Fractional Chern insulators in twisted bilayer graphene

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LPMMC (Grenoble), CNRS

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Les Diablerets, May 11 2023

LPMMC laboratoire de physique et de modélisation des milieux condensés

UGA Université Grenoble Alpes

cnrs

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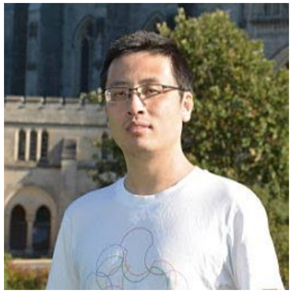
T. Senthil
(MIT)

CR, T. Senthil Phys. Rev. Research 2020

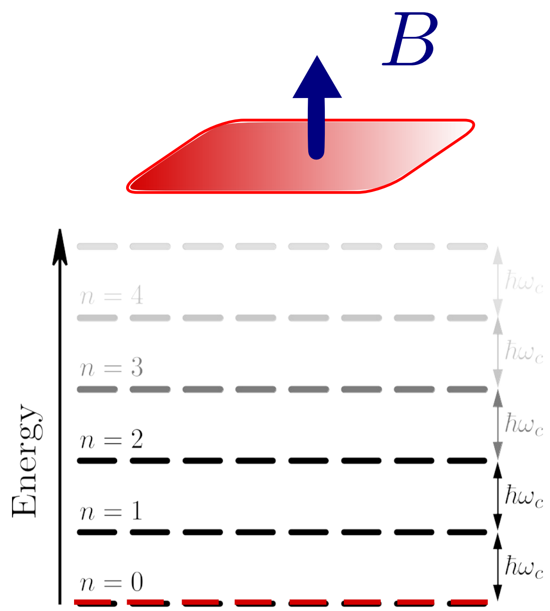
CR, Z. Dong, Y. Zhang, T. Senthil Phys. Rev. Lett. 2020

Zhihuan Dong
(MIT)



Ya-Hui Zhang
(MIT → Johns Hopkins)

Chern bands in 2D lattices : anomalous Hall effect



Landau levels

Lowest LL filling

$$\nu = p/q$$

Insulator with $\sigma_H = \frac{p}{q}\frac{e^2}{h}$

Fractional Quantum Hall effect

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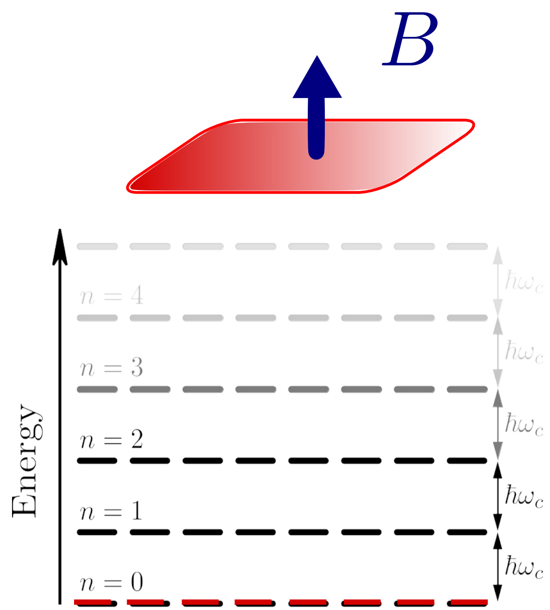

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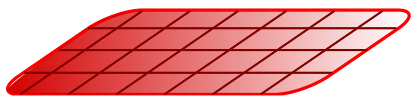

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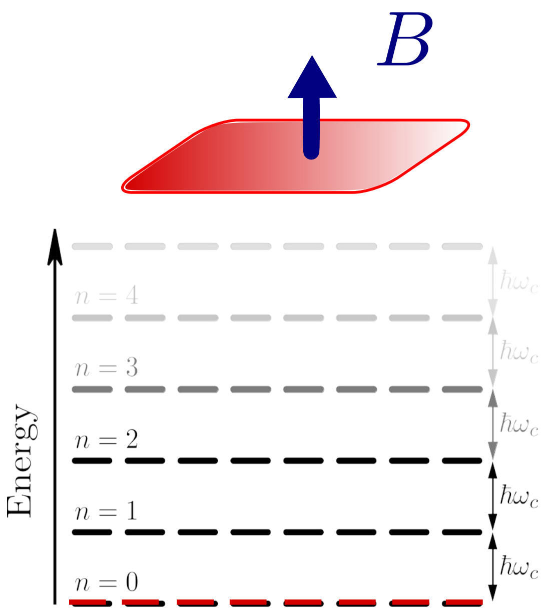

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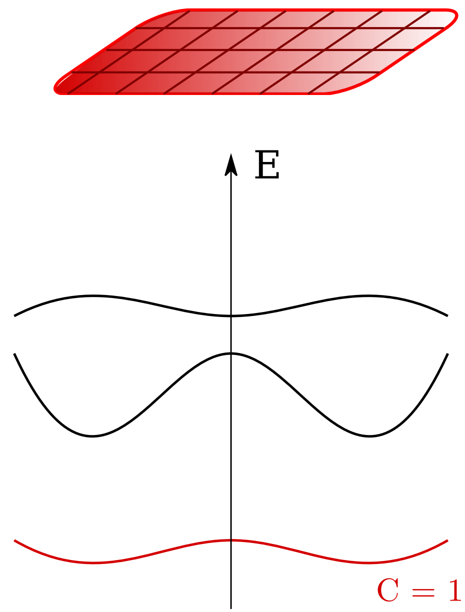

Band structure (Chern insulator)

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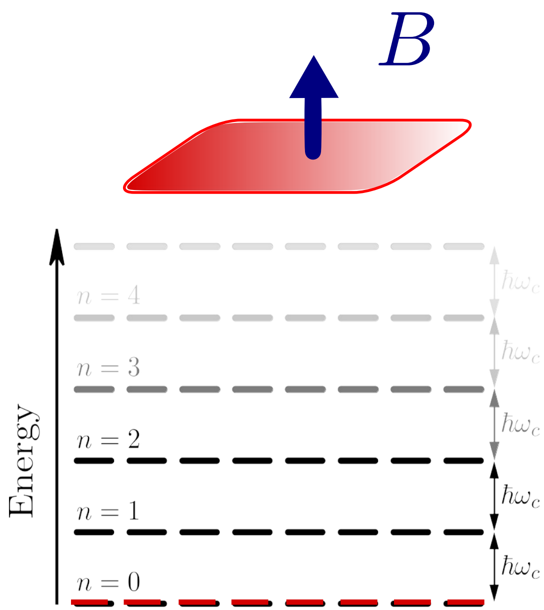

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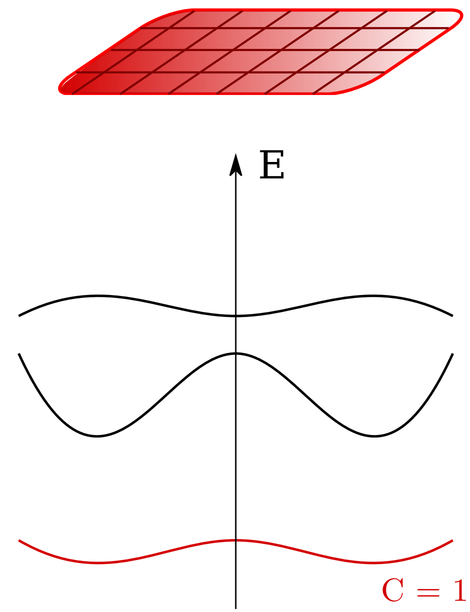

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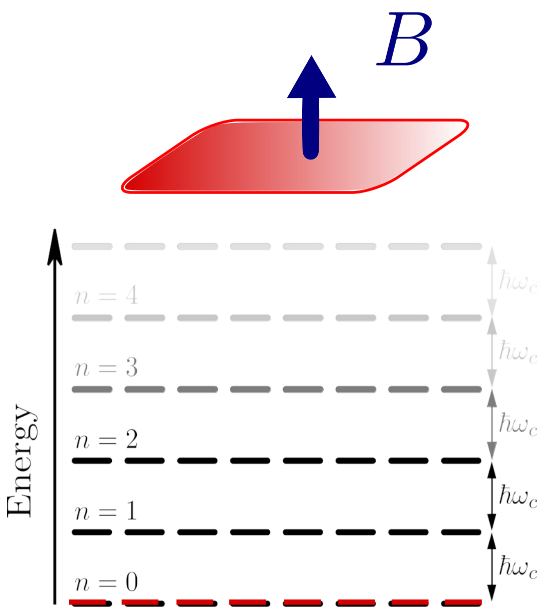

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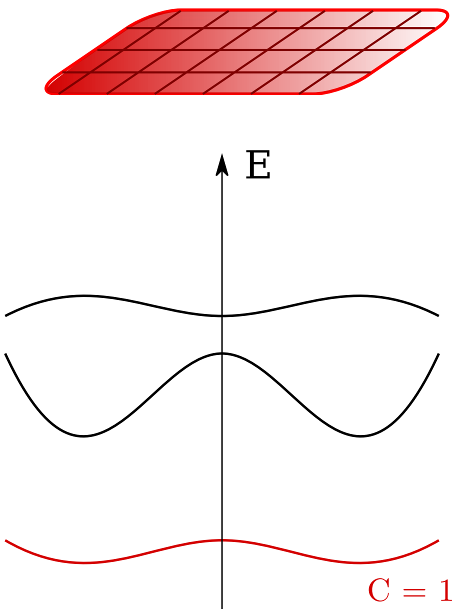

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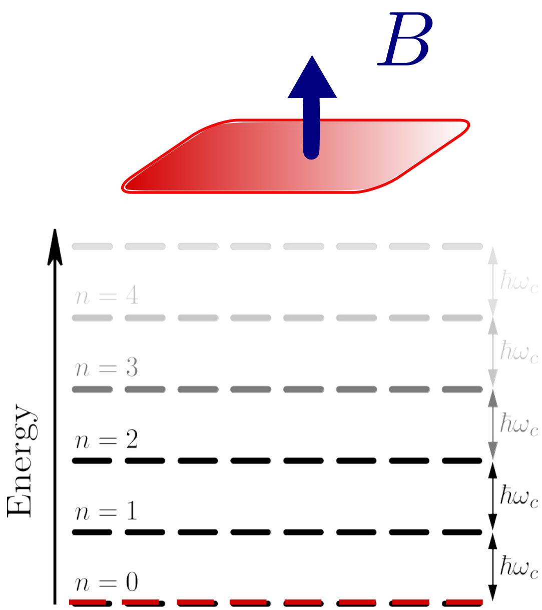

