Lectures on the supersymmetric Higgs boson(s)

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Abstract

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1 Introduction

These notes will introduce you to the properties of Higgs bosons in supersymmetric theories, and what we can learn from them. This also necessarily involves the properties of the electroweak sector, including the W boson mass. I will focus mainly on the CP-conserving MSSM, but especially nowadays we should keep an open mind about models of low-energy supersymmetry, and I will try to show how results generalise.

There have been many good reviews written over the years on the properties of supersymmetric models. I try to stick to the conventions of Stephen Martin's supersymmetry primer [1] for the basics. There are the two tomes by Djouadi for a comprehensive discussion in the SM [2] and MSSM [3] circa 2005. For a more recent summary of (supersymmetric) Higgs production and decays see the LHC Higgs cross-section working group report from 2016 [4]. For an overview of recent developments in precision calculations of the masses of Higgs bosons, see [5].

2 The Higgs bosons in the MSSM at tree level

I'll start by going through the standard stuff about the Higgs bosons at tree level in the MSSM.

2.1 Potential

For our supersymmetric model we have two Higgs doublets, so its potential is somewhat complicated because there are scalars that mix. At tree level, the Higgs scalar potential is given in components by

$$\begin{aligned} V =& V_F + V_D + V_{\rm soft} \\ =& (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\ &+ [B_\mu\,(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}] \\ &+ \frac{1}{8}(g_Y^2 + g_2^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{1}{2} g_2^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 \end{aligned}$$

In terms of just the neutral components this gives

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (B_\mu\, H_u^0 H_d^0 + \text{c.c.}) + \frac{1}{8}(g_Y^2 + g_2^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

The first thing that we note is that the quartic coupling is given by the gauge couplings! This is a drastic and important difference compared to the SM: the Higgs quartic coupling (at tree level) is not a parameter, but a *prediction* of the theory! To work out the Higgs masses we need to find the minimum conditions. But we also find that the potential must obey some conditions in order to be at a true minimum, since we now have more field directions.

We need the potential to have a minimum and not a runaway at infinity; at large H_u, H_d this is true except perhaps when $H_u = H_d = H$. Along that (D-flat) line, we have

$$V \to (m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2)|H|^2 \to m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2 > 0.$$

Similarly, at the origin of field space, taking the second derivatives wrt H_u^0, H_d^0 we find the mass matrix

$$\mathcal{M}_h^2\Big|_{H_u^0=H_d^0=0} = \begin{pmatrix} m_{H_u}^2+\mu^2 & -B_\mu \\ -B_\mu & m_{H_d}^2+\mu^2 \end{pmatrix}.$$

We see that if $(m_{H_u}^2+\mu^2)(m_{H_d}^2+\mu^2) < B_\mu^2$ the origin of field space is only a saddle point, and so the true electroweak vacuum can be at nonzero H_u^0, H_d^0.

2.1.1 Goldstones and fields

When we break the $SU(2)\times U(1)_Y$ symmetry down to $U(1)_{\rm em}$ we find would-be goldstone bosons. However, determining which fields they are is not quite so straightforward since we still have only three goldstones but we now have 8 real scalar degrees of freedom. Let us define

$$\langle H_u^0\rangle = \frac{1}{\sqrt{2}}v_u \equiv \frac{1}{\sqrt{2}}v\sin\beta, \qquad \langle H_d^0\rangle = \frac{1}{\sqrt{2}}v_d \equiv \frac{1}{\sqrt{2}}v\cos\beta. \tag{2.1}$$

If we write

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix},$$

then we can expand the kinetic terms to find the mass of the Z and W bosons, similar to the Standard Model:

$$\begin{aligned}\mathcal{L} \supset & |(\partial_\mu + i\frac{1}{2}g_Y B_\mu + ig_2 T^a W_\mu^a)H_u|^2 + |(\partial_\mu - \frac{1}{2}g_Y B_\mu + ig_2 T^a W_\mu^a)H_d|^2 \\ \supset & \frac{|v|^2}{8}\left[\left(g_Y B_\mu - g_2 W_\mu^3\right)^2 \sin^2\beta + \left(g_Y B_\mu - g_2 W_\mu^3\right)^2\cos^2\beta\right] + \frac{v^2}{4}g_2^2|W_\mu^+|^2\sin^2\beta + \frac{v^2}{4}g_2^2|W^-|^2\cos^2\beta \\ = & \frac{(g_Y^2+g_2^2)v^2}{4}\times\frac{1}{2}Z_\mu Z^\mu + \frac{g_2^2v^2}{4}W_\mu^+W^{-\,\mu}.\end{aligned} \tag{2.2}$$

So we see that the combination of generators $g_Y Y + g_2 T^3$ corresponds to the broken, Z, direction, and an orthogonal combination proportional to $g_2 Y - g_Y T^3$ is unbroken, corresponding to the photon. The fact that the (tree-level) predictions about the mass of the W and the Z bosons are identical to the prediction of the SM (once we defined $v_u = \sin\beta v, v_d = \cos\beta v$) is guaranteed because the

Now we can use some facts about Goldstone bosons: if we define the field transformations under the (broken) symmetries to be

$$\delta\phi_i = \alpha_i^a \equiv T_{ij}^a\phi_j, \qquad \langle\delta\phi_i\rangle \neq 0, \tag{2.3}$$

then the Goldstone directions are given by $G^a \propto \alpha_i^a\phi_i$ because $\langle\alpha_i\frac{\partial^2 V}{\partial\phi_i\partial_{\phi_j}}\rangle = 0$. To see this, note that the potential of the theory is invariant under these transformations, so $V(\phi_i+\delta\phi_i) = V(\phi_i)$ implying

$$0 = \delta\phi_i\frac{\partial V}{\partial\phi_i}. \tag{2.4}$$

This is true for all ϕ_i so we can differentiate it:

$$0 = \frac{\partial \delta\phi_i}{\partial \phi_j}\frac{\partial V}{\partial \phi_i} + \delta\phi_i \frac{\partial^2 V}{\partial\phi_i \partial\phi_j}. \tag{2.5}$$

The first term vanishes if we are at the minimum of the potential, and the second is the mass matrix multiplied by a non-vanishing vector for the Goldstone directions.

Now for the case above we have broken $SU(2)$ generators $\frac{1}{2}\sigma^1, \frac{1}{2}\sigma^2$ and the broken transformation associated with the Z-boson $g_Y Y + g_2 T^3$. Fortunately we know that the charged and neutral components cannot mix, so for σ^1, σ^2 we have two real Goldstones that make one complex one: G^+ must be some combination of $H_u^+, (H_d^-)^*$. Following the Goldstone procedure we can determine the broken generators just by making group tranformations acting on the expectation values of H_u, H_d. Writing the broken transformations as

$$ia^1 T^1 + ia^2 T^2 + ia^Z \frac{1}{\sqrt{g_Y^2 + g_2^2}}(g_Y Y + g_2 T^3),$$

we can write

$$\begin{pmatrix} \delta H_u^+ \\ \delta H_u^0 \end{pmatrix} = \frac{i}{2\sqrt{2}} v \sin\beta \begin{pmatrix} a^1 - ia^2 \\ \alpha^Z(\frac{g_Y - g_2}{\sqrt{g_Y^2+g_2^2}}) \end{pmatrix}, \qquad \begin{pmatrix} \delta H_d^0 \\ \delta H_d^- \end{pmatrix} = \frac{i}{2\sqrt{2}} v \cos\beta \begin{pmatrix} -a^Z(\frac{g_Y - g_2}{\sqrt{g_Y^2+g_2^2}}) \\ a^1 + ia^2 \end{pmatrix}. \tag{2.6}$$

So if we write

$$H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}[s_\beta v + h_u^r + ih_u^i] \end{pmatrix}, \qquad H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}[c_\beta v + h_d^r + ih_d^r] \\ H_d^- \end{pmatrix},$$

and defining $a^+ \equiv ia^1 + a^2, a^- \equiv ia^1 - a^2 = -(a^+)^*$, then we can rewrite the above as

$$\begin{pmatrix} \delta H_u^+ \\ \delta h_u^i \end{pmatrix} = \sin\beta \begin{pmatrix} N^\pm a^+ \\ N^0 a^Z \end{pmatrix}, \qquad \begin{pmatrix} \delta h_d^i \\ \delta H_d^- \end{pmatrix} = \cos\beta \begin{pmatrix} -N^0 a^Z \\ N^\pm a^- \end{pmatrix} = \cos\beta \begin{pmatrix} -N^0 a^Z \\ -N^\pm (a^+)^* \end{pmatrix},$$

where $N^0, N^\pm$ are real normalisation constants. By that token, we can isolate the neutral Goldstone boson as

$$h_u^i = N^0 \sin\beta G^0 + ..., h_d^i = -N^0 \cos\beta G^0 + ... \tag{2.7}$$

The sign of N^0 is entirely a convention (its modulus must be unity to ensure that the fields are canonically normalised.) and differs among references. H_u^0, H_d^0 contain together 4 real scalars, one of which is our would-be Goldstone boson G^0. The other three must include the Higgs boson – but we have two additional Higgses! If we neglect CP violation (which must be small in the Higgs sector anyway) then B_μ, μ must be real. Then the real and imaginary parts of H_d^0, H_u^0 do not mix – they are split into two scalars and two pseudoscalars. Then equation (2.7) is enough to determine the pseudoscalars, because we have only two and 2×2 matrices are easy: by the conventions of [6] we take $N^0 = 1$ and

$$h_u^i = \sin\beta G^0 + \cos\beta A, \qquad h_d^i = -\cos\beta G^0 + \sin\beta A. \tag{2.8}$$

For the charged bosons, we can either expand the real and imaginary parts, or write in terms of complex fields where $H_u^+, (H_d^-)^*$ mix to give a charged Goldstone G^+ (and where $G^- = (G^+)^*$):

$$H_u^+ = N^\pm \sin\beta G^+ + ..., (H_d^-)^* = -N^\pm \cos\beta G^+ + ... \tag{2.9}$$

by the same convention we take $N^\pm = 1$ and

$$H_u^+ = \sin\beta G^+ + \cos\beta H^+, \qquad (H_d^-)^* = -\cos\beta G^+ + \sin\beta H^+. \tag{2.10}$$

Finally, the real parts of the neutral bosons will also mix. However, their mixing is *not* determined by the symmetries; so we introduce a new mixing angle α.

$$\begin{pmatrix} h_u^r \\ h_d^r \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix} \equiv R_\alpha \begin{pmatrix} h \\ H \end{pmatrix} \tag{2.11}$$

By defining

$$R_\beta \equiv \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \tag{2.12}$$

we can write, compactly,

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \frac{v}{\sqrt{2}} \begin{pmatrix} \sin\beta \\ \cos\beta \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h \\ H \end{pmatrix} + \frac{i}{\sqrt{2}} R_\beta \begin{pmatrix} G \\ A \end{pmatrix} \tag{2.13}$$

$$\begin{pmatrix} H_u^+ \\ (H_d^-)^* \end{pmatrix} = R_\beta \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}. \tag{2.14}$$

(recall $G^- = \overline{G^+}, H^- = \overline{H^+}$).

We therefore see that the Higgs sector has decomposed into:

- One neutral and two charged would-be Goldstone bosons.
- Three real scalars h, H, A. If CP is preserved then h, H are scalars that can mix with each other, and A is a pseudoscalar that cannot mix with the other states (if we allow for CP violation then we need to introduce more phases, see later).
- A charged Higgs $H^\pm$.

It now remains to determine the masses of these states!

2.2 Minimum condition and mass matrices

Taking the first derivatives of the potential w.r.t. H_u^0, H_d^0 in the absence of CP violation we find

$$\begin{aligned} 0 =& v \sin\beta \bigg[m_{H_u}^2 + \mu^2 - B_\mu \cot\beta - \frac{1}{2} M_Z^2 \cos 2\beta \bigg] \\ 0 =& v \cos\beta \bigg[m_{H_d}^2 + \mu^2 - B_\mu \tan\beta + \frac{1}{2} M_Z^2 \cos 2\beta \bigg]. \end{aligned} \tag{2.15}$$

To satisfy these equations we must eliminate two quantities. Clearly in the MSSM the most convenient are either $\{m_{H_u}^2, m_{H_d}^2\}$ or $\{\mu^2, B_\mu\}$. Neither relate to observable quantities, but in many traditional models there is a prediction for μ/B_μ so we typically solve for the first pair, and I shall do so here.

I note in passing that in other SUSY theories, sometimes *trilinear* couplings enter in the tadpole equations (or even singlet tadpole terms!) and it can therefore be convenient to eliminate those. This can apparently be simpler because they affect the masses less but have different complications.

