Adiabatic Deformations of Quantum Hall Droplets

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arXiv:2212.12935 with B. Estienne
arXiv:2301.01726 with B. Lapierre, P. Moosavi, J.M. Stéphan, B. Estienne

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See also talks by **Per Moosavi** and **Andrea Cappelli**

Intro

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...to prove **emergent diffeo invariance** in cond-mat ?

- We'll focus on quantum Hall droplets

MOTIVATION

quantum Hall droplets

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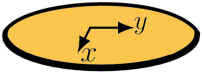

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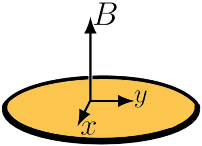

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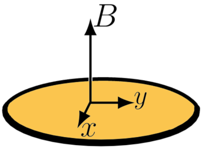

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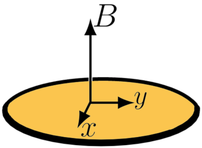

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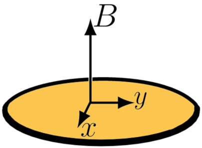

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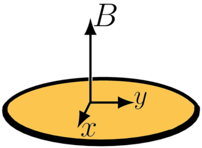

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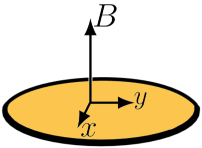

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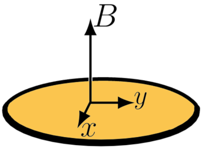

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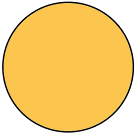

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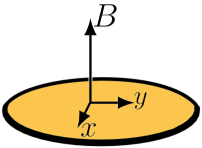

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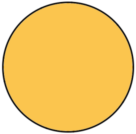

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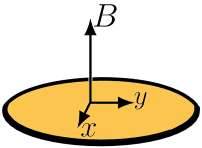

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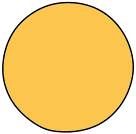

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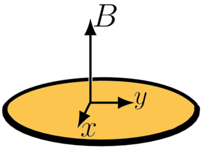

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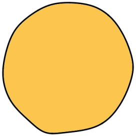

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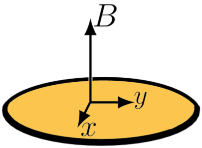

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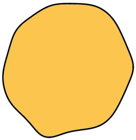

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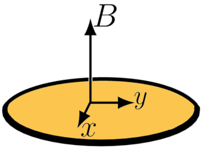

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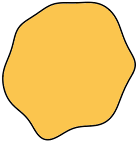

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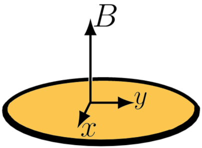

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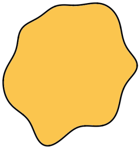

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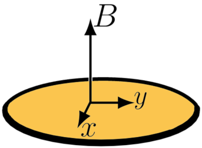

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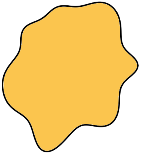

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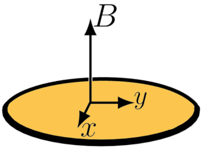

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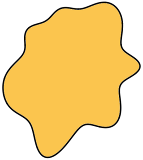

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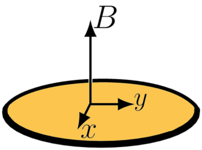

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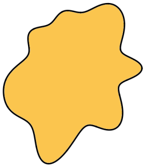

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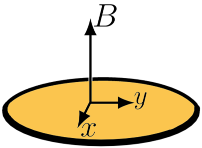

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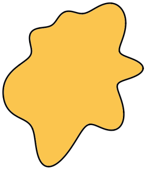

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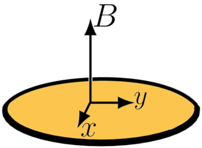

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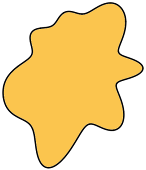

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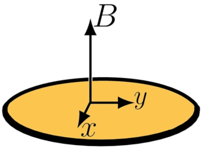

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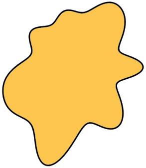

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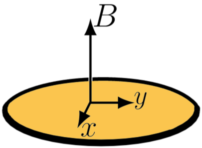

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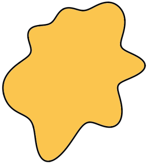

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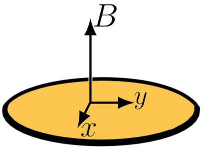

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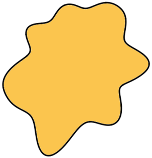

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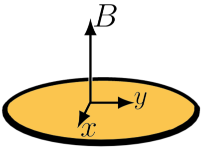

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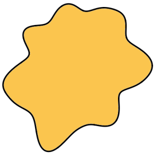

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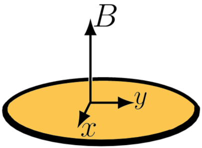

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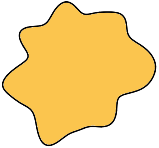

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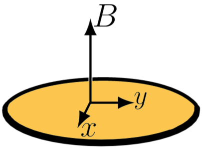

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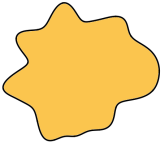

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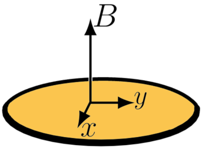

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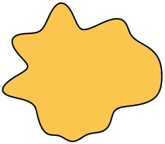

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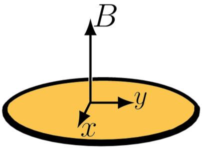

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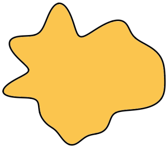

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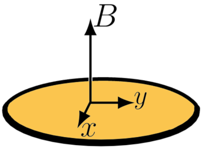

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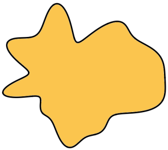

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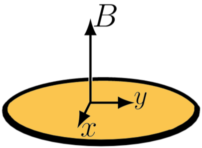

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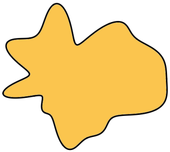

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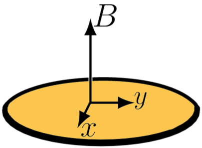

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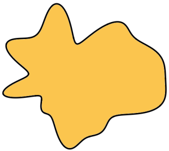

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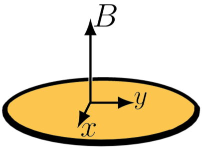

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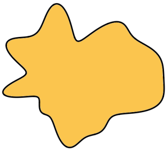

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Diffeos are crucial for **quantum Hall droplets** :

['80's–'90's]

- Incompressible quantum fluids
- Symmetry under area-preserving diffeos
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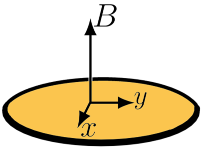

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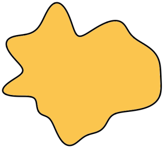

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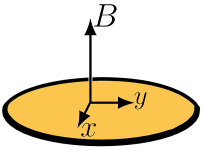

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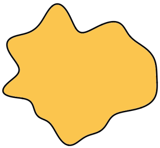

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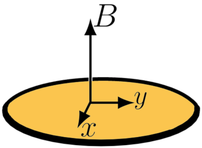

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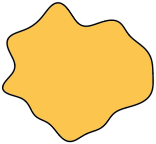

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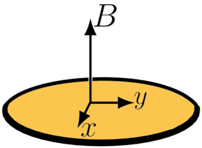

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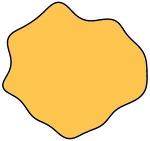

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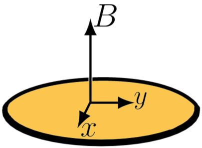

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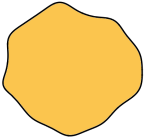

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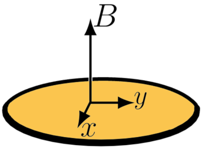

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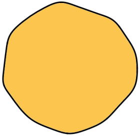

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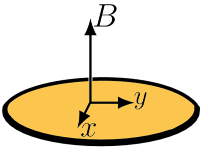

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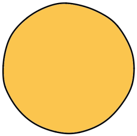

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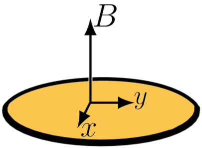

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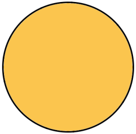

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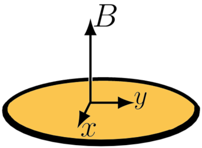

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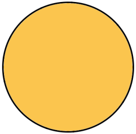

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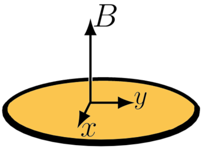

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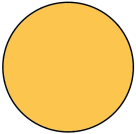

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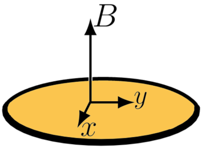

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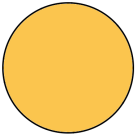

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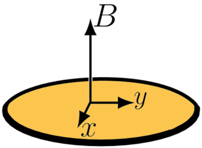

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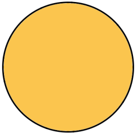

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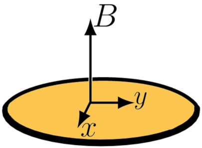

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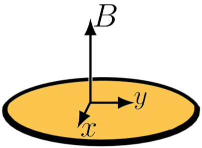

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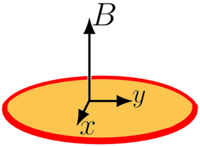

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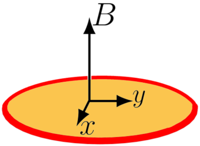

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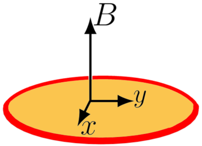

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- Generalize Hall viscosity
- Probe edge modes
- Gauge symm in **bulk** but global symm on **edge** ?

