Flavored Gauge-Mediated Supersymmetry Breaking Models with Discrete Non-Abelian Symmetry

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Based on:

Everett, Garon	1610.09024
Everett, Garon, Rock	1812.10811
Everett, Garon, Rock	1912.12938
Eu, Everett, Garon, Leonard	**2110.05599**

Gauge mediated SUSY breaking (GMSB)

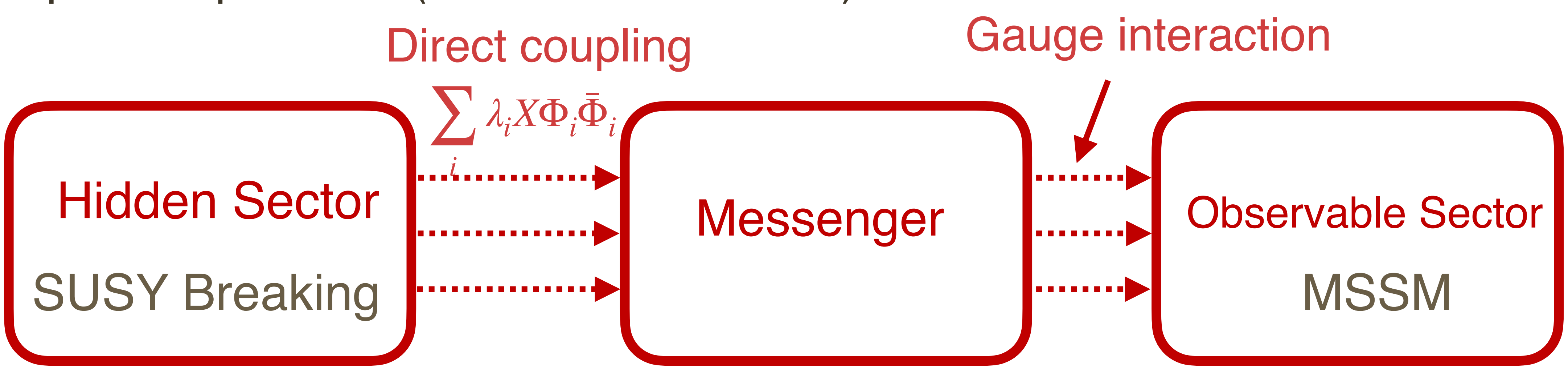
SUSY is a **broken** symmetry in the vacuum state

SUSY cannot be broken spontaneously at tree-level (renormalizable)

Supertrace theorem: $\mathcal{S}\mathrm{Tr}(m^2) = \mathrm{Tr}(m_S^2) - 2\mathrm{Tr}(m_f^2) + 3\mathrm{Tr}(m_V^2) = 0$

Dimopoulos, Georgi (1981)

Require loop effects (non-renormalizable)



SM Singlet X

~~SUSY~~ $\langle F_X \rangle$

Φ_i , $\bar{\Phi}_i$

(has SM quantum number)

Dine, Fischeler, Srednicki (1981)
Dine, Nelson (1993)
Dine, Nelson, Shirman (1996)
Giudice, Rattazzi (1999)

Flavored Gauge Mediation (FGM)

Motivation:

Minimal Gauge mediation : Higgs mass of $\sim 125\,\mathrm{GeV}$ requires heavy stops/ maximal mixing

$$\Delta m_h^2 = \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left(\ln\frac{\tilde{m}_{t1}\tilde{m}_{t2}}{m_t^2} + \frac{X_t^2}{\tilde{m}_{t1}\tilde{m}_{t2}}\left(1 - \frac{X_t^2}{12\tilde{m}_{t1}\tilde{m}_{t2}}\right)\right)$$

$$X_t = A_t - \mu\cot\beta$$

A terms are zero, stops must have masses $> \mathcal{O}(10\,\mathrm{TeV})$

One possible extension: Flavored gauge mediation

- Idea: $SU(2)_L$ doublet messengers mix with MSSM Higgs $H_{u,d}$
- New messenger Yukawa superpotential coupling terms eg. $Y_u Q\bar{u}H_u + Y_u' Q\bar{u}M_u$
- This Higgs-messenger mixing is governed by an imposed symmetry
  eg. $U(1)$ benchmark model by Ierushalmi et al. (2016)

FGM with discrete non-Abelian symmetry $\mathcal{S}_3$

- More constraining and thus more predictive
- $\mathcal{S}_3$ is often used in generation of fermion masses

Perez, Ramond, Zhang (2012)

Extend PRZ'12 work for 2-family scenario to 3 families:

- $\mathcal{S}_3$: Higgs-messenger symmetry + part of family symmetry

	$\mathcal{H}_u^{(2)}$	$\mathcal{H}_u^{(1)}$	$\mathcal{H}_d^{(2)}$	$\mathcal{H}_d^{(1)}$	$Q_{\mathbf{2}}$	$Q_{\mathbf{1}}$	$\bar{u}_{\mathbf{2}}$	$\bar{u}_{\mathbf{1}}$	$\bar{d}_{\mathbf{2}}$	$\bar{d}_{\mathbf{1}}$	$L_{\mathbf{2}}$	$L_{\mathbf{1}}$	$\bar{e}_{\mathbf{2}}$	$\bar{e}_{\mathbf{1}}$	X_H
$\mathcal{S}_3$	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**

- Extension of Higgs-messenger sector: μ and $B\mu$ can be tuned separately
- In the basis

$$Q = (Q_{\mathbf{2}}, Q_{\mathbf{1}})^T = ((Q_{\mathbf{2}})_1, (Q_{\mathbf{2}})_2, Q_{\mathbf{1}})^T, \qquad \overline{u} = (\overline{u}_{\mathbf{2}}, \overline{u}_{\mathbf{1}})^T = ((\overline{u}_{\mathbf{2}})_1, (\overline{u}_{\mathbf{2}})_2, \overline{u}_{\mathbf{1}})^T$$

Superpotential (eg. up quarks)

$$W^{(u)} = \tilde{y}_u Q^T \begin{pmatrix} \mathcal{H}_{u1}^{(2)} & \beta_{1u}\mathcal{H}_u^{(1)} & \beta_{2u}\mathcal{H}_{u2}^{(2)} \\ \beta_{1u}\mathcal{H}_u^{(1)} & \mathcal{H}_{u2}^{(2)} & \beta_{2u}\mathcal{H}_{u1}^{(2)} \\ \beta_{3u}\mathcal{H}_{u2}^{(2)} & \beta_{3u}\mathcal{H}_{u1}^{(2)} & \beta_{4u}\mathcal{H}_u^{(1)} \end{pmatrix} \bar{u}$$

