Jet Cross Sections, Shapes and Substructure

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PITT PAC

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- Why and where are there jets?
- The measures and structures of jets.

  We'll try and point out ways in which QCD jets are unique, yet part of a universal phenomenon in field theory.

What we're going to try and get across in Part 1: Why and where are there jets?

A. The intuition behind particle jets, and a sketch of their history in experiment.

B. Challenges at very high energy: why and how soft and collinear enhancements arise in long-time behavior

C. Why energy flow is a guide to calculable cross sections: infrared safety

D. How jets are found and their cross sections computed

E. Inside jets I: jet shapes, their resummations in and beyond perturbation theory

Two recent and useful reviews:

– A.J. Larkoski, I. Moult, B. Nachman, 1709.044642.
– S. Marzani, G. Soyez, M. Spannowsky, 1901.10342.

A. The intuition behind particle jets, and a sketch of their history in experiment.

Outline

- Quantitative comparisons of QCD to experiment began with fully inclusive processes.
- In a seeming paradox, inclusive cross sections can be related to elastic scattering of quarks (the parton model). Asymptotic freedom makes this plausible
- Electron positron annihilation to hadrons is dominated by two-jet events that clearly reflect quark pair creation. The observable called "thrust" helps identify jets and justify the use of the term jet.
- High energy accelerators, at energies far above (light) quark masses, all produce events consistent with this interpretation.

Prehistory of jets: the 1950's – 1960's

- The first observations of particle "jets" was in cosmic ray detection.

  Particle jets in cosmic rays . . .

  "The average transverse momentum resulting from our measurements is p_T=0.5 BeV/c for pions . . . Table 1 gives a summary of jet events observed to date . . ." (B. Edwards et al, Phil. Mag. 3, 237 (1957))

- The era of high energy physics and the discovery of the Standard Model

  Once asymptotic freedom explained scaling (Feynman, Bjorken)

$$\sigma^{\rm incl}_{e\,{\rm proton}}\left(Q, x = \frac{Q^2}{2p\cdot q}\right) \to \sigma^{\rm excl}_{e\,{\rm parton}}(Q) \times F_{\rm proton}(x)\,,$$

- the question arose: what happens to partons in the final state?
  (Feynman, Bjorken & Paschos, Drell, Levy & Yan, 1969)
  Do "the hadrons 'remember' the directions along which the bare constituents were emitted? . . . "the observation of such 'jets' in colliding beam processes would be most spectacular." (Bjorken & Brodsky, 1969) Or does confinement forbid a it?

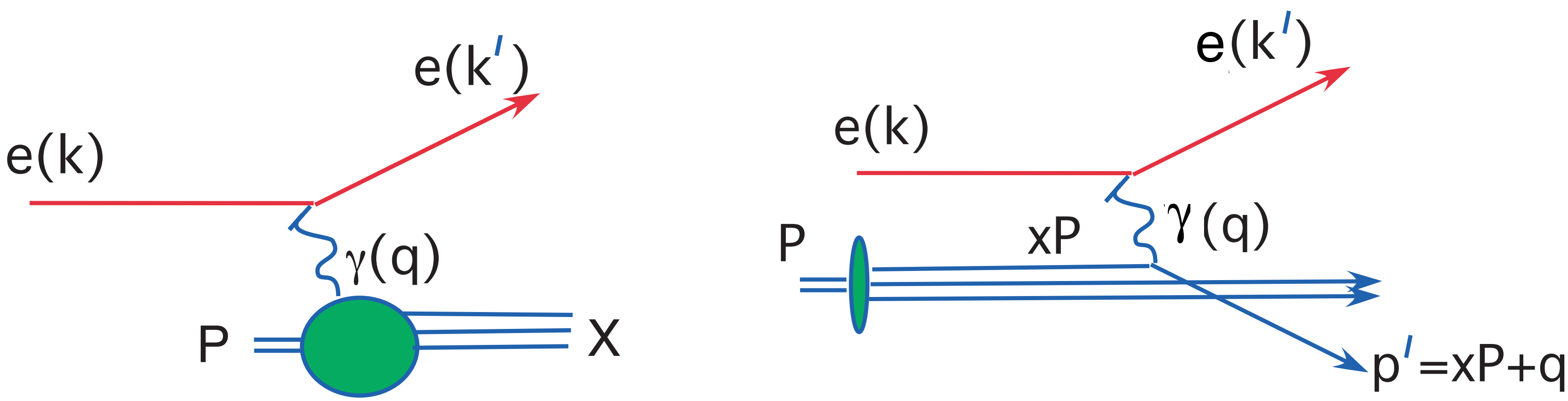
- The inclusive DIS cross section is described by exclusive partonic scattering. Could something similar happen in a less inclusive observable?

- **To make this long story short: Quantum Chromodynamics (QCD) reconciled the irreconcilable. Here was the problem.**

  1. Quarks and gluons explain spectroscopy, but aren't seen directly – confinement.

  2. In highly ("deep") inelastic, electron-proton scattering, the inclusive cross section was found to well-approximated by lowest-order elastic scattering of point-like (spin-1/2) particles (="partons" = quarks here) a result called "scaling":

$$\frac{d\sigma_{e+p}(Q, p\cdot q)}{dQ^2}\Big|_{\rm inclusive} \quad \propto \quad F\left(x = \frac{Q^2}{2p\cdot q}\right) \frac{d\sigma^{\rm free}_{e+{\rm spin}\,\frac{1}{2}}}{dQ^2}\Big|_{\rm elastic}$$



- **If the "spin-$\frac{1}{2}$ is a quark, how can a confined quark scatter freely?**

- This paradoxical combination of confined bound states at long distances and nearly free behavior at short distances was explained by asymptotic freedom: In QCD, the force between quarks behaves at short distances like

$$f(r) \sim \frac{\alpha_s(r)}{r^2}, \qquad \alpha_s(r^2) = \frac{4\pi}{\ln\left(\frac{1}{r^2\Lambda^2}\right)}$$

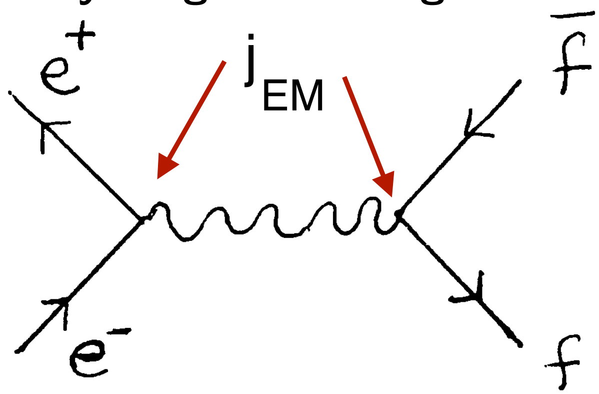
where $\Lambda \sim 0.2$ GeV. For distances much less than $1/(0.2GeV) \sim 10^{-13}$cm the force weakens. These are distances that began to be probed in deep inelastic scattering experiments at SLAC in the 1970s.

- The short explanation of DIS: Over the times $ct \leq \hbar/GeV$ it takes the electron to scatter from a quark-parton, the quark really does seem free. Later, the quark is eventually confined, but by then it's too late to change the probability for an event that has already happened.

- The function $F(x)$ is interpreted as the probability to find quark of momentum xP in a target of total momentum P – a parton distribution.

- **To explore further, SLAC used the quantum mechanical credo: anything that can happen, will happen.**

- **Quarks have electric charge, so if they are there to be produced, they will be. This can happen when colliding electron-positron pairs annihilate to a virtual photon, which (ungratefully) decays to just anything with charge.**



- **Of course because of confinement it's not that. But more generally, we believe that a virtual photon decays at a point through a local operator: $j_{\rm em}(x)$.**

- **This enables translating measurements into correlation functions . . . In fact, the cross section for electron-positron annihilation probes the vacuum with an electromagnetic current.**

- On the one hand, all final states are familiar hadrons, with nothing special about them to tell the tale of QCD, $|N\rangle = |\mathrm{pions,\ protons}\ldots\rangle$,

$$\sigma_{\mathrm{e^+e^-\to\ hadrons}}(Q) \ \propto\ \sum_N |\langle 0|j^\mu_{\mathrm{em}}(0)|N\rangle|^2\, \delta^4(Q-p_N)$$

- On the other hand, $\sum_N |N\rangle\langle N| = 1$, and using translation invariance this gives

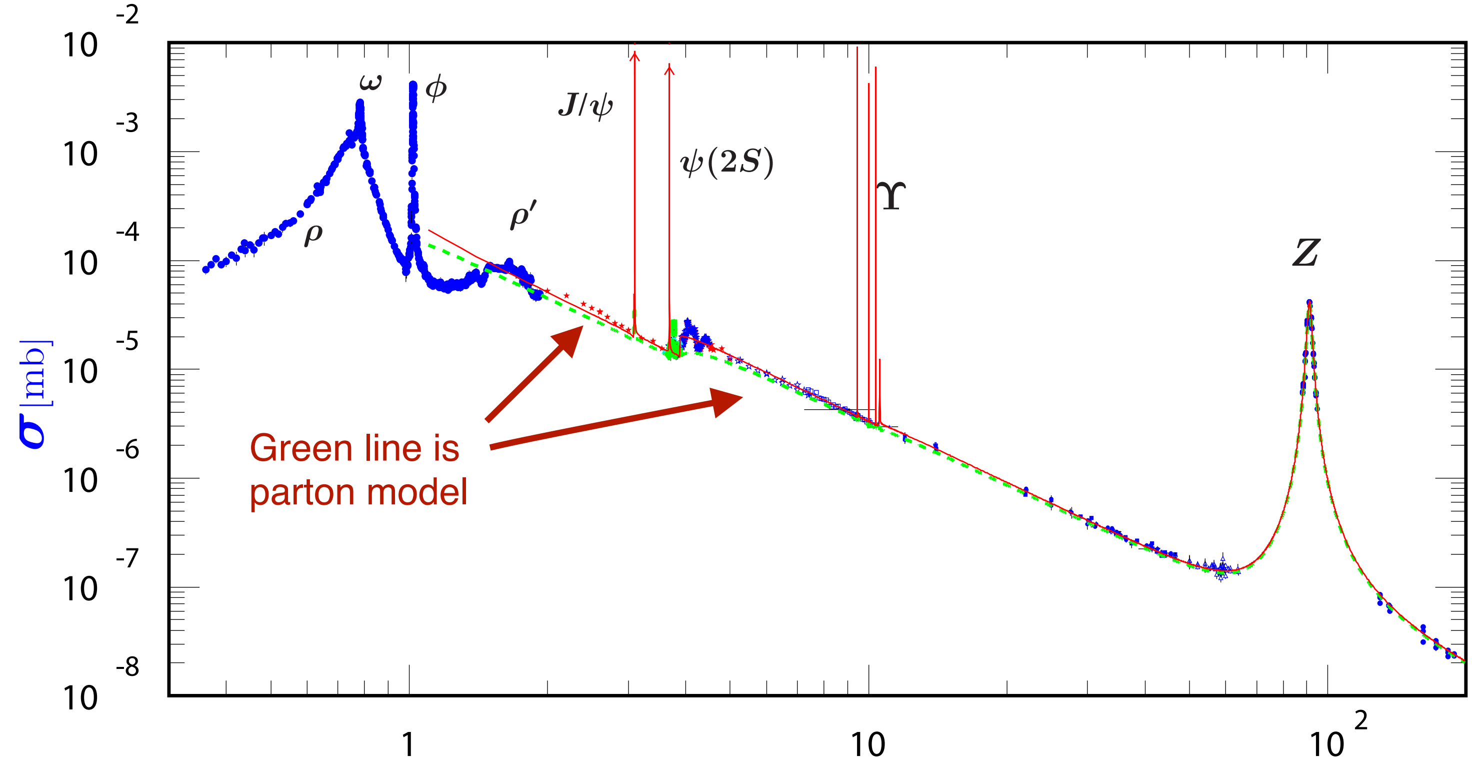
$$\sigma_{\mathrm{e^+e^-\to\ hadrons}}(Q) \ \propto\ \int d^4x\, e^{-iQ\cdot x}\ \langle 0|j^\mu_{\mathrm{em}}(0)\ j^\mu_{\mathrm{em}}(x)|0\rangle$$

- We are probing the vacuum at short distances, imposed by the Fourier transform as $Q\to\infty$. The currents are only a distance $1/Q$ apart.