Plan

1. **Berry phases and 1D diffeos**
2. 2D deformations of metric and potential [BO, Estienne]
3. 2D deformations of potential alone [BO, Lapierre, Moosavi, Stéphan, Estienne]

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C. Berry phases measure central charges

Berry ϕ in general

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What Berry phase is produced by sample diffeos ?

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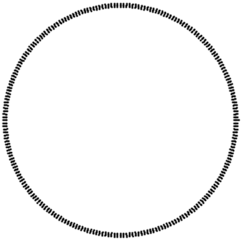

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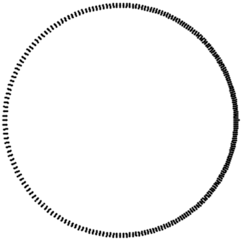

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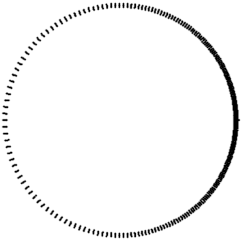

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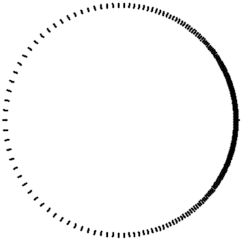

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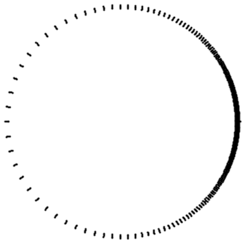

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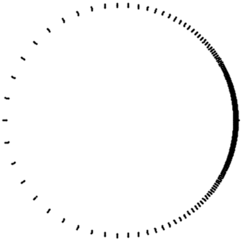

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- Unitary Diff S^1 action : $(\mathcal{U}[g]\psi)(\varphi) = \sqrt{\bar{g}'(\varphi)} \quad \psi(\bar{g}(\varphi))$

Choose adiabatic, cyclic $g_t(\varphi)$

- Assume $\psi(\varphi) \propto e^{ij\varphi}$
- Berry phases ?
- Berry $= i\int \mathrm{d}t \, \langle\psi|\mathcal{U}[g_t]^{-1}\frac{\partial}{\partial t}\mathcal{U}[g_t]|\psi\rangle = j\int \mathrm{d}t \, \mathrm{d}\varphi \, \frac{\dot{g}}{g'}$

Berry ϕ and central charges

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[Alekseev-Shatashvili 1989]
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Questions :

- Analogue for **2D electron droplets** ?

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Questions :

- Analogue for **2D electron droplets** ?
- Measure edge central charge ?

2. Berry phases from 2D quantomorphisms

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A. Area-preserving diffeos

2. Berry phases from 2D quantomorphisms

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Plane $\mathbb{R}^2$

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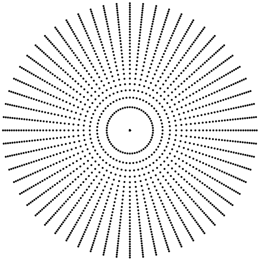

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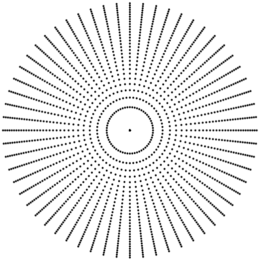

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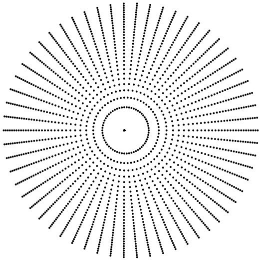

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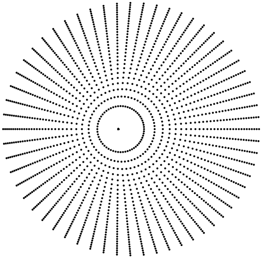

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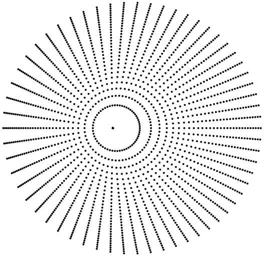

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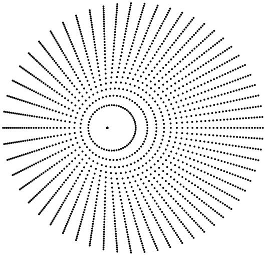

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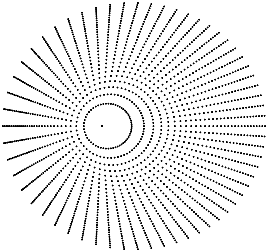

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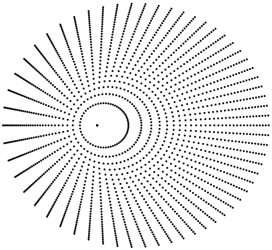

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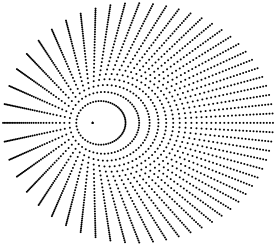

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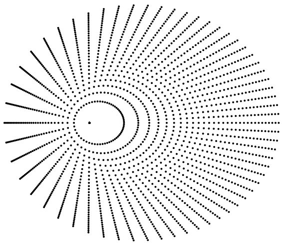

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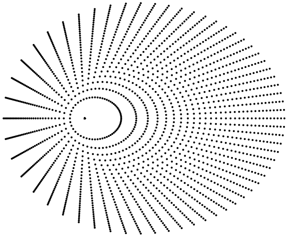

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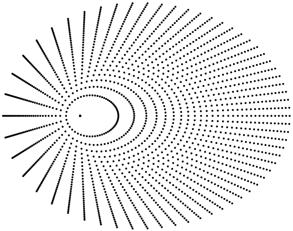

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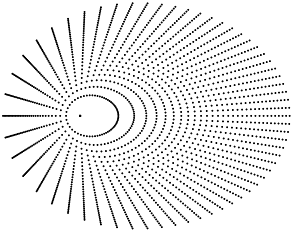

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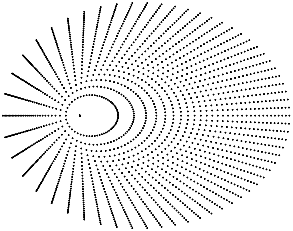

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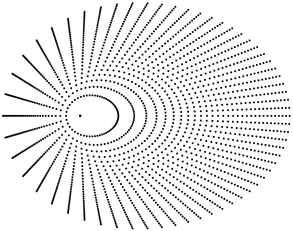

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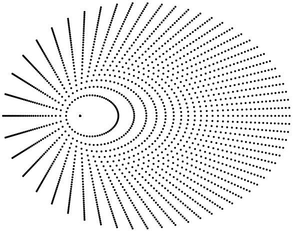

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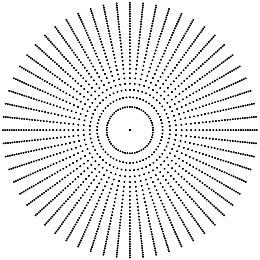

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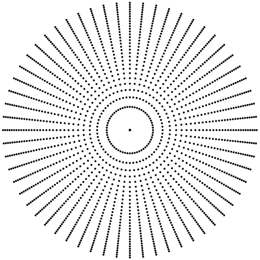

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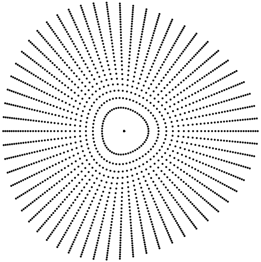

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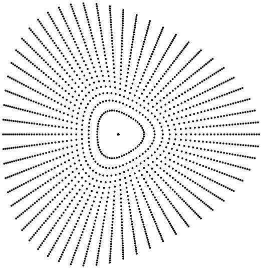

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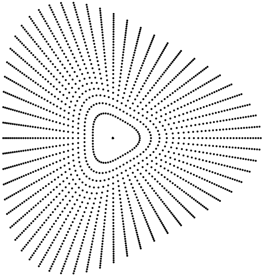

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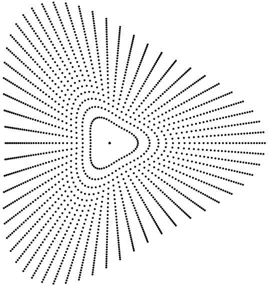