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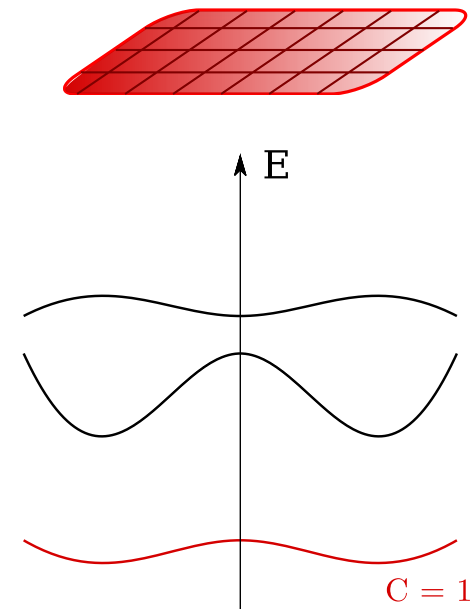

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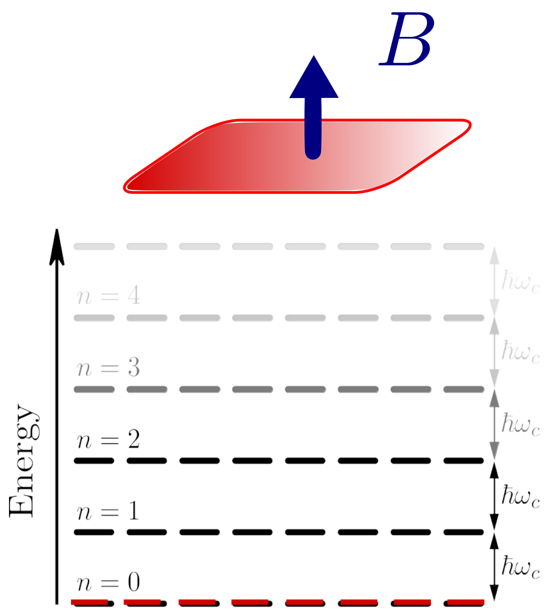

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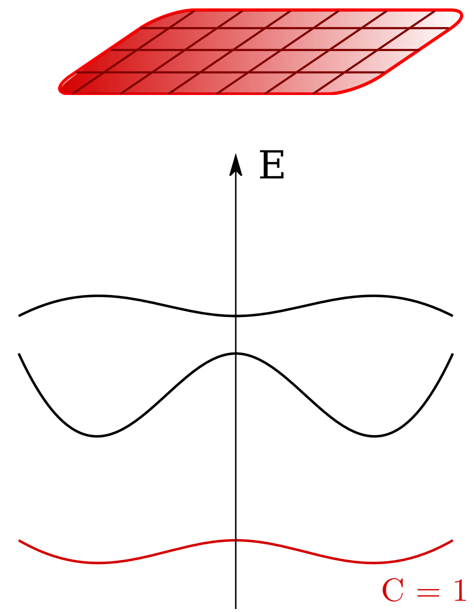

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Fractional Quantum Hall effect

Fractional Chern insulator

Quantum Hall physics in moire materials, without a magnetic field ?

Magic angle twisted bilayer graphene : single-particle properties

Quantized (integer) anomalous Hall effect in twisted bilayer graphene

Fractional Chern insulator in TBG : numerical evidence

Spin phase transition of a fractional Chern insulator

Magic angle twisted bilayer graphene : single-particle properties

Band structure properties : energetics

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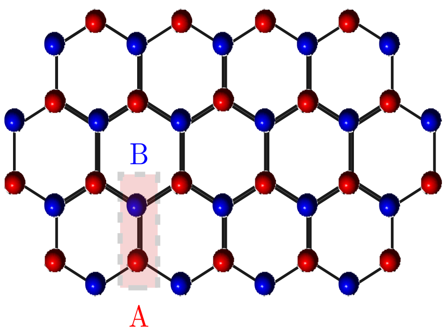
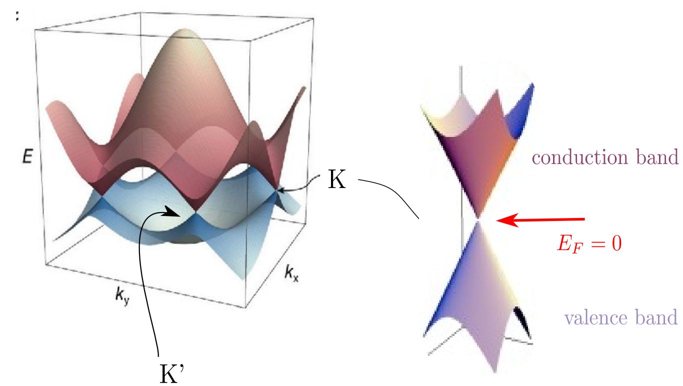
Monolayer graphene





Dirac cone

→ 2 valleys (K, K')

→ 2 spins

Band structure properties : energetics

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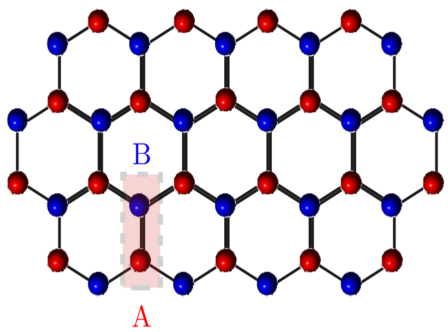

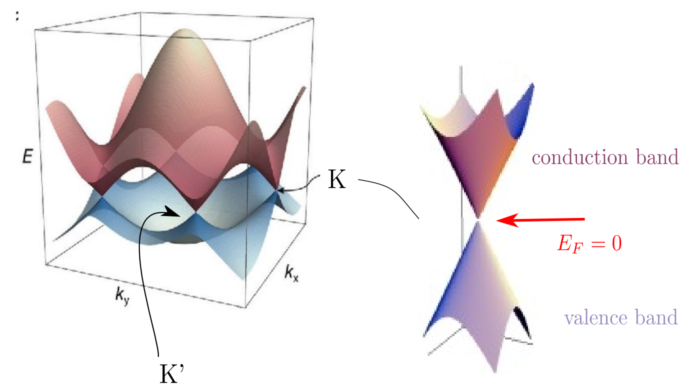

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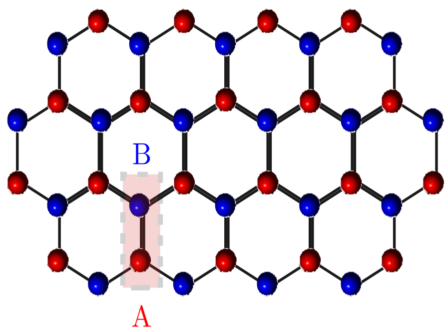

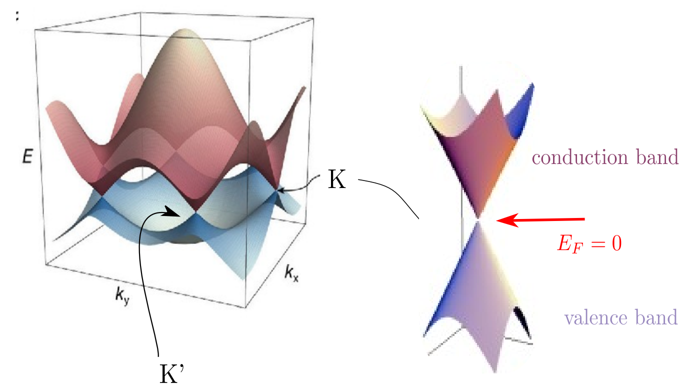

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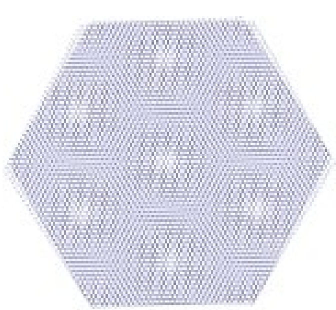

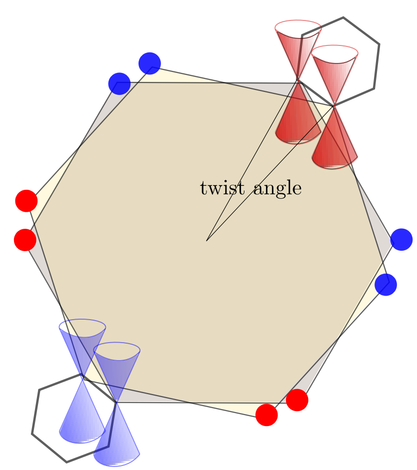

Moire Brillouin zone = mini BZ

Flattening the band: finding the magic angle

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Folding of Brillouin zone → several bands

Interlayer tunneling → level repulsion

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Focus on valence and conduction bands

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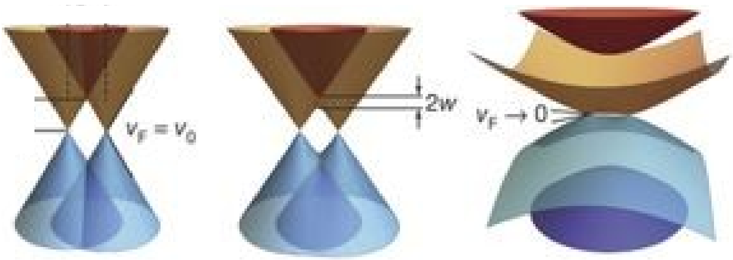
Cao, Jarillo-Herrero, Nature 2018



decreasing θ

Magic angle $\theta_m \simeq 1.1°$

Magic angle condition:

Fermi velocity v_F vanishes

Bistritzer, MacDonald, PNAS 2011

Also:

Suarez-Morell et al PRB 2010
Lopes dos Santos et al PRB 2012
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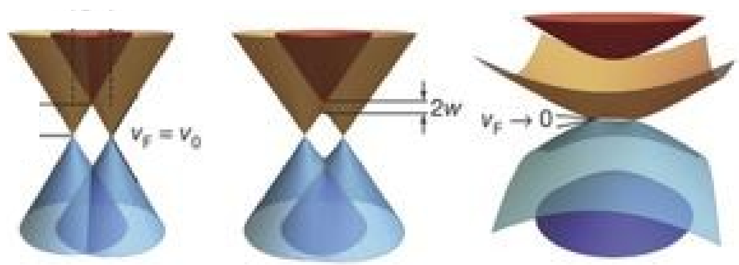