- Asymptotic freedom suggests a “free” result: QCD at lowest order (“quark-parton model”) at cm. energy Q and angle θ

$$\sigma^{tot}_{e^+e^-\to\mathrm{hadrons}} \ =\ \frac{4\pi\alpha^2_{\mathrm{EM}}}{3Q^2}$$

- **This works for σ_{tot} to quite a good approximation (with calculable corrections)**



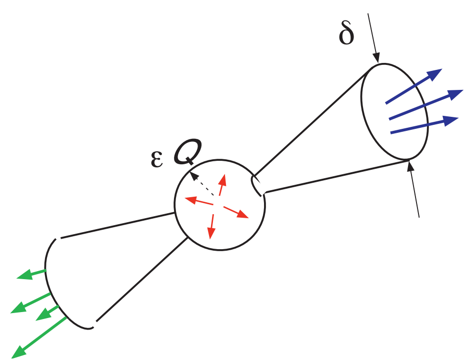
- **So the "free" theory again describes the inclusive sum over confined (nonperturbative) bound states – another "paradox".**

- Is there an imprint on these states of their origin? Yes. What to look for? The spin of the quarks is imprinted in their angular distribution:

$$\frac{d\sigma(Q)}{d\cos\theta} = \frac{\pi\alpha_{\rm EM}^2}{2Q^2}\left(1+\cos^2\theta\right)$$

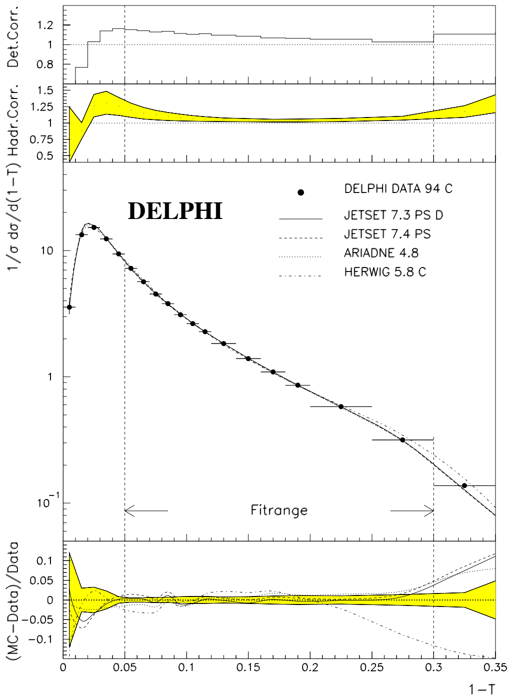
- It's not quarks, but can look for a back to back flow of energy by finding an axis that maximizes the projection of particle momenta ("thrust") measuring a "jet-like" structure

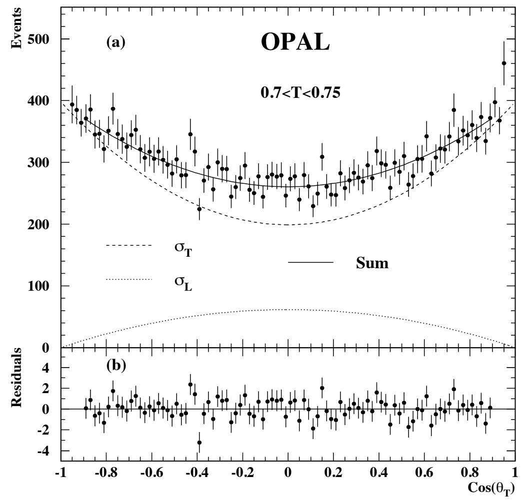
$$\frac{d\sigma_{\rm e^+e^-\to\ hadrons}(Q)}{dT} \propto \sum_N |\langle 0|j^\mu_{\rm em}(0)|N\rangle|^2\,\delta^4(Q-p_N)\;\delta\left(T-\frac{1}{Q}\max_{\hat{\rm n}}\sum_{i\in N}|\vec{p}_i\cdot\hat{\rm n}|\right)$$

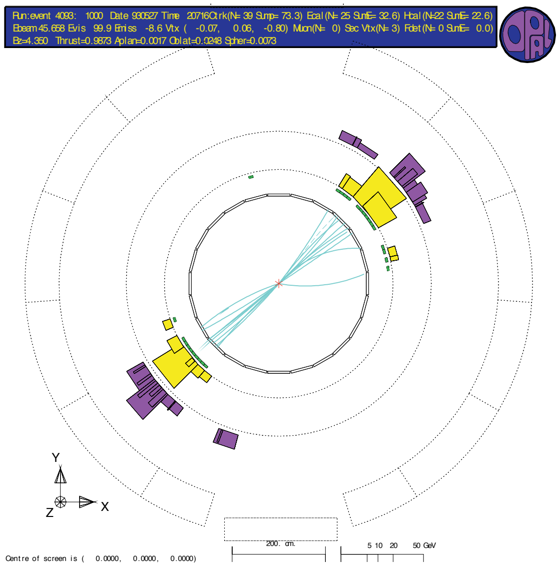


- When the particles all line up $T\to 1$ (neglecting masses). So what happens?

• **Here's what was found (from a little later, at LEP):**







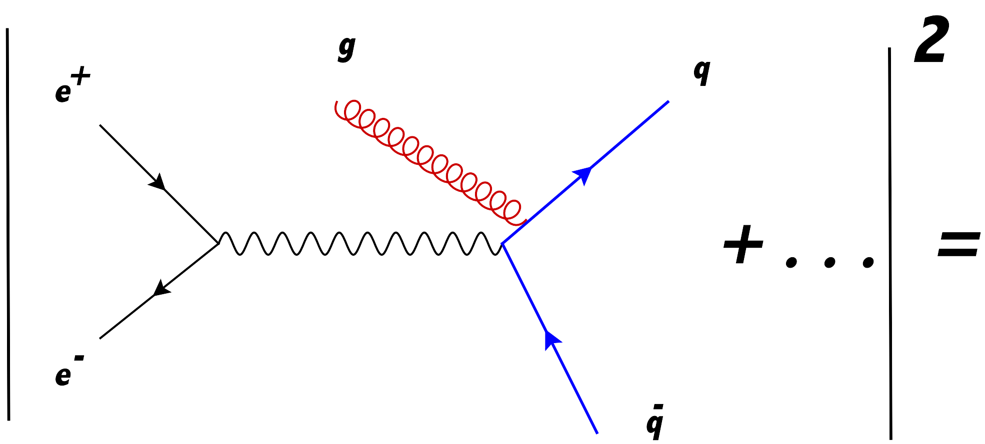
• **Thrust is peaked near unity and follows the $1 + \cos^2\theta$ distribution – reflecting the production of spin $\frac{1}{2}$ particles – back-to-back. All this despite confinement. Quarks have been replaced by "jets" of hadrons. What could be better? But what's going on? How can we understand persistence of short-distance structure into the final state, evolving over many many orders of magnitude in time?**

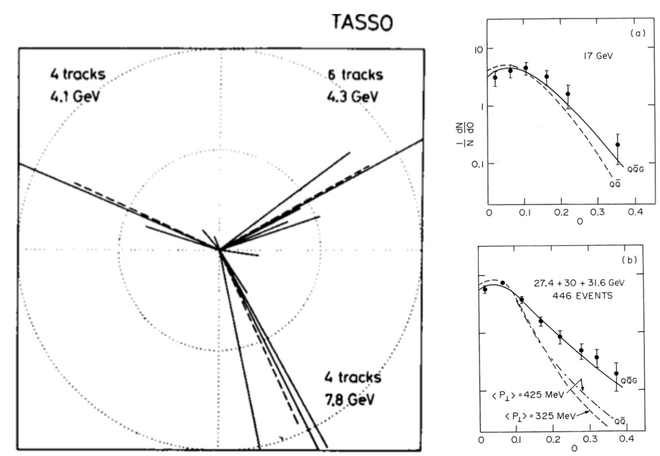
• **Back to the Timeline . . . 1975 -1980: the first quark and gluon jets**

• **As we've seen: in electron-positron annihilation to hadrons, the angular distribution for energy flow follows the lowest-order ("Born") cross section for the creation of spin-1/2 pairs of quarks and antiquarks** (As first seen by Hanson *et al*, at SLAC in 1975)

• **Jets are "rare" because the high momentum transfer scattering of partons is rare (but calculable), but in e^+e^- annihilation to hadrons the "rarity" is in the likelihood of annihilation. Once that takes places, jets are nearly always produced.**

• **And then (**Ellis, Gaillard, Ross (1976) Ellis, Karliner (1979)**): hints of three gluons in Upsilon decay, and then unequivocal gluon jets at Petra (1979)** (S.L. Wu (1984))



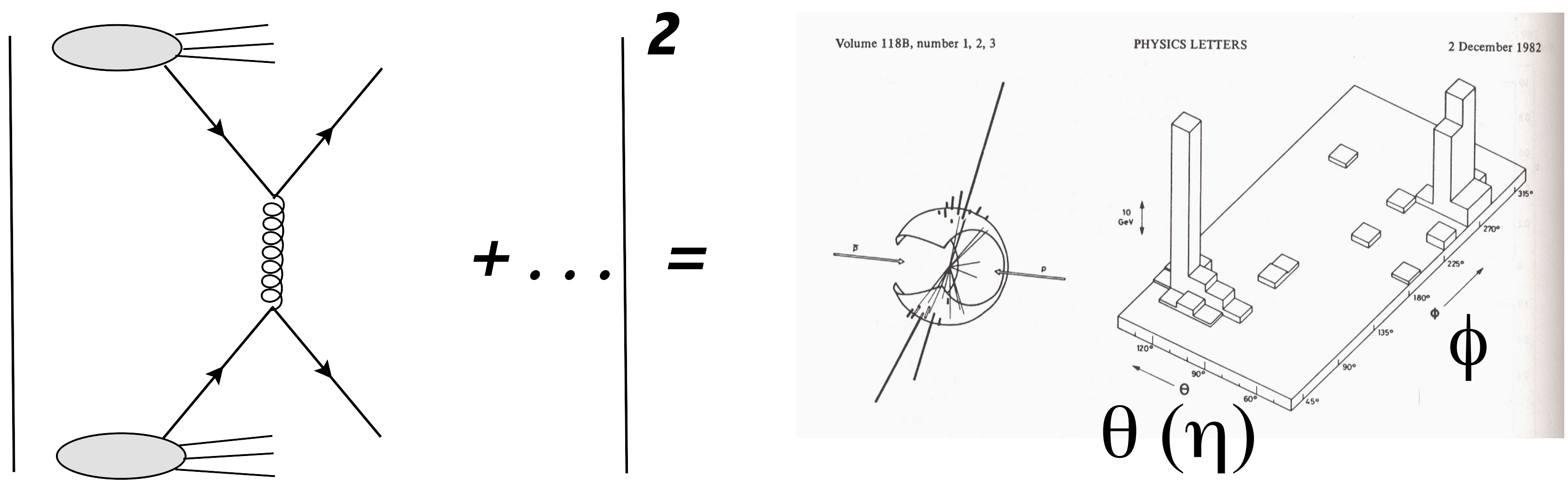


(a) (b)

