A SHORT INTRODUCTION TO SUPERSTRING THEORY

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OUTLINE

(CLOSED) SUPERSTRINGS IN D=10 1

OPEN STRINGS AND D-BRANES 2

COMPACTIFICATION, DUALITIES, … 3

SUPERSYMMETRY BREAKING 4

OPEN STRINGS

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\, \partial_\alpha X \cdot \partial^\alpha X$$

eqs. of motion

$$\Box X^\mu = 0$$

Bdry conditions $\sigma = 0, \pi$

$$\partial_\sigma X = 0 \qquad \text{or} \qquad \partial_\tau X = 0$$

Neumann — Dirichlet

NN

$$X(\sigma,\tau) = x + 2\alpha' p\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n}{n}\, e^{-in\tau}\, \cos(n\sigma)$$

x: centre of mass position

$2\alpha' p\tau$: centre of mass momentum

DD

$$X(\sigma,\tau) = x + \delta\,\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n}{n}\, e^{-in\tau}\, \sin(n\sigma)$$

x: position $\sigma = 0$

δ: length

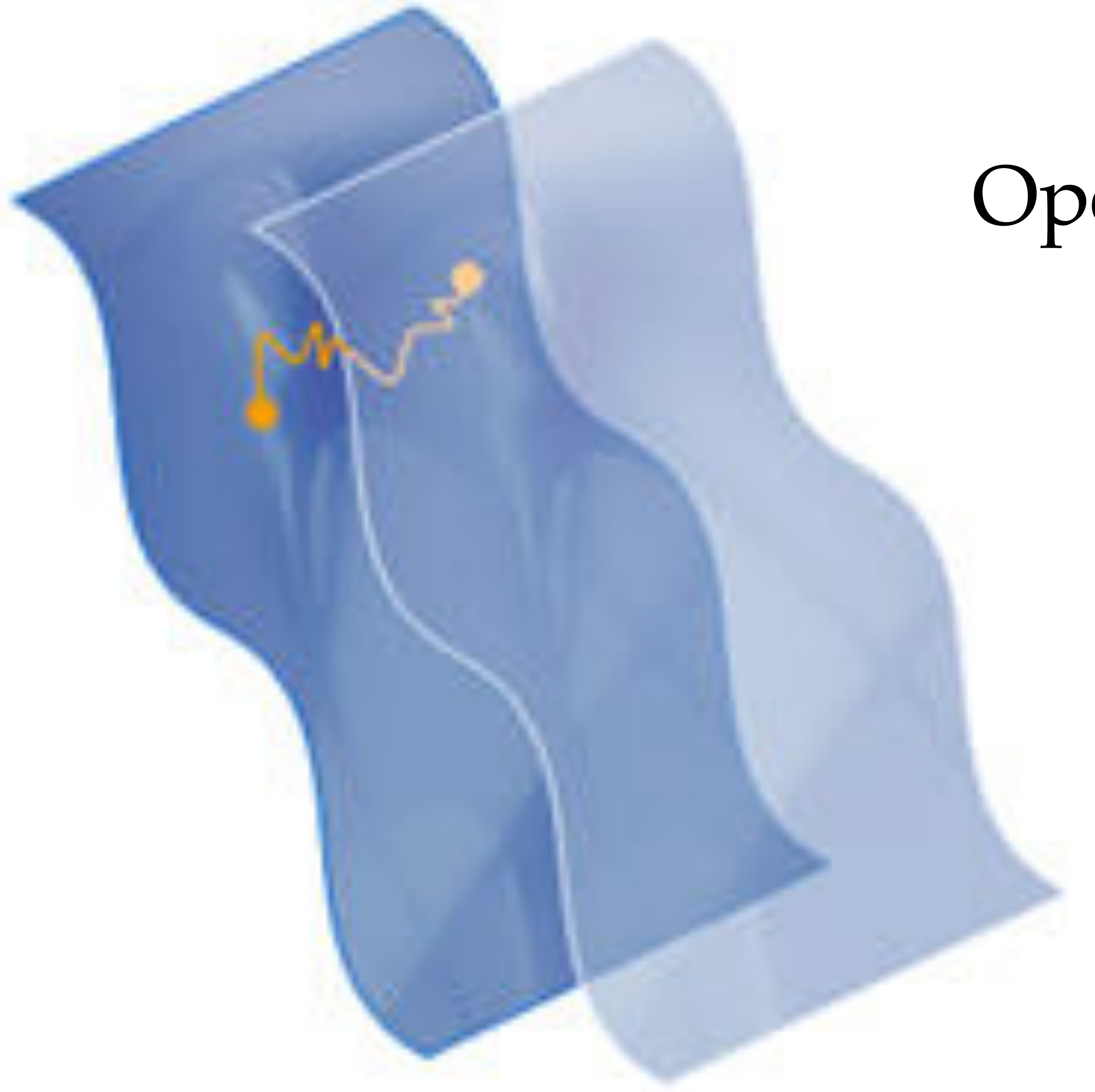
D-BRANES

NN: $X^\mu(\sigma,\tau) = x^\mu + 2\alpha' p^\mu \tau + \ldots$

$\mu = 0,\ldots,p$

DD: $X^i(\sigma,\tau) = x^i + \delta^i\,\sigma + \ldots$

$i = p+1,\ldots,D-1$



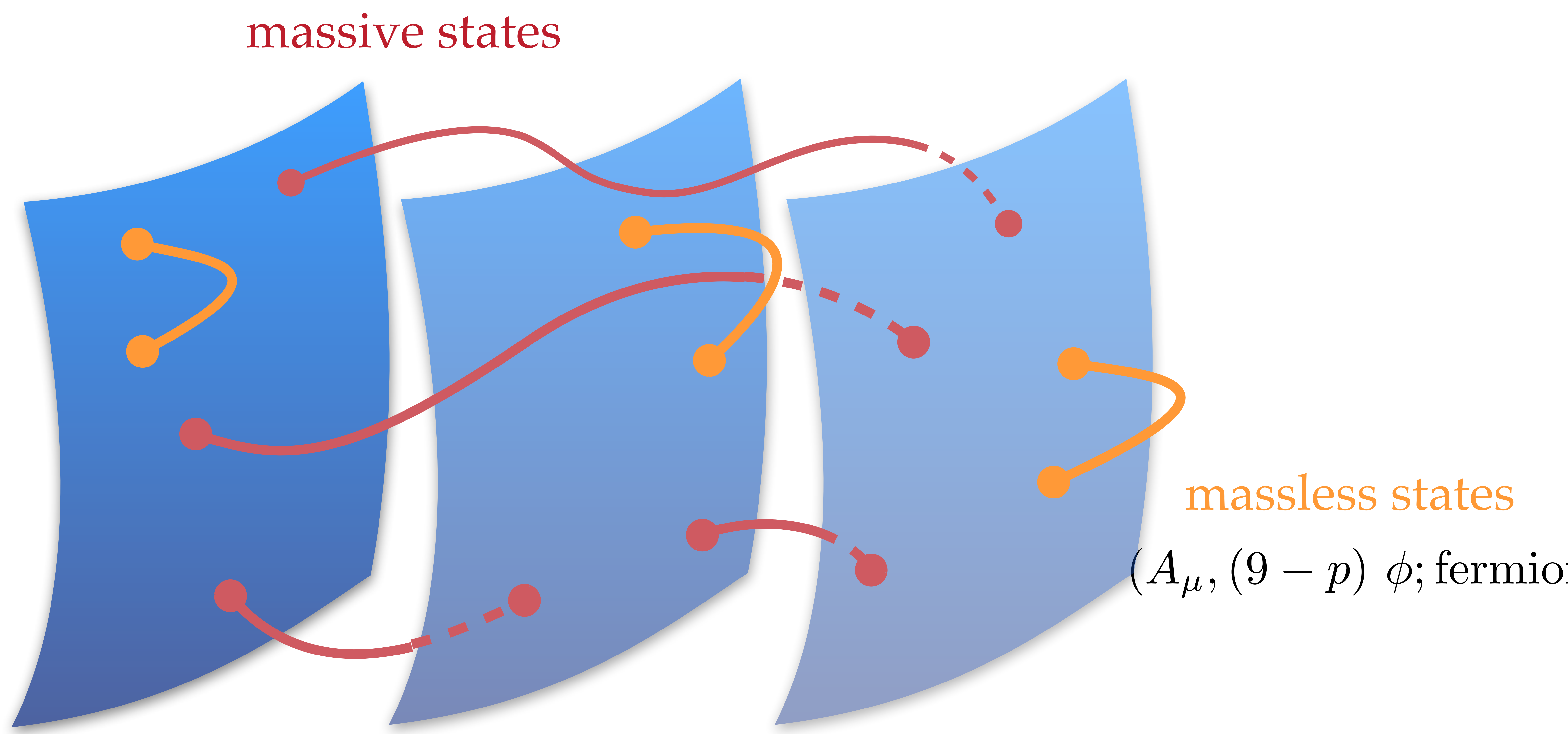
Open strings propagate only on a (p+1)-dimensional hypersurface in the target space

LIGHT EXCITATIONS ON A D-BRANE $(D = 10)$

$$M^2 = \frac{1}{\alpha'}\left[N_X + N_\psi + \Delta\right] + \delta^i \delta^i$$

