Underdamped Axionic Blue Isocurvature Perturbations

Sai Chaitanya Tadepalli

Advisor: Daniel J. H. Chung

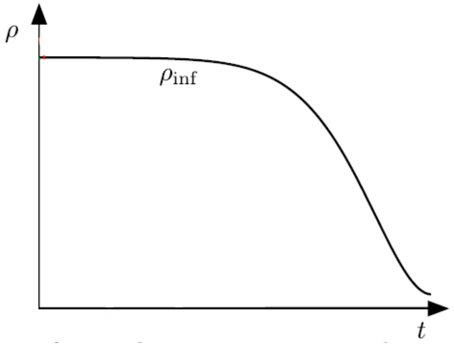
University of Wisconsin-Madison

30th April 2022

based on arxiv:2110.02272

Layout of the talk

1. Isocurvature fluctuations
2. Axion model and blue power spectrum
3. Underdamped axionic system
4. Key results
5. Prospects



Our story begins during inflation…

The simplest model: a single scalar “inflaton” field

Quantum fluctuations

Adiabatic

$$\Phi(k, \eta \to 0) = \Phi^i(k)$$
$$\delta_S(k, \eta \to 0) = 0$$

$$\delta\phi \to \delta n_\gamma$$
$$\delta\phi \to \delta n_\chi$$

Isocurvature

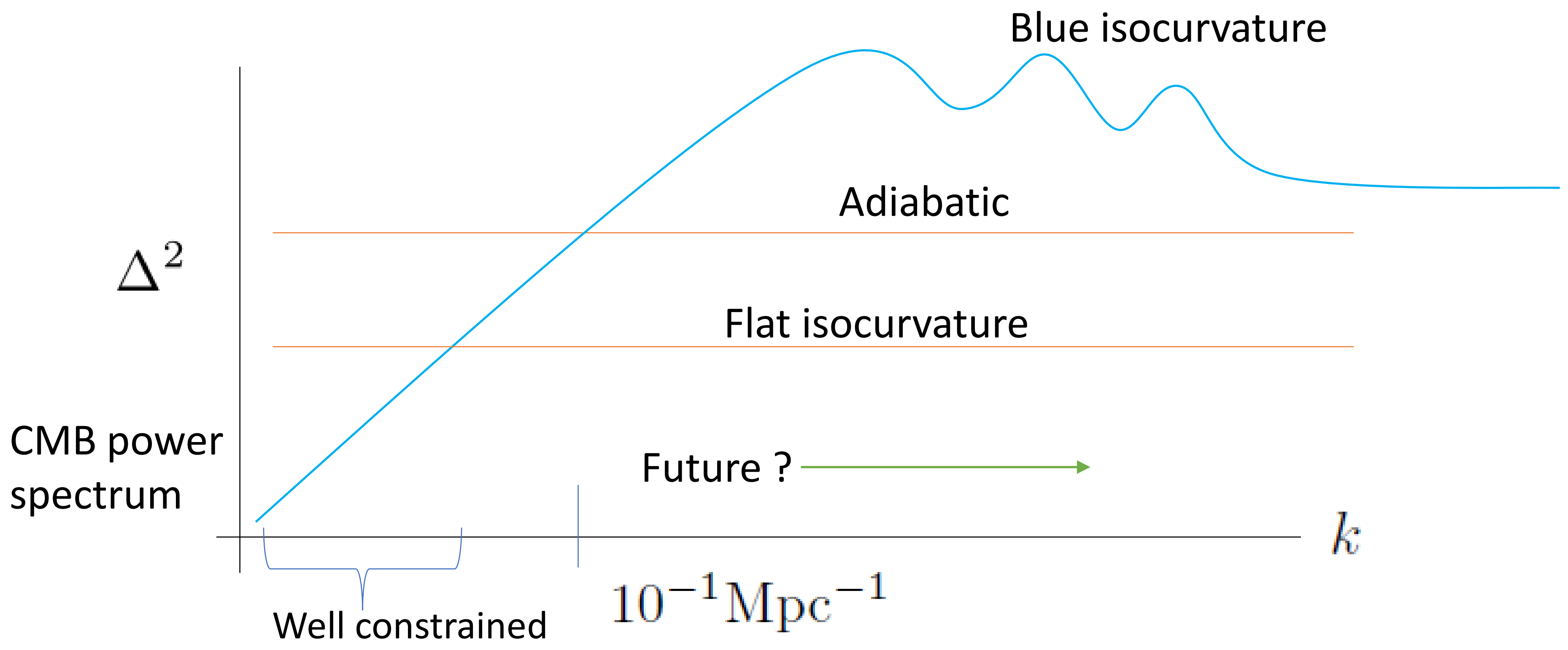
$$\Phi(k, \eta \to 0) = 0$$
$$\delta_S(k, \eta \to 0) = \delta_S^i$$

$$S_{XY} = \frac{\delta n_X}{n_X} - \frac{\delta n_Y}{n_Y}$$

Baumman notes, Dodelson, Kolb-Turner, Weinberg etc

Current bounds on IC perturbations

scale invariant isocurvature perturbations are observationally constrained to be less than 2% on large (CMB) scales at k=0.05/Mpc. [1807.06211]



Axions

1. The PQ solution to Strong CP problem (by Peccei-Quinn) -> elevate θ to a dynamical field associated with a U(1) symmetry.

