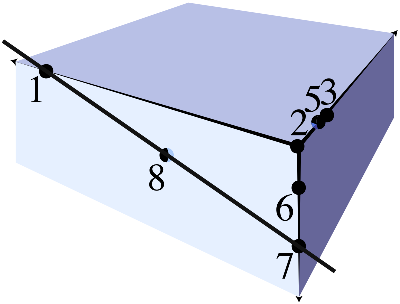

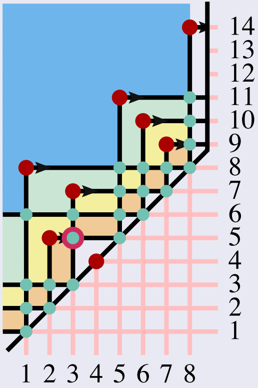

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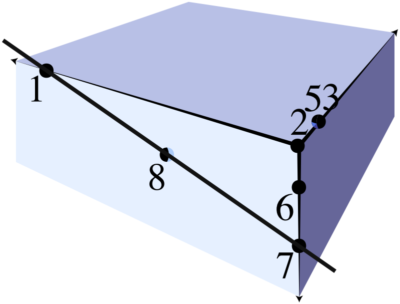

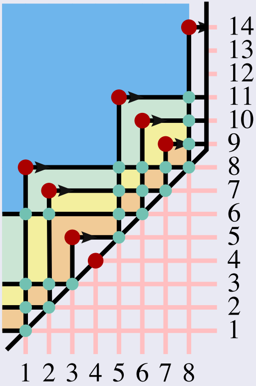

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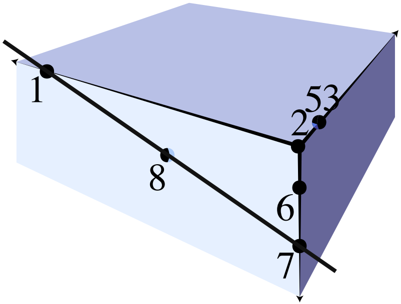

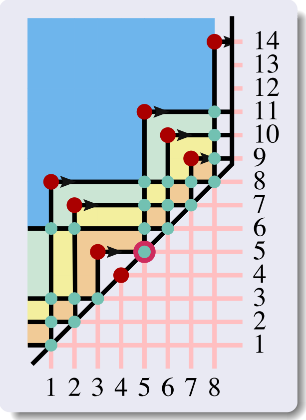

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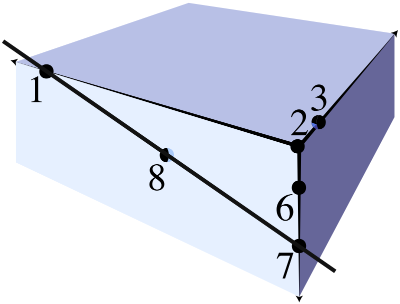

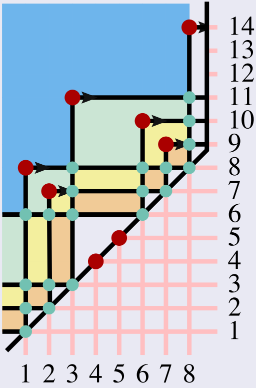

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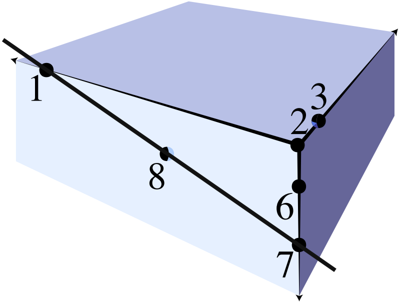

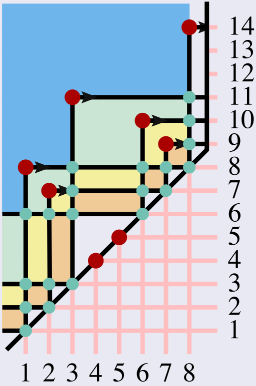

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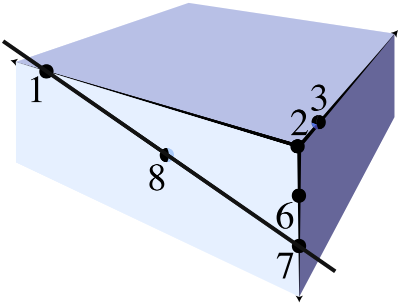

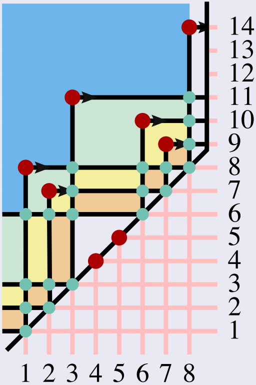

Geometric meaning of the permutation $\sigma(a)=b$

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Permutation Label

$\{8, 7, 11, 4, 5, 10, 9, 14\}$

Equivalence with On-Shell Diagram Permutation Labels

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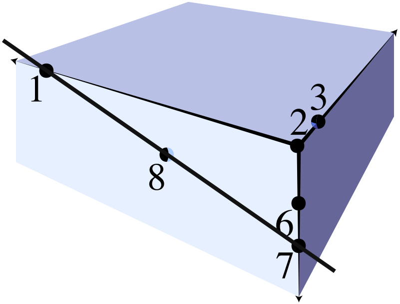

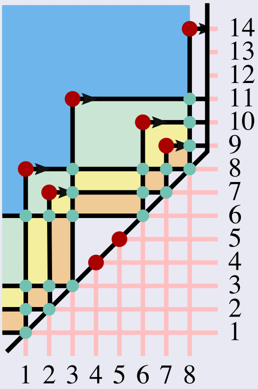

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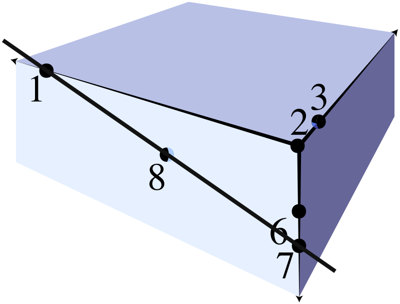

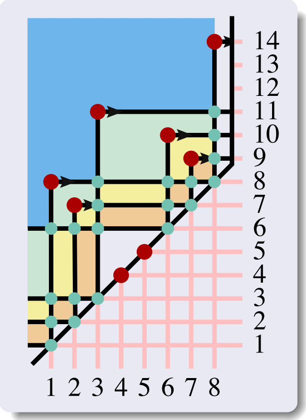

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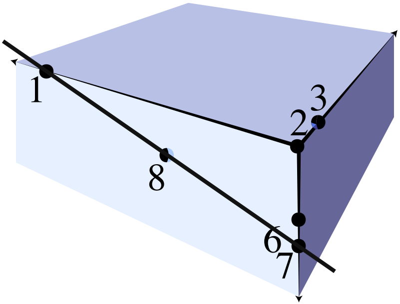

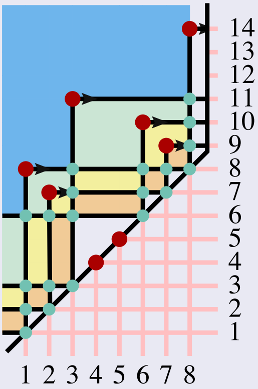

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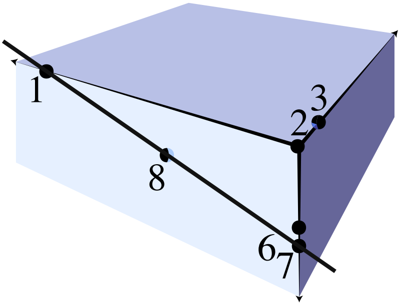

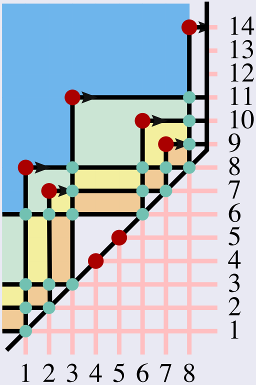

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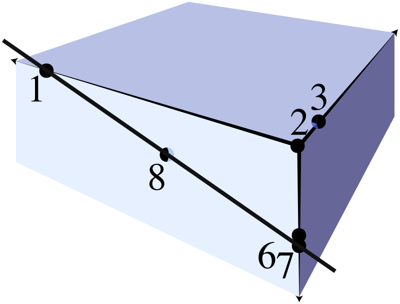

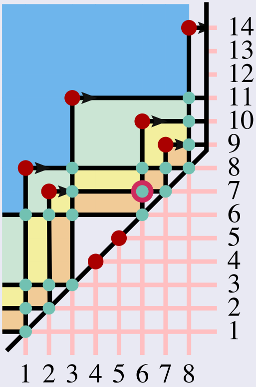

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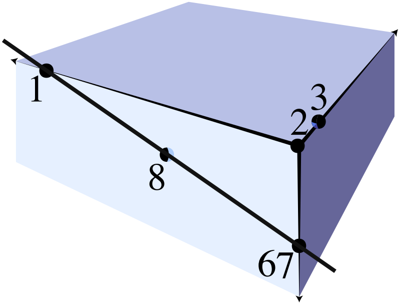

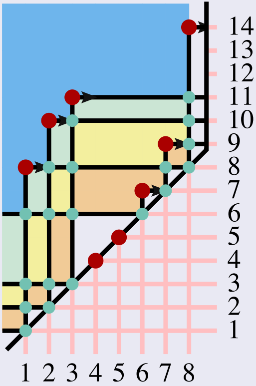

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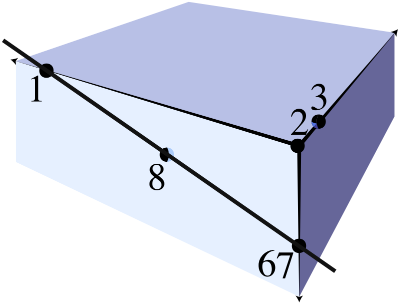

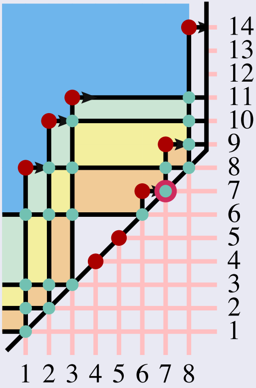

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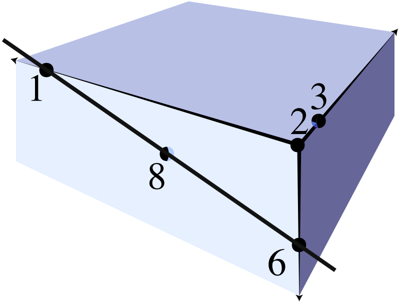

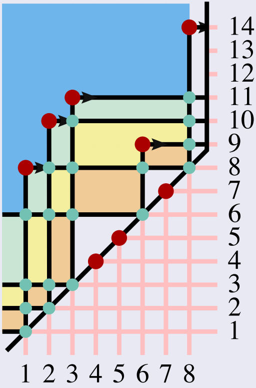

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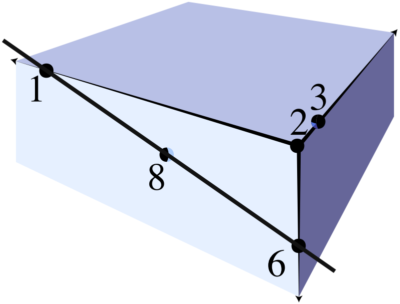

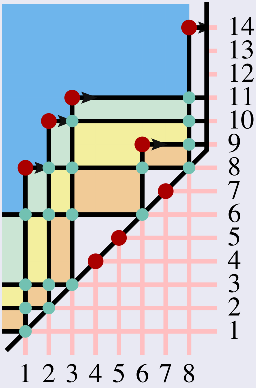

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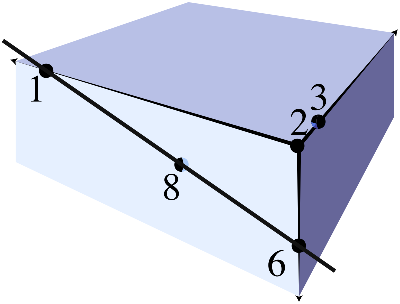

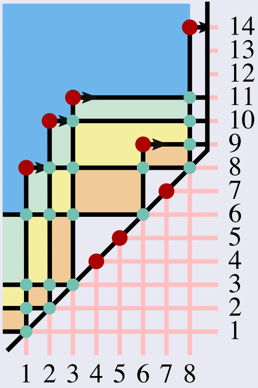

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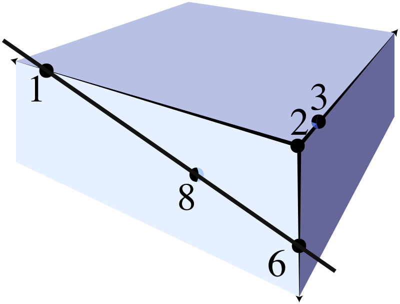

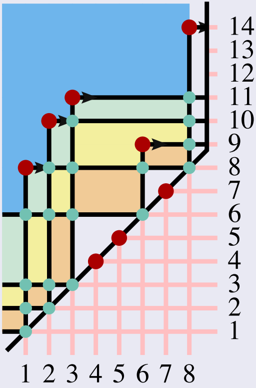

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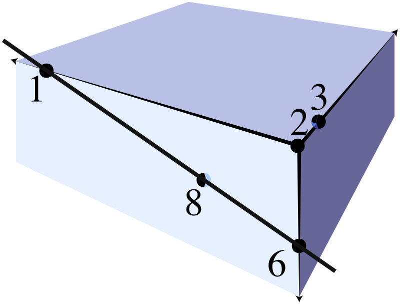

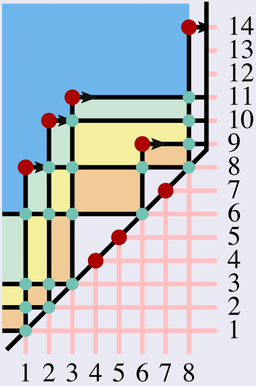

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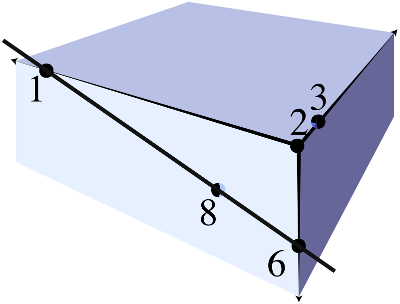

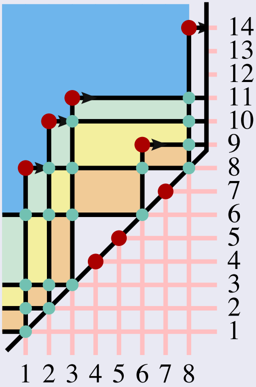

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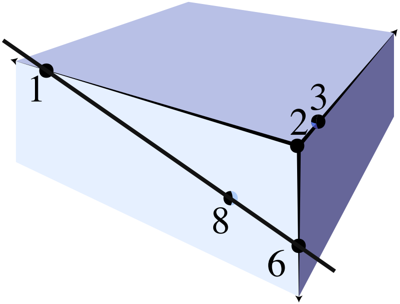

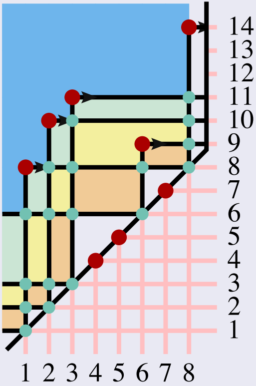

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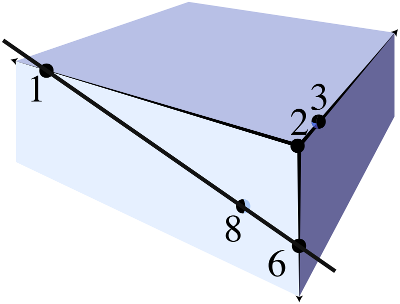

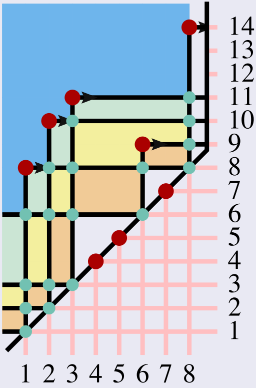

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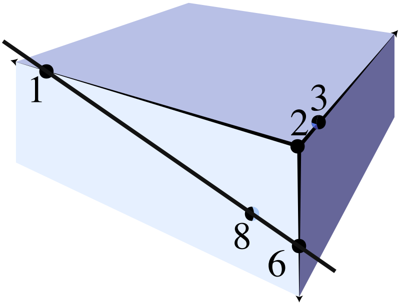

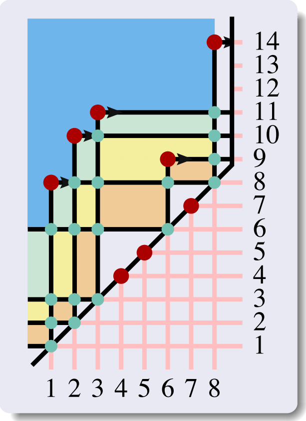

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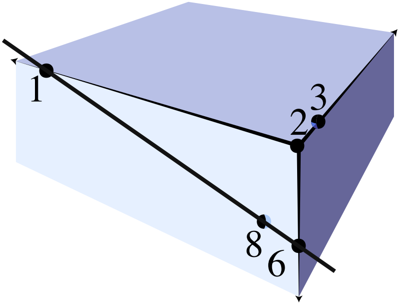