To derive the mass matrices, it is most convenient to stick to the original basis and write

$$H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}[s_\beta v + h_u^r + i h_u^i] \end{pmatrix}, \qquad H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}[c_\beta v + h_d^r + i h_d^r] \\ H_d^- \end{pmatrix}.$$

Then let us write the mass matrices for the three classes of fields. We start with the pseudoscalars (h_d^i, h_u^i) :

$$\begin{aligned} \mathcal{M}_A^2 =& \begin{pmatrix} m_{H_u}^2 + \mu^2 - \frac{1}{2} M_Z^2 \cos 2\beta & B_\mu \\ B_\mu & m_{H_d}^2 + \mu^2 + \frac{1}{2} M_Z^2 \cos 2\beta \end{pmatrix} \\ =& \begin{pmatrix} B_\mu \cot\beta & B_\mu \\ B_\mu & B_\mu \tan\beta \end{pmatrix} \end{aligned} \tag{2.16}$$

where on the second line we used the minimum conditions. We then see that we have the Goldstone boson (in Landau gauge – in general we need an R_ξ gauge which gives a mass to the Goldstone) and a pseudoscalar A of mass

$$M_A^2 = B_\mu (\cot\beta + \tan\beta) = \frac{2 B_\mu}{\sin 2\beta}. \tag{2.17}$$

This allows us to rewrite the minimisation conditions as

$$\begin{aligned} \mu^2 =& -\frac{M_Z^2}{2} + \frac{1}{\tan^2\beta - 1} (m_{H_d}^2 - \tan^2\beta m_{H_u}^2) \\ M_A^2 =& m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 > 0. \end{aligned}$$

We can also write the charged Higgs mass matrix in the basis (H_u^+, H_d^-), recalling that $M_W^2 = \frac{1}{4} v^2 g_2^2$, as

$$\begin{aligned} \mathcal{M}_{H^\pm}^2 =& \begin{pmatrix} m_{H_u}^2 + \mu^2 - \frac{1}{2} M_Z^2 \cos 2\beta + M_W^2 \cos^2\beta & B_\mu + M_W^2 \sin\beta\cos\beta \\ B_\mu + M_W^2 \sin\beta\cos\beta & m_{H_d}^2 + \mu^2 + \frac{1}{2} M_Z^2 \cos 2\beta + M_W^2 \sin^2\beta \end{pmatrix} \\ =& (B_\mu + M_W^2 \sin\beta\cos\beta) \begin{pmatrix} \cot\beta & 1 \\ 1 & \tan\beta \end{pmatrix}, \end{aligned} \tag{2.18}$$

so again we have the would-be Goldstone boson, and the charged Higgs of mass

$$M^2_{H^+} = M^2_A + M^2_W. \tag{2.19}$$

Finally, in the basis (h^r_u, h^r_d) the Higgs mass matrix is

$$\begin{aligned}\mathcal{M}^2_H =& \begin{pmatrix} m^2_{H_u} + \mu^2 - \frac{1}{2}M^2_Z \cos 2\beta + M^2_Z \sin^2\beta & -B_\mu - M^2_Z \cos\beta\sin\beta \\ -B_\mu - M^2_Z\cos\beta\sin\beta & m^2_{H_d} + \mu^2 + \frac{1}{2}M^2_Z\cos 2\beta + M^2_Z\cos^2\beta \end{pmatrix} \\ =& M^2_A \cos\beta\sin\beta \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix} + M^2_Z \cos\beta\sin\beta \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix}. \end{aligned} \tag{2.20}$$

These now do not give a zero eigenvalue. However, it is common to diagonalise the piece proportional to M^2_A first by making the transformation

$$\begin{pmatrix} h^r_u \\ h^r_d \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} \tilde{h} \\ \tilde{H} \end{pmatrix}$$

because in the basis $(\tilde{h}, \tilde{H})$ the matrix simplifies a little to

$$\mathcal{M}^2_h \to \begin{pmatrix} M^2_Z \cos^2 2\beta & -M^2_Z \sin 2\beta \cos 2\beta \\ -M^2_Z \sin 2\beta\cos 2\beta & M^2_A + M^2_Z \sin^2 2\beta \end{pmatrix}. \tag{2.21}$$

Since we expect $M^2_A > M^2_Z$ we can then treat the diagonalisation of the mass matrix perturbatively; in the limit $M^2_A \gg M^2_Z$ the perturbations become very small – the heavy higgs *decouples* – and we must have

$$\sin\alpha = -\cos\beta, \qquad \cos\alpha = \sin\beta, \tag{2.22}$$

or equivalently $\alpha = \beta - \pi/2$. In that case, the Higgs bosons *align* with the would-be Goldstone boson rotations, so we could separate the entire complex fields into a “SM-like” and a “heavy” one (which we will discuss more later). Alternatively, it is also possible to have “alignment without decoupling” if the angles coincide without M_A being very heavy, and this might also be important.

We note that in the case of alignment we can write

$$H^0_u \to \frac{1}{\sqrt{2}}(v+h)\sin\beta + ..., H^0_d \to \frac{1}{\sqrt{2}}(v+h)\cos\beta$$

and so the state h is really a Standard-Model-like Higgs. However, we find that this is the maximal value for the light Higgs mass; we can write (at tree level)

$$\begin{aligned} M^2_A + M^2_Z =& M^2_h + M^2_H \to M^2_h = M^2_A + M^2_Z - M^2_H \\ M^2_{h,H} =& \frac{1}{2}\left(M^2_A + M^2_Z \mp \sqrt{(M^2_A - M^2_Z)^2 + 4M^2_Z M^2_A \sin^2 2\beta}\right) \\ \frac{s_{2\alpha}}{s_{2\beta}} =& -\frac{M^2_h + M^2_H}{M^2_H - M^2_h}, \qquad \frac{\tan 2\alpha}{\tan 2\beta} = \frac{M^2_A + M^2_Z}{M^2_A - M^2_Z}. \end{aligned} \tag{2.23}$$

By convention then we take $\beta \in [0, \pi/2], \alpha \in [-\pi/2, 0]$.

At any fixed value of $\tan\beta$, we can calculate the derivative of M_h^2 with respect to M_A^2:

$$\frac{dM_h^2}{dM_A^2} = 1 - \frac{M_A^2 - M_Z^2 + 2M_Z^2 s_{2\beta}^2}{\sqrt{(M_A^2 - M_Z^2)^2 + 4M_A^2 M_Z^2 s_{2\beta}^2}} = 1 - \frac{M_A^2 - M_Z^2 + 2M_Z^2 s_{2\beta}^2}{\sqrt{(M_A^2 - M_Z^2 + 2M_Z^2 s_{2\beta}^2)^2 + 4M_Z^4 s_{2\beta}^2 c_{2\beta}^2}}, \quad (2.24)$$

which is only zero as $M_A^2 \to \infty$ and is positive everywhere else.

From the above we can conclude:

- At tree level, $m_h^2 \leq M_Z^2 \cos^2 2\beta$!
- Therefore in the MSSM loop corrections are large:

$$(125 \text{ GeV})^2 - (91 \text{ GeV})^2 \simeq (86 \text{ GeV})^2$$

- Since this corresponds to the maximal tree-level mass, *in general the loop corrections are at least as large, and often larger than, the tree mass.*

The loop corrections to the Higgs mass are dominated by the stop squarks, which couple via the top Yukawa coupling; these can easily give the required boost. However, it means that the two-loop corrections to the Higgs mass are significant: they can give a mass shift of up to ~ 10 GeV – so there has been/is a lot of work in understanding these.

2.3 CP violation

I will now make a few brief comments about the theory with CP violation. In general, we should allow for a phase between the expectation values of H_d and H_u. This is easily taken care of by defining and overall phase, η:

$$H_d \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + \phi_d + i\sigma_d) \\ H_d^- \end{pmatrix}, \qquad H_u \equiv e^{i\eta} \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}(v_u + \phi_u + i\sigma_u) \end{pmatrix}. \quad (2.25)$$

The fact that the same phase rotates v_u and H_u^+ is necessary for the charged Goldstone boson. We can treat η as an input parameter in the same way as we previously treated $v, \tan\beta$ as input parameters, even though strictly speaking they should be determined from the minimisation of the potential.

The real scalars $\{\sigma_u, \sigma_d, \phi_u, \phi_d\}$ (which includes the longitudinal component of the Z) do not have a definite CP phase any more so they can all mix together; we no longer deal with 2×2 but now 4×4 matrices, which are best handled numerically.

As regards the minimisation conditions, we have an additional phase that enters from the holomorphic Higgs mass term $\mathcal{L} \supset -B_\mu H_u \cdot H_d$; we put $B_\mu \equiv e^{i\varphi_{B_\mu}} |B_\mu|$ and extract φ_{B_μ} from the tadpole equations. But now we can take the derivative of the potential with respect to four fields:

$$\begin{aligned}
\frac{\partial \Delta V}{\partial \phi_d}\bigg|_{\phi_{u,d}=\sigma_{u,d}=0} =& 0 = v c_\beta [m_{H_d}^2 + \frac{1}{2} c_{2\beta}^2 M_Z^2 + |\mu|^2 - t_\beta |B_\mu| \cos(\eta + \varphi_{B_\mu})] \\
\frac{\partial \Delta V}{\partial \phi_u}\bigg| =& 0 = v s_\beta [m_{H_u}^2 - \frac{1}{2} c_{2\beta}^2 M_Z^2 + |\mu|^2 - \frac{|B_\mu|}{t_\beta} \cos(\eta + \varphi_{B_\mu})] \\
\frac{\partial \Delta V}{\partial \sigma_d}\bigg| =& 0 = v s_\beta [\sin(\eta + \varphi_{B_\mu}) |B_\mu|] \\
\frac{\partial \Delta V}{\partial \sigma_u}\bigg| =& 0 = v c_\beta [\sin(\eta + \varphi_{B_\mu}) |B_\mu|]
\end{aligned} \tag{2.26}$$

The last two equations are not independent due to the gauge symmetries. They also show that η and φ_{B_μ} are not independent: at tree level $\eta = -\varphi_{B_\mu}$, but this can be modified once loop corrections are taken into account.

3 Higgs boson couplings

The mixing matrices determine everything as far as the phenomenology is concerned: for searches, we will be interested in the couplings to SM particles – as well as any possible invisible or exotic particles. The SM couplings are easily written down just from the lagrangian and the mixing matrices.

3.1 Gauge boson couplings

For the gauge boson-Higgs couplings, for a generic theory involving scalars and vectors the possible terms are

$$\mathcal{L} \supset \frac{1}{2} g_{S_i V_a V_b} S_i V_{a,\mu} V_b^\mu + \frac{1}{4} g_{S_i S_j V_a V_b} S_i S_j V_{a,\mu} V_b^\mu + g_{S_i S_j V_a} (S_i \partial_\mu S_j - S_j \partial_\mu S_i) V_a. \tag{3.1}$$

For the first type of term, we have expanding the kinetic terms:

$$\begin{aligned}
\mathcal{L} \supset & \bigg[\frac{(g_Y^2 + g_2^2)}{4} Z_\mu Z^\mu + \frac{g_2^2}{2} W_\mu \overline{W}^\mu\bigg] \bigg(|H_u^0|^2 + |H_d^0|^2\bigg) \\
\supset & \bigg[\frac{1}{2} m_Z^2 Z_\mu Z^\mu + m_W^2 W_\mu \overline{W}^\mu\bigg] \bigg[(\sin\beta + \cos\alpha h + \sin\alpha H)^2 + (\cos\beta - h \sin\alpha + H\cos\alpha)^2\bigg].
\end{aligned} \tag{3.2}$$

We therefore find the couplings

$$\begin{aligned}
g_{hZZ} =& m_Z^2 \sin(\beta - \alpha), \qquad g_{HWW} = m_W^2 \sin(\beta - \alpha) \\
g_{HZZ} =& m_Z^2 \cos(\beta - \alpha), \qquad g_{HWW} = m_W^2 \cos(\beta - \alpha).
\end{aligned} \tag{3.3}$$