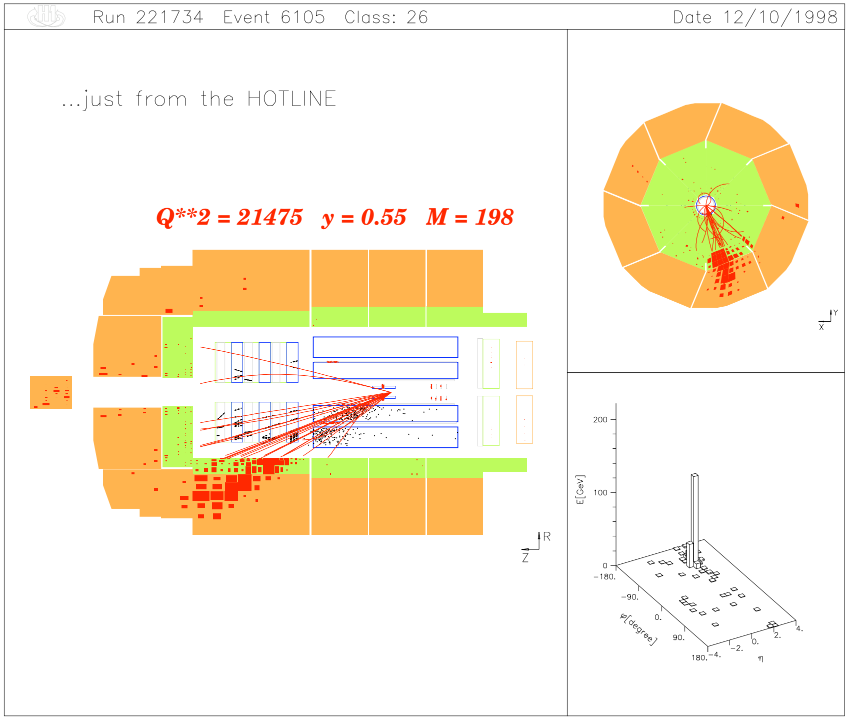
(On the right, O is oblateness, which measures the spread of energy in a plane.)

• **confirmed color as a dynamical variable.**

- **Jets at hadron colliders . . .**
- **80's: direct and indirect 'sightings' of scattered parton jets at Fermilab and the ISR at CERN, often in the context of single-particle spectra. Overall, however, an unsettled period until the SPS large angular coverage makes possible (UA2) 'lego plots' in terms of energy flow, and leads to the unequivocal observation of high-p_T jet pairs that represent scattered partons.**

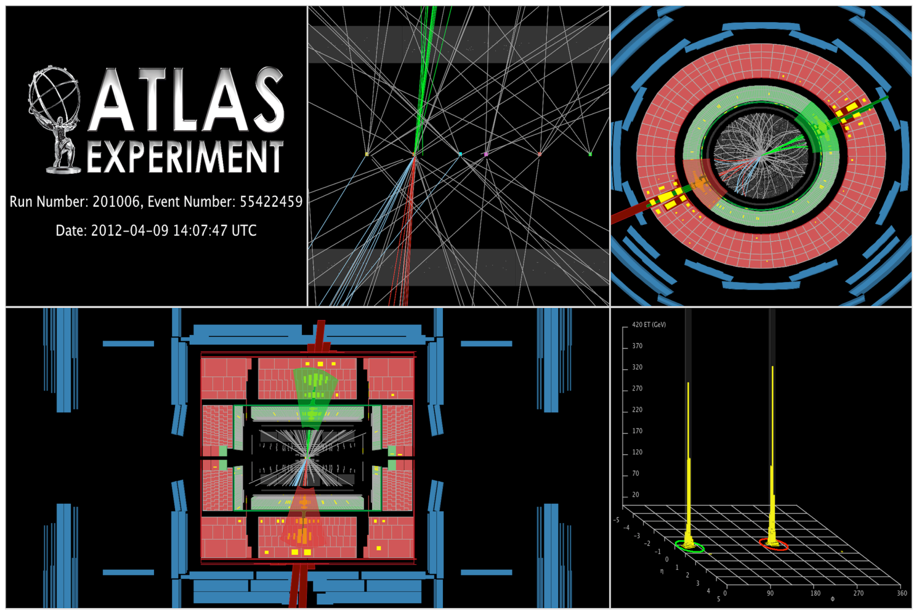


- **1990's – 2005: The great Standard Model machines: HERA, the Tevatron Run I, and LEP I and II provided jet cross sections over multiple orders of magnitude. The scattered quark appears.**



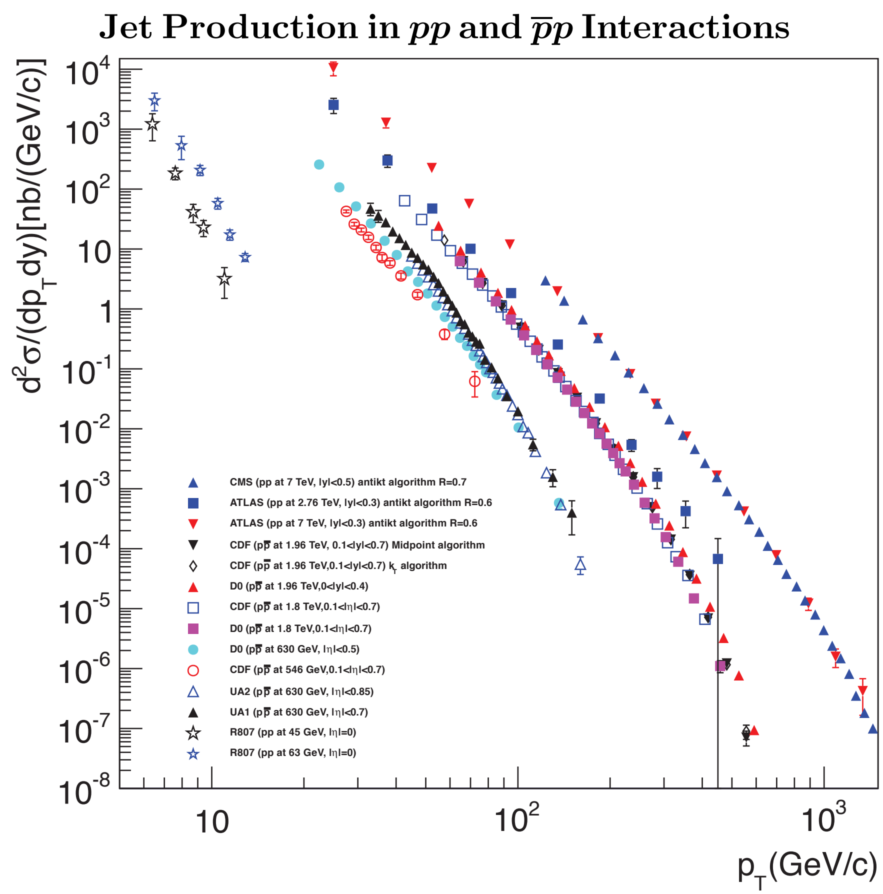
- **And now . . . the era of jets at the anticipated limits of the SM, ushered in by Tevatron Run II, on to the LHC: 2 $\rightarrow$ 7 $\rightarrow$ 8 $\rightarrow$ 13 TeV .**

- **Events at the scale $\delta x \sim \frac{\hbar}{1 \text{ TeV}} \sim 2 \times 10^{-19}$ meters . . . observed about 10 meters away.**



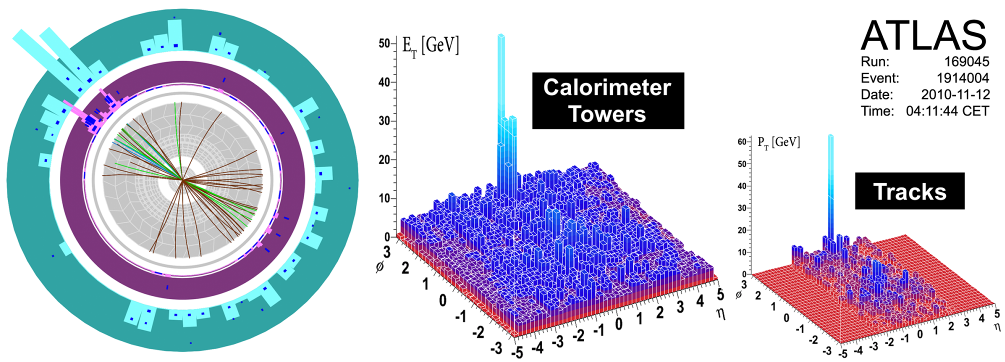
- **These jets can be remarkably narrow in an energy histogram, even if surrounded by a concentration of much softer particle tracks. This suggests a relation to QED bremsstrahlung.**

“REVIEW OF PARTICLE PROPERTIES” FIGURE: TEV JETS AND BEYOND



In brief, in their other life: shining from the inside, jets are probe of new phases of strongly-interacting matter in nuclear collisions at RHIC and the LHC,

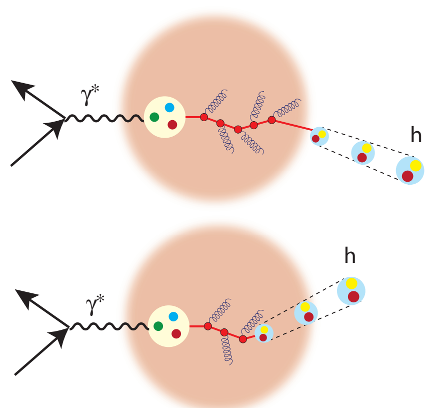
(Bjorken (1983) . . .)



(From 1011.6182)

And of "cold nuclei" in electron-ion collisions,

(A. Arccadi *et al.*, **Electron-ion Collider White Paper (1212.1701))**



Summary, Part A

- Quantitative comparisons of QCD to experiment began with fully inclusive processes.
- In a seeming paradox, inclusive cross sections can be related to elastic scattering of quarks (the parton model). Asymptotic freedom makes this plausible
- Electron positron annihilation to hadrons is dominated by two-jet events that clearly reflect quark pair creation. The observable “thrust” helps identify and justify the use of the term jet.
- High energy accelerators, at energies far above (light) quark masses, all produce events consistent with this interpretation.

B: Challenges at very high energy: why and how soft and collinear enhancements arise in long-time behavior

Outline

- In QCD, long-time dynamics is not accessible to perturbation theory.

- The example of QED suggests that partially inclusive cross sections can be calculable perturbatively by eliminating infrared divergences.

- When energies are much larger than masses, divergences appear in scattering amplitudes when lines in virtual states become collinear as well as soft.

- Time-ordered perturbation theory provides a convenient picture of how an amplitude develops in time. It gives insight into both UV and IR behavior.

- At large times, the effects of interactions between high energy particles vanish, except for those between collinear-moving and/or soft particles.

How to use perturbation theory in QCD?