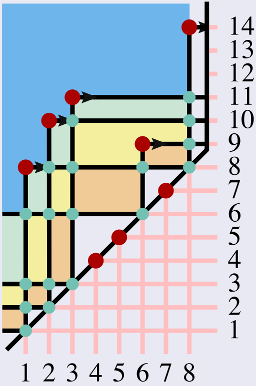

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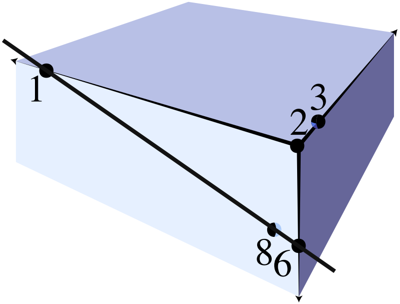

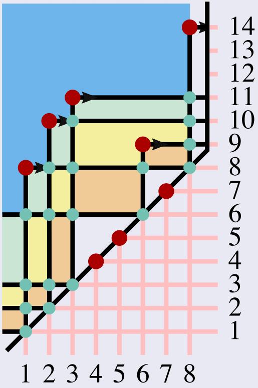

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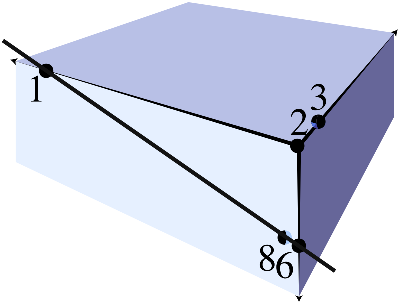

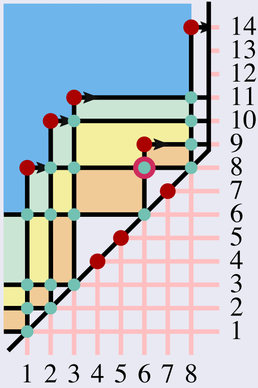

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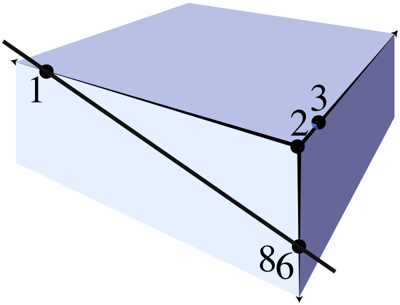

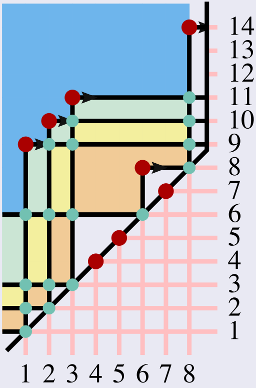

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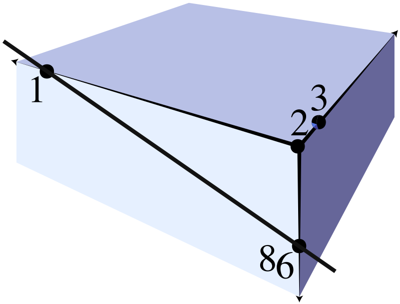

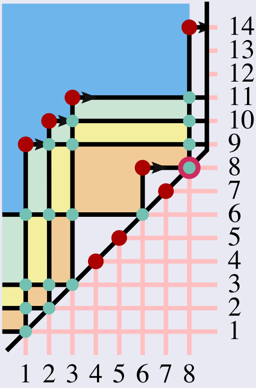

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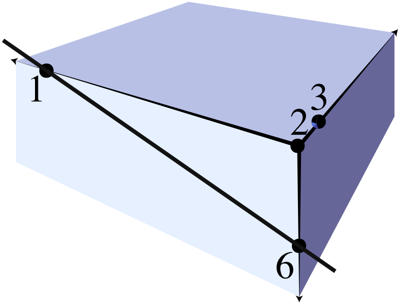

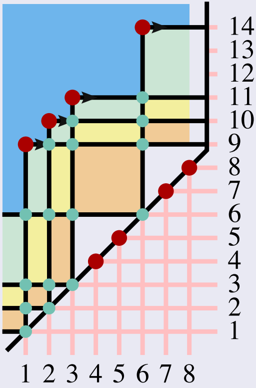

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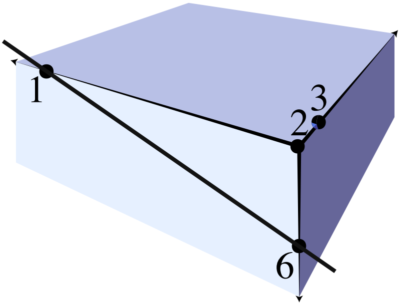

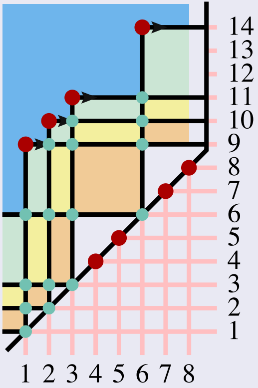

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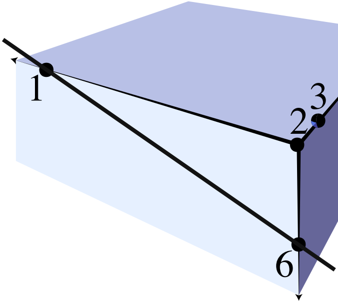
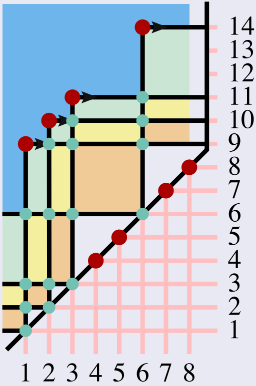


$$C \equiv \begin{array}{c} \begin{array}{cccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \end{array} \\ \left(\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right) \end{array}$$

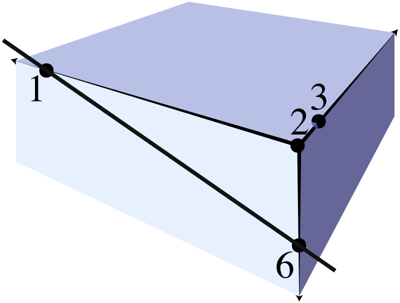


Permutation Label

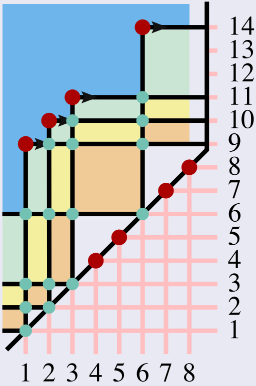
$\{9, 10, 11, 4, 5, 14, 7, 8\}$

Equivalence with On-Shell Diagram Permutation Labels

As for permutations labeling on-shell diagrams, we can decompose σ into adjacent transpositions:



$$C\equiv\begin{array}{c}\begin{array}{cccccccc}c_1&c_2&c_3&c_4&c_5&c_6&c_7&c_8\end{array}\\\left(\begin{array}{cccccccc}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&0&1&0&0\end{array}\right)\end{array}$$



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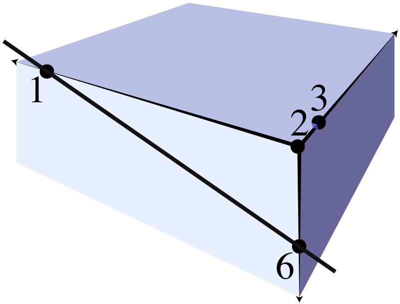

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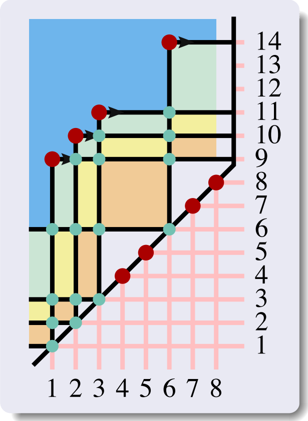

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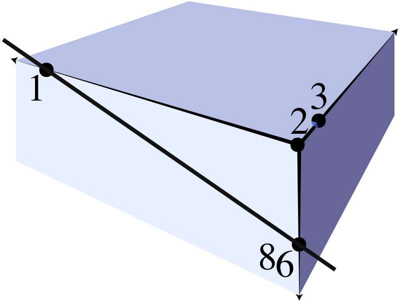

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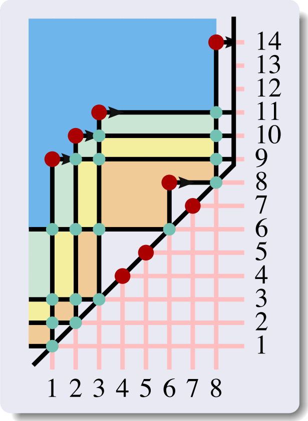
$(6\,8)$: $c_8 \mapsto c_8 + \alpha_9\, c_6$



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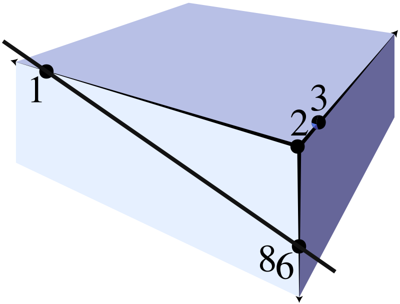

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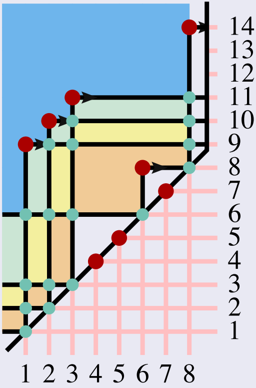

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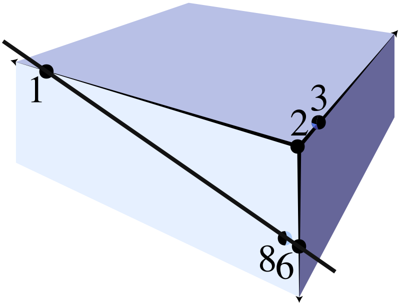

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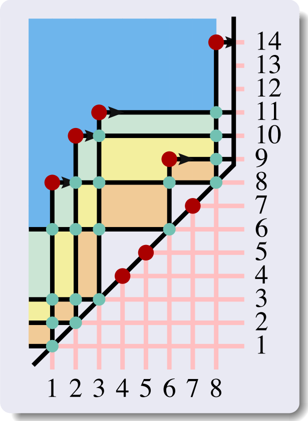

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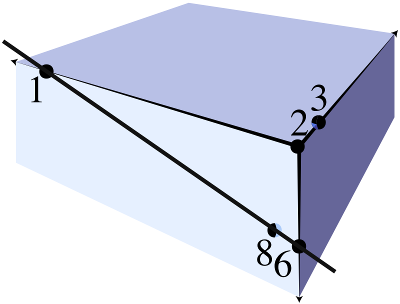

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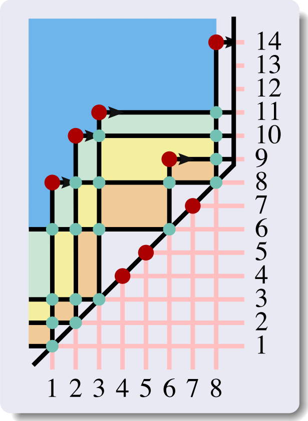

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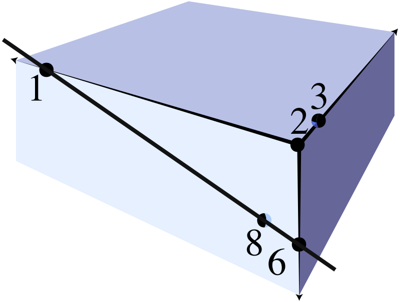

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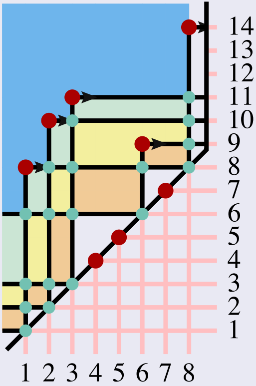

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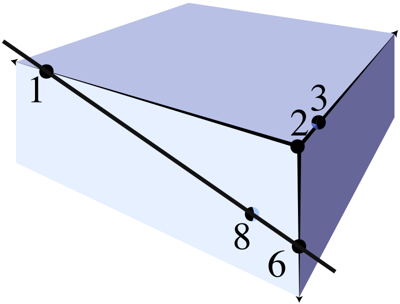

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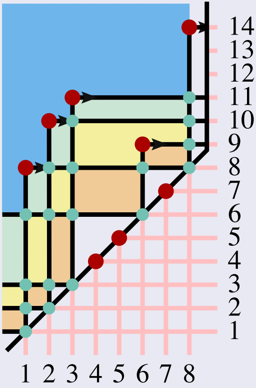

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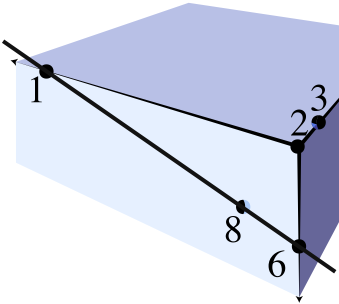

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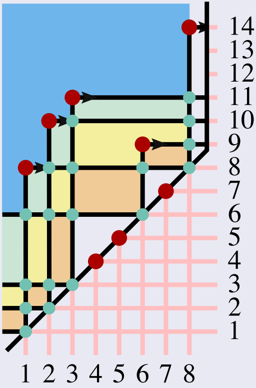

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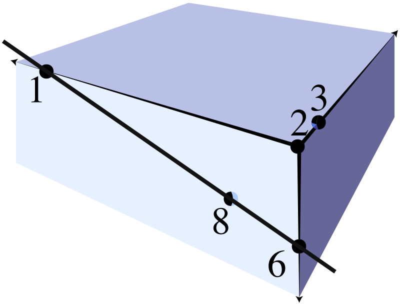

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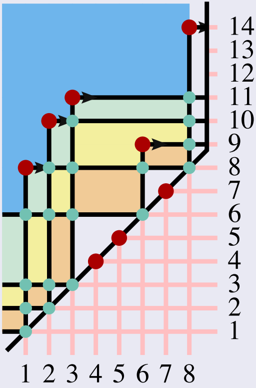

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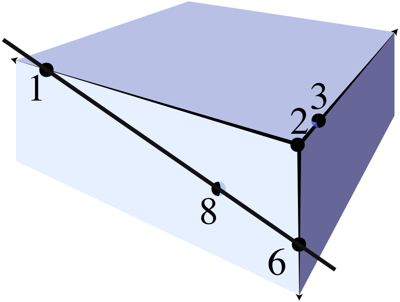

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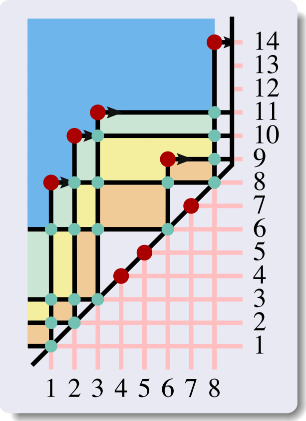

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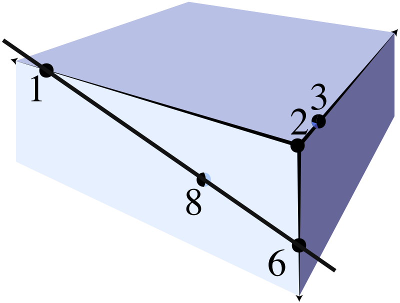

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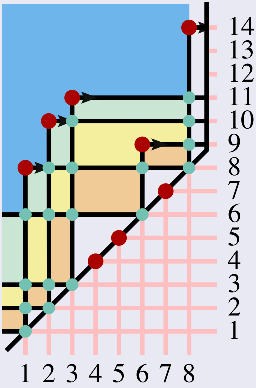

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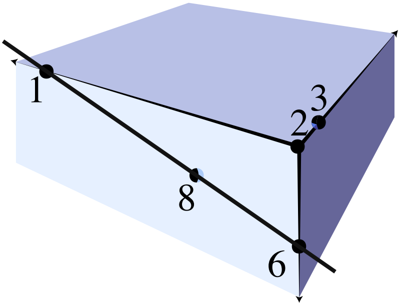

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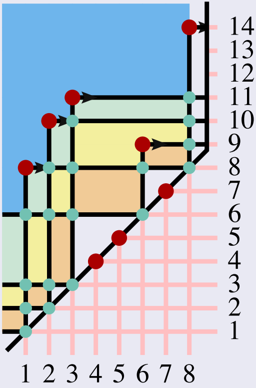

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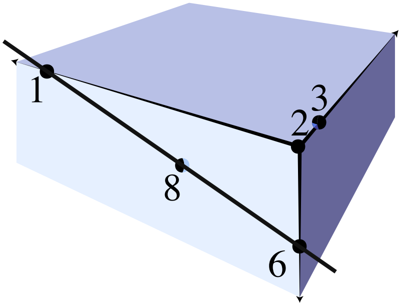

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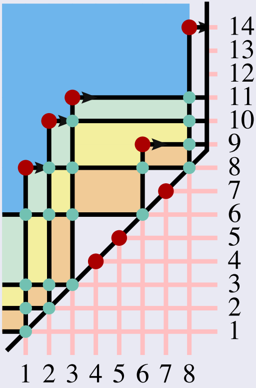

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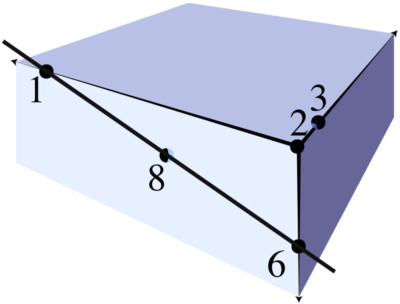