Bearing in mind that when we have alignment we have that $\alpha = \beta - \pi/2$ and hence $\sin(\beta - \alpha) = 1, \cos(\beta - \alpha) = 0$ we see that when H becomes heavy, or in the alignment limit, its coupling via the mass term of the gauge bosons vanishes. To derive the other trilinear, we have

$$D_\mu = \partial_\mu - ieQA_\mu - \frac{ie}{c_W s_W} Z_\mu (T_3 - s_W^2 Q) - \frac{ig_2}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} \tag{3.4}$$

and so

$$D_\mu H_u = \begin{pmatrix} \partial_\mu H_u^+ \\ \partial_\mu H_u^0 \end{pmatrix} - ieA_\mu \begin{pmatrix} H_u^+ \\ 0 \end{pmatrix} - \frac{ie}{c_W s_W} Z_\mu \begin{pmatrix} (\frac{1}{2} - s_W^2) H_u^+ \\ -\frac{1}{2} H_u^0 \end{pmatrix} - \frac{ig_2}{\sqrt{2}} \begin{pmatrix} W_\mu^+ H_u^0 \\ W_\mu^- H_u^+ \end{pmatrix},$$

$$D_\mu H_d = \begin{pmatrix} \partial_\mu H_d^0 \\ \partial_\mu H_d^- \end{pmatrix} + ieA_\mu \begin{pmatrix} 0 \\ H_d^- \end{pmatrix} - \frac{ie}{c_W s_W} Z_\mu \begin{pmatrix} \frac{1}{2} H_d^0 \\ -(\frac{1}{2} + s_W^2) H_d^- \end{pmatrix} - \frac{ig_2}{\sqrt{2}} \begin{pmatrix} W_\mu^+ H_d^- \\ W_\mu^- H_d^0 \end{pmatrix}. \quad (3.5)$$

This gives us

$$\begin{aligned} |D_\mu H_u|^2 + |D_\mu H_d|^2 \supset & \frac{-ie}{2c_W s_W} Z^\mu [\partial_\mu H_u^0 (H_u^0)^* - H_u^0 \partial_\mu (H_u^0)^* - \partial_\mu H_d^0 (H_d^0)^* + H_d^0 \partial_\mu (H_d^0)^*] \\ \supset & \frac{-ie}{2c_W s_W} Z^\mu [\cos(\beta-\alpha) Z^\mu (A\partial_\mu h - h\partial_\mu A) - \sin(\beta - \alpha)(A\partial_\mu H - H\partial_\mu A)]. \end{aligned} \quad (3.6)$$

Similarly we find

$$g_{W^\pm H H^\mp} \propto \sin(\beta - \alpha), \qquad g_{W^\pm h H^\mp} \propto \cos(\beta - \alpha).$$

All of these couplings are proportional to $\cos(\beta - \alpha)$ or $\sin(\beta - \alpha)$.

We can also derive the couplings to Goldstone bosons (we have not discussed gauge fixing, but it is done in the standard way for R_ξ gauges). However, it can be shown that for general renormalisable theories – not just the MSSM or the SM – all would-be Goldstone boson couplings can be related to the gauge couplings and gauge boson masses.

3.2 Fermion couplings

The couplings to fermions are easier to derive. We just start from the Yukawa couplings:

$$\mathcal{W}_{\text{Yukawa}} = \overline{u}_i (Y_u)_{ij} Q_j \cdot H_u - \overline{d}_i (Y_d)_{ij} Q_j \cdot H_d - \overline{e}_i (Y_e)_{ij} L_j \cdot H_d. \quad (3.7)$$

Once we diagonalise the fermion mass matrices, we also diagonalise the Yukawa couplings *for the neutral scalar higgses* h, H, A:

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} \supset & - \frac{\cos\alpha}{\sin\beta} \frac{m_t}{v} \bar{t} t h - \frac{\sin\alpha}{\sin\beta} \frac{m_t}{v} \bar{t} t H - i \cot\beta \frac{m_t}{v} \bar{t} \gamma_5 t A \\ & + \frac{\sin\alpha}{\cos\beta} \frac{m_b}{v} \bar{b} b h - \frac{\cos\alpha}{\cos\beta} \frac{m_b}{v} \bar{b} b H - i \tan\beta \frac{m_t}{v} \bar{b} \gamma_5 b A \\ & + \frac{\sqrt{2}}{v} V_{ud} \Big(H^+ \overline{u} [m_d \tan\beta P_L + m_u \cot\beta P_R] d + h.c. \Big). \end{aligned} \quad (3.8)$$

We see that the couplings to charged Higgses are not diagonal; and those with the pseudoscalars contain factors of γ_5. However, more pertinently, we see that certain couplings can have large enhancements compared to the SM.

One possible explanation for the hierarchy between the up-type quark masses and the down-type ones is that the up-type Higgs vev is larger. We have $m_t = 172.69 \pm 0.3$ GeV from the PDG from direct measurements, where the uncertainty is hard to quantify, or 172.5 ± 0.7 from direct measurements.

There are very large corrections from strong coupling effects which mean that the $\overline{\rm MS}$ mass parameter in the Lagrangian is closer to 160 GeV. Similarly, the pole mass of the bottom quark is difficult to define, but the $\overline{MS}$ mass defined at the scale m_b – usually quoted as $m_b(m_b)$ – is 4.18 ± 0.03 GeV. There is thus a factor of 40. While the hierarchy between the other quarks is smaller, the fact that the masses of the leptons are similar to the down-type quarks in scale fits nicely with the structure of the MSSM/2HDM since the scale of both is determined by $v_d = v \cos\beta$.

The upshot is that we expect $v_u/v_d = \tan\beta$ to be large, or at least larger than 1. If $\tan\beta$ is small then the top Yukawa coupling becomes large, and the bottom small, since

$$y_t = \frac{\sqrt{2}}{v \sin\beta} m_t + ..., \qquad y_b = \frac{\sqrt{2}}{v \cos\beta} m_b + ... \tag{3.9}$$

where in the ellipsis we include quantum corrections. We find in practice that values below 1 for $\tan\beta$ are hard to realise, and complicate running of the RGEs among other problems. But we see if $\tan\beta$ is large, then $\sin\beta \sim 1, \cos\beta \sim 1/\tan\beta$ and we can have y_b of $\mathcal{O}(1)$ for $\tan\beta \sim 40$. Hence values $\tan\beta$ are typically considered in this range.

However, when we have such large values, we have enhancements of the bottom quark and tau lepton Yukawa couplings. We also see very large enhancements to the coupling of the pseudoscalar A to the bottom quarks and tau leptons.

4 Production and decays

Following the discovery of the SM-like Higgs boson, its couplings have been measured with impressive precision. This means that we can constrain SUSY models *both* from direct searches for the heavy Higgs bosons *and* from deviations of the properties of the SM-like one.

4.1 Couplings of the SM-like Higgs

The SM Higgs couples strongly to the massive gauge bosons, and to top and bottom quarks. The branching ratios for the important decay modes are

$$\begin{aligned}
BR_{SM}(h \to \bar{b}b) =& 5.81 \times 10^{-1} \\
BR_{SM}(h \to WW^*) =& 2.15 \times 10^{-1} \\
BR_{SM}(h \to gg) =& 8.18 \times 10^{-2} \\
BR_{SM}(h \to \tau\tau) =& 6.3 \times 10^{-2} \\
BR_{SM}(h \to \bar{c}c) =& 2.88 \times 10^{-2} \\
BR_{SM}(h \to ZZ^*) =& 2.64 \times 10^{-2} \\
BR_{SM}(h \to \mu\mu) =& 2.2 \times 10^{-4} \\
BR_{SM}(h \to \gamma\gamma) =& 2.27 \times 10^{-3} \\
BR_{SM}(h \to Z\gamma) =& 1.54 \times 10^{-3}.
\end{aligned} \tag{4.1}$$

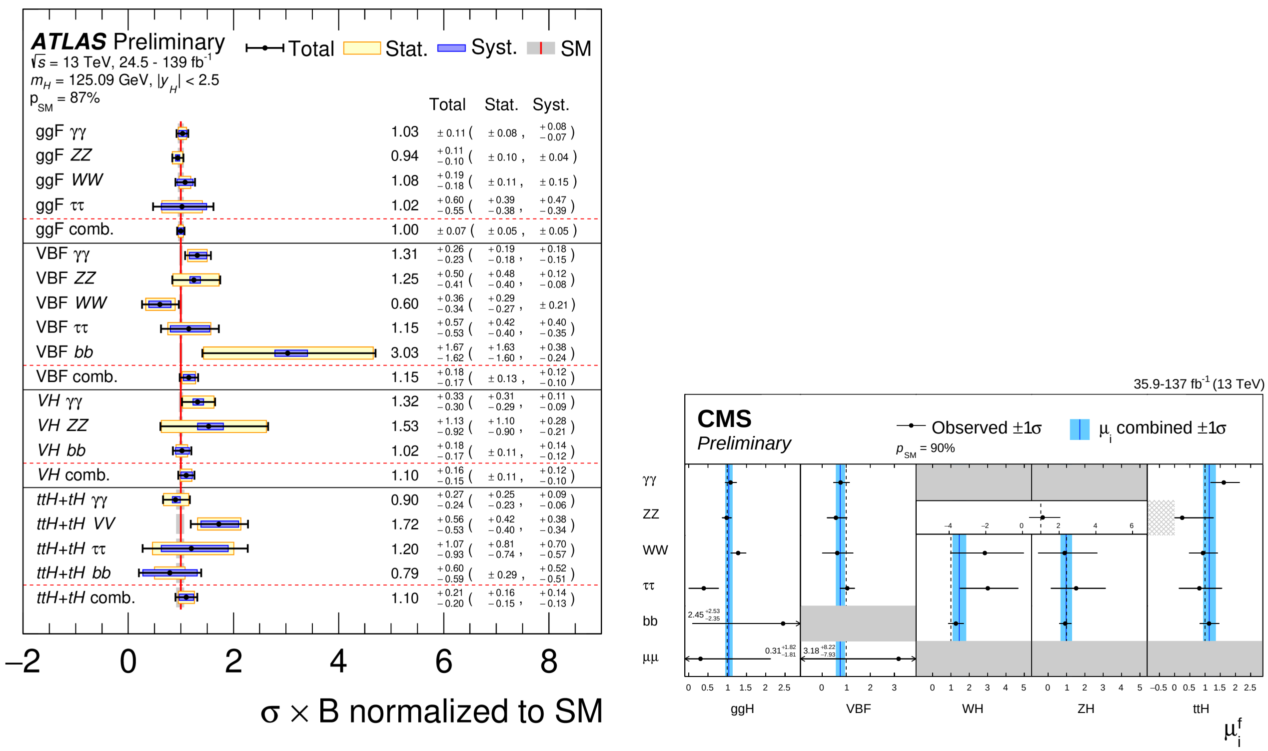

Figure 1: ATLAS [?] (left) and CMS [?] (right) full run 2 data for Higgs production and decays.

A comparison with the full run 2 data is shown in figure 1.

Any BSM theory could modify these predictions. To parametrise the deviations *in the measurements* we can define

$$\mu_X \equiv \frac{\sigma(X \to h)}{\sigma_{SM}(X \to h)}, \qquad \mu^Y \equiv \frac{Br(h \to Y)}{Br_{SM}(h \to Y)}, \tag{4.2}$$

and therefore construct for each process a new observable

$$\mu \equiv \mu_X \mu^Y, \tag{4.3}$$

which is the ratio of $\sigma \times \mathrm{Br}$ for the production mode times branching ratio (which is the quantity that can actually be measured). These have been measured by both ATLAS and CMS rather precisely, and are in good agreement with the SM, as can be seen from figure 1. Each experiment also produces a combination of the weighted average μ over all of its channels; as of writing (in 2022) the values are [**?**, **?**]:

$$\mu = 1.06 \pm 0.07 \text{ (ATLAS)}, \qquad \mu = 1.02^{+0.07}_{-0.06} \text{ (CMS)}. \tag{4.4}$$

We could expect that a slightly stronger bound on generic new physics would be found from combining the two, but this has so far not been done for the full Run 2 dataset.

As a simple example of how constraining the Higgs couplings are, if we imagine that we have some other inert singlet field S (e.g. in the NMSSM) that mixes with the Higgs so that only $\tilde{h}$ couples to the Standard Model fermions and gauge bosons, but then the mass eigenstates are h, s mixing via

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ -S_{12} & S_{11} \end{pmatrix} \begin{pmatrix} \tilde{h} \\ S \end{pmatrix} \tag{4.5}$$

then we will find that

$$\mu = |S_{11}|^2 \leq 1. \tag{4.6}$$

Hence if we allow a 3σ deviation from the ATLAS result, we require

$$1 - |S_{11}|^2 \leq 0.15. \tag{4.7}$$

Finally, the LHC has put limits on the Higgs decay to invisible particles, of [**?**, 7]:

$$\mathrm{BR}(h \to \text{invisible}) < \begin{cases} 0.145 & \text{ATLAS} \\ 0.18 & \text{CMS} \end{cases} \tag{4.8}$$

These constrain SUSY theories in the case that there is a light neutralino, because the large phase space typically leads to large branching ratios. However, this is a rather model-dependent statement.

