WORKSHOP "ANALYTIC TECHNIQUES IN DYNAMICS AND GEOMETRY" SCHEDULE

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00–10:00	Pollicott	Hintz	Faure	Paternain	Weich
10:00–10:30	coffee/tea	coffee/tea	coffee/tea	coffee/tea	coffee/tea
10:30-11:30	Forni	Nonnenmacher	Tsujii	Stefanov	Li
11:30–12:30	Giulietti	Vasy	Zhang	Guillarmou	Chaubet
12:30–14:00	lunch	lunch	lunch	lunch	lunch
14:00–16:00	break	break	break (hike)	break	
16:00–16:30	coffee/tea	coffee/tea	coffee/tea (hike)	coffee/tea	
16:30–17:30	Burns	Macià	free (hike)	Oksanen	
17:30–18:30	Rivière	T. de Lizaur	free (hike)	Tzou	
19:00–20:30	dinner	dinner	dinner	raclette and wine	
20:30–21:30			open problems		

Titles and Abstracts

Mark Pollicott (University of Wawrick): "Zeta functions for Anosov diffeomorphisms, flows and actions".

Abstract: We will review some of the classical approaches to zeta functions for Anosov diffeomorphisms and Anosov flows (and their metamorphic extensions). We will discuss some extensions to Anosov actions and related settings.

Giovanni Forni (University of Maryland): "Renormalization for nilflows and Weyl sums".

Abstract: We will review results on effective equidistribution of nilflows from the self-similar (horospherical) case to "non-renormalizable" examples related to higher dimensional toral skew-shifts. Effective equidistribution of such flows is related to bounds on Weyl sums in analytic number theory. We will explain how dynamical ideas based on renormalization can recover most of the results on Weyl sums. The analytical tools are based on unitary representation theory for nilpotent Lie groups.

Paolo Giulietti (University of Pisa): "Random-like properties of chaotic forcing".

Abstract: We prove that skew systems with a sufficiently expanding base have approximate exponential decay of correlations, meaning that the exponential rate is observed modulo an error. The fiber maps are only assumed to be Lipschitz regular and to depend on the base in a way that guarantees diffusive behavior on the vertical component. The assumptions do not imply an hyperbolic picture and one cannot rely on the spectral properties of the transfer operators involved. The approximate nature of the result is the inevitable price one pays for having so mild assumptions on the dynamics on the vertical component. However, the error in the approximation goes to zero when the expansion of the base tends to infinity. The result can be applied beyond the original setup when combined with acceleration or conjugation arguments, as our examples show. Joint work with M. Tanzi (LPSM) and S. Marmi (SNS).

Keith Burns (Northwestern University): "Uniqueness of the measure of maximal entropy for geodesic flows on surfaces with caps".

Abstract: The class of surfaces in this talk was introduced in the 1980s by Donnay in order to exhibit a smooth Riemannian metric on the two sphere with ergodic geodesic flow with respect to the smooth Liouville measure. Recent joint work with Todd Fisher and Rachel McEnroe has shown that the geodesic flows for these surfaces have unique (and therefore ergodic) measures of maximal entropy.

Gabriel Rivière (University of Nantes): "Poincaré series and linking of Legendrian knots".

Abstract: On a negatively curved compact surface, I will explain that Poincaré series associated with geodesic arcs joining two given points have a meromorphic extension to the whole complex plane. I will also show that the value at 0 is given by the inverse of the Euler characteristic by interpreting it as the linking of two Legendrian knots and that these

results remain true more generally for geodesic arcs that are orthogonal to two given closed geodesics of the surface. This is a joint work with N.V. Dang.

Peter Hintz (ETH): "Mode stability and shallow quasinormal modes of Kerr-de Sitter black holes".

Abstract: The Kerr-de Sitter metric describes a rotating black hole with mass m and specific angular momentum a in a universe, such as our own, with cosmological constant $\Lambda > 0$. I will explain a proof of mode stability for the scalar wave equation on Kerr-de Sitter spacetimes in the following setting: fixing Λ and the ratio $|a/m| < 1$ (related to the subextremality of the black hole in question), mode stability holds for sufficiently small black hole mass m. We also obtain estimates for the location of quasinormal modes (resonances) σ in any fixed half space $\Im(\sigma) > -C$. Our results imply that solutions of the wave equation decay exponentially in time to constants, with an explicit exponential rate. The proof is based on careful uniform estimates for the spectral family in the singular limit $m \to 0$ in which, depending on the scaling, the Kerr-de Sitter spacetime limits to a Kerr or the de Sitter spacetime.

Stéphane Nonnenmacher (University of Paris-Saclay): "Delocalization of the Laplace eigenmodes on Anosov surfaces".

Abstract: The eigenmodes of the Laplace-Beltrami operator on a smooth compact Riemannian manifold (M, g) can exhibit various localization properties in the high frequency regime, which strongly depend on the properties of the geodesic flow. We will focus on situations where this flow is Anosov, e.g. if the sectional curvature of (M, g) is negative. The Quantum Ergodicity theorem then states that almost all the eigenmodes become equidistributed on M in the the high frequency limit.