$\Delta_{\text{NS-NS}} = -\frac{1}{2}$

$\Delta_{\text{R-R}} = 0$



$(A_\mu, (9-p)\ \phi; \text{fermions})$

LIGHT EXCITATIONS ON A D-BRANE

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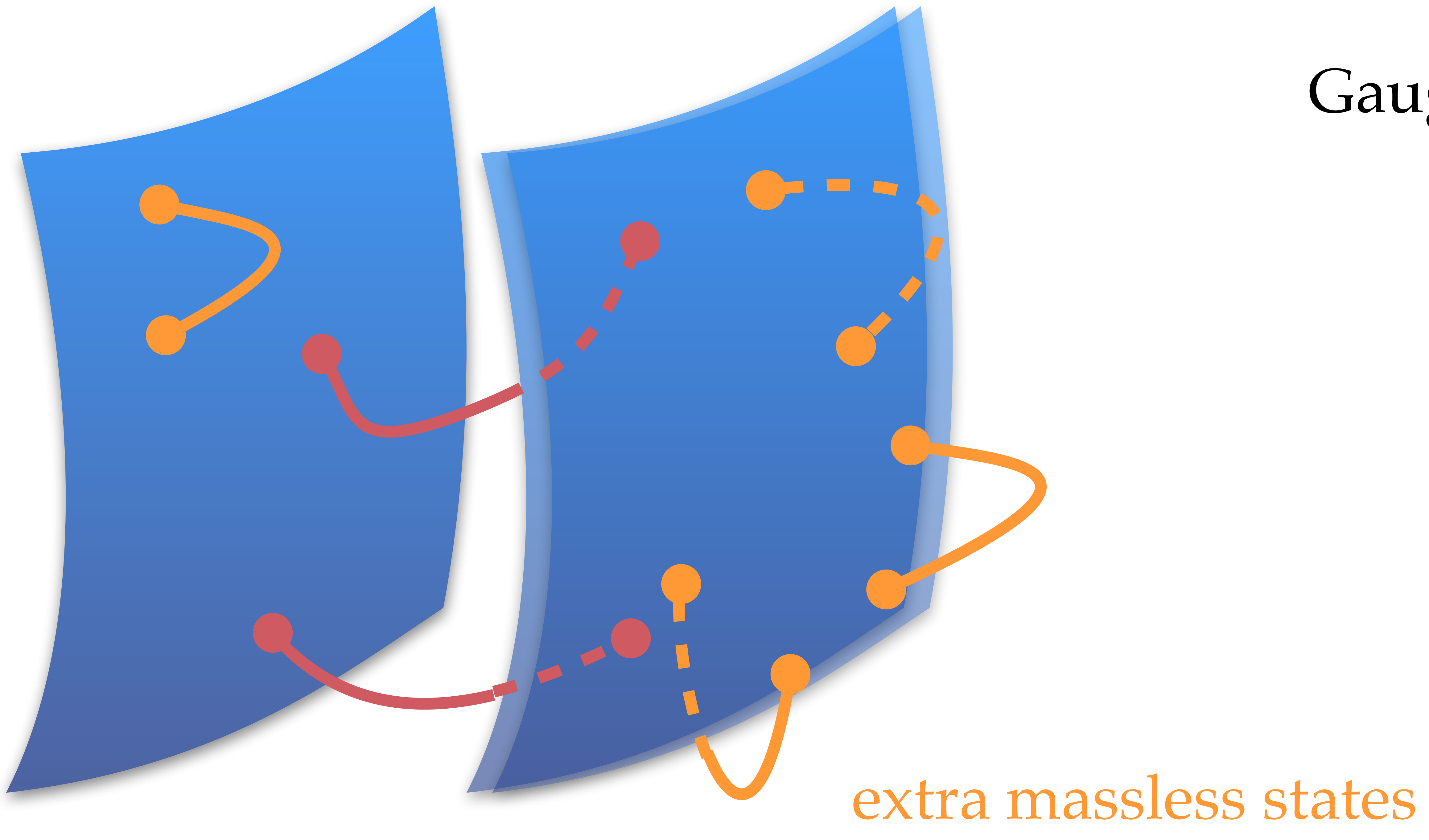


Gauge symmetry enhancement

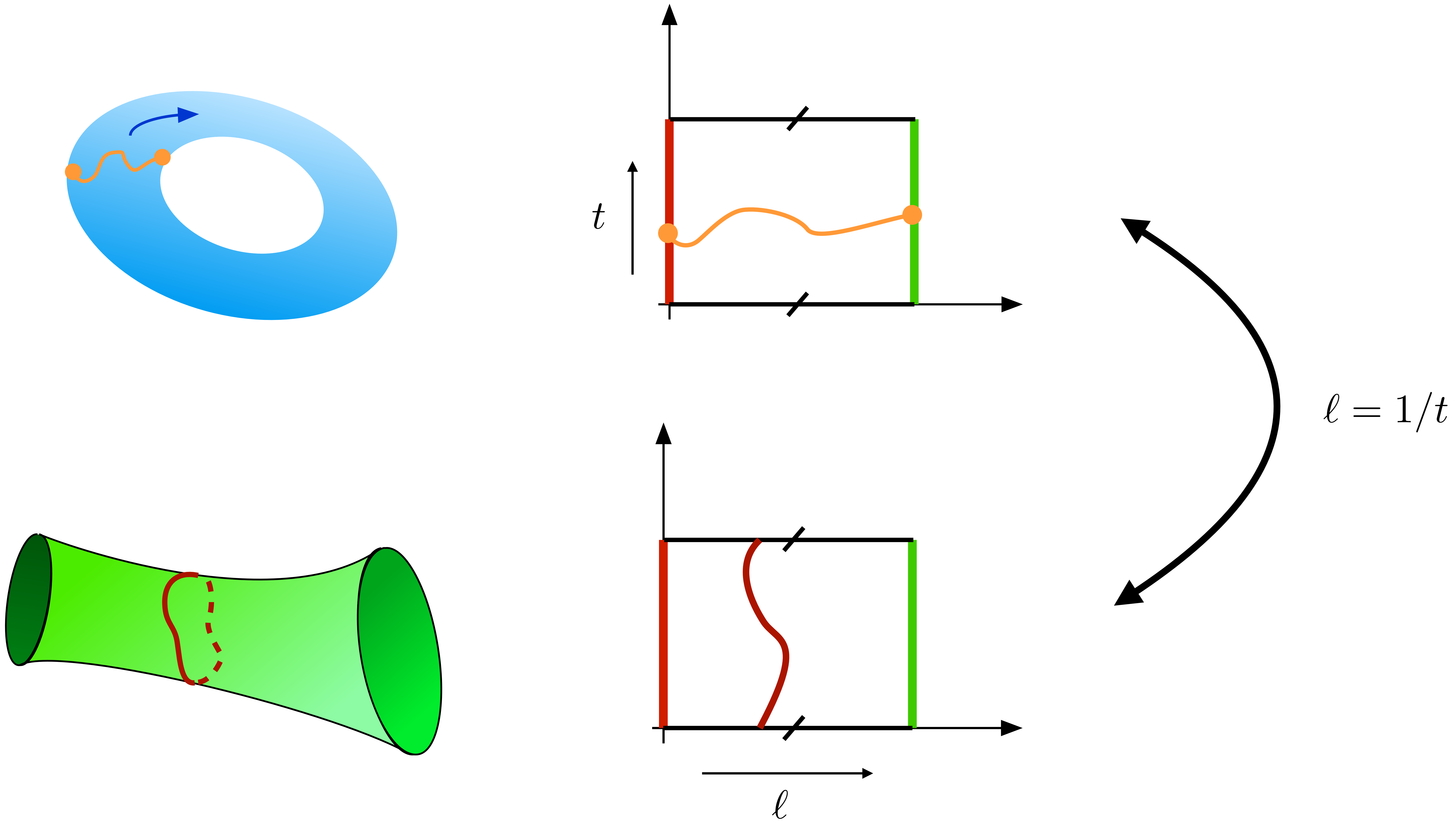
$$\mathrm{U}(1) \times \mathrm{U}(1) \to \mathrm{U}(2)$$

