A SHORT INTRODUCTION TO SUPERSTRING THEORY

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OUTLINE

(CLOSED) SUPERSTRINGS IN D=10 1

OPEN STRINGS AND D-BRANES 2

COMPACTIFICATION 3

SUPERSYMMETRY BREAKING 4

CIRCLE COMPACTIFICATION: FIELD THEORY

Kaluza-Klein dimensional reduction $\mathcal{M}_{1,4} = \mathcal{M}_{1,3} \times S^1(R)$

$$\int d^5x \, \left(\tfrac{1}{2}\partial_M \Phi \, \partial^M \Phi - \tfrac{1}{2} M_5^2 \Phi^2\right)$$

$$\Phi(x,y) = \sum_{k\in\mathbb{Z}} \phi_k(x) e^{ikx/R}$$

… in four dimensions

$$M_4^2(k) = M_5^2 + \left(\frac{k}{R}\right)^2$$

CIRCLE COMPACTIFICATION: FIELD THEORY

A spin s_5 field gives rise to several fields with spin $s_4 \leq s_5$

$$SO(1,4) \to SO(1,3)$$

For instance …

$$M = (\mu, 5)$$

spin 1

$$A_M = (A_\mu\,,\; A_5)$$

spin 0

spin 2

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & g_{\mu\,5} \\ g_{5\,\nu} & g_{55} \end{pmatrix}$$

spin 1

spin 0

CIRCLE COMPACTIFICATION: FIELD THEORY

5 dimensional diffeomorphisms

$$\delta g^{MN} = \xi^P \partial_P g^{MN} + g^{PN} \partial_P \xi^M + g^{MP} \partial_P \xi^N \qquad \Rightarrow \qquad \delta g^{\mu 5} = \partial^\mu \xi^5$$

The KK excitations are charged with respect to this U(1)

$$\delta \Phi = \xi^M \partial_M \Phi \qquad \Rightarrow \qquad \delta \phi_k(x) = i \frac{k}{R} \xi^5$$

CIRCLE COMPACTIFICATION: STRING THEORY

Closed strings can wind around the internal circle

$$X(\sigma+\pi,\tau) = X(\sigma,\tau) + 2n\pi R$$

The lower-dimensional mass operator

$$M^2 = \frac{4}{\alpha'}\left[N_X + \tilde{N}_X - 2\right] + \left(\frac{k}{R}\right)^2 + \left(\frac{nR}{\alpha'}\right)^2$$

T-duality : $R \to \alpha'/R$

CIRCLE COMPACTIFICATION: STRING THEORY

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level matching

$$N_X-\tilde{N}_X=-kn$$

T-duality : $R\to\alpha'/R$

CIRCLE COMPACTIFICATION: STRING THEORY

Massless fields …

$$\begin{tikzcd}
\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0,\tilde{0}\rangle_{0,0} & \alpha^{\mu}_{-1}\tilde{\alpha}^{5}_{-1}|0,\tilde{0}\rangle_{0,0} & \alpha^{5}_{-1}\tilde{\alpha}^{\nu}_{-1}|0,\tilde{0}\rangle_{0,0} & \alpha^{5}_{-1}\tilde{\alpha}^{5}_{-1}|0,\tilde{0}\rangle_{0,0} \\
g_{\mu\nu}\,,\;\; B_{\mu\nu}\,,\;\; \phi \arrow[u] & g_{\mu\,5}\,,\;\; B_{\mu\,5} \arrow[u] \arrow[ur] & & g_{55} \arrow[u]
\end{tikzcd}$$

$$G = \mathrm{U}(1)\times\mathrm{U}(1)$$

$$\delta B_{MN} = \partial_M \Lambda_N - \partial_N \Lambda_M \qquad \Rightarrow \qquad \delta B_{\mu\,5} = \partial_\mu \Lambda_5$$

CIRCLE COMPACTIFICATION: STRING THEORY

New states allowed by level-matching: $N_X - \tilde{N}_X = -kn$

$\alpha^{\mu}_{-1}|0,\tilde{0}\rangle_{\pm1,\mp1}$ $\tilde{\alpha}^{\mu}_{-1}|0,\tilde{0}\rangle_{\pm1,\pm1}$

$$M^2 = \left(\frac{1}{R} - \frac{R}{\alpha'}\right)^2$$

$$M^2 = \frac{4}{\alpha'}\left[N_X + \tilde{N}_X - 2\right] + \left(\frac{k}{R}\right)^2 + \left(\frac{nR}{\alpha'}\right)^2$$

For $R = \sqrt{\alpha'}$ these four vectors become massless!

