Anisotropic quantum Hall droplets

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Geometric and analytic aspects of the Quantum Hall effect
SwissMAP Research Station in Les Diablerets

May 9, 2023

Joint work arXiv:2301.01726 with
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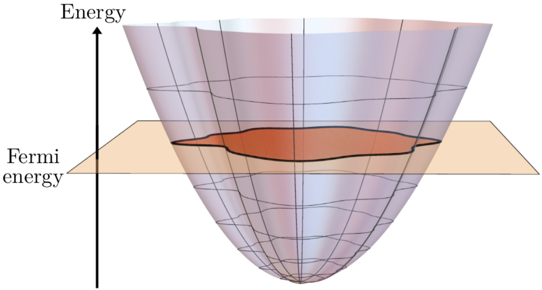
Non-interacting 2D electrons

$$H_{\text{1-body}} = \frac{1}{2M}(\mathbf{p} - q\mathbf{A})^2 + V(\mathbf{x})$$

in strong magnetic field $\mathbf{B} = \mathrm{d}\mathbf{A}$ with anisotropic potential $V(\mathbf{x})$:



Restrict to the lowest Landau level (LLL). Our objectives:

- ◇ One-particle spectrum and wave functions.
- ◇ Many-body observables and effective edge theory.

Class of edge-deformed potentials

Assumption: $V(\mathbf{x})$ is smooth and "monotonous" (i.e., has unique global minimum surrounded by nested level curves).

Can always find an area-preserving diffeomorphism of the plane that sends each level curve on a circle. [Izosimov, Khesin, Mousavi, Ann. Inst. Fourier (2016)]

Important special case in polar coordinates: Edge deformation

$$(r, \varphi) \mapsto \left(\frac{r}{\sqrt{f'(\varphi)}}, f(\varphi) \right)$$

given by orientation-preserving circle diffeomorphism $f(\varphi)$.

Restrict to edge-deformed potentials

$$V(r, \varphi) = V_0 \left(\frac{r^2}{2f'(\varphi)} \right).$$

Setup and eigenvalue problem

Magnetic length

$$\ell \equiv \sqrt{\hbar/qB}.$$

Assumption: $V(\mathbf{x})$ weak relative to $\mathbf{B}$ and slowly varying on the scale ℓ.

LLL-projection of $H_{\text{1-body}}$ yields eigenvalue problem:

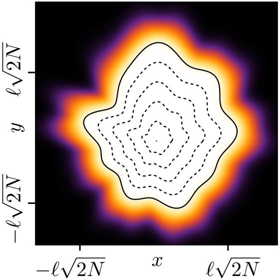
$$PVP|\psi_m\rangle = E_m|\psi_m\rangle.$$

Generally to be solved using semiclassical expansion in ℓ^2.

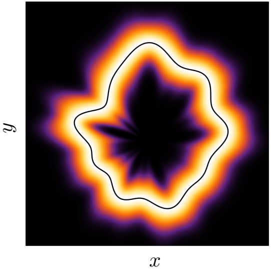
See e.g.: [Trugman, PRB (1983)], [Joynt, Prange, PRB (1984)], [Champel, Florens, PRB (2007)], [Champel, Florens, Canet, PRB (2008)], [Charles, CMP (2003); Commun. Partial. Differ. Equ. (2003)]

Claim: $|\psi_m\rangle$ labeled by large quantum number $m \in \mathbb{N}$, generalizing angular momentum, localized on level curves γ_m of $V(\mathbf{x})$ with area given by the Bohr-Sommerfeld condition $\oint_{\gamma_m} x\,\mathrm{d}y = 2\pi\ell^2 m$.