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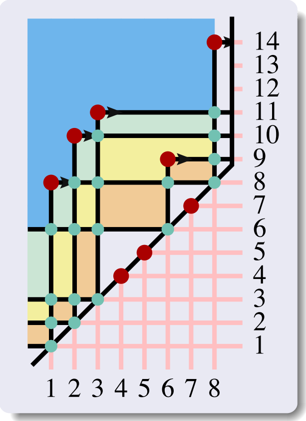

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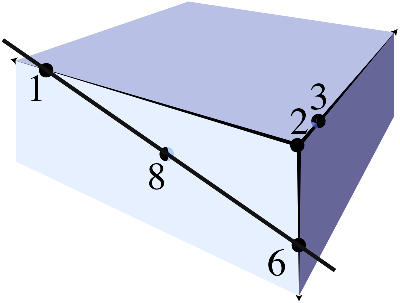

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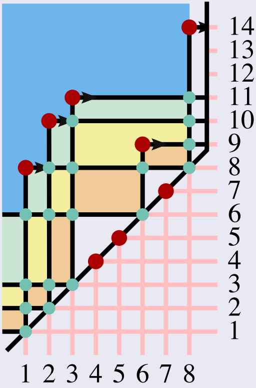

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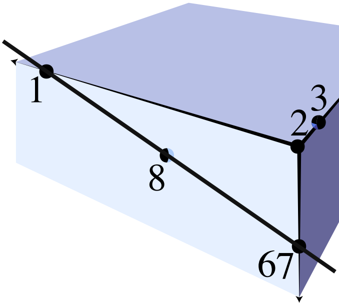

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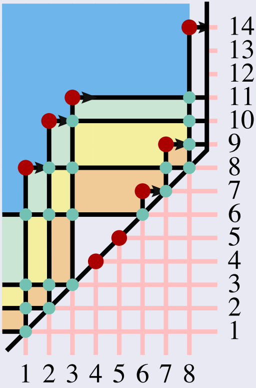

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Equivalence with On-Shell Diagram Permutation Labels

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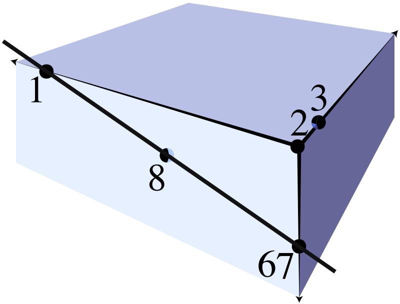
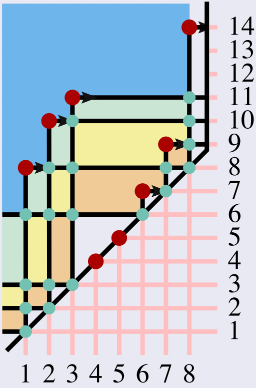


$$C\equiv\left(\begin{array}{cccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8\\ \hline 1 & 0 & 0 & 0 & 0 & \alpha_8 & \alpha_7\alpha_8 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & \alpha_7 & \alpha_9 \end{array}\right)$$

$$(6\,7):\quad c_7\mapsto c_7+\alpha_7\,c_6$$



Permutation Label

$\{8, 10, 11, 4, 5, 7, 9, 14\}$

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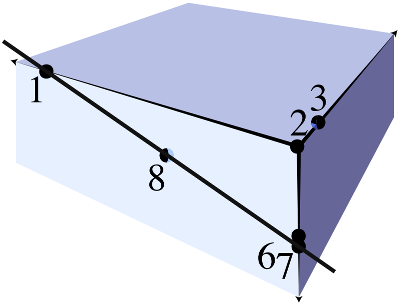

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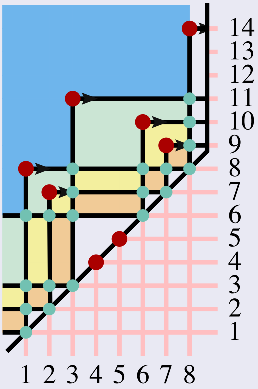
$(2\,6)$: $c_6 \mapsto c_6 + \alpha_6\, c_2$



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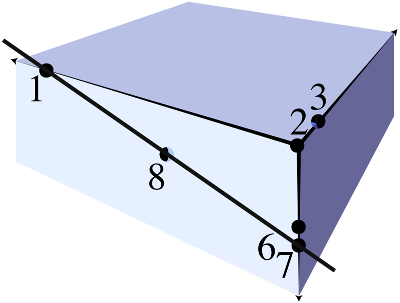

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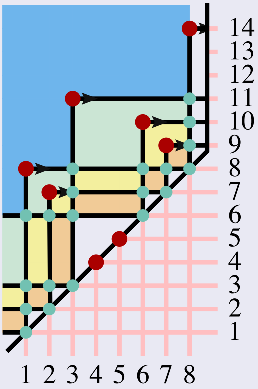

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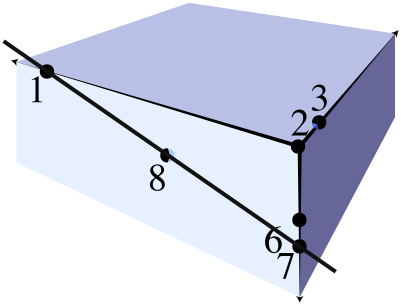

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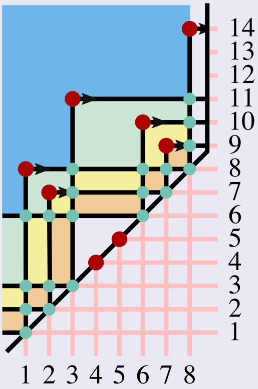

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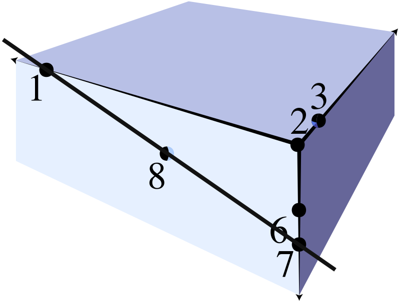

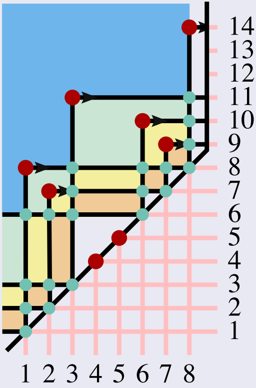

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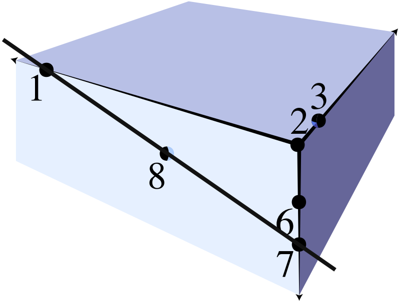

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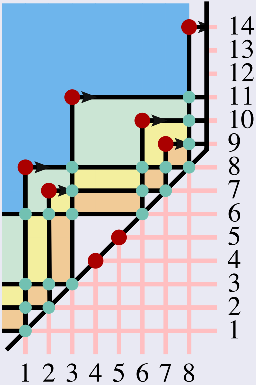

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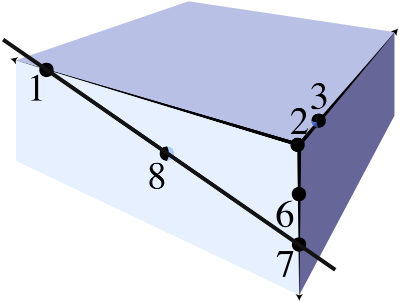

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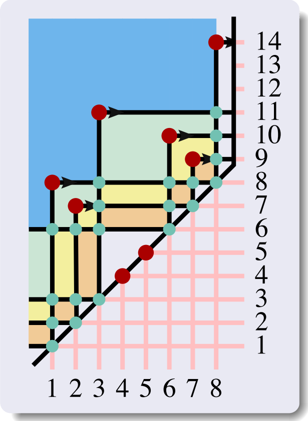

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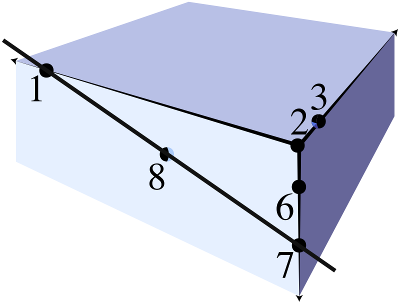

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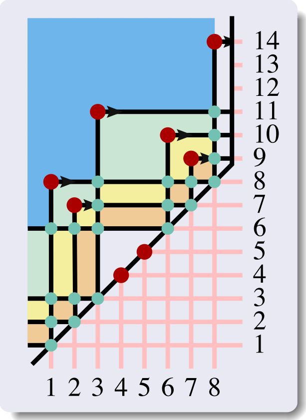

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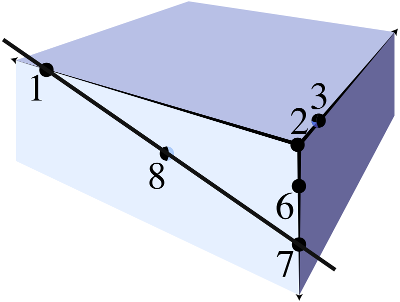

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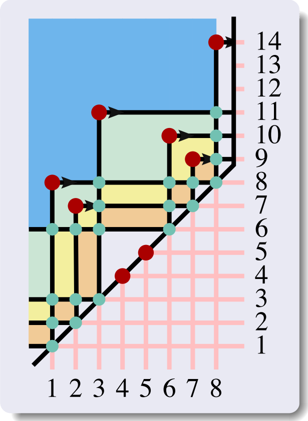

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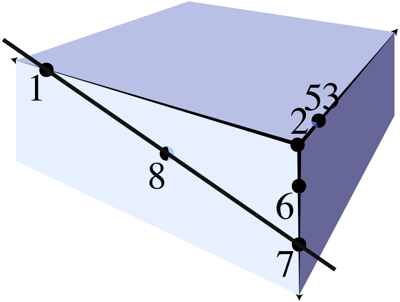

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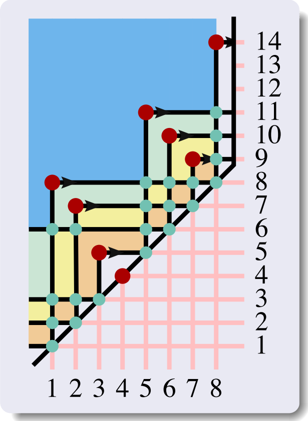
$(3\,5)$: $\quad c_5 \mapsto c_5 + \alpha_5\, c_3$



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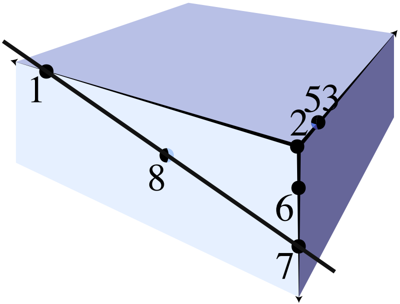

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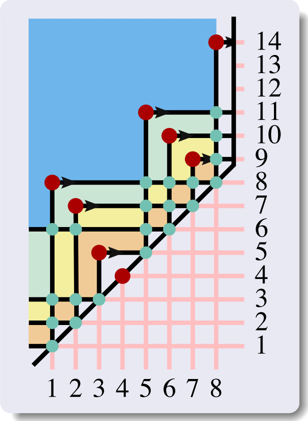

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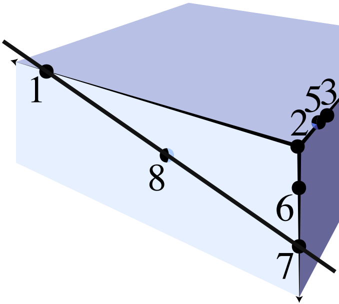

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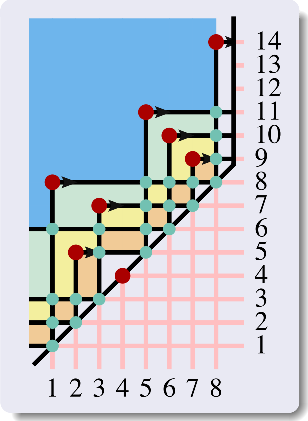
$$(2\,3): \quad c_3 \mapsto c_3 + \alpha_4\, c_2$$



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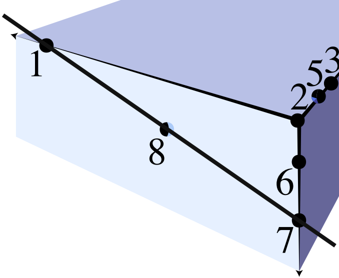

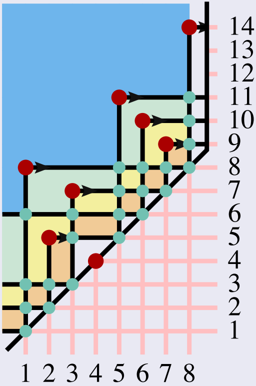

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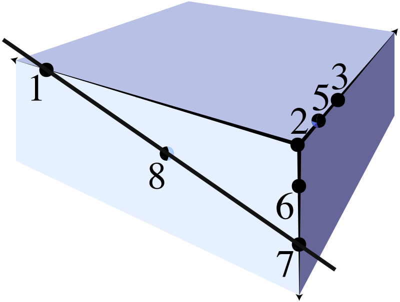

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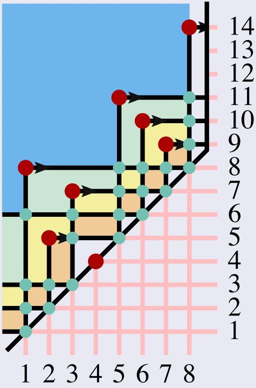

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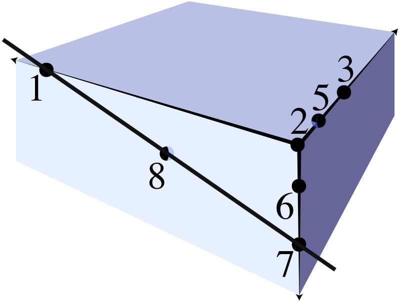

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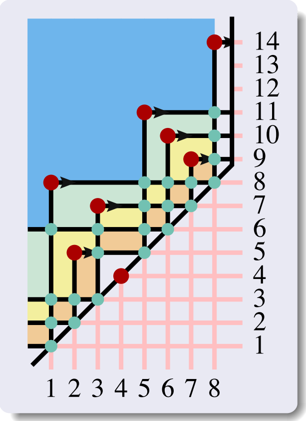

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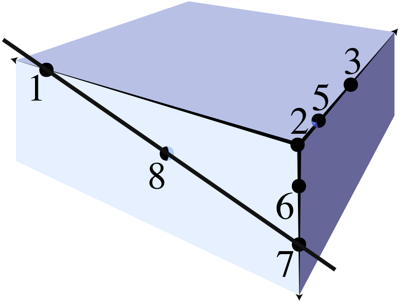

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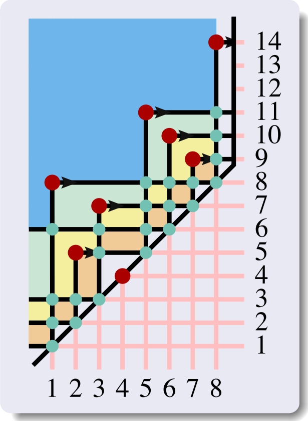

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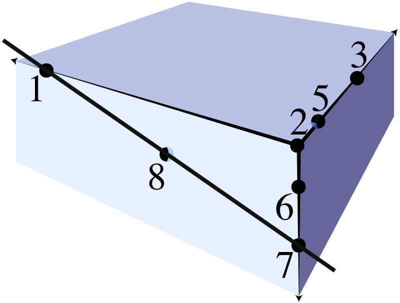

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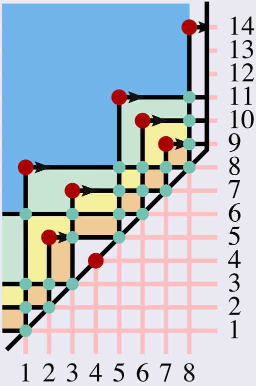

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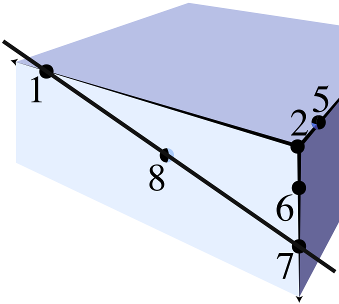

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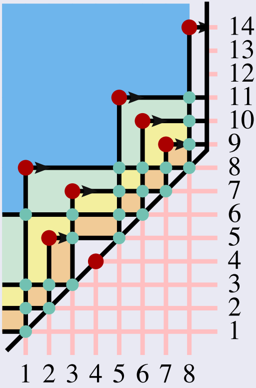
$(2\,3)$: $\quad c_3 \mapsto c_3 + \alpha_4\, c_2$

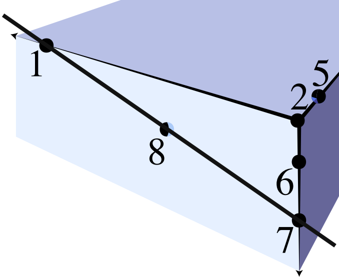


Permutation Label

$\{8, 5, 7, 4, 11, 10, 9, 14\}$

Equivalence with On-Shell Diagram Permutation Labels

As for permutations labeling on-shell diagrams, we can decompose σ into adjacent transpositions:



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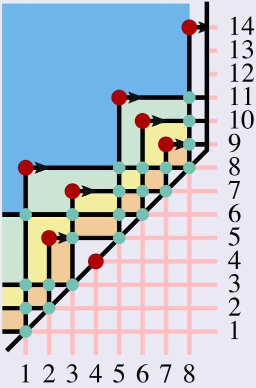