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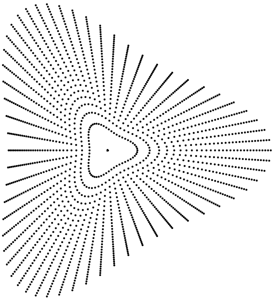

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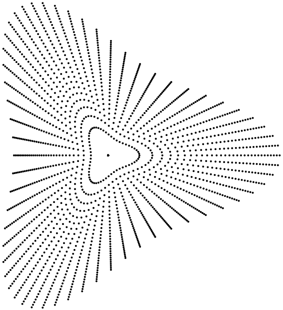

AREA-PRESERVING DIFFEOS

Plane $\mathbb{R}^2$, potential $\mathbf{A} = A_i(\mathbf{x})\mathrm{d}x^i$

- Magnetic field $\mathbf{B} = \mathrm{d}\mathbf{A}$
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Example :

$$(r^2, \varphi) \longmapsto \left(\frac{r^2}{g'(\varphi)}, \boldsymbol{g(\varphi)}\right)$$

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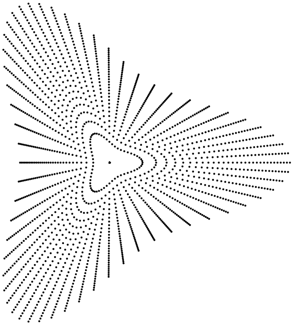

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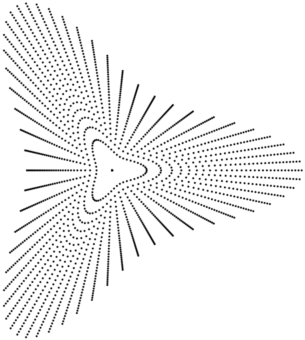

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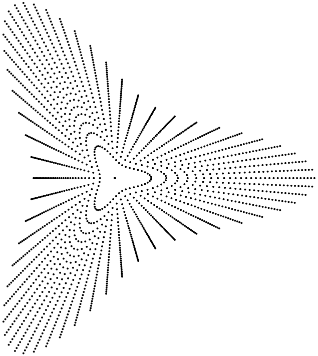

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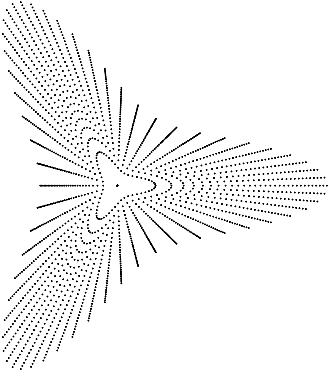

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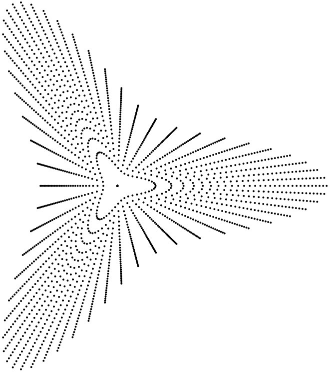

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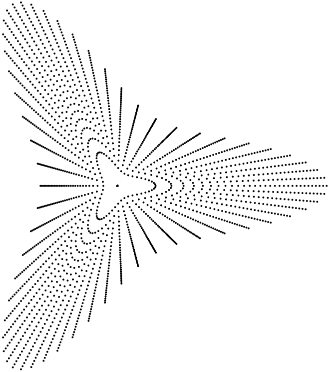

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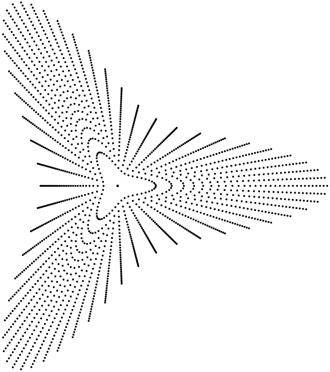

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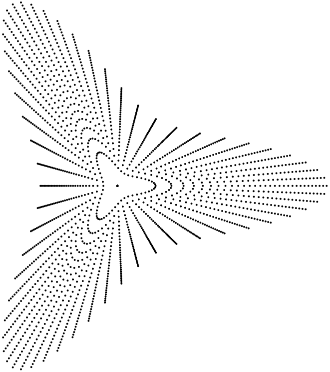

QUANTOMORPHISMS

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Electron in $\mathbb{R}^2$

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- Hilbert space $L^2(\mathbb{R}^2)$

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Action on Hamiltonian ?

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- **Deform both metric and potential**

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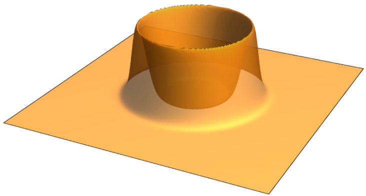

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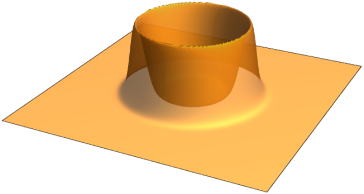

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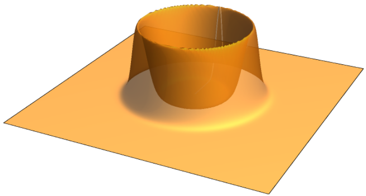

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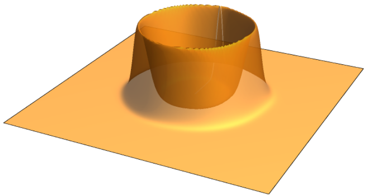

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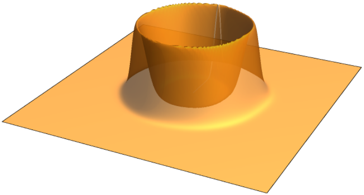

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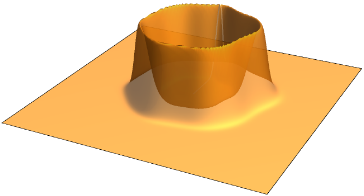

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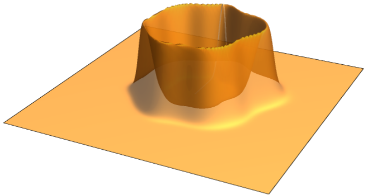

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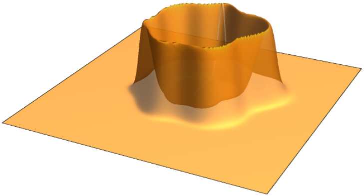

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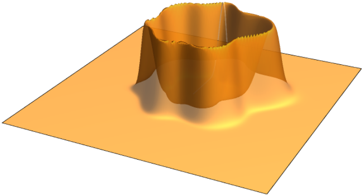

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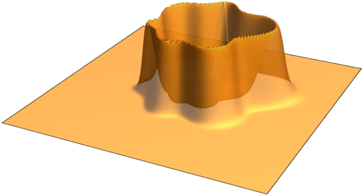

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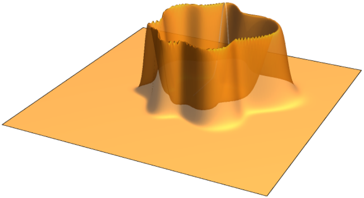

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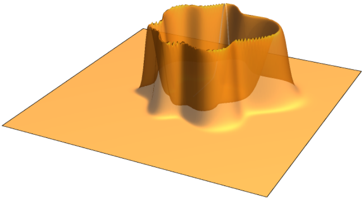

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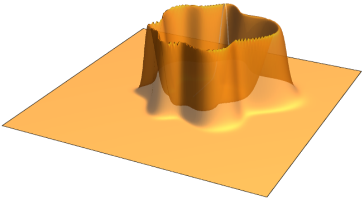

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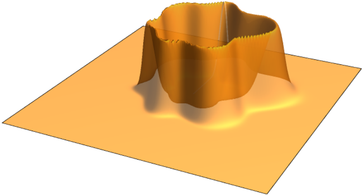

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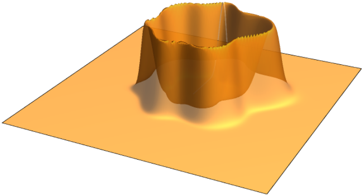

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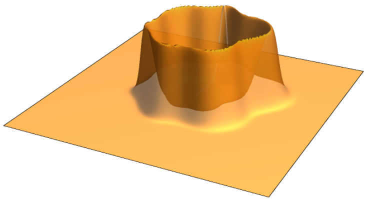

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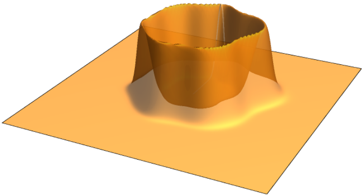

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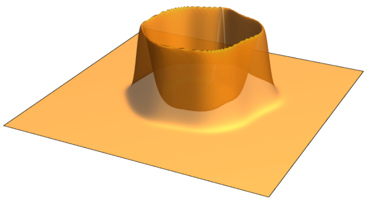