2. Axial direction remains flat giving rise to PNG boson: **axions**

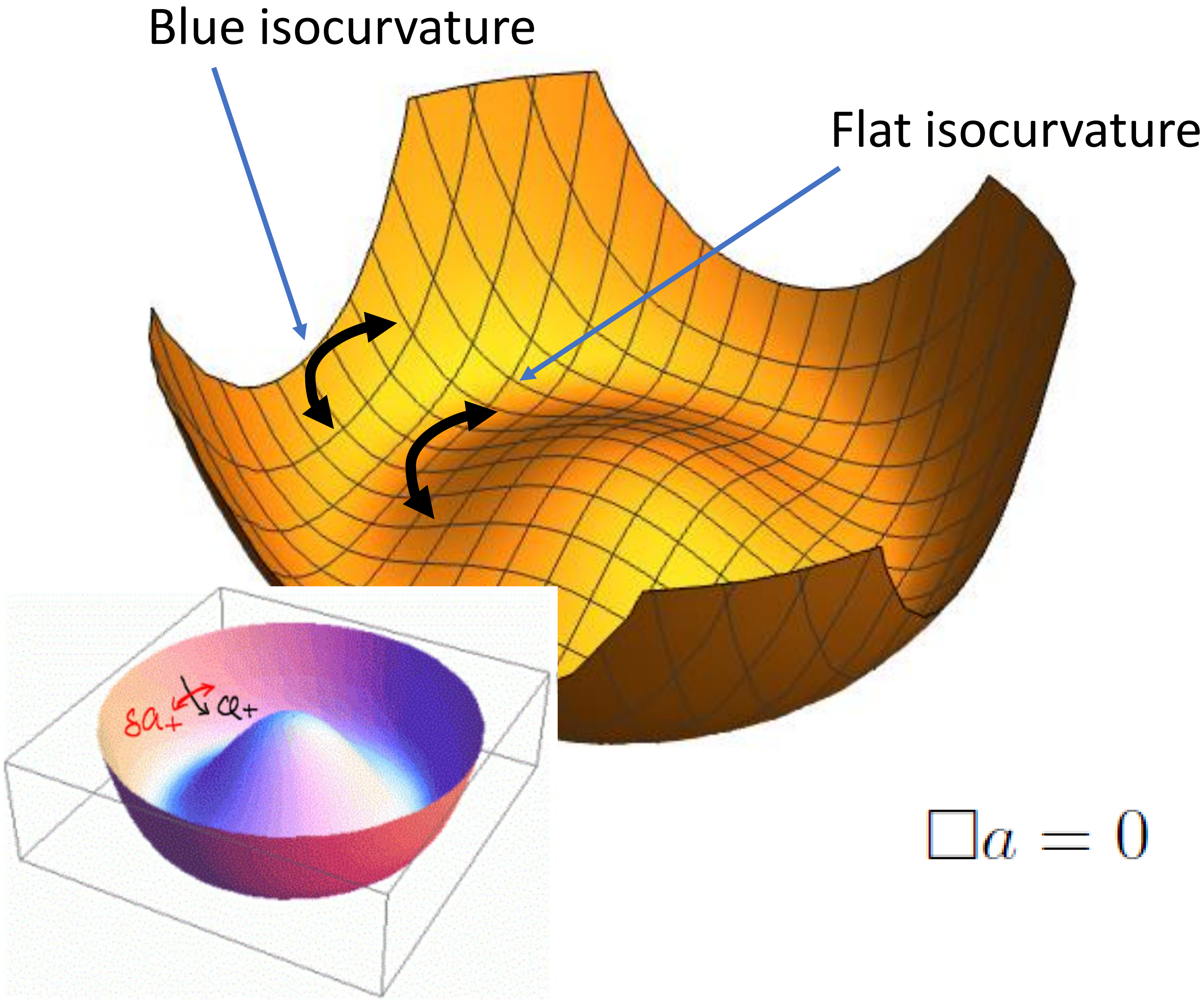
$$\phi = |\phi| e^{i\theta_a} = |\phi| e^{ia/\eta}.$$

$$\mathcal{L} = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a},$$

$$\mathcal{L} = \frac{g_s^2}{32\pi^2} \frac{a}{F_a} G^a_{\mu\nu} \tilde{G}^{\mu\nu a},$$

How to generate blue-tilted spectra ?