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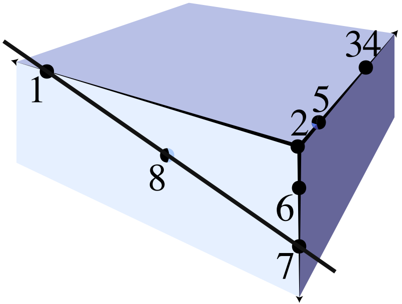

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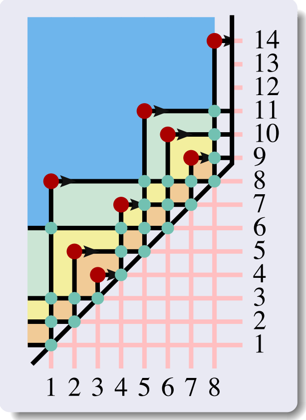
$(3\,4)$: $c_4 \mapsto c_4 + \alpha_3\, c_3$



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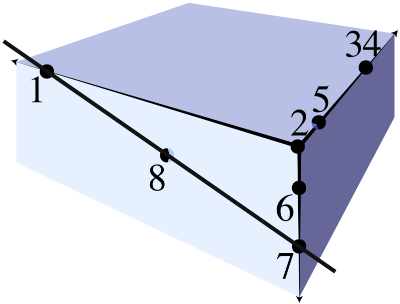

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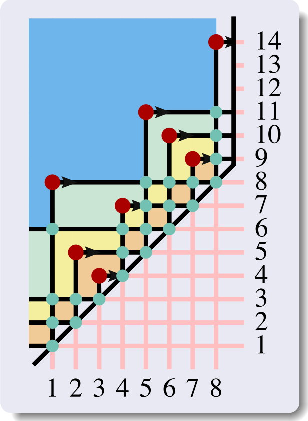

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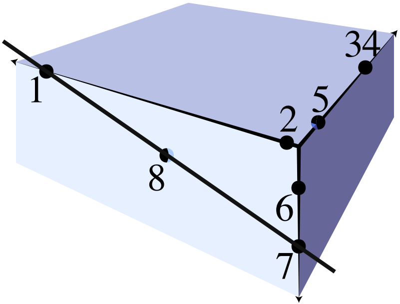

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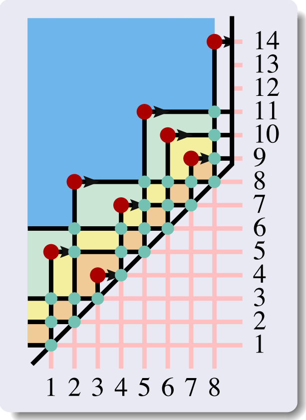
$(1\,2)$: $\;c_2\mapsto c_2+\alpha_2\,c_1$



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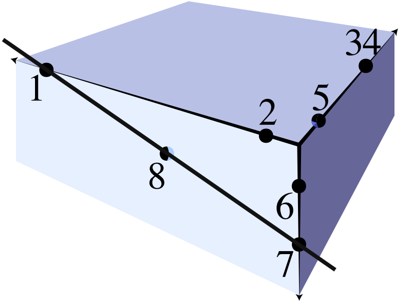

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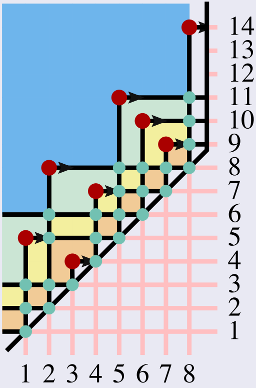

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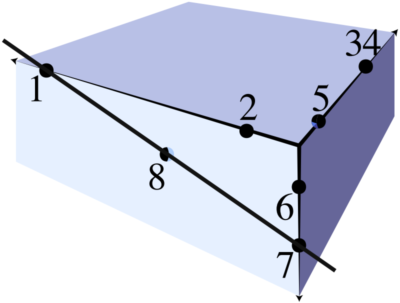

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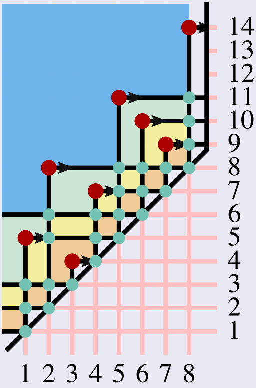

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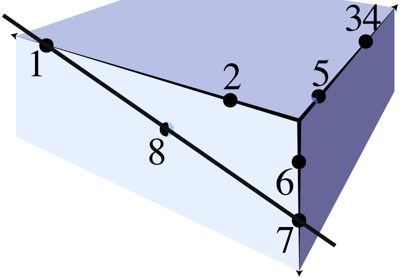

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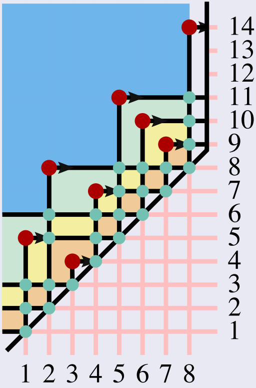

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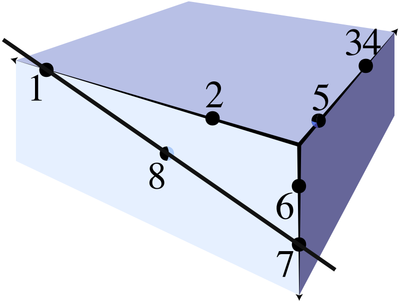

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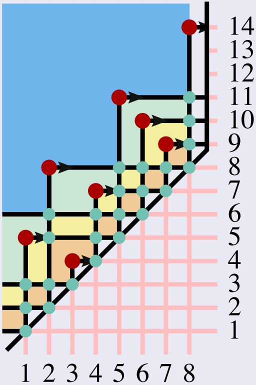

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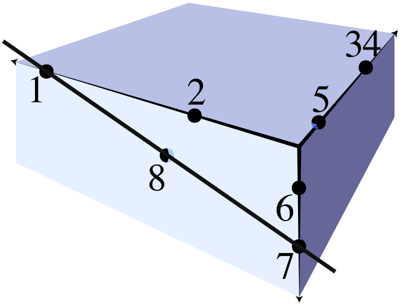

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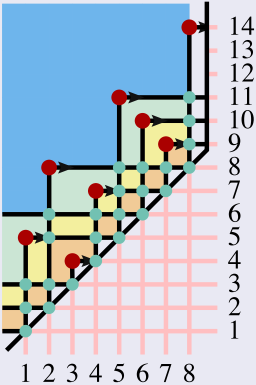

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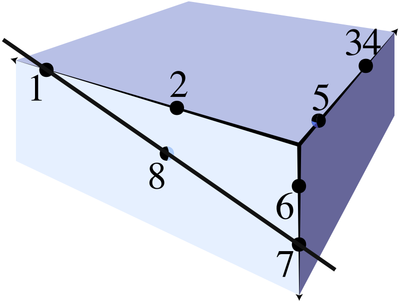

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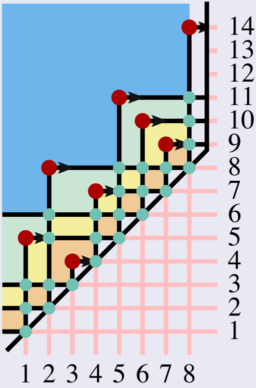

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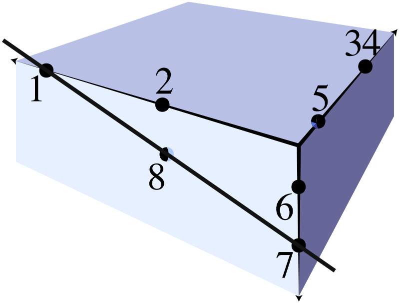

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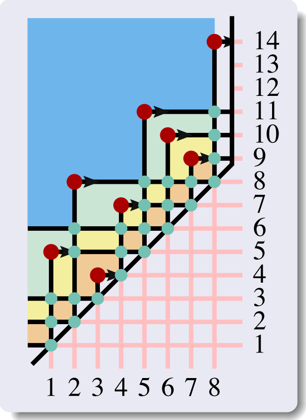

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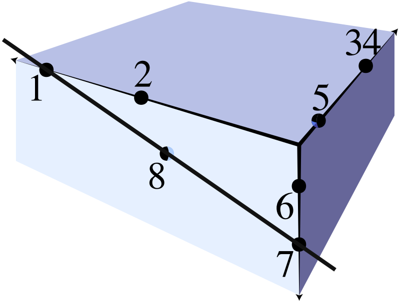

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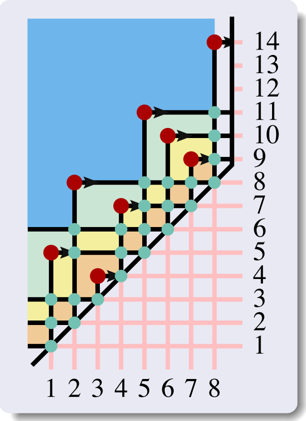

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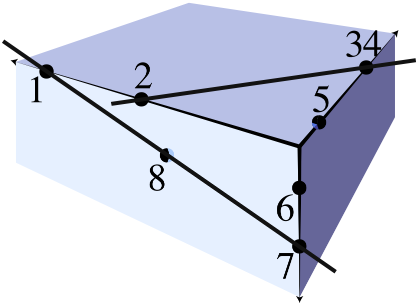

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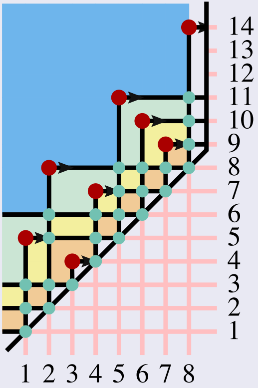

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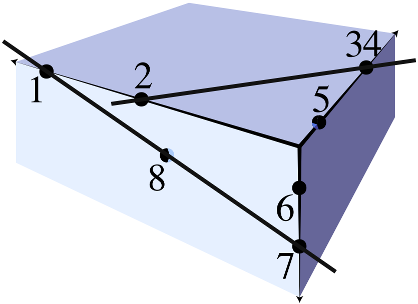

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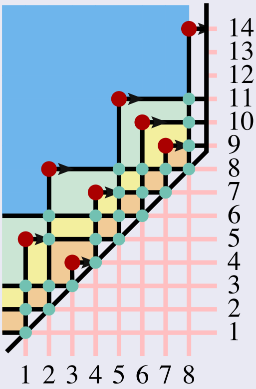

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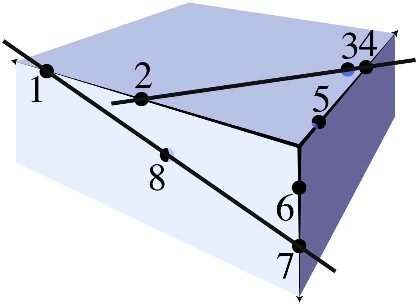

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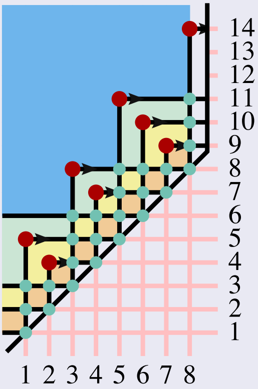

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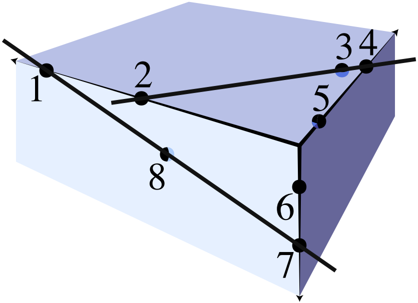

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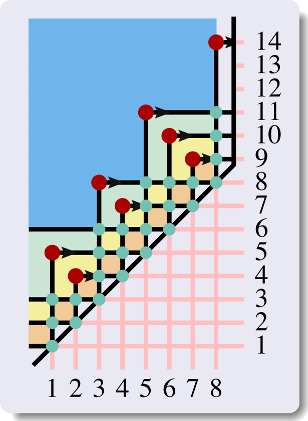

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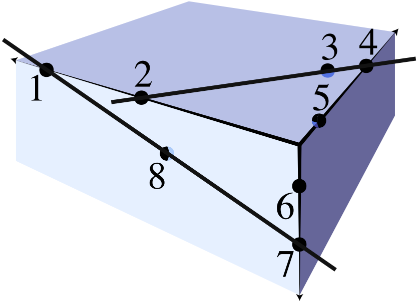

$$C \equiv \begin{array}{c} \\ \end{array}\begin{array}{cccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \\ \hline \end{array}$$

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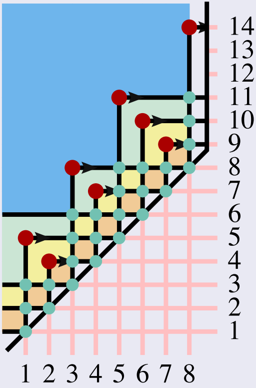
$(2\,3)$: $\;c_3 \mapsto c_3 + \alpha_1\, c_2$



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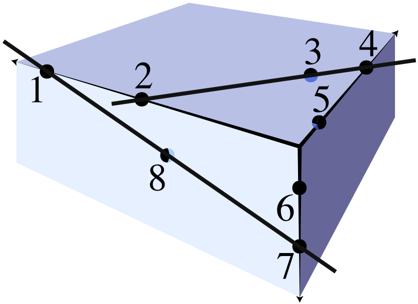

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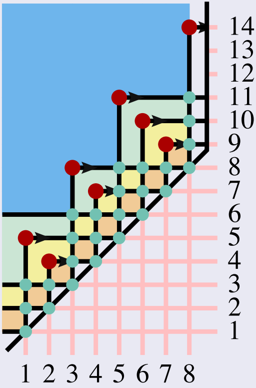

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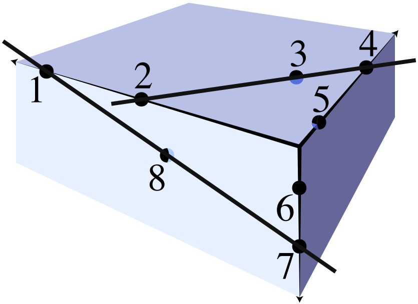

$$
C \equiv \begin{array}{c}
\begin{array}{cccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \end{array} \\
\left(\begin{array}{cccccccc}
1 & \alpha_2 & \alpha_1\alpha_2 & 0 & 0 & \alpha_8 & \alpha_7\alpha_8 & 0 \\
0 & 1 & \alpha_1+\alpha_4 & \alpha_3\alpha_4 & 0 & \alpha_6 & 0 & 0 \\
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\end{array}\right)
\end{array}
$$

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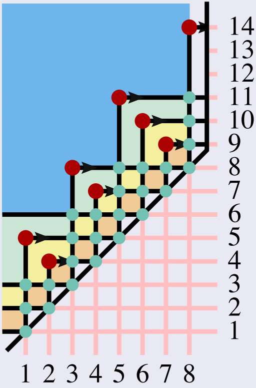

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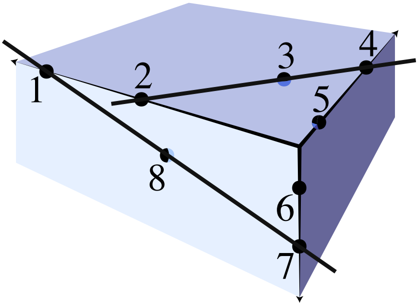

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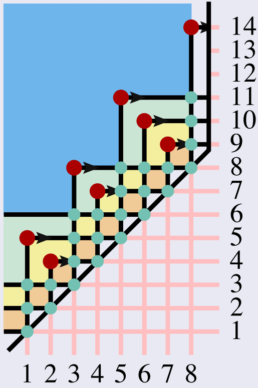

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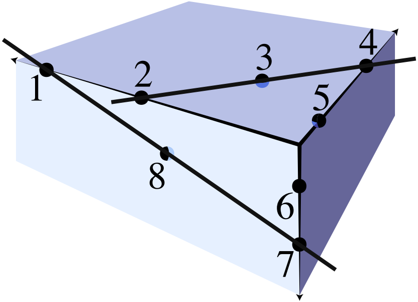

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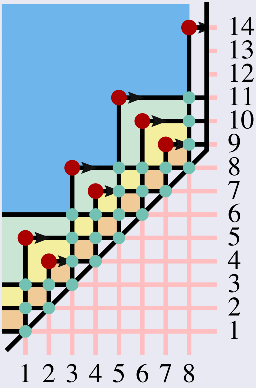

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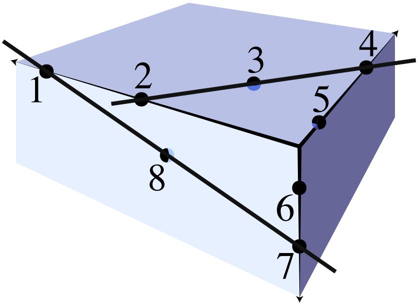

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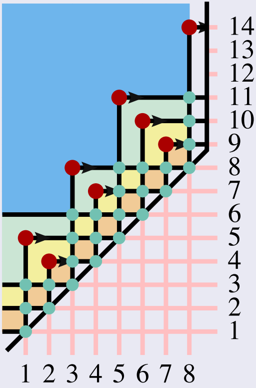

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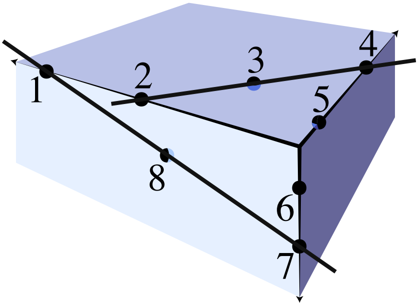