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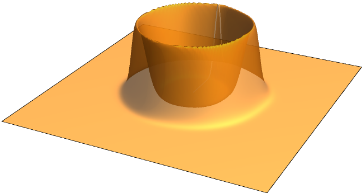

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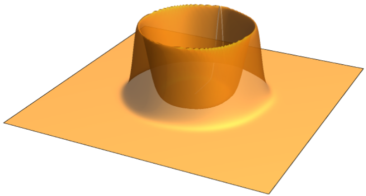

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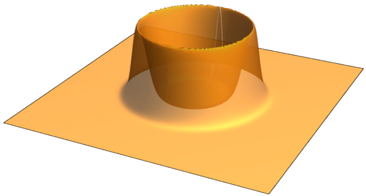

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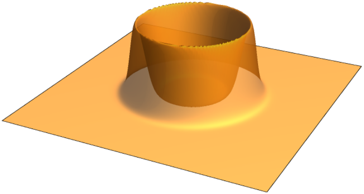

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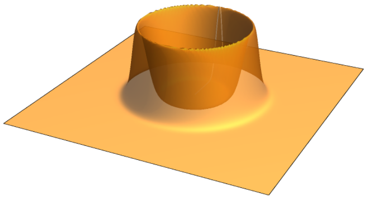

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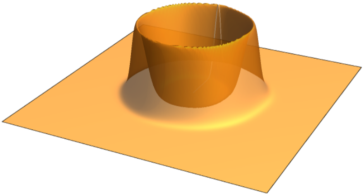

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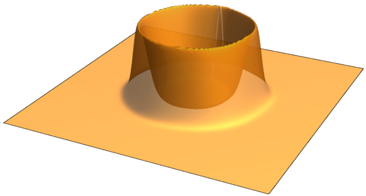

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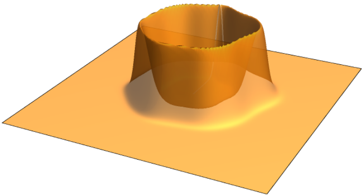

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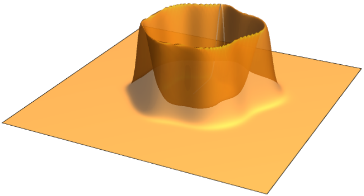

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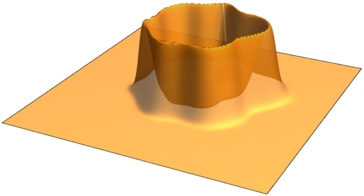

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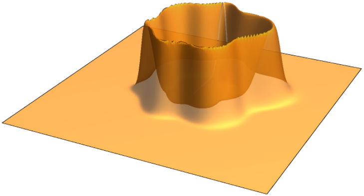

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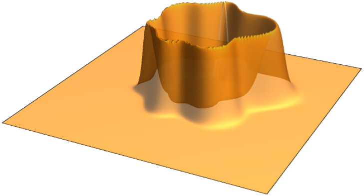

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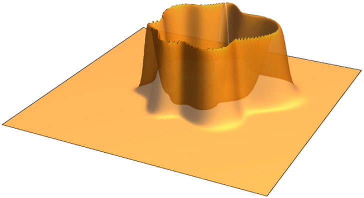

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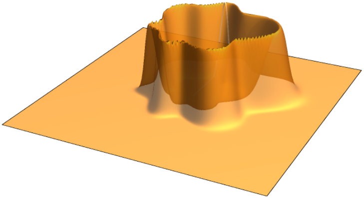

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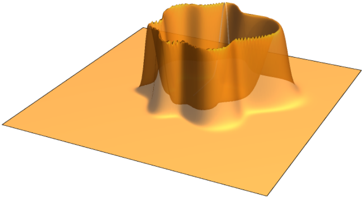

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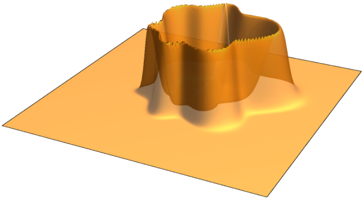

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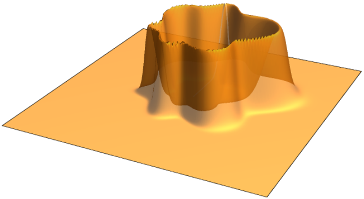

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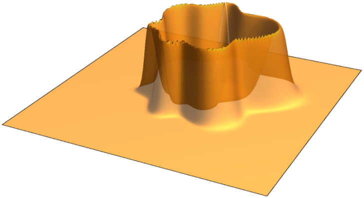

BERRY ϕ FROM 2D DIFFEOS

Neutral preliminary

Berry ϕ from 2D diffeos

Neutral preliminary : $(\mathcal{U}[g]\psi)(\mathbf{x}) \equiv \psi(\bar{g}(\mathbf{x}))$

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$$\text{Berry} = \text{Current} \times \text{Velocity} + \text{Density} \times \text{Aharonov-Bohm}$$

Berry ϕ from 2D diffeos

- Applies to any wave function !

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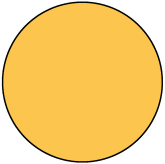

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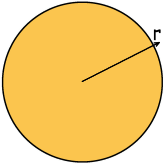

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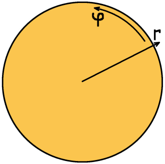

Berry = Current × Velocity + Density × Aharonov-Bohm

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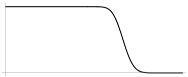

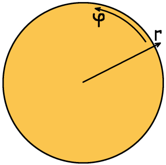

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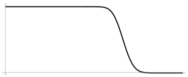

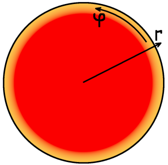

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- Applies to any wave function !
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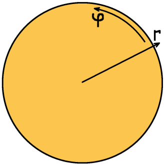

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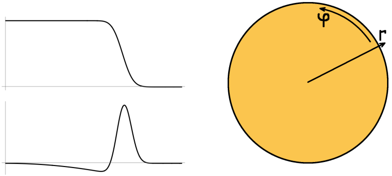

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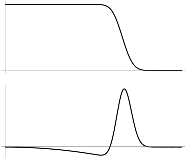

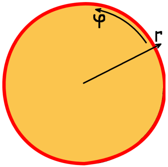

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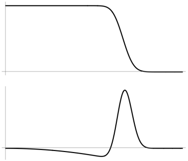

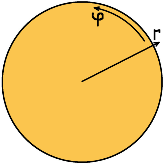

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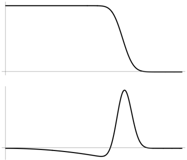

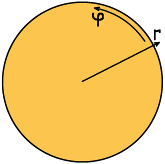

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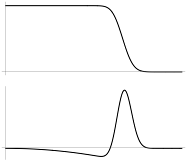

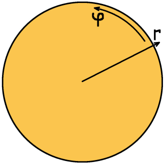

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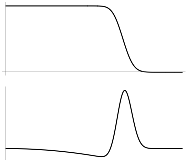

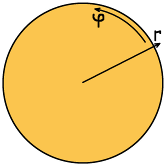

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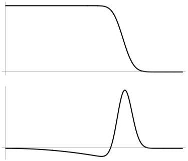

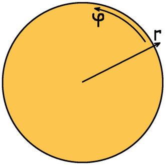

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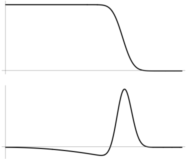

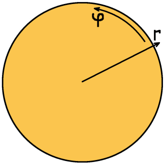

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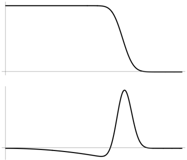

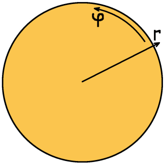

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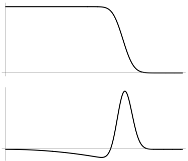

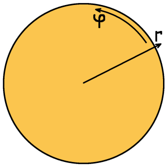

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[BO, Estienne 2022]

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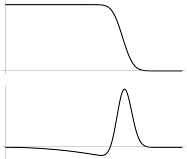

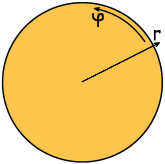

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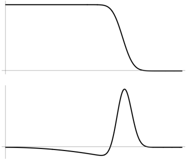

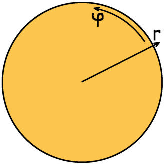

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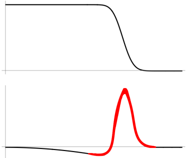

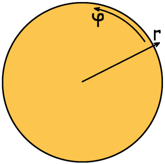

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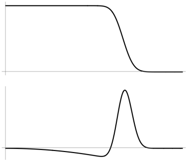

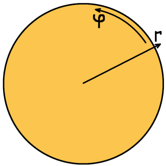

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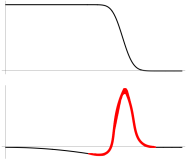

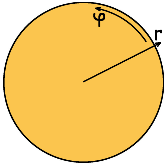

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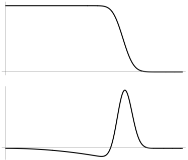