These observations are rather constraining and force us to be rather near the alignment limit for the MSSM Higgs bosons.

The good news is that at *tree level* in the alignment limit the branching ratios of the SM-like Higgs boson will also be *identical in SUSY models*. However, quantum effects in SUSY theories are very important and might (in future) be relevant to spoil even this case.

4.2 Diphotons and digluons

The first important observation is that the decays to diphotons (and digluons) only take place at loop level. So any new charged particle that couples to the Higgs will modify it at the same order. This channel, although a small branching ratio, is very clean and was used for the Higgs discovery. To compute it, we will need the couplings to the fermions, the W boson, and any light SUSY particles.

The partial widths for any scalar decaying to diphotons and gluons at LO are given by

$$\begin{aligned} \Gamma(\Phi \to \gamma\gamma)_{\mathrm{LO}} &= \frac{G_F \alpha^2(0) m_\Phi^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c^f Q_f^2 r_f^\Phi A_f(\tau_f) + \sum_s N_c^s r_s^\Phi Q_s^2 A_s(\tau_s) + \sum_v N_c^v r_v^\Phi Q_v^2 A_v(\tau_v) \right|^2 \\ \Gamma(\Phi \to gg)_{\mathrm{LO}} &= \frac{G_F \alpha_s^2(\mu) m_\Phi^3}{36\sqrt{2}\pi^3} \left| \sum_f \frac{3}{2} D_2^f r_f^\Phi A_f(\tau_f) + \sum_s \frac{3}{2} D_2^s r_s^\Phi A_s(\tau_s) + \sum_v \frac{3}{2} D_2^v r_v^\Phi A_v(\tau_v) \right|^2 . \end{aligned} \tag{4.9}$$

Here, the sums are over all fermions f, scalars s and vector bosons v which are charged or coloured and which couple to the scalar Φ. Q is the electromagnetic charges of the fields, N_c are the colour factors and D_2 is the quadratic Dynkin index of the colour representation which is normalised to $\frac{1}{2}$ for the fundamental representation. We note that the electromagnetic fine structure constant α must be taken at the scale $\mu = 0$, since the final state photons are real. In contrast, α_s is evaluated at $\mu = m_\Phi$. r_i^Φ are the so-called reduced couplings, the ratios of the couplings of the scalar Φ to the particle i normalised to SM values. These are calculated as

$$r_f^\Phi = \frac{v}{2M_f}(C^L_{\bar{f}f\Phi} + C^R_{\bar{f}f\Phi}), \tag{4.10}$$

$$r_s^\Phi = \frac{v}{2M_s^2} C_{ss^*\Phi}, \tag{4.11}$$

$$r_v^\Phi = -\frac{v}{2M_v^2} C_{vv^*\Phi}. \tag{4.12}$$

Here, v is the electroweak VEV and C are the couplings between the scalar and the different fields with mass M_i $(i = f, s, v)$. Furthermore,

$$\tau_x = \frac{m_\Phi^2}{4m_x^2} \tag{4.13}$$

holds and the loop functions are given by

$$A_f = 2(\tau + (\tau - 1)f(\tau))/\tau^2, \tag{4.14}$$

$$A_s = -(\tau - f(\tau))/\tau^2, \tag{4.15}$$

$$A_v = -(2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau))\tau^2, \tag{4.16}$$

with

$$f(\tau) = \begin{cases} \arcsin^2\sqrt{\tau} & \text{for } \tau \leq 1, \\ -\frac{1}{4}\left(\log\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right)^2 & \text{for } \tau > 1. \end{cases} \tag{4.17}$$

the loop functions have the limiting values

$$\begin{aligned} A_0(0) &= -7, & A_{1/2}(0) &= \frac{4}{3}, & A_0(0) &= \frac{1}{3} \\ A_1(1/\tau) &= -2 + \mathcal{O}(1/\tau), & A_{1/2}(1/\tau) &= \mathcal{O}(1/\tau), & A_0(1/\tau) &= \mathcal{O}(1/\tau). \end{aligned} \tag{4.18}$$

The Standard Model values are

$$A_1(\tau_W) \simeq -8.32, \qquad A_{1/2}(\tau_t) \simeq 1.38. \tag{4.19}$$

The limit $\tau = 0$ corresponds to $m_x \to \infty$, which is useful for the top relative to the SM Higgs, and for heavy SUSY fields. In that case though, unless the mass of the scalar/fermion scales with v, in the limit $m_x \to \infty$ the amplitude vanishes – the heavy particles decouple.

For a pure pseudo-scalar state only fermions contribute, i.e. the LO widths read

$$\Gamma(A \to \gamma\gamma)_{\mathrm{LO}} = \frac{G_F \alpha^2 m_A^3}{32\sqrt{2}\pi^3} \left| \sum_f N_c^f Q_f^2 r_f^A A_f^A(\tau_f) \right|^2, \tag{4.20}$$

$$\Gamma(A \to gg)_{\mathrm{LO}} = \frac{G_F \alpha_s^2 m_A^3}{36\sqrt{2}\pi^3} \left| \sum_f 3D_2^f r_f^A A_f^A(\tau_f) \right|^2, \tag{4.21}$$

where

$$A_f^A = f(\tau)/\tau\,, \tag{4.22}$$

and r_f^A takes the same form as r_f^Φ in (4.10), simply replacing $C^{L,R}_{\bar f f\Phi}$ by $C^{L,R}_{\bar f f A}$. The leading higher-order effects in QCD for these are known for the SM and can be extended to SUSY theories, but we cannot do better than that at the moment.

In the SM, the W boson and the top quark contribute significantly to the diphoton rate. But we see that if any charged SUSY particle is light then it could contribute to the diphoton rate. We see that (in contrast to the early days of the LHC) a 3σ deviation of the diphoton rate would correspond to about 30% currently.

4.3 Higgs production

At 125.09 GeV the production cross-sections at 13 TeV (as listed on the CERN yellow pages) are

$$\begin{aligned}\sigma_{SM}(pp\to h) =& 48.5^{+4.6\%}_{-6.7\%}\text{pb} \qquad \text{gluon fusion}\\ &+3.779 \pm 2.1\%\text{pb} \qquad \text{vector boson fusion}\\ &+1.369 \pm 1.9\%\text{pb} \qquad \text{WH process}\\ &+0.8824 \pm 4.1\%\text{pb} \qquad \text{ZH process}\\ &+0.5065 \pm 9.9\%\text{pb} \qquad \text{ttH process}\end{aligned} \tag{4.23}$$

We see that it is dominated by gluon fusion, which is given at LO by the same amplitude computed above for $\Gamma(\Phi\to gg)$; we could even use the Breit-Wigner formula:

$$\sigma(gg\to\Phi) = \frac{\pi}{8m_h}\frac{\hat s\Gamma(\Phi\to gg)/m_h}{(\hat s - m_h)^2 + (\hat s\Gamma_{\rm tot}/m_h)^2} \simeq \frac{\pi^2}{m_h}\Gamma(\Phi\to gg)\delta(\hat s - m_h^2) \tag{4.24}$$

which forces us to be around $\hat s = m_h^2$, which is the centre of mass of the gluon system. For proton-proton collisions We define $\tau_H \equiv m_h^2/s$ where s is now centre of mass energy of the protons, and integrate over the parton distribution functions of the gluons $g(x,\mu)$:

$$\sigma_{LO}(pp\to\Phi) \simeq \sigma_0^H \tau_H \int_{\tau_H}^1 \frac{dx}{x} g(x,\mu_F) g(\tau_H/x,\mu_F). \tag{4.25}$$

If we are near the alignment limit (as we argued above that we should be) we have

$$\begin{aligned}\mathcal{L}_{\rm Yukawa} \sim & -\frac{m_t}{v}\bar t t h + \cot\beta\frac{m_t}{v}\bar t t H - i\cot\beta\frac{m_t}{v}\bar t\gamma_5 t A\\ & -\frac{m_b}{v}\bar b b h - \tan\beta\frac{m_b}{v}\bar b b H - i\tan\beta\frac{m_t}{v}\bar b\gamma_5 b A\\ & + \frac{\sqrt2}{v}V_{ud}\bigg(H^+\overline{u}[m_d\tan\beta P_L + m_u\cot\beta P_R]d + h.c.\bigg).\end{aligned} \tag{4.26}$$

We see that there are $\tan\beta$-enhancements for the couplings of H/A to bottom quarks – and therefore also tau leptons in the decays. These also enhance the bottom contribution to the gluon fusion

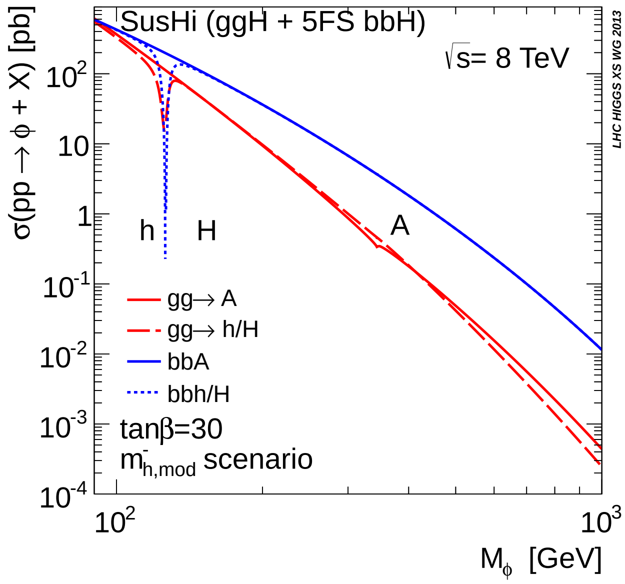

Figure 2: SUSY Higgs cross sections at 8 TeV.

diagrams, while the top contribution is suppressed. At low $\tan\beta$, the production rate of H/A is therefore small, but even when $\tan\beta$ reaches ~ 10, the enhancement to the bottom quark coupling means that the gluon fusion rate becomes large! In addition, at large $\tan\beta$ the bbH process can even dominate for H/A. See figure 2 for an example at $\tan\beta = 30$ at 8 TeV.

At NLO, the process $pp \to h$ is not infra-red safe: real gluon emission must be included. Hence cross-sections are quoted as $pp \to h + X$. In the SM, the leading NNNLO corrections have been computed, at least in the infinite top mass limit. For the MSSM and THDM, some contributions have been computed and combined with the SM-like ones. These are included in the code `SusHi` [], available at `https://sushi.hepforge.org`. Going beyond the MSSM is more tricky. However, even in that case, while any new *colourful* SUSY particles can modify the SM Higgs production, the current limits on such particles imply that their contribution should be small. In which case rescaling the production cross-sections for a SM-like Higgs (and possibly including some leading NLO QCD effects that are available for gluon fusion in certain approximations) should be good enough:

$$\sigma(pp \to \Phi + X)_{\text{channel i}} \simeq \frac{\Gamma(\Phi \to i)}{\Gamma(h \to i)_{\text{SM}, m_h = m_\Phi}} \times \sigma(pp \to h + X)_{\text{SM, channel i}}. \tag{4.27}$$

And we can substitute the width for the coupling squared. Interpolating functions can be constructed for the SM-like cross-section. This is the approach taken in `HiggsBounds`/`HiggsTools` `https://higgsbounds.hepforge.org`, `https://gitlab.com/higgsbounds/higgstools` which can be used to place limits on *any* new Higgs-like bosons, if you give it the couplings to SM fields and the decay widths into SM and BSM particles. Such a calculation is automatically included in `SARAH` `https://sarah.hepforge.org` for *any* model.

4.4 Searches for Heavy Higgs bosons

It is not possible to give model-independent bounds on supersymmetric Higgs bosons. There are therefore three approaches we can take:

1. Consider scenarios with simplifying assumptions. E.g. in the early days of the LHC the CMSSM was much used. I will discuss the hMSSM in a moment.
2. Produce benchmark points to compare with experiments. A set of these was produced recently [8] and is being used.
3. Attempt to generalise the computation of limits so that *any* given scenario in a supersymmetric model can be compared to data. This is now possible with the automatic tools available that I already mentioned above.

The first two approaches are most useful for the experiments. For theorists – especially thos interested in new scenarios or new models – the latter approach is necessary. Fortunately, a significant amount of work has gone into automation in recent years to make this job easier.