N coincident branes

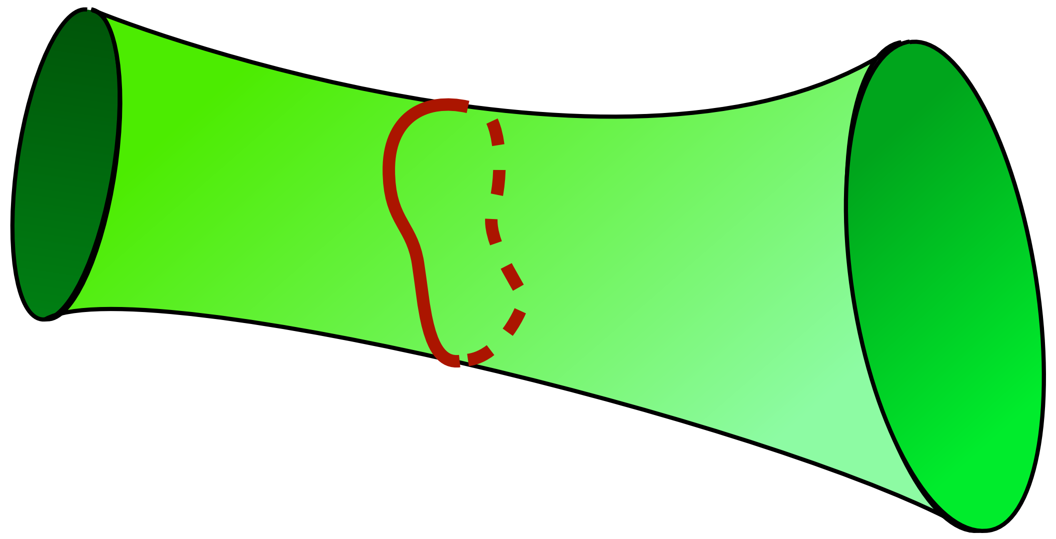
$$\mathrm{U}(1)^N \to \mathrm{U}(N)$$

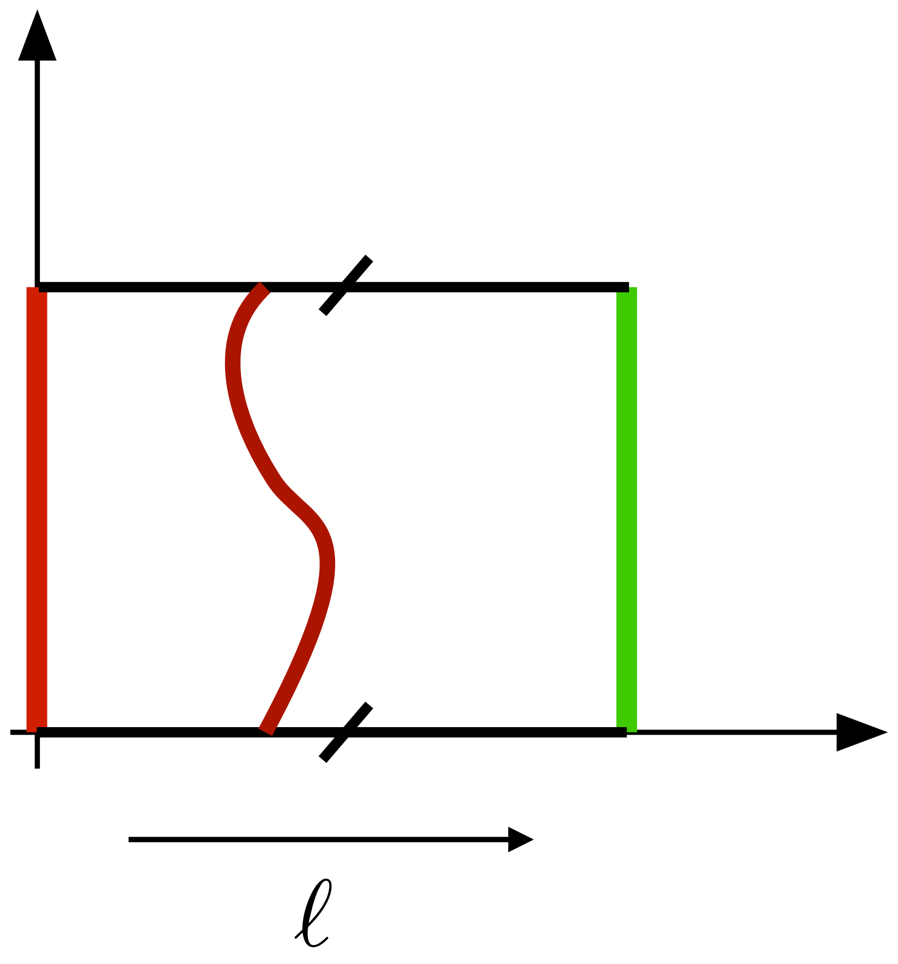
THE OPEN STRING PARTITION FUNCTION: OPEN CHANNEL

$$\mathcal{A} = \int_0^\infty \frac{dt}{t}\,\mathrm{tr}\left(e^{\pi t(\Box - M^2)}\right)$$



THE OPEN STRING PARTITION FUNCTION: CLOSED CHANNEL

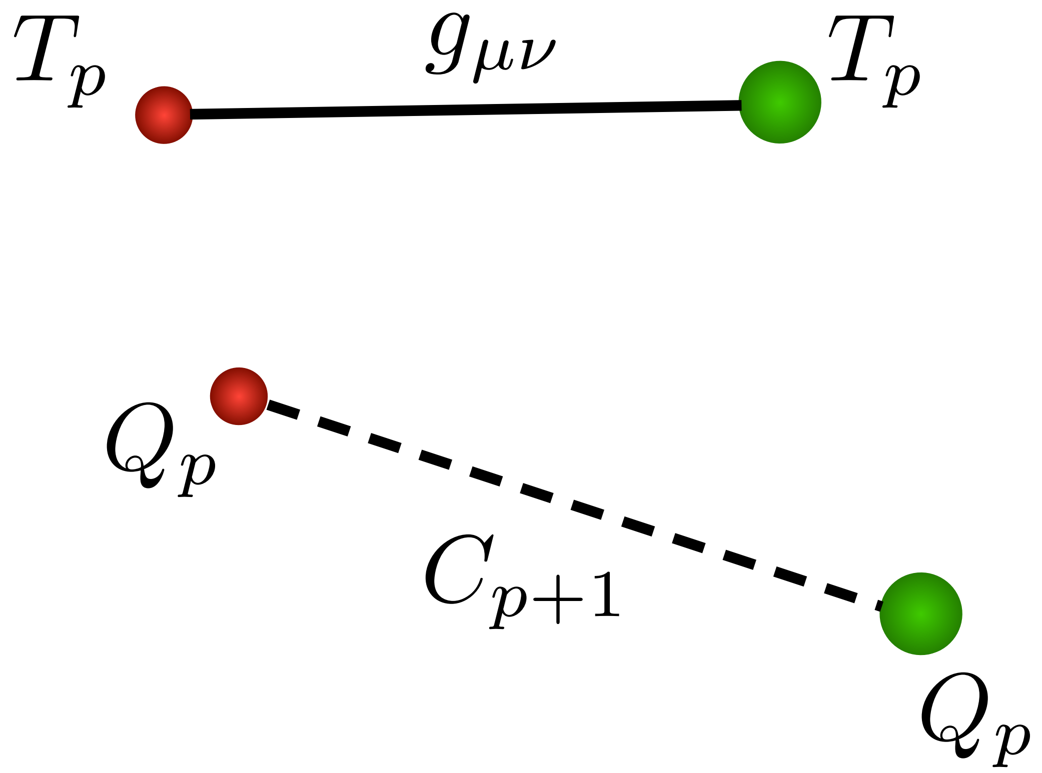




$$\tilde{\mathcal{A}} = \int_0^\infty d\ell \sum_{\substack{\text{closed}\\ \text{states}}} B(n)\, \Delta_n(\ell)\, B(n)$$

$\ell \to \infty$

only massless states propagate



D-BRANES

Hypersurfaces defined by the boundary conditions.

Open string describe the excitations of these hypersurfaces.

Solitonic objects sourcing the closed-string fields:

Graviton, dilaton, RR C_p form field

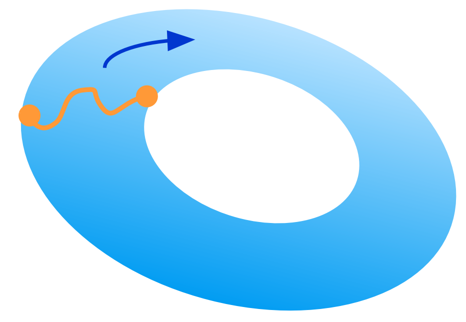
ORIENTIFOLD CONSTRUCTION

Modding out by world-sheet parity

$$\Omega : \qquad \sigma \to -\sigma$$

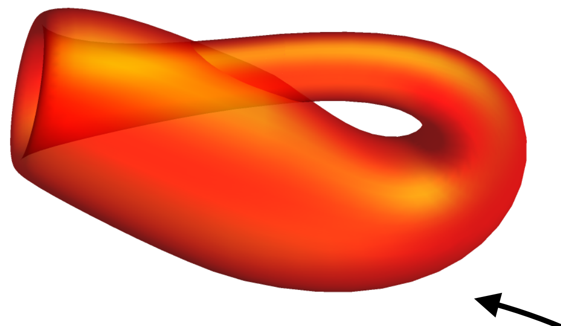
The invariant spectrum

$$\mathrm{tr}_{\mathrm{closed}} \left(\frac{1+\Omega}{2} \, q^{M_c^2} \right) \qquad \mathrm{tr}_{\mathrm{open}} \left(\frac{1+\Omega}{2} \, q^{M_o^2} \right)$$