Gauge symmetry enhancement

$$\mathrm{U}(1) \times \mathrm{U}(1) \to \mathrm{SU}(2) \times \mathrm{SU}(2)$$

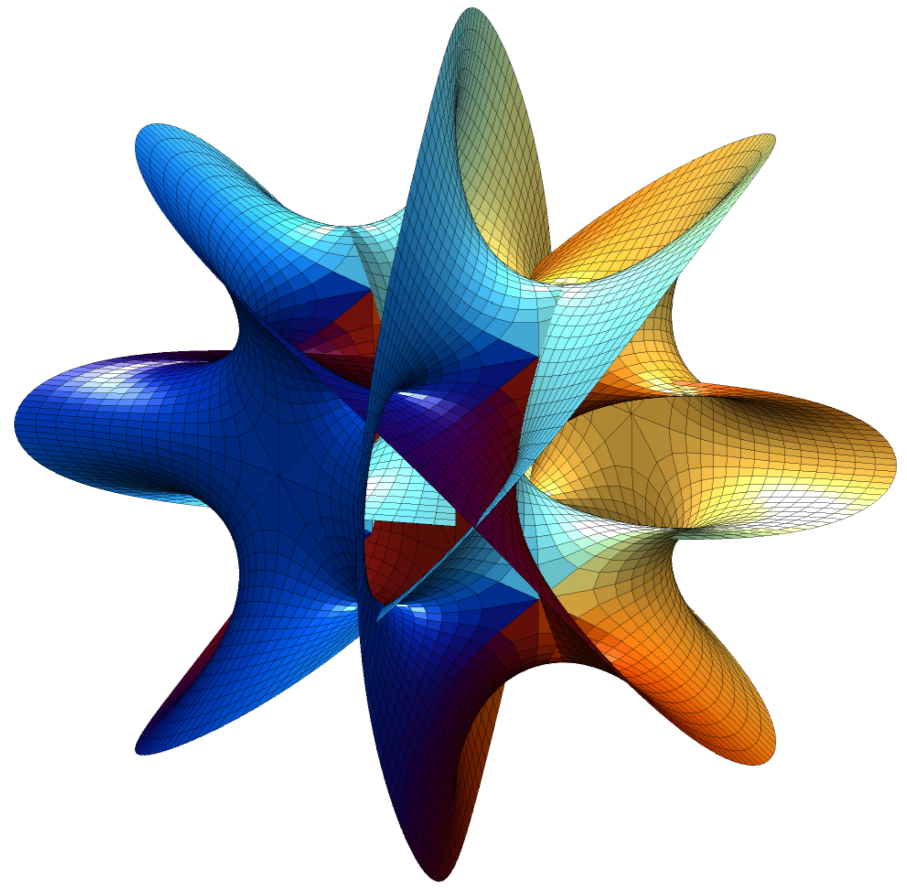
COMPACTIFICATION vs CHIRALITY

Circle (or toroidal) compactifications preserve
the maximal number of killing spinors

As a result the light (fermionic) spectrum is not chiral!

To improve the scenario we need more complicated space,
which admit a smaller number of killing spinors

Calabi-Yau spaces



ORBIFOLD COMPACTIFICATION

combine the

success of Calabi-Yau compactifications

calculability of toroidal compactifications

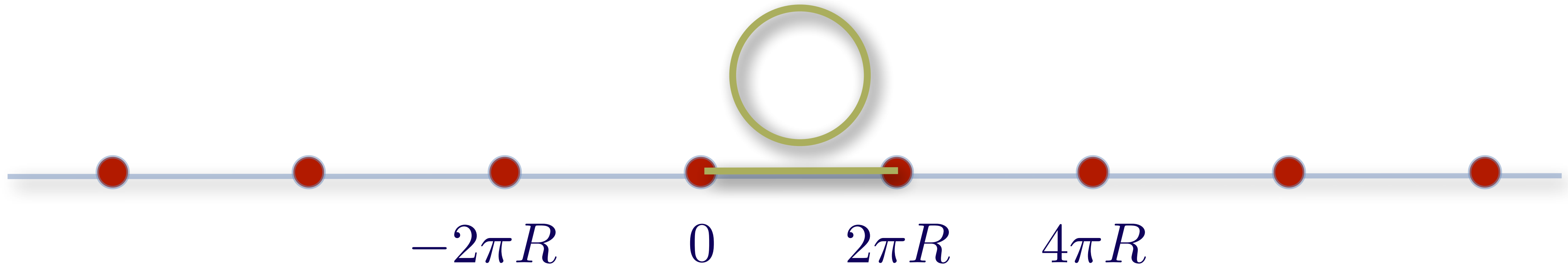
An orbifold is the quotient of a (smooth) manifold by a discrete symmetry group