A brief and very incomplete list of tools that can be used:

- `SusHi` `https://sushi.hepforge.org`.
- `HiggsSignals`/`HiggsBounds`/`HiggsTools` `https://higgsbounds.hepforge.org`, `https://gitlab.com/higgsbounds/higgstools`.
- `Lilith` `https://lpsc.in2p3.fr/projects-th/lilith/`. Compares model to constraints on the SM-like Higgs couplings.
- `FeynHiggs` `http://www.feynhiggs.de/`. Computes spectrum, decays and production cross-sections for MSSM Higgs bosons with state-of-the-art precision.
- `SUSY-HIT` `https://www.itp.kit.edu/~maggie/SUSY-HIT/` which includes `HDECAY` to compute the spectrum of particles (see later) and decays.
- `NMSSMCALC` `https://www.itp.kit.edu/~maggie/NMSSMCALC/` does the same for the MSSM. Most advanced code for the NMSSM.
- `FlexibleSUSY` `https://flexiblesusy.hepforge.org/`. Creates a spectrum generator *any* model linked to the `SoftSUSY` library (`https://softsusy.hepforge.org/`) using expressions from `SARAH`. Now includes LO Higgs decays.
- `SARAH` `https://sarah.hepforge.org`. Creates a spectrum generator for *any* model linked to the `SPheno` library (`https://spheno.hepforge.org/`), including Higgs masses and decays with state-of-the-art precision, Higgs cross-sections, and output files for `HiggsBounds/HiggsSignals/HiggsTools`.

4.4.1 The hMSSM

Here I will consider one important and simple scenario which is used as a benchmark for seaches. The idea is to attempt to use the Higgs mass as an input. We will see in the next section that this is

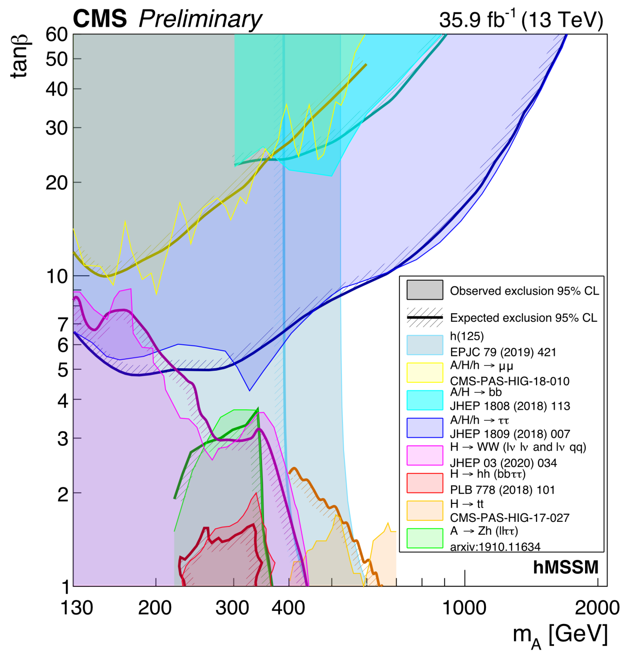

Figure 3: Latest constraints from CMS on the hMSSM.

complicated, because there are substantial quantum corrections to the tree-level masses. But suppose that we assume that the dominant contribution is to correct the up-type Higgs, so in the basis H_u^0, H_d^0 we have

$$\mathcal{M}_h^2 = (\mathcal{M}_h^2)_{\text{tree}} + \begin{pmatrix} 0 & 0 \\ 0 & \Delta M_{22}^2 \end{pmatrix}. \tag{4.28}$$

If we rotate to the $\tilde{h}, \tilde{H}$ basis this gives

$$m_{h,H}^2 = \begin{pmatrix} M_Z^2 c_{2\beta}^2 + \epsilon s_\beta^4 & -M_Z^2 s_{2\beta} c_{2\beta} + s_\beta^3 c_\beta \epsilon \\ -M_Z^2 s_{2\beta} c_{2\beta} + s_\beta^3 c_\beta \epsilon & m_A^2 + M_Z^2 s_{2\beta}^2 + s_\beta^2 c_\beta^2 \epsilon \end{pmatrix}. \tag{4.29}$$

By assuming that the lightest eigenvalue is $(125\,\text{GeV})^2$, we can solve this for ΔM_{22}^2 as a function of $\tan\beta$ and M_A^2! In turn, we can use this to determine α and M_H^2 as a function of these:

$$\begin{aligned} M_H^2 =& \frac{(M_A^2 + M_Z^2 - m_h^2)(M_Z^2 \cos^2\beta + M_A^2 \sin^2\beta) - M_A^2 M_Z^2 \cos^2 2\beta}{M_Z^2 \cos^2\beta + M_A^2 \sin^2\beta - m_h^2} \\ \tan\alpha =& -\frac{(M_Z^2 + M_A^2)\cos\beta\sin\beta}{M_Z^2 \cos^2\beta + M_A^2 \sin^2\beta - m_h^2}. \end{aligned} \tag{4.30}$$

In the hMSSM, we ignore the quantum corrections to $M_{H^\pm}^2 = M_A^2 + M_W^2$ which are therefore nearly degenerate. Hence the entire phenomenology of the Higgs sector is reduced to the parameters $\tan\beta$ and M_A. The latest constraints on this scenario are given in figure 3.

One conclusion is that alignment without decoupling is not possible in the hMSSM. If $\tan\alpha = -\cot\beta$ then

$$M_Z^2 \cos^2\beta + M_A^2 \sin^2\beta - m_h^2 = (M_Z^2 + M_A^2)\sin^2\beta \longrightarrow m_h^2 = M_Z^2 \cos 2\beta \tag{4.31}$$

which is not possible. Hence in the hMSSM the measurements of the *light* Higgs' couplings constrain the $\tan\beta - -M_A$ parameter space. This can certainly be relaxed in in more general scenarios or models beyond the MSSM.

Of course, this approximation can break down in many ways. It assumes that all other SUSY particles do not change the situation, so must be heavy; yet at the same time it assumes that the tree-level expressions for the couplings of the Higgs in terms of the gauge couplings etc should not be changed. This therefore ignores, among other things, running of the gauge couplings. So the predictions should not be considered precise, but it can be a useful guide.

4.4.2 Remark on rare decays

Finally, while I will discuss precision computations in the next section, it is also necesary to mention the decay $B \to s\gamma$. This has a contribution from loops involving the charged Higgs, and, as determined in [9], it bounds the charged Higgs mass to be heavier than 580 GeV independent of the value of $\tan\beta$ (which in turn bounds the mass of the pseudoscalar Higgs to be above around 568 GeV).

5 Precision corrections

In the previous section I hinted at ways in which loop corrections are important in SUSY theories. There has been a very significant industry to bring the precision of supersymmetric models up to the level of the MSSM. Notably this includes computations of decays and production as we have discussed, but in recent years there has been a concerted effort to improve the precision of the prediction of the Higgs *mass*, to promote it to the level of an electroweak precision observable. This should also be accompanied by predictions for the W boson mass, which are less sensitive to (very) heavy particles.

5.1 The Higgs potential in the SM

I'll start with a recap about the Higgs potential in the SM, to show the similarities and differences when we go to the SUSY case.

In the Standard Model, we write the Higgs potential as

$$V_{\rm SM} = \mu^2|H|^2 + \lambda|H|^4 = \mu^2(|G^+|^2 + |H^0|^2) + \lambda(|G^+|^2 + |H^0|^2)^2,$$

We usually write $H^0 = \frac{1}{\sqrt{2}}(v + h + iG)$; the real scalar G and the charged complex Higgs scalar G^+ are the would-be goldstone bosons of the broken symmetries and are eaten by the Z and W bosons respectively. We can then minimise the potential by taking derivatives; we find

$$\left.\frac{\partial V}{\partial h}\right|_{h=G=0} = v(\mu^2 + \lambda v^2) = 0,$$

and then find the Higgs mass to be

$$\left.\frac{\partial^2 V}{\partial h^2}\right|_{h=G=0} = \mu^2 + 3\lambda v^2 = 2\lambda v^2.$$

The electroweak vev v is not something that we measure directly, but it has a very good proxy: G_F! At tree level, we predict G_F from the decay of the muon $\mu \to e\nu_\mu \overline{\nu}_e$ to be:

$$G_F = \frac{1}{\sqrt{2}v^2}. \tag{5.1}$$

Hence we can extract the SM quartic coupling λ from the measurements of the Higgs mass and G_F! At tree level this gives us

$$\lambda = \frac{m_h^2}{\sqrt{2}} G_F \simeq 0.129. \tag{5.2}$$

A lot of work has gone into refining this calculation to include loop effects over recent years. At loop level, we have a correction to the Higgs mass from the self energy:

$$m_h^2 = \mu^2 + 3\lambda v^2 + \Pi_{hh}(m_h^2) \tag{5.3}$$

but we also have a correction to the parameter μ^2! At loop level, we must sit at the minimum of the *effective potential* instead of the tree-level potential. There are different ways of dealing with this, but the conventional one is to take the electroweak vev v to be defined as the value at the minimum of the full effective potential $V + \Delta V$. Recall that, at one loop, we have

$$\Delta V^{(1)} = \frac{1}{64\pi^2} \sum_i (-1)^{2s_i}(2s_i - 1) m_i^4 \left(\log \frac{m_i^2}{Q^2} - c_i \right) \tag{5.4}$$

where s_i is the spin of the particle ($\in \{0, 1/2, 1\}$) and the constants c_i depend on the spin and the renormalisation scheme. For $\overline{\text{MS}}$ ("minimal subtraction") they are $\{3/2, 3/2, 5/6\}$; in $\overline{\text{DR}}$ ("dimensional reduction," where we keep gauge boson/vector Lorentz indices in 4 dimensions) we have $c_i = 3/2$ for all. We see that $\overline{\text{DR}}$ is well-suited to SUSY theories, respecting the symmetries between the components of multiplets.

Then we must satisfy

$$0 = (\mu^2 + \lambda v^2)v + \left.\frac{\Delta V}{\partial h}\right|_{h=0}. \tag{5.5}$$

We can either compute the right-hand term by taking derivatives of the effective potential (think the Coleman-Weinberg potential at one loop) or diagrammatically as *tadpole diagrams*. We then solve this equation as before for μ, and obtain

$$m_h^2 = 2\lambda v^2 + \Pi_{hh}(m_h^2) - \frac{1}{v}\left.\frac{\Delta V}{\partial h}\right|_{h=0} \equiv 2\lambda v^2 + \Delta M^2 \tag{5.6}$$

We can then invert this as before to solve for λ. The complication now is that *the loops depend on λ and v* (and also μ^2 ...) so this is usually done iteratively.

The biggest effect is from loops involving the top quark, because it couples very strongly to the Higgs boson:

$$\mathcal{L} \supset -y_t Q_3 \cdot H U_3 + h.c. \supset -\frac{y_t}{\sqrt{2}}(h+v) t_L t_R + h.c. \tag{5.7}$$

If we take the tree-level values $v = 246.22$ GeV and use the pole mass of the top quark $m_t = 172.83 \pm 0.28 \pm 0.52$ GeV then we obtain $y_t = 0.993$ but in this case the loop corrections are very large and we have to be careful about what *scheme* we use.

The contribution to the effective potential at one loop of the top quark is

$$\Delta V^{(\text{tops})} \supset -12 \frac{m_t^4(h)}{64\pi^2}\left[\log\frac{m_t^2(h)}{Q^2} - \frac{3}{2}\right] \tag{5.8}$$

where $m_t(h) = \frac{y_t}{\sqrt{2}}(v+h)$. A common approximation to the Higgs mass is to ignore the momentum in the loop. This is justified by the fact that the Higgs quartic is smaller than the top Yukawa coupling or the strong gauge coupling squared, so we drop terms of order λ:

$$\begin{aligned}\Pi_{hh}(m_h^2) =& \Pi_{hh}(0) + m_h^2 \Pi'_{hh}(0) + ... \\ =& \frac{\partial^2 \Delta V}{\partial h^2} + \mathcal{O}(\lambda).\end{aligned} \tag{5.9}$$

In the SM it is not necessarily a very good approximation, but it is *much* better in SUSY theories where the Higgs mass is much smaller than the heavy SUSY particles. In this case, we have

$$(\Delta M^2)^{(\text{tops})} \approx -\frac{3}{4\pi^2} y_t^2 m_t^2 \log\frac{m_t^2}{Q^2}. \tag{5.10}$$

Extracting λ this gives

$$\begin{aligned}\lambda =& \frac{m_h^2}{2v^2} + \frac{(\Delta M^2)^{(\text{tops})}}{2v^2} \\ \equiv& \lambda_{\text{tree}} + \Delta\lambda.\end{aligned} \tag{5.11}$$

If we choose $Q = m_h$ then we find

$$\frac{\Delta\lambda(Q = m_h)}{\lambda_{\text{tree}}} = 9\%$$

which is a substantial shift; as a result, since this is the largest effect, we typically choose $Q = m_t$, so the shift from tops is zero (in this approximation). On the other hand, if we were to try to extract λ at, say, 1 TeV – the scale where there might be SUSY partners – we will find an enormous shift in λ of $\Delta\lambda/\lambda = -50\%$!