ORIENTIFOLD CONSTRUCTION

Klein-bottle amplitude

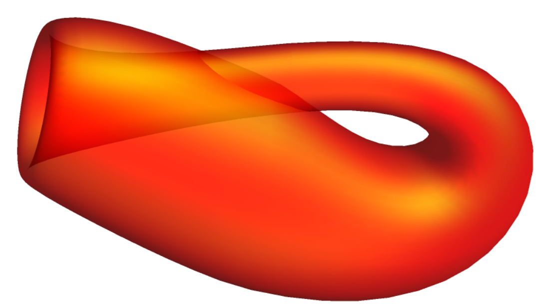


$$\mathrm{tr}_{\mathrm{closed}} \left(\frac{1 + \Omega}{2} \, q^{M_c^2} \right)$$

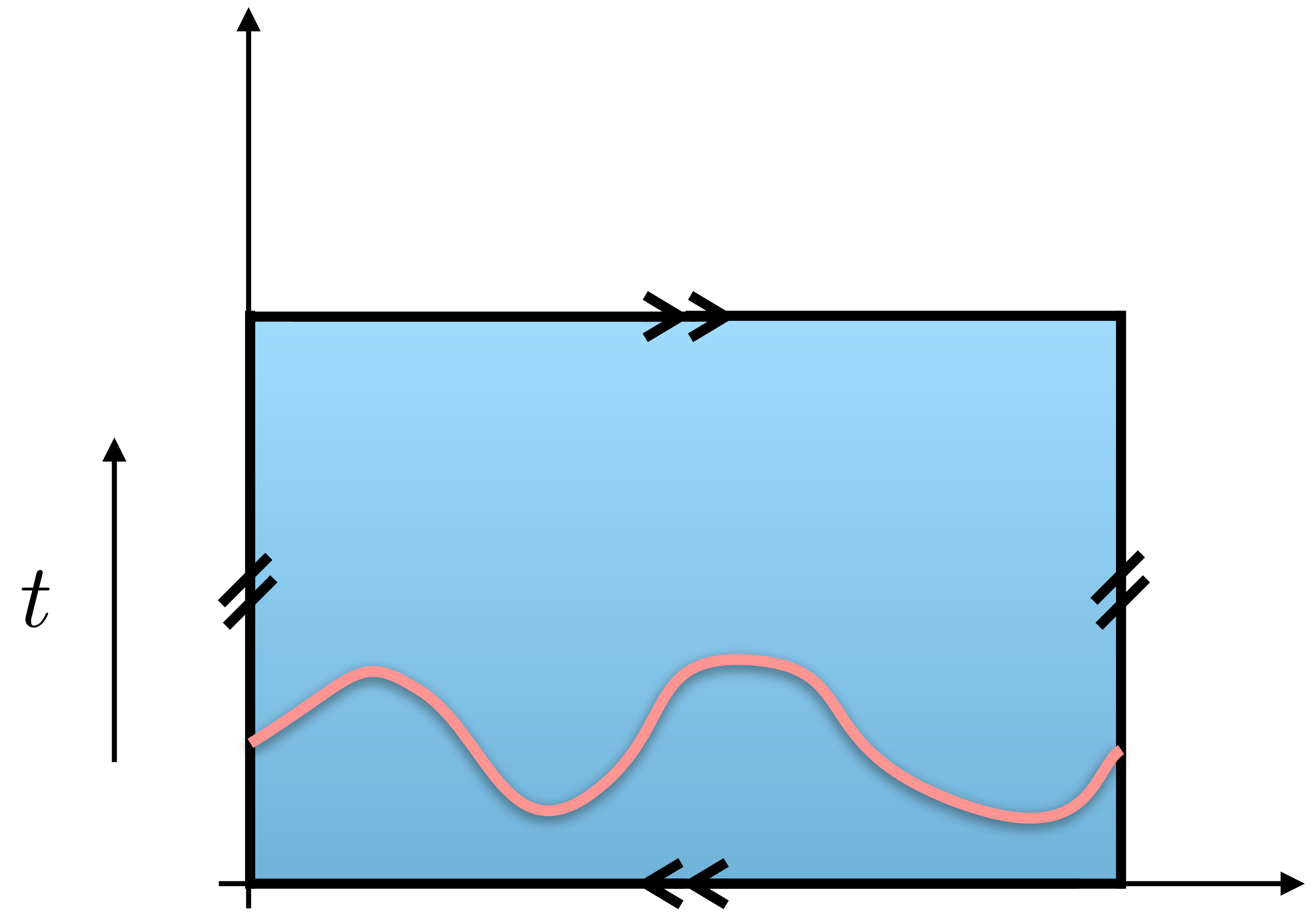
$$\mathrm{tr}_{\mathrm{open}} \left(\frac{1 + \Omega}{2} \, q^{M_o^2} \right)$$