$$\mathcal{O} = \mathcal{M}/\Gamma$$

SIMPLEST ORBIFOLD

the circle …

$$\mathbb{R}/\delta \qquad \delta : \quad x \simeq x + 2\pi m R$$

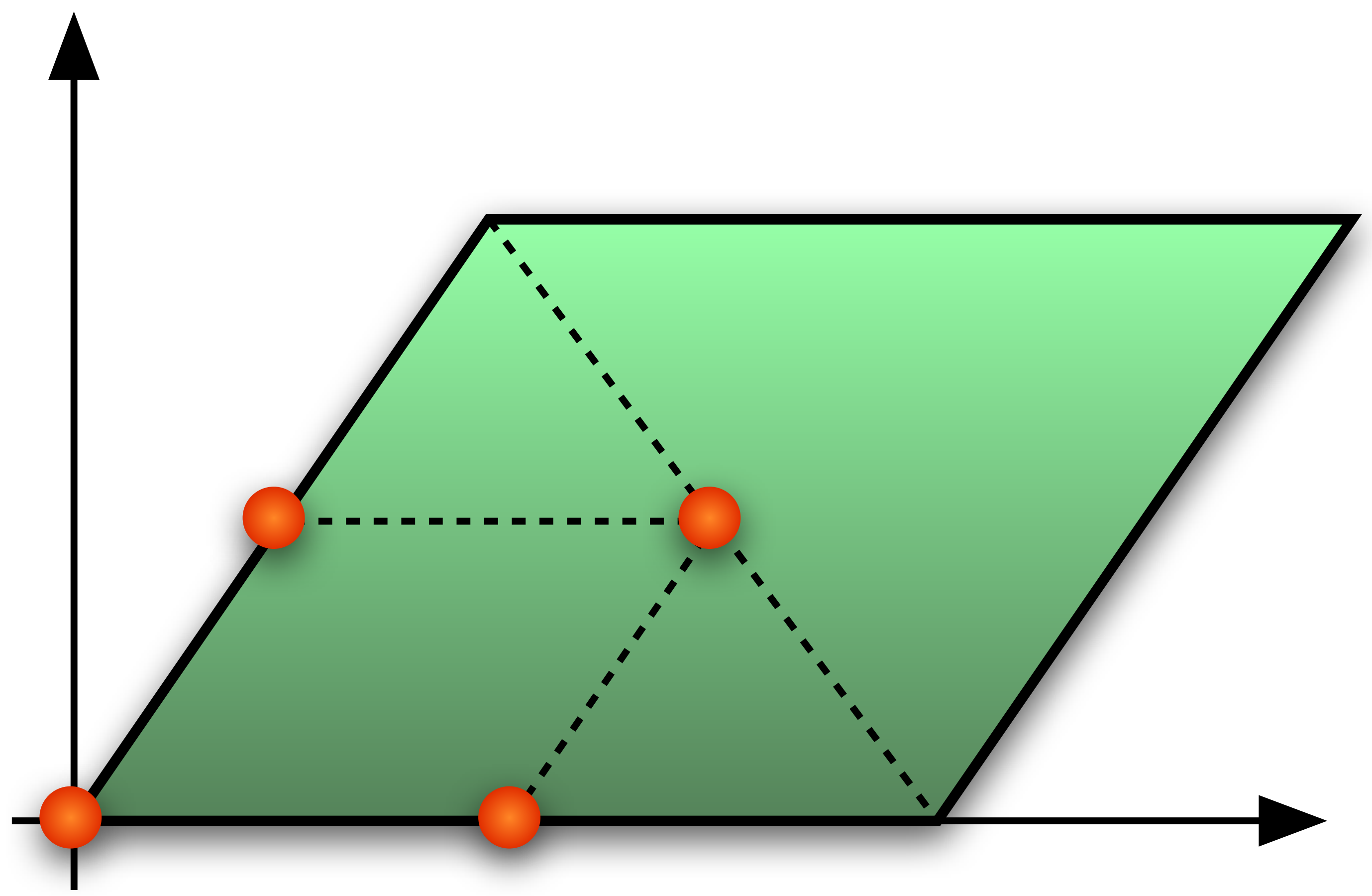


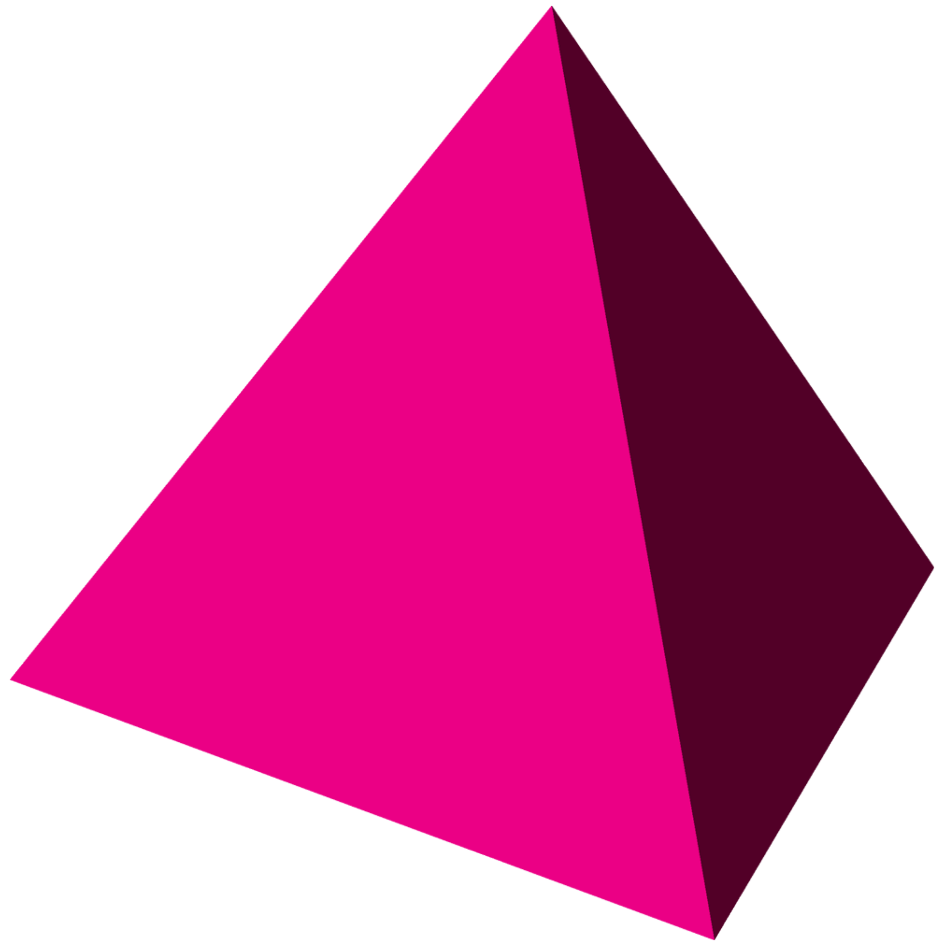
A MORE INTERESTING ORBIFOLD

the tetrahedron ...

$T^2/\mathbb{Z}_2$ $\quad$ $\mathbb{Z}_2 \, : z \sim -z$ $\quad$ $z \sim z + m + n\tau$

fixed points: $(0,0)$ $\quad$ $(\frac{1}{2}, 0)$ $\quad$ $(0, \frac{1}{2})$ $\quad$ $(\frac{1}{2}, \frac{1}{2})$





STRINGS ON ORBIFOLDS

Restrict the Hilbert space to the invariant fields

$$\mathcal{Z} = \mathrm{tr}\left(\frac{1 + g + g^2 + \cdots + g^{N-1}}{N}\, q^{M_{\mathrm{L}}^2}\, \bar{q}^{M_{\mathrm{R}}^2} \right)$$

Massless fields ... e.g. $T^2/\mathbb{Z}_2$

$\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0,\tilde{0}\rangle_{0,0}$ $\alpha^{\mu}_{-1}\tilde{\alpha}^{j}_{-1}|0,\tilde{0}\rangle_{0,0}$ $\alpha^{i}_{-1}\tilde{\alpha}^{\nu}_{-1}|0,\tilde{0}\rangle_{0,0}$ $\alpha^{i}_{-1}\tilde{\alpha}^{j}_{-1}|0,\tilde{0}\rangle_{0,0}$