The Quantum Unique Ergodicity conjecture claims that this behaviour admits no exception. This conjecture remaining inaccessible, a less ambitious goal is to constrain the possible departure from equidistribution.

I will present a recent progress in the case of surfaces with Anosov geodesic flow: we pave that all the eigenmodes fully elocalize across all of M in the high frequency limit: for any open set Ω on M, the L^2 mass on Ω of all eigenstates is uniformly bounded from below. This is in contrast with, e.g., the case of the round sphere.

The proof, which generalizes a previous work by Dyatlov-Jin in the constant curvature case, uses various methods of semiclassical analysis, the structure of stable and unstable manifolds, and a recent Fractal Uncertainty Principle due to Bourgain-Dyatlov.

Joint work with S. Dyatlov and L. Jin.

Andras Vasy (Stanford University): "The Feynman propagator and self-adjointness".

Abstract: In this talk I will discuss the Feynman and anti-Feynman inverses for wave operators on certain Lorentzian manifolds; these are two inverses which from a microlocal analysis perspective are more natural than the standard causal (advanced/retarded) ones. For instance, for the spectral family of the wave operator, these are the natural inverses when the spectral parameter is non-real. Indeed, I will explain that these connect to the self-adjointness of the wave operator, and the positivity properties that follow.

Fabricio Macià (Universidad Politécnica de Madrid): "Factorization of the Calderón problem through the Born approximation".

Abstract: We address the problem of reconstructing a scalar conductivity from the Dirichlet-to-Neumann map on the boundary of a domain in Euclidean space (the reconstruction aspect of the Calderón problem). It is well-known that, under suitable assumptions on the conductivity, this problem can be reduced to the analysis and reconstruction of the potential of a Schrödinger operator $\Delta + V$ on the sphere. This problem is rather involved in general, from both the analytical and numerical points of view. Here we introduce an object that is obtained in terms of certain matrix elements of the Dirichlet-to-Neumann map – the Born approximation – which is reminiscent of an approximation for the potential that has been extensively studied in the context of inverse scattering theory. We will show a number of interesting analytical properties of the Born approximation, in particular how it can approximate in a suitable sense the potential in the Calderón problem and recover its singularities. This is based on joint works with Juan Antonio Barceló, Carlos Castro, Thierry Daudé, Cristóbal Meroño, François Nicoleau and Daniel Sánchez-Mendoza.

Francisco Torres de Lizaur (University of Sevilla): "Quasiperiodicity and chaos in the incompressible Euler equation".

Abstract: The incompressible Euler equation is a nonlinear PDE describing the motion of an inviscid, incompressible fluid. It can be viewed as a first order ODE on the infinite dimensional space of divergence-free vector fields in the fluid domain. In this talk I will review recent results on existence and dynamics of finite dimensional invariant manifolds of the Euler equation. These are families of divergence-free fields, parametrized by some manifold N (for example, a torus), with the property that the solutions of the Euler equation with initial condition in the family exist and remain there for all time, defining a finite-dimensional ODE on N. Based on joint work with A. Enciso and D. Peralta-Salas.

Frédéric Faure (Institut Fourier, Université de Grenoble Alpes): "Some relationship between classical and quantum dynamics".

Abstract: In geometric quantization (Kostant Souriau 70'), we start from a Hamiltonian dynamical system. We consider (1) its contact R-extension which is a naturally defined dynamical system (called pre-quantum). The step (2) called quantization consists in projecting it on the quotient space by an arbitrary non-invariant Lagrangian integrable distribution (called polarization). This construction gives the Schrödinger equation in physics but it is not unique. In this talk, it will be shown and discussed that for an Anosov geodesic flow, the pre-quantum dynamics already naturally and dynamically defines a quantization where the polarization corresponds to the stable invariant distribution. Work in collaboration with Masato Tsujii.

Masato Tsujii (Kyushu University): "Virtually expanding dynamics".

Abstract: We propose a class of discrete dynamical systems that we call virtually expanding. This is an open subset of self-covering maps on a closed manifold which contains all expanding maps and some partially hyperbolic volume-expanding maps. We show that the Perron-Frobenius operator is quasi-compact on a Sobolev space of positive order for such class of

dynamical systems. We conjecture that generic volume-expanding maps on a closed manifold are virtually expanding.

Zhiyuan Zhang (CNRS, LAGA - Université Paris 13 Nord): "Prevalence of exponential mixing of Anosov flows".

Abstract: We show that for a dense set of transitive Anosov flow, the flow is exponentially mixing with respect to its unique SRB measure. We will show that this set is prevalent in certain sense. We will also discuss the situation for other equilibrium measures. This is a joint work with Masato Tsujii.

Gabriel P. Paternain (University of Cambridge): "Resonant forms at zero for dissipative Anosov flows".