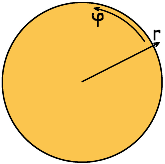

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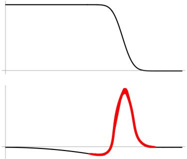

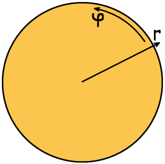

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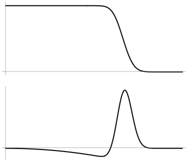

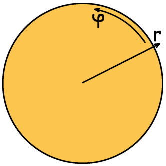

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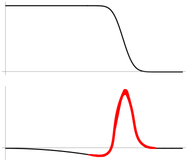

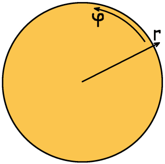

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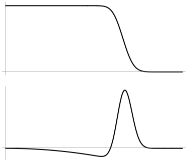

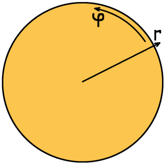

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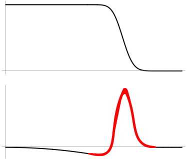

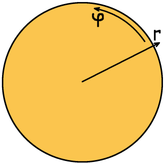

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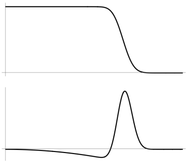

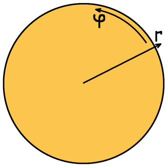

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[BO, Estienne 2022]

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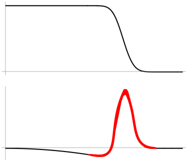

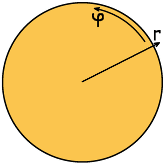

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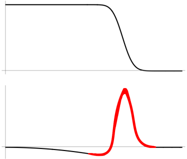

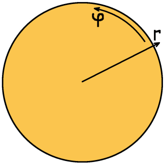

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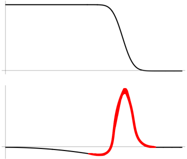

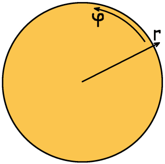

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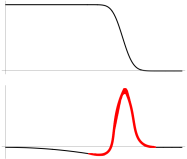

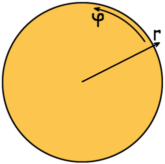

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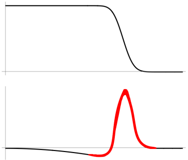

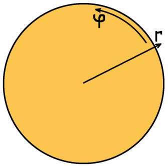

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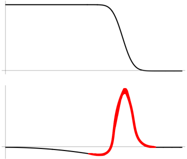

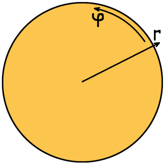

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$\downarrow$

$\propto$ Hall viscosity from edge !

[BO, Estienne 2022]

3. Berry phases from deformed potentials

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A. Deformed potentials

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B. Semiclassical wave functions

3. Berry phases from deformed potentials

A. Deformed potentials

B. Semiclassical wave functions

C. Berry phases and edge central charge ? [in progress]

DEFORMED POTENTIALS

Setup : Electrons in 2D plane + magnetic field

Deformed potentials

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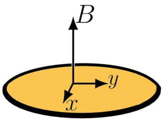

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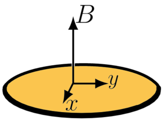

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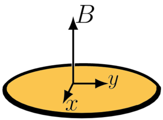

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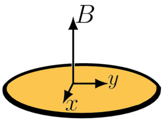

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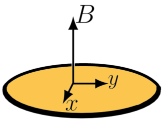

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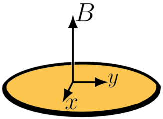

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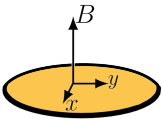

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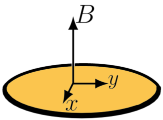

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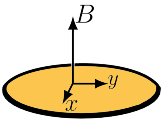

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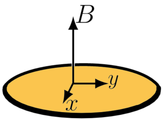

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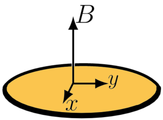

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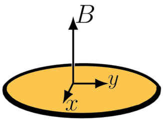

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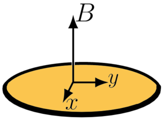

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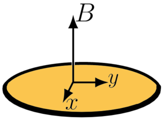

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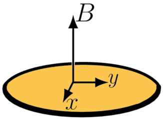

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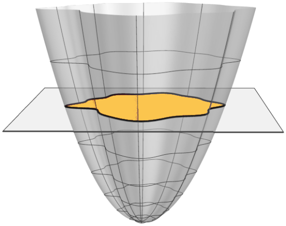

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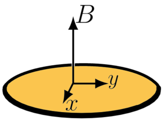

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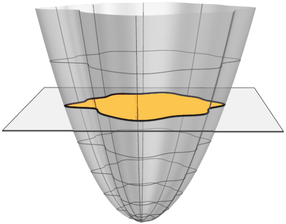

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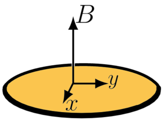

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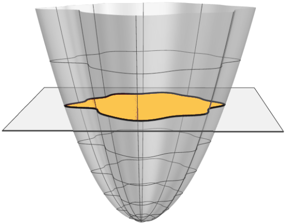

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  $\Rightarrow$ work in **lowest Landau level** !

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APPROXIMATE EIGENSTATES

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- Potential = effective Hamiltonian

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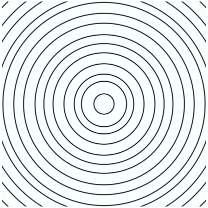

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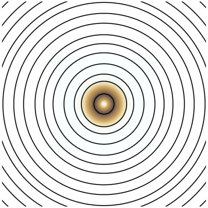

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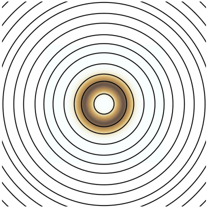

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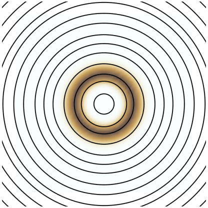

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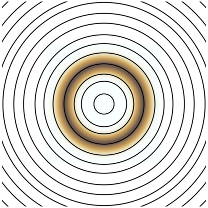

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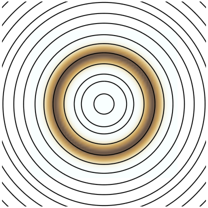

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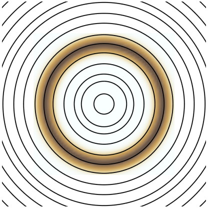

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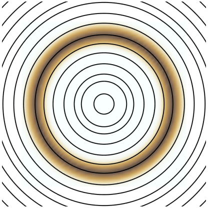

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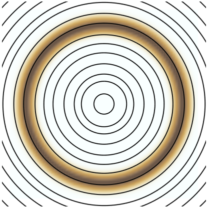

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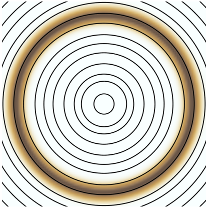

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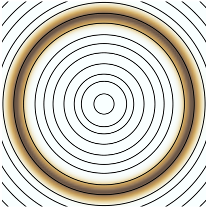

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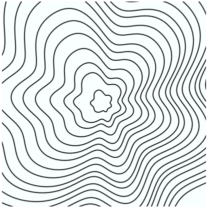

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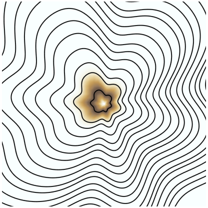

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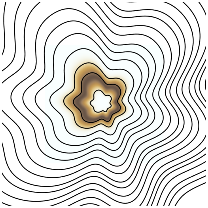

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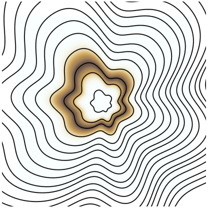

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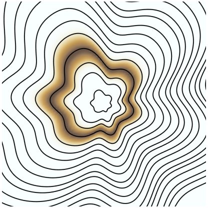

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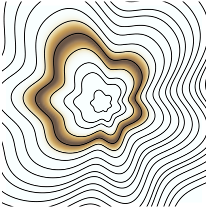

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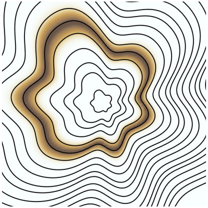

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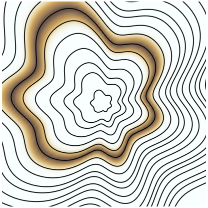

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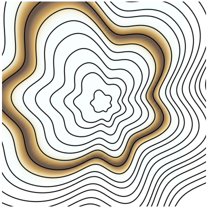

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Build localized eigenfunctions in **semiclassical regime**

Approximate eigenstates

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Build localized eigenfunctions at **small ℓ, large m**

Approximate eigenstates

WKB ansatz for ψ_m :

Approximate eigenstates

WKB ansatz for ψ_m :