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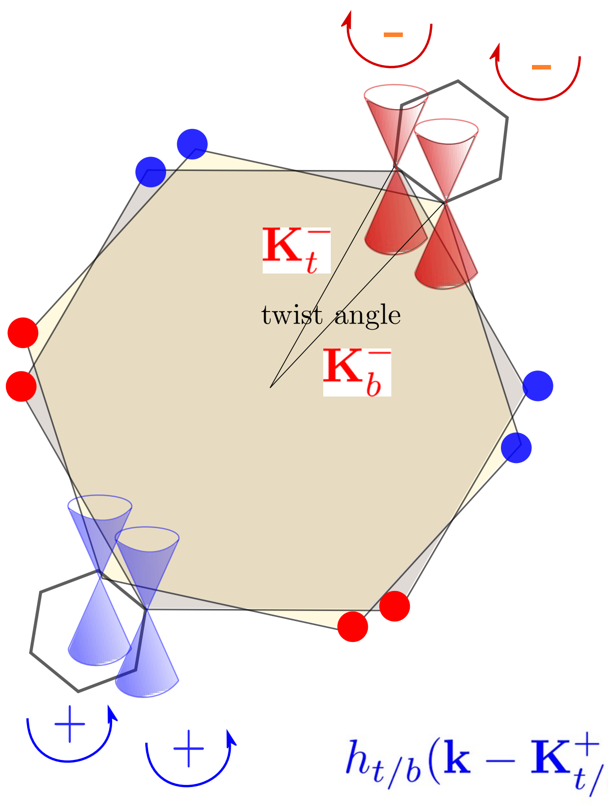
Flat band

Symmetries and band topology in TBG



Dirac cones with the same chirality at the corner of mBZ, protected by $C_2\mathcal{T}$

$$h_{t/b}(\mathbf{k}-\mathbf{K}^-_{t/b}) = \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}$$

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Symmetries and band topology in TBG

Conservation of U(1) valley charge

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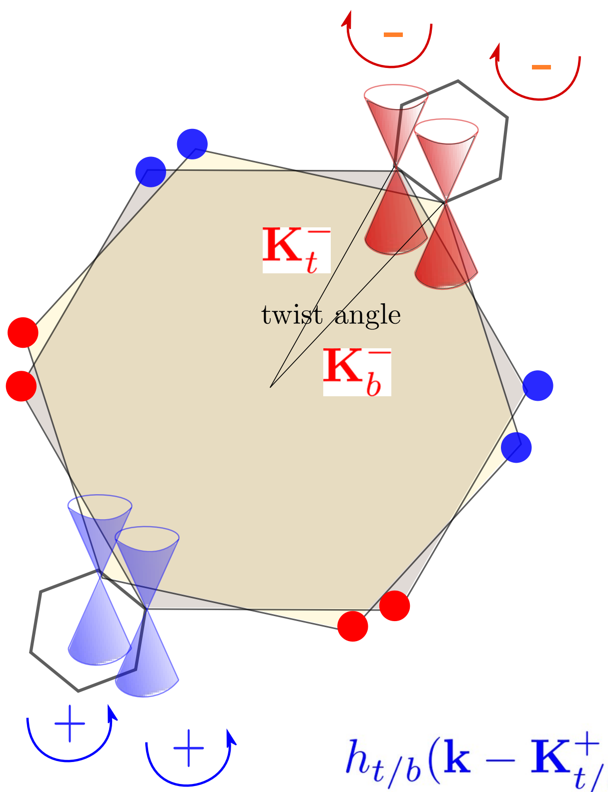

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Symmetries and band topology in TBG

Conservation of U(1) valley charge
SU(2) spin symmetry in each valley

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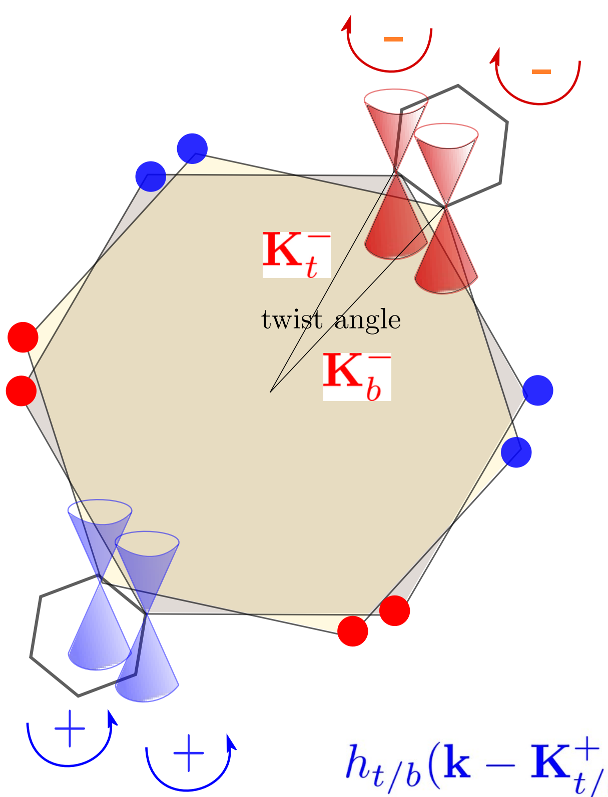

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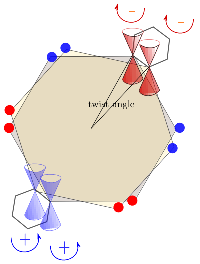
Continuous model for TBG aligned with hBN



Valley K+ Hamiltonian for TBG :

$$H_{TBG}^{K+} = \sum_{\mathbf{k}} \left(\Psi_{\mathbf{k},t}^{\dagger}, \Psi_{\mathbf{k},b}^{\dagger}\right) \begin{pmatrix} h(R_{\theta/2}\mathbf{k}) & 0 \\ 0 & h(R_{-\theta/2}\mathbf{k}) \end{pmatrix} \begin{pmatrix} \Psi_{\mathbf{k},t}^{\dagger} \\ \Psi_{\mathbf{k},b}^{\dagger} \end{pmatrix}$$

$$+ \sum_{\mathbf{k},j=1,2,3} \left(\Psi_{\mathbf{k}+\mathbf{G}_j,t}^{\dagger}, \Psi_{\mathbf{k}+\mathbf{G}_j,b}^{\dagger}\right) \begin{pmatrix} 0 & T_j \\ T_j* & 0 \end{pmatrix} \begin{pmatrix} \Psi_{\mathbf{k},t}^{\dagger} \\ \Psi_{\mathbf{k},b}^{\dagger} \end{pmatrix}$$

$h(\mathbf{k})$ Hamiltonian of single layer graphene

[Bistritzer, MacDonald, PNAS ‘11]

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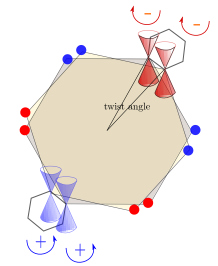

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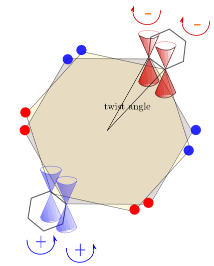

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Alignment with hBN : breaks $C_{2z}T$

effectively adds a mass $\Delta_{AB}\sigma_z$ to the Dirac fermions in one layer .

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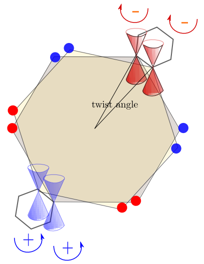

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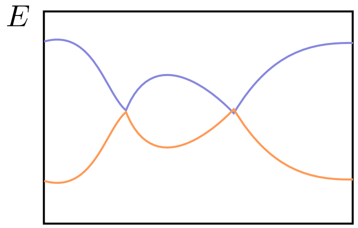
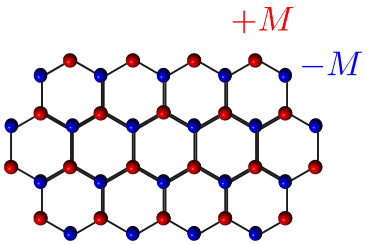
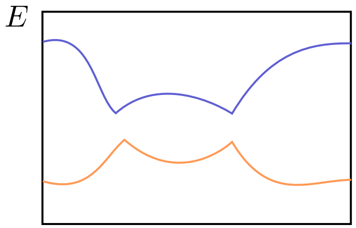


Dirac touching
protected by symmetry





graphene aligned with hBN
$\rightarrow$ broken sublattice symmetry

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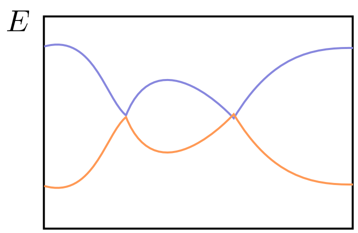

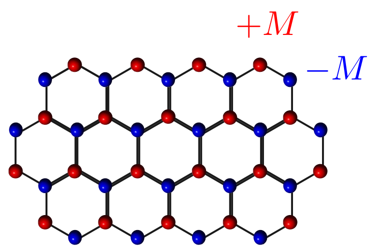

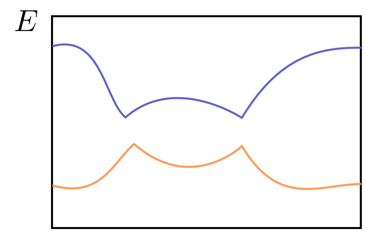

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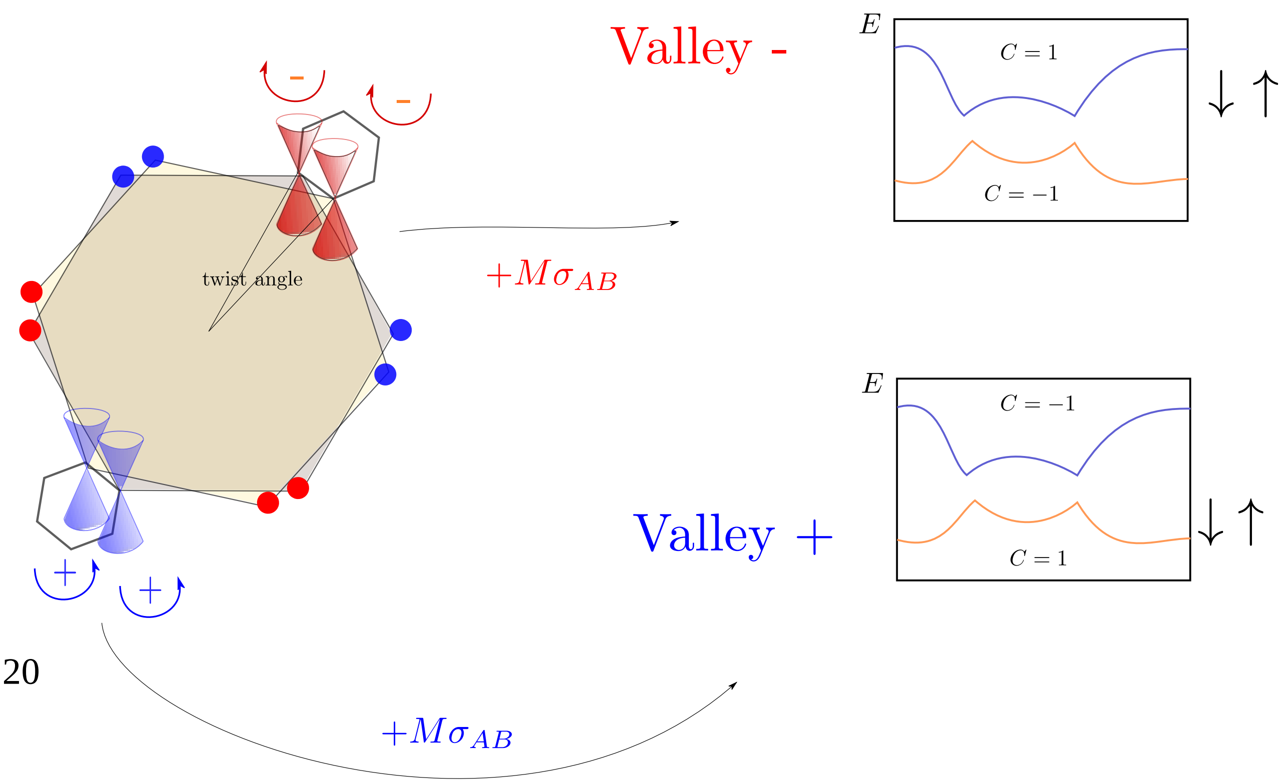