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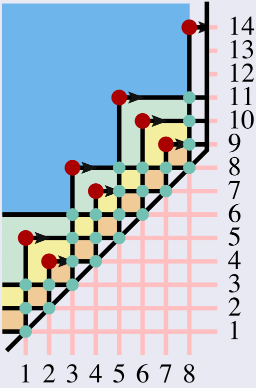

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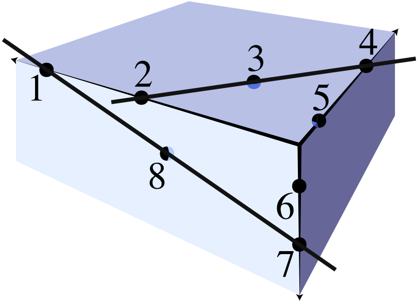

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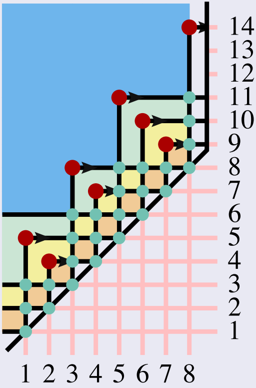

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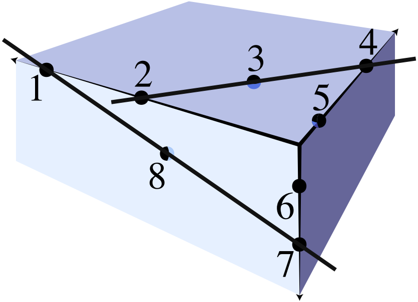

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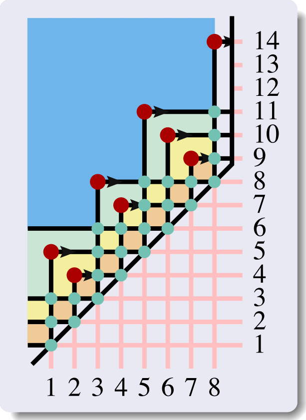

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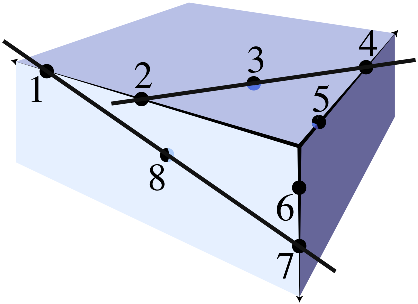

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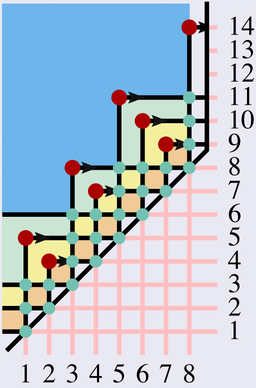

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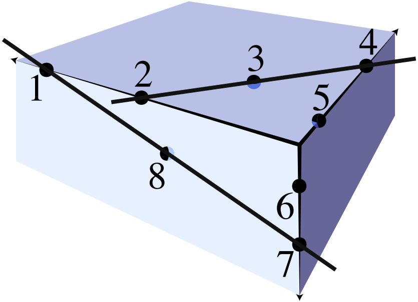

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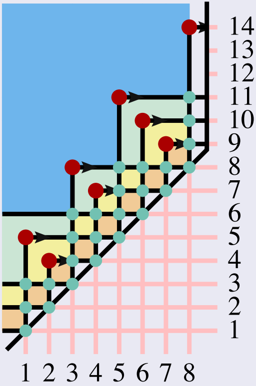

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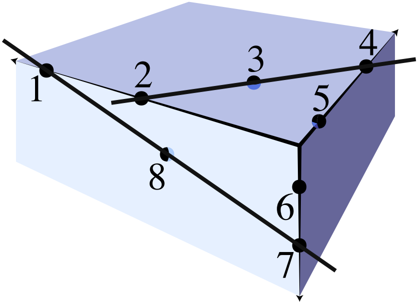

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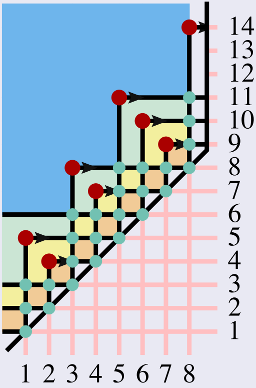

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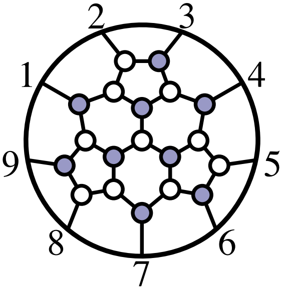
The Combinatorics and Geometry of On-Shell Physics



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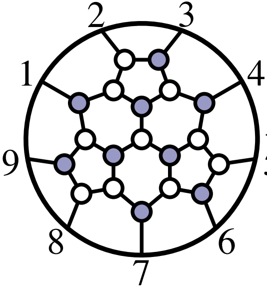

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ & & & & & & & & \\ & & & & & & & & \end{pmatrix}$$

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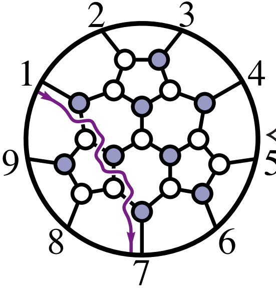

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & & & & & & & & \\ 7 & & & & & & & & \end{pmatrix}$$

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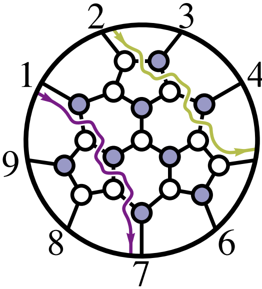

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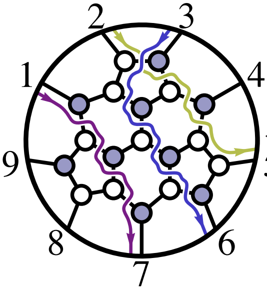

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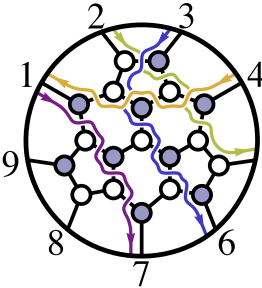

$$\Leftrightarrow \left(\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & & & & & \\ 7 & 5 & 6 & 10 & & & & & \end{array}\right)$$

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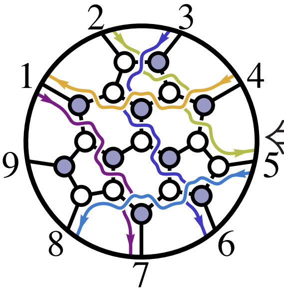

$$\Leftrightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & & \\ 7 & 5 & 6 & 10 & 8 & & & & \end{pmatrix}$$

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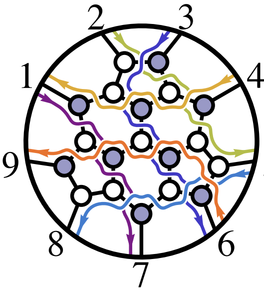

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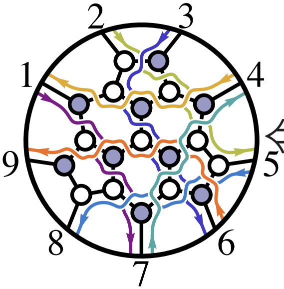

$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & & \end{pmatrix}$$

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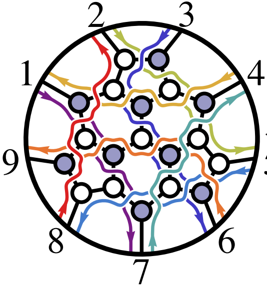

$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\ \downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\\ 7&5&6&10&8&9&13&11&\end{pmatrix}$$

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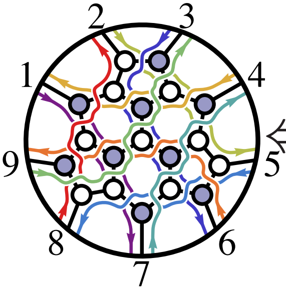
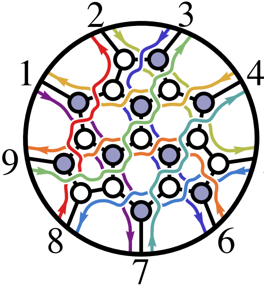


$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\left(\begin{array}{ccccccccc}1&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&1\end{array}\right)\in G_+(4,9)$$

$\swarrow\!\!\!\nearrow$ $\Updownarrow$



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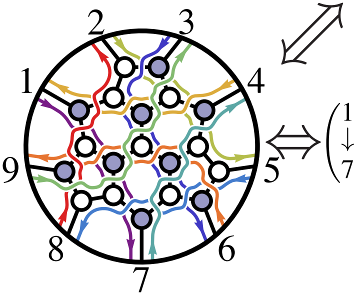
The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

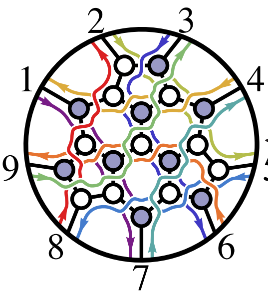


$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&0&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

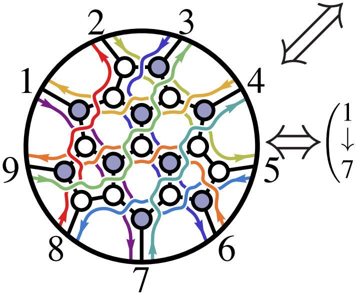


$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & 0 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

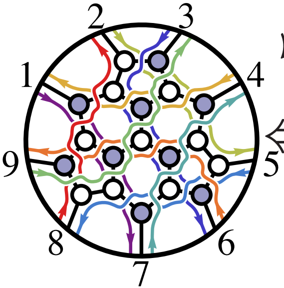


$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&0&0&0&0&0&0&0&0\\0&0&0&1&0&\alpha_4&0&0&0\\0&0&0&0&0&0&1&0&\alpha_3\\0&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&0&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

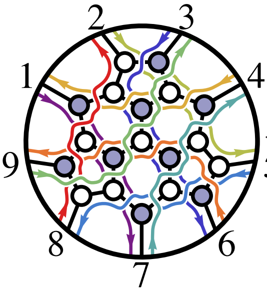


$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & 0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \alpha_3 \\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & 0 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

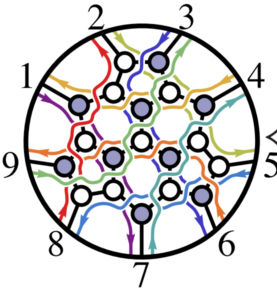


$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & 0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & \alpha_4 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3\\ 0 & 0 & \alpha_1 & 0 & 0 & -\alpha_2\,\alpha_1 & 0 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

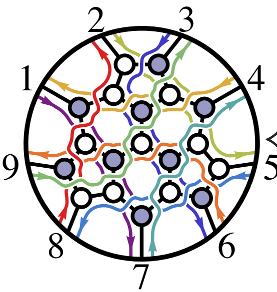


$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&0&\alpha_5&0&0&0&0&0&0\\0&0&0&1&0&\alpha_4&\alpha_7\,\alpha_4&0&0\\0&0&0&0&0&0&1&\alpha_6&\alpha_3\\0&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

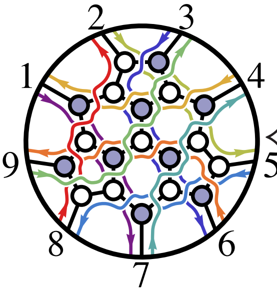


$$\Longleftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\left(\begin{array}{ccccccccc}1&\alpha_8&\alpha_5&0&0&0&0&0&0\\0&0&0&1&0&\alpha_4&\alpha_7\,\alpha_4&0&0\\0&0&0&0&0&0&1&\alpha_6&\alpha_3\\0&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{array}\right)\in G_{+}(4,9)$$

$\Updownarrow$

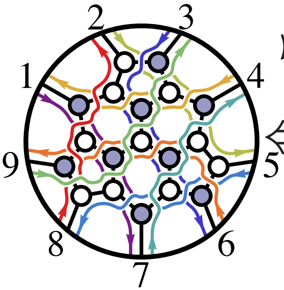


$$\Longleftrightarrow\left(\begin{array}{ccccccccc}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{array}\right)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&\alpha_5&0&0&0&0&0&0\\0&0&0&1&0&\alpha_4&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6&\alpha_3\\-\alpha_9&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Nearrow\qquad\qquad\Updownarrow$$

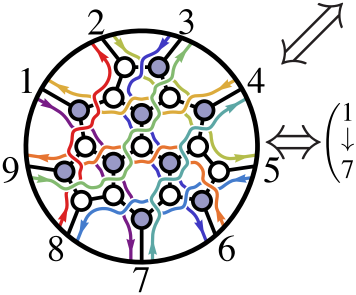


$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&\alpha_5&0&0&0&0&0&0\\0&0&0&1&\alpha_{10}&\alpha_4&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6&\alpha_3\\-\alpha_9&0&\alpha_1&0&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$

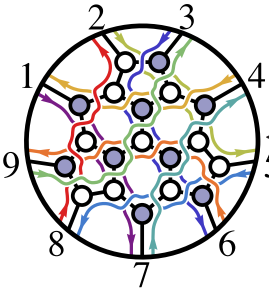


$$\Longleftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & \alpha_5 & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & \alpha_3\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\nearrow\!\!\!\!\swarrow \qquad\qquad \Updownarrow$$

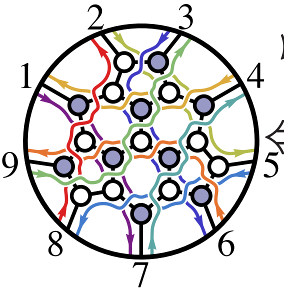


$$\Longleftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & \alpha_5 & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & \alpha_4 & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6\,(\alpha_3+\alpha_{12}\,\alpha_6) \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

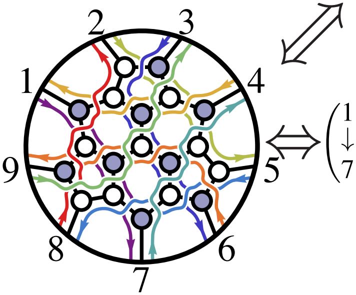
$$\Updownarrow$$



$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & \alpha_5 & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6) \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



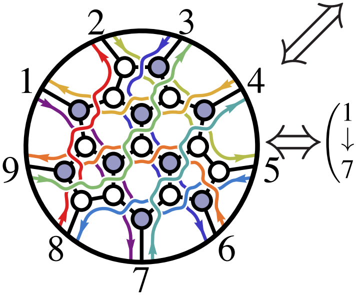
$$\Updownarrow$$

$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\left(\begin{array}{ccccccccc}1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1\end{array}\right)\in G_+(4,9)$$

$$\Updownarrow$$

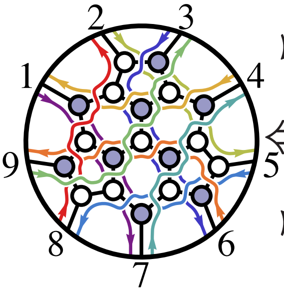


$$\Leftrightarrow\left(\begin{array}{ccccccccc}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12\end{array}\right)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&(\alpha_5+\alpha_{14}\,\alpha_8)&\alpha_{11}\,\alpha_5&0&0&0&0&0\\0&0&0&1&\alpha_{10}&(\alpha_4+\alpha_{13}\,\alpha_{10})&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6&(\alpha_3+\alpha_{12}\,\alpha_6)\\-\alpha_9&0&\alpha_1&\alpha_{11}\,\alpha_1&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$



$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

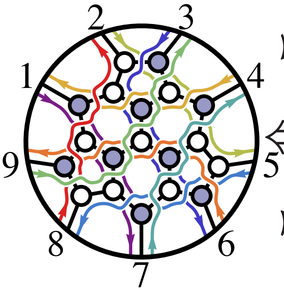
$$\Updownarrow$$

$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\;\delta^{k\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{k\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\Updownarrow$$



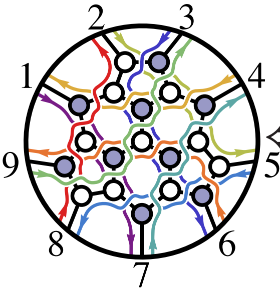
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}$$

$$\Updownarrow$$

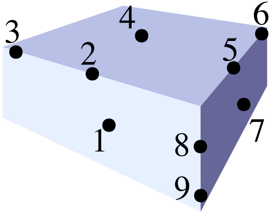
$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\;\delta^{k\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{k\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&(\alpha_5+\alpha_{14}\,\alpha_8)&\alpha_{11}\,\alpha_5&0&0&0&0&0\\0&0&0&1&\alpha_{10}&(\alpha_4+\alpha_{13}\,\alpha_{10})&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6&(\alpha_3+\alpha_{12}\,\alpha_6)\\-\alpha_9&0&\alpha_1&\alpha_{11}\,\alpha_1&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$



$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

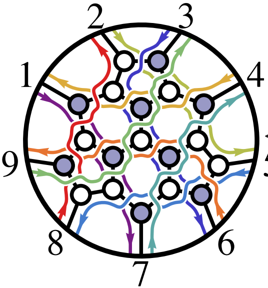


$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\;\;\delta^{k\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{k\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