- **How to go beyond totally inclusive cross sections in QCD? Can quarks and gluons be of help? At lowest order, $\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\bar{\mathrm{q}}$ is easy to calculate, but what can we do with $\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\bar{\mathrm{q}}\mathrm{g}$? It is divergent when the energy of the gluon vanishes, and has logs of quark mass over total energy. (We'll see why.)**

- **And what to do about the running of the asymptotically free QCD coupling? If low-energy divergences imply sensitivity to long distances, doesn't the coupling blow up, making the entire process nonperturbative?**

- **Very analogous questions were phrased for strong interactions at high energy (think cosmic rays) in the 1930s, even before renormalization was invented. And back then the analysis of Bloch and Nordseick for QED was recognized as a possible way forward . . .**

- **The glorious example of QED: At lowest order, electron-electron scattering is finite, but at next to leading order it is IR divergent for both virtual corrections and photon emission. But in a partially inclusive sum over soft photon emission only, the divergences cancel, and we derive a finite cross section.**

- **How? We introduce an "energy resolution", ϵE, below which we count all photons. Then divergences are replaced by factors $\alpha \ln(E_e/\epsilon E)$, and this "inclusive" cross section is well-approximated by the lowest order (again). Schematically, for n-photon emission:**

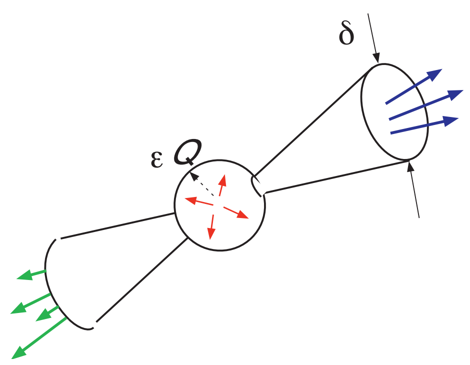
$$\frac{d\sigma_n^{(\mathrm{IR})}}{d\Omega} \sim \frac{d\sigma_0}{d\Omega} \times \frac{1}{n!} \left(\frac{\alpha_{\mathrm{EM}}}{\pi} \ln\left(\frac{E_e}{m_e}\right) \ln\left(\frac{\epsilon E_{\mathrm{tot}}}{m_\gamma}\right)\right)^n \exp\left[- \frac{\alpha_{\mathrm{EM}}}{\pi} \ln\left(\frac{E_e}{m_e}\right) \ln\left(\frac{E_e}{m_\gamma}\right)\right]$$

- **For $|\ln \epsilon| \ll 137$, the sum over n is very close to the Born ($n = 0$) cross section. All the higher orders cancel (corrected by well-behaved terms we've omitted here). The paradoxical lesson: "the more inclusive, the closer to the lowest order."**

- **Once QCD was invented, QED served an inspiration for the treatment of strong interactions in the limit when energies and momentum transfers are much larger than masses.**

- **For QCD, at very high energy we had to introduce an energy resolution and another, "angular" resolution. We'll see why below, and how to generalize to a much larger set of observables.**

- **From now on, all our particles will be massless. Particles whose masses are of the order of the energy/momentum transfer scale can be treated at the same time, but require special attention. (Aside – this is treating QCD as though it were a conformal theory, with no intrinsic mass scale.) The picture:**



- **With ϵQ the energy resolution, an δ an angular resolution. Defines a "cone jet".**

- **Looks promising, but how does it work? First, we have to isolate the problem, then show how the jet approach solves it.**

- **Let's remember what we'd like to calculate. It's a general "transition probability", or cross section, summed over final states "f", which we'll represent as**

$$
\begin{aligned}
P[S] &= \sum_f S[f] \, |\langle m_f | m_0 \rangle|^2 \\
&= \sum_f S[f] \sum_{n',n} \langle m_0 | m_f \rangle^{(n')} \langle m_f | m_0 \rangle^{(n)}
\end{aligned}
$$

The function $S[f]$ defines the cross section. It includes all the normalizations, and otherwise can be unity for some states, zero for others, or in between. Generally, we'll assume it's a smooth function.

- **To calculate $P[S]$, we'll start with the amplitude $\langle m_f | m_0 \rangle^{(n)}$ at fixed perturbative order (n) in QCD or some other theory. This is "just" a bunch of Feynman diagrams, but we'll consider a variation of this route.**

Perturbation theory “from the beginning”

- It really just follows from Schrödinger equation for mixing of free particle states $|m\rangle$,

$$i\hbar\,\frac{\partial}{\partial t}|\psi(t)>=\left(H^{(0)}+V\right)|\psi(t)>$$

Usually with free-state “IN” boundary condition :

$$|\psi(t=-\infty)>=|m_0>=|p_1^{\rm IN},p_2^{\rm IN}\rangle$$

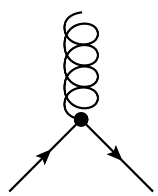
- Notation : $V_{ji}=\langle m_j|V|m_i\rangle$ (vertices)

- Theories differ in their list of particles and their (hermitian) Vs.

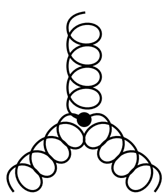
For QCD, the Lagrange density

$$\begin{aligned} \mathcal{L}_{\mathrm{QCD}} &= \overline{\psi}_i (i\gamma^\mu \partial_\mu - m)\psi_i - \tfrac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} - g_{\mathrm{s}}\, \overline{\psi}_i \lambda^a_{ij} \psi_j\, \gamma^\mu A^a_\mu \\ F_a^{\mu\nu} &= \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - 2 g_{\mathrm{s}}\, f_{abc} A_b^\mu A_c^\nu \end{aligned}$$

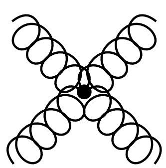
And vertices



$g_{\mathrm{s}}\, \bar{\psi}_i \lambda^a_{ij} \psi_j \gamma^\mu A^a_\mu$ quark-gluon vertex



$g_{\mathrm{s}}\, (\partial^\mu A_a^\nu - \partial^\nu A_a^\mu) f_{abc} A^b_\mu A^c_\nu$ 3-gluon vertex



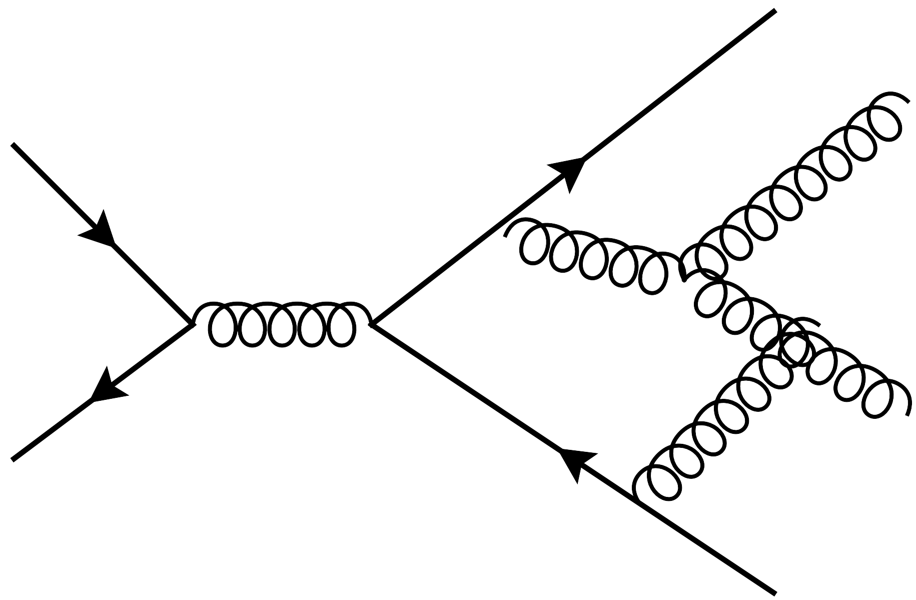
$g_{\mathrm{s}}^2\, f_{abc} A_b^\mu A_c^\nu\, f_{ade} A^d_\mu A^e_\nu$ 4-gluon vertex

- **Solutions to the Schrödinger equation are sums of ordered time integrals. "Old-fashioned perturbation theory."**

$$\langle m_F | m_0 \rangle^{(n)} = \sum_{\tau \text{ orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1$$
$$\times \prod_{\text{loops } i} \int \frac{d^3 \ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2E_j} \times \prod_{\text{vertices } a=1}^{n} i V_{a-1 \to a}$$
$$\times \exp \left[i \sum_{\text{states } m=1}^{n-1} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \right) (\tau_m - \tau_{m+1}) - i E_0 \tau_1 \right]$$

- **Perturbative QFT in a nutshell: integrals are divergent in QFT from:**
- $\tau_i \to \tau_j$ **(UV) and** $\tau_i \to \infty$ **(IR).**
- **Renormalization takes care of coinciding times. We'll just assume this is done.**

Each term in this expansion corresponds to a "time-ordered" diagram



Here the vertices are ordered at different times. Sums of orderings give (topologically equivalent) "Feynman diagrams", which exhibit the Lorentz invariance manifestly.

The integrals over loop momenta are exactly the sums over all virtual states.

- Once renormalized, infinities only come from large times in ... (same formula)

$$\langle m_n | m_0 \rangle = \sum_{\tau \text{ orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1 \times \prod_{\text{loops } i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2E_j} \times \prod_{\text{vertices } a=1}^{n} iV_{a-1\to a} \times \exp\left[i \sum_{\text{states } m=1}^{n-1} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \right) (\tau_m - \tau_{m+1}) - iE_0\tau_1 \right]$$

- Divergences from $\tau_i \to \infty$ are "Infrared=IR". In some sense, their "solution" is jets,

- because – it's not as bad as it looks. Time integrals extend to infinity, but usually oscillations damp them and answers are finite. Long-time, "infrared" divergences (logs) come about when phases vanish and the time integrals diverge.

- **When does this happen?** **Here's the phase:**

$$\exp\left[\, i \sum_{\text{states}\, m=1}^{n-1} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \right) (\tau_m - \tau_{m+1}) \right] =$$

$$\exp\left[\, i \sum_{\text{vertices}\, m=1}^{n} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \; - \sum_{j \text{ in } m-1} E(\vec{p}_j) \right) \; \tau_m \right]$$

- **Divergences for $\tau_i \to \infty$ requires two things:**

i) (RHS) the phase must vanish $\leftrightarrow$ "degenerate states"

$$\sum_{j \in m} E(\vec{p}_j) = \sum_{j \in m+1} E(\vec{p}_j)\,, \quad \text{and}$$

ii) (LHS) the phase must be stationary in loop momenta (sums over states):

$$\frac{\partial}{\partial \ell_{i\mu}} \,[\,\text{phase}\,] = \sum_{\text{states}\, m} \; \sum_{j \text{ in } m} (\pm \beta_j^{\mu})(\tau_m - \tau_{m-1}) = 0$$

where the β_js are normal 4-velocities:

$$\beta_j = \pm \partial E_j / \partial \ell_i \,.$$

- Condition of stationary phase:

$$\sum_{\text{states } m} \sum_{j \text{ in } m} (\pm\beta_j^\mu)(\tau_m - \tau_{m-1}) = 0$$

- $\beta^\mu \Delta\tau = x^\mu$ is a classical translation. For IR divergences, there must be free, classical propagation as $t \to \infty$. Easy to satisfy if all the β_j's are equal.

- Whenever fast partons (quarks or gluons) emerge from the same point in space-time,

  they will rescatter for long times only with collinear partons.