This discussion is very important for the following reasons. Firstly, the very stability of the Higgs potential is at stake. In figure 4 I show how the value of λ runs with the renormalisation scale – the

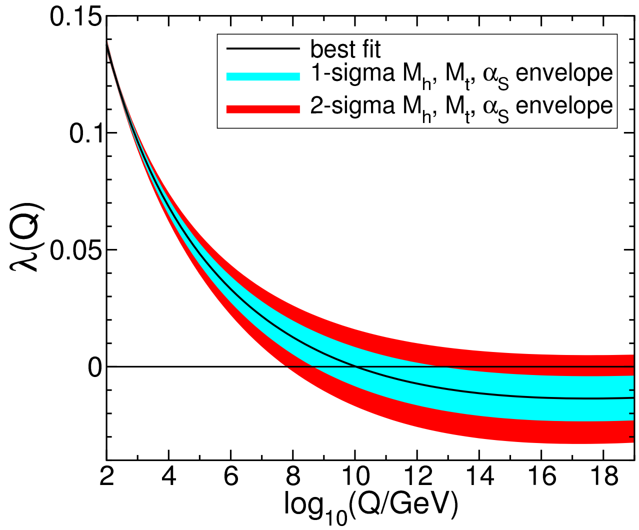

Figure 4: Running of the Higgs quartic coupling λ. Taken from [6], which uses central values from 2019 ($m_t = 173.1$ GeV, $m_h = 125.1$ GeV.)

central value is negative at high energies, indicating that the potential is not absolutely stable. It turns out that it is metastable, although there is still some substantial uncertainty due to the mass of the top quark.

Some people have tried to turn the requirement of $\lambda = 0$ (and possibly $\frac{d\lambda}{d\log Q} = 0$ at high energies into an axiom of some high-energy theory. This generally is called asymptotic safety. In minimal SUSY theories, $\lambda \geq 0$ is a (tree-level) prediction of the theory at the scale of superpartners, so we could instead use the point at which $\lambda = 0$ as an *upper* limit on the SUSY scale: this generally goes under the name of high-scale SUSY.

5.2 Higgs masses in supersymmetric models

The observation that the tree-level Higgs mass is too small was already problematic in the early 90s. Around the early 2000s, LEP placed bounds on the mass of the Higgs to be greater than 114 GeV. From the perspective of a phenomenologist until the Higgs discovery, supersymmetric partners should be not far above the electroweak scale in order to preserve naturalness, and then there was a tradeoff between these two, having heavy enough but not too heavy stops. It was then very important to have as precise as possible a fixed-order computation of the Higgs mass in SUSY theories. The expectation was that we would be able to measure the masses of the stops and *predict the mass of the Higgs when it was discovered afterwards!*

With what we know now, the colourful superpartners, if present in nature, are very likely somewhat above a TeV in mass at leas. Although they may yet be light and hiding, we may never see them. Nonetheless, we can still infer as much as we can from them – and the other SUSY partners – from their influence on the Higgs, especially its mass. Or alternatively, we can turn the computation around, like in the SM, and extract the effective quartic coupling – and infer the SUSY scale!

5.2.1 Traditional one-loop computation of the Higgs mass

The traditional computation involves the loop contributions from the stop squarks and the top quarks, which dominate at one loop because they couple directly to the Higgs via the top Yukawa coupling. Generalising the discussion that we had for the SM, in the MSSM (conserving CP for simplicity) we recall that we have two tadpole equations to solve for two parameters, equation (**??**); at loop level we will need to modify

$$\begin{aligned} m_{H_u}^2 &\rightarrow (m_{H_u}^2)^{\text{tree}} - \frac{1}{v\sin\beta}\frac{\partial \Delta V}{\partial h_u^r}\bigg| \\ m_{H_d}^2 &\rightarrow (m_{H_d}^2)^{\text{tree}} - \frac{1}{v\cos\beta}\frac{\partial \Delta V}{\partial h_d^r}\bigg| . \end{aligned} \tag{5.12}$$

Hence we have to include these shifts along with the self energies for the Higgs bosons. Note that these shifts affect not just the masses of the neutral scalar Higgs bosons, but also those of the pseudoscalar and charged Higgses!

Consider now the effective potential approximation for the MSSM. In the SM, we argued that λ was smaller than y_t^2 so we could neglect momenta. In the MSSM this approximation is even better, because the light Higgs mass is proportional to the Z boson mass – it is thus proportional to the electroweak gauge couplings only! The effective potential for the stop-top sector at one loop is

$$\begin{aligned} \Delta V^{(1),\text{stops/tops}} =& \frac{3}{16\pi^2}\Big[2f(m_{\tilde{t}_1}^2) + 2fm_{\tilde{t}_2}^2) - 4f(m_t^2)\Big] \\ f(x) \equiv& \frac{1}{4}x^2(\log\frac{x}{Q^2} - \frac{3}{2}) \end{aligned} \tag{5.13}$$

where Q is the renormalisation scale. To obtain the shifts to the neutral scalar masses in this approximation we can write

$$\begin{aligned} \mathcal{L}_{\text{stop masses}} =& -(\ \widetilde{t}_L^* \quad \widetilde{t}_R\ )\mathbf{m}^{\mathbf{2}}_{\tilde{\mathbf{t}}} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R^* \end{pmatrix} \\ \rightarrow \mathbf{m}^{\mathbf{2}}_{\tilde{\mathbf{t}}} =& \begin{pmatrix} m_Q^2 + m_t^2 + M_Z^2(\frac{1}{2} - \frac{2}{3}s_W^2)c_{2\beta} & m_t^*(A_t^* - \mu^*\cot\beta) \\ m_t(A_t - \mu\cot\beta) & m_{t_R}^2 + m_t^2 + \frac{2}{3}M_Z^2 s_W^2 c_{2\beta} \end{pmatrix}. \end{aligned}$$

There are different strategies we can then take; we can rewrite the effective potential in terms of *traces of matrices* instead of eigenvalues and differentiate, or we can use the fact that we deal with only 2×2 matrices:

$$\begin{aligned} \tilde{X} \equiv& y_t(A_t H_u^0 - \mu \overline{H}_d^0), \qquad \tilde{m}_t \equiv y_t H_u^0, \\ m_{\tilde{t}_1,\tilde{t}_2}^2 =& \frac{1}{2}\Big[m_Q^2 + m_U^2 + 2|\tilde{m}_t|^2 \pm \sqrt{(m_Q^2 - m_U^2)^2 + 4|\tilde{X}|^2}\Big] + \mathcal{O}(\alpha). \end{aligned} \tag{5.14}$$

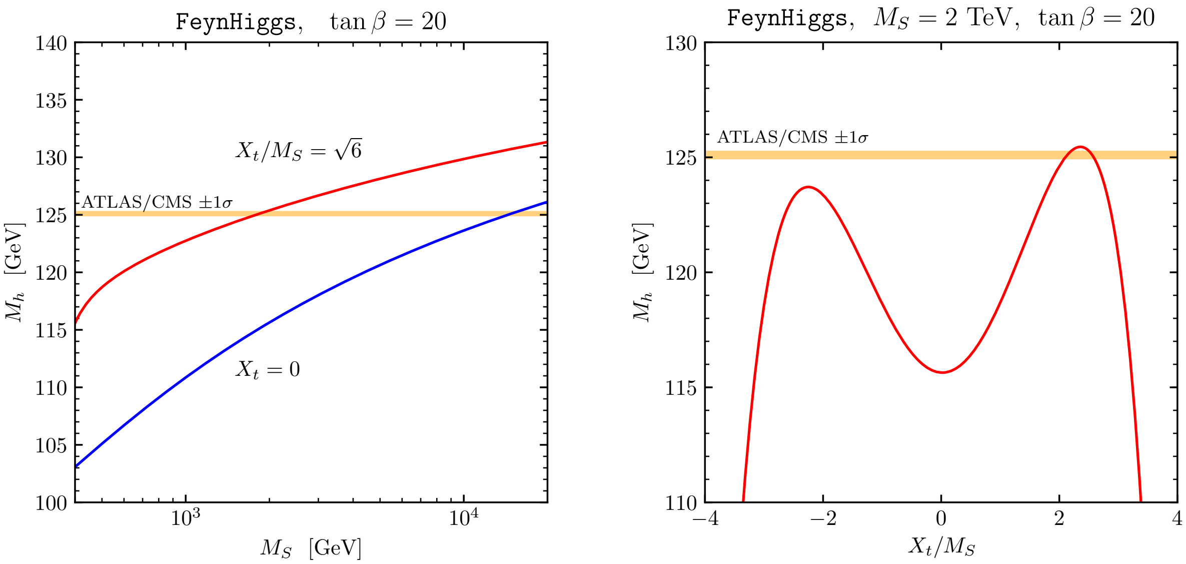

Figure 5: Plot of the Higgs mass using FeynHiggs for left: maximal stop mixing, varying M_S; right: fixed M_S, varying stop mixing.

If we are interested in the full mass matrix, we can differentiate with respect to H_u^0, H_d^0. This technique was used to compute the gaugeless-limit two-loop corrections in the CP-conserving MSSM, and was the state-of-theart for a long time.

If we want to just look at the decoupling/alignment limit, we can differentiate with respect to the vev v! In that case we obtain

$$(\Delta m_h^2)^{1-loop} \simeq \frac{3m_t^4}{2\pi^2 v^2}\left(\log\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12M_S^4}\right) \tag{5.15}$$

where now

$$X_t \equiv A_t - \mu\cot\beta, \qquad M_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}. \tag{5.16}$$

This computation leads to two important conclusions: stop mixing can *greatly* enhance the Higgs mass; and there is still an upper bound on the Higgs mass in the MSSM. The maximal mass occurs for $X_t/M_S = \pm\sqrt{6}$; I give some illustrative plots in figure 5, which include all available corrections in `FeynHiggs` but still display the behaviour dominated by the above.

In traditional texts, it was argued then that the Higgs mass was bounded from above in SUSY theories by about 130 to 140 GeV, by putting $M_S = 1$ TeV or so in the above formula as the maximum acceptable value from naturalness. Nowadays naturalness is less persuasive – we almost certainly have a little hierarchy. This then leads to a different reasoning and extra complications.

5.2.2 EFT computation

We can instead consider the SM as an effective field theory below the scale of heavy superpartners; it is most reasonable to take this to be the geometric mean of the stop masses. The computation of the Higgs mass is then equivalent to computing the Higgs quartic coupling in the SM. This is interesting and very different to most of the recent literature on EFTs, which is chiefly concerned with computing, or the effects of, higher-dimensional operators: we actually need to compute precision corrections to *renormalisable* couplings.

There are different ways that this can be done. The first is to directly compute the relevant diagrams in the unbroken phase of the theory (with $v = 0$). In the MSSM at one loop this means box, penguin and bubble diagrams. Since $m_h^2 = 2\lambda v^2$ it is straightforward to see that

$$\delta\lambda = \frac{3m_t^4}{4\pi^2 v^4}\left(\frac{X_t^2}{M_S^2} - \frac{X_t^4}{12M_S^4}\right). \tag{5.17}$$

What about the logarithmic term? That we should reproduce from the running from M_S down to m_t, which we take to be the scale at which we compute the Higgs mass in the SM (as we argued above):

$$\frac{d\lambda}{d\log Q^2} = -\frac{3y_t^4}{16\pi^2} + ... \longrightarrow \lambda(m_t) = \lambda(M_S) - \frac{3y_t^4}{16\pi^2}\log\frac{m_t^2}{M_S^2} = \frac{3y_t^4}{4\pi^2}\log\frac{M_S^2}{m_t^2}. \tag{5.18}$$

Putting $m_h^2 = 2\lambda(m_t)v^2$ then gives the same result in both approaches.

The advantage of the EFT approach becomes apparent when M_S starts to become large: in the fixed order calculation, what values should we take for y_t, m_t etc? In older computations, these were extracted at M_Z or m_t ... it also led to a large amount of discussion about choices of schemes: whether we should use the quarks on-shell, and even the stops on-shell (since we were supposed to discover them). All of these choices are at two-loop order in the fixed order computation, yet can have very large effects; so we need at least a partial two-loop computation of the Higgs mass in a spectrum generator.