- Pick equipotential

Approximate eigenstates

WKB ansatz for ψ_m :

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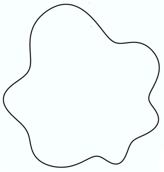

Approximate eigenstates

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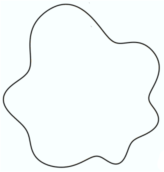

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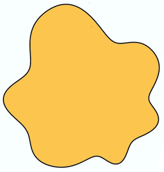

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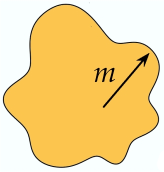

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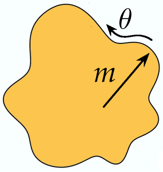
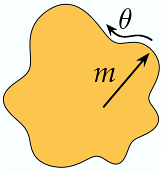


APPROXIMATE EIGENSTATES



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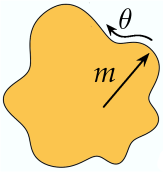

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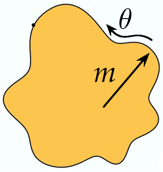

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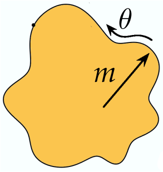

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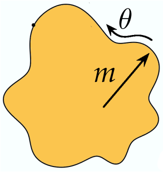

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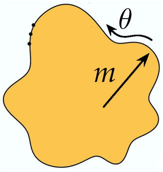

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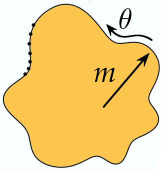

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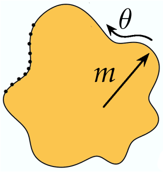

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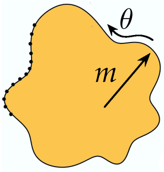

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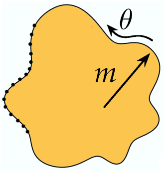

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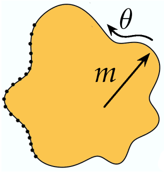

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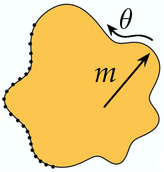

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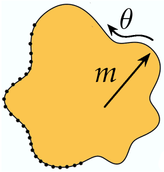

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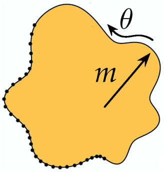

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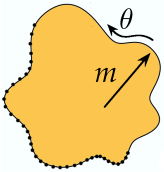

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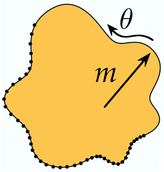

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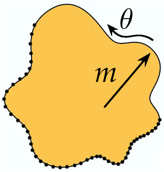

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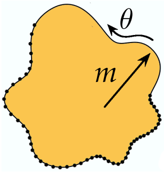

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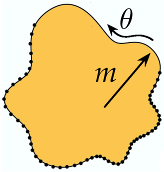

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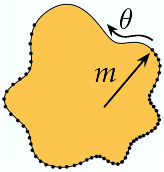

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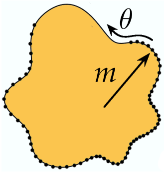

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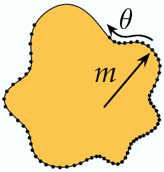

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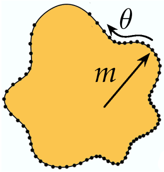

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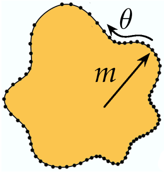

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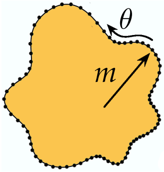

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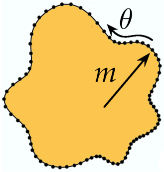

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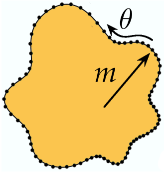

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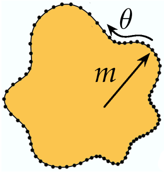

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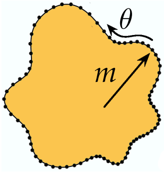

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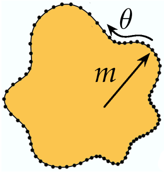

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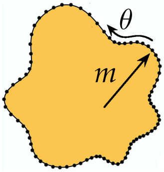

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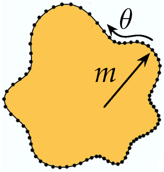

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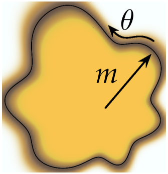

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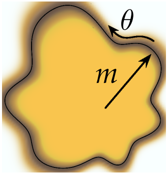

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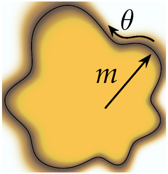

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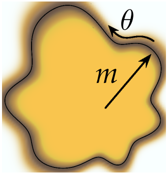

Example : **Edge-deformed traps**

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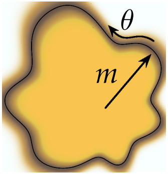

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$(r^2, \varphi) \mapsto \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$

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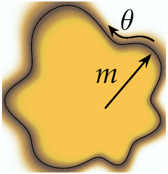

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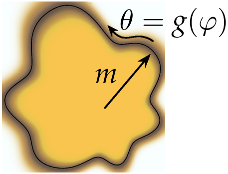

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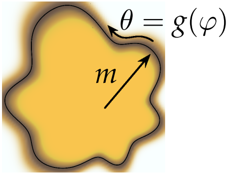

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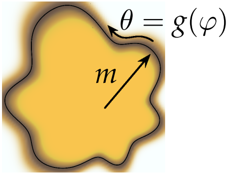

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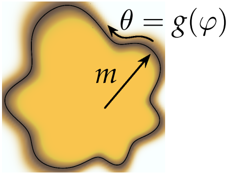

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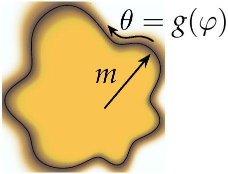

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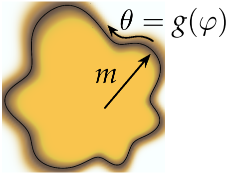

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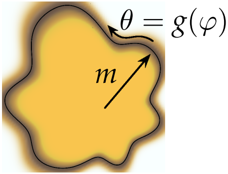

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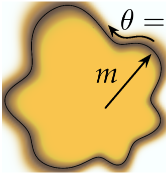

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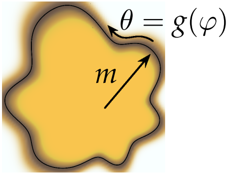

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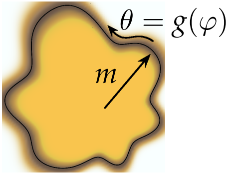

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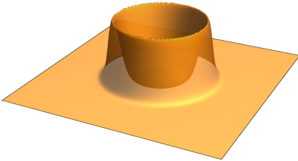

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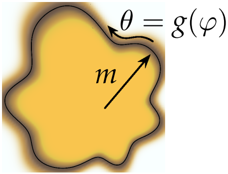

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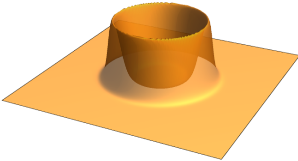

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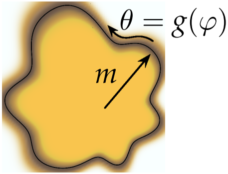

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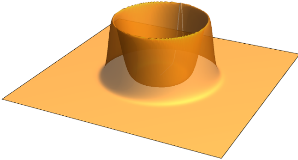

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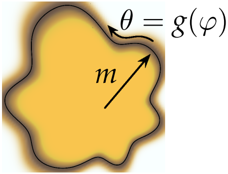

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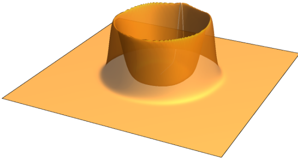

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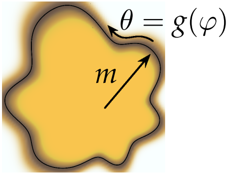

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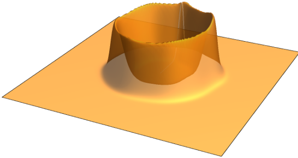

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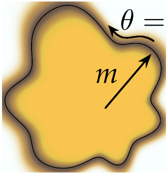

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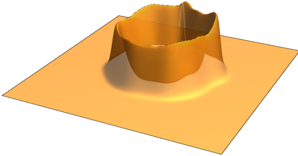

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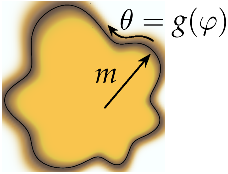

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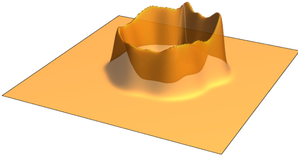

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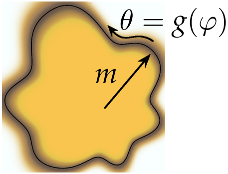

