Searching for Lorentz violation and spin-gravity couplings in experiments

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Based on: V. A. Kostelecký, Z. Li, PRD 104,044054(2021).

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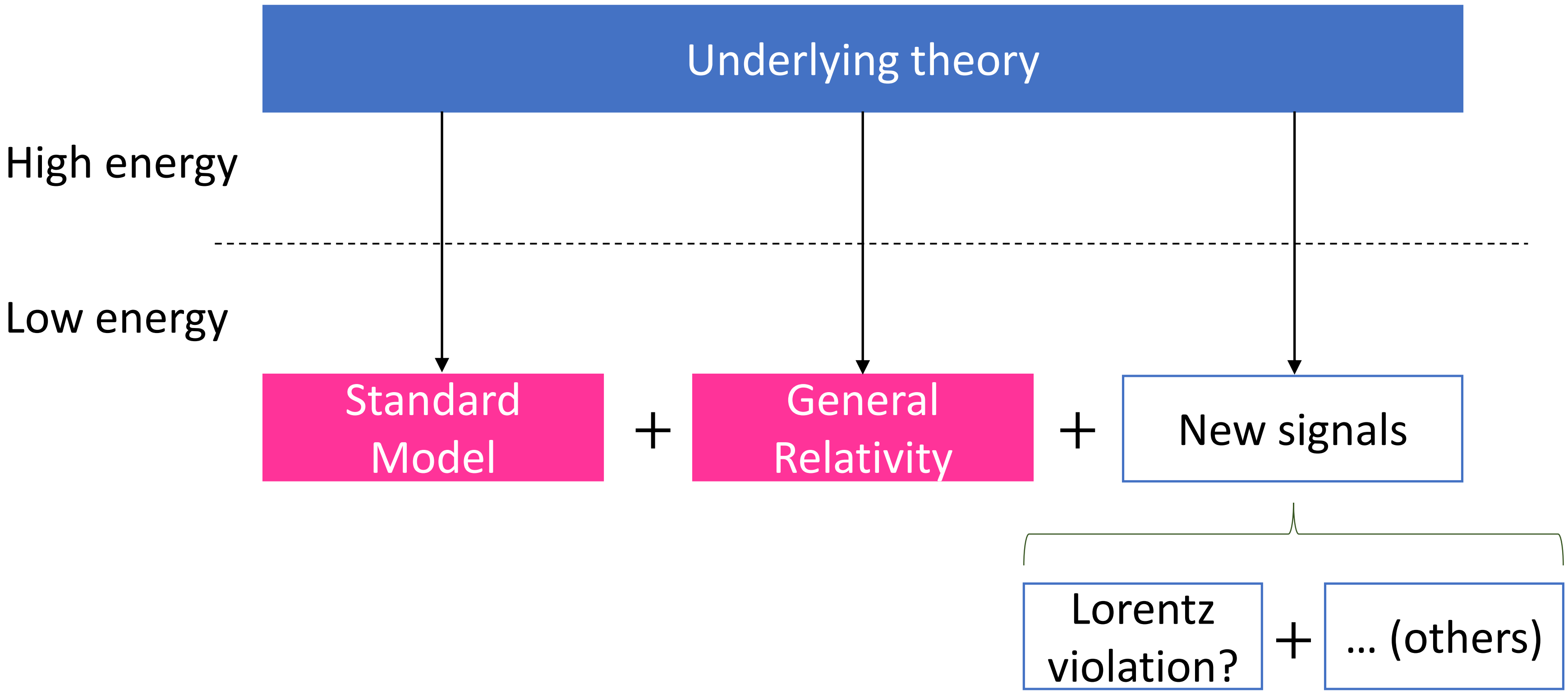
Outline

- Introduction to Lorentz violation
- Standard-Model Extension in curved spacetime
- Experiment (1): free-dropping experiments
- Experiment (2): gravitational interferometer experiments
- Experiment (3): gravitational bound-state experiments

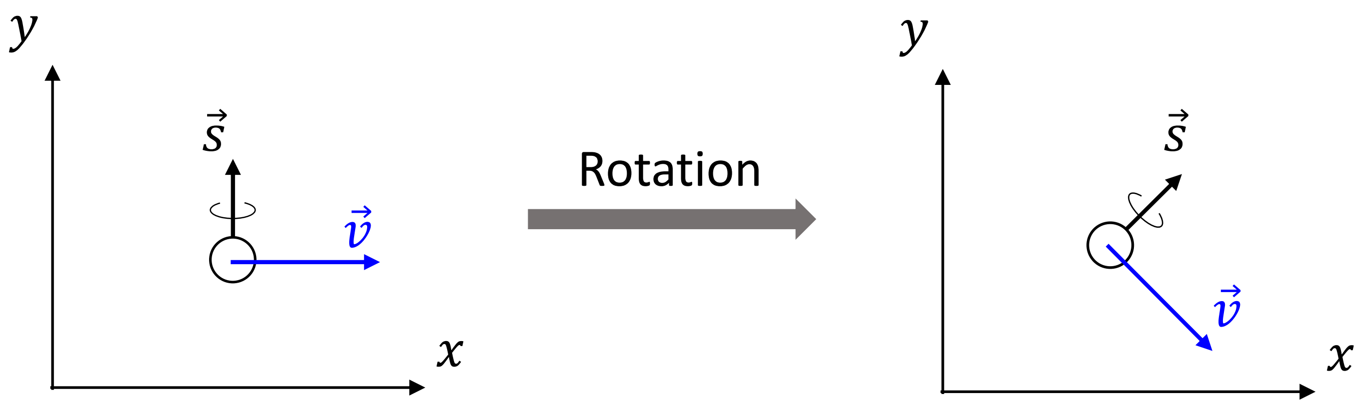
Unification of the Standard Model and General Relativity

- Testing Lorentz violation can shed light on the underlying theory



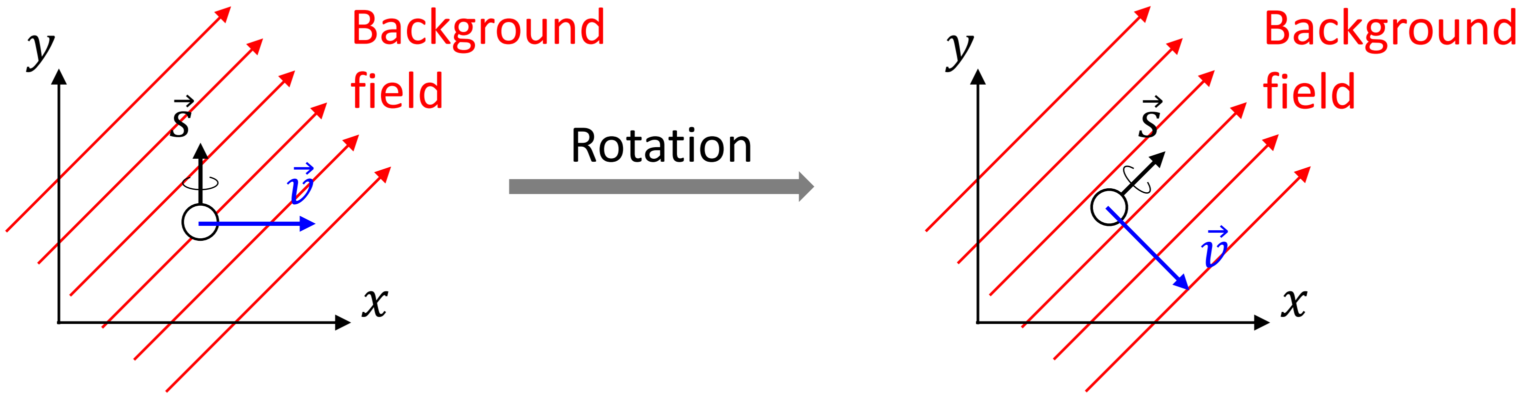
Lorentz symmetry

- Experimental results are independent of the orientation and velocity of the laboratory.
- Lorentz transformation: 3 rotations and 3 boosts
- A fundamental symmetry in physics
- Must be precisely tested in experiments

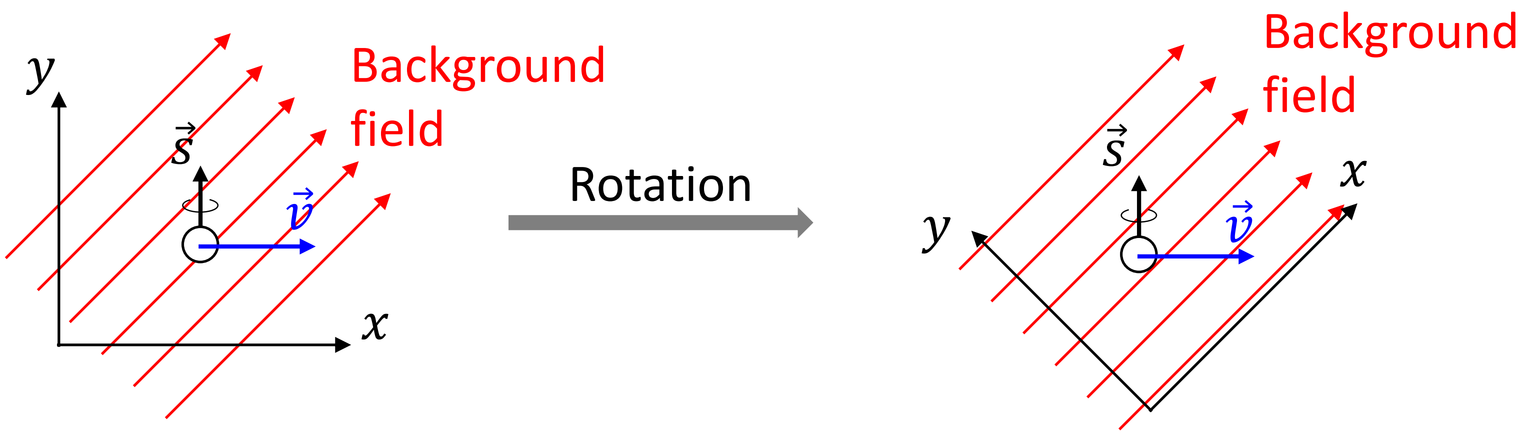


Lorentz violation (LV)

- Particle Lorentz transformation (violation)



- Observer Lorentz transformation (invariance)



Standard-Model Extension (SME)

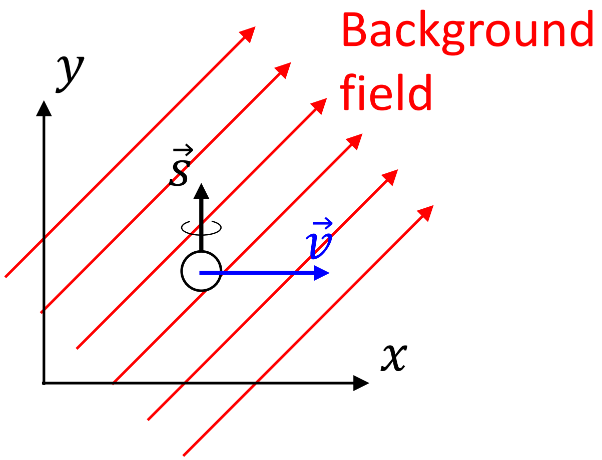
- Studies Lorentz violation in the context of effective field theory