FGM with $\mathcal{S}_3$

$$W^{(u)} = \tilde{y}_u Q^T \begin{pmatrix} \mathcal{H}_{u1}^{(2)} & \beta_{1u}\mathcal{H}_u^{(1)} & \beta_{2u}\mathcal{H}_{u2}^{(2)} \\ \beta_{1u}\mathcal{H}_u^{(1)} & \mathcal{H}_{u2}^{(2)} & \beta_{2u}\mathcal{H}_{u1}^{(2)} \\ \beta_{3u}\mathcal{H}_{u2}^{(2)} & \beta_{3u}\mathcal{H}_{u1}^{(2)} & \beta_{4u}\mathcal{H}_u^{(1)} \end{pmatrix} \bar{u} \qquad Y_u = \frac{\tilde{y}_i}{\sqrt{3}} \begin{pmatrix} 1 & \beta_{1u} & \beta_{2u} \\ \beta_{1u} & 1 & \beta_{2u} \\ \beta_{3u} & \beta_{3u} & \beta_{4u} \end{pmatrix}$$

- Our goal: Achieve realistic quark mass hierarchy at leading order

Need extra structures—relations among β_{iu}

Classification: Different paths to hierarchy

Case 1:

- Singlet-dominated limit

$$\beta_{1u} = 1, \quad \beta_{2u}\beta_{3u} = \beta_{4u}$$

- Democratic limit

All $\beta_{iu} = 1$

Case 2:

- Doublet-dominated limit

$$|\beta_{1u}| \gg \beta_{2u,3u} \gg \beta_{4u} = 0$$

Two different orderings:

$$\beta_{3u} > \beta_{2u}$$
$$\beta_{2u} > \beta_{3u}$$

Case 1: Democratic limit

- All coefficients are equal: $\beta_{1i} = \beta_{2i} = \beta_{3i} = \beta_{4i} = 1$
- MSSM Yukawa matrix: $$Y_i = \frac{\tilde{y}_i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$ Everett, Garon (2018)
- At leading order: 2 vanishing eigenvalues + an $\mathcal{O}(1)$ eigenvalue (3rd gen.)
- Flavor democratic mass matrix with $\mathcal{S}_{3L} \times \mathcal{S}_{3R}$ symmetry

Eu, Everett, Garon, Leonard (2021)

Generate non-zero 1st and 2nd gen. fermion masses:

$$Y_i^{(\mathrm{corr})} = \frac{\tilde{y}_i \epsilon_i}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{\tilde{y}_i \sigma_i}{\sqrt{3}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

Xing (1996)
Fritzsch, Xing (2000)

3 non-vanishing eigenvalues

$$\mathcal{S}_{3L} \times \mathcal{S}_{3R} \Rightarrow \mathcal{S}_{2L} \times \mathcal{S}_{2R} \Rightarrow \mathcal{S}_{1L} \times \mathcal{S}_{1R}$$

These terms can be generated via renormalizable& non-renormalizable superpotential couplings

Estimation of relative strength of $\epsilon_{u,d,e}$ and $\sigma_{u,d,e}$

- Diagonalizing MSSM Yukawa using biunitary diagonalization:

$$(Y_u)^2 = \begin{pmatrix} y_u^2 & 0 & 0 \\ 0 & y_c^2 & 0 \\ 0 & 0 & y_t^2 \end{pmatrix} = \begin{pmatrix} \frac{3\sqrt{3}\sigma_u^2}{2\epsilon_u} & 0 & 0 \\ 0 & -\frac{2\epsilon_u}{3\sqrt{3}} - \frac{8\epsilon_u^2}{27\sqrt{3}} + \frac{56\epsilon_u^3}{243\sqrt{3}} - \frac{3\sqrt{3}\sigma_u^2}{2\epsilon_u} & 0 \\ 0 & 0 & \sqrt{3} + \frac{5\epsilon_u}{3\sqrt{3}} + \frac{8\epsilon_u^2}{27\sqrt{3}} - \frac{56\epsilon_u^3}{243\sqrt{3}} \end{pmatrix}$$

Keep to appropriate subleading orders

- Quark masses: Yukawa couplings multiplied by the appropriate Higgs VEV

$$m_t = \frac{y_t v_u}{\sqrt{2}} = \frac{y_t v \sin\beta}{\sqrt{2}} \qquad m_b = \frac{y_b v_d}{\sqrt{2}} = \frac{y_b v \cos\beta}{\sqrt{2}}$$

where $v_u^2 + v_d^2 = v^2 = (246\ \text{GeV})^2, \quad \tan\beta = \frac{v_u}{v_d}$

- Use the known fermion masses to find the relative strength of the parameters and examine their effects on sparticle spectra

Estimation of CKM matrix elements

$$\epsilon_u \approx 3 \times 10^{-2},\ \sigma_u \approx 10^{-3},\ \epsilon_d \approx 0.1,\ \sigma_d \approx 9 \times 10^{-3},\ \epsilon_e \approx 0.3,\ \sigma_e \approx 8 \times 10^{-3}$$

- Using the approximation of unitary matrices up to order $\epsilon^4\sigma^2$

$$|U_{\text{CKM}}| \approx \begin{pmatrix} 0.99 & 0.17 & 0 \\ 0.17 & 0.99 & 0.02 \\ 0.01 & 0.02 & 1 \end{pmatrix}$$

- Reasonable estimate compared to experimental data:

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97370 \pm 0.00014 & 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\ 0.221 \pm 0.004 & 0.987 \pm 0.011 & 0.0410 \pm 0.0014 \\ 0.0080 \pm 0.0003 & 0.0388 \pm 0.0011 & 1.013 \pm 0.030 \end{bmatrix}$$

PDG (2020)