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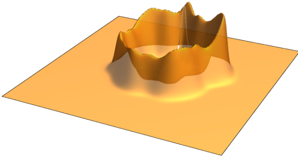

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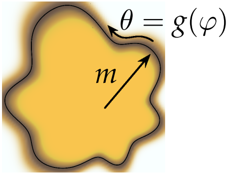

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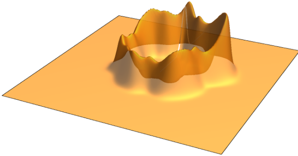

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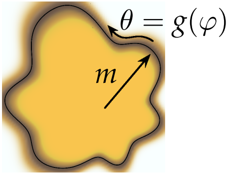

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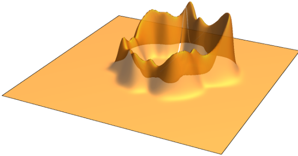

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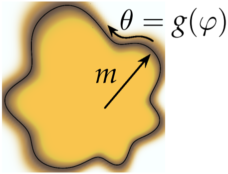

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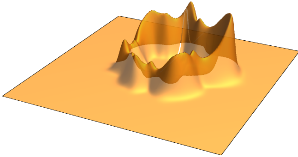

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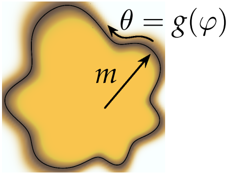

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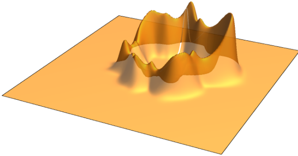

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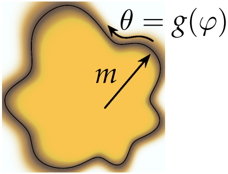

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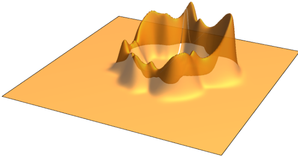

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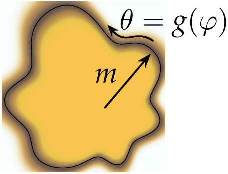

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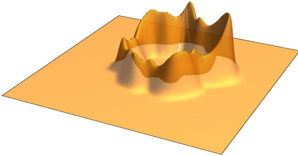

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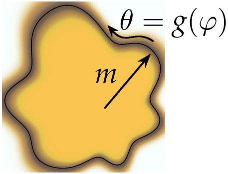

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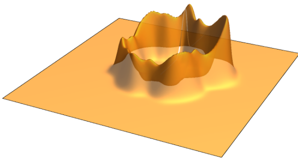

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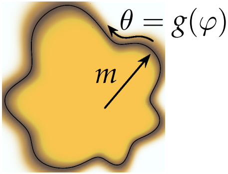

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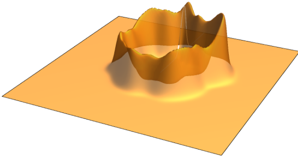

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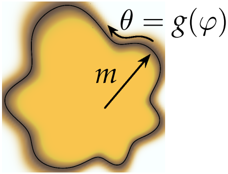

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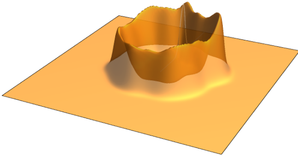

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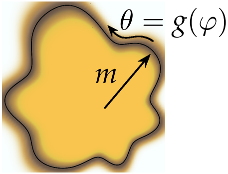

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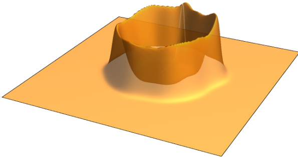

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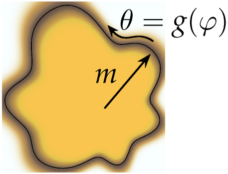

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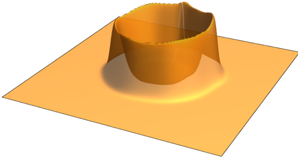

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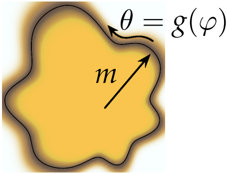

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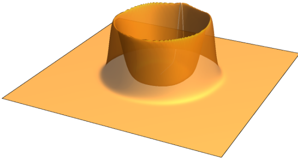

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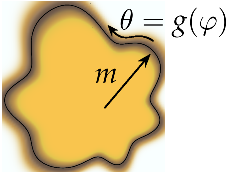

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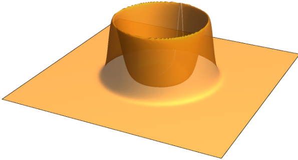

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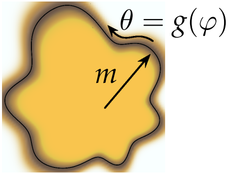

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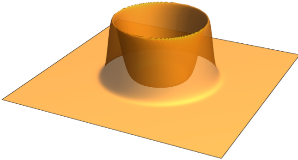

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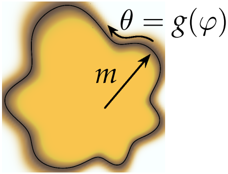

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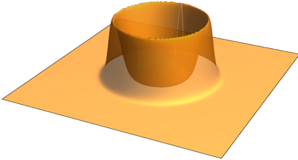

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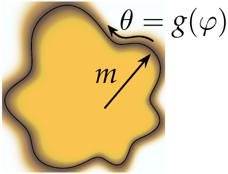

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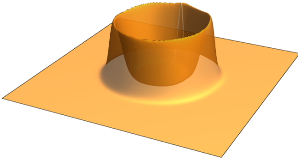

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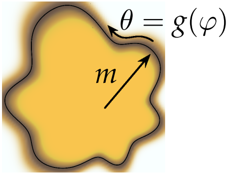

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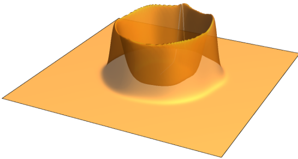

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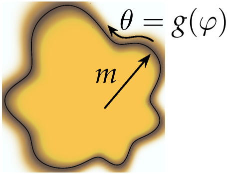

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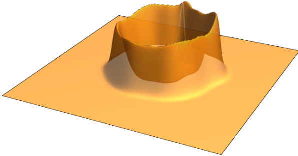

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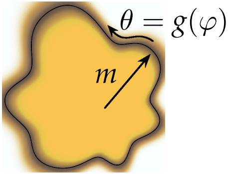

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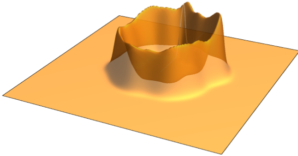

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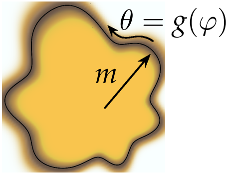

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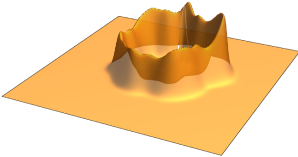

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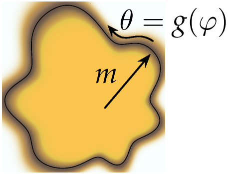

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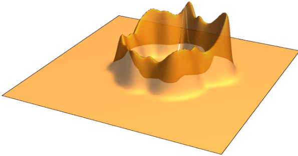
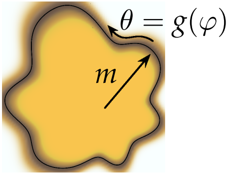


APPROXIMATE EIGENSTATES



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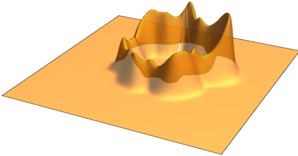

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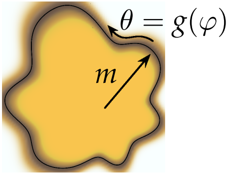

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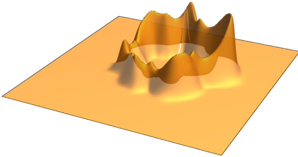

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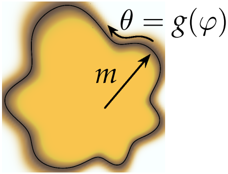

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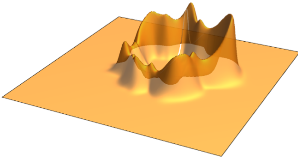

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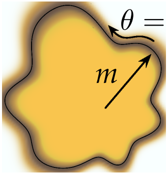

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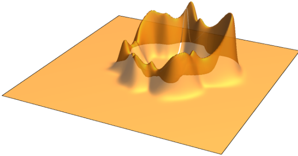

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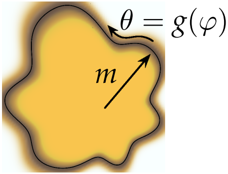

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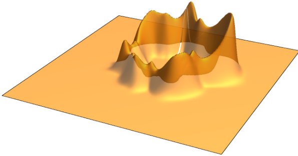

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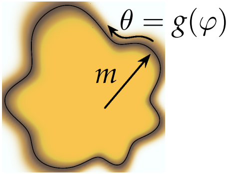