SME	**=**	**Standard Model coupled to General Relativity**	**+**	**All possible modifications**
$\mathcal{L}$	**=**	$\mathcal{L}_0$	**+**	$\mathcal{L}'$

Example: $\mathcal{L}' \supset -a^\mu \overline{\psi}\gamma_\mu\psi$

$a^\mu \rightarrow$ background field
coefficients for Lorentz violation
coupling constants
control the size of Lorentz violation

$\overline{\psi}\gamma_\mu\psi \rightarrow$ dynamical field
fermion sector
free propagating term



Colladay, Kostelecký, PRD 55,6760(1997)

Experiments to test Lorentz violation

Electron and/or proton sectors:
- Penning trap
- Atom spectroscopy
- K/He magnetometer
- Atomic clock comparison
- 1S-2S transition
- Electron kinematics
- Nuclear binding energy

Photon sector:
- Astrophysical birefringence
- Astrophysical dispersion
- CMB polarization
- Laser interferometry
- Microwave, optical resonators
- Cavity oscillators
- Compton scattering

Gravity sector:
- Gravitational waves
- Short-range gravity
- Atom interferometry
- Binary pulsars
- Gravimetry
- Cosmic ray
- Torsion pendulum

Neutrino sector:
- Neutrino oscillation
- Double beta decay
- Neutrino time-of-flight
- Čerenkov radiation
- Tritium decay

And more…

Kostelecky, Russell, *Data Tables for Lorentz and CPT Violation*, Rev.Mod.Phys. 83.11(2011) updated as arXiv:0801.0287v15(2022)

Theoretical framework

Gauge-covariant spacetime-tensor operators

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Full Lagrange density in curved spacetime

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Linearized Lagrange density in weak gravity field

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Relativistic hamiltonian in weak gravity field

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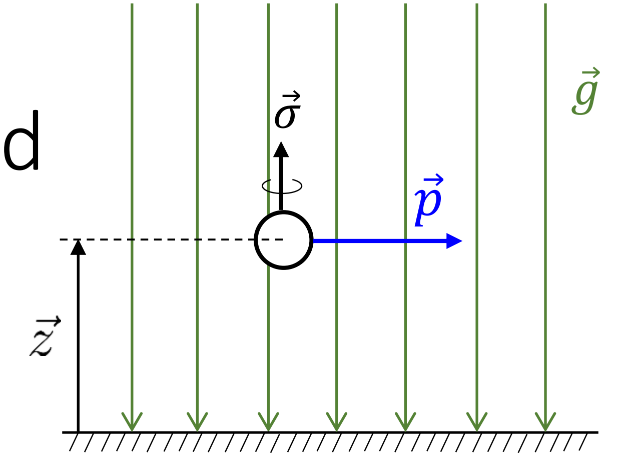
Nonrelativistic hamiltonian in weak gravity field

Kostelecký, Li, PRD 103,024059(2021); PRD 104,044054(2021)

Nonrelativistic hamiltonian in the Earth's gravitational field



- Conventional term: $H_0 = -m\vec{g}\cdot\vec{z}$
- Corrections from the SME:

Spin-Independent

$$H_\phi = (k^{\rm NR}_\phi)\vec{g}\cdot\vec{z} + (k^{\rm NR}_{\phi p})^j \tfrac{1}{2}(p^j\vec{g}\cdot\vec{z} + \vec{g}\cdot\vec{z}\,p^j) + (k^{\rm NR}_{\phi pp})^{jk}\tfrac{1}{2}(p^jp^k\vec{g}\cdot\vec{z} + \vec{g}\cdot\vec{z}\,p^jp^k)$$

$$H_g = (k^{\rm NR}_g)^j g^j + (k^{\rm NR}_{gp})^{jk}p^jg^k + (k^{\rm NR}_{gpp})^{jkl}p^jp^kg^l$$

Spin-dependent

$$H_{\sigma\phi} = (k^{\rm NR}_{\sigma\phi})^j\sigma^j\vec{g}\cdot\vec{z} + (k^{\rm NR}_{\sigma\phi p})^{jk}\tfrac{1}{2}\sigma^j(p^k\vec{g}\cdot\vec{z} + \vec{g}\cdot\vec{z}\,p^k) + (k^{\rm NR}_{\sigma\phi pp})^{jkl}\tfrac{1}{2}\sigma^j(p^kp^l\vec{g}\cdot\vec{z} + \vec{g}\cdot\vec{z}\,p^kp^l)$$

$$H_{\sigma g} = (k^{\rm NR}_{\sigma g})^{jk}\sigma^jg^k + (k^{\rm NR}_{\sigma gp})^{jkl}\sigma^jp^kg^l + (k^{\rm NR}_{\sigma gpp})^{jklm}\sigma^jp^kp^lg^m$$

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TABLE III. Correspondence between nonrelativistic and linearized coefficients.

NR coefficient	Linearized coefficient
$(k_{\phi}^{\rm NR})$	$2(m'^{\rm L})^{ss} - 2(a^{\rm L})^{tss} + 2m(e_h^{\rm L})^{tss} - 2m(c_h^{\rm L})^{ttss} + 2m^2(m_h^{(5){\rm L}})^{ttss} - 2m^2(a_h^{(5){\rm L}})^{tttss}$
$(k_{\phi p}^{\rm NR})^j$	$\frac{2}{m}(a^{\rm L})^{jss} - 2(e_h^{\rm L})^{jss} + 2(c_h^{\rm L})^{jtss} + 2(c_h^{\rm L})^{tjss} - 4m(m_h^{(5){\rm L}})^{jtss} + 2m(a_h^{(5){\rm L}})^{jttss} + 4m(a_h^{(5){\rm L}})^{tjtss}$
$(k_{\phi pp}^{\rm NR})^{jk}$	$-\frac{1}{m}[(c_h^{\rm L})^{jkss} + (c_h^{\rm L})^{kjss}] + 2(m_h^{(5){\rm L}})^{jkss} - 2(a_h^{(5){\rm L}})^{tjkss} - 2[(a_h^{(5){\rm L}})^{jktss} + (a_h^{(5){\rm L}})^{kjtss}]$
	$-\delta^{jk}\left[\frac{1}{m^2}(m'^{\rm L})^{ss} + \frac{1}{m}(c_h^{\rm L})^{ttss} - (m_h^{(5){\rm L}})^{ttss} + 2(a_h^{(5){\rm L}})^{tttss}\right]$
$(k_{\sigma\phi}^{\rm NR})^j$	$-2(b^{\rm L})^{jss} + \epsilon^{jkl}(H^{\rm L})^{klss} - 2m(d_h^{\rm L})^{jtss} + m\epsilon^{jkl}(g_h^{\rm L})^{kltss} - 2m^2(b_h^{(5){\rm L}})^{jttss} + m^2\epsilon^{jkl}(H_h^{(5){\rm L}})^{klttss}$
$(k_{\sigma\phi p}^{\rm NR})^{jk}$	$2(d_h^{\rm L})^{jkss} - \epsilon^{jmn}(g_h^{\rm L})^{mnkss} + 4m(b_h^{(5){\rm L}})^{jktss} - 2m\epsilon^{jmn}(H_h^{(5){\rm L}})^{mnktss}$
	$+\delta^{jk}\left[\frac{2}{m}(b^{\rm L})^{tss} + 2(d_h^{\rm L})^{ttss} + 2m(b_h^{(5){\rm L}})^{tttss}\right] - \epsilon^{jkl}\left[\frac{2}{m}(H^{\rm L})^{tlss} + 2(g_h^{\rm L})^{tltss} + 2m(H_h^{(5){\rm L}})^{tlttss}\right]$