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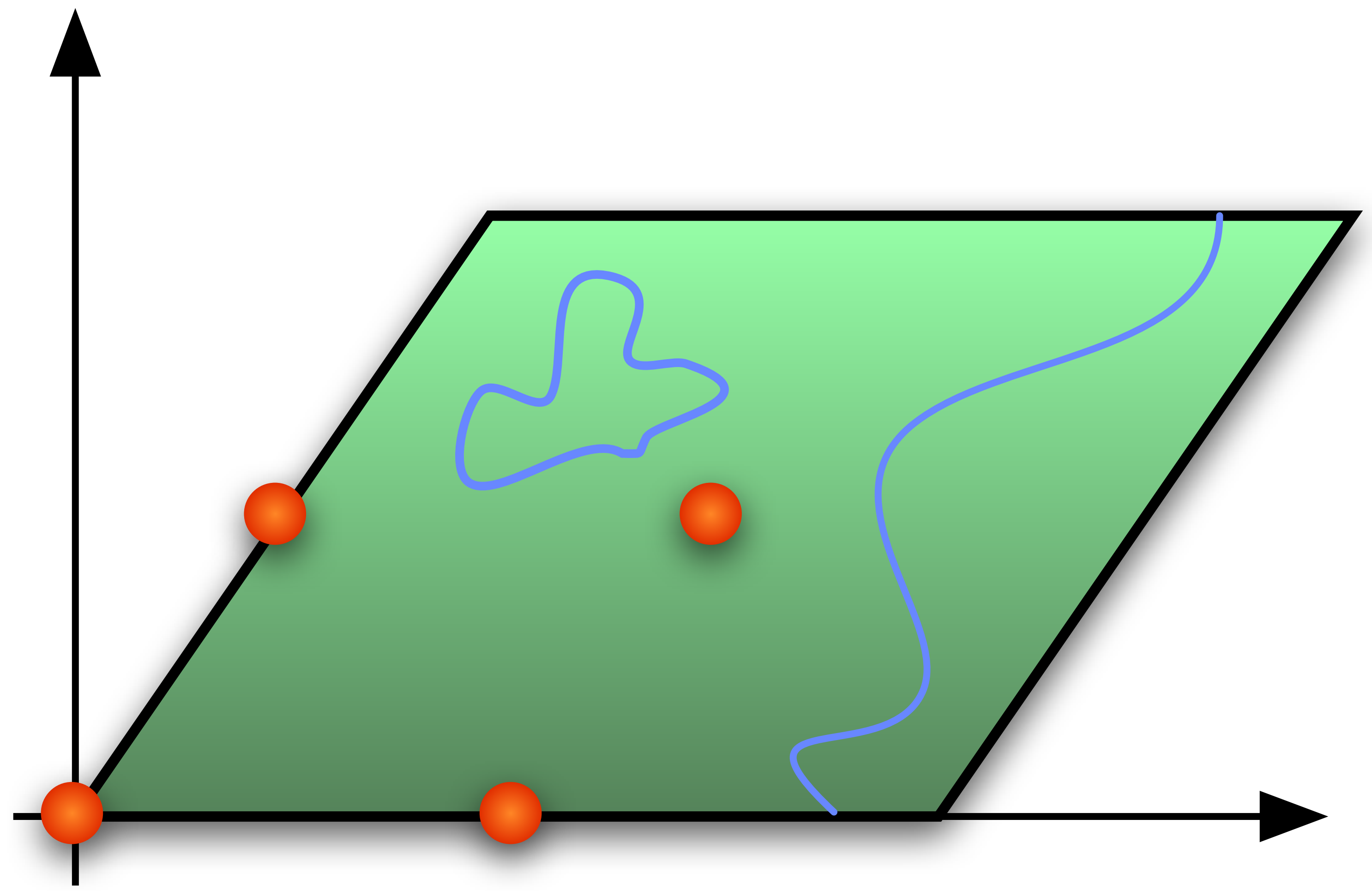