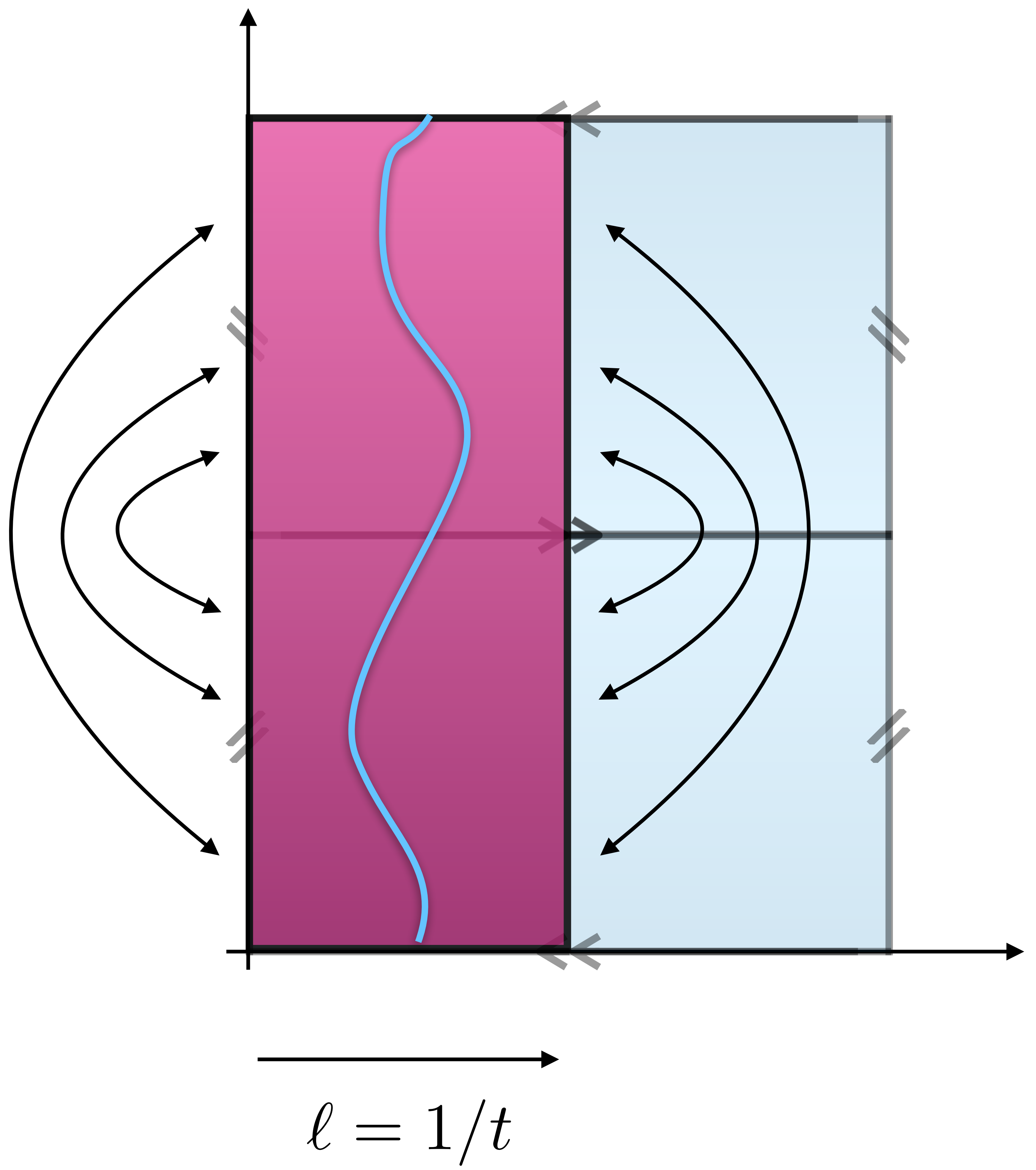
Möbius-strip amplitude

ORIENTIFOLD CONSTRUCTION



Klein-bottle amplitude





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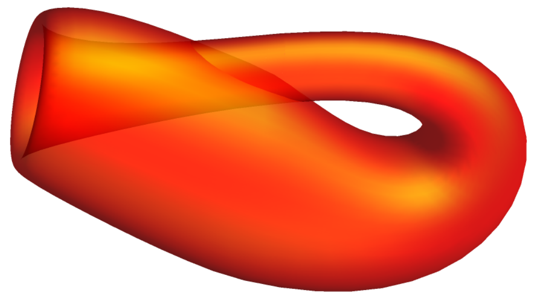
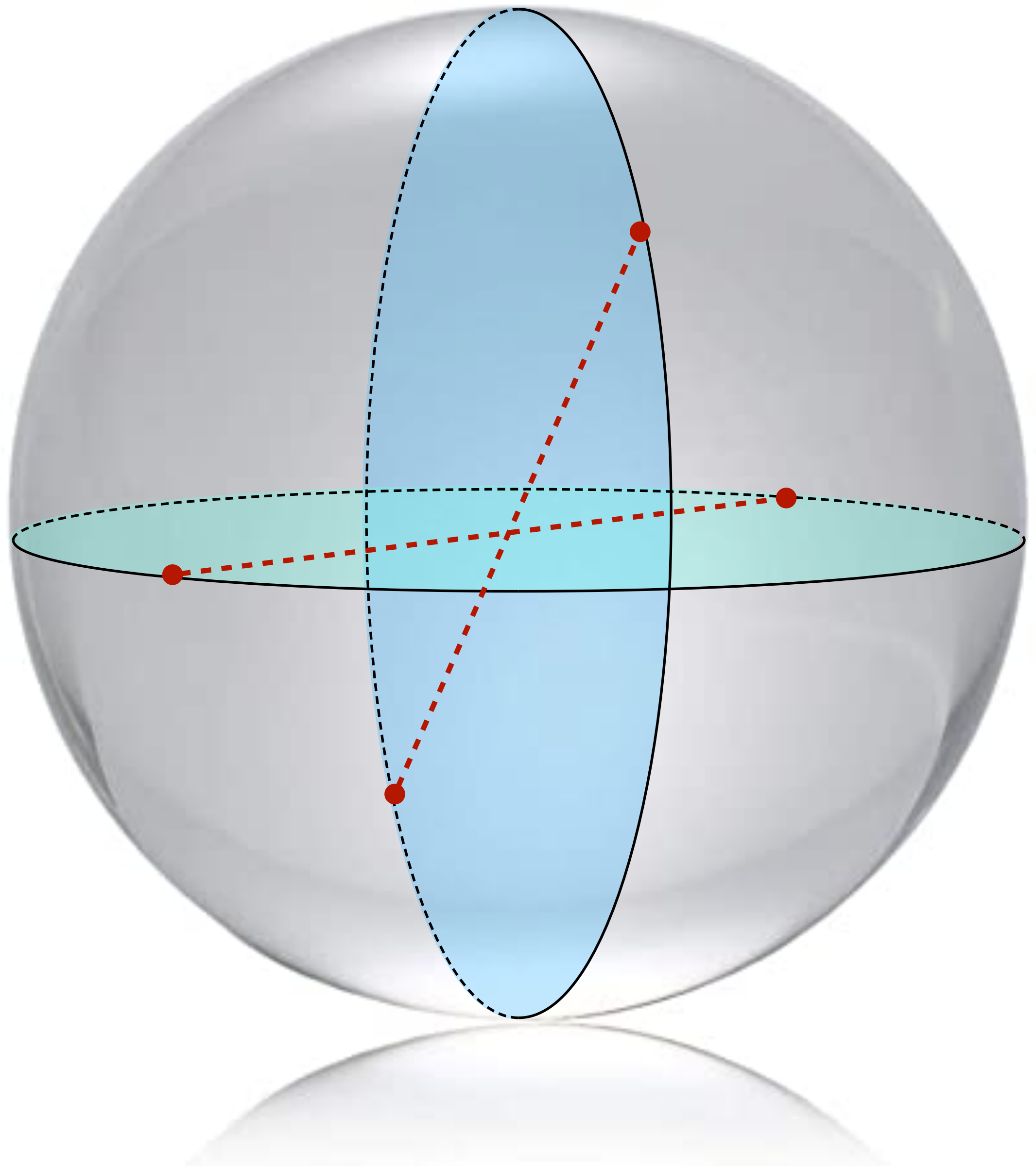
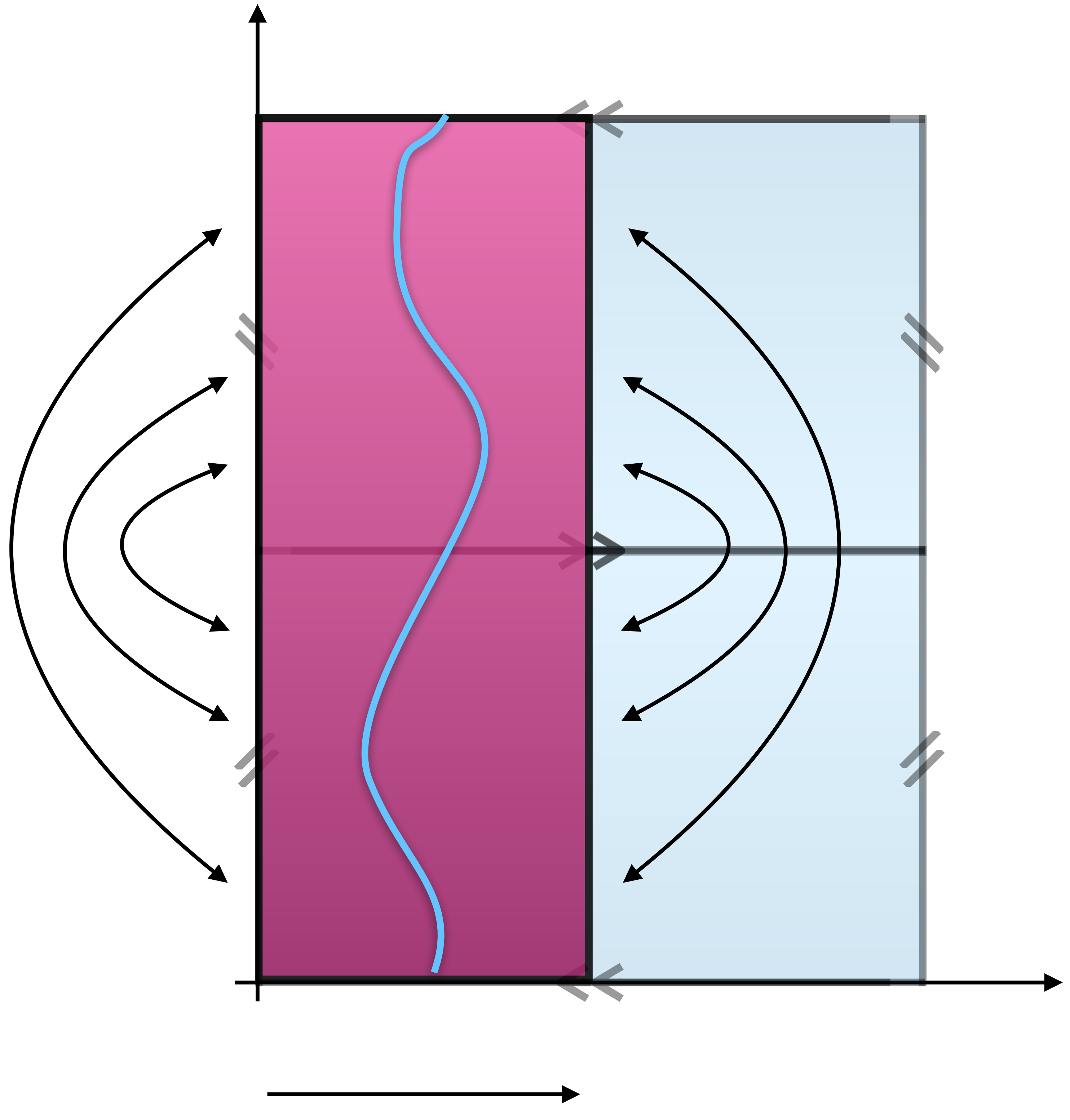