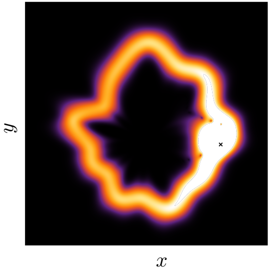
Many-body observables for droplet with N electrons



Density



Current



Correlations

Remark: Chiral inhomogeneous conformal field theory (CFT)

Hamiltonian $(\hbar = 1)$

$$H = \int_{-L/2}^{L/2} \mathrm{d}x \, v(x) T(x)$$

with smooth x-dependent propagation velocity $v(x) = v(x+L) > 0$ and stress-energy-tensor component $T(x) = T(x+L)$ satisfying

$$\left[T(x), T(y)\right] = -2\mathrm{i}\delta'(x-y)T_{\pm}(y) + \mathrm{i}\delta(x-y)T'_{\pm}(y) + \frac{c}{24\pi}\mathrm{i}\delta'''(x-y).$$

[Dubail, Stéphan, Viti, Calabrese, SciPost Phys. (2017)], [Dubail, Stéphan, Calabrese, SciPost Phys. (2017)]
[Gawędzki, Langmann, P.M., J. Stat. Phys. (2018)], [Langmann, P.M., PRL (2019)], [P.M., AHP (2021)]

Diffeomorphism $f(x) = \int_0^x \mathrm{d}y \, \frac{v_0}{v(y)}$ with $\frac{1}{v_0} = \frac{1}{L} \int_{-L/2}^{L/2} \frac{\mathrm{d}y}{v(y)}$ can be used to flatten out $v(x)$:

$$U(f) H U(f)^{-1} = \int_{-L/2}^{L/2} v_0 T(x) + \text{const}$$

using projective unitary representation $U(f)$ on the Hilbert space.

Outline

- Review: Isotropic potentials
- Edge-deformed potentials
- One-particle results
- Many-body observables
- Effective edge theory

Review: Isotropic potentials

Wave functions

For strong **B**, wave functions for isotropic potential $V(\mathbf{x}) = V_0(r^2/2)$ are standard LLL wave functions in symmetric gauge:

$$\phi_m(\mathbf{x}) = \frac{1}{\sqrt{2\pi\ell^2}} \frac{z^m}{\sqrt{m!}} \, \mathrm{e}^{-|z|^2/2}$$

labeled by angular momentum m and expressed in complex coordinate

$$z \equiv \frac{x + \mathrm{i}y}{\sqrt{2}\ell}.$$

$\Longrightarrow$ Gaussian decay away from maximum on circles γ_m: $|z| = \sqrt{m}$.

LLL-projection operator $P \equiv \sum_{m=0}^{\infty} |\phi_m\rangle\langle\phi_m|$ with kernel

$$\langle z, \bar{z}|P|w, \bar{w}\rangle = \frac{1}{2\pi\ell^2} \, \mathrm{e}^{-(|z|^2+|w|^2)/2} \, \mathrm{e}^{z\bar{w}}.$$

One-particle spectrum

Exact eigenvalues

$$E_m = \langle \phi_m | V | \phi_m \rangle = \frac{1}{m!} \int_0^\infty \mathrm{d}t\, t^m\, \mathrm{e}^{-t}\, V_0(\ell^2 t).$$

By saddle-point approximation for $m \gg 1$ keeping $\ell^2 m$ fixed:

$$E_m = V_0(\ell^2 m) + \ell^2 \left(\Omega_m + \frac{\Gamma_m}{2} \right) + O(\ell^4)$$

with $\Omega_m \equiv V_0'(\ell^2 m) > 0$ and $\Gamma_m \equiv \ell^2 m\, V_0''(\ell^2 m)$.

$\Longrightarrow$ Example of semiclassical expansion in ℓ^2.

1-body density matrix

Consider a droplet of N electrons:

$$C(\mathsf{x}_1, \mathsf{x}_2) \equiv \sum_{m=0}^{N-1} \overline{\phi_m(\mathsf{x}_1)}\phi_m(\mathsf{x}_2) = \frac{1}{2\pi\ell^2}\frac{\Gamma(N, \bar{z}_1 z_2)}{\Gamma(N)}\mathrm{e}^{-(|z_1|^2+|z_2|^2)}\mathrm{e}^{\bar{z}_1 z_2}$$

using the upper incomplete Gamma function $\Gamma(s, x) = \int_x^\infty \mathrm{d}t\, t^{s-1}\mathrm{e}^{-t}$:

$$\Gamma(N, x) = \Gamma(N)\mathrm{e}^{-x}\sum_{m=0}^{N-1}\frac{x^m}{m!} \quad \text{for integer } s = N.$$