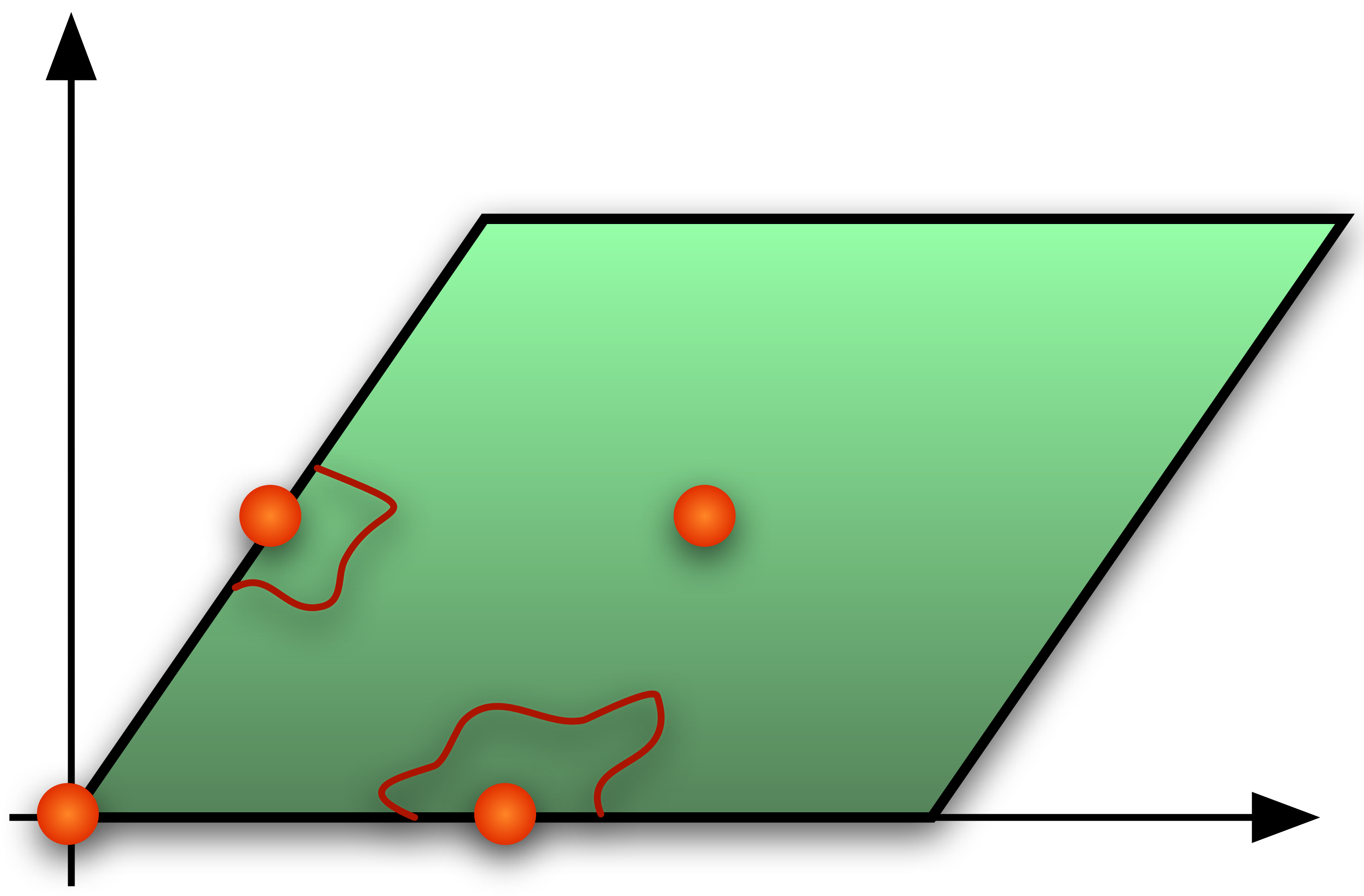
STRINGS ON ORBIFOLDS

… can be closed modulo che action of the orbifold group

$$X^i(\sigma+\pi,\tau) = g \cdot X^i(\sigma,\tau)$$

The mode expansion involves non-integer frequencies

The zero-mode is associated to the position of the fixed points

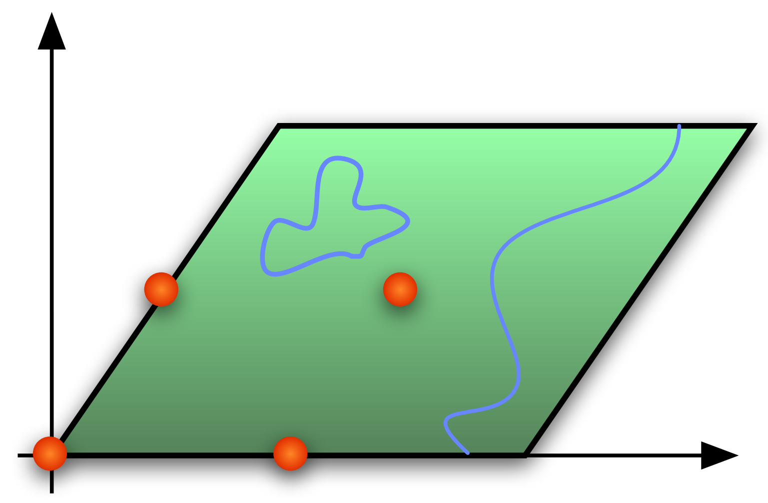


STRINGS ON ORBIFOLDS

untwisted states

Invariant subset of the original states

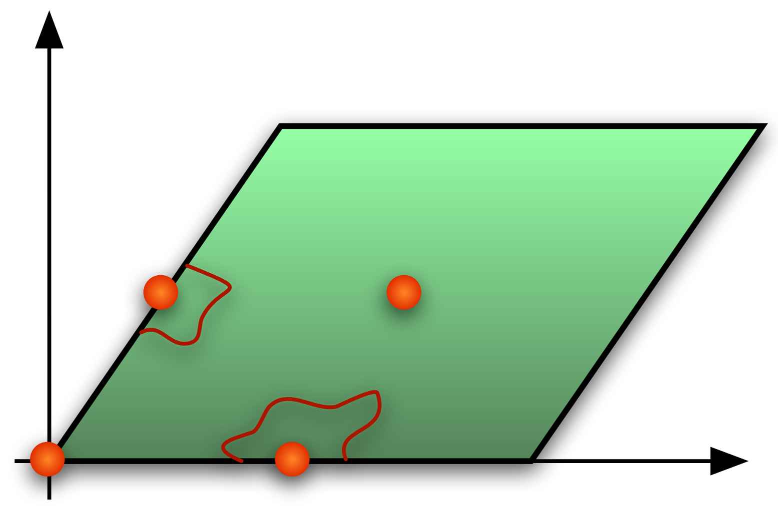
They have a (quantised) momentum along the compact directions and there are free to propagate