S. Kasuya, M. Kawasaki [0904.3800]



$$\phi = |\phi| e^{i\theta_a} = |\phi| e^{ia/\eta}.$$

$$\frac{\delta a}{a} \sim \frac{H/2\pi}{\Phi\theta}$$

$$\Phi : O(M_{\rm Pl}) \to f_{\rm PQ}$$

Dynamical non-equilb mass

$$\Box a = 0 \xrightarrow{\text{1501.05618}} \left(\Box - \frac{\Box\varphi_+}{\varphi_+}\right) a = 0$$

Radial potential $\phi_\pm$

$$\phi = |\phi| e^{i\theta_a} = |\phi| e^{ia/\eta}.$$

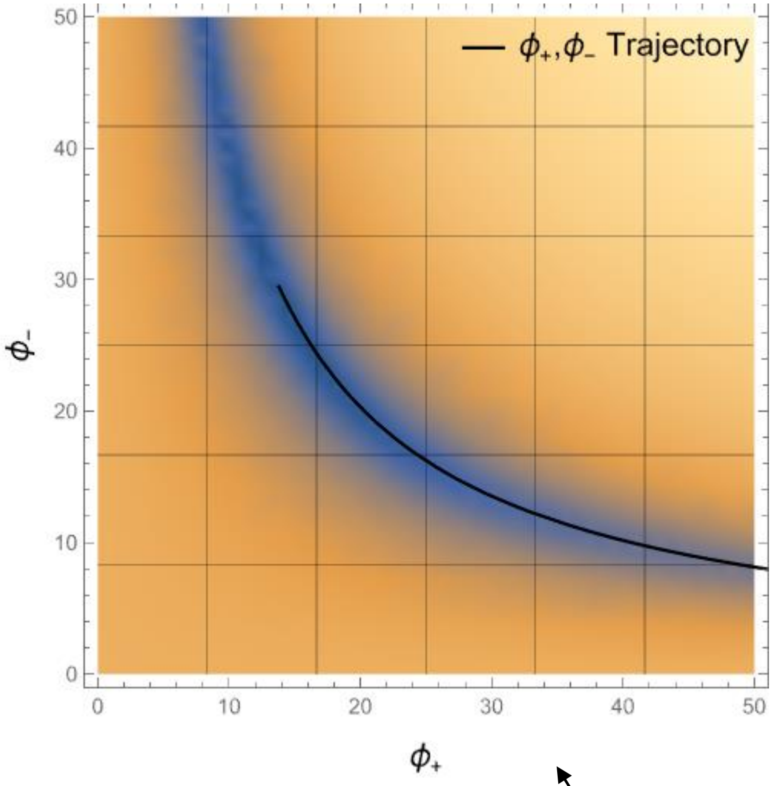
$$V = \frac{1}{2} c_+ H^2 \left|\Phi_+\right|^2 + \frac{1}{2} c_- H^2 \left|\Phi_-\right|^2 + \frac{1}{2} \left|\Phi_+ \Phi_- - F_a\right|^2$$

c+ > 9/4 is the underdamped situation

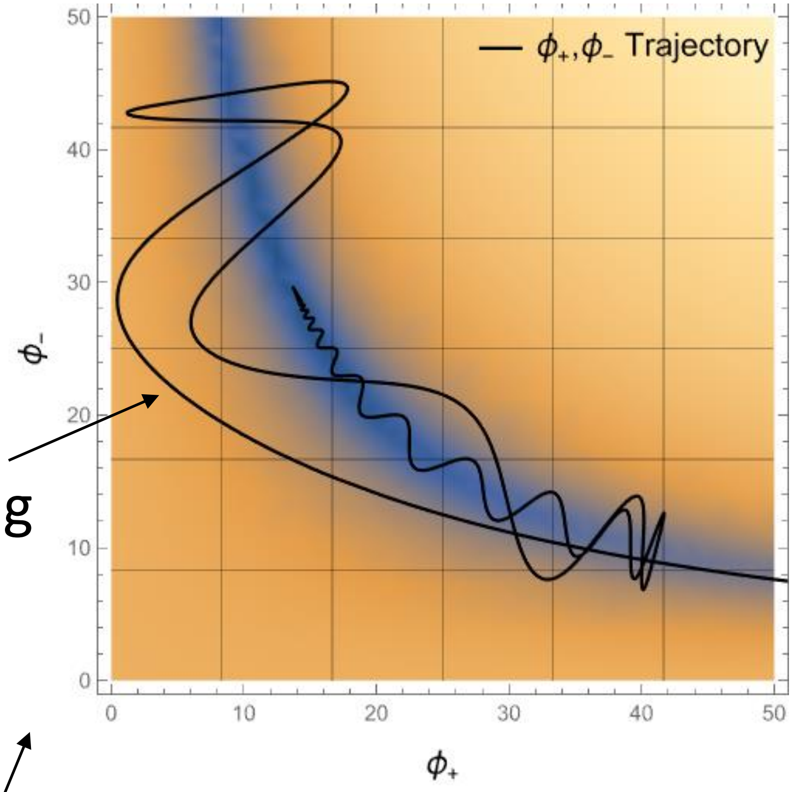
<u>Cubic non-linearity</u>

Complicated dynamical behaviour: highly non-trivial

Overdamped
c+ = 9/4 – 0.1



Underdamped
c+ = 9/4 + 0.1



overshooting

$$\phi_+ \sim \exp\left(-\frac{3}{2}T + T\sqrt{\frac{9}{4} - \frac{m_a^2}{H^2}}\right)$$

