Effective theories in classical and quantum particle systems

SwissMAP Research Station Les Diablerets

19-23 June 2023

PROGRAM

Monday 19.06

09:20-09:30: Opening

09:30-10:30: François Golse (École Polytechnique)

Quantum Klimontovich Solutions and Applications to the Mean-Field Limit

10:30-11:00: Coffee break

11:00-12:00: Peter Pickl (University of Tübingen)

Beyond Bogoliubov Dynamics

12:00-14:00: Lunch break

14:00-15:00: Maria Colombo (EPFL)

Instability and non-uniqueness for the Euler and Navier-Stokes equations

15:00-16:00: Laurent Desvillettes (Université Paris Diderot)

Landau and Landau-Fermi Dirac operators

16:00-16:30: Coffee break

16:30-17:30: Mikaela Iacobelli (ETHZ)

Stability and singular limits in plasma physics

17:45-18:45: Welcome apero

19:00: Dinner

Tuesday 20.06

09:30-10:30: Robert Seiringer (IST Austria)

Ground state energy and effective mass of a strongly coupled polaron

10:30-11:00: Coffee break

11:00-12:00: Emanuela L. Giacomelli (LMU Munich)

On the low density Fermi gas in three dimensions

12:00-14:00: Lunch break

14:00-15:00: Arnaud Triay (LMU Munich)

The free energy of dilute Bose gases at low temperatures

15:00-16:00: Nikolai Leopold (University of Basel)

Renormalized Bogoliubov Theory for the Nelson Model

16:00-16:30: Coffee break

16:30-17:30: Thomas Chen (University of Texas at Austin)

On the emergence of Quantum Boltzmann dynamics in boson and fermion gases

19:00: Dinner

Wednesday 21.06

09:30-10:30: Dario Bambusi (University of Milan)

KAM tori for the Nonlinear Klein Gordon equation in the non relativistic limit

10:30-11:00: Coffee break

11:00-12:00: Mitia Duerinckx (Université Libre de Bruxelles)

Effective description of particle suspensions in fluids

12:00-14:00: Lunch break

Free afternoon

19:00: Dinner

Thursday 22.06

09:30-10:30: Thierry Bodineau (IHES)

On the fluctuating Boltzmann equation

10:30-11:00: Coffee break

11:00-12:00: Alessia Nota (University of L'Aquila)

On the Smoluchowski equation for aggregation phenomena under non-equilibrium conditions

12:00-14:00: Lunch break

14:00-15:00: Christian Hainzl (LMU Munich)

TBA

15:00-16:00: Niels Benedikter (University of Milan)

Interacting Fermions beyond Hartree-Fock theory

16:00-16:30: Coffee break

16:30-17:30: Michele Correggi (Politecnico di Milano)

Quasi-classical Limit and Ultraviolet Renormalization in the Nelson Model

19:00: Dinner

Friday 23.06

09:30-10:30: Antti Knowles (University of Geneva)

Euclidean field theories and interacting Bose gases

10:30-11:00: Coffee break

11:00-12:00: Benjamin Schlein (University of Zürich)

Dynamics of extended Fermi gases at high densities

12:00-12:10: Concluding remarks

ABSTRACTS OF THE TALKS

Speaker: Dario Bambusi

Title: *KAM tori for the Nonlinear Klein Gordon equation in the non relativistic limit*

Abstract: It is well known that solutions of the Nonlinear Klein Gordon equation should be well approximate in the non relatistic limit ($c \to \infty$) by solutions of the Nonlinear Schrödinger equation. In the present talk I will present discuss some well known results on the problem of justfying the above informal stetement, then I will concentrate on the case of 1-d space periodic solutions and present a preliminary result ensuring existence of invariant KAM tori for the NKG equation, uniformly in the nonrelativistic limit. The result also ensures that such tori depend smoothly on the speed of light and converge as $c \to \infty$. Joint work with Andrea Belloni and Filippo Giuliani.

Speaker: Niels Benedikter

Title: *Interacting Fermions beyond Hartree-Fock theory*

Abstract: Almost 100 years after the publication of the Schrödinger equation, the quantum many-body problem is still a source of mathematical challenges. In certain scaling limits, the challenges in the computation of physically observable predictions may be overcome by deriving effective evolution equations. I will discuss the coupled mean-field and semi-classical scaling limit for high-density fermionic systems, and sketch the derivation of the time-dependent Hartree-Fock equation as the simplest effective evolution equation. I will then present more recent results based on bosonization that can be seen as a next-order correction to Hartree-Fock theory.