$(k_{\sigma\phi pp}^{\rm NR})^{jkl}$	$-2(b_h^{(5){\rm L}})^{jklss} + \epsilon^{jmn}(H_h^{(5){\rm L}})^{mnklss} + \delta^{kl}\left[\frac{1}{m^2}(b^{\rm L})^{jss} + \frac{1}{2m}\epsilon^{jmn}(g_h^{\rm L})^{mntss} - (b_h^{(5){\rm L}})^{jttss} + \epsilon^{jkl}(H_h^{(5){\rm L}})^{mnttss}\right]$
	$+\frac{1}{2}\Big[\Big(- \delta^{jk}\left[\frac{1}{m^2}(b^{\rm L})^{lss} + \frac{1}{2m^2}\epsilon^{lmn}(H^{\rm L})^{mnss} + \frac{2}{m}(d_h^{\rm L})^{tlss} + \frac{1}{m}(d_h^{\rm L})^{ltss} + \frac{1}{2m}\epsilon^{lmn}(g_h^{\rm L})^{mntss}\right.$
	$\left.+4(b_h^{(5){\rm L}})^{ttlss} + (b_h^{(5){\rm L}})^{lttss} + \frac{1}{2}\epsilon^{lmn}(H_h^{(5){\rm L}})^{mnttss}\right] + \epsilon^{jkm}\left[\frac{2}{m}(g_h^{\rm L})^{tmlss} + 4(H_h^{(5){\rm L}})^{tmltss}\right]\Big) + (k \leftrightarrow l)\Big]$
$(k_{g}^{\rm NR})^j$	$\frac{1}{m}(H^{\rm L})^{tjss} + 2(e_{\partial h}^{\rm L})^{jss} - 2(c_{\partial h}^{\rm L})^{tjss} + (g_h^{\rm L})^{tjtss} + 2m(m_{\partial h}^{(5){\rm L}})^{tjss} - 2m(a_{\partial h}^{(5){\rm L}})^{ttjss} + m(H_h^{(5){\rm L}})^{tjttss}$
$(k_{gp}^{\rm NR})^{jk}$	$\frac{2}{m}(c_{\partial h}^{\rm L})^{jkss} - \frac{1}{m}(g_h^{\rm L})^{tkjss} - 2(m_{\partial h}^{(5){\rm L}})^{jkss} + 2(a_{\partial h}^{(5){\rm L}})^{tjkss} + 2(a_{\partial h}^{(5){\rm L}})^{jtkss} - 2(H_h^{(5){\rm L}})^{tkjtss}$
	$-\epsilon^{jkl}[\frac{1}{2m^2}(b^{\rm L})^{lss} + \frac{1}{2m}(d_h^{\rm L})^{ltss} + \frac{1}{2}(b_h^{(5){\rm L}})^{lttss}] - \epsilon^{jkl}\epsilon^{lmn}[\frac{1}{4m^2}(H^{\rm L})^{mnss} + \frac{1}{4m}(g_h^{\rm L})^{mntss} + \frac{1}{4}(H_h^{(5){\rm L}})^{mnttss}]$
$(k_{gpp}^{\rm NR})^{jkl}$	$-\frac{1}{m}(a_{\partial h}^{(5){\rm L}})^{jklss} - \frac{1}{m}(a_{\partial h}^{(5){\rm L}})^{kjlss} + \frac{1}{m}(H_h^{(5){\rm L}})^{tljkss}$
	$-\delta^{jk}\left[\frac{1}{m^2}(e_{\partial h}^{\rm L})^{lss} - \frac{1}{2m^2}(g_h^{\rm L})^{tltss} + \frac{1}{m}(a_{\partial h}^{(5){\rm L}})^{ttlss} - \frac{1}{m}(H_h^{(5){\rm L}})^{tlttss}\right]$
	$+\epsilon^{jlm}[\frac{1}{2m^2}(d_h^{\rm L})^{mkss} + \frac{1}{m}(b_h^{(5){\rm L}})^{mtkss}] + \epsilon^{jlm}\epsilon^{mnr}[\frac{1}{4m^2}(g_h^{\rm L})^{nrkss} + \frac{1}{2m}(H_h^{(5){\rm L}})^{nrtkss}]$
$(k_{\sigma g}^{\rm NR})^{jk}$	$-2(d_{\partial h}^{\rm L})^{jkss} + \epsilon^{jmn}(g_{\partial h}^{\rm L})^{mnkss} - 2m(b_{\partial h}^{(5){\rm L}})^{jtkss} + m\epsilon^{jmn}(H_{\partial h}^{(5){\rm L}})^{mntkss}$
	$-\delta^{jk}\left[\frac{1}{m}(m_5^{\rm L})^{ss} + (f_h^{\rm L})^{tss} + m(m_{5h}^{(5){\rm L}})^{ttss}\right] + \epsilon^{jkl}\left[\frac{1}{m}(a^{\rm L})^{lss} + (c_h^{\rm L})^{ltss} + m(a_h^{(5){\rm L}})^{lttss}\right]$
$(k_{\sigma gp}^{\rm NR})^{jkl}$	$2(b_{\partial h}^{(5){\rm L}})^{jklss} - \epsilon^{jmn}(H_{\partial h}^{(5){\rm L}})^{mnklss} + \epsilon^{jkl}\left[\frac{1}{2m^2}(m'^{\rm L})^{ss} + \frac{1}{2m^2}(a^{\rm L})^{tss} + \frac{1}{2m}(e_h^{\rm L})^{tss} + \frac{1}{2m}(c_h^{\rm L})^{ttss}\right.$
	$\left.+\frac{1}{2}(m_h^{(5){\rm L}})^{ttss} + \frac{1}{2}(a_h^{(5){\rm L}})^{tttss}\right] + \delta^{jk}\left[\frac{2}{m}(d_{\partial h}^{\rm L})^{tlss} + 2(b_{\partial h}^{(5){\rm L}})^{ttlss}\right] + \delta^{jl}\left[\frac{1}{m}(f_h^{\rm L})^{kss} + 2(m_{5h}^{(5){\rm L}})^{ktss}\right]$
	$-\epsilon^{jkm}\left[\frac{2}{m}(g_{\partial h}^{\rm L})^{tmlss} + 2(H_{\partial h}^{(5){\rm L}})^{tmtlss}\right] - \epsilon^{jlm}\left[\frac{1}{m}(c_h^{\rm L})^{mkss} + 2(a_h^{(5){\rm L}})^{mktss}\right]$
$(k_{\sigma gpp}^{\rm NR})^{jklm}$	$-\delta^{jm}\delta^{kl}\left[\frac{2}{m^2}(f_h^{\rm L})^{tss}\frac{1}{m}(m_{5h}^{(5){\rm L}})^{ttss}\right] - \delta^{jm}\frac{1}{m}(m_h^{(5){\rm L}})^{klss} + \delta^{kl}\left[\frac{1}{m^2}(d_{\partial h}^{\rm L})^{jmss} + \frac{1}{2m}\epsilon^{jnr}(H_{\partial h}^{(5){\rm L}})^{nrtmss}\right]$
	$+\delta^{kl}\epsilon^{jmn}\left[\frac{1}{2m^2}(c_h^{\rm L})^{ntss} + \frac{1}{m}(a_h^{(5){\rm L}})^{nttss}\right] + \epsilon^{jmn}\frac{1}{m}(a_h^{(5){\rm L}})^{nklss}$
	$-\frac{1}{2}\Big[\Big(\delta^{jk}\left[\frac{1}{m^2}(d_{\partial h}^{\rm L})^{lmss} + \frac{1}{2m^2}\epsilon^{lnr}(g_{\partial h}^{\rm L})^{nrmss} + \frac{2}{m}(b_{\partial h}^{(5){\rm L}})^{tlmss} + \frac{1}{m}(b_{\partial h}^{(5){\rm L}})^{ltmss} + \frac{1}{2m}\epsilon^{lnr}(H_{\partial h}^{(5){\rm L}})^{nrtmss}\right]$
	$+\epsilon^{jkm}\left[\frac{1}{2m^2}(e_h^{\rm L})^{lss} + \frac{1}{2m^2}(c_h^{\rm L})^{tlss} + \frac{1}{m}(m_h^{(5){\rm L}})^{ltss} + \frac{1}{m}(a_h^{(5){\rm L}})^{tltss}\right] - \epsilon^{jkn}\frac{2}{m}(H_{\partial h}^{(5){\rm L}})^{tnlmss}\Big) + (k \leftrightarrow l)\Big]$