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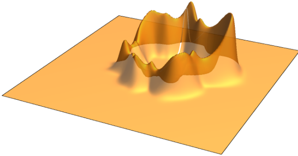

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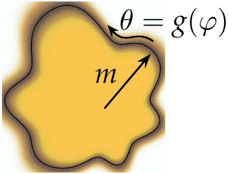

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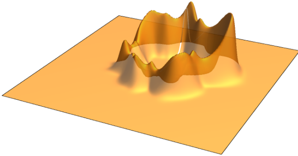

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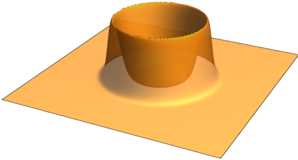

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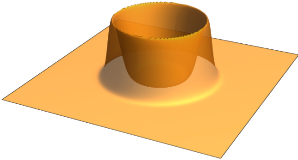

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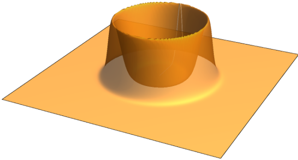

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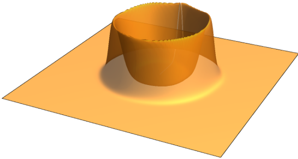

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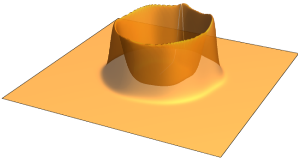

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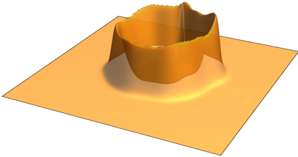

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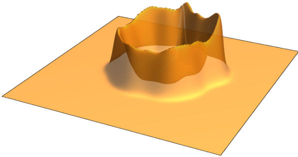

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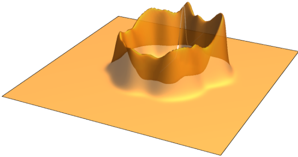

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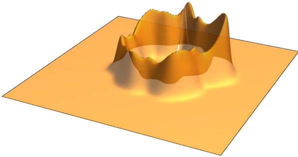

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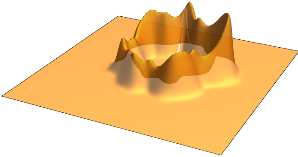

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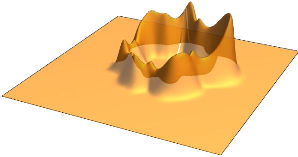

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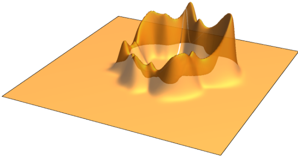

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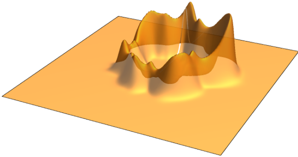

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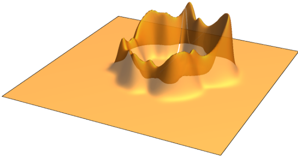

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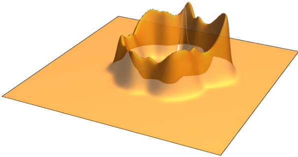

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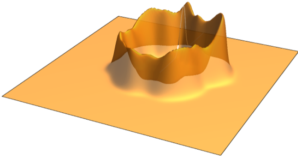

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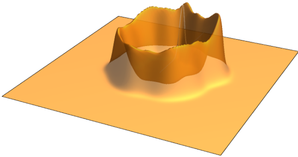

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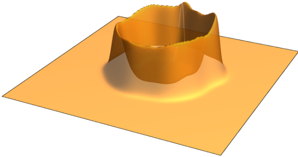

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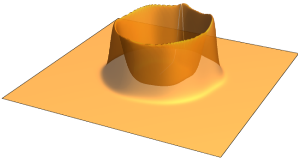

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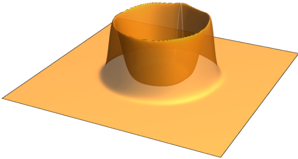

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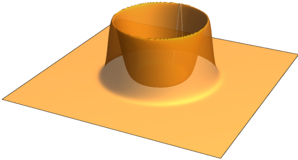

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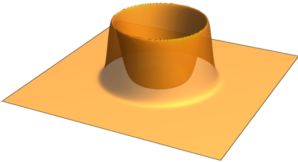

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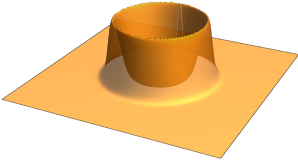

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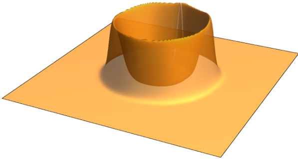

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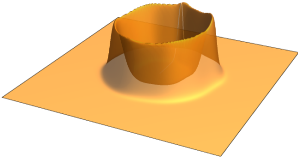

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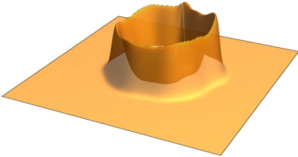

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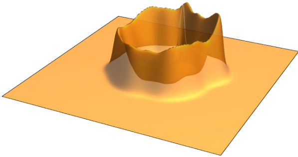

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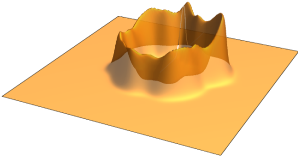

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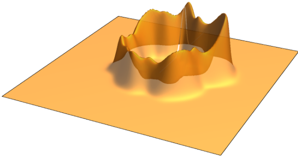

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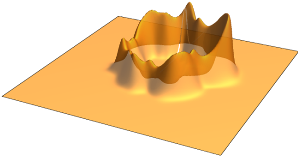

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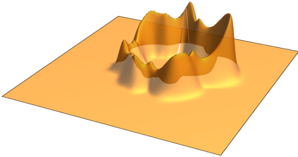

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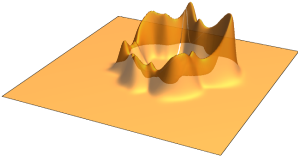

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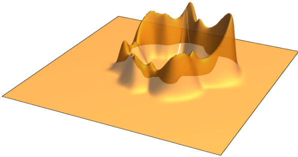

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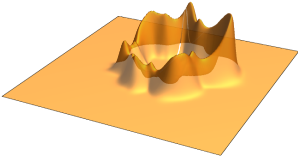

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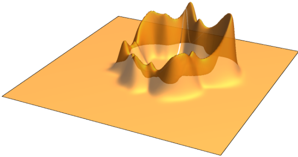

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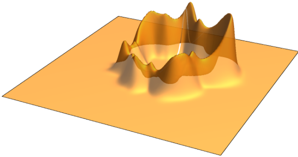

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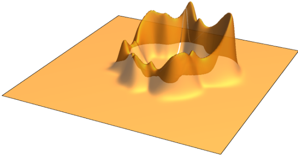

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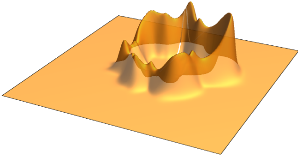

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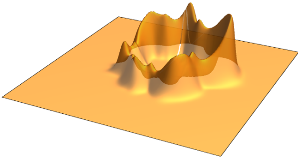

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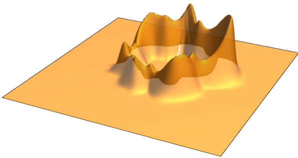

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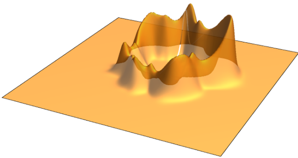

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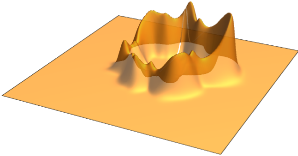

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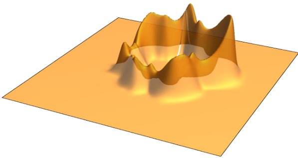

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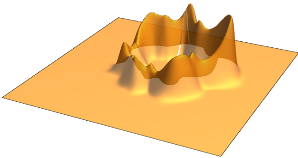

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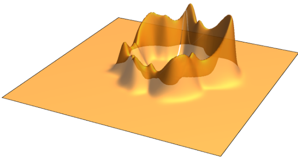

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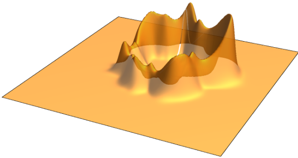

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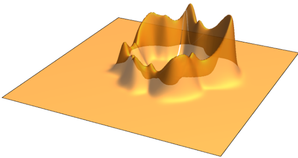

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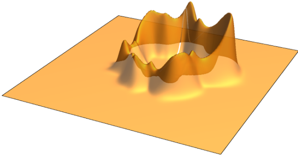

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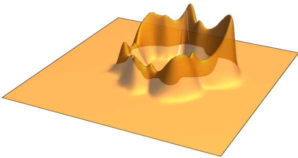

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[with Beauvillain, Estienne, Goldman, Lapierre, Moosavi, Petropoulos, Stéphan,...

...Stay tuned !]


Thanks !