Effects of perturbations

$\sigma_{u,d,e} : \Delta m_{\tilde{d}_{1,2}} = 0.01 \rightarrow 1\text{GeV}$

$\sigma_{u,d,e} : \Delta m_{\tilde{u}_{4,5}} = 0.01 \rightarrow 1\text{GeV}$

$\epsilon_u : \Delta m_{\tilde{u}_{1,2}} = 0.02 \rightarrow 70\text{GeV}$

$\epsilon_u : \Delta m_{\tilde{d}_{1,2}} = 0.02 \rightarrow 25\text{GeV}$

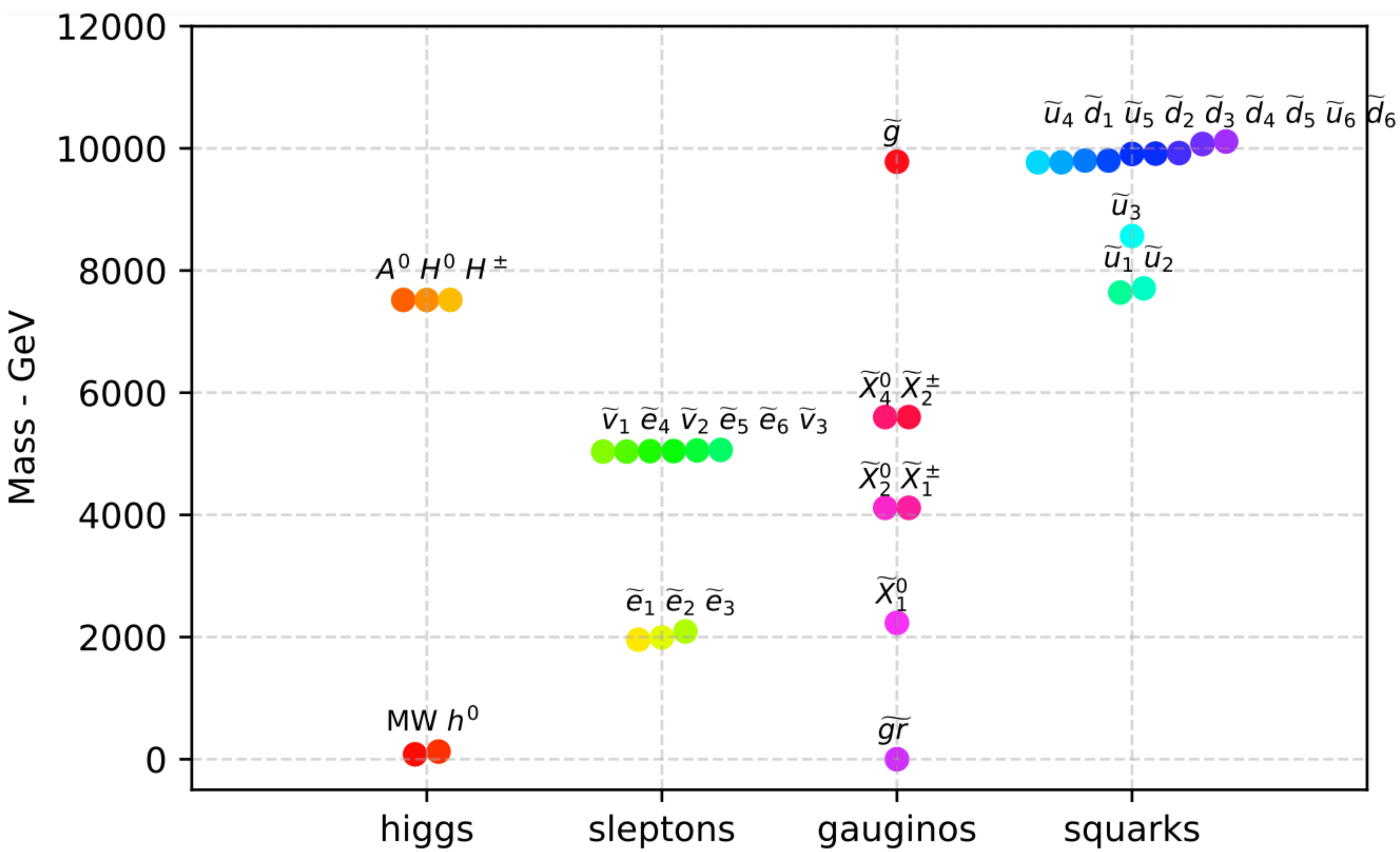
[GeV]	No perturbation	Nonzero $\sigma_{u,d,e}$	Nonzero $\sigma_{u,d,e}$, ϵ_u	Nonzero $\sigma_{u,d,e}$, $\epsilon_{u,d,e}$
M_{mess}	10^{12}	10^{12}	10^{12}	10^{12}
Λ	7.7×10^5	7.7×10^5	7.7×10^5	7.7×10^5
$\tan\beta$	10	10	10	10
h^0	125.14	125.14	125.20	124.97
H^0	7175.6	7175.6	7176.6	7176.2
A^0	7175.6	7175.5	7176.6	7176.1
g	9334.0	9334.0	9334.0	9334.0
$\tilde{\chi}_1^0$	2120.9	2120.9	2120.9	2120.9
$\tilde{\chi}_2^0$	3914.7	3914.7	3914.7	3914.7
$\tilde{\chi}_3^0$	-5353.3	-5353.3	-5354.8	-5354.8
$\tilde{\chi}_4^0$	5356.0	5355.8	5357.3	5357.3
$\tilde{\chi}_1^\pm$	3914.9	3914.9	3914.9	3914.9
$\tilde{\chi}_2^\pm$	5356.1	5356.0	5357.6	5357.6
$\tilde{e}_1$	1876.8	1873.9	1873.9	1858.0
$\tilde{e}_2$	1876.9	1879.8	1879.7	1893.7
$\tilde{e}_3$	1985.7	1985.8	1985.7	1986.7
$\tilde{e}_4$	4799.3	4798.8	4798.8	4795.1
$\tilde{e}_5$	4799.3	4799.9	4799.9	4802.6
$\tilde{e}_6$	4812.0	4812.0	4812.0	4812.7
$\tilde{\nu}_1$	4798.3	4797.8	4797.8	4794.7
$\tilde{\nu}_2$	4798.4	4798.9	4798.9	4801.6
$\tilde{\nu}_3$	4820.8	4820.8	4820.8	4820.9
$\tilde{u}_1$	7334.8	7334.8	7299.9	7299.9
$\tilde{u}_2$	7334.8	7334.8	7365.9	7365.9
$\tilde{u}_3$	8164.4	8164.4	8166.5	8166.5
$\tilde{u}_4$	9338.3	9337.8	9324.3	9323.0
$\tilde{u}_5$	9338.3	9338.8	9350.2	9351.2
$\tilde{u}_6$	9601.3	9601.3	9602.8	9602.8
$\tilde{d}_1$	9338.5	9338.0	9324.8	9323.4
$\tilde{d}_2$	9338.5	9339.0	9350.5	9351.4
$\tilde{d}_3$	9456.6	9445.5	9445.4	9453.2
$\tilde{d}_4$	9456.6	9457.6	9457.5	9458.6
$\tilde{d}_5$	9466.4	9466.5	9466.4	9466.7
$\tilde{d}_6$	9641.4	9641.4	9642.5	9642.5