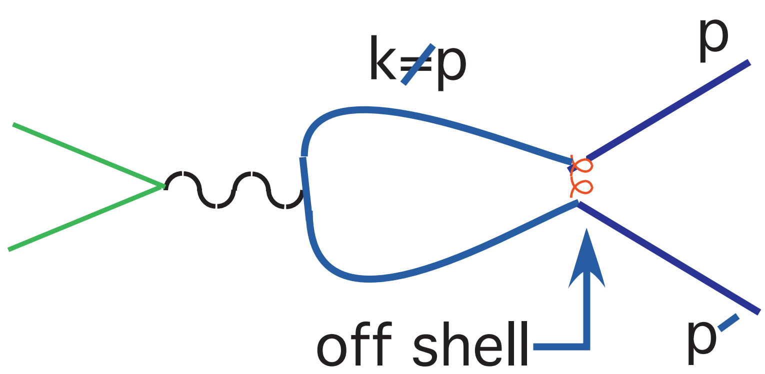
  Of course, radiating or absorbing zero momentum particles also don't affect the phase.

  Note, all the states we can reach by rescattering or zero momentum interactions describe the same energy flow.

  When we get to cross sections, this is where the conditions for infrared safety will come from.

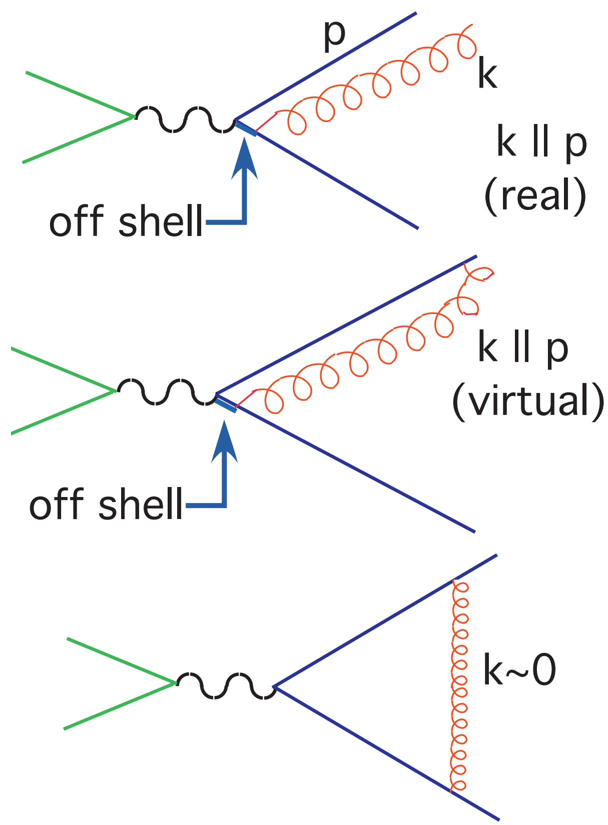
- Let's illustrate the role of classical propagation.

- Example 1: degenerate states that cannot give long-time divergences:



- This makes identifying enhancements a lot simpler!

- **RESULT: For particles emerging from a local scattering, (only) collinear or soft lines can give long-time behavior and enhancement.** **Example:**



- **This generalizes to any order, and any field theory, but gauge theories alone have soft ($k \to 0$) divergences.**

- **These are what we can't compute in pQCD (as physical processes). And we didn't want to, because they are never produced! Let's find out what we can compute.**

Summary, Part B

– **In QCD, long-time dynamics is not accessible to perturbation theory.**

– **The example of QED suggests that partially inclusive cross sections can be calculable perturbatively by eliminating infrared divergences.**

– **At very high energies, divergences appear when lines become collinear as well as soft.**

– **Time-ordered perturbation theory provides a convenient picture of how an amplitude develops in time. It gives insight into both UV and IR behavior.**

– **At large times, the effects of interactions among high energy particles vanish, except for those between collinear-moving and/or soft particles.**

C. Why energy flow is a guide to calculable cross sections: infrared safety

Outline

– The integral of the largest time controls IR behavior.

– Particle emission or absorption requires a characteristic formation time, which diverges is the collinear limit.

– The momentum flow evolution of each jet is independent of the others.

– Time-ordered emissions provide ordered branching pictures.

– In cross sections, a free sum over states always cancels long-time behavior by use of the largest time equation.

– Infrared safe weight functions can provide perturbative cross sections, and properties of jets.

– Energy correlations offer a window into energy flow.

- **The role of the largest time:**

$$\langle m_F | m_0 \rangle^{(n)} = \sum_{\text{orders } \mathrm{m_1 \ldots m_n}} \prod_{\text{loops } i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2E_j}$$

$$\times \prod_{\text{vertices } a=1}^{n} \int_{-\infty}^{\tau_{a+1}} iV_{a-1\to a} \exp\left[i \left(\sum_{j \text{ in } a-1} E(\vec{p}_j) \right) (\tau_{a-1} - \tau_a) - iE_0\tau_1 \right]$$

$$= \sum_{\text{orders } \mathrm{m_1 \ldots m_n}} \prod_{\text{loops } i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2E_j}$$

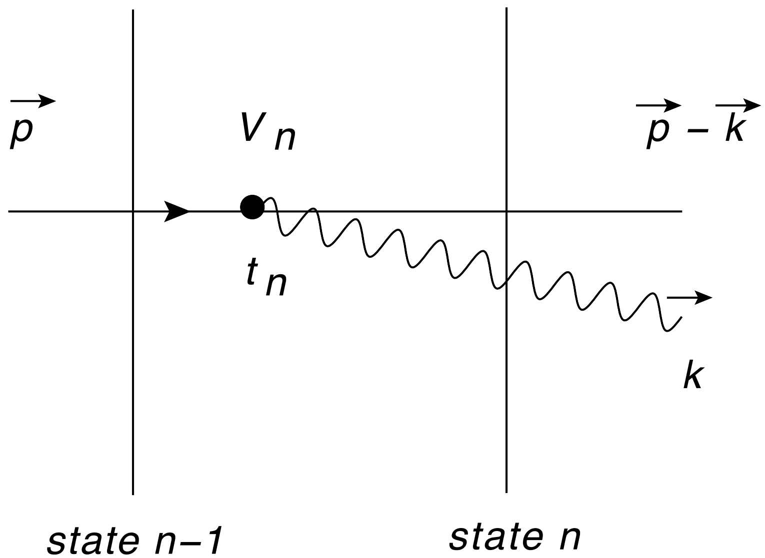
$$\times \prod_{\text{vertices } a=1}^{n} \int_{-\infty}^{\tau_{a+1}} iV_{a-1\to a} \exp\left[i \left(\sum_{j \text{ in } a} E(\vec{p}_j) - \sum_{j \text{ in } a-1} E(\vec{p}_j) \right) \tau_a \right]$$

With $\tau_{n+1} = \infty$.

- **So large times are controlled by the τ_n integral ("n=F"): the "largest time".**

$$\int_{\tau_{n-1}}^{\infty} iV_{n-1\to n} \exp\left[i \left(\sum_{j \text{ in } F} E(\vec{p}_j) - \sum_{j \text{ in } n-1} E(\vec{p}_j) \right) \tau_n \right]$$

Say the final interaction is the splitting of one particle into two, all treated as massless:



Here state n = the final state F
All the other energies cancel, and the largest time integral is

$$\int_{\tau_{n-1}}^{\infty} d\tau_n i V_{n-1\to F} e^{i\left(\sum_{j\,\mathrm{in}\,n} E(\vec{p}_j)\ -\ \sum_{j\,\mathrm{in}\,n-1} E(\vec{p}_j)\right)\tau_n}$$

$$= \int_{\tau_{n-1}}^{\infty} d\tau_n i V_{n-1\to F} e^{i\Delta_n \tau_n}$$

Relabel: $p \to k_1,\ k \to k_2$:

$$\Delta_n \ = \ E(\vec{k}_1 - \vec{k}_2) + E(\vec{k}_2) - \ E(\vec{k}_1)$$

Can use the $i\epsilon$ prescription $\Delta \to \Delta + i\epsilon$ to make the integral converge. Or, we can observe that most of this integral cancels out "oscillation by oscillation" . Say $\tau_{n-1} \to 0$:

$$\begin{aligned}
\int_0^\infty d\tau_n e^{i\Delta_n \tau_n} &= \frac{1}{\Delta_n}\int_0^\infty dx\,[\cos x + i\sin x] \\
&= \frac{1}{\Delta_n}\int_0^\infty dx \frac{d}{dx}\,[\sin x - i\cos x] \\
&= -\frac{1}{\Delta_n}\,[\sin 0 - i\cos 0] \\
&= \frac{i}{\Delta_n}\int_0^{\pi/2} dx \sin x
\end{aligned}$$

– Only times smaller than $\pi/2\Delta_n$ really contribute to the amplitude.

– $1/\Delta_n$ is called the "formation time" of state n.

What is Δ_n and when does it vanish? When it does, we're going to have problems!

$$\Delta_n \ = \ E(\vec{k}_1 - \vec{k}_2) + E(\vec{k}_2) - \ E(\vec{k}_1)$$

– **Kinematics**

$$\vec{k}_1 \ = \ (P, \vec{0}_T)\,, \ \vec{k}_2 \ = \ (zP, \vec{k}_T)\,, \ k_T \leq zP \ll P$$

– **Then**

$$\Delta_n \ = \ \frac{k_T^2}{2zP} \ \Leftrightarrow \ \frac{1}{\Delta_n} \ = \ \frac{2zP}{k_T^2}$$

– **Formation time grows for $k_T \to 0$ at fixed z (collinear radiation) and when $z \to 0$ but $k_T \sim zP$ (soft radiation).**

– **In terms of the angle: $k_T = zP \sin\theta$, for small θ,**

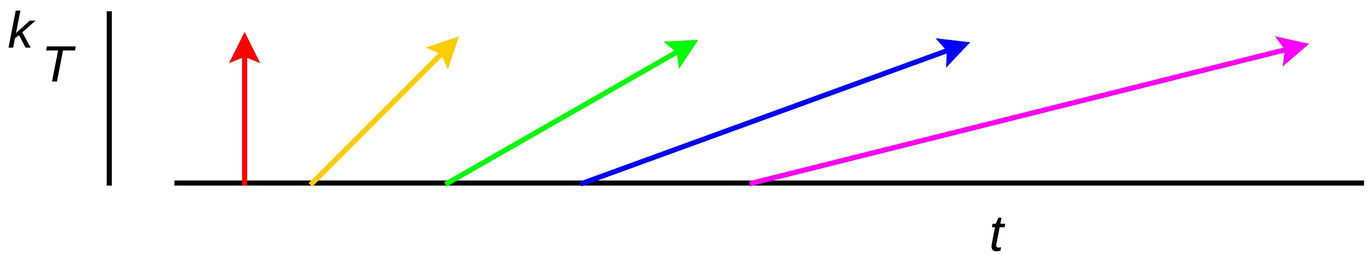
$$\frac{1}{\Delta_n} \sim \ \frac{1}{\theta^2 zP} \ \sim \ \frac{1}{\theta k_T}$$

– **At fixed k_T, formation time increases as radiation becomes more forward.**

– This is a very general picture of the formation of final states.

– Because the final time limits all other integrals, particles produced at earlier times can only involve shorter formation times – wider angles and/or larger k_T.