Abstract: The Ruelle zeta function is a natural function associated with the periods of closed orbits of an Anosov flow, and it is known to have a meromorphic extension to the whole complex plane. The order of vanishing of the Ruelle zeta function at zero is expected to carry interesting topological and dynamical information and can be computed in terms of certain resonant spaces of differential forms for the action of the Lie derivative on suitable spaces with anisotropic regularity. In this talk I will explain how to compute these resonant spaces for any transitive Anosov flow in 3D, with particular emphasis in the dissipative case, that is, when the flow does not preserve any absolutely continuous measure. A prototype example is given by the geodesic flow of an affine connection with torsion and we shall see that for such a flow the order of vanishing drops by 1 in relation to the usual geodesic flow due to the Sinai-Ruelle-Bowen measure having non-zero winding cycle. This is joint work with Mihajlo Cekić.

Plamen Stefanov (Purdue University): "The Lorentzian scattering rigidity problem and rigidity of stationary metrics".

Abstract: We study scattering rigidity in Lorentzian geometry: recovery of a Lorentzian metric from the scattering relation known on a lateral boundary. We show that, under a non-conjugacy assumption, every defining function $r(x, y)$ of the submanifold of pairs of boundary points which can be connected by a lightlike geodesic plays the role of the boundary distance function in the Riemannian case in the following sense. Its linearization is the light ray transform of tensor fields of order two which are the perturbations of the metric. Next, we study scattering rigidity of stationary metrics in time-space cylinders and show that it can be reduced to boundary rigidity of magnetic systems on the base; a problem studied previously. This implies several scattering rigidity results for stationary metrics.

Colin Guillarmou (CNRS, University of Paris-Saclay): "Marked length spectrum rigidity for surfaces with Anosov geodesic flows".

Abstract: we explain how to prove that the marked length spectrum (MLS) of Riemannian closed surfaces with Anosov geodesic flow determines the isometry class of the surface. This extends Otal result in negative curvature to the class of Anosov flows in dimension 2. The proof relies on the determination of the complex structure from the MLS, via the properties of invariant distributions for the flow. This is joint work with Lefeuvre and Paternain.

Lauri Oksanen (University of Helsinki): "Lorentzian Calderón problem under curvature bounds".

Abstract: We introduce a method of solving inverse boundary value problems for wave equations on Lorentzian manifolds, and show that zeroth order coefficients can be recovered under certain curvature bounds. The set of Lorentzian metrics satisfying the curvature bounds has a non-empty interior in the sense of sufficiently smooth perturbations of the metric, and the interior contains the Minkowski metric. On the contrary, all previous results on this problem impose conditions on the metric that force it to be real analytic with respect to a suitably defined time variable. The analogous problem on Riemannian manifolds is called the Calderón problem, and in this case the known results require the metric to be independent of one of the variables. In particular, the Riemannian version of the problem is open near the Euclidean metric. Our approach is based on a new unique continuation result in the exterior of the double null cone emanating from a point. The approach shares features with the classical Boundary Control method, and can be viewed as a generalization of this method to cases where no real analyticity is assumed.

The talk is based on joint work with Spyros Alexakis and Ali Feizmohammadi.

Leo Tzou (University of Amsterdam): "Geodesic Levy Flight and Foraging Hypothesis"

Abstract: The Levy flight hypothesis postulates that Levy flight search patterns are more efficient than searches based on Brownian motions. However, in this talk I will present some recent results suggesting that this may not always be the case.

These insights are based on obtaining detailed properties of the infinitesimal generators for geodesic Levy processes, which are not always fractional Laplacians as in the Euclidean case. Instead, depending on the behaviour of geodesic flows, these infinitesimal generators can take on various forms, including in some cases displaying propagation type behaviours like fundamental solutions of wave equation. We will discuss the ramification of these unexpected properties on the underlying stochastic process.

Tobias Weich (University of Paderborn): "Ruelle-Taylor resonances for higher rank Anosov actions".

Abstract: The aim of this talk is to explain how the spectral theory of Ruelle resonances for Anosov flows can be generalized to Anosov actions of higher rank. As an application of this theory I would like to explain how we obtain the existence of SRB measures as well as a Bowen type formula for these measures.

Jialun Li (CNRS, École Polytechnique): "Exponential mixing of frame flows for geometrically finite hyperbolic manifolds".

Abstract: Let M be a geometrically finite hyperbolic manifold, that is, a hyperbolic manifold with a fundamental domain consisting of a finitely-sided polyhedron. There exists a unique measure on the unit tangent bundle invariant under the geodesic flow with maximal entropy, and we consider its lift to the frame bundle. In joint work with Pratyush Sarkar and Wenyu Pan, we proved that the frame flow is exponentially mixing with respect to this measure. To establish exponential mixing, we base on the countable coding of the flow and

a version of Dolgopyat's method, à la Sarkar-Winter and Tsujii-Zhang. To overcome the difficulty of the fractal structure in applying Dolgopyat's method, we show a large deviation property for symbolic recurrence to the large subsets.

Yann Chaubet (University of Cambridge): "Closed geodesics and intersection numbers".

Abstract: On a closed negatively curved surface, Margulis gave the asymptotic growth of the number of closed geodesics of bounded lengths, when the bound goes to infinity. In this talk, we will investigate such an asymptotic growth for closed geodesics of which certain intersection numbers are prescribed.