Example mass spectrum

$M_{\text{Mess}} = 10^{12}\text{GeV},\ \Lambda = 8.1 \times 10^5\text{GeV},\ \tan\beta = 10$

$\epsilon_u = 0.033,\ \epsilon_d = 0.108,\ \epsilon_e = 0.281$

$\sigma_u = 0.001,\ \sigma_d = 0.009,\ \sigma_e = 0.008$

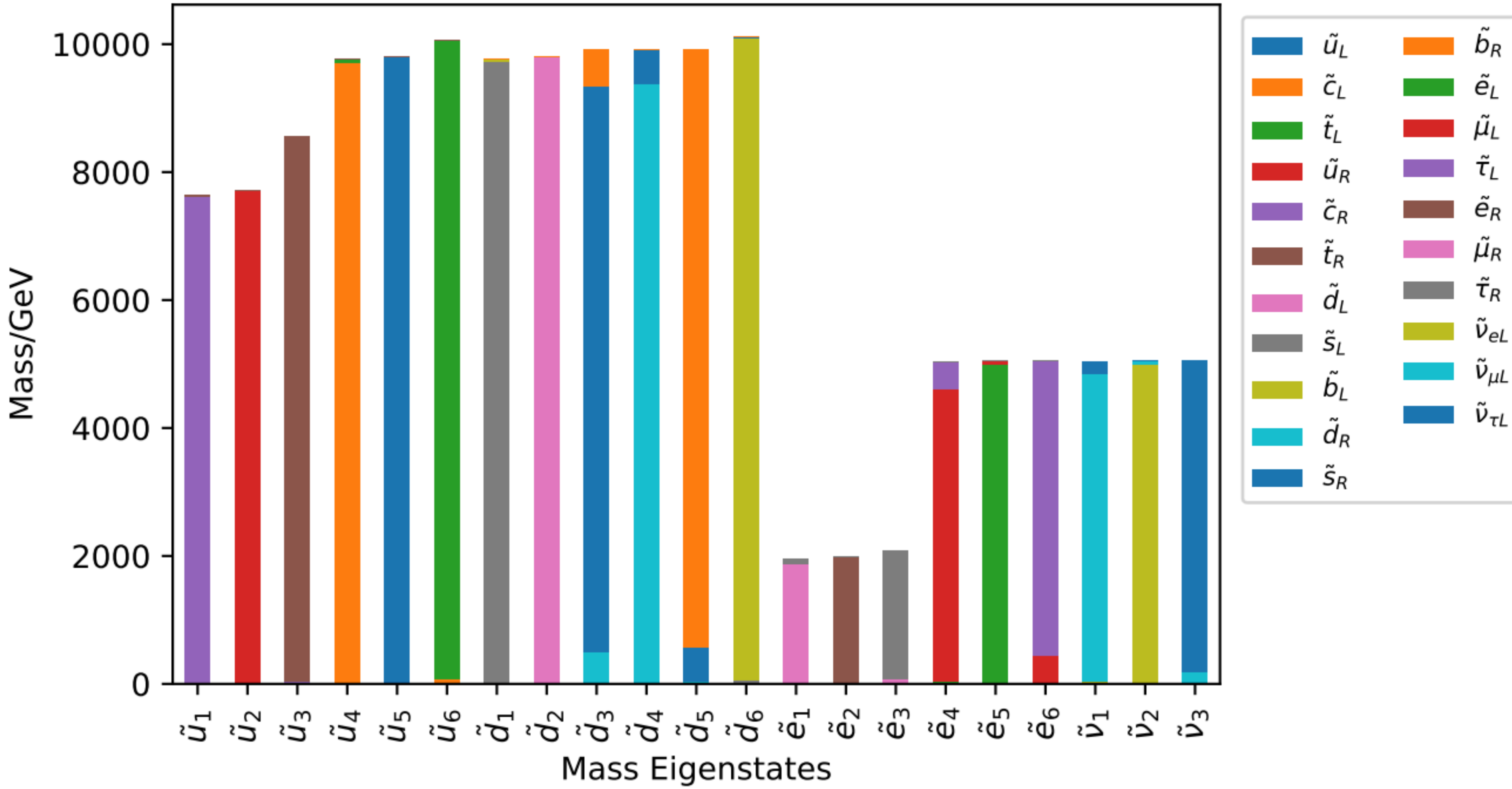


Example mass spectrum (continued)

$M_{\rm Mess} = 10^{12}{\rm GeV},\ \Lambda = 8.1\times 10^{5}{\rm GeV},\ \tan\beta = 10$