Restricting x_1 and x_2 to the edge, i.e., $z_j = \sqrt{N}\mathrm{e}^{\mathrm{i}\theta_j}$, yields correlations

$$C(\mathsf{x}_1, \mathsf{x}_2) \sim \frac{\mathrm{i}}{2\sin([\theta_1 - \theta_2]/2)}$$

by saddle-point analysis. [Cappelli, Dunne, Trugenberger, Zemba, Nucl. Phys. B (1993)]

Edge-deformed potentials

Action-angle coordinates

Recall

$$V(r,\varphi) \equiv V_0\left(\frac{r^2}{2f'(\varphi)}\right).$$

Canonical action-angle coordinates

$$\big(\ell^2 K,\, \theta\big) = \big(r^2/2f'(\varphi),\; f(\varphi)\big)$$

so $V(r,\varphi) = V_0(\ell^2 K)$. Dim.less action $K\big|_{\gamma_m} = m$ on level curves γ_m.

Cartesian coordinates

$$x = \sqrt{\frac{2\ell^2 K}{(f^{-1})'(\theta)}}\,\cos(f^{-1}(\theta)) \equiv X(K,\theta),$$

$$y = \sqrt{\frac{2\ell^2 K}{(f^{-1})'(\theta)}}\,\sin(f^{-1}(\theta)) \equiv Y(K,\theta).$$

WKB ansatz

Quantum state perfectly localized on γ_m:

$$|\Psi_m\rangle \equiv 2\pi\ell^2 \oint \mathrm{d}\theta\, n(\theta)\, \mathrm{e}^{\mathrm{i}m\theta} \big| X(m,\theta), Y(m,\theta) \big\rangle$$

for unknown complex function $n(\theta)$ satisfying $n(2\pi) = n(0)$.

LLL-projected wave functions:

$$\psi_m(z,\bar{z}) \equiv \langle z, \bar{z}|P|\Psi_m\rangle$$

that after change of variable to $\varphi = f^{-1}(\theta)$ yields

$$\psi_m(z,\bar{z}) = \mathrm{e}^{-|z|^2/2} \oint \mathrm{d}\varphi\, f'(\varphi)\, n(f(\varphi))\, \mathrm{e}^{\mathrm{i}mf(\varphi) - \frac{1}{2}mf'(\varphi) + z\sqrt{mf'(\varphi)}\,\mathrm{e}^{-\mathrm{i}\varphi}}.$$

$\Longrightarrow$ Gaussian decay away from maximum on level curves γ_m.

Analogy with standard WKB theory

Recall standard WKB ansatz:

$$\psi \sim \mathrm{e}^{\mathrm{i}\left[\frac{1}{\hbar}S_0+S_1+O(\hbar)\right]}$$

with $S_1 = \operatorname{Re} S_1 + \mathrm{i} \operatorname{Im} S_1$, where $\operatorname{Re} S_1 = O(\hbar)$ can be omitted.

First-order quantum correction S_1 given by transport equation:

$$2\nabla(\mathrm{i}S_1)\cdot\nabla(\mathrm{i}S_0) + \nabla^2(\mathrm{i}S_0) = 0.$$

Compare with our ansatz:

$$\mathrm{e}^{\mathrm{i}mf(\varphi)} \leftrightarrow \mathrm{e}^{\mathrm{i}\frac{1}{\hbar}S_0}, \qquad n(f(\varphi)) \leftrightarrow \mathrm{e}^{\mathrm{i}S_1} = \mathrm{e}^{-\operatorname{Im} S_1 + O(\hbar)}.$$

Projected eigenvalue equation

Recall

$$PVP|\psi_m\rangle = E_m|\psi_m\rangle$$

where

$$E_m = E_m^0 + \ell^2 E_m^1 + O(\ell^4)$$

with $E_m^0 = V(\gamma_m) = V_0(\ell^2 m)$ and unknown E_m^1.

Q: How to understand PVP?