Speaker: Thierry Bodineau

Title: *On the fluctuating Boltzmann equation*

Abstract: We consider a hard sphere dynamics at equilibrium and show that in the dilute gas limit, the fluctuations are described by the fluctuating Boltzmann equation. The convergence is valid for long times so that the fluctuating Fourier and Stokes equations can be also derived in a hydrodynamic scaling. This is joint work with I. Gallagher, L. Saint-Raymond, S. Simonella

Speaker: Thomas Chen

Title: *On the emergence of Quantum Boltzmann dynamics in boson and fermion gases*

Abstract: We present some recent results regarding the emergence of Quantum Boltzmann dynamics in interacting quantum manybody systems. This phenomenon is studied in the case of bosons near a BEC in joint work with Michael Hott. In the case of fermions, we present recent joint work with Esteban Cardenas which describes a subtle interplay between Boltzmann type collisions and interactions with bosonized electron-hole pairs.

Speaker: Maria Colombo

Title: *Instability and non-uniqueness for the Euler and Navier-Stokes equations*

Abstract: In his seminal work, Leray demonstrated the existence of global weak solutions, with nonincreasing energy, to the Navier-Stokes equations in three dimensions. In this talk we exhibit two distinct Leray solutions with zero initial velocity and identical body force. The starting point of our construction is Vishik's answer to another long-standing problem in fluid dynamics, namely whether the Yudovich uniqueness result for the 2D Euler system can be extended to the class of L^p-integrable vorticity. Building on Vishik's work, we construct a 'background' solution which is unstable for the 3D Navier-Stokes dynamics in similarity variables; the second solution from the same initial datum is a trajectory on the unstable manifold associated to the background solution, in accordance with the predictions of Jia and Sverak.

Speaker: Michele Correggi

Title: *Quasi-classical Limit and Ultraviolet Renormalization in the Nelson Model*

Abstract: We review the quasi-classical limit of the Nelson model, describing nucleons interacting with a scalar bosonic field, i.e., when the field degrees of freedom becomes classical while the nucleons retain their quantum nature. It is well known that such a model admits a simple energy renormalization of the ultraviolet divergence via the so-called dressing transformation. We then investigate the interplay between such a renormalization and the quasi-classical limit in both the stationary and dynamical pictures.

Speaker: Laurent Desvillettes

Title: *Landau and Landau-Fermi Dirac operators*

Abstract: The Landau operator describes the effect of collisions on the charged particles

of a plasma. It is possible to take into account the Pauli exclusion principle when the particles are fermions, the operator then becomes the Landau-Fermi Dirac operator. In this talk, we explain how to get in the simplest possible way entropy dissipation inequalities for those two models, which enable to improve what is known on the large time behavior of the corresponding spatially homogeneous equations. The talk is based in part on two papers in collaboration with Ricardo Alonso, Véronique Bagland and Bertrand Lods.

Speaker: Mitia Duerinckx

Title: *Effective description of particle suspensions in fluids*

Abstract: This talk will be devoted to the effective large-scale behavior of suspensions of rigid particles in Stokesian fluids. Such systems are ubiquitous both in nature and in practical applications, and are well known to give rise to complex rheological behaviors, in particular non-Newtonian effects. The main source of complexity is that particles are not point-like and that their interactions via the surrounding fluid flow are thus multibody in nature. We will mainly focus on the derivation of so-called Doi models in a suitable dilute mean-field regime, and on limitations to such effective descriptions.

Speaker: Emanuela L. Giacomelli

Title: *On the low density Fermi gas in three dimensions*

Abstract: In recent decades, the study of many-body systems has been an active area of research in both physics and mathematics. In this talk, we will consider a system of N spin 1/2 interacting fermions confined in a box in the dilute regime, with a particular focus on the correlation energy which is defined as the difference between the ground state energy and that of the free Fermi gas. We will discuss some recent results about a first order asymptotics for the correlation energy in the thermodynamic limit where the number of particles and the size of the box are sent to infinity keeping the density fixed. In particular, we will present a new upper bound for the correlation energy, which is consistent with the well-known Huang-Yang formula from 1957.

Speaker: François Golse

Title: *Quantum Klimontovich Solutions and Applications to the Mean-Field Limit (joint work with T. Paul and I. Ben Porat)*

Abstract: Klimontovich observed that, if a system of interacting point particles satisfies the system of differential equations corresponding to Newton's second law of motion written for each particle, the phase space empirical measure of this particle system is an exact solution in the sense of distributions of the Vlasov equation with the self-consistent, mean-field interaction potential. The purpose of this talk is (1) to define a quantum analogue of the notion of Klimontovich solution, and (2) to discuss applications of this notion to the mean-field limit in quantum mechanics.

Speaker: Christian Hainzl

Title: *TBA*

Abstract: TBA

Speaker: Mikaela Iacobelli

Title: *Stability and singular limits in plasma physics*

Abstract: In this talk, we will present two kinetic models that are used to describe the evolution of charged particles in plasmas: the Vlasov-Poisson system and the Vlasov-Poisson system with massless electrons. These systems model respectively the evolution of electrons, and ions in a plasma. We will discuss the well-posedness of these systems, the stability of solutions, and their behavior under singular limits. Finally, we will introduce a new class of Wasserstein-type distances specifically designed to tackle stability questions for kinetic equations.