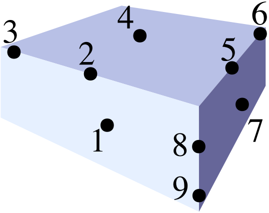
The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$

$$\swarrow\!\nearrow \qquad \Updownarrow \qquad \nwarrow\!\searrow$$



$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$

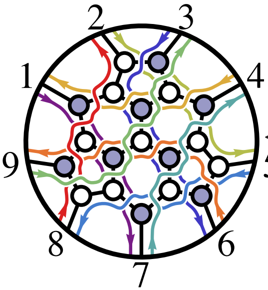


$$\nwarrow\!\searrow \qquad \Updownarrow$$

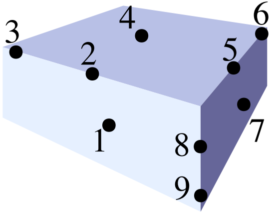
$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\;\delta^{k\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{k\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0\\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6)\\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



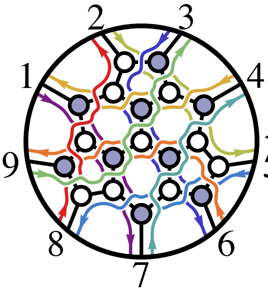
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow\\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$



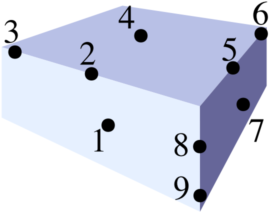
$$f_\sigma\equiv\int\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\;\delta^{k\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{k\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times(n-k)}\big(\lambda\cdot C(\alpha)^{\perp}\big)$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix} 1 & \alpha_8 & (\alpha_5+\alpha_{14}\,\alpha_8) & \alpha_{11}\,\alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha_{10} & (\alpha_4+\alpha_{13}\,\alpha_{10}) & \alpha_7\,\alpha_4 & 0 & 0 \\ -\alpha_9\,\alpha_3 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_6 & (\alpha_3+\alpha_{12}\,\alpha_6) \\ -\alpha_9 & 0 & \alpha_1 & \alpha_{11}\,\alpha_1 & 0 & -\alpha_2\,\alpha_1 & -\alpha_7\,\alpha_2\,\alpha_1 & 0 & 1 \end{pmatrix}\in G_+(4,9)$$



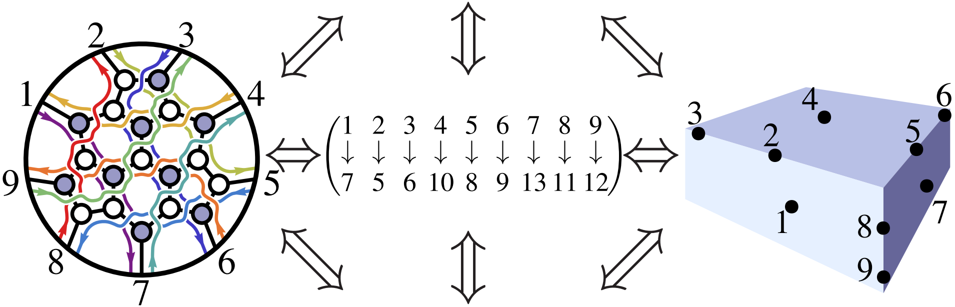
$$\Leftrightarrow\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 5 & 6 & 10 & 8 & 9 & 13 & 11 & 12 \end{pmatrix}\Leftrightarrow$$



$$f_\sigma\equiv\oint_\sigma\frac{d^{4\times9}C}{GL(4)}\,\frac{\delta^{k\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{k\times2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C^\perp\big)}{\langle1234\rangle\langle2345\rangle\langle3456\rangle\langle4567\rangle\langle5678\rangle\langle6789\rangle\langle7891\rangle\langle8912\rangle\langle9123\rangle}$$

The Combinatorics and Geometry of On-Shell Physics

$$C(\alpha)\equiv\begin{pmatrix}1&\alpha_8&(\alpha_5+\alpha_{14}\,\alpha_8)&\alpha_{11}\,\alpha_5&0&0&0&0&0\\0&0&0&1&\alpha_{10}&(\alpha_4+\alpha_{13}\,\alpha_{10})&\alpha_7\,\alpha_4&0&0\\-\alpha_9\,\alpha_3&0&0&0&0&0&1&\alpha_6&(\alpha_3+\alpha_{12}\,\alpha_6)\\-\alpha_9&0&\alpha_1&\alpha_{11}\,\alpha_1&0&-\alpha_2\,\alpha_1&-\alpha_7\,\alpha_2\,\alpha_1&0&1\end{pmatrix}\in G_+(4,9)$$



$$\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}$$

$$f_\sigma\equiv\oint\limits_\sigma\frac{d^{4\times9}C}{GL(4)}\frac{\delta^{k\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{k\times2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C^\perp\big)}{\langle1234\rangle\langle2345\rangle\langle3456\rangle\langle4567\rangle\langle5678\rangle\langle6789\rangle\langle7891\rangle\langle8912\rangle\langle9123\rangle}$$

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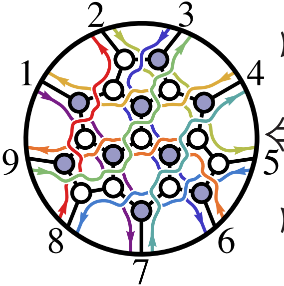
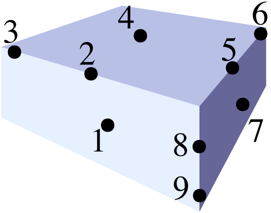
$$\Updownarrow$$



$$\Leftrightarrow\begin{pmatrix}1&2&3&4&5&6&7&8&9\\\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow&\downarrow\\7&5&6&10&8&9&13&11&12\end{pmatrix}\Leftrightarrow$$



$$\Updownarrow$$

$$f_\sigma\equiv\oint_\sigma\frac{d^{4\times9}C}{GL(4)}\,\frac{\delta^{k\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{k\times2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C^{\perp}\big)}{\langle1234\rangle\langle2345\rangle\langle3456\rangle\langle4567\rangle\langle5678\rangle\langle6789\rangle\langle7891\rangle\langle8912\rangle\langle9123\rangle}$$

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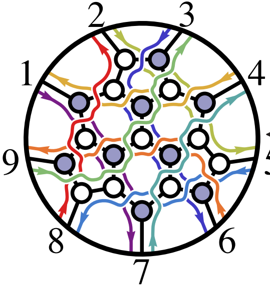
⇗ ⇕ ⇘



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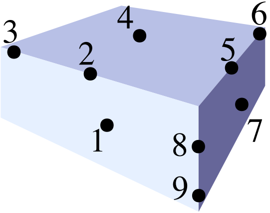

⇘ ⇕ ⇗

$$f_\sigma\equiv\oint_\sigma\frac{d^{4\times9}C}{GL(4)}\,\frac{\delta^{k\times4}\big(C\cdot\widetilde{\eta}\big)\,\delta^{k\times2}\big(C\cdot\widetilde{\lambda}\big)\,\delta^{2\times(n-k)}\big(\lambda\cdot C^{\perp}\big)}{\langle1234\rangle\langle2345\rangle\langle3456\rangle\langle4567\rangle\langle5678\rangle\langle6789\rangle\langle7891\rangle\langle8912\rangle\langle9123\rangle}$$

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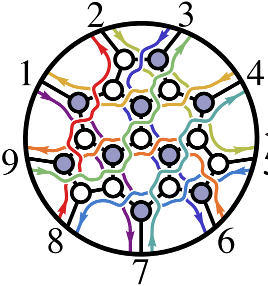

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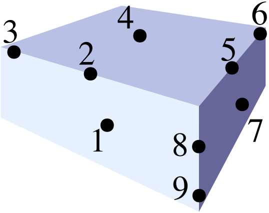

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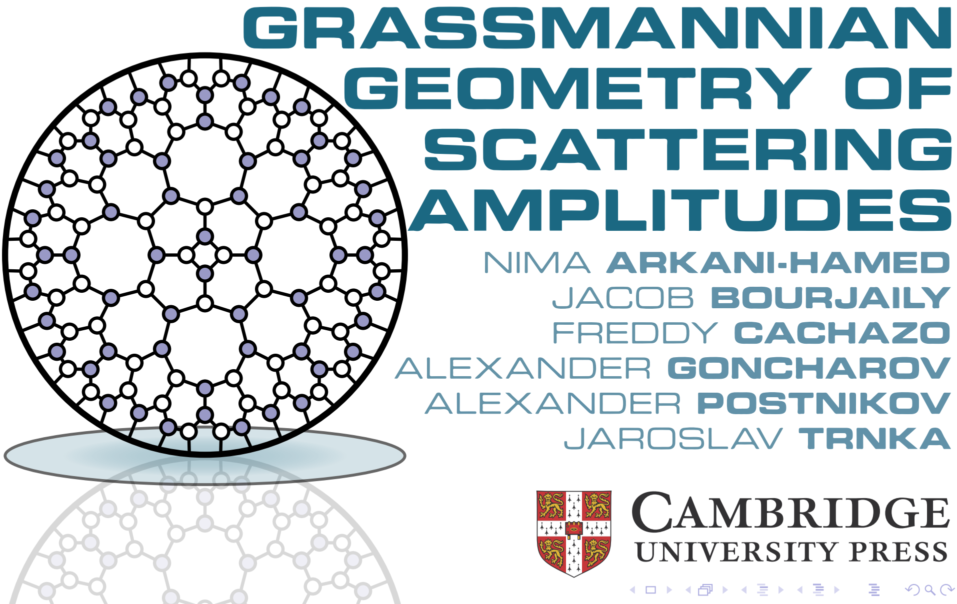

GRASSMANNIAN
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FREDDY **CACHAZO**
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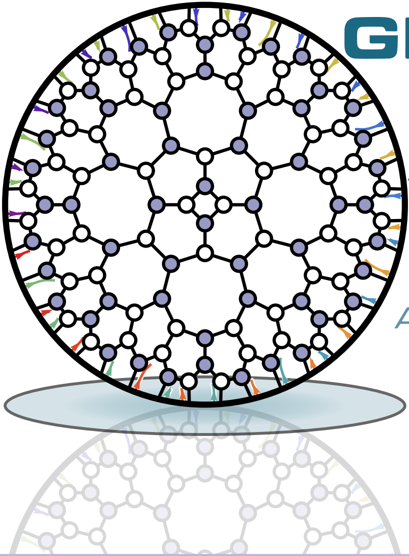

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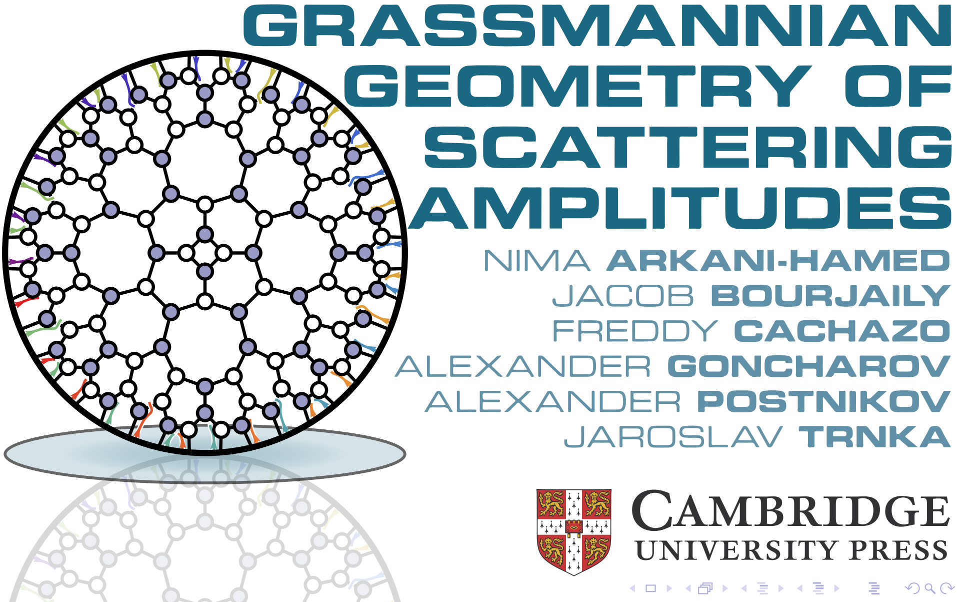

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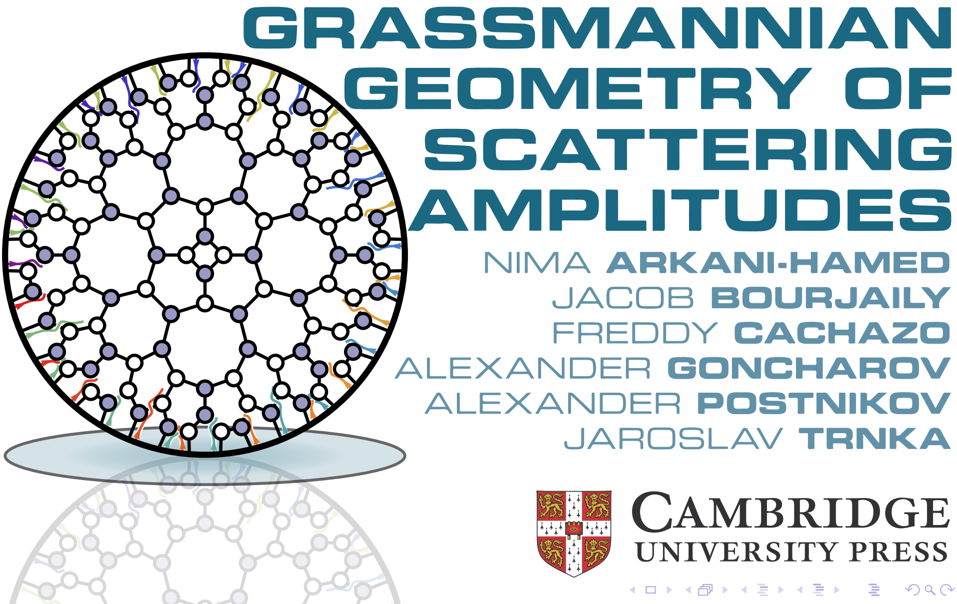

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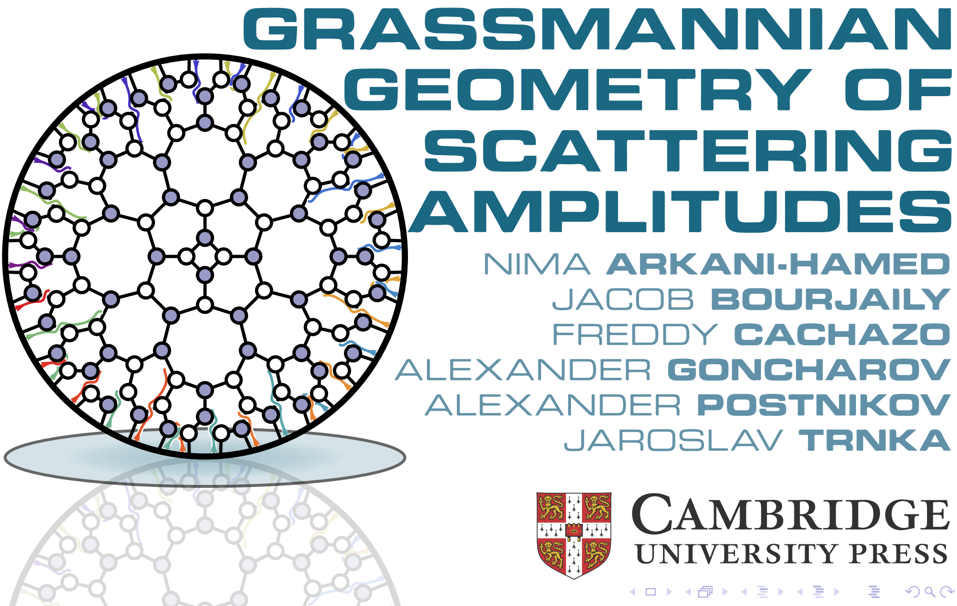

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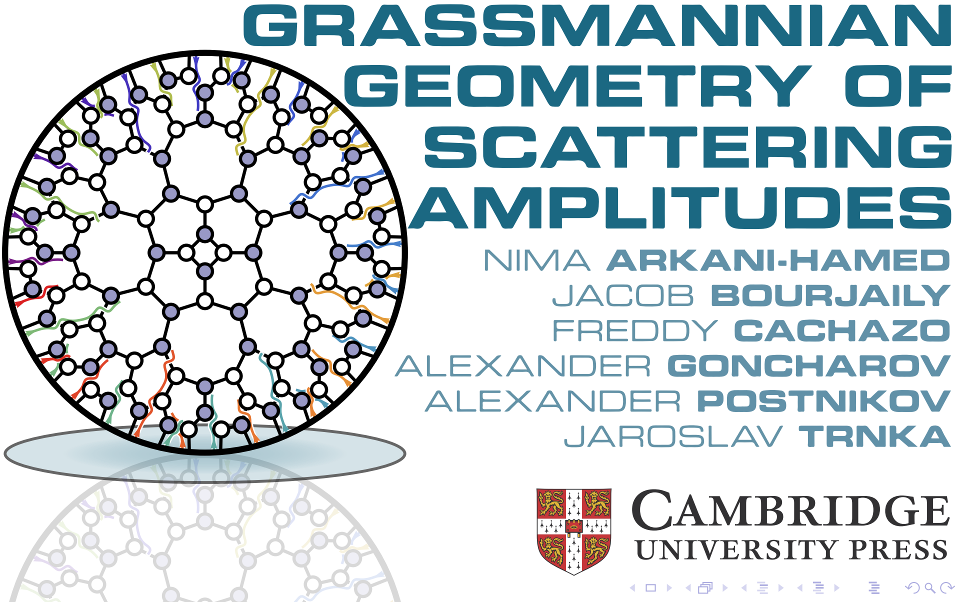