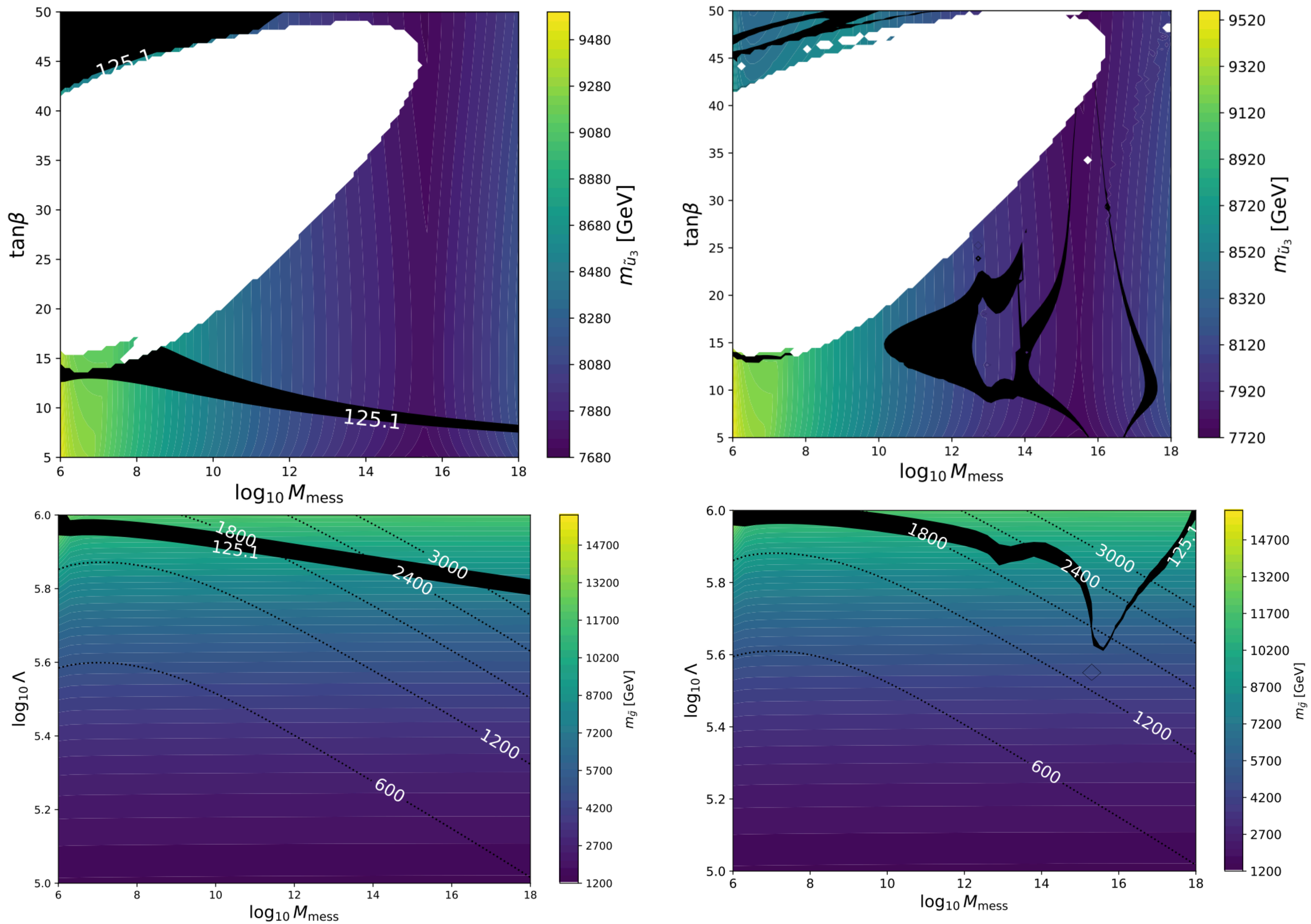
$\epsilon_u = 0.033,\ \epsilon_d = 0.108,\ \epsilon_e = 0.281$

$\sigma_u = 0.001,\ \sigma_d = 0.009,\ \sigma_e = 0.008$



- No mixing of flavor eigenstates when there is no perturbation added
- Generational mixing& small left-right mixing

Full Parameter Space Scans



Higgs mass (black shading)

Lighter gluino& $\tilde{e}_1$ masses with perturbations

Flavor Constraints (SUSY Flavor problem)

- Flavor changing mixing in sfermion mass matrices $\Rightarrow$ FCNC

Mass Insertion Approximation (MIA):

$$(\delta_Q^{IJ})_{XY} = \frac{(\Delta_Q^{IJ})_{XY}}{(m_{QI})_{XX}(m_{QJ})_{YY}}$$

eg. Super-CKM squark mass squared matrix

$$(M_{\tilde{U}}^2)_{LL} = \begin{pmatrix} (m_{\tilde{U}1}^2)_{LL} & (\Delta_{\tilde{U}}^{12})_{LL} & (\Delta_{\tilde{U}}^{13})_{LL} \\ (\Delta_{\tilde{U}}^{21})_{LL} & (m_{\tilde{U}2}^2)_{LL} & (\Delta_{\tilde{U}}^{23})_{LL} \\ (\Delta_{\tilde{U}}^{31})_{LL} & (\Delta_{\tilde{U}}^{32})_{LL} & (m_{\tilde{U}3}^2)_{LL} \end{pmatrix}$$

I, J : quark flavor
Q : up/ down quark superfield sector
X, Y: superfield chirality

- Non-degenerate squark masses but not strongly hierarchical $\Rightarrow$ MIA ✓
- $|(\delta_Q)_{XY}^{IJ}|$ predicted in our models are well bounded

  Loose bounds since the constraints scale with squark masses (heavy squarks)

  Mass insertion is proportional to mass difference between squarks which are small

Summary

So far we have …

- Built models with 3 massive quarks consistent with SM quark masses, with enhanced $\mathcal{S}_{3L} \times \mathcal{S}_{3R}$ symmetry.
- Achieved reasonable estimation of CKM in Case 1 democratic model
- Explored SUSY parameter space in Case 1 democratic model
- Related SUSY breaking and flavor symmetry breaking with the same symmetry group $\mathcal{S}_3$
- Shown that our models with flavor mixing satisfy FCNC constraints
- For Case 1 democratic model, spectra remain heavy
- Gravitino can be plausible dark matter candidate


Thank you!
Questions?

Backup: S3 symmetry

- Permutation Group on three objects
- Three irreducible representations:

  The singlet $\mathbf{1}$, a one-dimensional representation $\mathbf{1}'$, a doublet $\mathbf{2}$

- Tensor products:

$$\mathbf{1} \otimes \mathbf{2} = \mathbf{2}, \qquad \mathbf{1}' \otimes \mathbf{2} = \mathbf{2}, \qquad \mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$$

- Possible singlet representation obtained:

$$(\mathbf{2} \otimes \mathbf{2})_{\mathbf{1}} = \left[\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}\right]_{\mathbf{1}} = a_1 b_2 + a_2 b_1 .$$

$$(\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2})_{\mathbf{1}} = \left[\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \otimes \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}\right]_{\mathbf{1}} = a_1 b_1 c_1 + a_2 b_2 c_2$$