A: Semiclassical expansion of Berezin-Toeplitz operator

$$\langle z,\bar{z}|PVP|w,\bar{w}\rangle \overset{\ell\ll 1}{\sim} \frac{1}{2\pi\ell^2}\,\mathrm{e}^{-\frac{|z-w|^2}{2}}\,\mathrm{e}^{\frac{z\bar{w}-\bar{z}w}{2}}\left[\mathcal{V}(z,\bar{w}) + \frac{\ell^2}{2}(\nabla^2 V)(z,\bar{w})\right]$$

with $\mathcal{V}(z,\bar{z}) \equiv V(x,y)$ and $(\nabla^2 V)(z,\bar{w}) = \frac{4}{2\ell^2}\partial_z\partial_{\bar{w}}\mathcal{V}$.

[Charles, CMP (2003); Commun. Partial. Differ. Equ. (2003)]

Projected eigenvalue equation

Recall

$$PVP|\psi_m\rangle = E_m|\psi_m\rangle$$

where

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Projected eigenvalue equation expanded in ℓ^2

Problem: Find $n(f(\varphi))$ and E_m^1 solving

$$0 = \oint \mathrm{d}\varphi\, f'(\varphi)\, n(f(\varphi))\, \mathrm{e}^{imf(\varphi) - \frac{1}{2}mf'(\varphi) + z\sqrt{mf'(\varphi)}\,\mathrm{e}^{-\mathrm{i}\varphi}} \times \left(\mathcal{V}(z,\bar{w}) + \frac{\ell^2}{2}\nabla^2 V(z,\bar{w}) - E_m^0 - \ell^2 E_m^1 \right)\Bigg|_{\bar{w}=\sqrt{mf'(\varphi)}\,\mathrm{e}^{-\mathrm{i}\varphi}}$$

for all z in the complex plane.

One-particle results

Projected eigenvalue equation expanded in ℓ^2

Recall

$$0 = \oint \mathrm{d}\varphi\, f'(\varphi)\, n(f(\varphi))\, \mathrm{e}^{\mathrm{i}mf(\varphi) - \frac{1}{2}mf'(\varphi) + z\sqrt{mf'(\varphi)}\,\mathrm{e}^{-\mathrm{i}\varphi}} \times \left(\mathcal{V}(z,\bar{w}) + \frac{\ell^2}{2}\nabla^2 V(z,\bar{w}) - E_m^0 - \ell^2 E_m^1 \right) \Bigg|_{\bar{w}=\sqrt{mf'(\varphi)}\,\mathrm{e}^{-\mathrm{i}\varphi}}$$

for all z.

Strategy: Evaluate for $z = \sqrt{mf'(\alpha)}\,\mathrm{e}^{\mathrm{i}\alpha}$, $\alpha \in [0, 2\pi)$ on γ_m for $m \gg 1$ and use holomorphicity to infer that end result is valid for all z.

Result: Transport equation for $n(f(\varphi)) = \mathcal{N}(\varphi)\,\mathrm{e}^{\mathrm{i}\Phi(\varphi)}$ split into real and imaginary parts (obtained by saddle-point analysis).

Real part of transport equation

ODE for the phase $\Phi(\varphi)$:

$$\Phi'(\varphi) = \frac{E_m^1}{\Omega_m} f'(\varphi) - \frac{1}{2}\left(\frac{\Gamma_m}{\Omega_m} + 1\right)\left(1 + \frac{f''(\varphi)^2}{4f'(\varphi)^2}\right) - \frac{1}{2} + \partial_\varphi\left(\frac{f''(\varphi)}{8f'(\varphi)}\right) + \frac{1}{2}\partial_\varphi \arctan\left(\frac{f''(\varphi)}{2f'(\varphi)}\right)$$

with $\Omega_m \equiv V_0'(\ell^2 m) > 0$ and $\Gamma_m \equiv \ell^2 m\, V_0''(\ell^2 m)$.

Result: First-order quantum correction to energies

$$E_m^1 = \frac{\Omega_m}{2} + \frac{\Gamma_m + \Omega_m}{2} \oint \frac{\mathrm{d}\varphi}{2\pi}\left(1 + \frac{f''(\varphi)^2}{4f'(\varphi)^2}\right).$$

Obtained by integrating the ODE and using $\Phi(2\pi) = \Phi(0)$.