twisted states

Strings closed modulo the action of the symmetry group

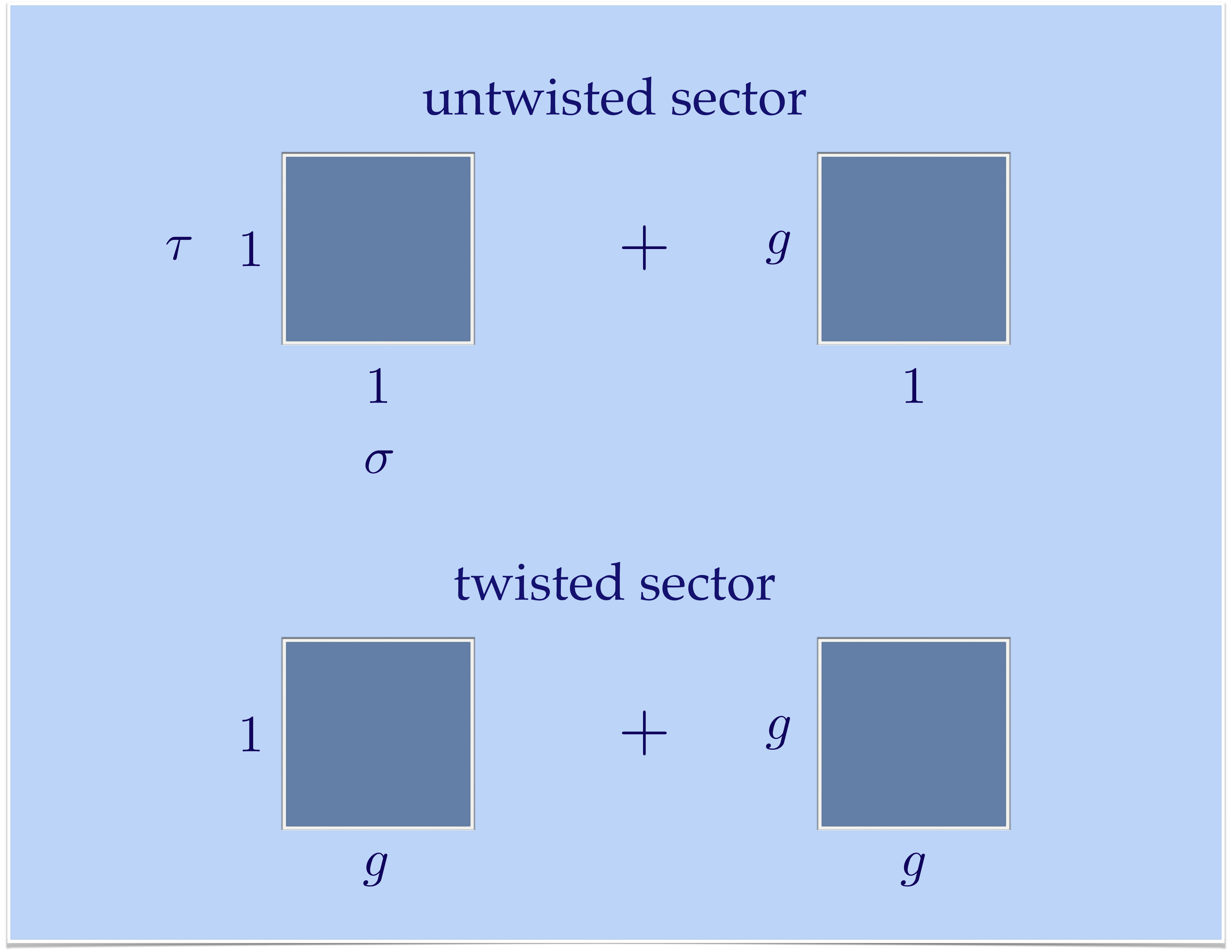
Don't have momentum and thus are not free to propagate. They may come in multiple families.



STRINGS ON ORBIFOLDS

The twisted sector is essential for the consistency of the theory

The propagation on singular spaces is now consistent



STRINGS ON ORBIFOLDS

… can yield lower-dimensional chirality

Under a rotation of angle θ on plane, a spinor of helicity η transforms as

$$s \to e^{i\eta\theta}\, s \qquad \eta = \pm\tfrac{1}{2}$$

Therefore, for the orbifold $T^4/\mathbb{Z}_3$ $\quad (\tfrac{2\pi}{3}, -\tfrac{2\pi}{3})$

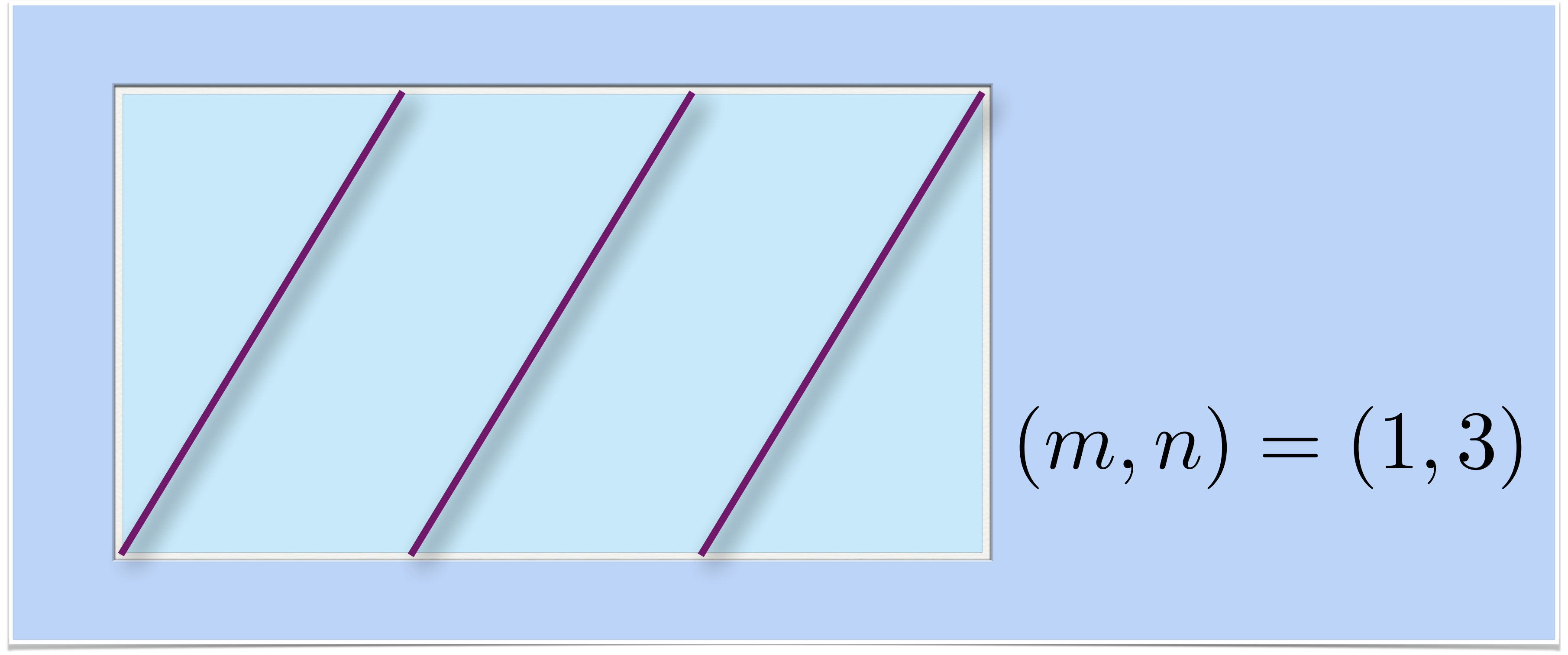
$$\underbrace{(\eta_1, \eta_2, \eta_3 | \eta_4, \eta_5)}_{\#(\text{-})=\text{even}} \to \underbrace{(\eta_1, \eta_2, \eta_3}_{\#(\text{-})=\text{even}} | \eta_4, \eta_5)\, e^{\frac{2i\pi}{3}(\eta_4 - \eta_5)}$$