Backup: Higgs Messenger Fields and Rotational Matrix

- Higgs-Messenger field with $\mathcal{S}_3$ quantum number

$$\mathcal{H}_u \equiv \begin{pmatrix} (\mathcal{H}_u^{(2)})_1 \\ (\mathcal{H}_u^{(2)})_2 \\ \mathcal{H}_u^{(1)} \end{pmatrix} \equiv \begin{pmatrix} \mathcal{H}_{u1}^{(2)} \\ \mathcal{H}_{u2}^{(2)} \\ \mathcal{H}_u^{(1)} \end{pmatrix} = \mathcal{R}_u \begin{pmatrix} H_u \\ M_{u1} \\ M_{u2} \end{pmatrix}$$

$$\mathcal{H}_d \equiv \begin{pmatrix} (\mathcal{H}_d^{(2)})_1 \\ (\mathcal{H}_d^{(2)})_2 \\ \mathcal{H}_d^{(1)} \end{pmatrix} \equiv \begin{pmatrix} \mathcal{H}_{d1}^{(2)} \\ \mathcal{H}_{d2}^{(2)} \\ \mathcal{H}_d^{(1)} \end{pmatrix} = \mathcal{R}_d \begin{pmatrix} H_d \\ M_{d1} \\ M_{d2} \end{pmatrix}$$

- The unitary rotational matrices: $\mathcal{R}_{u,d} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \mp\frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right) & \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right) \\ \frac{1}{\sqrt{3}} & \pm\frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right) & -\frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right) \\ \frac{1}{\sqrt{3}} & \pm\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$

- Higgs-messenger fields in terms of MSSM Higgs doublets and messenger doublets:

$$\begin{pmatrix} \mathcal{H}_{u1}^{(2)} \\ \mathcal{H}_{u2}^{(2)} \\ \mathcal{H}_u^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}}H_u - \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right)M_{u1} + \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right)M_{u2} \\ \frac{1}{\sqrt{3}}H_u + \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right)M_{u1} - \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right)M_{u2} \\ \frac{1}{\sqrt{3}}(H_u + M_{u1} + M_{u2}) \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{H}_{d1}^{(2)} \\ \mathcal{H}_{d2}^{(2)} \\ \mathcal{H}_d^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}}H_d + \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right)M_{d1} + \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right)M_{d2} \\ \frac{1}{\sqrt{3}}H_d - \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right)M_{d1} - \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right)M_{d2} \\ \frac{1}{\sqrt{3}}(H_d - M_{d1} + M_{d2}) \end{pmatrix}$$

Backup: Addressing μ/b problem

$$
\begin{aligned}
W_H &= \mathcal{H}_u^\top \mathbb{M} \mathcal{H}_d + \theta^2 \mathcal{H}_u^\top \mathbb{F} \mathcal{H}_d \\
&= \lambda(X_H \mathcal{H}_u^{(2)} \mathcal{H}_d^{(2)}) + \lambda'(X_H \mathcal{H}_u^{(1)} \mathcal{H}_d^{(2)}) + \lambda''(X_H \mathcal{H}_u^{(2)} \mathcal{H}_d^{(1)}) + \kappa M(\mathcal{H}_u^{(2)} \mathcal{H}_d^{(2)}) + \kappa' M(\mathcal{H}_u^{(1)} \mathcal{H}_d^{(1)})
\end{aligned}
$$

Use the expression for Higgs-messenger fields and:

$$
\langle \lambda X_H \rangle = M \begin{pmatrix} \sin\phi \\ \cos\phi \end{pmatrix} + \theta^2 F \begin{pmatrix} \sin\xi \\ \cos\xi \end{pmatrix}
$$

We can get:

$$
\mathbb{M} = M \begin{pmatrix} \sin\phi & \kappa & \epsilon'\cos\phi \\ \kappa & \cos\phi & \epsilon'\sin\phi \\ \epsilon''\cos\phi & \epsilon''\sin\phi & \kappa' \end{pmatrix}, \qquad \mathbb{F} = F \begin{pmatrix} \sin\xi & 0 & \epsilon'\cos\xi \\ 0 & \cos\xi & \epsilon'\sin\xi \\ \epsilon''\cos\xi & \epsilon''\sin\xi & 0 \end{pmatrix}.
$$

For simplicity, choose $\epsilon'' = \epsilon, \epsilon' = 1$:

$$
[\mathbb{M}, \mathbb{F}] = 0 \Rightarrow \kappa' = \kappa = \frac{\sin(\phi - \xi)}{\cos\xi - \sin\xi}
$$

$$
\mathbb{M} = M\cos\phi \begin{pmatrix} \tan\phi & \frac{\tan\phi - \tan\xi}{1-\tan\xi} & 1 \\ \frac{\tan\phi - \tan\xi}{1-\tan\xi} & 1 & \tan\phi \\ 1 & \tan\phi & \frac{\tan\phi - \tan\xi}{1-\tan\xi} \end{pmatrix}, \qquad \mathbb{F} = F\cos\xi \begin{pmatrix} \tan\xi & 0 & 1 \\ 0 & 1 & \tan\xi \\ 1 & \tan\xi & 0 \end{pmatrix}
$$

Backup: Addressing μ/b problem (cont.)

Eigenvalues of $\mathbb{M}$ and $\mathbb{F}$:

$$F_1 = F(\cos\xi + \sin\xi), \qquad F_{2,3} = \mp F\sqrt{1-\sin\xi\cos\xi}.$$

$$M_1 = M\left(\frac{\cos(\xi+\phi) - 2\sin(\xi-\phi)}{\cos\xi - \sin\xi}\right), \qquad M_{2,3} = \mp M\left(\frac{\cos\phi - \sin\phi}{\cos\xi - \sin\xi}\right)\sqrt{1-\sin\xi\cos\xi}.$$

Build in eigenvalues hierarchy:

$$F_1 \equiv b \ll F_{2,3} \quad \text{and} \quad M_1 \equiv \mu \ll M_{2,3}$$

Choosing

$$\xi = -\pi/4 + \eta, \quad \phi = \xi + \rho$$

We get:

$$\frac{b}{F} \equiv F_1 = \sqrt{2}\eta + O(\eta^2), \qquad \frac{F_{2,3}}{F} = \sqrt{\frac{3}{2}} + O(\eta^2)$$

$$\frac{\mu}{M} \simeq \sqrt{2}\eta + \frac{3}{\sqrt{2}}\rho, \qquad \frac{M_{2,3}}{M} \simeq \sqrt{\frac{3}{2}}$$

$$B_\mu = \frac{b}{\mu} = \frac{F}{M}\frac{2\eta}{3\rho} \sim \frac{1}{16\pi^2}\frac{F}{M} \sim m_{\text{soft}}$$