Zhang, Mao, Senthil PRR ‘19
Bultinck, Chatterjee, Zaletel PRL ‘20

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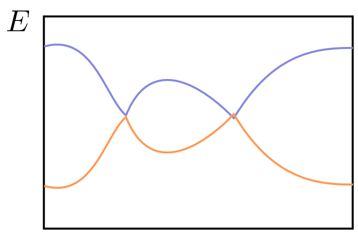

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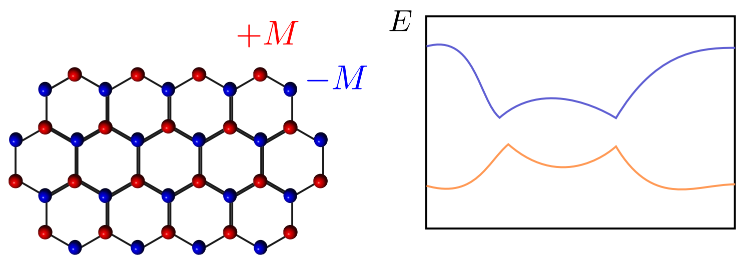

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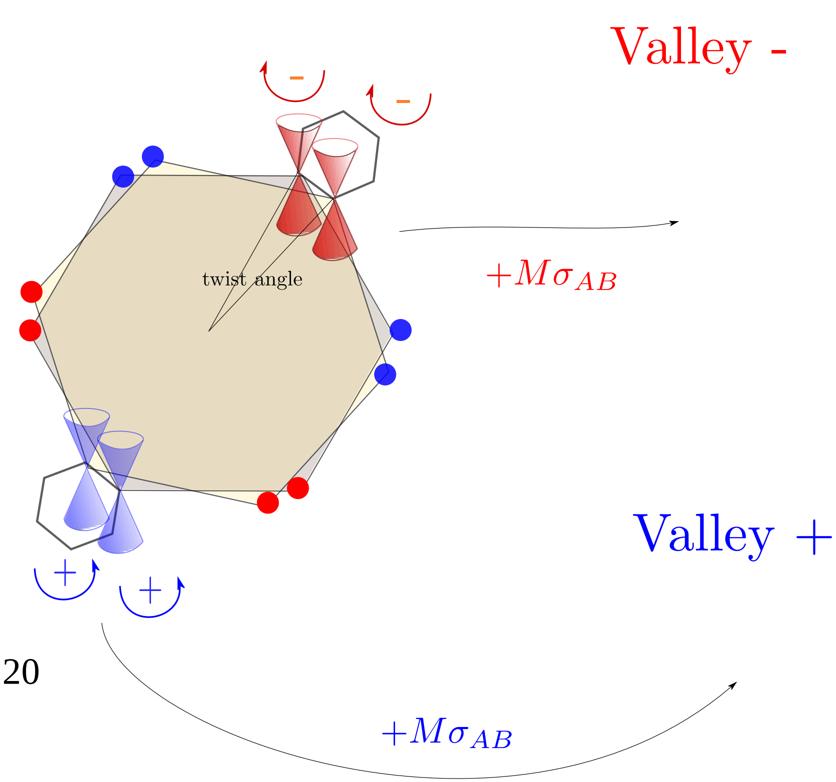

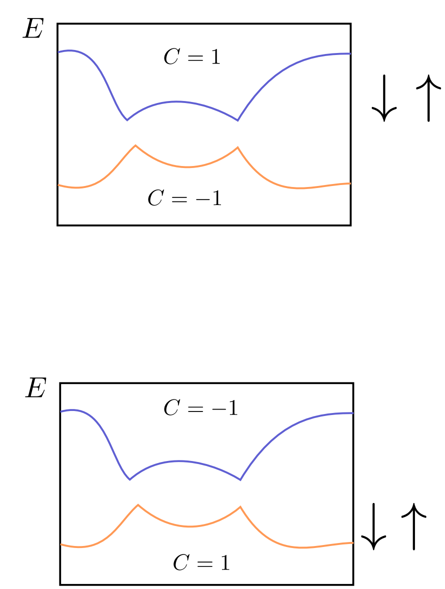

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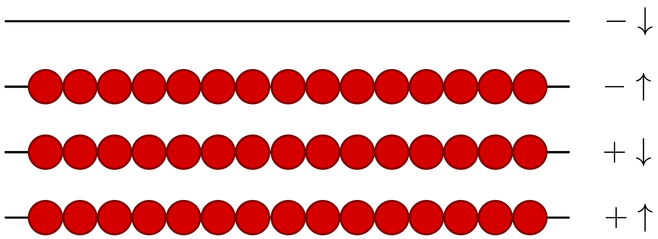
Flat Chern band

Magic angle twisted bilayer graphene : integer filling

Flat band ferromagnetism

We now turn on electron-electron interactions

Projected Hamiltonian : $H = \sum_{\mathbf{q}} V(\mathbf{q}) \tilde{\rho}_{\mathbf{q}} \; \tilde{\rho}_{-\mathbf{q}}$



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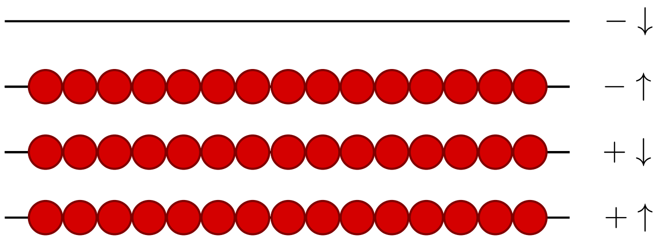


Projected Hamiltonian : $H = \sum_{\mathbf{q}} V(\mathbf{q}) \tilde{\rho}_{\mathbf{q}} \, \tilde{\rho}_{-\mathbf{q}}$

$V(\mathbf{q}) =$ screened Coulomb interaction

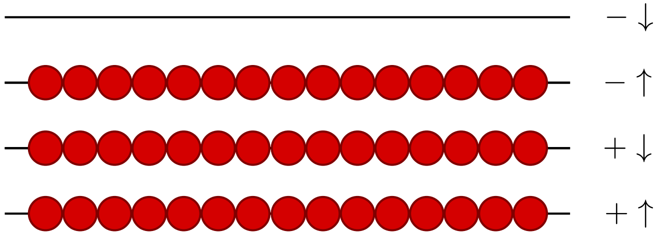
$$\tilde{\rho}_{\mathbf{q}} = \tilde{\rho}_{\mathbf{q}}^{\uparrow +} + \tilde{\rho}_{\mathbf{q}}^{\uparrow -} + \tilde{\rho}_{\mathbf{q}}^{\downarrow +} + \tilde{\rho}_{\mathbf{q}}^{\downarrow -}$$

Projected density operator

$$\tilde{\rho}_{\mathbf{q}}^{\tau} = \sum_{\mathbf{k}} \langle u_{\mathbf{k}+\mathbf{q}} | u_{\mathbf{k}} \rangle c_{\tau,\mathbf{k}+\mathbf{q}}^{\dagger} c_{\tau,\mathbf{k}}$$

Flat band ferromagnetism

We now turn on electron-electron interactions



Projected Hamiltonian : $H = \sum_{\mathbf{q}} V(\mathbf{q}) \tilde{\rho}_{\mathbf{q}} \, \tilde{\rho}_{-\mathbf{q}}$

$V(\mathbf{q}) =$ screened Coulomb interaction

$$\tilde{\rho}_{\mathbf{q}} = \tilde{\rho}_{\mathbf{q}}^{\uparrow +} + \tilde{\rho}_{\mathbf{q}}^{\uparrow -} + \tilde{\rho}_{\mathbf{q}}^{\downarrow +} + \tilde{\rho}_{\mathbf{q}}^{\downarrow -}$$ Projected density operator

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$|\Psi_{\text{polarized}}\rangle$ satisfies $\tilde{\rho}_{\mathbf{q}} |\Psi_{\text{polarized}}\rangle = 0$ $\quad q \neq$ reciprocal lattice vector

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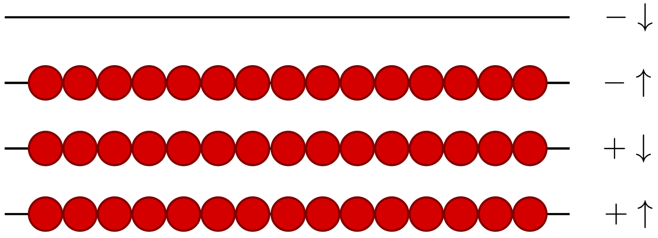

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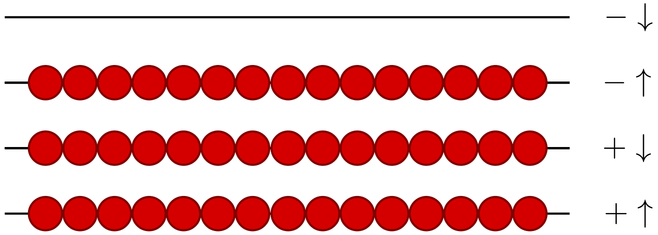

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Analogy with quantum Hall ferromagnetism at $\nu = 1$

Experiments at integer filling $\nu = 3$

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Quantized anomalous Hall effect $\sigma_H = -e^2/h$

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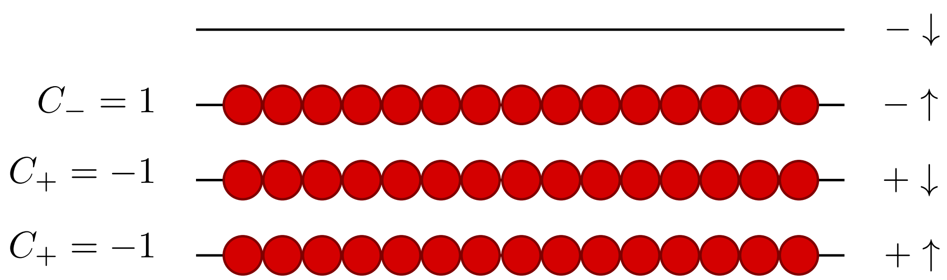
→ spontaneous valley polarization