ANOTHER WAY FOR CHIRALITY

Branes at angles

$$\partial_\sigma \left(\cos\theta\, X^1 + \sin\theta\, X^2\right) = 0$$
$$\partial_\tau \left(-\sin\theta\, X^1 + \cos\theta\, X^2\right) = 0$$

On the torus the angle must be quantised

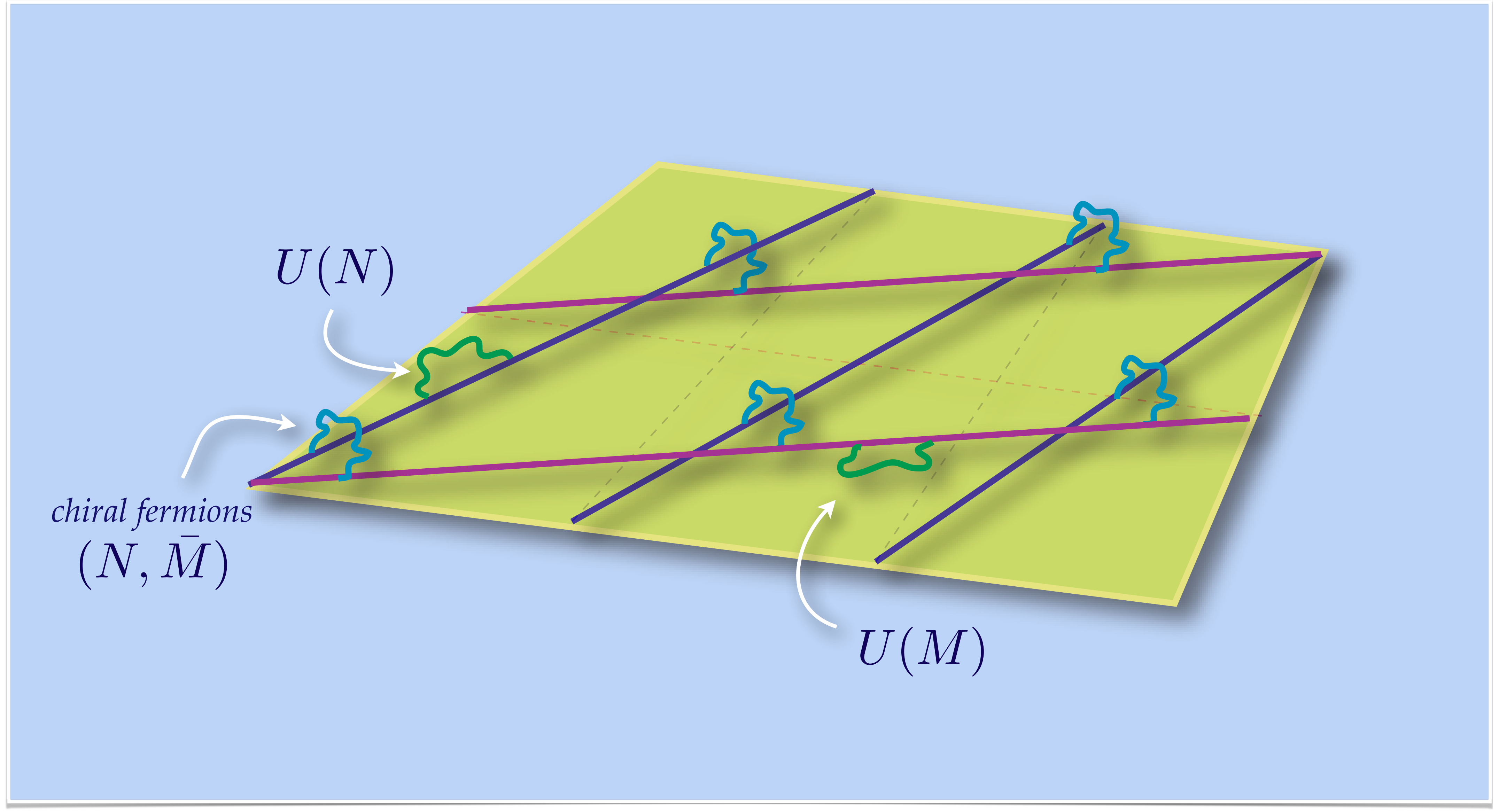


A D-brane is characterised by the wrapping numbers

$$(m, n)$$

ANOTHER WAY FOR CHIRALITY

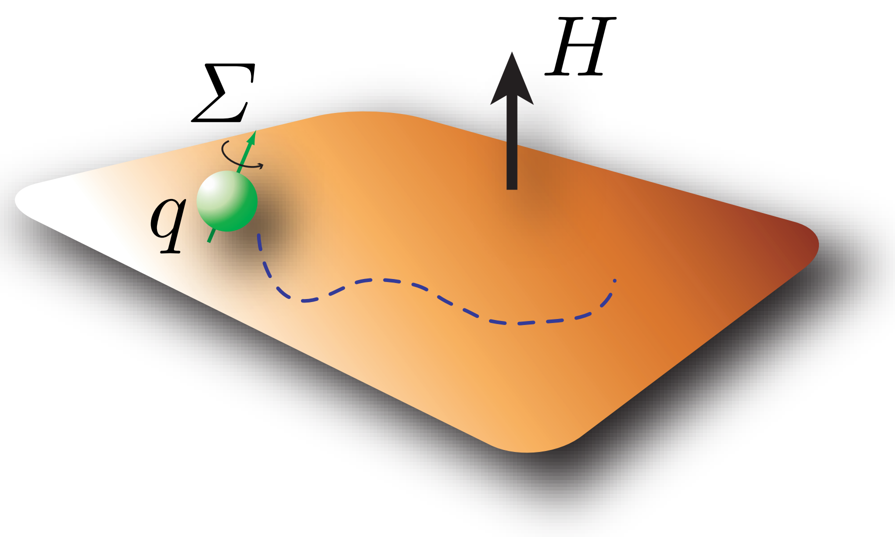
In the presence of multiple branes rotated by different angles