Imaginary part of transport equation

ODE for the norm $\mathcal{N}(\varphi)$:

$$\frac{\mathcal{N}'(\varphi)}{\mathcal{N}(\varphi)} = \frac{1}{4}\partial_\varphi \log\left[\frac{1}{f'(\varphi)}\left(1+\frac{f''(\varphi)^2}{4f'(\varphi)^2}\right)\right].$$

Result: First-order quantum correction $n(f(\varphi)) = \mathcal{N}(\varphi)\,\mathrm{e}^{\mathrm{i}\Phi(\varphi)}$ with

$$\mathcal{N}(\varphi) = N_0\left[\frac{1}{f'(\varphi)}\left(1+\frac{f''(\varphi)^2}{4f'(\varphi)^2}\right)\right]^{1/4} = N_0\sqrt{\sigma(\varphi)}$$

using the width

$$\sigma(\varphi)^2 \equiv \frac{1}{f'(\varphi)}\left(1+\frac{f''(\varphi)^2}{4f'(\varphi)^2}\right)$$

for some normalization N_0.

Gaussian wave functions

Recall our WKB ansatz for $m \gg 1$:

$$\psi_m(z,\bar{z}) = \mathrm{e}^{-|z|^2/2} \oint \mathrm{d}\varphi\, f'(\varphi)\, n(f(\varphi))\, \mathrm{e}^{\mathrm{i}mf(\varphi)-\frac{1}{2}mf'(\varphi)+z\sqrt{mf'(\varphi)}\,\mathrm{e}^{-\mathrm{i}\varphi}}$$

with maximum on level curves γ_m.

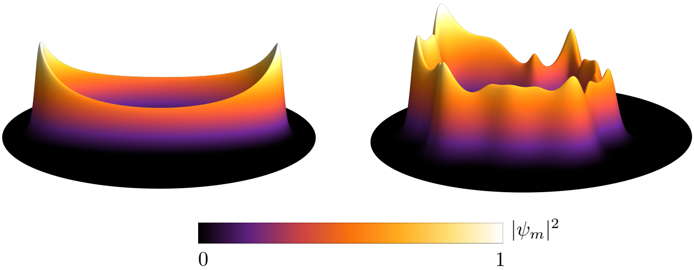
By saddle-point approximation for $z = \left(\sqrt{m}+a\right)\sqrt{f'(\alpha)}\,\mathrm{e}^{\mathrm{i}\alpha}$ near γ_m:

$$\psi_m(z,\bar{z}) \sim \frac{\mathrm{e}^{\mathrm{i}\Theta_m(\mathbf{x})}}{\sqrt{2\pi\ell^2\sigma(\alpha)}} \frac{\mathrm{e}^{-a^2/\sigma(\alpha)^2}}{(2\pi m)^{1/4}}$$

with phase $\Theta_m(\mathbf{x}) = mf(\alpha) + \Phi(\alpha) - \dfrac{a^2 f''(\alpha)}{2f'(\alpha)\sigma(\alpha)^2}$.

Normalization $N_0 = \frac{1}{2\pi\ell}\left(\frac{m}{2\pi}\right)^{1/4}$ so that $\int \mathrm{d}^2\mathbf{x}|\psi_m|^2 = 1$.

Plots of $|\psi_m|^2$



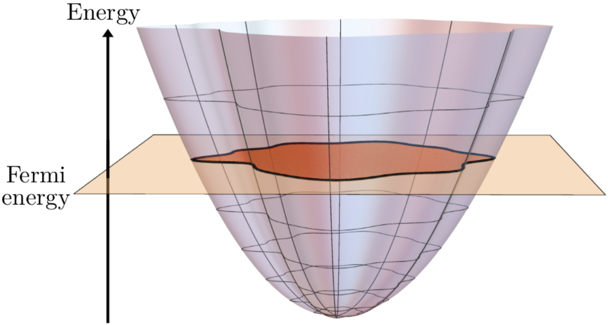
For elliptic harmonic potential

For potential in previous figs.