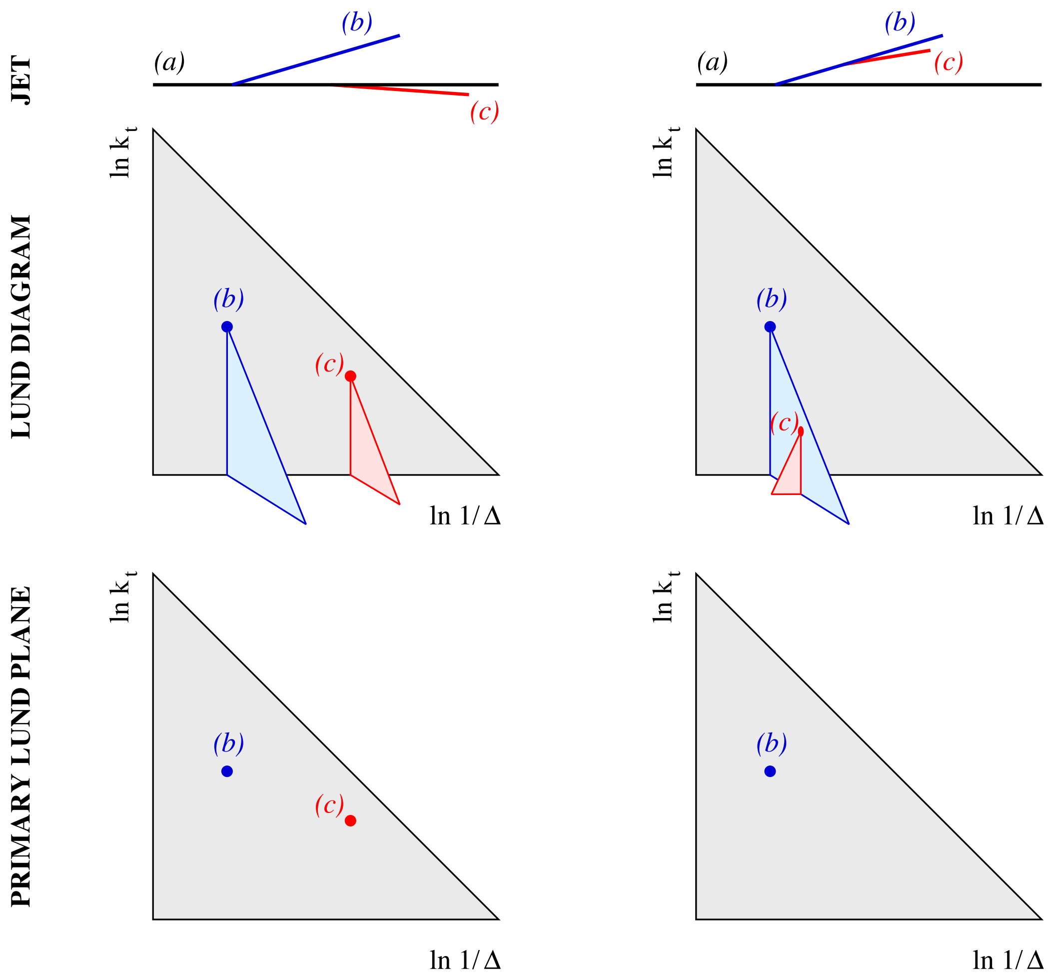
– Gives a “branching” picture of radiation. At fixed k_T it starts with soft wide-angle, and moves on to smaller and smaller angles.



– When we probe smaller and smaller angle radiation, we look at states that took longer and longer to produce.

– As time increases, particle emission of each jet becomes more collimated. The jets evolve independently.

– **Another popular way of representing radiation "branches": the "Lund Plane". Each branch is a point.**



from Dryer, Salam, Soyez 1807.04578

– In the presence of massless particles, we encounter a divergent time integral whenever we find a $\Delta_n = 0$.

– The point $\Delta_n = 0$ is exactly a point of stationary phase in k_T.

$$\int d^2k_T \int^\infty d\tau_n e^{i\Delta_n \tau_n} = \int d^2k_T \int^\infty d\tau_n e^{i\tau_n k_T^2/2zzP}$$
$$\sim 2\pi z P \int^\infty \frac{d\tau_n}{\tau_n}$$

– $\Delta_n = 0$ when $z = 0$ and/or $k_T = 0$: Soft or collinear radiation.

– Now we can motivate the construction of IR finite cross sections.

Finite-time cross sections and what they represent. Consider the probability for a sum over states f, each weighted by $S[f]$,

$$P[S] \; = \; \sum_f S[f] \sum_{n',n} \langle m_0 | m_f \rangle^{(n')} \langle m_f | m_0 \rangle^{(n)}$$

- Each matrix element and complex conjugate is a sum of ordered time integrals
- In any term of $P[S]$, as we integrate over times, there is a largest time.
- The largest time may be in the amplitude, or in the complex conjugate. We combine these two possibilities. Inside the sum over states, we find

$$\ldots \times \int_{\tau'_{n-2}}^{\tau'_n} e^{i\Delta_{n-1}\tau_{n-1}} (-iV'_{f-2\to f-1}) e^{-i\Delta_{n-1}\tau'_{n-1}} \quad \Leftarrow \text{in } \langle m_0 | m_f \rangle$$

$$\times \int_{\tau_{f-1}}^{\infty} d\tau_n V_{f-1\to f} \left\{ i e^{i\Delta_n \tau_n} S[f] - i e^{-i(-\Delta_n)\tau_n} S[f-1] \right\}$$

$$\text{in } \langle m_f | m_0 \rangle \Rightarrow \quad \times \int_{\tau_{n-2}}^{\tau_n} e^{i\Delta_{n-1}\tau_{n-1}} i V_{f-2\to f-1} e^{i\Delta_{n-1}\tau_{n-1}} \times \ldots$$

- When $S[f] = S[f-1]$ this vanishes! This is called the "largest time equation". It is an expression of unitarity – the sum of all probabilities has to be one.
- All that matters is the difference due to the last interaction: $V_{f-1\to f}$. When this produces a difference in $S[f]$, the result is nonzero.

- As in the introductory lectures, we define a set of smooth (symmetric) functions which depend only on the flow of energy, and not particle content:

$$S_{n+1}(p_1 \ldots (1-z)p_n, zp_n) \;=\; S_{n+1}(p_1 \ldots p_n)$$

In our examples, whenever $\Delta_n \to 0$, we only need

$$S_{n+1}[f] - S_n[f-1] \;\sim\; k_\perp^b s_f$$

for some constant s_f with $b > 0$. Then

$$\int d\tau_n \; e^{i\Delta_n\tau_n} \left(S_{n+1}[f] - S_n[f-1]\right) \;\to\; s_f \int d\tau_n \; k_\perp^b e^{i\Delta_n\tau_n}$$

- There is now suppression for large times:

$$s_f \int d^2k_T \, k_\perp^b \int^\infty d\tau_n e^{i\Delta_n\tau_n} \;\; = \; \pi s_f \Gamma(1+b) \int^\infty \frac{d\tau_n}{\tau_n^{1+b/2}}$$

- and the perturbative integral will be finite. The largest time integral converges, and so must the smaller ones,
  Our calculations now give predictions, rather than infinities. This is infrared safety.

- In summary, For any $S[f]$ that respects energy flow, we compute the cross section

$$\boldsymbol{P}[\boldsymbol{S}] \;=\; \sum_f \boldsymbol{S}[\boldsymbol{f}]\; |\langle \boldsymbol{m}_f | \boldsymbol{m}_0 \rangle|^2$$

- The same applies to jet cross sections themselves if they are designed to respect the flow of energy. Here, S[f] is chosen to be unity for states that obey certain conditions in jet finding algorithms – which depend only on energy flows,

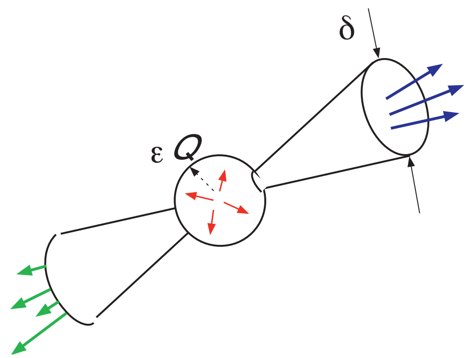
$$\boldsymbol{\sigma}[\boldsymbol{S}_{\rm n-jet}] \;=\; \sum_f \boldsymbol{\theta}(\boldsymbol{S}_{\rm n-jet}[\boldsymbol{f}])\; |\langle \boldsymbol{m}_f | \boldsymbol{m}_0 \rangle|^2$$

- Once we have identified a set of jets, we can then explore their properties by using weight functions $w_{\rm n-jet}[f]$ that reveal their structure,

$$\langle \boldsymbol{w}_{\rm n-jet} \rangle \;=\; \frac{\sum_f \boldsymbol{w}_{\rm n-jet}[\boldsymbol{f}]\, \boldsymbol{\theta}(\boldsymbol{S}_{\rm n-jet}[\boldsymbol{f}])\; |\langle \boldsymbol{m}_f | \boldsymbol{m}_0 \rangle|^2}{\sum_f \boldsymbol{\theta}(\boldsymbol{S}_{\rm n-jet}[\boldsymbol{f}])\; |\langle \boldsymbol{m}_f | \boldsymbol{m}_0 \rangle|^2}$$

- These are what we can compute.

- An example is the cross section for a cone jet with a given energies,



- The smaller (larger) the "resolutions" ϵ and δ, the more (less) sensitivity to long times. We follow the story only to times like $1/Q\delta$.

  Other fundamental choices: radiation pattern and and energy-energy correlation

$$
\begin{aligned}
S_{\rm rad}[\hat{n}] &= \sum_i E_i\, \delta^2\left(\hat{n} - \hat{n}(\vec{k}_i)\right) \\
S_{\rm EEC}(\hat{n}_1, \hat{n}_2) &= \sum_{i,j} E_i\, E_j\, \delta^2\left(\hat{n}_1 - \hat{n}(\vec{k}_i)\right) \delta^2\left(\hat{n}_2 - \hat{n}(\vec{k}_i)\right) .
\end{aligned}
$$

  Perhaps surprisingly, we can treat the delta functions as if they were smooth, and if we integrate over $\hat{n}_1 \ldots$, we can generate any weight function.