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TABLE II. Relationships between coefficients in $\mathcal{L}_\psi^{\rm L}$ and in $\mathcal{L}_\psi$.

$\mathcal{L}_\psi^{\rm L}$	$\mathcal{L}_\psi$
$(m'^{\rm L})^{\mu\nu}$	$(\overline{m}'^{\rm L})^{\mu\nu} + \frac{1}{2}\overline{m}'_{\rm asy}\eta^{\mu\nu}$
$(m_5^{\rm L})^{\mu\nu}$	$(\overline{m}_5^{\rm L})^{\mu\nu} + \frac{1}{2}\overline{m}_{5{\rm asy}}\eta^{\mu\nu}$
$(a^{\rm L})^{\kappa\mu\nu}$	$(\overline{a}^{\rm L})^{\kappa\mu\nu} + \frac{1}{2}\overline{a}^{\kappa}_{\rm asy}\eta^{\mu\nu} + \frac{1}{4}(\overline{a}^{\mu}_{\rm asy}\eta^{\nu\kappa} + \overline{a}^{\nu}_{\rm asy}\eta^{\mu\kappa})$
$(b^{\rm L})^{\kappa\mu\nu}$	$(\overline{b}^{\rm L})^{\kappa\mu\nu} + \frac{1}{2}\overline{b}^{\kappa}_{\rm asy}\eta^{\mu\nu} + \frac{1}{4}(\overline{b}^{\mu}_{\rm asy}\eta^{\nu\kappa} + \overline{b}^{\nu}_{\rm asy}\eta^{\mu\kappa})$
$(H^{\rm L})^{\kappa\lambda\mu\nu}$	$(\overline{H}^{\rm L})^{\kappa\lambda\mu\nu} + \frac{1}{2}\overline{H}^{\kappa\lambda}_{\rm asy}\eta^{\mu\nu} + \frac{1}{4}[(\overline{H}^{\mu\lambda}_{\rm asy}\eta^{\kappa\nu} + \overline{H}^{\nu\lambda}_{\rm asy}\eta^{\kappa\mu}) - (\kappa \leftrightarrow \lambda)]$
$(c_h^{\rm L})^{\kappa\mu\nu\rho}$	$(\overline{c}^{\rm L})^{\kappa\mu\nu\rho} + \frac{1}{2}\overline{c}^{\kappa\mu}_{\rm asy}\eta^{\nu\rho} + \frac{1}{4}(\overline{c}^{\nu\mu}_{\rm asy}\eta^{\rho\kappa} + \overline{c}^{\rho\mu}_{\rm asy}\eta^{\nu\kappa})$
$(d_h^{\rm L})^{\kappa\mu\nu\rho}$	$(\overline{d}^{\rm L})^{\kappa\mu\nu\rho} + \frac{1}{2}\overline{d}^{\kappa\mu}_{\rm asy}\eta^{\nu\rho} + \frac{1}{4}(\overline{d}^{\nu\mu}_{\rm asy}\eta^{\rho\kappa} + \overline{d}^{\rho\mu}_{\rm asy}\eta^{\nu\kappa})$
$(e_h^{\rm L})^{\mu\nu\rho}$	$(\overline{e}^{\rm L})^{\mu\nu\rho} + \frac{1}{2}\overline{e}^{\mu}_{\rm asy}\eta^{\nu\rho}$
$(f_h^{\rm L})^{\mu\nu\rho}$	$(\overline{f}^{\rm L})^{\mu\nu\rho} + \frac{1}{2}\overline{f}^{\mu}_{\rm asy}\eta^{\nu\rho}$
$(g_h^{\rm L})^{\kappa\lambda\mu\nu\rho}$	$(\overline{g}^{\rm L})^{\kappa\lambda\mu\nu\rho} + \frac{1}{2}\overline{g}^{\kappa\lambda\mu}_{\rm asy}\eta^{\nu\rho} + \frac{1}{4}[(\overline{g}^{\nu\lambda\mu}_{\rm asy}\eta^{\kappa\rho} + \overline{g}^{\rho\lambda\mu}_{\rm asy}\eta^{\kappa\nu}) - (\kappa \leftrightarrow \lambda)]$
$(c_{\partial h}^{\rm L})^{\kappa\mu\nu\rho}$	$\frac{1}{8}(\overline{d}^{\alpha\nu}_{\rm asy}\eta_{\alpha\beta}\epsilon^{\beta\mu\rho\kappa} + \overline{d}^{\alpha\rho}_{\rm asy}\eta_{\alpha\beta}\epsilon^{\beta\mu\nu\kappa})$
$(d_{\partial h}^{\rm L})^{\kappa\mu\nu\rho}$	$\frac{1}{8}(\overline{c}^{\alpha\nu}_{\rm asy}\eta_{\alpha\beta}\epsilon^{\beta\mu\rho\kappa} + \overline{c}^{\alpha\rho}_{\rm asy}\eta_{\alpha\beta}\epsilon^{\beta\mu\nu\kappa})$
$(e_{\partial h}^{\rm L})^{\mu\nu\rho}$	$\frac{1}{4}(\overline{g}^{\mu\nu\rho}_{\rm asy} + \overline{g}^{\mu\rho\nu}_{\rm asy})$
$(f_{\partial h}^{\rm L})^{\mu\nu\rho}$	$-\frac{1}{8}(\overline{g}^{\alpha\beta\nu}_{\rm asy}\eta_{\alpha\gamma}\eta_{\beta\delta}\epsilon^{\gamma\delta\mu\rho} + \overline{g}^{\alpha\beta\rho}_{\rm asy}\eta_{\alpha\gamma}\eta_{\beta\delta}\epsilon^{\gamma\delta\mu\nu})$
$(g_{\partial h}^{\rm L})^{\kappa\lambda\mu\nu\rho}$	$\frac{1}{8}[(\overline{e}^{\nu}_{\rm asy}\eta^{\kappa\mu}\eta^{\lambda\rho} + \overline{e}^{\rho}_{\rm asy}\eta^{\kappa\mu}\eta^{\lambda\nu}) - (\kappa \leftrightarrow \lambda)] + \frac{1}{8}(\overline{f}^{\nu}_{\rm asy}\epsilon^{\kappa\lambda\mu\rho} + \overline{f}^{\rho}_{\rm asy}\epsilon^{\kappa\lambda\mu\nu})$
$(m_h^{(5)\rm L})^{\mu\nu\rho\sigma}$	$(\overline{m}^{(5)\rm L})^{\mu\nu\rho\sigma} + \frac{1}{2}(\overline{m}^{(5)}_{\rm asy})^{\mu\nu}\eta^{\rho\sigma}$
$(m_{5h}^{(5)\rm L})^{\mu\nu\rho\sigma}$	$(\overline{m}_5^{(5)\rm L})^{\mu\nu\rho\sigma} + \frac{1}{2}(\overline{m}^{(5)}_{5{\rm asy}})^{\mu\nu}\eta^{\rho\sigma}$
$(a_h^{(5)\rm L})^{\kappa\mu\nu\rho\sigma}$	$-(\overline{a}^{(5)\rm L})^{\kappa\mu\nu\rho\sigma} - \frac{1}{2}(\overline{a}^{(5)}_{\rm asy})^{\kappa\mu\nu}\eta^{\rho\sigma} - \frac{1}{4}[(\overline{a}^{(5)}_{\rm asy})^{\rho\mu\nu}\eta^{\kappa\sigma} + (\overline{a}^{(5)}_{\rm asy})^{\sigma\mu\nu}\eta^{\kappa\rho}]$
$(b_h^{(5)\rm L})^{\kappa\mu\nu\rho\sigma}$	$-(\overline{b}^{(5)\rm L})^{\kappa\mu\nu\rho\sigma} - \frac{1}{2}(\overline{b}^{(5)}_{\rm asy})^{\kappa\mu\nu}\eta^{\rho\sigma} - \frac{1}{4}[(\overline{b}^{(5)}_{\rm asy})^{\rho\mu\nu}\eta^{\kappa\sigma} + (\overline{b}^{(5)}_{\rm asy})^{\sigma\mu\nu}\eta^{\kappa\rho}]$
$(H_h^{(5)\rm L})^{\kappa\lambda\mu\nu\rho\sigma}$	$(\overline{H}^{(5)\rm L})^{\kappa\lambda\mu\nu\rho\sigma} + \frac{1}{2}(\overline{H}^{(5)}_{\rm asy})^{\kappa\lambda\mu\nu}\eta^{\rho\sigma} + \frac{1}{4}[[(\overline{H}^{(5)}_{\rm asy})^{\rho\lambda\mu\nu}\eta^{\kappa\sigma} + (\overline{H}^{(5)}_{\rm asy})^{\sigma\lambda\mu\nu}\eta^{\kappa\rho}] - (\kappa \leftrightarrow \lambda)]$
$(m_{\partial h}^{(5)\rm L})^{\mu\nu\rho\sigma}$	$\frac{1}{2}[(\overline{H}^{(5)}_{\rm asy})^{\nu\sigma\mu\rho} + (\overline{H}^{(5)}_{\rm asy})^{\nu\rho\mu\sigma}]$
$(m_{5\partial h}^{(5)\rm L})^{\mu\nu\rho\sigma}$	$-\frac{1}{4}[(\overline{H}^{(5)}_{\rm asy})^{\alpha\beta\mu\rho}\eta_{\alpha\gamma}\eta_{\beta\delta}\epsilon^{\gamma\delta\nu\sigma} + (\overline{H}^{(5)}_{\rm asy})^{\alpha\beta\mu\sigma}\eta_{\alpha\gamma}\eta_{\beta\delta}\epsilon^{\gamma\delta\nu\rho}]$
$(a_{\partial h}^{(5)\rm L})^{\kappa\mu\nu\rho\sigma}$	$\frac{1}{4}[(\overline{b}^{(5)}_{\rm asy})^{\alpha\mu\rho}\eta_{\alpha\beta}\epsilon^{\beta\nu\sigma\kappa} + (\overline{b}^{(5)}_{\rm asy})^{\alpha\mu\sigma}\eta_{\alpha\beta}\epsilon^{\beta\nu\rho\kappa}]$
$(b_{\partial h}^{(5)\rm L})^{\kappa\mu\nu\rho\sigma}$	$\frac{1}{4}[(\overline{a}^{(5)}_{\rm asy})^{\alpha\mu\rho}\eta_{\alpha\beta}\epsilon^{\beta\nu\sigma\kappa} + (\overline{a}^{(5)}_{\rm asy})^{\alpha\mu\sigma}\eta_{\alpha\beta}\epsilon^{\beta\nu\rho\kappa}]$
$(H_{\partial h}^{(5)\rm L})^{\kappa\lambda\mu\nu\rho\sigma}$	$\frac{1}{4}[[(\overline{m}^{(5)}_{\rm asy})^{\mu\rho}\eta^{\nu\kappa}\eta\sigma\lambda + (\overline{m}^{(5)}_{\rm asy})^{\mu\sigma}\eta^{\nu\kappa}\eta^{\rho\lambda}] - (\kappa \leftrightarrow \lambda)] + \frac{1}{4}[(\overline{m5}^{(5)}_{\rm asy})^{\mu\rho}\epsilon^{\kappa\lambda\nu\sigma} + (\overline{m5}^{(5)})^{\mu\sigma}\epsilon^{\kappa\lambda\nu\rho}]$
$(m_{\partial\partial h}^{(5)\rm L})^{\mu\nu\rho\sigma}$	$\frac{1}{2}[(\overline{m}^{(5)}_{R,{\rm asy}})^{\mu\rho\sigma\nu} + (\overline{m}^{(5)}_{R,{\rm asy}})^{\nu\rho\sigma\mu}] + \frac{1}{4}(\overline{m}^{(5)}_{\rm asy})^{\rho\sigma}\eta^{\mu\nu} - \frac{1}{8}[[(\overline{m}^{(5)}_{\rm asy})^{\mu\rho}\eta^{\nu\sigma} + (\overline{m}^{(5)}_{\rm asy})^{\mu\sigma}\eta^{\nu\rho}] + (\mu \leftrightarrow \nu)]$
$(m_{5\partial\partial h}^{(5)\rm L})^{\mu\nu\rho\sigma}$	$\frac{1}{2}[(\overline{m}^{(5)}_{5R,{\rm asy}})^{\mu\rho\sigma\nu} + (\overline{m}^{(5)}_{5R,{\rm asy}})^{\nu\rho\sigma\mu}] + \frac{1}{4}(\overline{m}^{(5)}_{5{\rm asy}})^{\rho\sigma}\eta^{\mu\nu} - \frac{1}{8}[[(\overline{m}^{(5)}_{5{\rm asy}})^{\mu\rho}\eta^{\nu\sigma} + (\overline{m}^{(5)}_{5{\rm asy}})^{\mu\sigma}\eta^{\nu\rho}] + (\mu \leftrightarrow \nu)]$
$(a_{\partial\partial h}^{(5)\rm L})^{\kappa\mu\nu\rho\sigma}$	$-\frac{1}{2}[(\overline{a}^{(5)}_{R,{\rm asy}})^{\kappa\mu\rho\sigma\nu} + (\overline{a}^{(5)}_{R,{\rm asy}})^{\kappa\nu\rho\sigma\mu}] - \frac{1}{4}(\overline{a}^{(5)}_{\rm asy})^{\kappa\rho\sigma}\eta^{\mu\nu} + \frac{1}{8}[[(\overline{a}^{(5)}_{\rm asy})^{\kappa\mu\rho}\eta^{\nu\sigma} + (\overline{a}^{(5)}_{\rm asy})^{\kappa\mu\sigma}\eta^{\nu\rho}] + (\mu \leftrightarrow \nu)]$
$(b_{\partial\partial h}^{(5)\rm L})^{\kappa\mu\nu\rho\sigma}$	$-\frac{1}{2}[(\overline{b}^{(5)}_{R,{\rm asy}})^{\kappa\mu\rho\sigma\nu} + (\overline{b}^{(5)}_{R,{\rm asy}})^{\kappa\nu\rho\sigma\mu}] - \frac{1}{4}(\overline{b}^{(5)}_{\rm asy})^{\kappa\rho\sigma}\eta^{\mu\nu} + \frac{1}{8}[[(\overline{b}^{(5)}_{\rm asy})^{\kappa\mu\rho}\eta^{\nu\sigma} + (\overline{b}^{(5)}_{\rm asy})^{\kappa\mu\sigma}\eta^{\nu\rho}] + (\mu \leftrightarrow \nu)]$
$(H_{\partial\partial h}^{(5)\rm L})^{\kappa\lambda\mu\nu\rho\sigma}$	$\frac{1}{2}[(\overline{H}^{(5)}_{R,{\rm asy}})^{\kappa\lambda\mu\rho\sigma\nu} + (\overline{H}^{(5)}_{R,{\rm asy}})^{\kappa\lambda\nu\rho\sigma\mu}] + \frac{1}{4}(\overline{H}^{(5)}_{\rm asy})^{\kappa\lambda\rho\sigma}\eta^{\mu\nu} - \frac{1}{8}[[(\overline{H}^{(5)}_{\rm asy})^{\kappa\lambda\mu\rho}\eta^{\nu\sigma} + (\overline{H}^{(5)}_{\rm asy})^{\kappa\lambda\mu\sigma}\eta^{\nu\rho}] + (\mu \leftrightarrow \nu)]$

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TABLE I. Terms containing operators of mass dimension $d \leq 5$ in the linearized fermion Lagrange density $\mathcal{L}^{\mathrm{L}}_{\psi}$.