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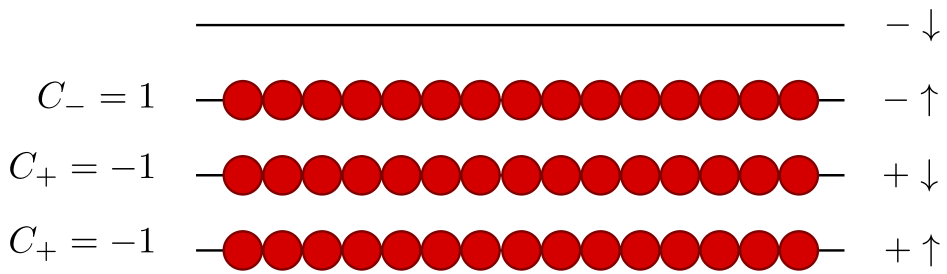
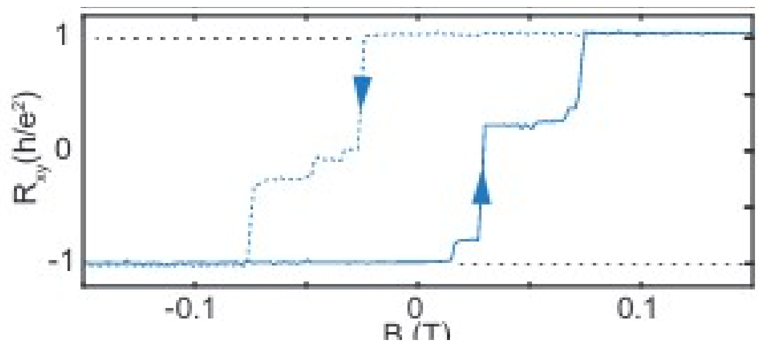


Orbital ferromagnetism

Hysteresis of the resistivity



[see also A. L. Sharpe, …, D.Goldhaber-Gordon, Science (2019), C.L. Tschirhart, …, A.F. Young Science (2021)]

Magic angle twisted bilayer graphene : fractional filling

Which fraction ?

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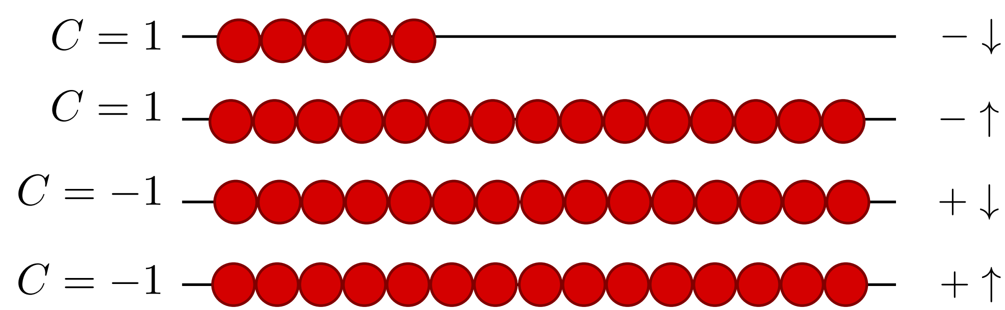

In the hole picture : Chern *C=1* band at filling $\nu = \frac{2}{3}$

Fractional Quantum Hall candidate wf at 2/3

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$\nu = \mathbf{1/3}$ $\quad \sigma_{\mathbf{H}} = \frac{\mathbf{1}}{\mathbf{3}} \frac{\mathbf{e^2}}{\mathbf{h}}$

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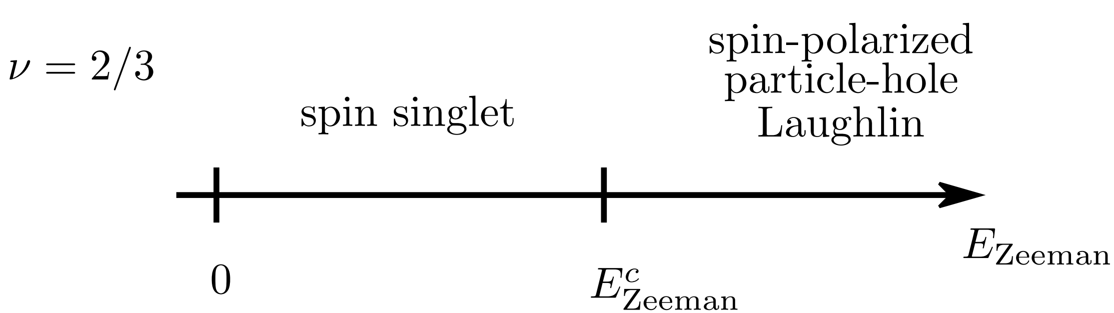

Numerics : [Xie, Guo, Zhang PRB ‘89, Wu, Dev, Jain PRL ‘93, MacDonald, Haldane PRB ‘96]

Experiments : [Clark et al PRL ’89, Eisenstein, Stormer, Pfeiffer, West PRB ‘90, Engel et al PRB ‘92]

Model and approximations

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Non-interacting continuum band model (Bistritzer-MacDonald + sublattice breaking term)

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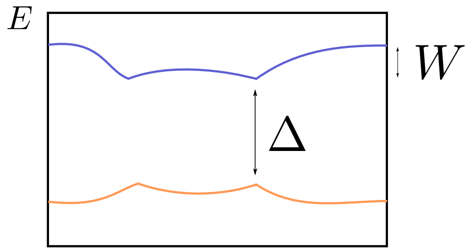

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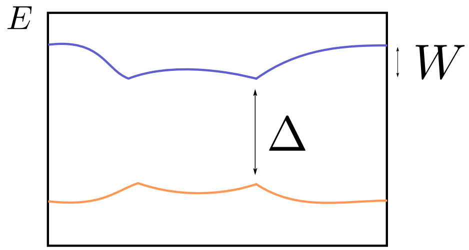

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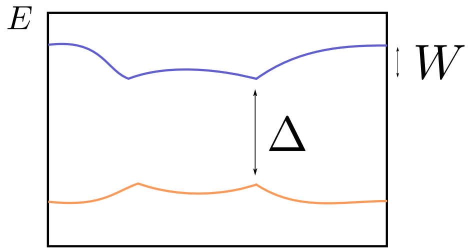

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Projected density operator

$u_{\mathbf{k}}$ Single-particle wave function from continuum model

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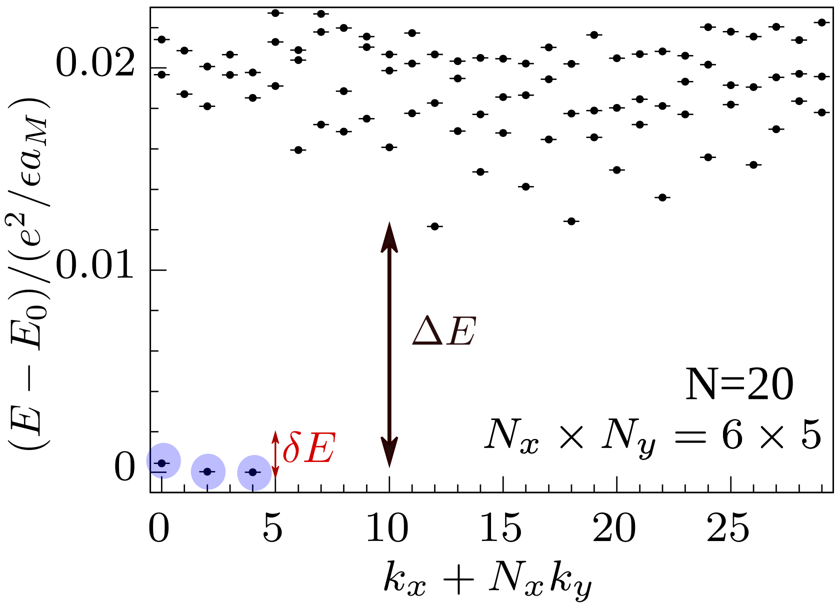

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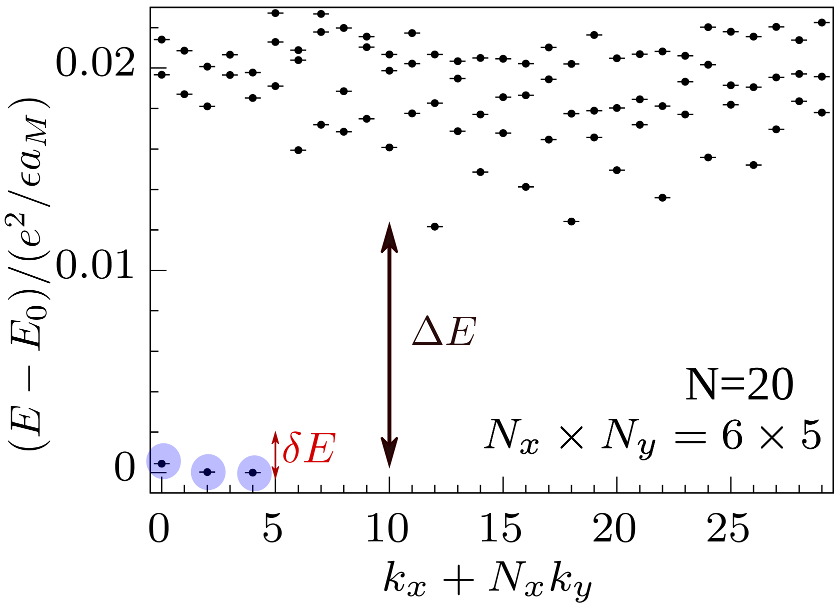

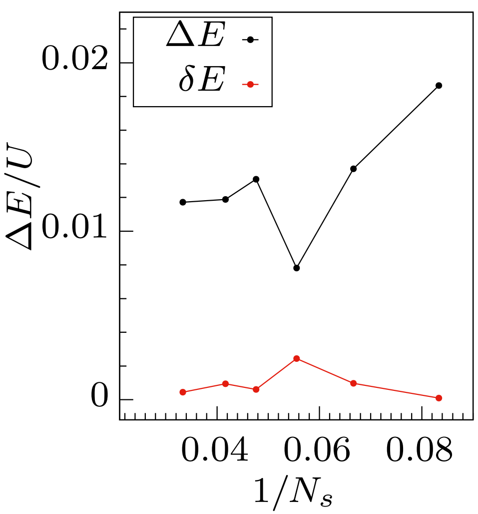

- Gap extrapolates to finite value $\Delta \simeq 0.2meV \simeq 2K$ in thermodynamic limit

[CR, Senthil PRR ‘20]

Role of the band properties

Numerical signature of a valley and spin polarized $\sigma_H = \frac{2}{3}\frac{e^2}{h}$ **FCI**

While realistic, our model has limits :

- Bands neglected in the projection (correction to the bandwidth)
- Disorder not taken into account
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Role of band geometry ?