- **Energy correlations link to QFT matrix elements, analogous to $\sigma_{\mathrm{tot}}^{\mathrm{e^+e^-}}$:**

$$\mathrm{ENC}(R_L) = \left(\prod_{k=1}^{N} \int d\Omega_{\vec{n}_k}\right) \delta(R_L - \Delta\hat{R}_L) \cdot \frac{1}{(E_{\mathrm{jet}})^N} \langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots\mathcal{E}(\vec{n}_N)\rangle$$

- **E2C $\equiv$ EEC**

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

- **A direct link also to the actual measurements done at colliders, and to theories beyond QCD.**

- **See: Chen, Moult, Zhang, Zhu (2004,11381); Komiske, Moult, Thaler, Zhu (2201.07800) & Lee, Mecal, Moult (2205.03414))**

- **At the LHC, energy flow is observed with calorimeters, aided by tracking of charged particles.**
- **For ATLAS:**

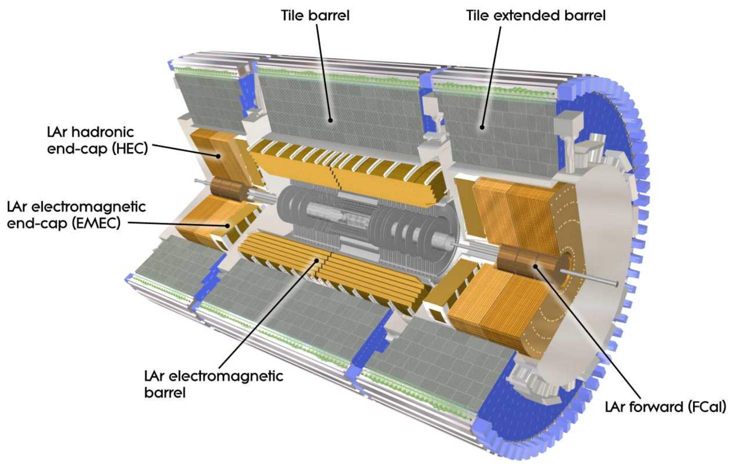
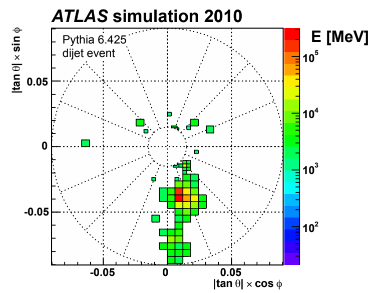
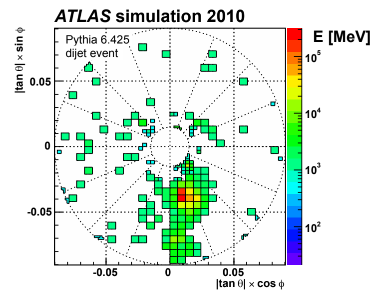


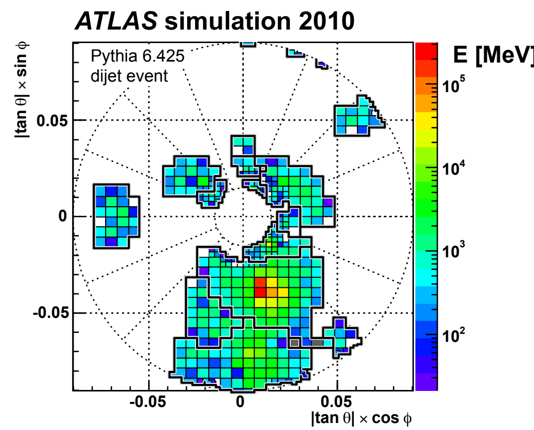
Figure 1: Cutaway view on the ATLAS calorimeter system.



(a) Cells passing selection in Eq. (3)

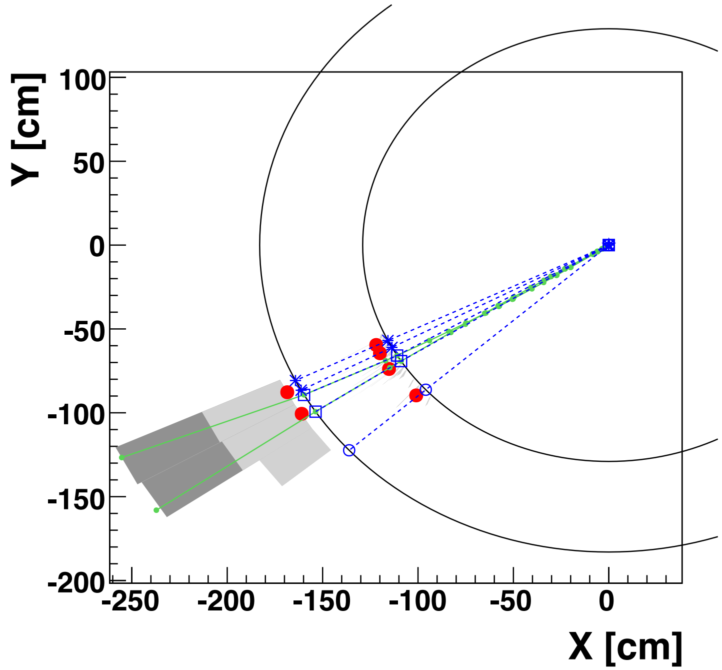


(b) Cells passing selection in Eq. (4)

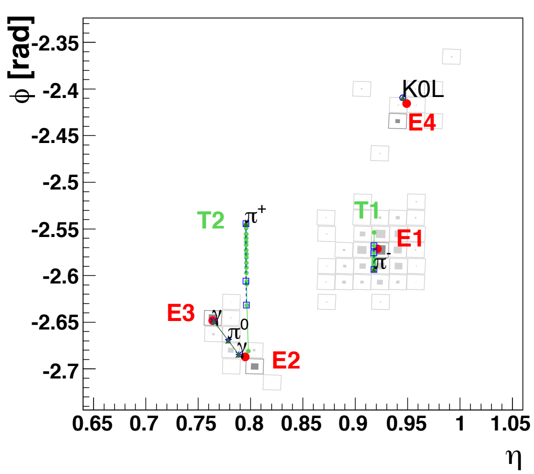


(c) All clustered cells

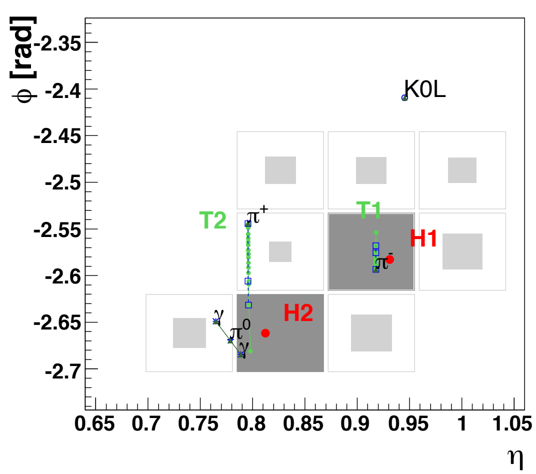
- **And for CMS:**



(a) The (x, y) view

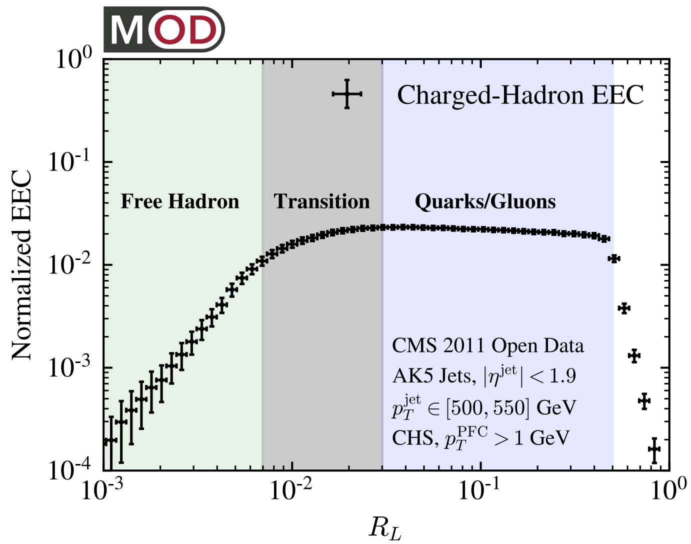


(b) The (η, φ) view on ECAL



(c) The (η, φ) view on HCAL

- **Example of a new window into the boundaries of pQCD: EEC as a function of anglular (and rapidity) separation (R_L).**



- **"Flat" portion is a direct result of $(p+k)^2 \sim p^0 k^0 \theta^2$ for 1 to 2 splitting. The "transition" shows the turnover from pQCD to hadronization.**

Summary, Part C

- The integral of the largest time controls IR behavior.
- Particle emission or absorption requires by a characteristic formation time, which diverges is the collinear limit.
- Jet evolution is independent.
- Time-ordered emissions provide angular-ordered branching pictures.
- In cross sections, a free sum over states always cancels long-time behavior by use of the largest time equation.
- Infrared safe weight functions can provide perturbative cross sections, and properties of jets.
- Energy correlations offer a window into energy flow

D. How jets are found and their cross sections computed

Outline

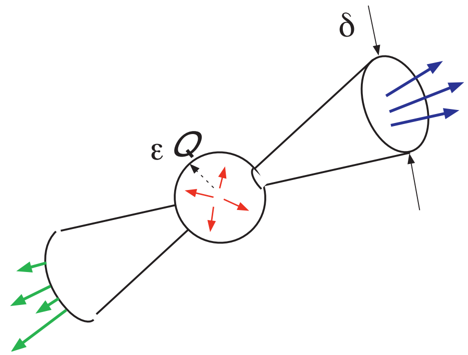
- The simplest example is the cone jet in e^+e^- annihilation. Predictions for hadrons from computations with partons.

- Thrust illustrates both jet finding and quantification by weight.

- Jet algortihms for hadronic collisions and N-jettiness can assemble and quantify hadronic jets.
- These methods are phenomenologically successful.

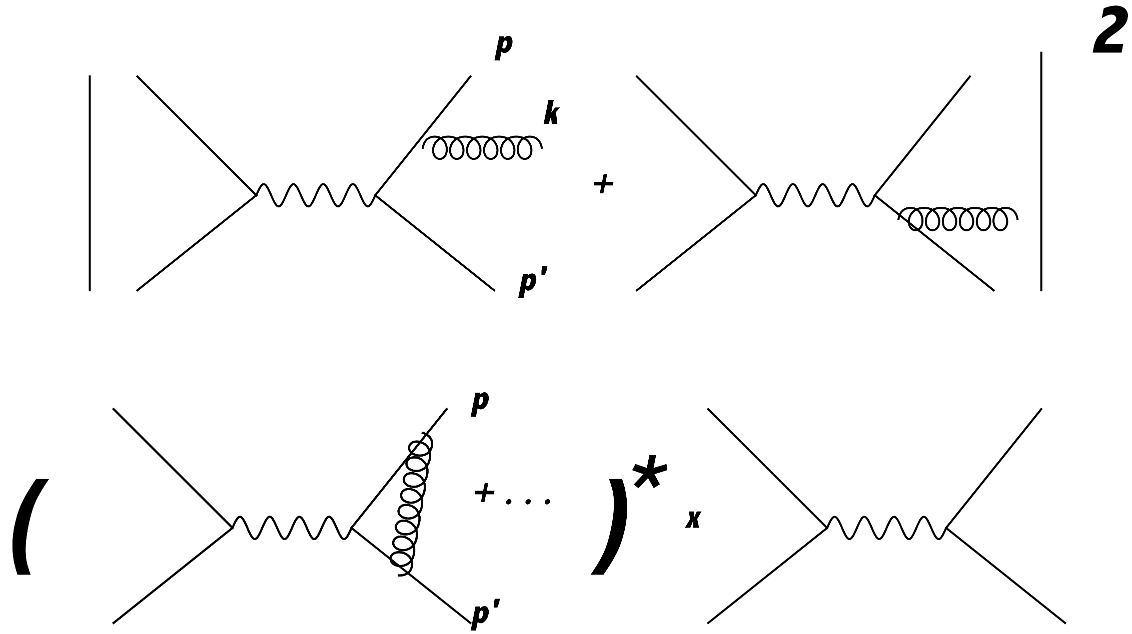
“Seeing” Quarks and Gluons With Jet Cross Sections

- **Simplest example: cone jets in e^+e^- annihilation. All but fraction ϵ of energy flows into cones of size δ. We proceed by calculating the jet cross section with quarks and gluons, imposing the jet constraints on partons, interpreting the result in terms of hadrons.**



- **Intuition: eliminating long-time behavior $\Leftrightarrow$ recognize the impossibility of resolving collinear splitting/recombination of massless particles.**
- **We can specify the “jet energy” flowing into the cone, or integrate subject to soft radiation outside the cone.**
- **We can keep the cone fixed in space, or calculate probability of events for which any cone receives a fixed energy.**
- **We can change ϵ or δ: There is no unique definition of the jet, but we can calculate how the probabilities change with the definition! This is the essence of QCD jet physics.**

Diagrams at order α_s:



- **The gluon can be collinear to either outgoing quark or antiquark or may be soft.**
- **For hadron-hadron scattering, more diagrams and gluon can be parallel to an incoming line.**
- **We can compute both the total and cone jet cross sections.**

At order α_s, the kind of integral we encounter, for both virtual and real gluon is:

$$4p\cdot p' \int \frac{d^3k}{2k}\,\frac{1}{2p_0k(1-\cos\theta_{pk})}\,\frac{1}{2p'_0k(1-\cos\theta_{p'k})}$$

For virtual gluon, go to overall c.m., where $\vec{p} = -\vec{p}'$ are back-to-back. Then $(Q \equiv p_0)$:

$$\text{virtual}: \quad = \ -\int_0^Q \frac{dk}{2k}\int_{-1}^{1} \frac{2\pi\, d\cos\theta}{(1\ -\ \cos^2\theta_{pk})}$$

For the real gluon, $\vec{p}$ and $\vec{p}' = -\vec{p} - \vec{k}$ are back-to-back when $\vec{k}$ is collinear to either $\vec{p}$ or $\vec{p}'$ or soft, so:

$$\text{real}: \quad = \ +\int_0^Q \frac{dk}{2k}\int_{-1}^{1} \frac{2\pi\, d\cos\theta}{(1\ -\ \cos^2\theta_{pk})}\ +\ \text{finite}$$

Singularities cancel even without IR regularization.