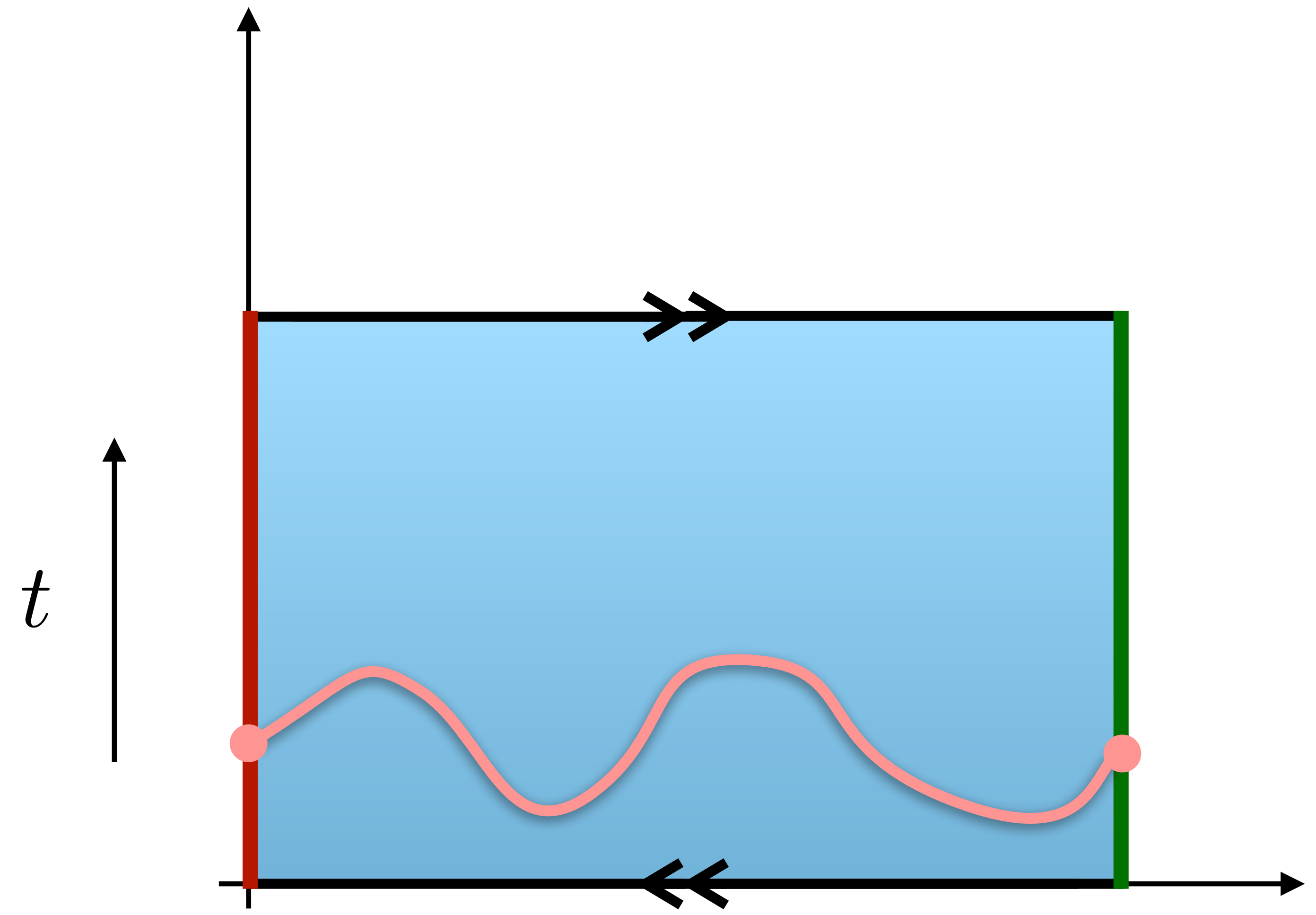
Klein-bottle amplitude

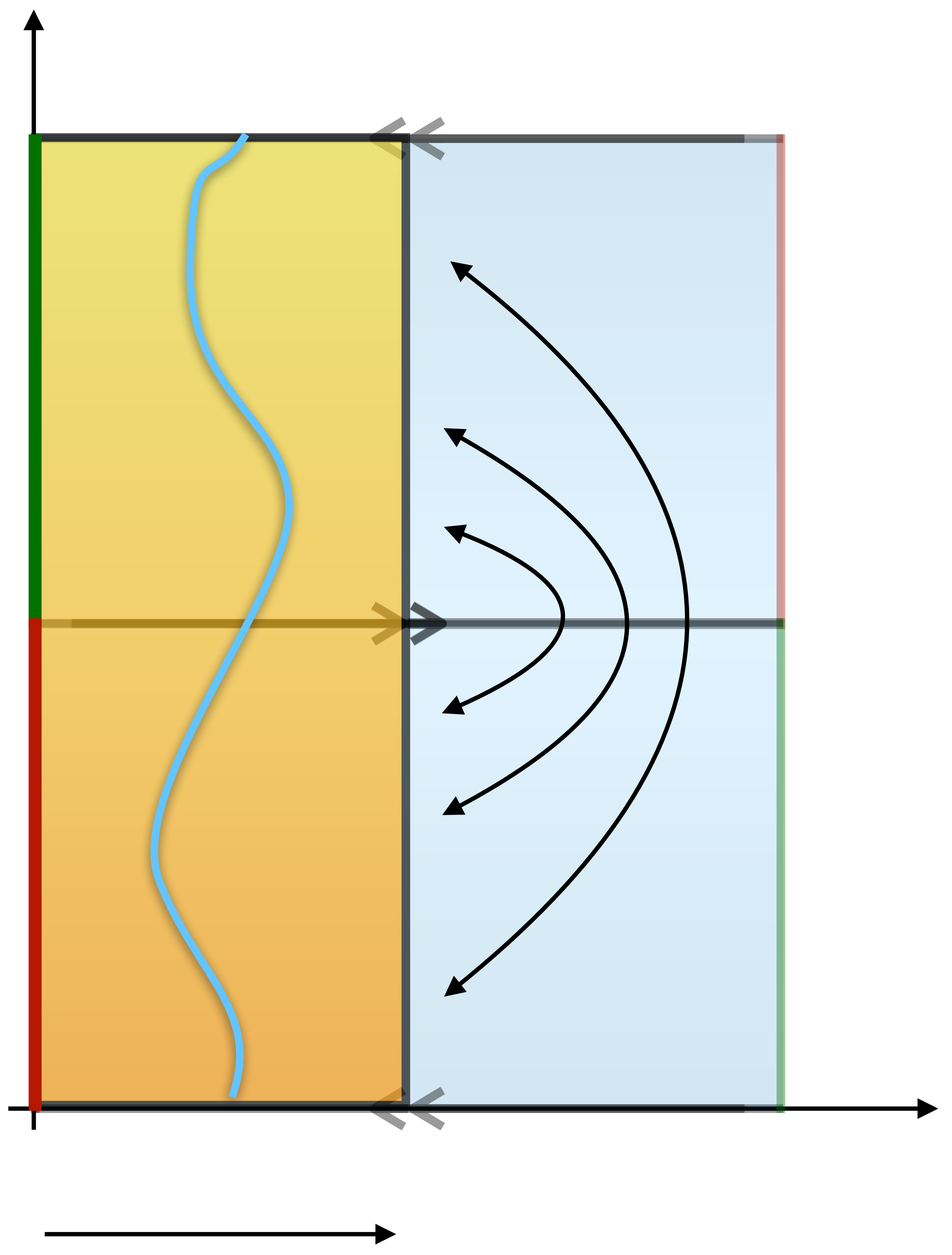




ORIENTIFOLD CONSTRUCTION

Möbius-strip amplitude





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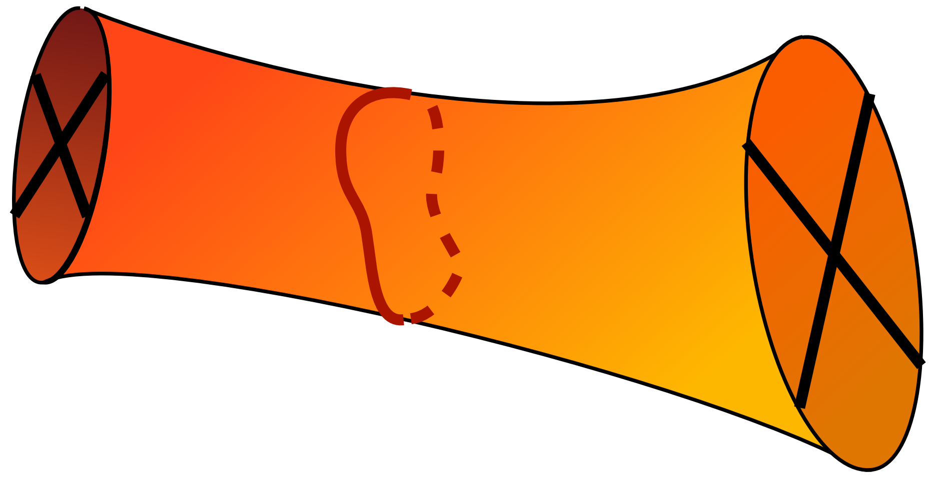
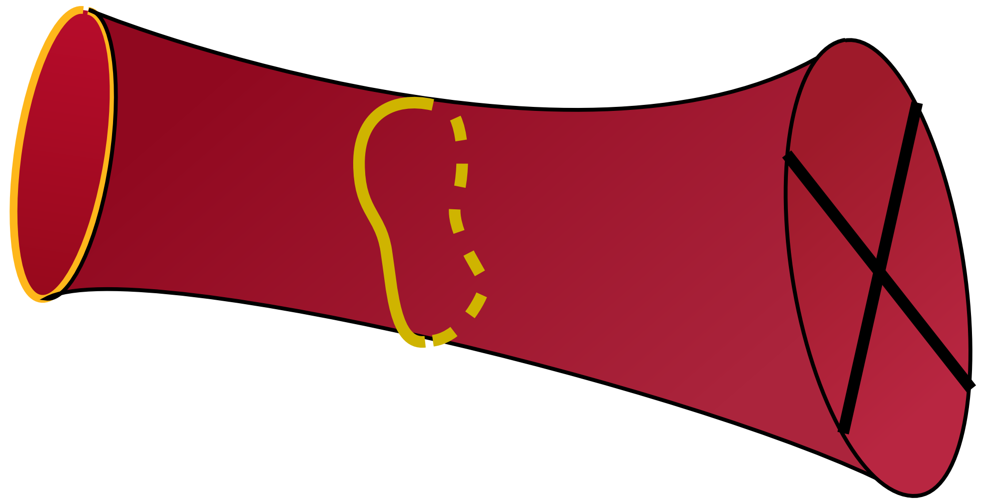
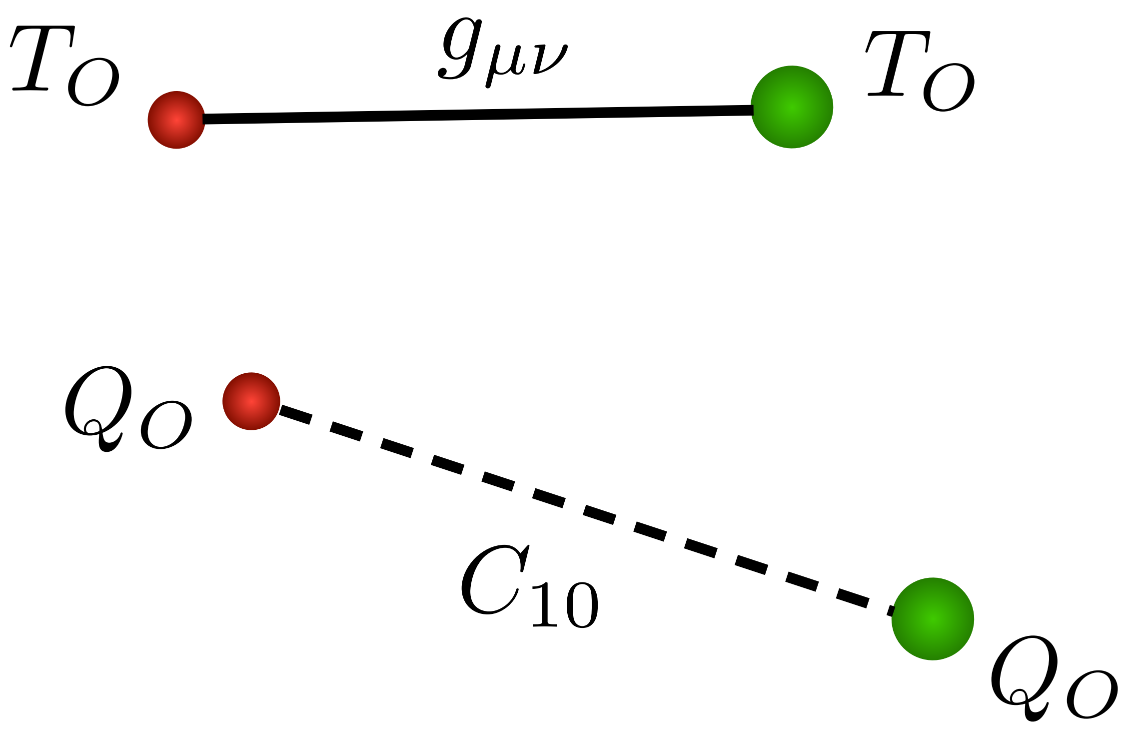
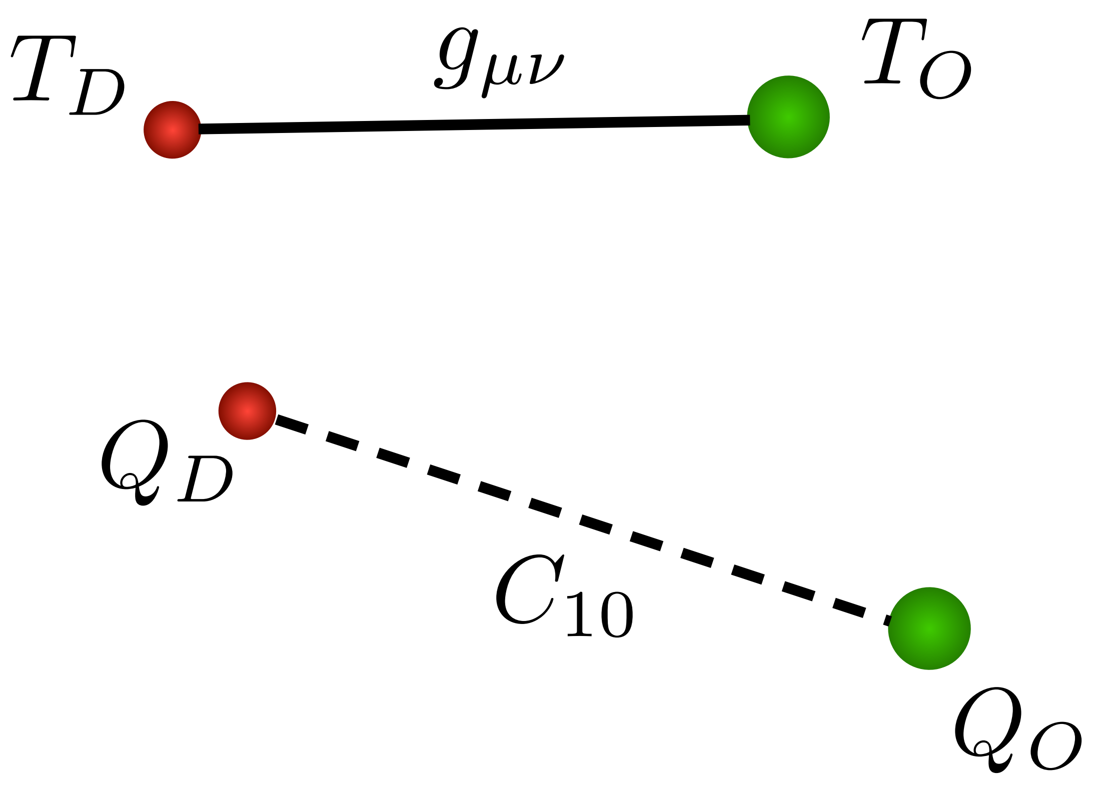




$$\tilde{\mathcal{K}} = \int_0^\infty d\ell \sum_{\substack{\text{closed}\\\text{states}}} \Gamma(n)\,\Delta_n(\ell)\,\Gamma(n)$$

$$\tilde{\mathcal{M}} = \int_0^\infty d\ell \sum_{\substack{\text{closed}\\\text{states}}} B(n)\,\Delta_n(\ell)\,\Gamma(n)$$





ORIENTIFOLD CONSTRUCTIONS

closed strings: $\mathcal{T} = \frac{1}{2}\, |V_8 - S_8|^2$ $\mathcal{K} = \frac{1}{2}(V_8 - S_8)$

open strings: $\mathcal{A} = \frac{1}{2}\, N^2\, (V_8 - S_8)$ $\mathcal{M} = -\frac{1}{2}\, N\, (V_8 - S_8)$

Tadpole conditions:

$$Q_O + Q_D = 0$$
$$T_O + T_D = 0$$

$N = 32$

$$\begin{cases} \mathcal{N} = (1,0) \quad \text{SUGRA} \quad \{g_{\mu\nu}\,,\, \phi\,,\, C_{\mu\nu}\,;\, \psi^\mu_{\rm L}\,,\, \lambda_{\rm R}\} \\ \\ \text{VECTOR} \quad \{A_\mu\,;\, \lambda_{\rm L}\} \quad G_{\rm CP} = \text{SO}(32) \end{cases}$$

THE LEEA FOR THE TYPE I SUPERSTRING

$$\chi = 2 - 2g - b$$

$$S = \int d^{10}x\,\sqrt{-g_{\rm I}}\,\left[e^{-2\phi_{\rm I}}\left(R(g_{\rm I}) + 4(\partial\phi_{\rm I})^2\right) - \tfrac{1}{12}(dC_{\rm I})^2 - \tfrac{1}{4}\,e^{-\phi_{\rm I}}\,{\rm Tr}F_{\rm I}^2 + {\rm fermions}\right)$$

in the *Einstein frame*

$$g_{{\rm I}\,\mu\nu} \to e^{\phi_{\rm I}/2}\,g_{{\rm I}\,\mu\nu}$$