Backup: Classification of models

- Yukawa matrix:

$$Y_i = \frac{y_i}{\sqrt{3}} \begin{pmatrix} 1 & \beta_{1i} & \beta_{2i} \\ \beta_{1i} & 1 & \beta_{2i} \\ \beta_{3i} & \beta_{3i} & \beta_{4i} \end{pmatrix}$$

- Biunitary diagonalization of its Hermitian combination:

$$U_{iL}^\dagger Y_i Y_i^\dagger U_{iL} = (Y_i Y_i^\dagger)_{\rm diag} = (Y_i^{(\rm diag)})^2, \qquad U_{iR}^\dagger Y_i^\dagger Y_i U_{iR} = (Y_i^\dagger Y_i)_{\rm diag} = (Y_i^{(\rm diag)})^2$$

- Eigenvalues found:

$$\lambda_{1u} = \frac{\tilde{y}_u^2}{3}(1-\beta_{1u})^2, \quad \lambda_{2u,3u} = \frac{\tilde{y}_u^2}{6}\left((1+\beta_{1u})^2 + 2(\beta_{2u}^2+\beta_{3u}^2) + \beta_{4u}^2 \mp \sqrt{\Lambda_u}\right)$$

$$\Lambda_u = (1+\beta_{1u})^4 + 4(\beta_{2u}^4+\beta_{3u}^4) + \beta_{4u}^4 + 4((1+\beta_{1u})^2+\beta_{4u}^2)(\beta_{2u}^2+\beta_{3u}^2) - 2(1+\beta_{1u})^2\beta_{4u}^2 - 8\beta_{2u}^2\beta_{3u}^2 + 16(1+\beta_{1u})\beta_{2u}\beta_{3u}\beta_{4u}$$

Case 1: $\lambda_{1,2} \ll \lambda_3$

$$\beta_1 \to 1, \ \beta_4 = \beta_2\beta_3$$

- democratic limit: $\text{All } \beta_{iu} = 1$
- singlet-dominated:

$$\beta_{1u} = 1, \quad \beta_{2u}\beta_{3u} = \beta_{4u}$$

Case 2: $\lambda_{2,3} \ll \lambda_1$

$$\beta_1 \to -1, \ \beta_{i=2,3,4} \ll 1, \ \Lambda \to 0$$

- Doublet dominated limit

Backup: Soft mass terms for democratic limit (no perturbation)

$$\begin{aligned}
\delta m^2_{Q_{11}} &= \delta m^2_{Q_{22}} = \frac{\Lambda^2}{(4\pi)^4}\left[6Y_t^4 + 6Y_b^4 + 2Y_b^2Y_t^2 + Y_b^2Y_\tau^2 - \tilde{g}_u^2Y_t^2 - \tilde{g}_d^2Y_b^2\right] \\
\delta m^2_{\bar{u}_{11}} &= \delta m^2_{\bar{u}_{22}} = \frac{\Lambda^2}{(4\pi)^4}\left[12Y_t^4 + 2Y_t^2Y_b^2 - 2\tilde{g}_u^2Y_t^2\right] \\
\delta m^2_{\bar{d}_{11}} &= \delta m^2_{\bar{d}_{22}} = \frac{\Lambda^2}{(4\pi)^4}\left[12Y_b^4 + 2Y_t^2Y_b^2 + 2Y_b^2Y_\tau^2 - 2\tilde{g}_d^2Y_b^2\right] \\
\delta m^2_{L_{11}} &= m^2_{L_{22}} = \frac{\Lambda^2}{(4\pi)^4}\left[4Y_\tau^4 + 3Y_b^2Y_\tau^2 - \tilde{g}_e^2Y_\tau^2\right] \\
\delta m^2_{\bar{e}_{11}} &= \delta m^2_{\bar{e}_{22}} = \frac{\Lambda^2}{(4\pi)^4}\left[8Y_\tau^4 + 6Y_b^2Y_\tau^2 - 2\tilde{g}_e^2Y_\tau^2\right] \\
\delta m^2_{H_u} &= \delta m^2_{H_d} = 0, \qquad \tilde{A}_u = \tilde{A}_d = \tilde{A}_e = 0
\end{aligned}$$

$$\tilde{g}_u^2 = \frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2, \qquad \tilde{g}_d^2 = \frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2, \qquad \tilde{g}_e^2 = 3g_2^2 + \frac{9}{5}g_1^2$$

- We have matrix structures of the following general form:

$$\begin{aligned}
\delta m^2_{\tilde{Q}} &= (\delta m^2_{\tilde{Q}})_{11}\mathrm{Diag}(1,1,0), \qquad \delta m^2_{\tilde{L}} = (\delta m^2_{\tilde{L}})_{11}\mathrm{Diag}(1,1,0) \\
\delta m^2_{\tilde{u},\tilde{d}} &= (\delta m^2_{\tilde{u},\tilde{d}})_{11}\mathrm{Diag}(1,1,0), \qquad \delta m^2_{\tilde{e}} = (\delta m^2_{\tilde{e}})_{11}\mathrm{Diag}(1,1,0) \\
\delta m^2_{H_u} &= \delta m^2_{H_d} = \tilde{A}_u = \tilde{A}_d = \tilde{A}_e = 0.
\end{aligned}$$

Backup: Soft mass terms for democratic limit (with perturbation)

- Difference: nonzero 23 and 32 entries (equal) generated at the first order of ϵ:
- eg. (only some soft mass terms with ϵ are shown here)

$$\delta m^2_{Q_{23}} = \delta m^2_{Q_{32}} = \frac{\Lambda^2}{(4\pi)^4}\left[\epsilon_u \left(\tfrac{8\sqrt{2}}{3} y_t^4 - \tfrac{4\sqrt{2}}{9}\tilde{g_u}^2 y_t^2 + \tfrac{4\sqrt{2}}{9} y_b^2 y_t^2\right) + \epsilon_d \left(\frac{8\sqrt{2}}{3} y_b^4 - \frac{4\sqrt{2}}{9}\tilde{g_d}^2 y_b^2 + \frac{4\sqrt{2}}{9} y_b^2 y_t^2 + \frac{4\sqrt{2}}{9} y_b^2 y_\tau^2\right) + \mathcal{O}(\epsilon^2)\right]$$