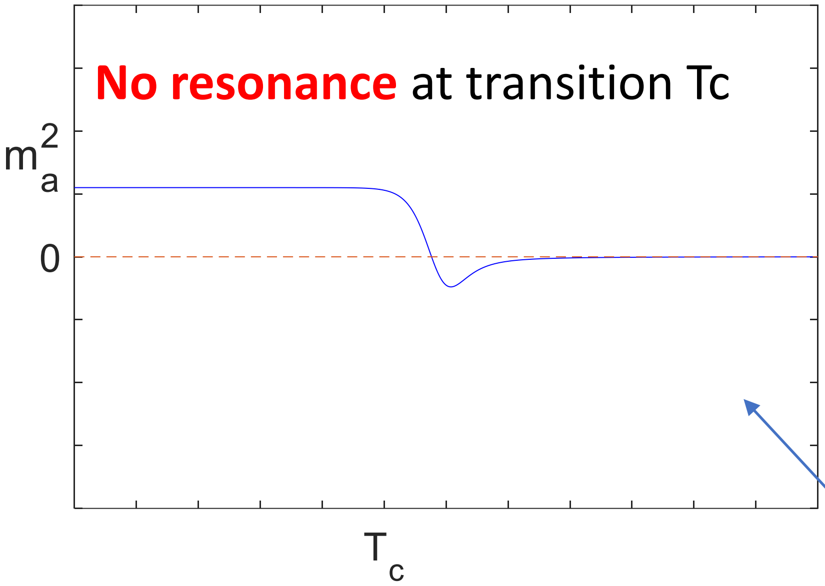
Gravity driven or Potential driven

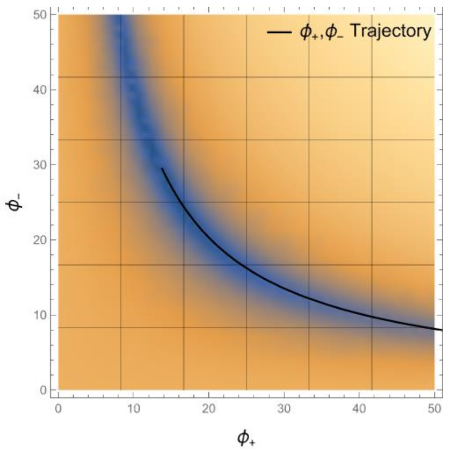
$$\phi_+ \sim \exp\left(-\frac{3}{2}T\right)\cos\left(T\sqrt{\frac{m_a^2}{H^2} - \frac{9}{4}} + \varphi\right)$$