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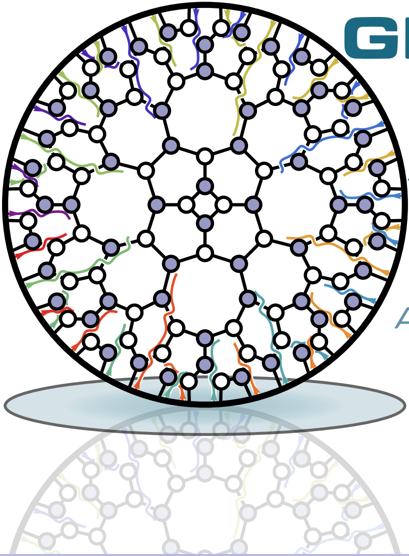

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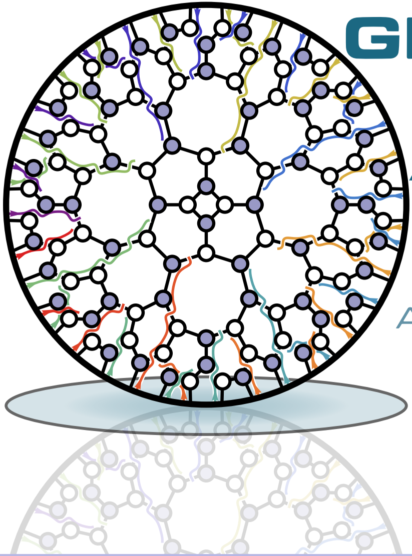

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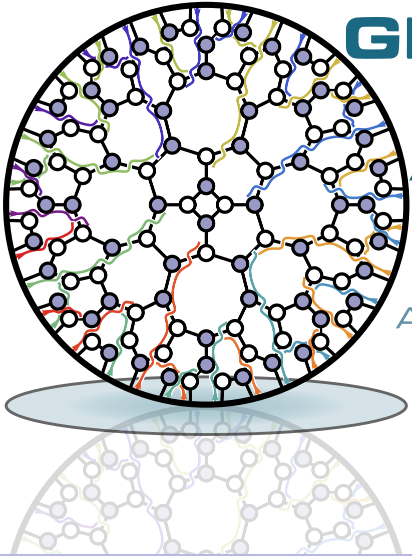

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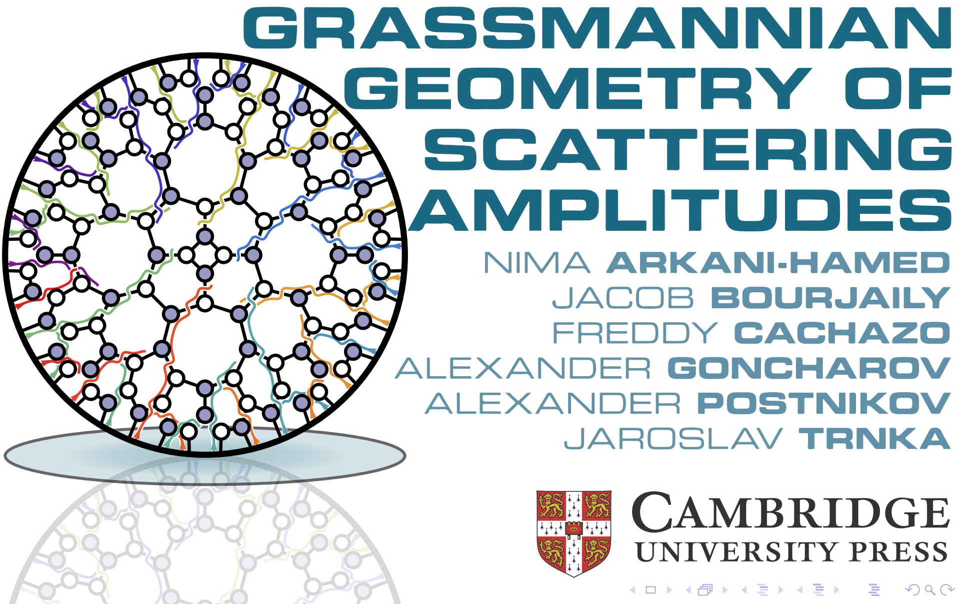

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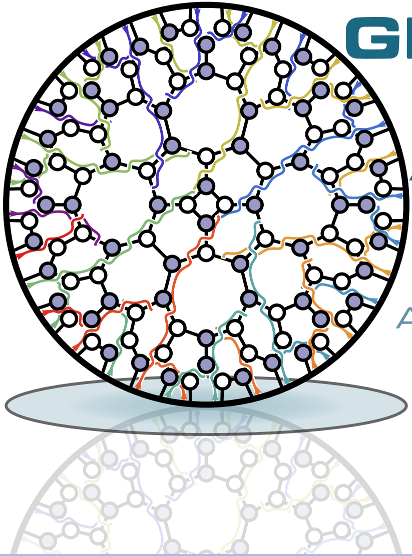

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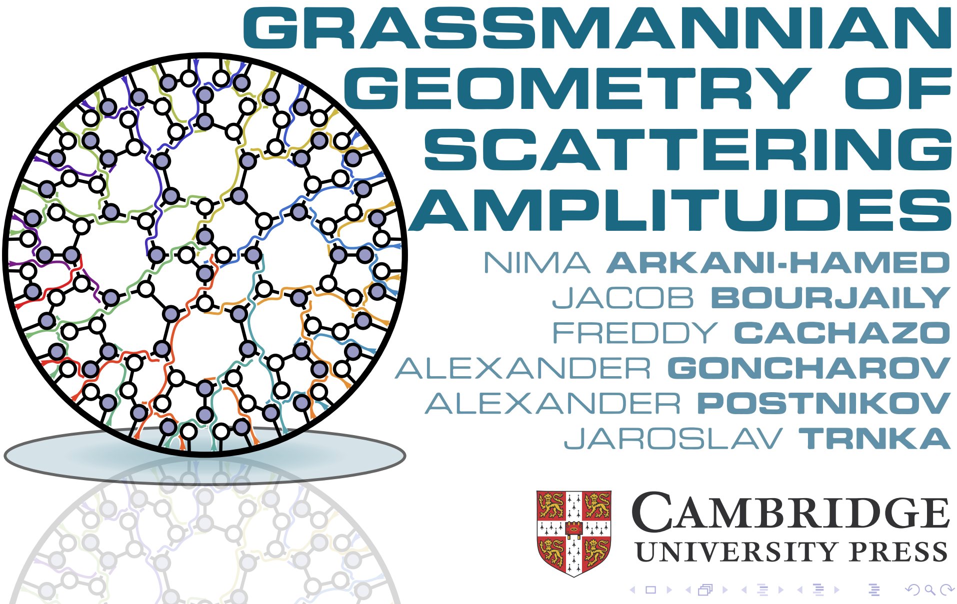

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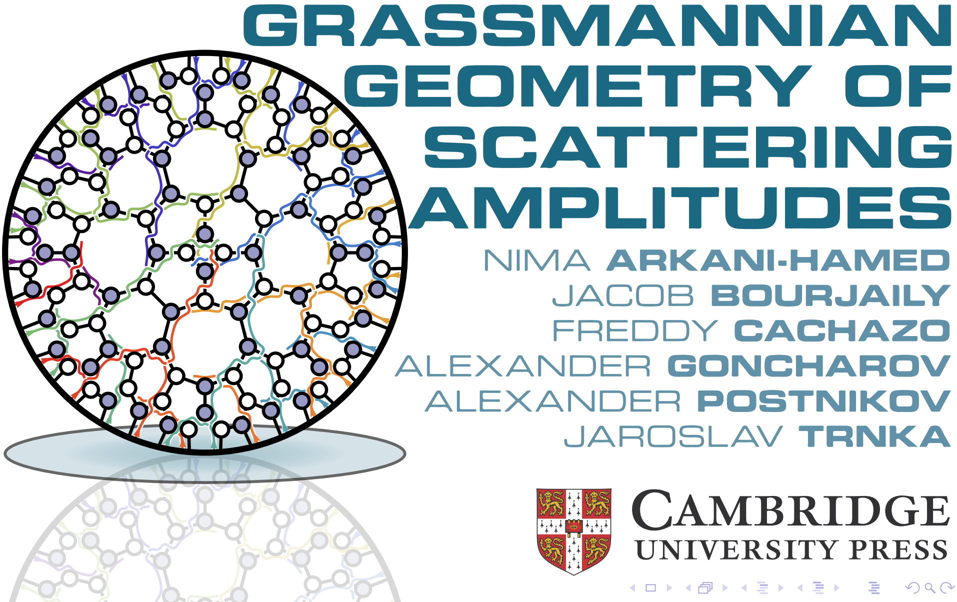

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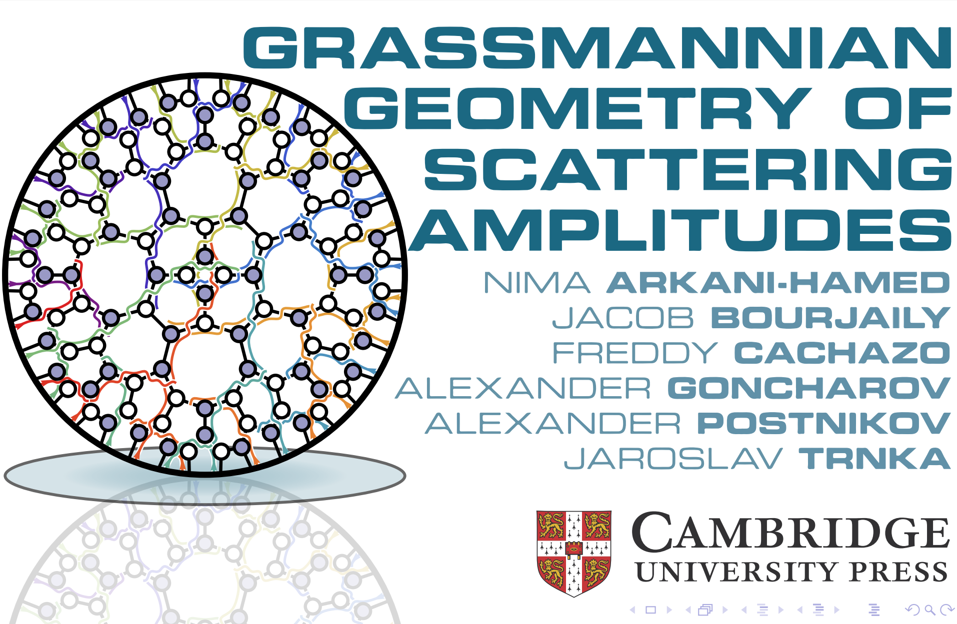

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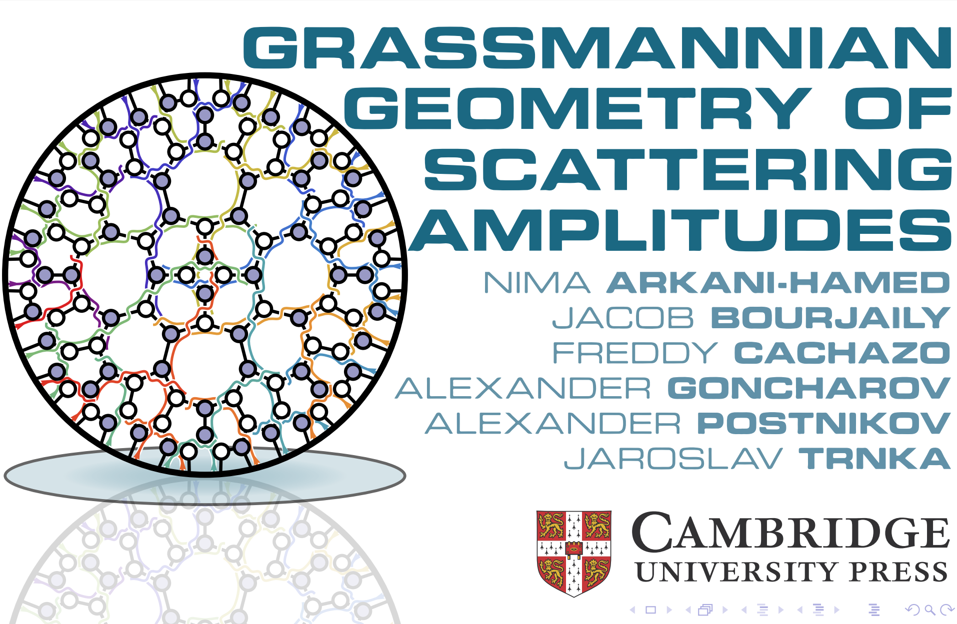

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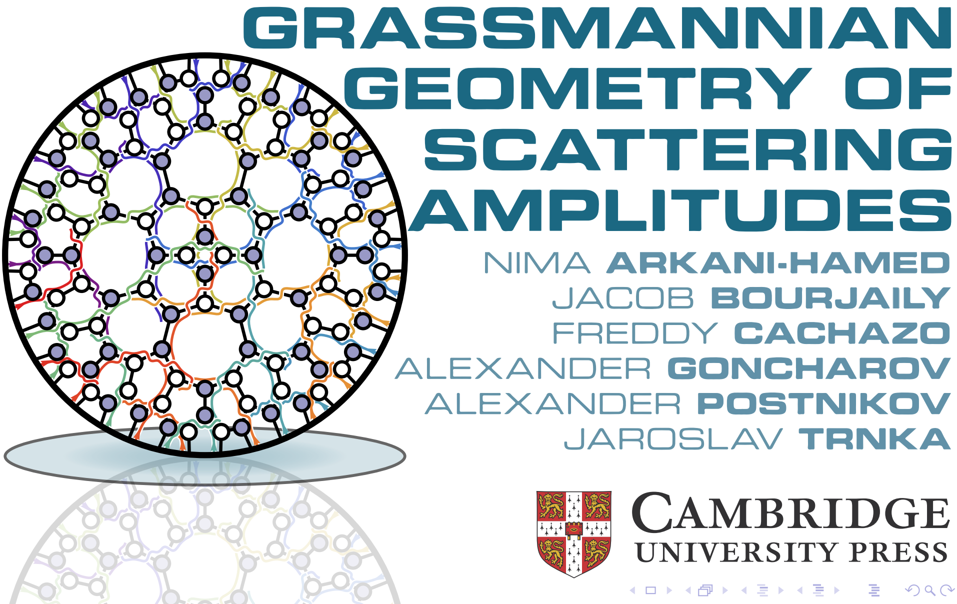

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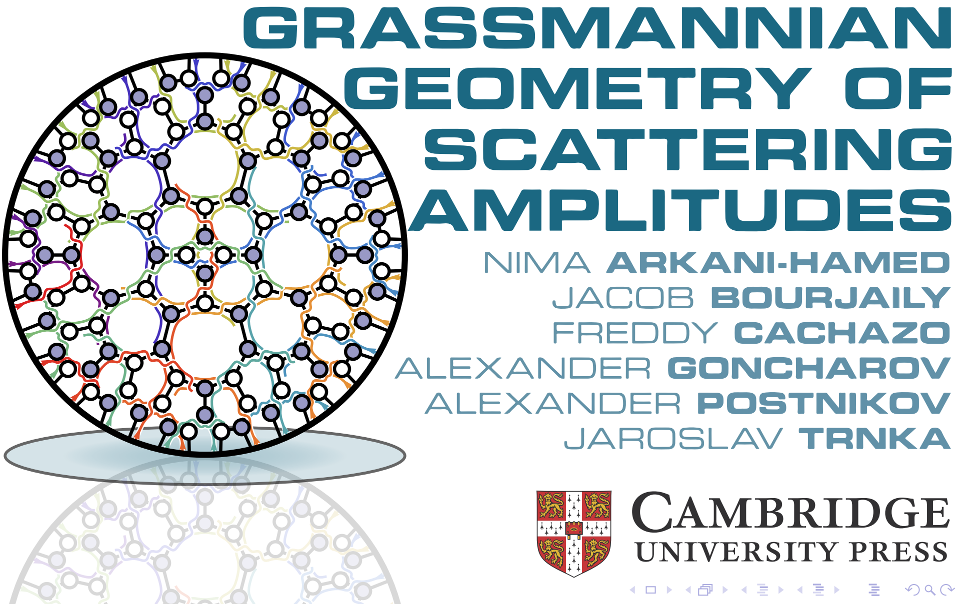

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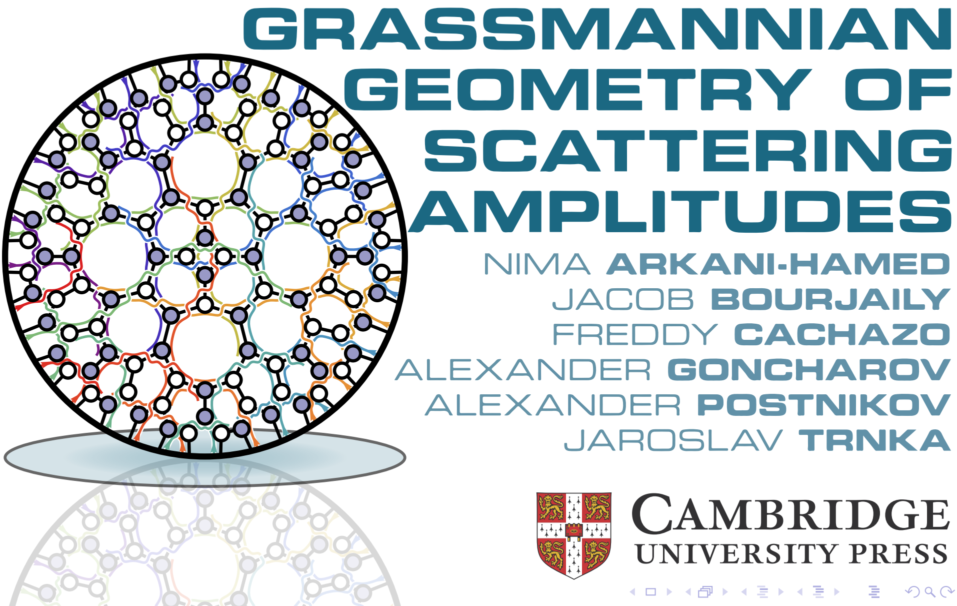