What is the robustness of the FCI state ?

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Small Berry curvature inhomogeneities $\tilde{F}$ in the BZ

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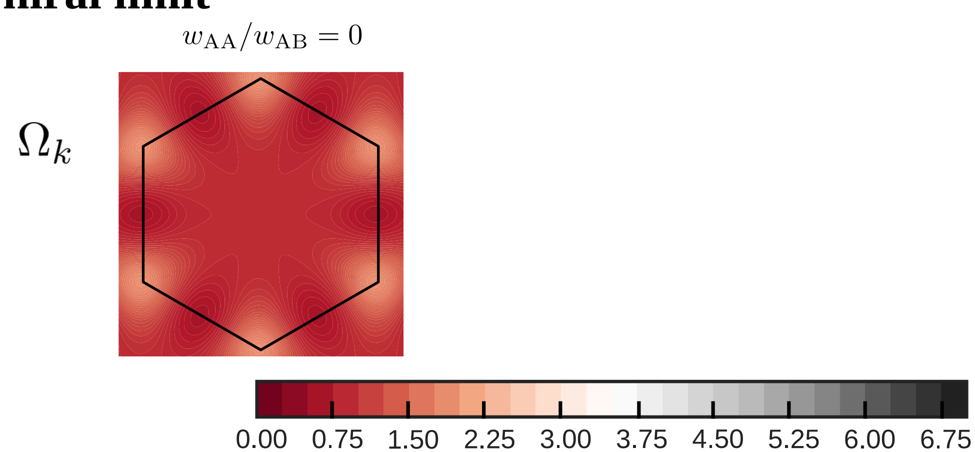

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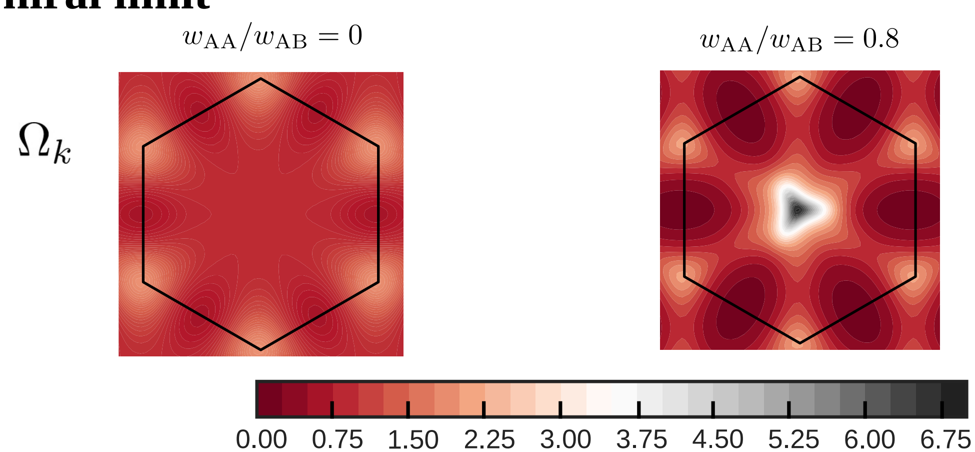

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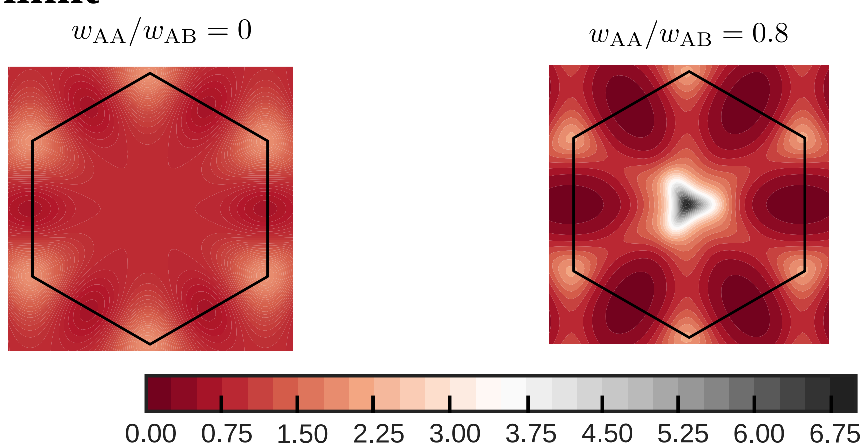

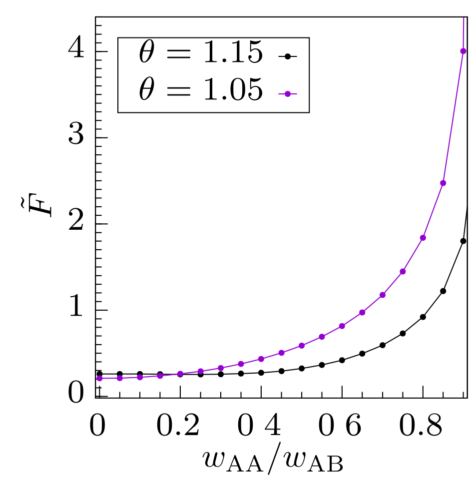

Role of the band properties (ED results)

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Role of the Berry curvature fluctuations

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Chiral limit

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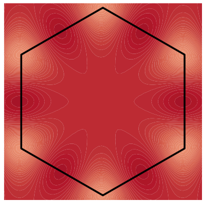
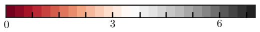


Berry fluctuations



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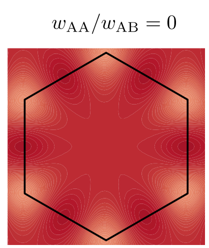

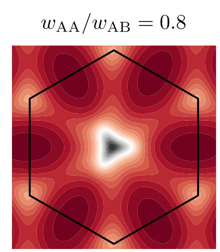

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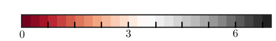

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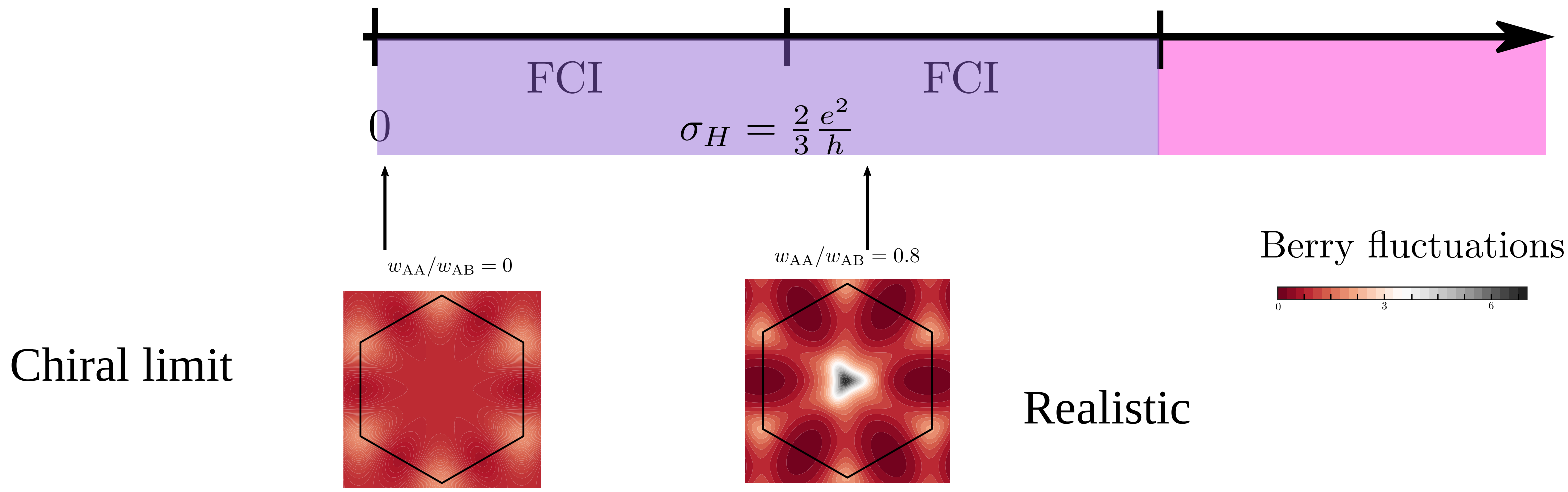

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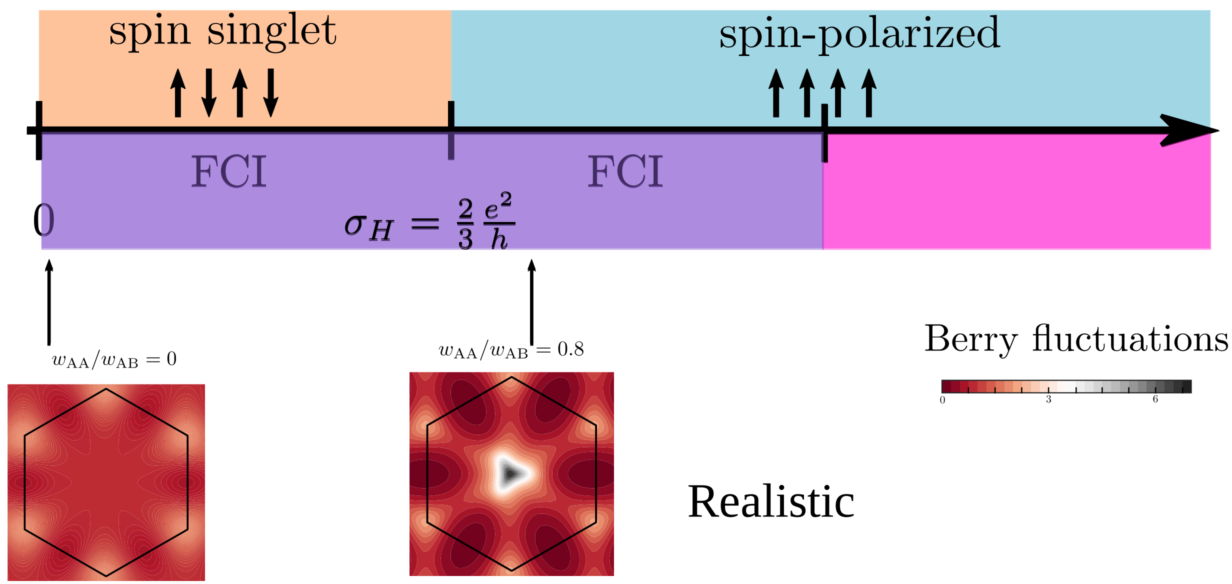
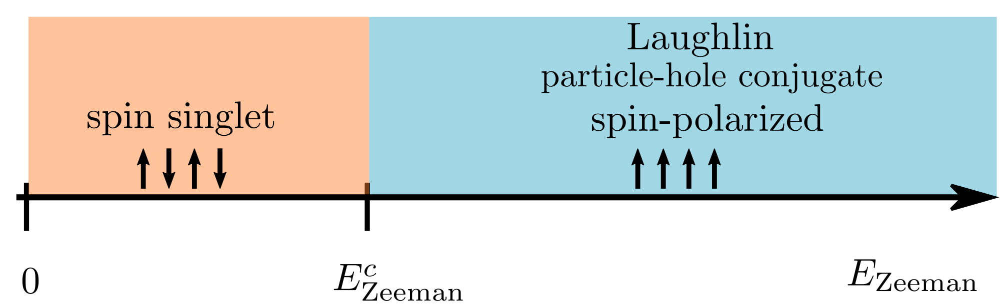