Large kinetic energy (due to cos function) makes a difference at transition

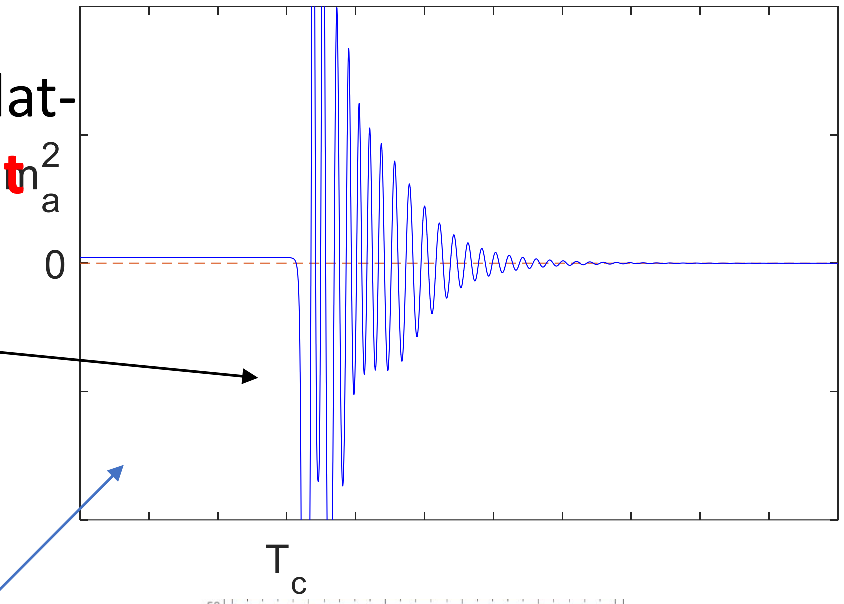
New resonant behavior

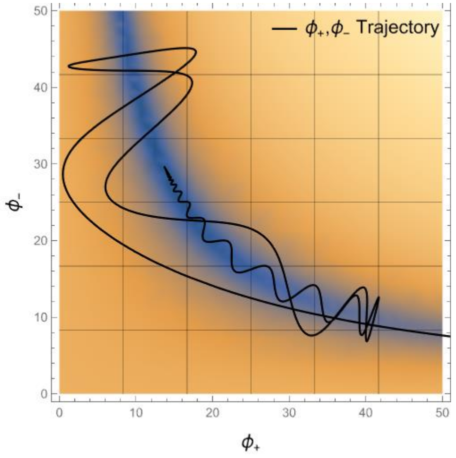
Overdamped
c+ = 9/4 – 0.1





Underdamped
c+ = 9/4 + 0.1





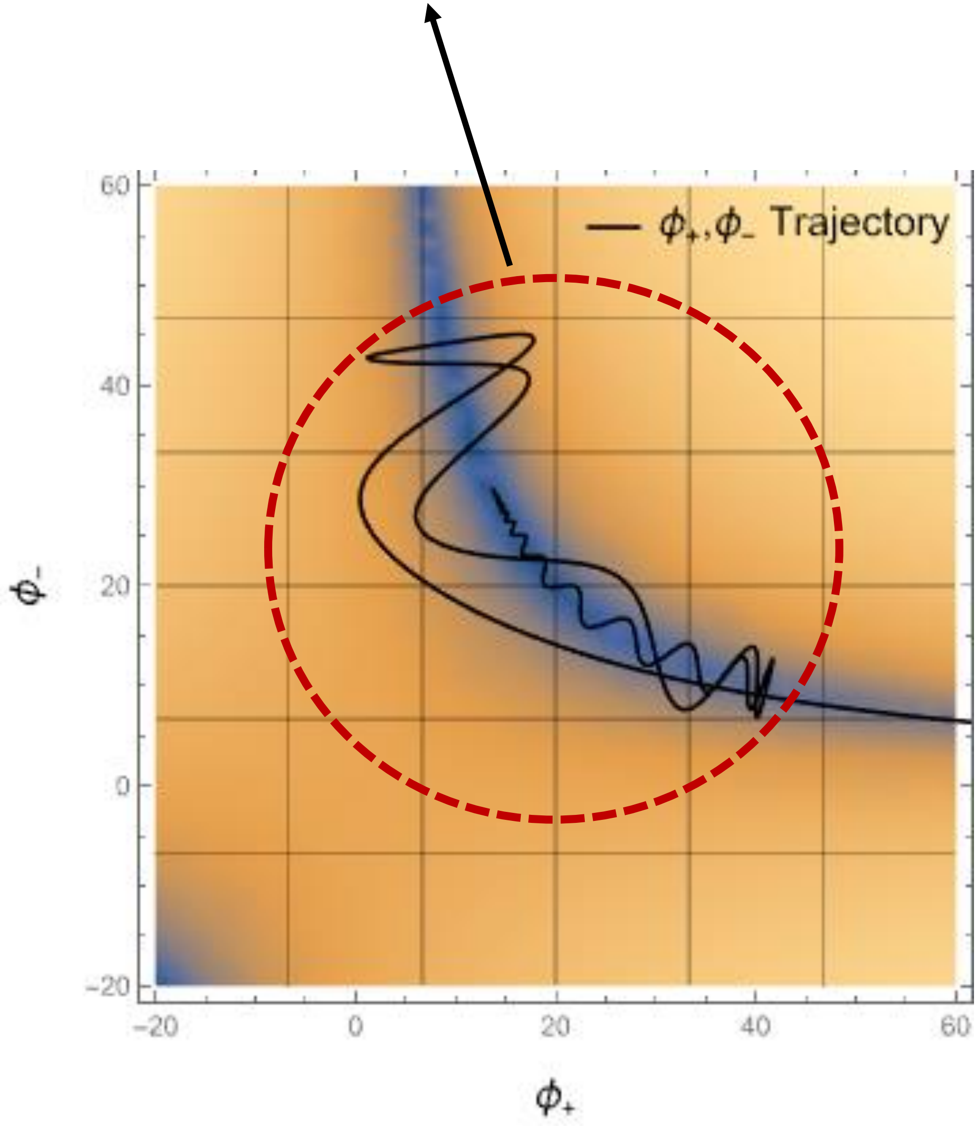
Significant deviation from flat-direction results in **resonant behavior** at transition Tc