Component	Expression
$\mathcal{L}^{\mathrm{L}}_{\psi,0}$	$\frac{1}{2}(\overline{\psi}\gamma^{\mu}i\partial_{\mu}\psi - m\overline{\psi}\psi) + \mathrm{h.c.}$
$\mathcal{L}^{\mathrm{L}}_{\psi,h}$	$\frac{1}{4}h\overline{\psi}\gamma^{\mu}i\partial_{\mu}\psi - \frac{1}{4}h^{\kappa\mu}\overline{\psi}\gamma_{\kappa}i\partial_{\mu}\psi - \frac{1}{4}mh\overline{\psi}\psi + \frac{1}{8}\epsilon^{\kappa\mu\nu\rho}(\partial_{\mu}h_{\nu\rho})\overline{\psi}\gamma_5\gamma_{\kappa}\psi + \mathrm{h.c.}$
$\mathcal{L}^{(3)\mathrm{L}}_{\psi}$	$-(m'^{\mathrm{L}})^{\mu\nu}h_{\mu\nu}\overline{\psi}\psi - i(m_5^{\mathrm{L}})^{\mu\nu}h_{\mu\nu}\overline{\psi}\gamma_5\psi - (a^{\mathrm{L}})^{\kappa\mu\nu}h_{\mu\nu}\overline{\psi}\gamma_{\kappa}\psi - (b^{\mathrm{L}})^{\kappa\mu\nu}h_{\mu\nu}\overline{\psi}\gamma_5\gamma_{\kappa}\psi - \frac{1}{2}(H^{\mathrm{L}})^{\kappa\lambda\mu\nu}h_{\mu\nu}\overline{\psi}\sigma_{\kappa\lambda}\psi$
$\mathcal{L}^{(4)\mathrm{L}}_{\psi h}$	$-\frac{1}{2}(c_h^{\mathrm{L}})^{\kappa\mu\nu\rho}h_{\nu\rho}\overline{\psi}\gamma_{\kappa}i\partial_{\mu}\psi - \frac{1}{2}(d_h^{\mathrm{L}})^{\kappa\mu\nu\rho}h_{\nu\rho}\overline{\psi}\gamma_5\gamma_{\kappa}i\partial_{\mu}\psi$ $-\frac{1}{2}(e_h^{\mathrm{L}})^{\mu\nu\rho}h_{\nu\rho}\overline{\psi}i\partial_{\mu}\psi - \frac{1}{2}i(f_h^{\mathrm{L}})^{\mu\nu\rho}h_{\nu\rho}\overline{\psi}\gamma_5 i\partial_{\mu}\psi - \frac{1}{4}(g_h^{\mathrm{L}})^{\kappa\lambda\mu\nu\rho}h_{\nu\rho}\overline{\psi}\sigma_{\kappa\lambda}i\partial_{\mu}\psi + \mathrm{h.c.}$
$\mathcal{L}^{(4)\mathrm{L}}_{\psi\partial h}$	$-(c_{\partial h}^{\mathrm{L}})^{\kappa\mu\nu\rho}(\partial_{\mu}h_{\nu\rho})\overline{\psi}\gamma_{\kappa}\psi - (d_{\partial h}^{\mathrm{L}})^{\kappa\mu\nu\rho}(\partial_{\mu}h_{\nu\rho})\overline{\psi}\gamma_5\gamma_{\kappa}\psi$ $-(e_{\partial h}^{\mathrm{L}})^{\mu\nu\rho}(\partial_{\mu}h_{\nu\rho})\overline{\psi}\psi - i(f_{\partial h}^{\mathrm{L}})^{\mu\nu\rho}(\partial_{\mu}h_{\nu\rho})\overline{\psi}\gamma_5\psi - \frac{1}{2}(g_{\partial h}^{\mathrm{L}})^{\kappa\lambda\mu\nu\rho}(\partial_{\mu}h_{\nu\rho})\overline{\psi}\sigma_{\kappa\lambda}\psi$
$\mathcal{L}^{(5)\mathrm{L}}_{\psi h}$	$-\frac{1}{2}(m_h^{(5)\mathrm{L}})^{\mu\nu\rho\sigma}h_{\rho\sigma}\overline{\psi}i\partial_{\mu}i\partial_{\nu}\psi - \frac{1}{2}i(m_{5h}^{(5)\mathrm{L}})^{\mu\nu\rho\sigma}h_{\rho\sigma}\overline{\psi}\gamma_5 i\partial_{\mu}i\partial_{\nu}\psi$ $-\frac{1}{2}(a_h^{(5)\mathrm{L}})^{\kappa\mu\nu\rho\sigma}h_{\rho\sigma}\overline{\psi}\gamma_{\kappa}i\partial_{\mu}i\partial_{\nu}\psi - \frac{1}{2}(b_h^{(5)\mathrm{L}})^{\kappa\mu\nu\rho\sigma}h_{\rho\sigma}\overline{\psi}\gamma_5\gamma_{\kappa}i\partial_{\mu}i\partial_{\nu}\psi$ $-\frac{1}{4}(H_h^{(5)\mathrm{L}})^{\kappa\lambda\mu\nu\rho\sigma}h_{\rho\sigma}\overline{\psi}\sigma_{\kappa\lambda}i\partial_{\mu}i\partial_{\nu}\psi + \mathrm{h.c.}$
$\mathcal{L}^{(5)\mathrm{L}}_{\psi\partial h}$	$-\frac{1}{2}(m_{\partial h}^{(5)\mathrm{L}})^{\mu\nu\rho\sigma}(\partial_{\nu}h_{\rho\sigma})\overline{\psi}i\partial_{\mu}\psi - \frac{1}{2}i(m_{5\partial h}^{(5)\mathrm{L}})^{\mu\nu\rho\sigma}(\partial_{\nu}h_{\rho\sigma})\overline{\psi}\gamma_5 i\partial_{\mu}\psi$ $-\frac{1}{2}(a_{\partial h}^{(5)\mathrm{L}})^{\kappa\mu\nu\rho\sigma}(\partial_{\nu}h_{\rho\sigma})\overline{\psi}\gamma_{\kappa}i\partial_{\mu}\psi - \frac{1}{2}(b_{\partial h}^{(5)\mathrm{L}})^{\kappa\mu\nu\rho\sigma}(\partial_{\nu}h_{\rho\sigma})\overline{\psi}\gamma_5\gamma_{\kappa}i\partial_{\mu}\psi$ $-\frac{1}{4}(H_{\partial h}^{(5)\mathrm{L}})^{\kappa\lambda\mu\nu\rho\sigma}(\partial_{\nu}h_{\rho\sigma})\overline{\psi}\sigma_{\kappa\lambda}i\partial_{\mu}\psi + \mathrm{h.c.}$
$\mathcal{L}^{(5)\mathrm{L}}_{\psi\partial\partial h}$	$-(m_{\partial\partial h}^{(5)\mathrm{L}})^{\mu\nu\rho\sigma}(\partial_{\mu}\partial_{\nu}h_{\rho\sigma})\overline{\psi}\psi - i(m_{5\partial\partial h}^{(5)\mathrm{L}})^{\mu\nu\rho\sigma}(\partial_{\mu}\partial_{\nu}h_{\rho\sigma})\overline{\psi}\gamma_5\psi$ $-(a_{\partial\partial h}^{(5)\mathrm{L}})^{\kappa\mu\nu\rho\sigma}(\partial_{\mu}\partial_{\nu}h_{\rho\sigma})\overline{\psi}\gamma_{\kappa}\psi - (b_{\partial\partial h}^{(5)\mathrm{L}})^{\kappa\mu\nu\rho\sigma}(\partial_{\mu}\partial_{\nu}h_{\rho\sigma})\overline{\psi}\gamma_5\gamma_{\kappa}\psi$ $-\frac{1}{2}(H_{\partial\partial h}^{(5)\mathrm{L}})^{\kappa\lambda\mu\nu\rho\sigma}(\partial_{\mu}\partial_{\nu}h_{\rho\sigma})\overline{\psi}\sigma_{\kappa\lambda}\psi$

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Dropping ^{87}Rb atoms

- ^{87}Rb with different spins
- Proton and electron sectors
- Sensitivity:

$$\Delta g/g \lesssim 10^{-7}$$

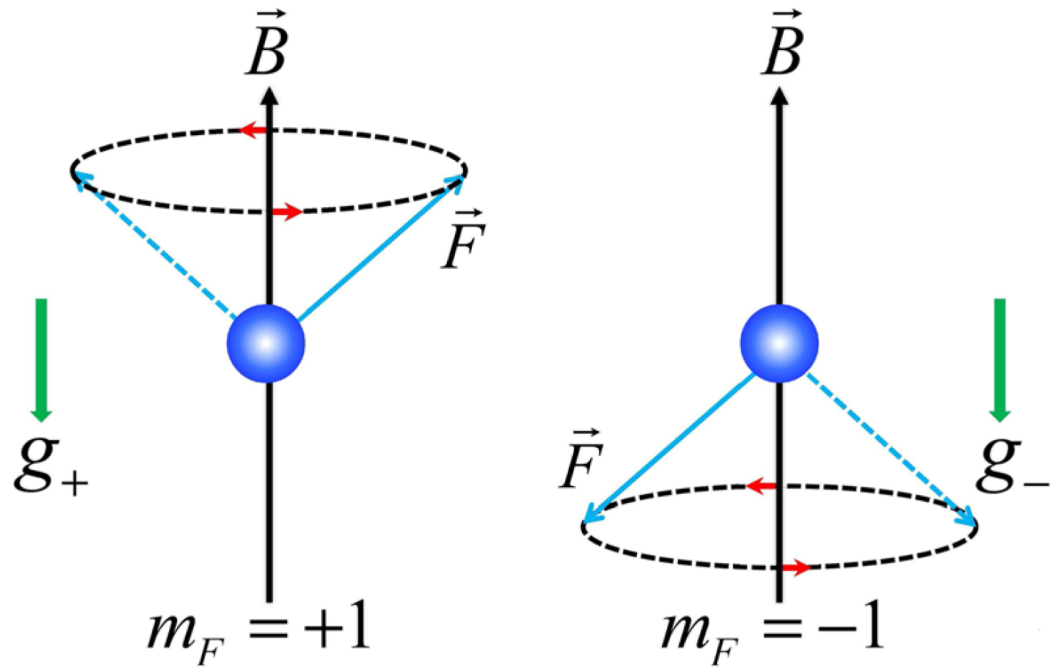


Image: Duan *et al.*, PRL'2016

- Constraints on coefficients for LV:

$$\left|(k^{\rm NR}_{\sigma\phi})^{Z}_{p} - 0.6(k^{\rm NR}_{\sigma\phi})^{Z}_{e}\right| < 2\times 10^{-5}\ \mathrm{GeV}$$

$$\left|(k^{\rm NR}_{\sigma\phi pp})^{ZJJ}_{p} + 0.3(k^{\rm NR}_{\sigma\phi pp})^{JJZ}_{p}\right| < 7\times 10^{-3}\ \mathrm{GeV}^{-1}$$

Duan *et al.*, PRL 117,023001(2016)
Kostelecký, Li, PRD 104,044054(2021)

Dropping ^{88}Sr and ^{87}Sr atoms

- ^{88}Sr: spin-0, bosonic
- ^{87}Sr: spin-9/2, fermionic
- Broadening of measured g for unpolarized ^{87}Sr
- Neutron sector
- Sensitivity: $\Delta g/g \lesssim 10^{-7}$

- Constraints on coefficients for LV:

$$\left|(k^{\rm NR}_{\sigma\phi})^{Z}_{n}\right| < 1\times 10^{-4}\ {\rm GeV}$$

$$\left|(k^{\rm NR}_{\sigma\phi pp})^{ZJJ}_{n} - 0.4(k^{\rm NR}_{\sigma\phi pp})^{ZZZ}_{n}\right| < 5\times 10^{-2}\ {\rm GeV}^{-1}$$

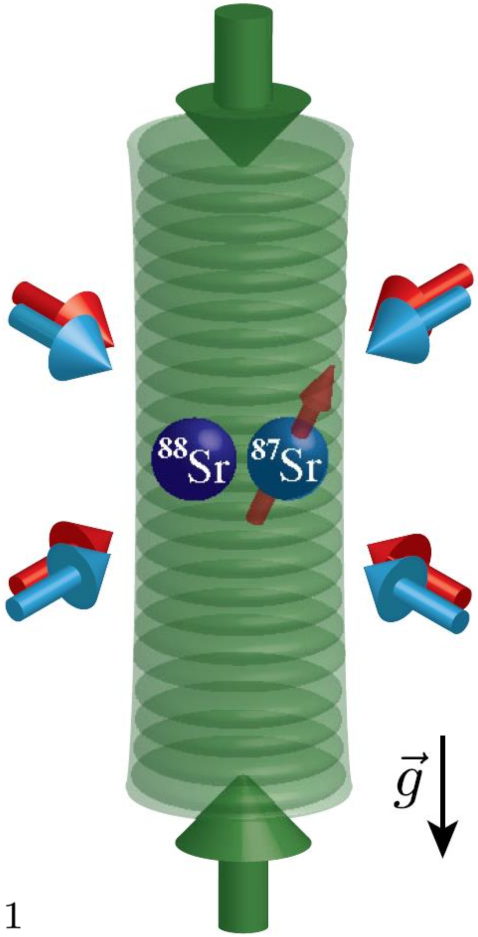


Image: Tarallo *et al.*, PRL'2014

Tarallo *et al.*, PRL 113,023005(2014)
Kostelecký, Li, PRD 104,044054(2021)