Chiral limit

Realistic

Reminiscent of Landau level FQH :

$\nu = 2/3$



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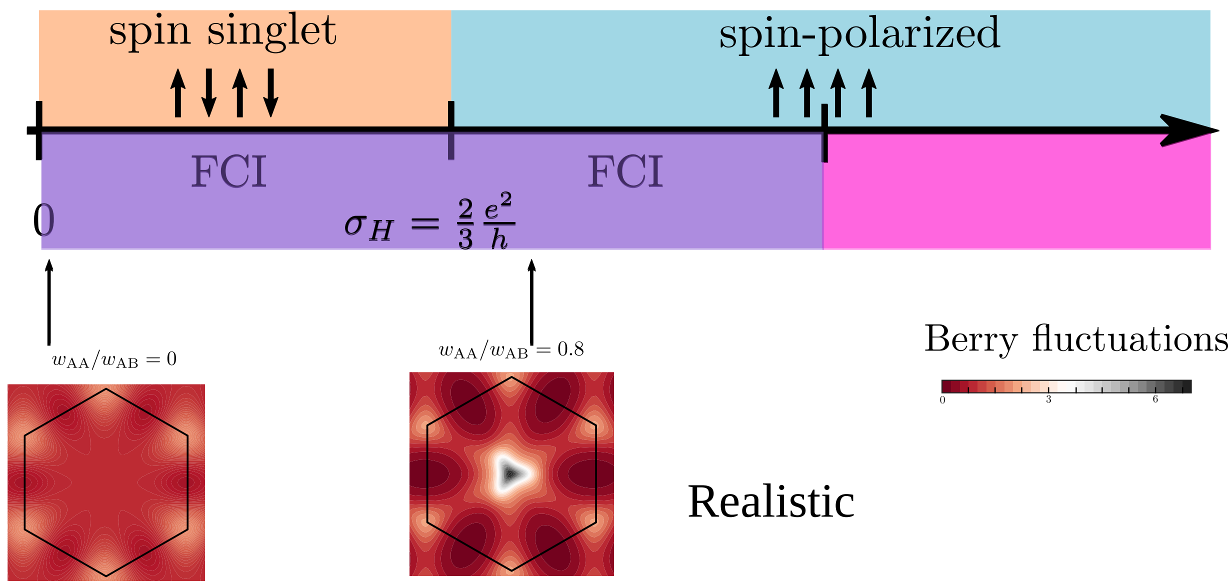
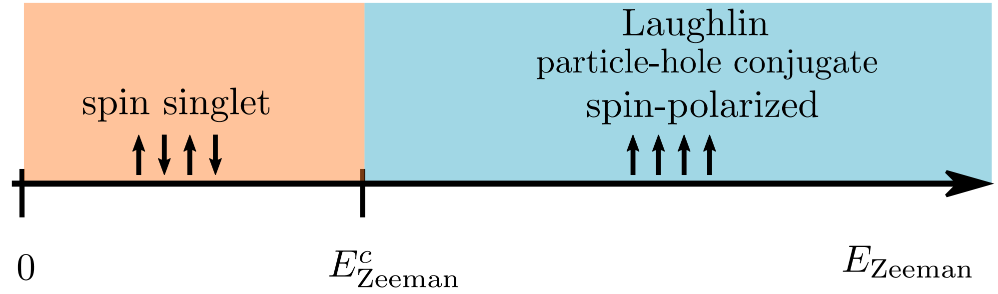


Chiral limit

Realistic

Reminiscent of Landau level FQH :

$\nu = 2/3$



Spin phase transition : an example of the unique role of band geometry in FQH physics

Observation of a FCI in MATBG/hBN + small B

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Yonglong Xie et al (Jarillo-Herrero and Yacoby groups) Nature 2021

Landau fan from local compressibility measurements

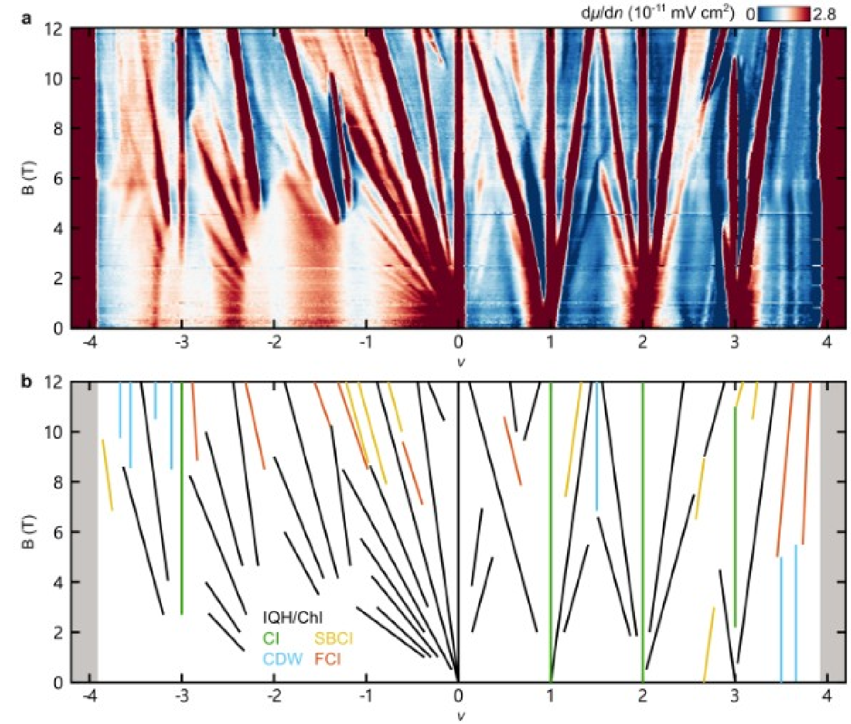
Streda formula (compressible state) :

$$\sigma_H = \frac{e^2}{h}\frac{\partial \rho}{\partial B}$$

$$\nu_T = 10/3 = 4 - 2/3$$

B = 0T : no FCI

B = 5T : FCI



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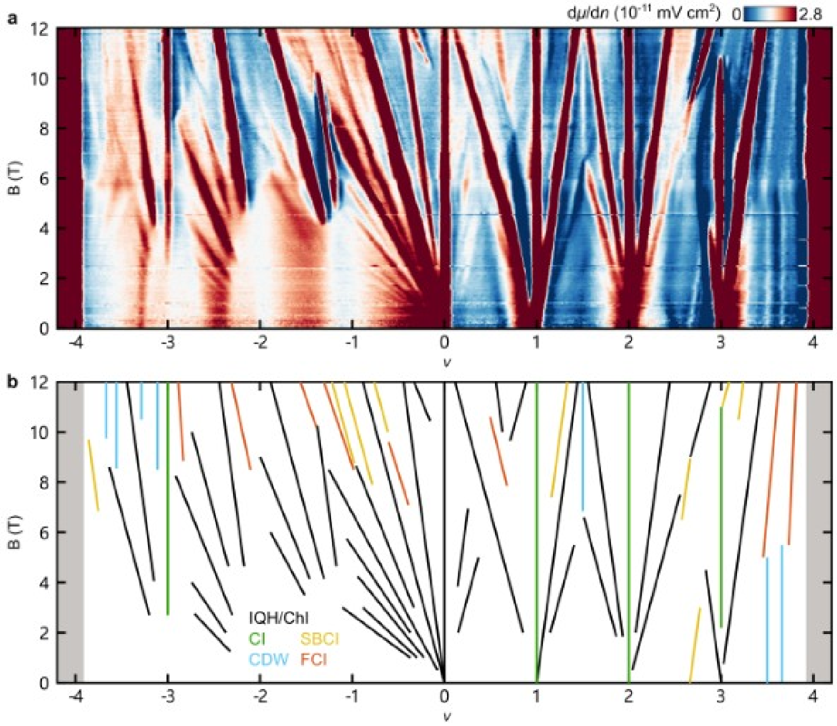

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Spin polarization in the experiment ?

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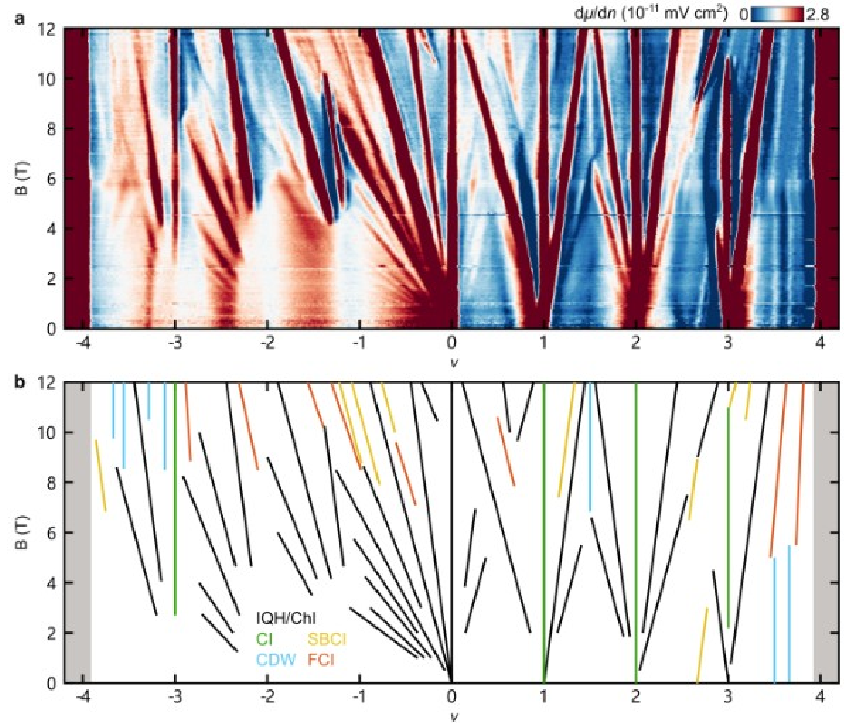

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Spin polarization in the experiment ?