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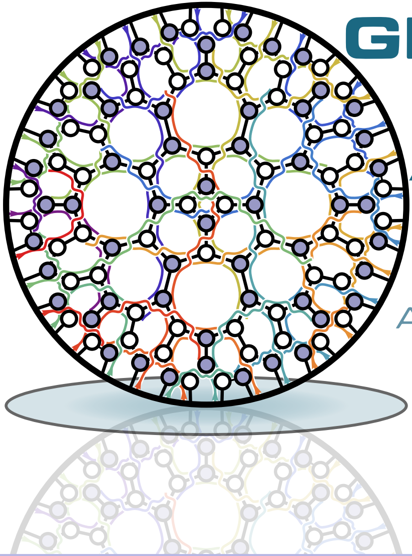

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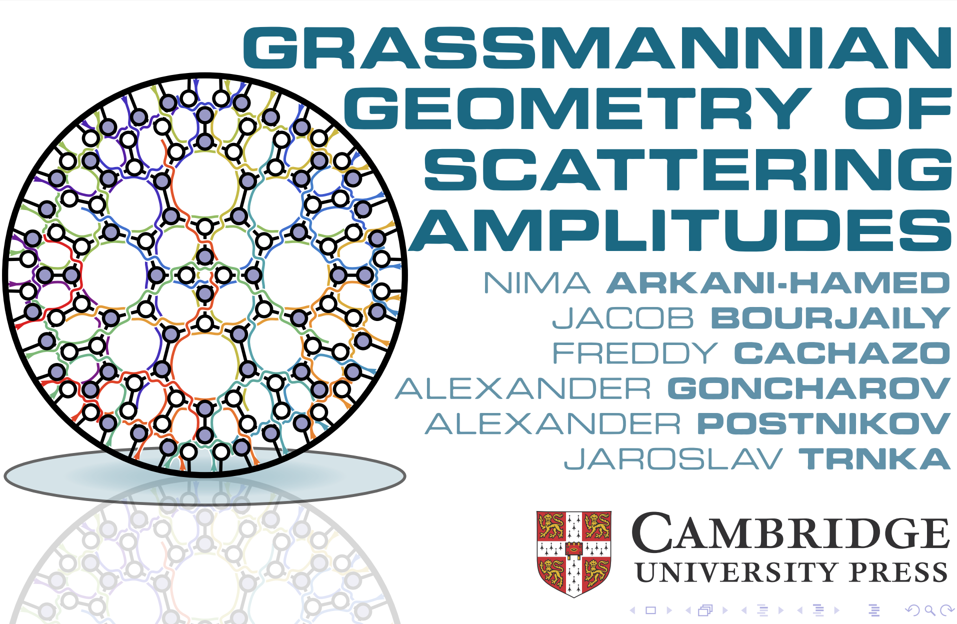

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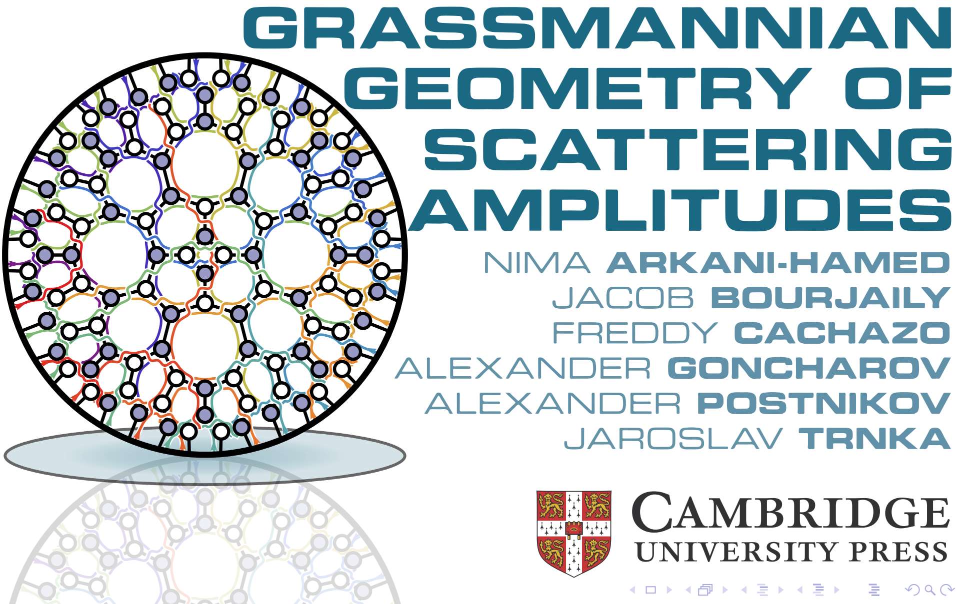

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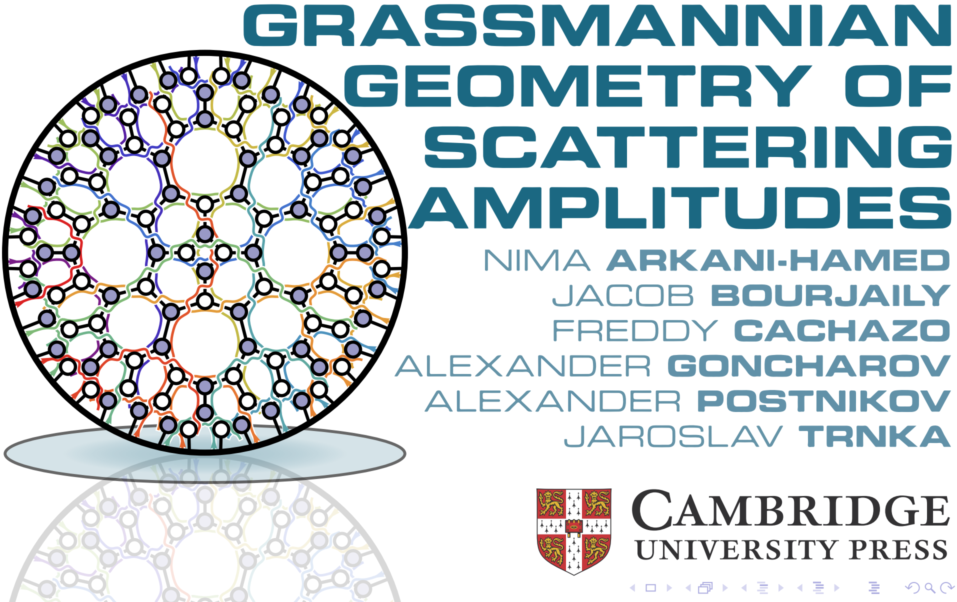

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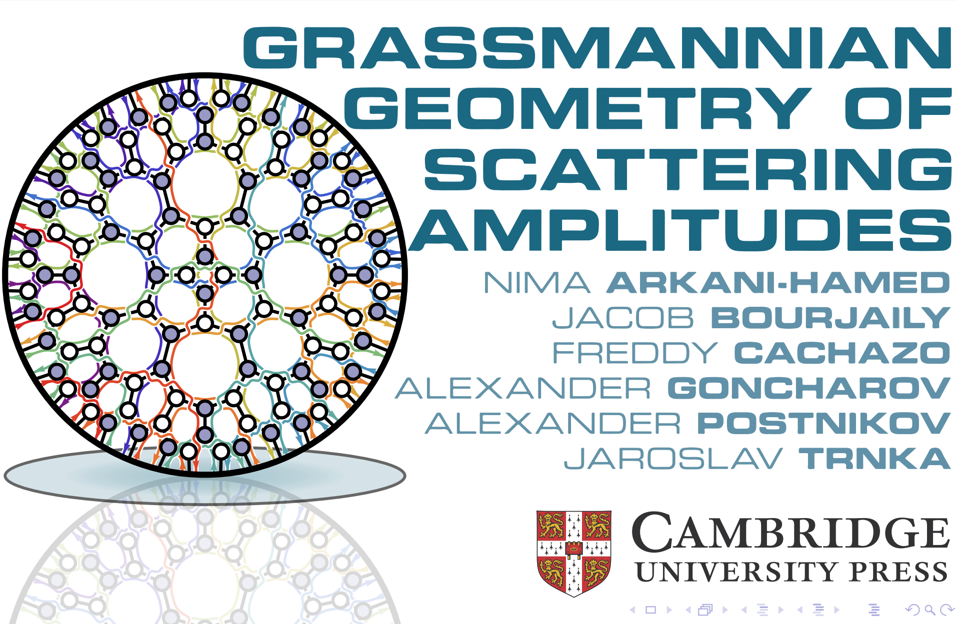

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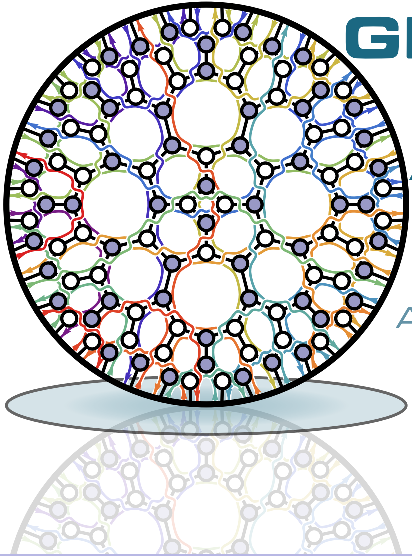

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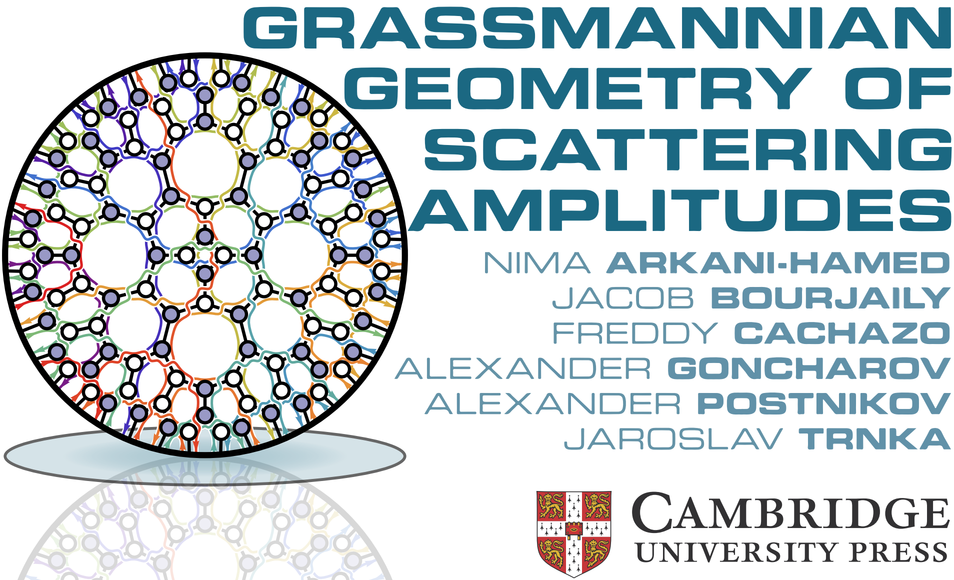

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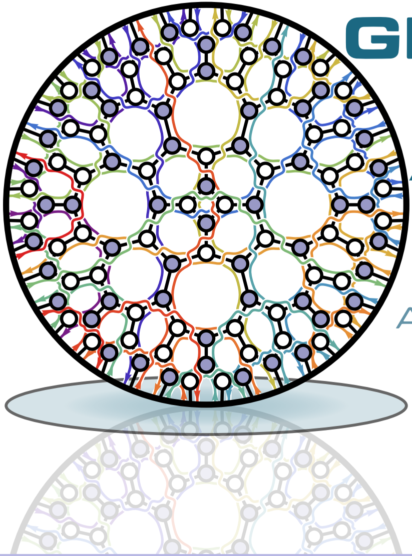

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JACOB **BOURJAILY**
FREDDY **CACHAZO**
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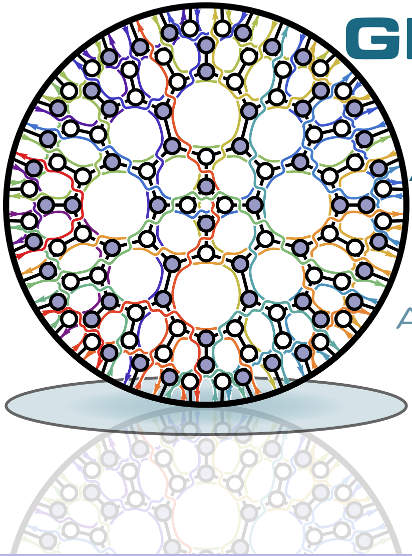

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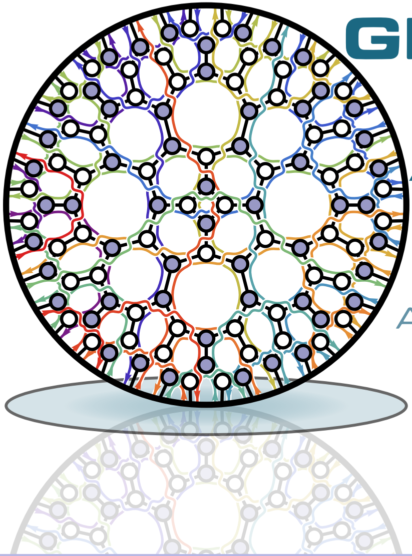

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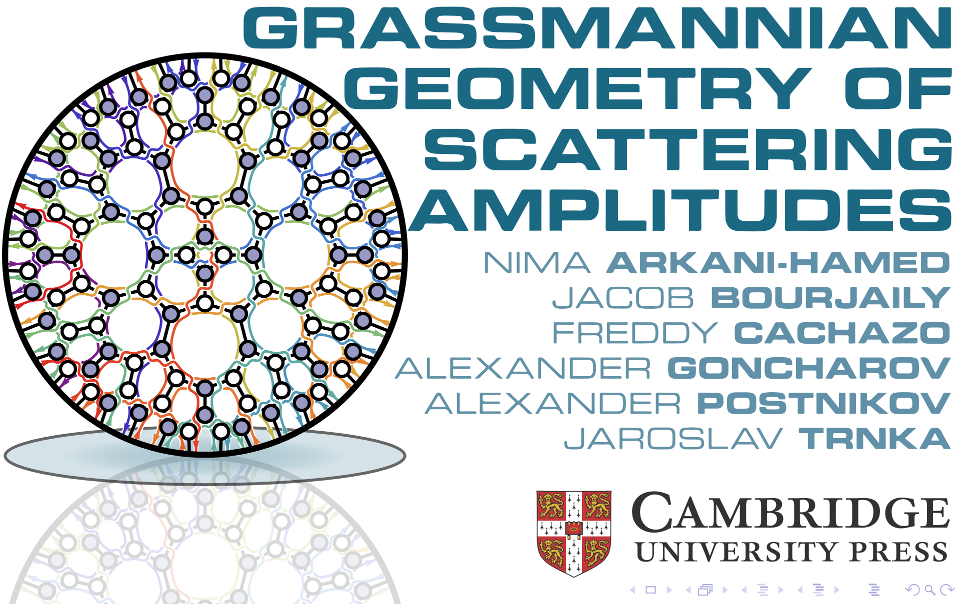

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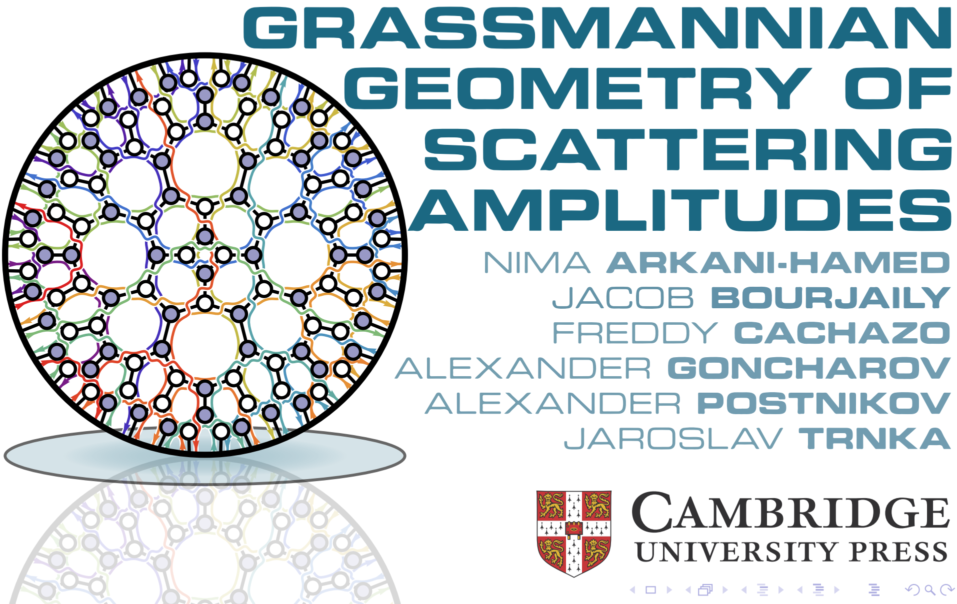

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