$$\left(\Box - \frac{\Box\varphi_+}{\varphi_+}\right) a = 0$$

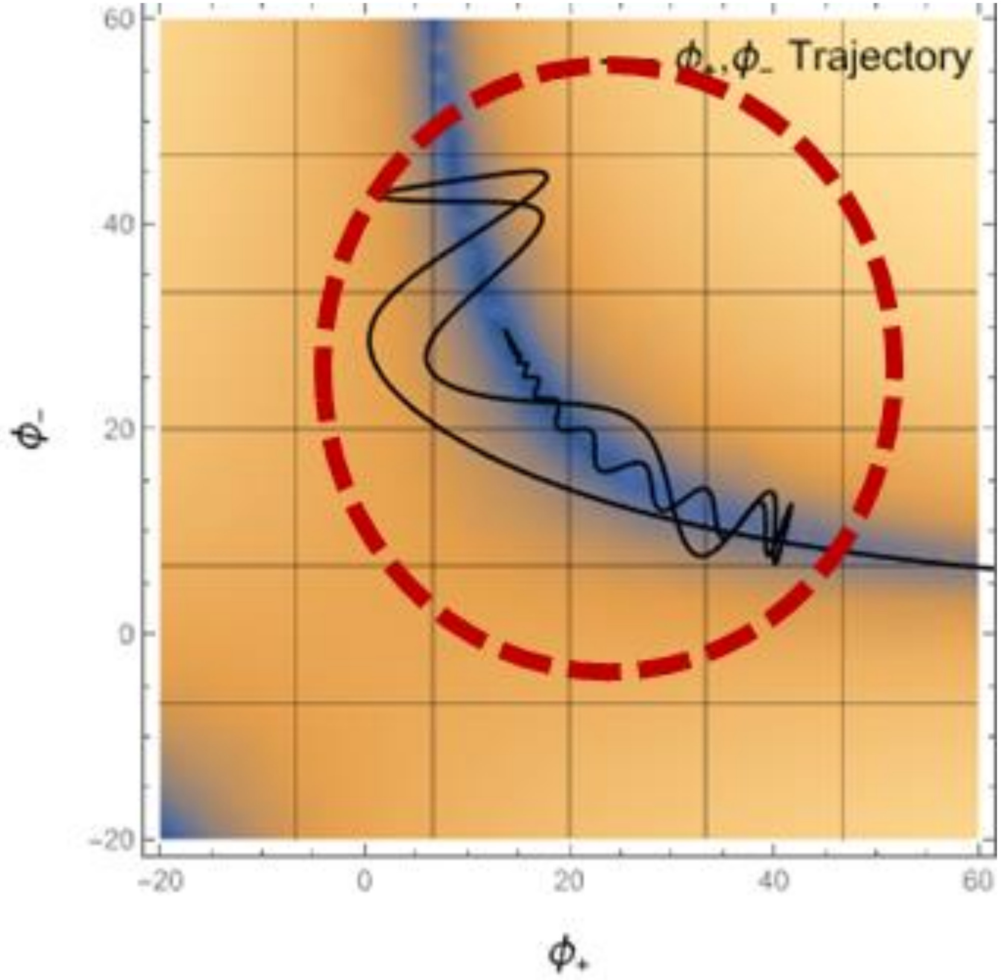
Analytic methods

- Decoupling
- Perturbation theory
- Polynomial analytic fits
- Nonlinear field redefinition
- Effective time-space potential (ETSP)
- Piecewise mass-model

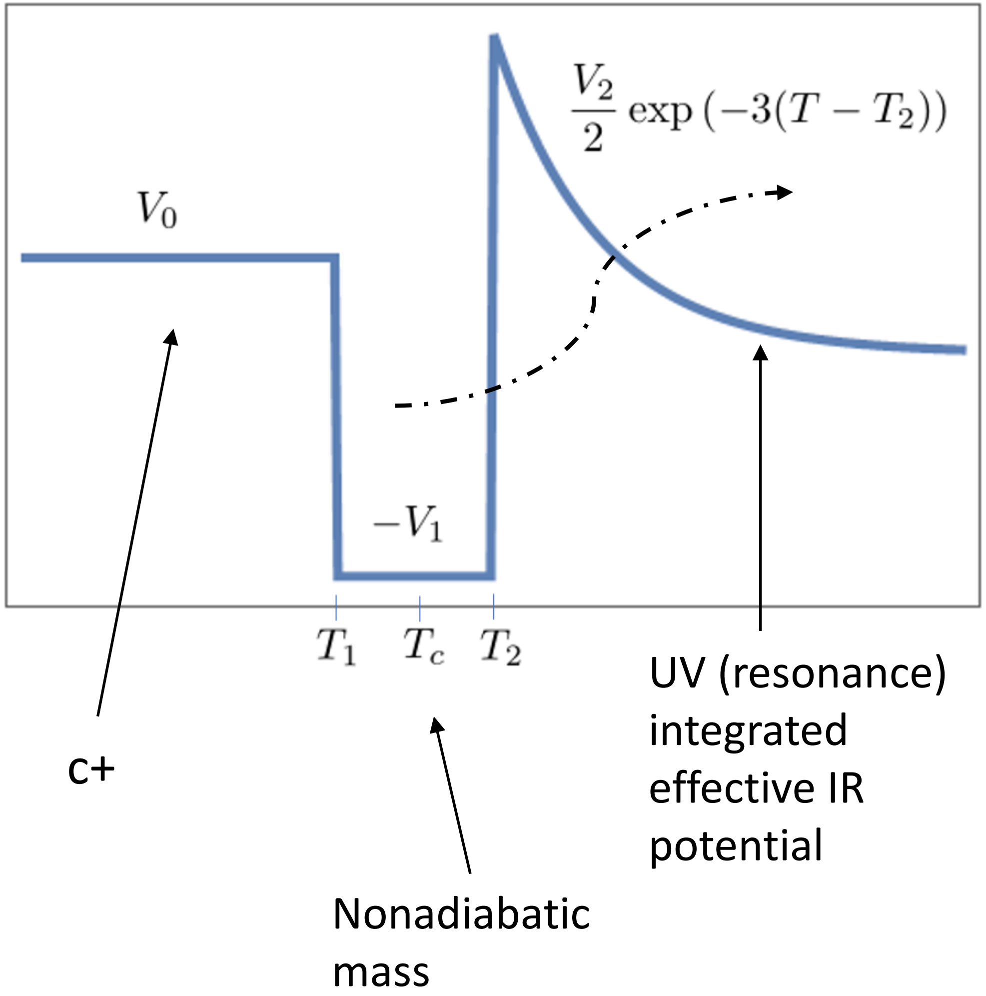
Complexity invites approximation for analytic solvability