$$S = \int d^{10}x\,\sqrt{-g_{\rm I}}\,\left(R(g_{\rm I}) + (\partial\phi_{\rm I})^2 - \tfrac{1}{12}e^{+\phi_{\rm I}}\,(dC_{\rm I})^2 - \tfrac{1}{4}\,e^{+\phi_{\rm I}/2}\,{\rm Tr}F_{\rm I}^2 + {\rm fermions}\right)$$

HETEROTIC SO(32) VS TYPE I SUPERSTRING

10D (1,0) Supergravity is unique

$$S = \int d^{10}x\,\sqrt{-g_{\rm H}}\,\left(R(g_{\rm H}) + (\partial\phi_{\rm H})^2 - \tfrac{1}{12}e^{-\phi_{\rm H}}\,H_{\rm H}^2 - \tfrac{1}{4}\,e^{-\phi_{\rm H}/2}\,{\rm Tr}F_{\rm H}^2 + \text{fermions}\right)$$

HETEROTIC

$$S = \int d^{10}x\,\sqrt{-g_{\rm I}}\,\left(R(g_{\rm I}) + (\partial\phi_{\rm I})^2 - \tfrac{1}{12}e^{+\phi_{\rm I}}\,(dC_{\rm I})^2 - \tfrac{1}{4}\,e^{+\phi_{\rm I}/2}\,{\rm Tr}F_{\rm I}^2 + \text{fermions}\right)$$

TYPE I

$$\phi_{\rm I} = -\phi_{\rm H} \qquad \Rightarrow \qquad g_{s\,{\rm I}} = \frac{1}{g_{s\,{\rm H}}}$$

strong-weak coupling duality

TYPE IIA SUPERSTRING VS 11D SUPERGRAVITY

Type IIA massless states: $\{g_{\mu\nu}\,,\,\phi\,,\,B_{\mu\nu}\,,\,C_{\mu}\,,\,C_{\mu\nu\rho}\,;\,\Psi^{\mu}\,,\,\lambda\}$ $\qquad \mu,\nu=0,\ldots,9$

Kaluza-Klein Ansatz: $$g_{MN}=\begin{pmatrix} g_{\mu\nu} & C_{\mu} \\ C_{\mu} & e^{4\phi/3}\end{pmatrix}$$

11D SUGRA $$A_{MNP}=\{C_{\mu\nu\rho}\,,\,B_{\mu\nu}\} \qquad M,N,=0,\ldots,10$$

$$\Psi^{M}=\{\Psi^{\mu}\,,\,\lambda\}$$

The strong coupling limit of type IIA superstring is M-theory

$$R=e^{2\phi/3}\to\infty \qquad \Rightarrow \qquad g_{s\,\mathrm{A}}\to\infty$$

THE MALDACENA CONJECTURE

… is based on the crucial observation that D-branes are …

Hypersurfaces defined by the boundary conditions.
Open string describe the excitations of these hypersurfaces.