FCI phase is in close competition with a $\sqrt{3} \times \sqrt{3}$ **CDW**

Wilhelm, Lang, Lauchli, PRB ‘21
Parker, Ledwith, Khalaf, Soejima, Hauschild, Xie, Pierce, Zaletel, Yacoby, Vishwanath, arXiv ‘21

Conclusion

- Fractional Chern insulators can emerge in MATBG : (gap ~ 2K)

  Other numerical predictions of FCIs in TBG :
  Abouelkomsan, Liu, Bergholtz PRL '20
  Wilhelm, Lang, Lauchli PRB '21

  Observed experimentally with small B *(Yonglong Xie et al Nature 2021)*

- **Quantum geometry plays a key role in spin polarization of FCIs**

  CR, Senthil. PRR '20

- Experimental evidence of FCIs in twisted bilayer TMD (MoTe2)
  Cai, Anderson, Xu et al arXiv:2304.08470
  Zeng, Xia, Mak, Shan et al arXiv:2305.00973

Degeneracy of quasihole excitations

From the ground state : entanglement spectrum

[Li, Haldane '08]

- Ground state Ψ: cut system in two parts A and B
- Reduced density matrix

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi| = \exp(-H_\xi)$$

- Entanglement spectrum = spectrum of

$$H_\xi$$

Particle entanglement spectrum : 'particle cut' $N = N_A + N_B$

$$|\Psi\rangle = \sum_i e^{-\xi_i/2} |\psi_i^A\rangle \otimes |\psi_i^B\rangle$$

[Sterdyniak, Bernevig, Regnault '11]

Equivalently :

$$\rho_A(x_1, ..., x_{N_A}, x'_1, ..., x'_{N_A}) = \int \left(\prod_{i=1}^{N_B} dx_{N_A+i} \right) \Psi^*(x_1, ..., x_N) \Psi(x'_1, ..., x'_N)$$

→ extract the degeneracy of bulk q.holes from the ground state

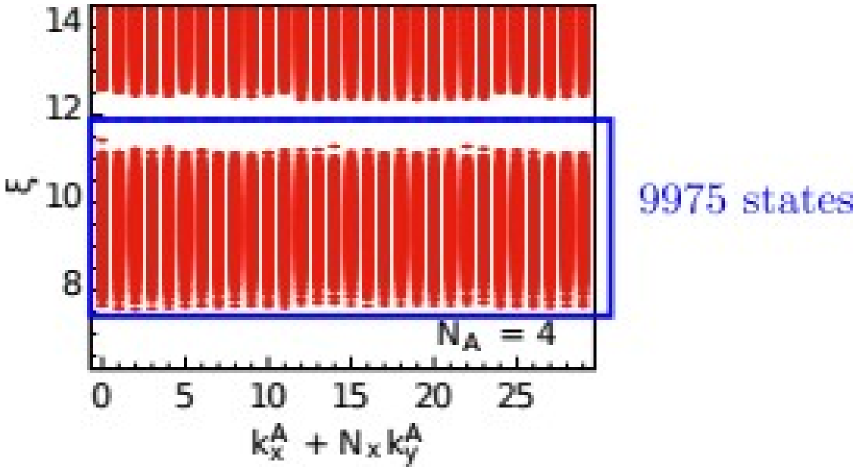
Degeneracy of quasihole excitations

From the particle entanglement spectrum

Particle entanglement spectrum

Lattice : 30 moire unit cells
N = 10 electrons

$N_A = 4$



Energy spectrum

FQH

$N_\phi = 30$

N = 4 electrons

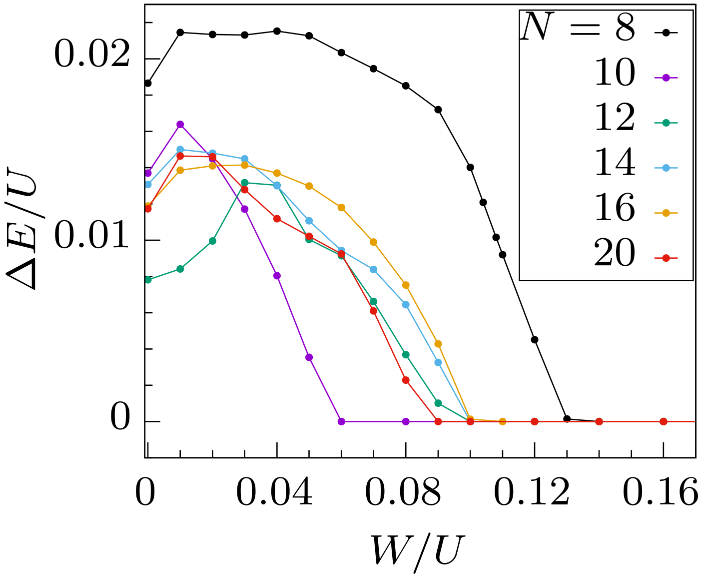
Same number of states below the 'gap'

Role of the band properties (ED results)

Role of the bandwidth

back to realistic $w_{\mathrm{AA}}/w_{\mathrm{AB}} = 0.8$

$W \gg U$ → kills fractional Chern insulators



Many-body gap of spin-polarized FCI

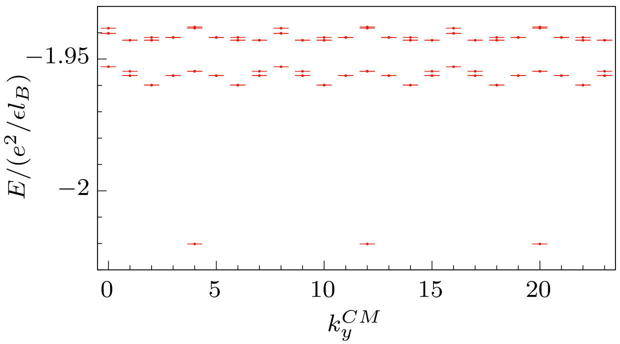
Maximum bandwith for FCI and for spin-polarization $W/U \simeq 0.1$

Degeneracy of quasihole excitations

Add one *e/3* q.h. by adding 1 flux quantum

$N = 8$ electrons

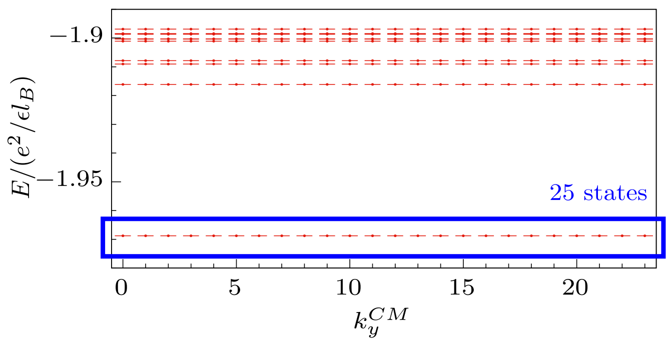
$N_\phi = 24$



$\nu = 1/3$

3 states

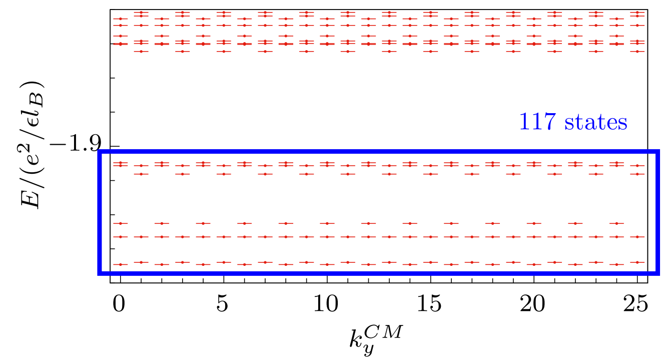
$N_\phi = 25$



+1 q.h.

25 states

$N_\phi = 26$



+2 q.h.

117 states

Q.h. spectrum is gapped
Number of states below the gap is a fingerprint of topological order

Degeneracy of quasihole excitations

<u>*Haldane's fractional exclusion principle :*</u>

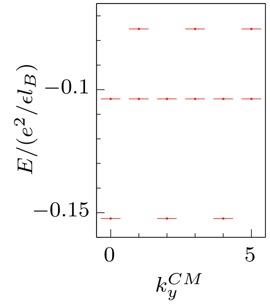
[Haldane PRL '91]

Degeneracy of a q.h. states is given by the number of configurations satisfying the generalized (k, r) Pauli principle : no more than k fermions in r consecutive orbitals.

Example : Laughlin $\nu = 1/3$ state : $(k = 1, r = 3)$

2-fermion ground state on the torus:
1 0 0 1 0 0
0 1 0 0 1 0
0 0 1 0 0 1

3 states

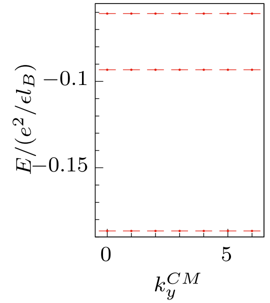


+ 1 orbital
(1 q.hole)

$N_\phi \to N_\phi + 1$

1 0 0 1 0 0 0
0 1 0 0 1 0 0
0 0 1 0 0 1 0
0 0 0 1 0 0 1
1 0 0 0 1 0 0
0 1 0 0 0 1 0
0 0 1 0 0 0 1

7 states



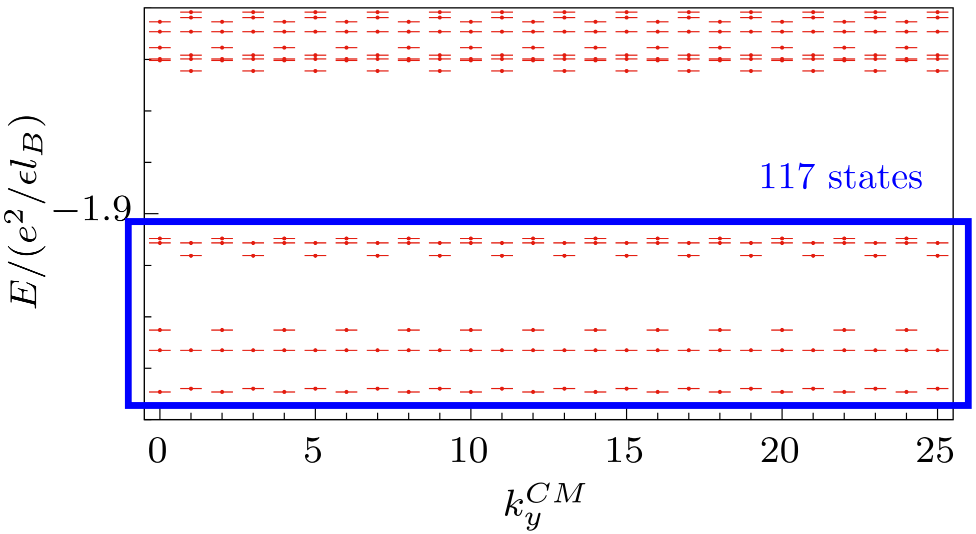
Thin-torus configurations are not FQH eigenstates

Extracting the degeneracy of quasihole excitations

From the energy spectrum

$N = 8$

$N_\phi = 26$



thin-torus configurations of 8 particles in 26 boxes with no more than 1 particle in 3 consecutive boxes : 117