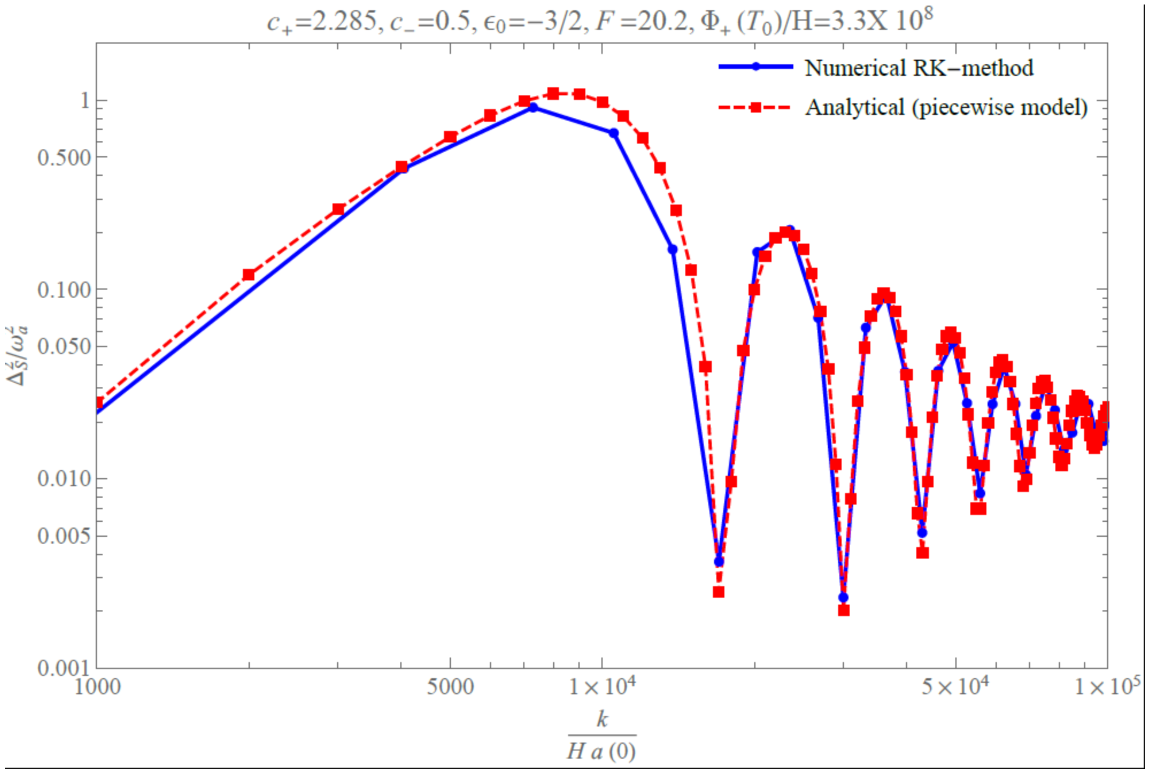
Key physics: Scattering of tachyonic quantum modes as they exit the well