Many-body observables

Quantum Hall droplet

Consider droplet with N electrons in an anisotropic potential:



Many-body density

Each electron contributes a single-particle probability density $|\psi_m(\mathbf{x})|^2$:

$$\rho(\mathbf{x}) = \sum_{m=0}^{N-1} |\psi_m(\mathbf{x})|^2 \sim \frac{1}{(2\pi)^{3/2}\ell^2} \sum_{m=0}^{N-1} \frac{\mathrm{e}^{-2a_m^2/\sigma(\alpha)^2}}{\sigma(\alpha)\sqrt{m}}$$

with $a_m = \frac{|z|}{\sqrt{f'(\alpha)}} - \sqrt{m}$.

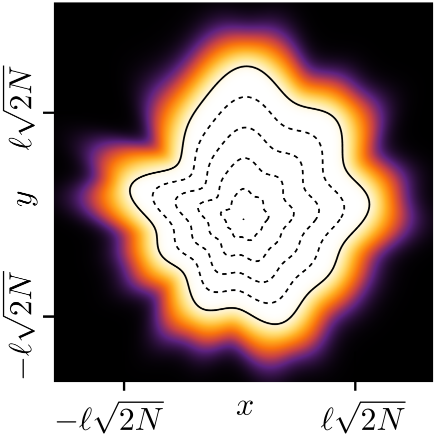
Parametrize $|z| = \sqrt{f'(\alpha)}(\sqrt{m_z} + a)$ for $m_z \gg 1$ and finite a, change summation to $k = m_z - m$, and use Euler-Maclaurin formula. Result:

$$\rho(\mathbf{x}) \sim \frac{1}{(2\pi)^{3/2}\ell^2} \sum_{k=m_z+1-N}^{m_z} \frac{\mathrm{e}^{-\frac{2}{\sigma(\alpha)^2}\left(a+k/2\sqrt{m_z}\right)^2}}{\sigma(\alpha)\sqrt{m_z}}$$

$$\sim \frac{1}{4\pi\ell^2}\,\mathrm{erfc}\left(\frac{\sqrt{2}}{\sigma(\alpha)}\left[a + \frac{m_z+1-N}{2\sqrt{m_z}}\right]\right)$$

constant $\frac{1}{2\pi\ell^2}$ in the bulk $m_z \ll N$ and decays near the edge $m_z \sim N$.

Plot of $\rho(\mathbf{x})$



Many-body current

Correspondingly:

$$\mathbf{J} = \sum_{m=0}^{N-1} \mathbf{j}_m, \qquad \mathbf{j} = \frac{1}{2\mathrm{i}}\left(\psi^* \mathrm{d}\psi - \psi \mathrm{d}\psi^* - 2\mathrm{i}\frac{q}{\hbar}\mathbf{A}|\psi|^2\right).$$

Work in symmetric gauge $\mathbf{A} = \frac{1}{2}Br^2\,\mathrm{d}\varphi = \frac{\hbar}{q}|z|^2\,\mathrm{d}\varphi$, let $|z| = \left(\sqrt{N} + a\right)\sqrt{f'(\varphi)}$, and proceed as before. Result: Near the edge,

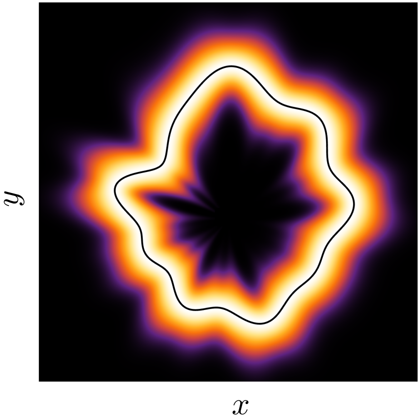
$$\mathbf{J}(r,\varphi) \sim -\frac{\mathrm{e}^{-\frac{2a^2}{\sigma(\varphi)^2}}}{(2\pi\ell^2)^{3/2}\sigma(\varphi)} \frac{\ell\sqrt{2Nf'(\varphi)}\,\mathrm{d}\varphi + \frac{f''(\varphi)}{2f'(\varphi)}\,\mathrm{d}r}{\sqrt{2f'(\varphi)}}$$

for $a = \left(r - \ell\sqrt{2Nf'(\varphi)}\right)/\ell\sqrt{2f'(\varphi)}$.