Solitonic objects sourcing the closed-string fields:
Graviton, dilaton, RR C_p form field

D-BRANE SOURCES

$$S_{\mathrm{SUGRA}} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{g}\,\left[e^{-2\phi}\left[R + 2(\partial\phi)^2\right) - \tfrac{1}{2}|F_{p+2}|^2\right]$$

$$\begin{aligned} S_{\text{D-brane}} = &-\frac{T_p}{2}\int d^{p+1}\xi\,\sqrt{\gamma}\left[e^{-\phi}\,\gamma^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} - (p-1)\right] \\ &+ \frac{(-1)^{p+1}\,\mu_p}{(p+1)!}\int d^{p+1}\xi\,\epsilon^{i_1\ldots i_{p+1}}\,A_{\mu_1\ldots\mu_{p+1}}\,\partial_{i_1}X^{\mu_1}\ldots\partial_{i_{p+1}}X^{\mu_{p+1}}\,, \end{aligned}$$

D-BRANE SOLUTION

$$ds^2 = H_p^{-1/2}(r)\, dx \cdot dx + H_p^{1/2}(r) \left(dr^2 + r^2\, d\Omega_{8-p}^2\right)$$

$$e^{\phi} = g_{\mathrm{s}}\, H_p^{(3-p)/4}(r)$$

$$C_{01\ldots p} = H_p(r)^{-1} - 1$$

$$H_p(r) = 1 + \left(\frac{r_p}{r}\right)^{7-p}$$

$$\left(\frac{r_p}{\ell_{\mathrm{s}}}\right)^{7-p} = (2\sqrt{\pi})^{5-p}\, \Gamma\left(\frac{7-p}{2}\right) g_{\mathrm{s}}\, N$$

D3-BRANE SOLUTION

$$e^{\phi} = g_{\rm s}$$

$$\begin{aligned} ds^2 &= H_3^{-1/2}(r)\, dx \cdot dx + H_3^{1/2}(r) \left(dr^2 + r^2\, d\Omega_{8-p}^2\right) \\ &\sim \left(\frac{r}{r_3}\right)^2 dx \cdot dx + \left(\frac{r_3}{r}\right)^2 \left(dr^2 + r^2 d\Omega_5^2\right) \\ &= r_3^3 \frac{dx \cdot dx + dz^2}{z^2} + r_3^2\, d\Omega_5^2 \end{aligned}$$

$r_3^4 = 4\pi\, g_{\rm s}\, N \alpha'^2$

$z = r_3^2/r$

Near horizon geometry: $\mathrm{AdS}_5 \times S^5$

Isometries: $\mathrm{SO}(4,2) \times \mathrm{SO}(6) \simeq \mathrm{SU}(2,2) \times \mathrm{SU}(4)$

This background preserves ***all*** 32 supercharges of type IIB

D3-BRANE DYNAMICS

The light degrees of freedom on the D3 branes propagate in D=4

$A_\mu \, , \, \phi^{a=1,\ldots,6} \, , \, \lambda^{i=1,\ldots,4}$ *N*=4 vector multiplet

$\mathrm{SO}(3,1) \times \mathrm{SU}(4)$ naïve global symmetries

The theory is conformally invariant at the quantum level $\mathrm{SO}(4,2) \times \mathrm{SU}(4)$

It has ***16 Q*** supercharges of *N*=4 plus ***16 S*** supercharges of the superconformal group

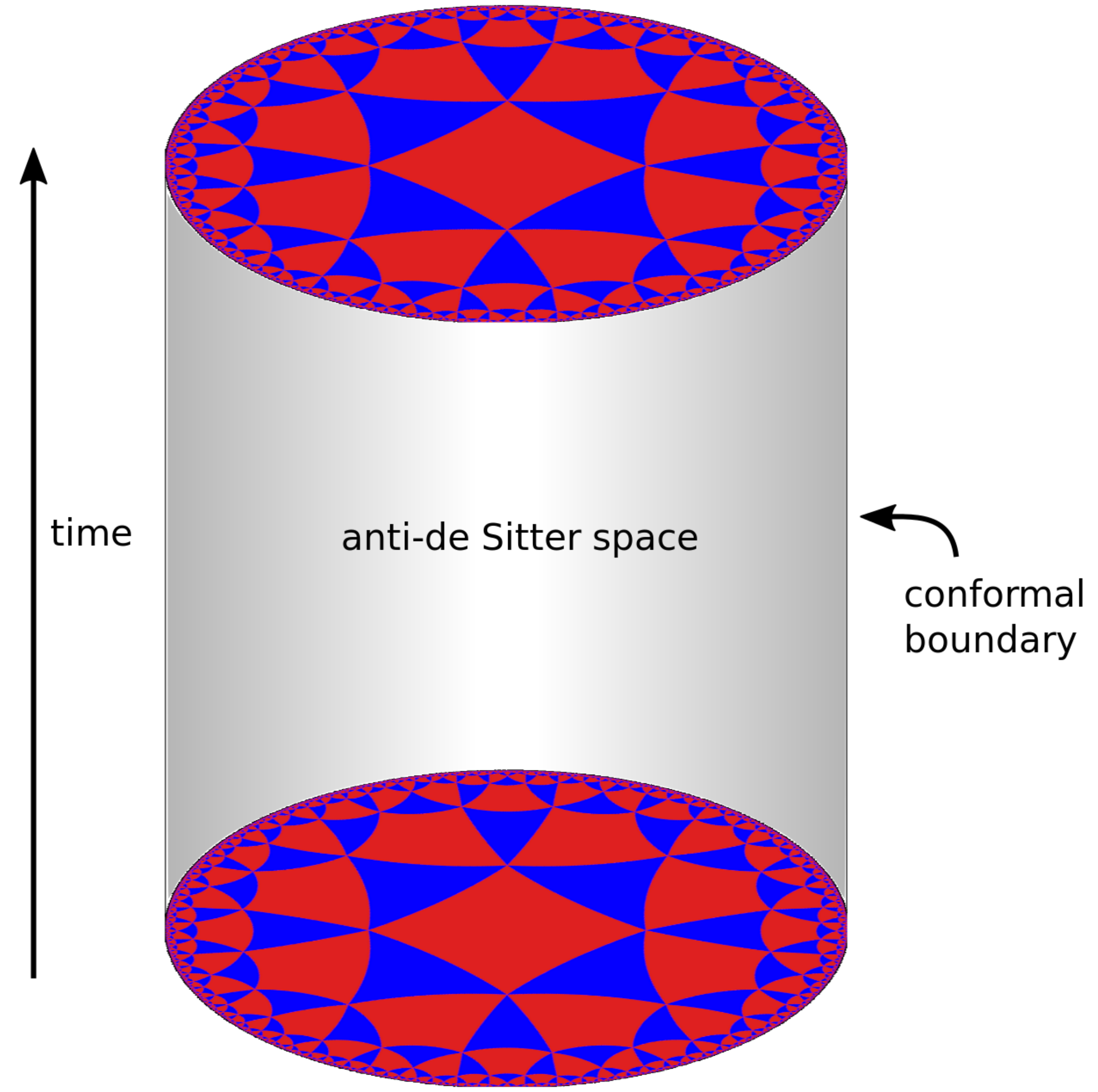
linearly realised non-linearly realised

THE AdS/CFT CORRESPONDENCE

Type IIB superstring theory on $\mathrm{AdS}_5 \times S^5$

is ***dual*** to

N=4 Super-Yang-Mills theory with U(N) gauge group



AdS$_5$ and S^5 radius

$$r_3 = \left(4\pi g_\mathrm{s} N\, \ell_\mathrm{s}^4\right)^{1/4} = \lambda^{1/4}\, \ell_\mathrm{s}$$

$$g_\mathrm{YM}^2 = 4\pi g_\mathrm{s}$$

$$\lambda = g_\mathrm{YM}^2\, N$$

String corrections under control for $r_3 \gg \ell_\mathrm{s}$