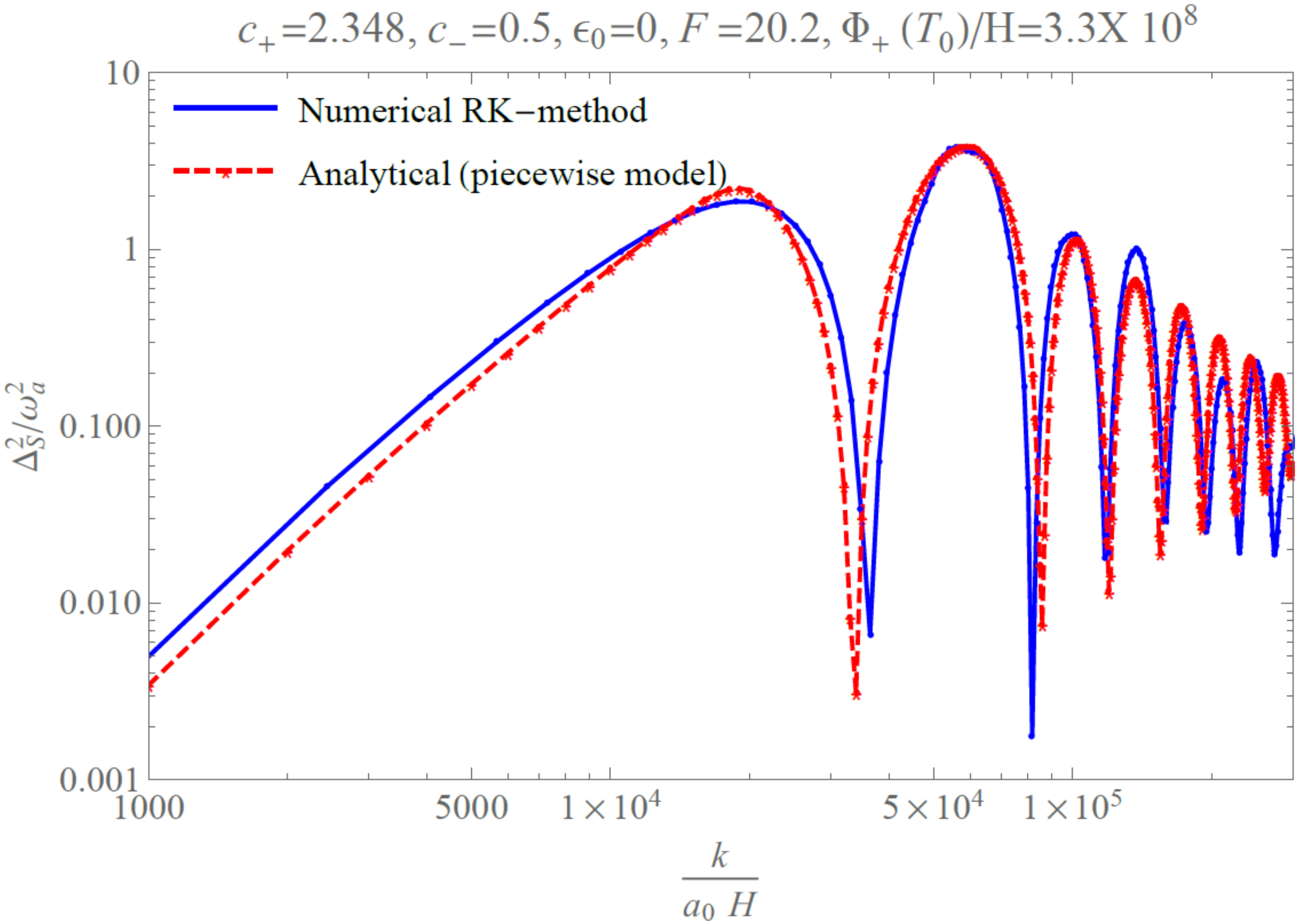


Analytical framework



Comparison of numerically generated and analytically solved axion isocurvature spectrum





Successful in generating key features of the spectrum.

Final results are intricate as expected (but stories can be told through them):

For resonant, not too energetic case:

$$\Delta_S^2(K) \approx |f_{\text{correction}}(K)|^2 \times \begin{cases} C_1 K^3 \left|1 - i\frac{\Gamma(i\omega)\Gamma(i\omega+1)}{\pi(1+i\cot(i\omega\pi))} e^{-2i\omega\ln\left(\frac{-K\tau_1}{2}\right)}\right|^2 & -K\tau_c \ll 1 \\ C_2\mathfrak{D}^2 \left|H^1_{i\omega}(-K\tau_1)\right|^2 (-K\tau_2)\left(\sin(-K\tau_2) \right. & \\ \left. + (3/2 + b\tanh[-b\Delta T])\left(\frac{\cos[-K\tau_2]}{-K\tau_2} - \frac{\sin[-K\tau_2]}{(-K\tau_2)^2}\right)\right)^2 & 0.5 \lesssim -K\tau_c < 3 \\ C_3\mathfrak{D}^2\cosh^2[b\Delta T] \times & \\ \left|(-ie^{iK\tau_2}) + \tanh[-b\Delta T] \times \right. & \\ \left.\left(\left(\frac{b}{-K\tau_2}\right)\cos[-K\tau_2] + \left(i\frac{-K\tau_2}{b}\right)\sin[-K\tau_2]\right)\right|^2 & 3 \lesssim -K\tau_c < K_2 \\ C_4 \times 1 & K > K_P \end{cases}$$

$$\omega \equiv \sqrt{c_+ - 9/4}$$

$$C_1 \approx C\mathfrak{D}^2 \frac{\pi}{8} e^{-\omega\pi} \cosh^2[b\Delta T] \frac{e^{-3T_2}}{3}\left(\frac{3}{2} - b\tanh[-b\Delta T]\right)^2 \left|\frac{1 + i\cot(i\omega\pi)}{\Gamma(i\omega+1)}\right|^2$$

$$C_2 \approx C\frac{\pi}{8} e^{-\omega\pi}\cosh^2[b\Delta T]$$

$$C_3 \approx C\frac{1}{4}$$

$$C_4 \approx \omega_a^2 \frac{h^2}{2\pi^2\theta_+^2 F^2}\left(\frac{r}{1+r^2}\right)$$

$$C = \omega_a^2 \frac{4}{\pi^2}\frac{r(1+r^4)}{(1+r^2)^3}\frac{h^2}{\theta_+^2 F^2}$$

$$r = \sqrt{c_+/c_-}$$

$$V_B \approx c_- + \frac{1}{(T_L - T_2)}\left(\frac{1063}{3072} + \frac{106793 c_-}{393216 c_+}\right)$$

$$\mathfrak{D} \approx \exp\left(\left(-\frac{3}{2} + \sqrt{\frac{9}{4} - V_B}\right)(T_B - \tilde{T})\right)$$

... more ... → https://pages.physics.wisc.edu/~stadepalli/Blue-Axion-IsoCurvSpec-Underdamped.nb

Blues prospects:

- A **2-sigma** hint from latest evaluations [**1711.06736, 1707.09354**]

- At 10 Mpc^{-1}, the isocurvature power can be 40 times larger than the adiabatic power

- Planck TT,TE,EE+lowE+lensing **[1807.06211]** gives **95 % CL** for a blue index 1.55 < $\mathbf{n_I}$ < 3.67 consistent with the recent findings (C&U, 2017).

- Large room for discovery with future expts like **SKA**, **LSST**, and **Pixie**.

Accessible from: **https://pages.physics.wisc.edu/~stadepalli/file.nb**

Thanks