Remarks:

- $\mathbf{J}(r,\varphi)$ is tangent to the droplet's edge (to leading order).
- $\mathbf{J}(r,\varphi)$ satisfies $\nabla \cdot \mathbf{J}(r,\varphi) = 0 + O(1/N)$.

Plot of $\mathbf{J}(\mathbf{x})$



Correlations

1-body density matrix

$$C(\mathsf{x}_1,\mathsf{x}_2) \equiv \sum_{m=0}^{N-1} \overline{\psi_m(\mathsf{x}_1)}\psi_m(\mathsf{x}_2).$$

Zoom in near the droplet's edge:

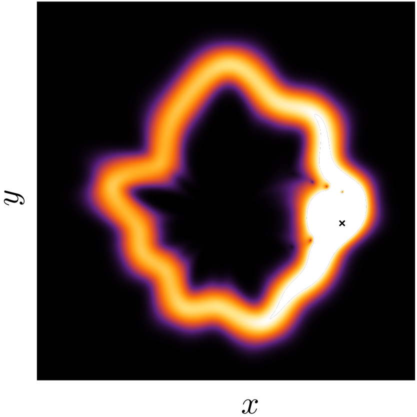
$$z_1 = \big(\sqrt{N}+a\big)\sqrt{f'(\varphi_1)}\,\mathrm{e}^{\mathrm{i}\varphi_1}, \qquad z_2 = \big(\sqrt{N}+b\big)\sqrt{f'(\varphi_2)}\,\mathrm{e}^{\mathrm{i}\varphi_2}.$$

Result: Near the edge,

$$C(\mathsf{x}_1,\mathsf{x}_2) \sim \frac{\mathrm{e}^{\mathrm{i}\Theta_N(\mathsf{x}_1,\mathsf{x}_2)}}{(2\pi)^{3/2}\ell^2\sqrt{N}}\,\frac{1}{\sqrt{\sigma(\varphi_1)\sigma(\varphi_2)}}\,\frac{\mathrm{i}\,\mathrm{e}^{-\frac{a^2}{\sigma(\varphi_1)^2}-\frac{b^2}{\sigma(\varphi_2)^2}}}{2\sin\big([f(\varphi_1)-f(\varphi_2)]/2\big)}$$

with phase $\Theta_N(\mathsf{x}_1,\mathsf{x}_2) = \Theta_N(\mathsf{x}_2) - \Theta_N(\mathsf{x}_1)$.

Plot of $|C(\mathbf{x}_1, \mathbf{x}_2)|$



$\mathbf{x}_2 = (x, y)$ and $\mathbf{x}_1 = (\ell\sqrt{2Nf'(0)}, 0)$ on the edge

Effective edge theory

Correlation functions

For correlations of effective edge excitations, need certain double radial integral of the 2(+1)D result $C(\mathsf{x}_1, \mathsf{x}_2)$ yielding 1+1D correlations

$$\frac{1}{2\pi} \frac{\mathrm{i}}{2 \sin\big([\theta_1 - \theta_2]/2\big)}$$

in terms of $\theta_1 = f(\varphi_1)$ and $\theta_2 = f(\varphi_2)$.

Agrees with correlations of 1+1D free fermion CFT

$$H = \hbar \omega_{\mathsf{F}} \oint \mathrm{d}\theta \, \Psi^\dagger(\theta) \, (-\mathrm{i}\partial_\theta) \Psi(\theta)$$

with $\{\Psi(\theta_1), \Psi^\dagger(\theta_2)\} = \delta(\theta_1 - \theta_2)$ and angular Fermi velocity $\omega_{\mathsf{F}} \equiv \dfrac{\ell^2 \Omega_N}{\hbar}$ from linearizing the dispersion relation near the edge.

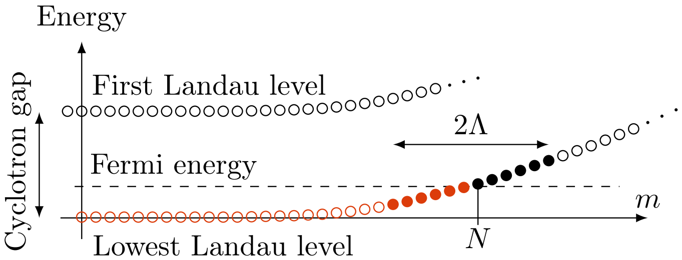
Different from inhomogeneous CFT which has φ-dependent prefactors also in the 1+1D correlations.