Degeneracy of the chiral spectrum = intersection number

$$I_{ab} = m_a n_b - m_b n_a$$

MAGNETIC FIELD ON A TORUS



On the torus, intersecting branes are T-dual to a magnetic field

$$\partial_\sigma X^1 + qF_{12}\,\partial_\tau X^2 = 0$$
$$\partial_\sigma X^2 - qF_{12}\,\partial_\tau X^1 = 0$$

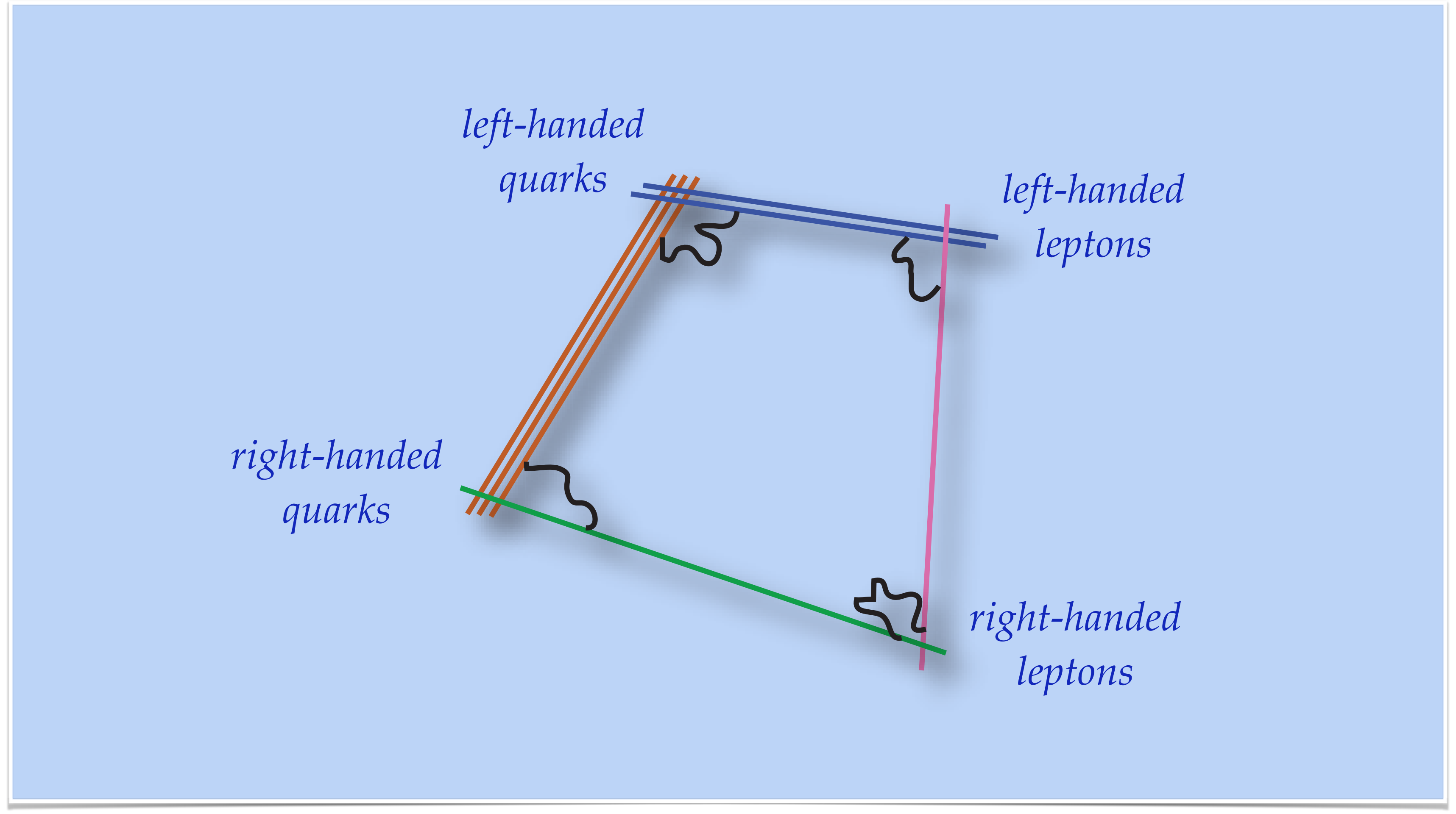
On the torus the flux of the magnetic field is quantised and the masses are shifted

$qF_{12}v_2 \in \mathbb{Z}$

$$\Delta M^2 = (2n+1)|QH| + 2\Sigma_{12}QH$$

The Landau levels are degenerate

ONE (PROMISING) ROAD TO THE STANDARD MODEL



$$U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$$

$$Q_Y = \tfrac{1}{6} Q_a - \tfrac{1}{2} Q_c + \tfrac{1}{2} Q_d$$

quarks: $(3, 2_+, 1, 1)$ $\quad 2 \times (3, 2_-, 1, 1)$ $\quad 3 \times (\bar{3}, 1, 1_-, 1)$ $\quad 3 \times (\bar{3}, 1, 1_+, 1)$

leptons: $3 \times (1, 2_-, 1, 1_-)$ $\quad 3 \times (1, 1, 1_+, 1_-)$ $\quad 3 \times (1, 1, 1_-, 1_-)$