In the EFT approach, the existing corrections have been "converted" and were used in the code `SUSYHD`. I give some examples of the predictions for the Higgs mass in different scenarios in **??**: if we take a common SUSY scale for all superpartners, we can bound the SUSY scale to be less than about 10^{12} GeV!

The other approach to EFT computations is sometimes dubbed 'hybrid' or 'pole mass matching': we match the pole mass in the high-energy theory with the computation in the SM, and solve

$$2\lambda v^2 = (m_h^2)^{\text{high energy theory}} - (\Delta M_h^2)^{SM}. \tag{5.19}$$

This is complicated by the need to also *match the vevs* between the two theories, which we can do by matching the Z boson masses: at one loop we can show

$$v_{SM}^2 = v_{HET}^2 + \frac{4}{g_Y^2 + g_2^2}\left[\Pi_{ZZ}^{\text{high energy theory}}(0) - \Pi_{ZZ}^{SM}(0)\right] + \mathcal{O}(v^4). \tag{5.20}$$

It can also be shown that this is equivalent to wavefunction renormalisation of the Higgs field in the $v = 0$ approach.

In recent years the state of the art of the fixed-order compuation has advanced so that in principle all two-loop scalar self-energies are known for generic theories. However, in spectrum generators only the gaugeless limit effective potential computation is available; the difference is now that hybrid approaches can take advantage of them to obtain a much more precise result for heavy SUSY. In principle, this is available for *any* theory (SUSY or non SUSY) in `SARAH` (via a diagrammatic calculation), although there is still much work to be done to improve the compuations there.

5.3 Prediction for the W mass

In the SM, the W mass is a *prediction* from measurable quantities. The fundamental parameters in the lagrangian are λ, g_Y, g_2, g_3, the Yukawa couplings, and either the Higgs mass-squared parameter μ^2 or the expectation value v. Of these, it is most logical to take v due to its relationship with G_F.

These should all be obtained from observations; clearly the Yukawa couplings, once diagonalised, are in one to one correspondence with the fermion masses. The strong gauge coupling is extracted from many different measurements independent from the electroweak sector. We saw that λ and the Higgs mass are interchangeable. But for g_Y, g_2, v we can use G_F, M_Z, M_W and $\alpha(0)$, the electromagnetic coupling meausured at low energies. Of these, $G_F, M_Z, \alpha(0)$ are very precisely measured, so everything else can be taken as a prediction of the SM, including the W mass.

We have

$$\begin{aligned} \alpha(0) =& 1/137.035999084(21) \\ G_F =& 1.1663788(6) \times 10^{-5} (\text{GeV})^{-2} \\ M_Z =& 91.1876(21)\,\text{GeV}. \end{aligned} \tag{5.21}$$

On the other hand, for the mass of the W there is now a dispute! The SM prediction is

$$M_W(\text{SM}) = 80352 \pm 6 \text{ MeV} \tag{5.22}$$

where the largest uncertainty is from the top quark mass. But with the latest measurement from CDF, we have:

$$M_W(\text{Tevatron} + \text{LEP}) = 80424.2 \pm 8.7 \text{ MeV}. \tag{5.23}$$

This contrasts with the LHC measurement from ATLAS of 80370 ± 19 MeV, closer to the SM but with larger uncertainty.

At loop level, there are different schemes we can employ; we can either treat the parameters above as $\overline{\text{MS}}$ or "on-shell" with counterterms that have finite parts. Earlier on, the two-loop computations in the SM were only available in an on-shell scheme, and this strongly influenced – and complicated

– matters for SUSY theories, for which the $\overline{\text{DR}}$ scheme is most appropriate. However, recently the calculations have been done in the SM in $\overline{\text{MS}}$ [6].

Of these, the simplest is

$$M_Z^2 = \frac{[g_2^2(M_Z) + g_Y^2(M_Z)]v^2}{4} - \Pi_{ZZ}(M_Z^2). \tag{5.24}$$

However, we typically instead trade the couplings for the electromagnetic gauge coupling and $\sin\theta_W \equiv s_W, \cos\theta_W \equiv c_W$, so we put

$$M_Z^2 = \frac{\pi\alpha(M_Z)v^2}{s_W^2 c_W^2} - \Pi_{ZZ}(M_Z^2). \tag{5.25}$$

In SUSY theories we actually use this to extract v!

For the electromagnetic coupling, we can write

$$\alpha(0) = \frac{g_2^2(M_Z)g_Y^2(M_Z)}{4\pi[g_2^2(M_Z) + g_Y^2(M_Z)]}\left[1 - \Delta\alpha^{(5)}_{\text{had}}(M_Z) - \Delta\alpha_{\text{pert}}\right] \equiv \alpha(M_Z)[1 - \Delta\alpha] \tag{5.26}$$

Of these, $\Delta\alpha_{\text{pert}}$ is the perturbative contribution from integrating out the heavy SM fields and running down to the masses of the leptons; it is sometimes computed including RG running. On the other hand, $\Delta\alpha^{(5)}_{\text{had}}$ involves the contributions of all the hadrons, which unfortunately include non-perturbative effects. In fact, these are very closely related to the same non-perturbative effects which are currently under scrutiny for the muon anomalous magnetic moment: lattice computations and experimentally-extracted ones disagree about their size. The R-ratio method defines it as

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z) = -\frac{M_Z^2}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{dq^2}{q^2(q^2 - M_Z^2)}R_{\text{had}}(q^2), \tag{5.27}$$

where

$$R_{\text{had}}(q^2) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \ell^+\ell^-)}. \tag{5.28}$$

The experimentally determined value is

$$\Delta\alpha^{(5)}_{\text{had}} = 276.1(1.1) \times 10^{-5}. \tag{5.29}$$

The total value of $\Delta\alpha \simeq 0.059$.

Finally, we can define

$$G_F \equiv \frac{1}{\sqrt{2}v^2}(1 + \Delta\tilde{r}). \tag{5.30}$$

when we define v to be the minimum of the full loop-corrected potential. We typically the corrections at one loop into the contribution from modifying the W propagator, which is

$$\sim \frac{1}{g_2^2 v^2/4 - \Pi_{WW}(0)} \sim \frac{1}{M_W^2 + \Pi_{WW}(M_W^2) - \Pi_{WW}(0)} \sim \frac{1}{M_W^2} + \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2},$$

and vertex/box diagram contributions δ_{VB}. But now we prefer to eliminate v from the right hand side. Let us put

$$s_W^2 = \frac{g_2^2(M_Z)}{g_2^2(M_Z) + g_Y(M_Z)}, \tag{5.31}$$

then

$$\begin{aligned} G_F =& \frac{\pi\alpha(M_Z)}{\sqrt{2}M_W^2 s_W^2}(1 + \Delta\hat{r}_W), \\ =& \frac{\pi\alpha(0)}{\sqrt{2}M_W^2 s_W^2}(1 + \Delta\alpha + \Delta\hat{r}_W) \\ \Delta r_W \equiv& \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2} + \delta_{VB}. \end{aligned} \tag{5.32}$$

However, an alternative formulation is to use

$$v^2 = \frac{s_W^2 c_W^2 M_Z^2}{\pi\alpha(M_Z)}(1 + \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}) \tag{5.33}$$

to write

$$\begin{aligned} G_F =& \frac{\pi\alpha(M_Z)}{\sqrt{2}s_W^2 c_W^2 M_Z^2}(1 - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW}(0)}{M_W^2} + \delta_{VB}) \\ =& \frac{\pi\alpha(0)}{\sqrt{2}s_W^2 c_W^2 M_Z^2}(1 + \Delta\alpha - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW}(0)}{M_W^2} + \delta_{VB}). \end{aligned} \tag{5.34}$$

Finally this allows us to extract s_W^2:

$$s_W^2 c_W^2 = \frac{\pi\alpha(0)}{\sqrt{2}G_F M_Z^2}(1 + \Delta\alpha - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW}(0)}{M_W^2} + \delta_{VB}). \tag{5.35}$$

Using

$$s_W^2 = \frac{1}{2} - \sqrt{\frac{1}{4} - s_W^2 c_W^2} \tag{5.36}$$

we have the tree-level value

$$\begin{aligned} \overline{s}_W^2 =& \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi\alpha(0)}{\sqrt{2}G_F M_Z^2}} \\ =& 0.21215 \end{aligned} \tag{5.37}$$

and

$$\overline{M}_W^2 = \frac{\pi\alpha(0)}{\sqrt{2}G_F \overline{s}_W^2} = (80938 \text{ MeV})^2 \tag{5.38}$$

which are both a very long way away from the correct values! Clearly precision corrections here are very important.

Instead, puting $s_W^2 = \overline{s}_W^2 + \delta s_W^2$, we can compute

$$\begin{aligned}\delta s_W^2 =& \frac{\delta(s_W^2 c_W^2)}{2\sqrt{\frac{1}{4} - s_W^2 c_W^2}} \\ =& \frac{s_W^2 c_W^2}{c_W^2 - s_W^2}(1 + \Delta\alpha - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW}(0)}{M_W^2} + \delta_{VB}).\end{aligned} \tag{5.39}$$

We can also invert the relations above to obtain M_W^2:

$$\begin{aligned}M_W^2 =& \frac{\pi\alpha(0)}{\sqrt{2}G_F \overline{s}_W^2}(1 + \Delta\alpha + \Delta r_W - \frac{\delta s_W^2}{s_W^2}) \\ \delta M_W^2 =& \frac{s_W^2 \overline{M}_W^2}{c_W^2 - s_W^2}\left[\frac{c_W^2}{s_W^2}\Delta\rho - \Delta r_W - \Delta\alpha\right]\end{aligned} \tag{5.40}$$

where we define

$$\Delta\rho \equiv \frac{M_W^2}{c_W^2 M_Z^2} - 1 = \Delta\rho_{\text{tree}} + \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2}. \tag{5.41}$$

Typically in BSM theories $\Delta\rho$ is the largest contribution to the shift in the mass of the W (while $\Delta\alpha$ is the largest in the SM). It is also closely related to the Peskin-Takeuchi T parameter:

$$\Delta\rho \approx \alpha(M_Z)T. \tag{5.42}$$

The fact that electroweak fits and many general expressions are expressed in terms of S, T, U is why so many papers focussed on those observables as proxies following the CDF result.

In the SM and MSSM, $\Delta\rho_{\text{tree}} = 0$, but in any theory with additional non-doublet fields that have expectation values – for example $SU(2)$ triplets in Dirac Gaugino models – its value is small and non-zero. Or in models with additional W' or Z' gauge bosons the mixing with their SM counterparts will modify the tree-level relations.

The loop-level contributions to $\Delta\rho$ arise from electroweak multiplets whose masses are split. In the SM there is an approximate custodial symmetry that prevents large corrections to $\Delta\rho$. The leading corrections must be either proportional to the gauge couplings, the difference of quark/lepton masses between members of each generation, and indeed we find from the top/bottom quarks

$$\begin{aligned}\Delta\rho^{t,b} =& \frac{3}{16\pi^2 v^2}F(m_t^2, m_b^2) \simeq \frac{3m_t^2}{16\pi^2 v^2} \simeq 0.009, \\ F(x,y) \equiv& x + y - \frac{2xy}{x-y}\log\frac{x}{y},\end{aligned} \tag{5.43}$$

and from the W/H sector

$$\Delta\rho^{W,H} \approx \frac{3g_Y^2}{64\pi^2}\log\frac{m_h^2}{M_W^2} \simeq 0.0005 \tag{5.44}$$

which is actually comparable to the experimental uncertainty. If we put these two contributions together with $\Delta\alpha \simeq 0.059$ then we get a more respectable value of $M_W \simeq 80600$ MeV, which is more respectable but still some way off.

In any BSM theory where there is a hierarchy between BSM and SM fields so that we can write the SM as an effective theory, then the BSM contributions to the shift in M_W^2 must be of order v^4/M^2, since the terms of order v^2 are just the SM. This means that the corrections in SUSY theories compared to the SM are typically small.