One-body Hamiltonian $H_{\text{1-body}} - \mu$ in second-quantized form:

$$H = \sum_{n,m \geq 0} (E_{n,m} - \mu) a^{\dagger}_{n,m} a_{n,m}$$

with

- $E_{n,m}$ one-particle eigenvalues and $\{a_{n,m}, a^{\dagger}_{n',m'}\} = \delta_{n,n'}\delta_{m,m'}$,
- Landau-level index $n \in \mathbb{N}$,
- action-variable quantum number $m \in \mathbb{N}$.



Derivation of the edge theory II

Observation: Low-energy excitations lie in LLL with approx. linear dispersion within window $[-\Lambda, \Lambda]$ around the Fermi momentum p_{F}.

Steps:

- Define $\epsilon(p) \equiv E_{0,p-1/2} = V_0(\ell^2 p) + \mathcal{O}(\ell^2)$ for $p \in \mathbb{N} + 1/2$.
- Introduce $p_{\mathsf{F}} \sim N$ defined by $\epsilon(p_{\mathsf{F}}) = \mu$.
- Shift $p \to p_{\mathsf{F}} + p$.

Then $\epsilon(p_{\mathsf{F}} + p) - \mu \sim \hbar\omega_{\mathsf{F}} p$ for $p \in \mathbb{Z} + 1/2$ so that $|p| \leq \Lambda$, and thus

$$H \sim \sum_{p \in [-\Lambda, \Lambda]} \hbar\omega_{\mathsf{F}}\, p :a_p^\dagger a_p:$$

with $a_p^{(\dagger)} \equiv a_{0,N+p-1/2}^{(\dagger)}$ satisfying $\{a_p, a_{p'}^\dagger\} = \delta_{p,p'}$.

Derivation of the edge theory III

Provided Λ is kept finite while taking $N \to \infty$, one can write

$$a_p^\dagger = \oint \frac{f'(\varphi)\,\mathrm{d}\varphi}{\sqrt{2\pi}}\,\mathrm{e}^{\mathrm{i}pf(\varphi)}\,\Psi^\dagger(f(\varphi))$$

for an 'edge field' $\Psi^\dagger(\theta)$ for $\theta = f(\varphi)$ (independent of p). It is given by a radial integral of 2D creation operators $c^\dagger(\mathsf{x})$ weighted by $\psi_N(\mathsf{x})$:

$$\Psi^\dagger(f(\varphi)) \equiv \frac{\sqrt{2\pi}}{f'(\varphi)}\mathrm{e}^{-\mathrm{i}f(\varphi)/2}\int_0^\infty r\,\mathrm{d}r\,c^\dagger(\mathsf{x})\,\psi_N(\mathsf{x}).$$

Remarks:

- $\{c(\mathsf{x}_1), c^\dagger(\mathsf{x}_2)\} = \delta^{(2)}(\mathsf{x}_1 - \mathsf{x}_2) \implies \{\Psi(\theta_1), \Psi^\dagger(\theta_2)\} = \delta(\theta_1 - \theta_2)$.
- Half-integer labels imply antiperiodic BCs for $\Psi^{(\dagger)}(\theta)$.

Removing the cutoff, $\Lambda \to \infty$, yields $H_{\mathsf{eff}} = \sum_p \hbar\omega_{\mathsf{F}} p :a_p^\dagger a_p:$ with $p \in \mathbb{Z} + 1/2$, or equivalently $H_{\mathsf{eff}} = \hbar \oint \mathrm{d}\theta\ :\Psi^\dagger(\theta)\,(-\mathrm{i}\omega_{\mathsf{F}}\partial_\theta)\Psi(\theta):$.

Summary

- ◇ Obtained one-particle spectrum and wave functions from solving transport equation derived from WKB theory.
- ◇ Computed many-body observables.
- ◇ Derived effective edge theory.

Thank you for your attention!