Colella-Overhauser-Werner (COW) experiment

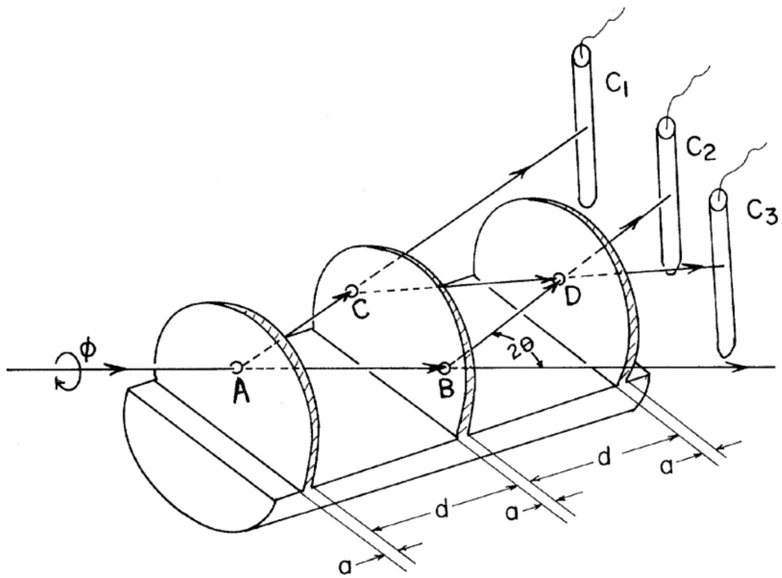


Image: Colella, Overhauser, Werner, PRL'1975

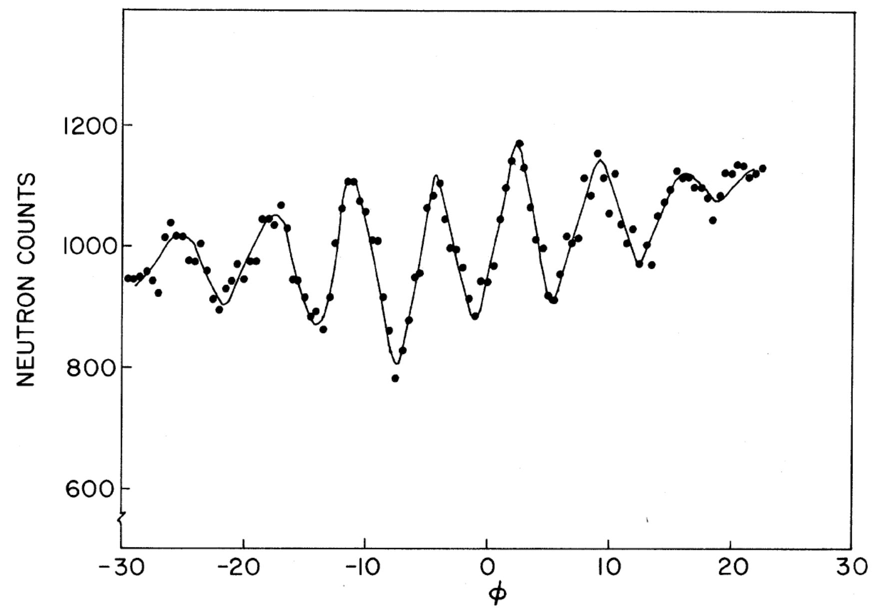


Image: Colella, Overhauser, Werner, PRL'1975

- Gravity-induced phase shift depending on coefficients for LV
- Sensitivity and constraints on coefficients for LV:

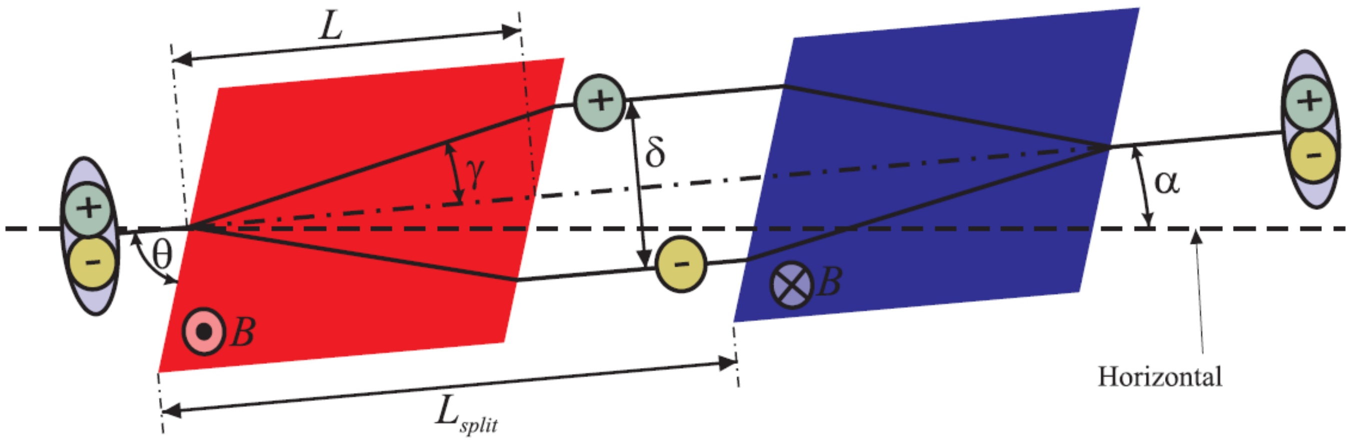
$$\Delta g/g = (k_\phi^{\mathrm{NR}})_n/m_n \qquad |(k_\phi^{\mathrm{NR}})_n| \lesssim 10^{-1}\ \mathrm{GeV}$$

↑

Spin-independent

Colella, Overhauser, Werner, PRL 34,23(1975)
Kostelecký, Li, PRD 104,044054(2021)

OffSpec experiment



- Neutron beams split by magnetic fields
- Phase shift depending on the coupling of gravity and spins
- Constraints on coefficients for LV:

$$\left|(k_{\phi}^{\mathrm{NR}})_n + (k_{\sigma\phi}^{\mathrm{NR}})_n^j \hat{s}^j\right| < 2.5\times 10^{-2}\ \mathrm{GeV}$$

Spin-independent

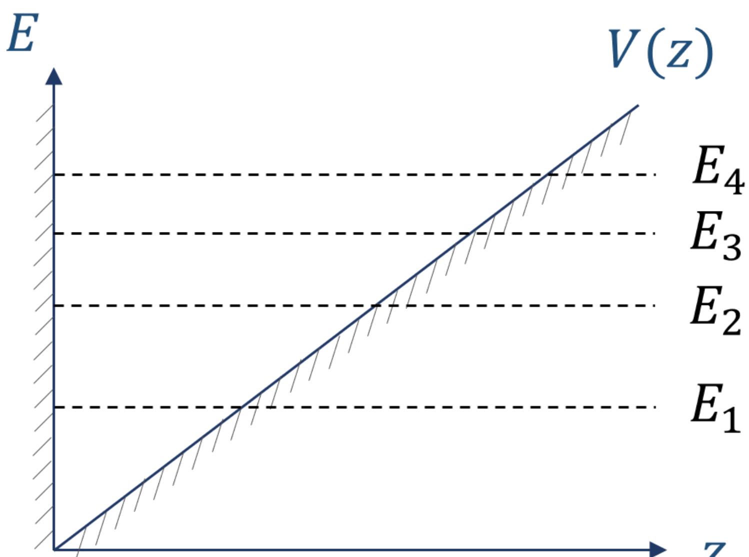
Spin-dependent

Initial spin of the neutron

de Haan *et al.*, PRA 89,063611(2014)
Kostelecký, Li, PRD 104,044054(2021)

Bound states of neutrons in the Earth's gravitational field

Linear potential



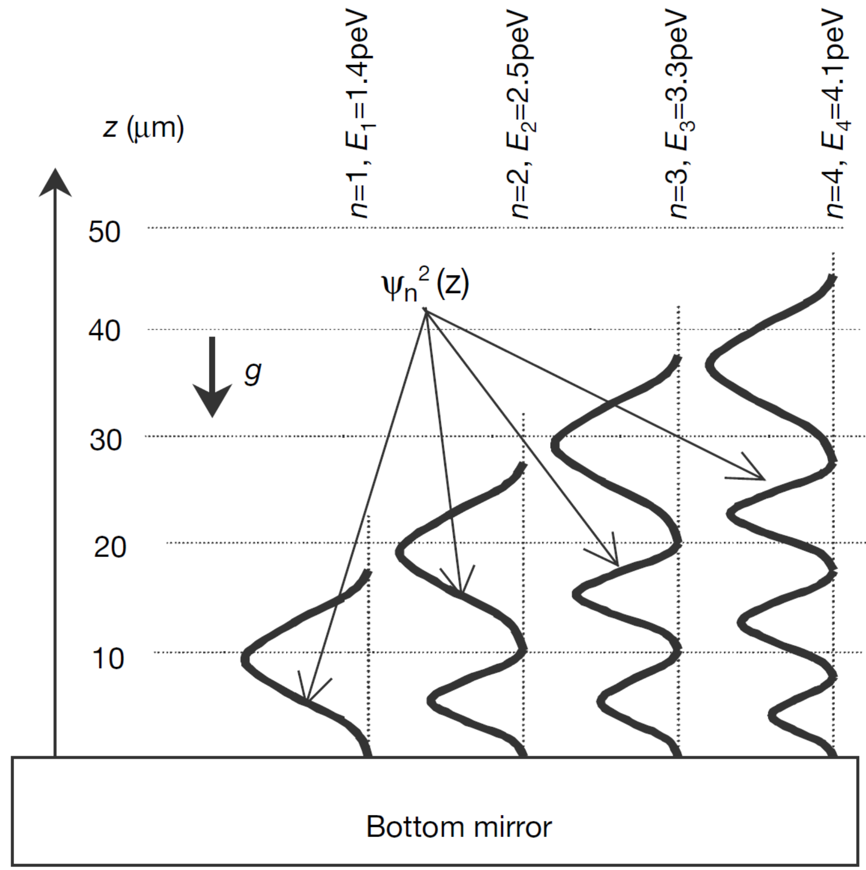
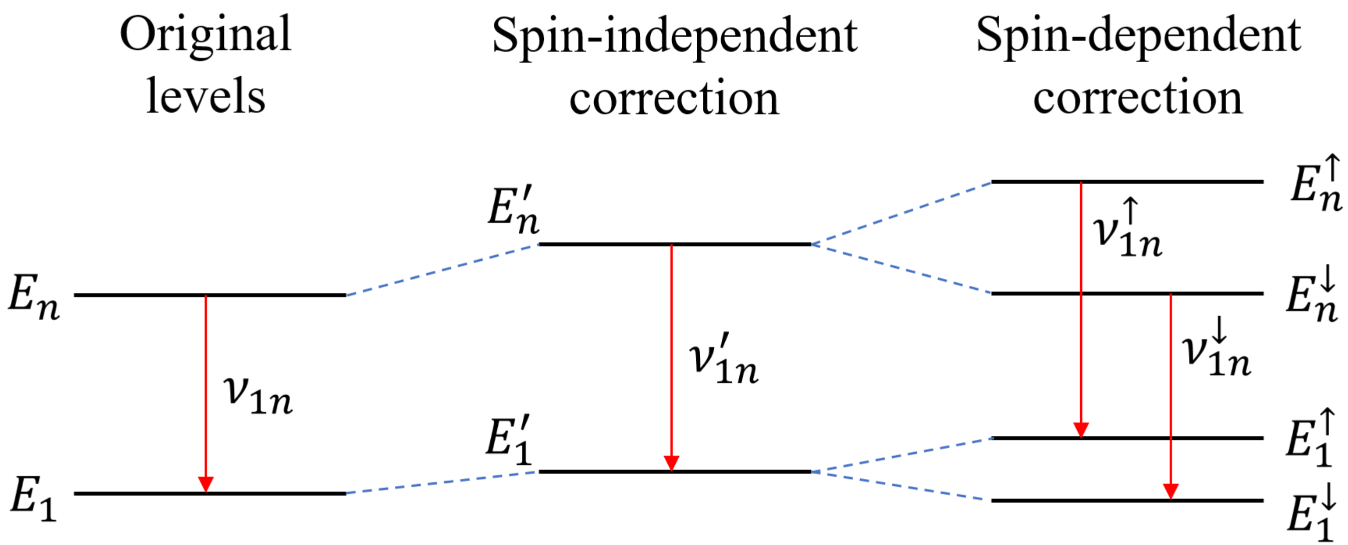