In the MSSM the largest contributions were long considered to be stops, because of the multiplicity (colours) and because it was assumed they would be light for reasons of naturalness. If we write the stop/sbottom mixing matrices in terms of mixing angles $\cos\theta_{\tilde{t}} \equiv c_t, \cos\theta_{\tilde{b}} \equiv c_b$ etc then we have the dominant contribution from

$$\Delta\rho^{\tilde{t},\tilde{b}} \simeq \frac{3}{16\pi^2 v^2}\bigg[- s_t^2 c_t^2 F(m_{\tilde{t}_1}^2, m_{\tilde{t}_1}^2) - s_t^2 c_t^2 F(m_{\tilde{t}_1}^2, m_{\tilde{t}_1}^2) + c_t^2 c_b^2 F(m_{\tilde{t}_1}^2, m_{\tilde{b}_1}^2) \\ - s_t^2 c_t^2 F(m_{\tilde{t}_1}^2, m_{\tilde{t}_1}^2) - s_t^2 c_t^2 F(m_{\tilde{t}_1}^2, m_{\tilde{t}_1}^2) + c_t^2 c_b^2 F(m_{\tilde{t}_1}^2, m_{\tilde{b}_1}^2)\bigg]. \tag{5.45}$$

The mass splitting between $m_{\tilde{t}_1}$ and $m_{\tilde{b}_1}$ depends on the vev because both are dominated by $m_{\tilde{Q}}^2$. As an example, suppose we take a common SUSY scale M_S, then we find

$$\Delta\rho^{\tilde{t},\tilde{b}} \simeq \frac{1}{160\pi^2 v^2 M_S^6}(m_t^2 X_t^2 - m_b^2 X_b)^2 \sim \mathcal{O}(v^2/M^2). \tag{5.46}$$

The electroweak corrections in SUSY theories are of course much more complicated than in the SM, so even for the MSSM only partial two loop results are available (see e.g. [10, 11]) and for models beyond the MSSM the complete one-loop computation exists only. However, since they contribute at subleading order in an expansion in v/M, the precision required for corrections to even moderately heavy SUSY theories is not as great as that for the Higgs mass.

6 Beyond the MSSM

6.1 Non-minimal models of low-energy SUSY

The MSSM is just the simplest choice that we can make to supersymmetrise our Standard Model; indeed, since we need such large loop corrections to the Higgs mass, it is natural to ask if this is always the case, and we find that it is not.

6.1.1 The NMSSM

The most popular extension of the MSSM is the NMSSM, the Next-to-Minimal Supersymmetric Standard Model. There we just add a new gauge singlet chiral superfield S. We then change the superpotential to

$$W = \lambda_S S H_u \cdot H_d + \frac{k}{3} S^3 + W_{Yukawa}.$$

We also add soft terms:

$$-\mathcal{L}_{\rm soft} \supset \frac{1}{3} A_K S^3 + A_S S H_u \cdot H_d + h.c. + m_S^2 |S|^2. \tag{6.1}$$

We could also add other terms but we usually take just these ones under the hypothesis of a $\mathbb{Z}_3$ discrete symmetry. This prevents a μ/B_μ term – but allows them to be dynamically generated by the expectation value of S! Putting $\langle S \rangle = \frac{1}{\sqrt{2}} v_S$, we have

$$\mu^{\rm eff} = \frac{1}{\sqrt{2}} \lambda_S v_S, \qquad B_\mu^{\rm eff} = \frac{1}{\sqrt{2}} v_S A_S + \frac{1}{2} k \lambda_S^* v_S^2. \tag{6.2}$$

While we add a scalar and pseudoscalar to the theory, and the associated neutral fermion (which mixes with the neutralinos), we also obtain a boost to the Higgs mass at tree level; we can see this if we look at the F-term potential, and go to the heavy M_A limit where $H_u = \frac{1}{\sqrt{2}}(v+h) s_\beta, H_d = \frac{1}{\sqrt{2}}(v+h) c_\beta$, and take the scalar S to be heavy:

$$V_F \supset |\lambda_S H_u \cdot H_d + k S^2|^2 \rightarrow \frac{1}{4} |\lambda_S|^2 (v+h)^4 c_\beta^2 s_\beta^2 \tag{6.3}$$

which increases the Higgs quartic coupling! We therefore find in this limit

$$M_h^2 = M_Z^2 c_{2\beta}^2 + \frac{1}{2} \lambda_S^2 v^2 s_{2\beta}^2. \tag{6.4}$$

We can therefore find $M_h \simeq 125$ GeV for small $\tan\beta \gtrsim 1$ and $\lambda_S \simeq 0.7$.

6.1.2 Dirac gauginos

One other possibility is that the gauginos have a *Dirac* mass rather than a Majorana one! This would mean adding an extra chiral multiplet in the adjoint representation for each gauge group. This has several advantages:

- We have the same λ_S coupling as in the NMSSM, but now also have a $W \supset \sqrt{2} \lambda_T H_u \cdot T H_d$ for the $SU(2)$ triplet T; now in the decoupling limit

$$M_h^2 = M_Z^2 c_{2\beta}^2 + \frac{1}{2} (\lambda_S^2 + \lambda_T^2) v^2 s_{2\beta}^2.$$

- The Dirac gaugino mass is *supersoft* $\rightarrow$ makes only finite corrections to stop and Higgs masses.
- Can therefore have a heavy gluino compared to stops.
- Lack of chirality-flip processes weakens bounds on light squarks and alleviates flavour constraints!

6.2 Split SUSY

Finally, one rather radical idea is to abandon the hierarchy problem: imagine that all of the SUSY scalars except for the SM Higgs are at a scale M_S, but keep the gauginos and higgsinos light, at the weak – TeV scale. This does barely affects the prediction of unification of gauge couplings! This is because the scalars are in complete GUT multiplets except for the Higgs, and the Higgs contributes very little to the RGEs.

To do this, we require an approximate R-symmetry to protect the gaugino masses against large corrections. We must also invoke anthropic tuning of the electroweak scale. This might not be so crazy, since only one parameter must be adjusted in the Higgs mass matrix:

$$\det \begin{pmatrix} m_{H_d}^2 & -B_\mu \\ -B_\mu & m_{H_u}^2 \end{pmatrix} \simeq 0 \to m_{H_u}^2 m_{H_d}^2 = B_\mu^2.$$

We now define β to be the mixing angle between the fields in this limit, so

$$\begin{pmatrix} \mathcal{H} \\ \mathcal{A} \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} -\epsilon H_d^* \\ H_u \end{pmatrix} \tag{6.5}$$

$$\begin{pmatrix} h_d^0 \\ h_u^0 \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix} \tag{6.6}$$

and so, since the mass matrix has a zero eigenvalue for h,

$$m_{H_d}^2 c_\beta - B_\mu s_\beta = 0 \to \tan\beta = \frac{m_{H_d}}{m_{H_u}}. \tag{6.7}$$

Some advantages are:

- Still have neutralino dark matter!
- Greatly ameliorate the flavour problem!
- Makes a prediction for the Higgs mass! The SM Higgs quartic coupling at the SUSY scale becomes

$$\lambda(M_S) = \frac{1}{4}(g^2 + (g')^2)\cos^2 2\beta + ...$$

A Custodial symmetry

Before EWSB the (pure) Higgs potential is only a function of $|H|^2$; writing H in terms of real scalars as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \longrightarrow |H|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2. \tag{A.1}$$

This is invariant under global $SO(4) \simeq SU(2)_L \times SU(2)_R$ rotations:

$$\mathcal{H} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_3 - i\phi_4 & \phi_1 + i\phi_2 \\ -(\phi_1 - i\phi_2) & \phi_3 + i\phi_4 \end{pmatrix} = (\epsilon H^\dagger, H) \tag{A.2}$$

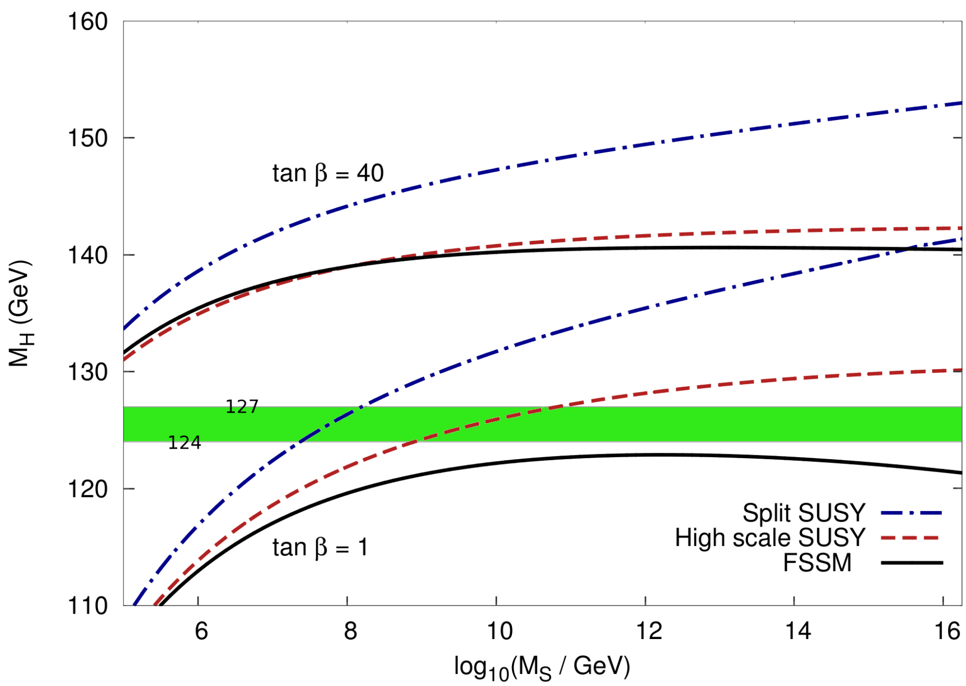

Figure 6: Prediction for the Higgs mass against supersymmetry scale in different SUSY scenarios. Taken from [12].

then transforms as

$$\mathcal{H} \to U_L \mathcal{H} U_R^\dagger. \tag{A.3}$$

There is one fly in the ointment already at this stage: the kinetic terms $|D_\mu|^2$ do not respect this symmetry (because the gauge group is not $SU(2)_L \times SU(2)_R$ with identical gauge couplings), which means that it will be violated at the quantum level. However, it remains exact even in the quantum theory when we take the limit $g_Y \to g_2 \to 0$. This seems like we throw the baby out with the bathwater but we will see how it is useful in a moment.

After EWSB we say ϕ_3 obtains a vev, so then $\langle \mathcal{H} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & \\ & v \end{pmatrix}$ which is still invariant under the diagonal $SU(2)_V \simeq SO(3)$.

This symmetry can be extended to include the quarks

$$\mathcal{L} \supset y_u Q \cdot H u_R - y_d Q H^\dagger d_R + h.c. \tag{A.4}$$

so in the case that $y_u = y_d = y$ we can put

$$\mathcal{L} \supset y(u_L \; d_L)\epsilon(\epsilon H^\dagger \; H) \begin{pmatrix} u_R \\ d_R \end{pmatrix}. \tag{A.5}$$

So we see that the custodial symmetry is nothing but isospin (if we only include one generation) or a copy of isospin for each generation! For the leptons, we can have the same symmetry by turning off the Yukawa couplings. After EWSB, to preseve the diagonal subgroup we would need to set $m_u = m_d$ and the lepton masses to zero.

In the end we see that custodial symmetry is violated at loop level by corrections proportional to the gauge couplings and also proportional to the differences between left/right quarks. How can this tell us something about $\Delta\rho$? If we compute

$$\frac{M_W^2}{c_W^2 M_Z^2} - 1 = \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2} + ... \tag{A.6}$$

we see that in the limit of vanishing gauge couplings we still have the associated currents:

$$\mathcal{L} \supset \frac{e}{c_W s_W} J_\mu^Z Z^\mu + g_2[J_\mu^W W^\mu + h.c] \tag{A.7}$$

and so we can replace the self energies by correlators of these currents:

$$\frac{\Pi_{ZZ}}{M_Z^2} \sim \frac{e^2}{c_W^2 s_W^2 M_Z^2} \langle J_\mu^Z J_\nu^Z \rangle \underset{e\to 0}{\longrightarrow} \propto \frac{1}{v^2} \langle J_\mu^Z J_\nu^Z \tag{A.8}$$

and similarly for the W mass term. Now what the custodial symmetry tells us is that in the limit of equal up and down quark masses, the lowest order contributions in α from Π_{ZZ} and Π_{WW} to $\Delta\rho$ must cancel. But we also know that

$$\Pi_{ZZ}(M_Z^2) \sim v^2, \qquad \Pi_{WW}(M_W^2) \sim v^2 \tag{A.9}$$

since they must vanish when electroweak symmetry is restored. This means that we must have

$$\lim \alpha \to 0\left[\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(M_W^2)}{M_W^2}\right] \propto \frac{(m_u^2 - m_d^2)}{v^2}. \tag{A.10}$$

This can potentially be large.

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