$$\delta m^2_{\bar{u}_{23}} = \delta m^2_{\bar{u}_{32}} = \frac{\Lambda^2}{(4\pi)^4}\left[\frac{8\sqrt{2}\epsilon_u}{9}\left(6y_t^4 + y_b^2 y_t^2 - \tilde{g_u}^2 y_t^2\right) + \mathcal{O}(\epsilon^2)\right]$$

$$\tilde{A}_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\tfrac{2\epsilon_u}{9}(3y_t^3 + y_t y_b^2) & \tfrac{4\sqrt{2}\epsilon_u}{9} y_t^3 + \tfrac{4\sqrt{2}\epsilon_d}{9} y_t y_b^2 \\ 0 & \tfrac{8\sqrt{2}\epsilon_u}{9} y_t^3 & \mathcal{O}(\epsilon^2) \end{pmatrix} \quad \tilde{A}_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\tfrac{2\epsilon_d}{9}(3y_b^3 + y_t^2 y_b) & \tfrac{4\sqrt{2}\epsilon_d}{9} y_b^3 + \tfrac{4\sqrt{2}\epsilon_u}{9} y_t^2 y_b \\ 0 & \tfrac{8\sqrt{2}\epsilon_d}{9} y_b^3 & \mathcal{O}(\epsilon^2) \end{pmatrix} \quad \tilde{A}_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\tfrac{2\epsilon_e}{3} y_\tau^3 & \tfrac{4\sqrt{2}\epsilon)e}{9} y_\tau^3 \\ 0 & \tfrac{8\sqrt{2}\epsilon_e}{9} y_\tau^3 & \mathcal{O}(\epsilon^2) \end{pmatrix}$$

- In actual codes, take into account relative strength between all perturbation parameters, subleading terms included:

$$\frac{\sigma_u}{\epsilon_u},\ \epsilon_u,\ \epsilon_d,\ \epsilon_d^2,\ \frac{\sigma_d}{\epsilon_d},\ \sigma_d,\ \epsilon_e,\ \epsilon_e^2,\ \epsilon_e^3,\ \epsilon_e^4,\ \sigma_e,\ \frac{\sigma_e}{\epsilon_e},\ \epsilon_u\epsilon_e,\ \epsilon_d\epsilon_e,\ \epsilon_d\epsilon_e^2$$

Backup: Generation of superpotential terms

- eg. ϵ perturbation for up quarks can be generated by renormalizable couplings:

$$\epsilon_u y_u [\beta_{2u} Q_{\mathbf{2}} \bar{u}_{\mathbf{1}} \mathcal{H}_u^{(2)} + \beta_{3u} Q_{\mathbf{1}} \bar{u}_{\mathbf{2}} \mathcal{H}_u^{(2)} + \beta_{4u} Q_{\mathbf{1}} \bar{u}_{\mathbf{1}} \mathcal{H}_u^{(1)}]$$

- eg. σ perturbation for up quarks can be generated by non-renormalizable couplings:

$$(\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2})_{\mathbf{1}} = \left[\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \otimes \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \otimes \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}\right]_{\mathbf{1}} = (a_1 b_2 \pm a_2 b_1)(c_1 d_2 \pm c_2 d_1)$$

Adding two flavons ϕ, ϕ_2 in doublets representation of $\mathcal{S}_3$: $\phi = \begin{pmatrix} \cos\theta \\ sin\theta \end{pmatrix}$, $\phi_2 = \begin{pmatrix} \cos\theta_2 \\ sin\theta_2 \end{pmatrix}$

$$\sigma_u y_u \left[\frac{1}{2}\left((Q_{\mathbf{2}} \phi \bar{u}_{\mathbf{2}} \mathcal{H}_u^{(2)})_+ + (Q_{\mathbf{2}} \phi \bar{u}_{\mathbf{2}} \mathcal{H}_u^{(2)})_-\right) + \beta_{2u} Q_{\mathbf{2}} \phi \bar{u}_{\mathbf{1}} \mathcal{H}_u^{(2)} + \beta_{3u} Q_{\mathbf{1}} \phi \bar{u}_{\mathbf{2}} \mathcal{H}_u^{(2)} + \frac{1}{2}\left((Q_{\mathbf{2}} \phi_2 \bar{u}_{\mathbf{2}} \mathcal{H}_u^{(2)})_+ + (Q_{\mathbf{2}} \phi_2 \bar{u}_{\mathbf{2}} \mathcal{H}_u^{(2)})_-\right) + \beta_{2u} Q_{\mathbf{2}} \phi_2 \bar{u}_{\mathbf{1}} \mathcal{H}_u^{(2)} + \beta_{3u} Q_{\mathbf{1}} \phi_2 \bar{u}_{\mathbf{2}} \mathcal{H}_u^{(2)}\right]$$

$$Y_u = \begin{pmatrix} \frac{\sigma\sin(\theta)}{\sqrt{3}} + \frac{\sigma\sin(\theta_2)}{\sqrt{3}} & 0 & \frac{\sigma\cos(\theta)}{\sqrt{3}} + \frac{\sigma\cos(\theta_2)}{\sqrt{3}} \\ 0 & \frac{\sigma\cos(\theta)}{\sqrt{3}} + \frac{\sigma\cos(\theta_2)}{\sqrt{3}} & \frac{\sigma\sin(\theta)}{\sqrt{3}} + \frac{\sigma\sin(\theta_2)}{\sqrt{3}} \\ \frac{\sigma\cos(\theta)}{\sqrt{3}} + \frac{\sigma\cos(\theta_2)}{\sqrt{3}} & \frac{\sigma\sin(\theta)}{\sqrt{3}} + \frac{\sigma\sin(\theta_2)}{\sqrt{3}} & 0 \end{pmatrix}$$

separately tune $\theta = \frac{\pi}{2}$, $\theta_2 = \pi$

Backup: Muon g-2

- Contribution coming from smuon-sneutrino and sneutrino-chargino loops (significant):

$$\delta a_\mu \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan\beta \simeq 15 \times 10^{-10} \left(\frac{100\mathrm{GeV}}{\tilde{m}}\right)^2 \tan\beta$$

Using $\tilde{m} \approx 1000$GeV, $\tan\beta = 50$

$$\delta a_\mu \approx 0.17 \times 10^{-10}$$

- Current experiment bound:

$$\delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$$

- Need detailed analysis including all relevant loop diagrams and other contributions including (χVH), ($\tilde{t}\gamma H$), ($\tilde{b}\gamma H$)…

To be done using GM2Calc

Stockinger (2006)
Carena et al (1996)