Image: Nesvizhevsky *et al.*, Nature'2002

Nesvizhevsky *et al.*, Nature 415,297(2002)

Transition frequencies



Spin-independent　Spin-dependent

- Corrections:

$$\Delta E_n - \Delta E_1 = \frac{2}{3}\frac{(k_\phi^{\mathrm{NR}})_n \pm \sqrt{\left[(k_{\sigma\phi}^{\mathrm{NR}})_n^j\right]^2}}{m}(E_n - E_1)$$

- Constraints on coefficients for LV:

$$\left|(k_\phi^{\mathrm{NR}})_n\right| < 10^{-3}\ \mathrm{GeV} \qquad \sqrt{\left[(k_{\sigma\phi}^{\mathrm{NR}})_n^j\right]^2} < 7.8\times 10^{-3}\ \mathrm{GeV}$$

Cronenberg *et al.*, Nat.Phys. 14,1022(2018)
Ivanov et al., PLB 136,640(2021)
Kostelecký, Li, PRD 104,044054(2021)

Constraints on coefficients for LV from various types of experiments

Spin-independent

Sector		Free-dropping	Interferometer	Bound-state
Proton sector	$(k_{\phi}^{\mathrm{NR}})_p$			
	$(k_{\sigma\phi}^{\mathrm{NR}})_p^J$	10^{-5}		
Neutron sector	$(k_{\phi}^{\mathrm{NR}})_n$		10^{-1}	10^{-3}
	$(k_{\sigma\phi}^{\mathrm{NR}})_n^J$	10^{-4}		10^{-2}
Electron sector	$(k_{\phi}^{\mathrm{NR}})_e$			
	$(k_{\sigma\phi}^{\mathrm{NR}})_e^J$	10^{-5}		

Spin-dependent

Kostelecký, Li, PRD 104,044054(2021)

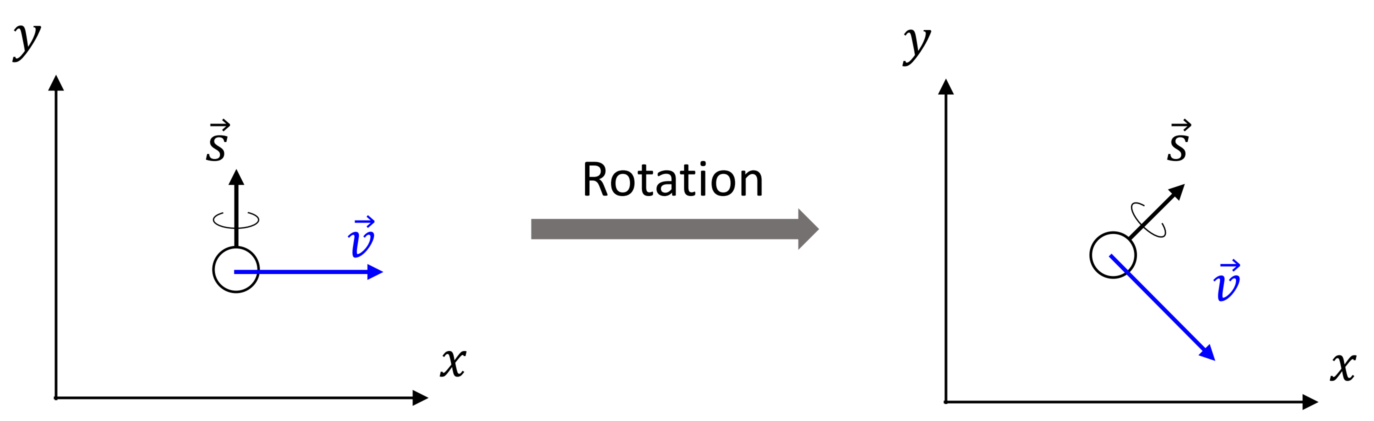
Ivanov et al., PLB 136,640(2021)

Summary

- The unification of the Standard Model and General Relativity
  may lead to small deviations from Lorentz symmetry.
- The Standard-Model Extension is developed
  to study Lorentz violation (LV) in the context of effective field theory.
- The general form of Lorentz-violating terms in curved spacetime
  and the limits in the Earth's gravitational field
  are obtained for the first time.
- Three types of experiments,
  free-dropping, interferometer, and bound-state experiments,
  are analyzed to extract first constraints
  on certain coefficients for LV and spin-gravity couplings.
- More experiments are needed for complete coverage of the coefficients.

Local Lorentz symmetry
and diffeomorphism invariance

- Physical rules are invariant under local Lorentz transformations and diffeomorphisms
- Experimental results are independent of the orientation and velocity of the laboratory
- Fundamental symmetries in known physics



Sun-centered frame and sidereal dependence

- Coefficients for LV are normally assumed approximately constant in the Sun-centered frame
  - Z axis: aligns with the rotational axis of the Earth
  - X axis: points from the Earth to the Sun at 2000 vernal equinox
  - Y axis: forms a right-handed system with X and Z axes
  - T axis: origin set at the 2000 vernal equinox
- Experimental results depend on the sidereal time

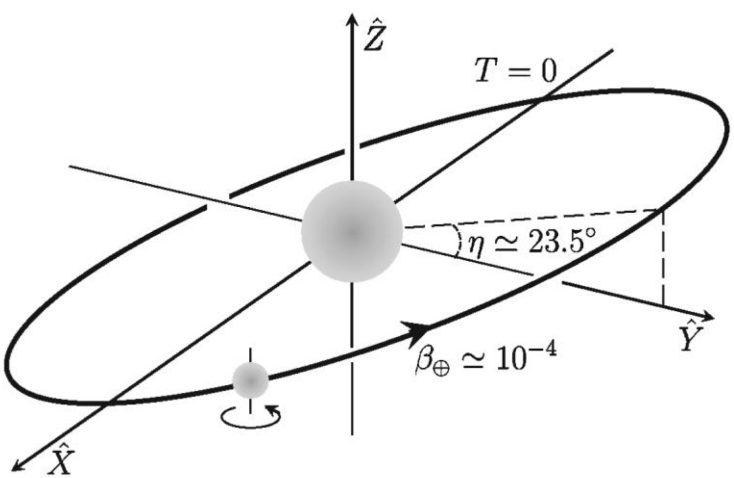


Image: Kostelecky, Russell, Rev.Mod.Phys. 83.11(2011)

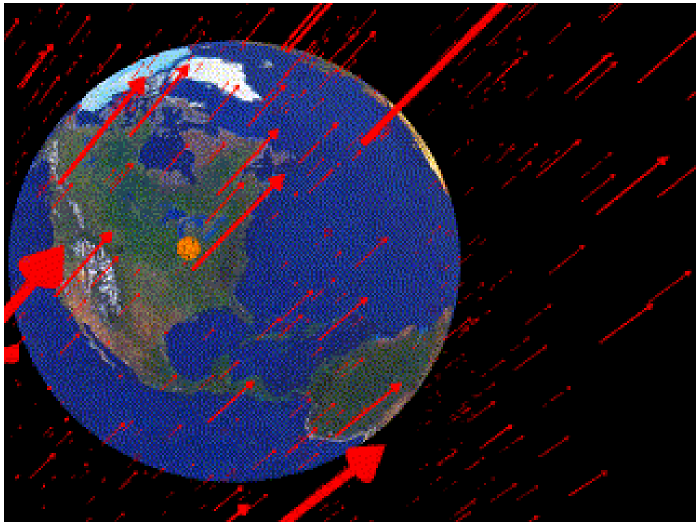


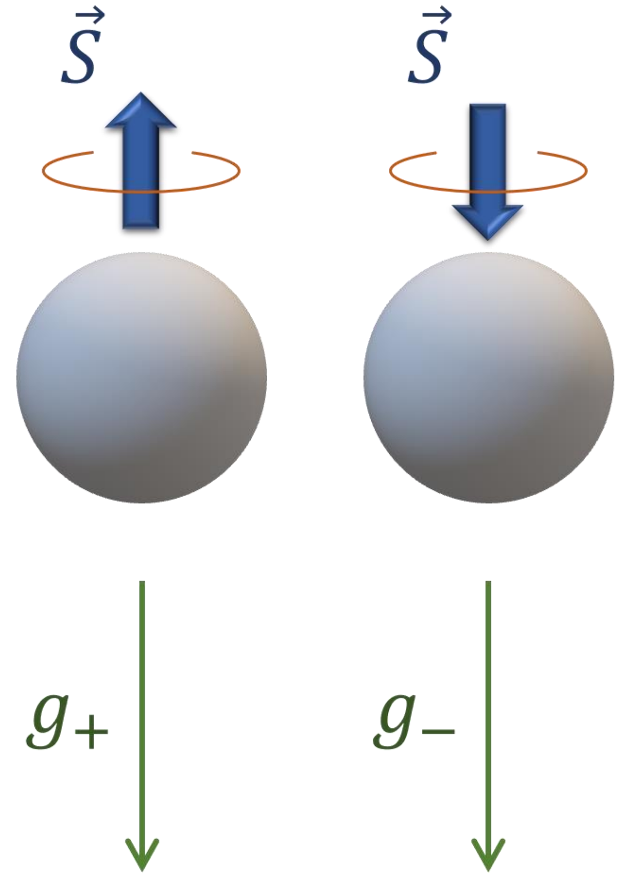
Image: www.physics.indiana.edu/~kostelec