Speaker: Antti Knowles

Title: *Euclidean field theories and interacting Bose gases*

Abstract: Euclidean field theories have been extensively studied in the mathematical literature since the sixties, motivated by high-energy physics and statistical mechanics. I explain how a complex scalar field theory with quartic interaction arises as a limit of an interacting Bose gas at positive temperature in a high-density regime. In dimensions higher than one, the field theory is supported on distributions of negative regularity, which requires a renormalization by divergent mass and energy counterterms. The proof is based on a new functional integral representation of the interacting Bose gas. Joint work with Jürg Fröhlich, Benjamin Schlein, and Vedran Sohinger.

Speaker: Nikolai Leopold

Title: *Renormalized Bogoliubov Theory for the Nelson Model*

Abstract: The renormalized Nelson model describes a quantum system of N non-relativistic identical particles linearly coupled to a quantized radiation field. In this talk, I will discuss its time evolution in a mean-field limit of many particles $N \gg 1$ with coupling constant proportional to $N^{-1/2}$. First, I will show that a Bose–Einstein condensate of particles and a coherent state of the field retain their structure during the time evolution and that the reduced densities can be approximated by solutions of the Schrödinger-Klein-Gordon equations. Second, I will introduce a renormalized Bogoliubov evolution which describes the quantum fluctuations around the Schrödinger-Klein-Gordon equations. This evolution allows to obtain an effective dynamics which approximates the time evolved many-body state in norm. The talk is based on a joint work with Marco Falconi, Jonas Lampart and David Mitrouskas.

Speaker: Alessia Nota

Title: *On the Smoluchowski equation for aggregation phenomena under non-equilibrium conditions*

Abstract: Smoluchowski's coagulation equation, an integro-differential equation of kinetic type, is a classical mean-field model for mass aggregation phenomena. In this talk I will present some recent results on the problem of existence or non-existence of stationary solutions, both for single and multi-component systems, under non-equilibrium conditions which are induced by the addition of a source term for small cluster sizes. The most striking feature of these stationary solutions is that, whenever they exist, they exhibit an unusual "spontaneous localization" phenomenon. This localization is a universal property of multicomponent systems and it has also been recently proved to occur in time dependent solutions to mass conserving coagulation equations. (Based on joint works with M.Ferreira, J.Lukkarinen and J. Velázquez)

Speaker: Peter Pickl

Title: *Beyond Bogoliubov Dynamics*

Abstract: Consider a system of N interacting bosons which are initially in a product state. It is well established that, in the mean-field scaling regime, product structure remains

approximately true under time evolution and that each particle is described by a non-linear wave equation, the Hartree equation. The next order corrections to this mean-field regime are, as proposed by Bogoliubov, given by pair-excitations which evolve naturally according to the Bogoliubov equation. Together with Lea Bossmann, Sören Petrat and Avy Soffer we could construct corrections to the Bogoliubov dynamics that approximate the true N-body dynamics in norm to arbitrary precision. The N-independent corrections are given in terms of the solutions of the Bogoliubov and Hartree equations. In this way, the complex problem of computing all n-point correlation functions for an interacting N -body system is essentially reduced to the problem of solving the Hartree equation and the PDEs for the Bogoliubov two-point correlation functions.

Speaker: Benjamin Schlein

Title: *Dynamics of extended Fermi gases at high densities*

Abstract: We consider systems of N fermions initially trapped in a volume V, at high density. We show that, for initial data close to Slater determinants exhibiting an appropriate semiclassical structure, the solution of the many-body Schrödinger equation can be approximated by the solution of the nonlinear Hartree equation, up to errors that are small, for large density, uniformly in N and V. This is joint work with L. Fresta and M. Porta.

Speaker: Robert Seiringer

Title: *Ground state energy and effective mass of a strongly coupled polaron*

Abstract: We explain recent bounds on the quantum corrections to the (classical) Pekar approximation of the ground state energy of the Fröhlich polaron model in the strong coupling limit, and their consequence on the existence of excited states and the polaron's effective mass.

Speaker: Arnaud Triay

Title: *The free energy of dilute Bose gases at low temperatures*

Abstract: In 1957 Lee Huang and Yang proposed a formula for the first eigenvalues of a Bose system in the dilute regime. I will present a recent work where we derive a lower bound on the free energy per unit volume of a dilute Bose gas that is in agreement with the LHY formula. Our result holds for temperatures for which the thermal contribution to the free

energy is of the same order as the famous zero-temperature LHY correction. This is joint work with Florian Haberberger, Christian Hainzl, Phan Thành Nam and Robert Seiringer.