Free-dropping experiments

- Atoms with different spins
- Gravitational accelerations (g)
- Precision measurement

Sensitive to

- Weak Equivalence Principle (WEP)
- Spin-gravity coupling
- Lorentz violation



Dropping hydrogen and antihydrogen

- Symmetry between particles and antiparticles
- Plans proposed by many groups
- Contributions from the SME:

spin

$$\frac{g_{\text{eff}}}{g} = 1 + \frac{1}{m} \sum_{\omega} \left[(k_{\phi}^{\text{NR}})_{\omega} + m_F (k_{\sigma\phi}^{\text{NR}})_{\omega}^{3} \right]$$

$\omega = e, p$

Different for hydrogen and antihydrogen

S. Aghion *et al.*, Nat. Commun. 5,4538(2014)
C. Amole *et al.*, Phys. Rev. Lett. 112,121102(2014)
P. Indelicato *et al.*, Nat. Commun. 4,1787(2013)
Kostelecký, Li, PRD 104,044054(2021)

Colella-Overhauser-Werner (COW) experiment

- Gravity-induced phase shift
- Quantum effect of gravity

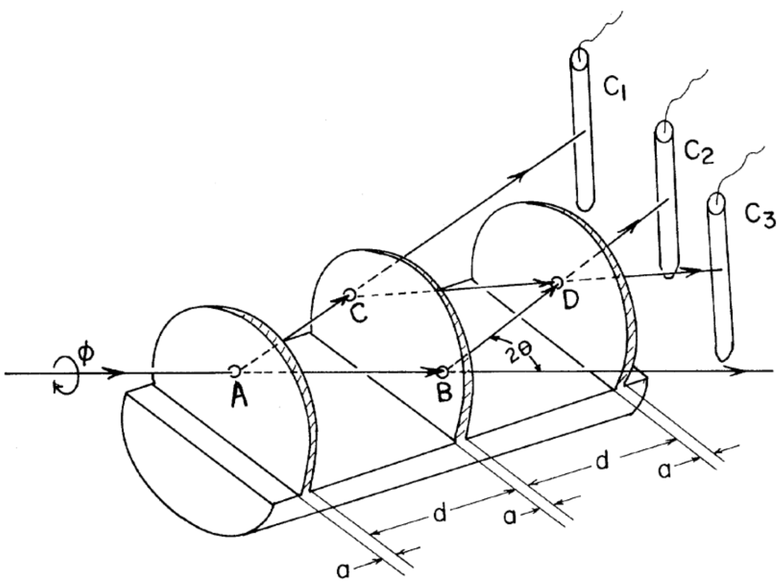


Image: Colella, Overhauser, Werner, PRL’1975

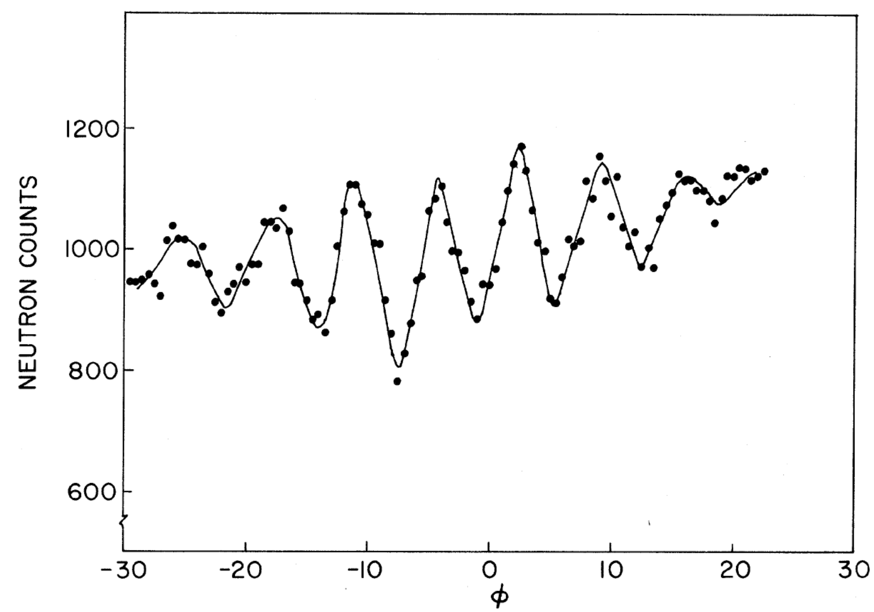
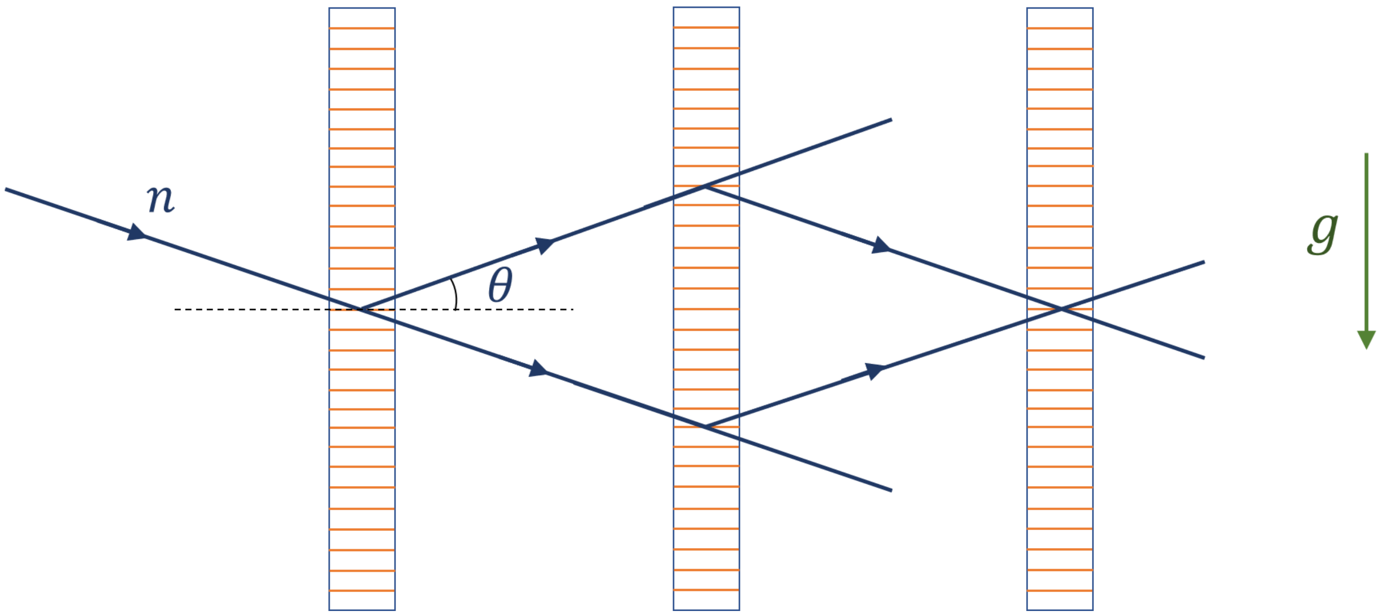


Image: Colella, Overhauser, Werner, PRL’1975

Colella, Overhauser, Werner, PRL 34,23(1975)

COW experiment



- Phase shift depending on coefficients for LV
- Sensitivity and constraints on coefficients for LV:

$$\Delta g/g = (k_\phi^{\mathrm{NR}})_n/m_n \qquad |(k_\phi^{\mathrm{NR}})_n| \lesssim 10^{-1}\ \mathrm{GeV}$$

↑
Spin-independent

Kostelecký, Li, PRD 104,044054(2021)

Critical heights

- Relation with energy:

$$mgz_n = E_n$$

- Corrections from the SME:

$$z'_n = \left(1 - \frac{(k_\phi^{\mathrm{NR}})_n}{3m} \mp \frac{\sqrt{\left[(k_{\sigma\phi}^{\mathrm{NR}})_n^j\right]^2}}{3m}\right) z_n$$

Spin-independent   Spin-dependent

- Constraints on coefficients for LV:

$$\left|(k_\phi^{\mathrm{NR}})_n\right| < 8.2 \times 10^{-1}\ \mathrm{GeV}$$

$$\sqrt{\left[(k_{\sigma\phi}^{\mathrm{NR}})_n^j\right]^2} < 5.4 \times 10^{-1}\ \mathrm{GeV}$$

Experimental setup

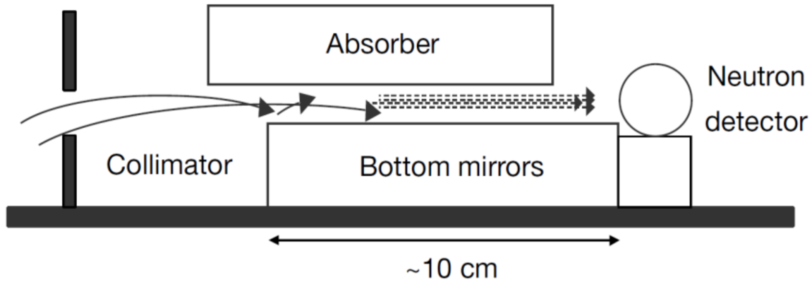


Image: Nesvizhevsky *et al.*, Nature'2002

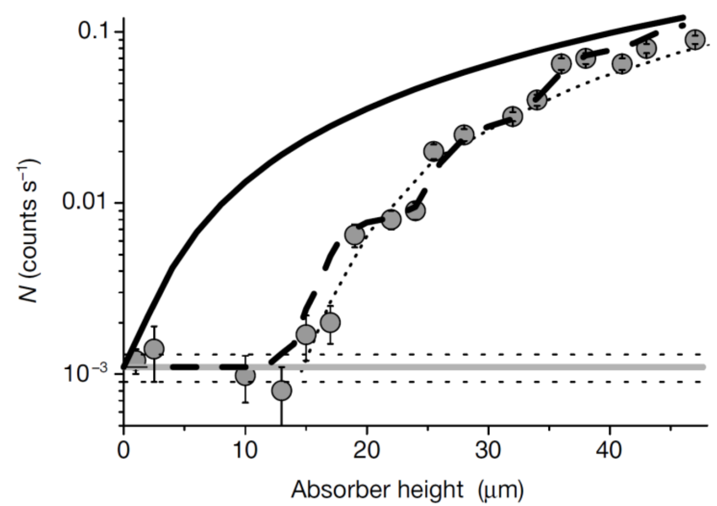


Image: Nesvizhevsky *et al.*, Nature'2002

Nesvizhevsky *et al.*, Nature 415,297(2002)
Kostelecký, Li, PRD 104,044054(2021)