- See how IR safety emerges with IR regularization: total $\mathrm{e}^+\mathrm{e}^-$ annihilation cross section to order α_s. Lowest order is $2 \to 2$, $\sigma_2^{(0)} \equiv \sigma_{\mathrm{LO}}$, σ_3 starts at order α_s.
  - Gluon mass regularization: $k^2 \to (k^2 - m_G^2)$

$$\sigma_3^{(m_G)} = \sigma_{\mathrm{LO}} \frac{4}{3}\frac{\alpha_s}{\pi}\left(2\ln^2\frac{Q}{m_g} - 3\ln\frac{Q}{m_g} - \frac{\pi^2}{6} + \frac{5}{2}\right)$$

$$\sigma_2^{(m_G)} = \sigma_{\mathrm{LO}}\left[1 - \frac{4}{3}\frac{\alpha_s}{\pi}\left(2\ln^2\frac{Q}{m_g} - 3\ln\frac{Q}{m_g} - \frac{\pi^2}{6} + \frac{7}{4}\right)\right]$$

which gives

$$\sigma_{\mathrm{tot}} = \sigma_2^{(m_G)} + \sigma_3^{(m_G)} = \sigma_{\mathrm{LO}}\left[1 + \frac{\alpha_s}{\pi}\right]$$

  - Pretty simple! (Cancellation of virtual (σ_2) and real (σ_3) gluon diagrams.)

– Dimensional regularization: change the area of a sphere of radius R from $4\pi R^2$ to $(4\pi)^{(1-\varepsilon)}\frac{\Gamma(1-\varepsilon)}{\Gamma(2(1-\varepsilon))}R^{2-2\varepsilon}$ with $\varepsilon = 2 - D/2$ in D dimensions.

$$\begin{aligned}
\sigma_3^{(\varepsilon)} &= \sigma_{\rm LO}\frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{(1-\varepsilon)^2}{(3-2\varepsilon)\Gamma(2-2\varepsilon)}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \\
&\times\left(\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}-\frac{\pi^2}{2}+\frac{19}{4}\right) \\
\sigma_2^{(\varepsilon)} &= \sigma_{\rm LO}\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{(1-\varepsilon)^2}{(3-2\varepsilon)\Gamma(2-2\varepsilon)}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\right. \\
&\left.\times\left(\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}-\frac{\pi^2}{2}+4\right)\right]
\end{aligned}$$

which gives again

$$\sigma_{\rm tot} = \sigma_2^{(m_G)} + \sigma_3^{(m_G)} = \sigma_0\left[1+\frac{\alpha_s}{\pi}\right]$$

- This illustrates IR Safety: σ_2 and σ_3 depend on regulator, but their sum does not.

At order α_s: the virtual has the same integral as for the total cross section:

$$\text{virtual}: \quad = -\int_0^Q \frac{dk}{2k} \int_{-1}^{1} d\cos\theta \; \frac{2\pi}{(1 \; - \; \cos^2\theta_{pk})}$$

Now the phase space for a real gluon is smaller, but still includes all regions where $\vec{p}$ and $\vec{p}' = -\vec{p} - \vec{k}$ are back-to-back when $\vec{k}$ is collinear to either or soft:

$$\text{real}: \quad \sim \; + \int_0^{\varepsilon Q} \frac{dk}{2k} \int_{-1+\delta^2/2}^{1-\delta^2/2} \frac{2\pi \, d\cos\theta}{(1 \; - \; \cos^2\theta_{pk})} + \int_0^Q \frac{dk}{2k} \left(\int_{1-\delta^2/2}^{1} + \int_{-1}^{-1+\delta^2/2} \right) \frac{2\pi \, d\cos\theta}{(1 \; - \; \cos^2\theta_{pk})}$$

Again singularities cancel even without IR regularization.

- **Finite, with no factors Q/m or $\ln(Q/m)$, a nice example of Infrared Safety.**

- **In this case,**

$$\sigma_{2J}(Q,\delta,\epsilon) \;=\; \frac{3}{8}\sigma_0(1+\cos^2\theta)\left(1-\frac{4\alpha_s}{\pi}\left[4\ln\delta\ln\varepsilon+3\ln\delta+\frac{\pi^2}{3}+\frac{5}{2}\right]\right)$$

- **Perfect for QCD: asymptotic freedom $\rightarrow d\alpha_s(Q)/dQ < 0$.**

- **No unique jet definition. $\leftrightarrow$ Each event a sum of possible histories.**

- **Relation to quarks and gluons always approximate but corrections to the approximation are computable.**

- **As we've seen, to identify and quantify back-to-back jets, we can use**

$$T = \frac{1}{s}\max_{\hat{n}}\ \sum_i |\hat{n}\cdot\vec{p}_i| = \frac{1}{s}\max_{\hat{n}}\ \sum_i E_i\,|\cos\theta_i|$$

with θ_i the angle of particle i to $\hat{n}$,, which we can define as a jet axis.

- **$T = 1$ for "back-to-back" jets, or**

$$\tau_0\ \equiv\ 1 - T\ \to\ 0$$

- **The thrust is IR safe precisely because it is insensitive to collinear emission (split energy at fixed θ_i) and soft emission ($E_i = 0$).**

- **Once jet direction is fixed, we can generalize thrust to any smooth weight function:**

$$\tau[f] = \sum_{\text{particles } i \text{ in jets}} E_i\, f(\theta_i)$$

and we will . . .

- **This approach, as above, with best available perturbative calculations and nonperturbative input, works really well (again $\tau = 1 - T$): (From R. Abbate *et al.* 1006.3080.)**

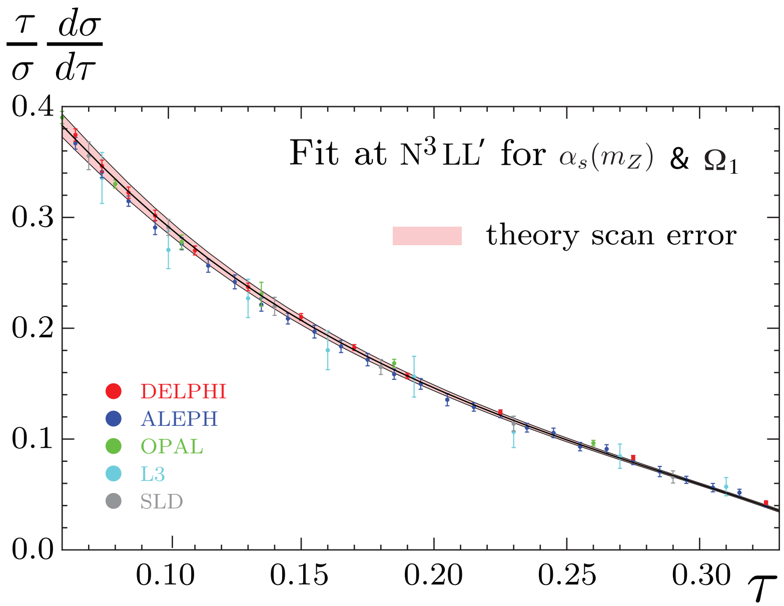


FIG. 13: Thrust distribution at N^{3}LL$'$ order and $Q = m_Z$ including QED and m_b corrections using the best fit values for $\alpha_s(m_Z)$ and Ω_1 in the R-gap scheme given in Eq. (68). The pink band represents the perturbative error determined from the scan method described in Sec. VI. Data from DELPHI, ALEPH, OPAL, L3, and SLD are also shown.

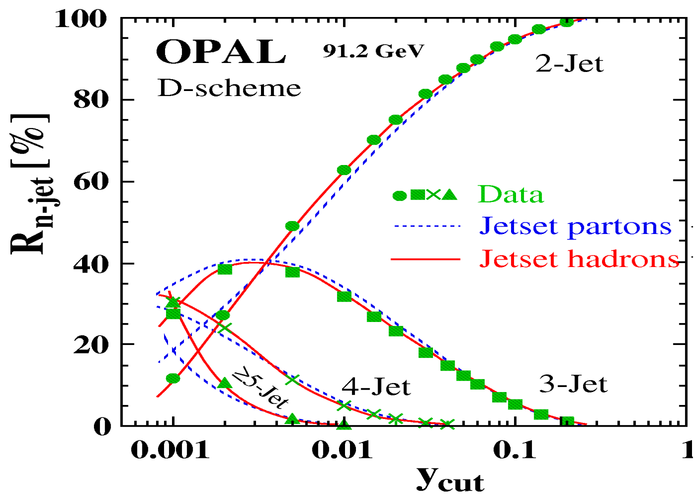
- **Of course, not every event qualifies as a two-jet event (large τ).**

For possibly multi-jet events, "cluster algorithms".

- $y_{\rm cut}$ Cluster Algorithm: Combine particles i and j into jets until all $y_{ij} > y_{\rm cut}$, where (e.g., "Durham alogrithm" for $\rm e^+e^-$):

$$y_{ij} = 2\min\left(E_i^2, E_j^2\right)(1 - \cos\theta_{ij})$$

- The number of jets depends on the variable $y_{\rm cut}$, and the dependence on the number of jets was an early application of jet physics. (Reproduced from Ali & Kramer, 1012)



- **To identify jets in hadronic collisions, jets are only well-defined away from the beam axis, so (instead of energy, E_i) use kinematic variables defined by the beam directions:**

transverse momentum, azimuthal angle and rapidity:

$$k_t$$

$$\phi$$

$$y = \frac{1}{2} \ln \left(\frac{E + p_3}{E - p_3} \right)$$

- **The beams define the '3-axis'.**

- Cluster variables for hadronic collisions:

$$d_{ij} = \min\left(k_{ti}^{2p}\,,\,k_{tj}^{2p}\right)\frac{\Delta_{ij}^2}{R^2}$$

$\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$. R is an adjustable parameter, analogous to the "cone size".

- The most common choices:
  - $p = 1$ k_t algorithm (the "classic")
  - $p = 0$ "Cambridge/Aachen"
  - $p = -1$ "anti-k_t" (probably the most common)

- For a given R, we effectively set $S_{\rm njet}[f] = 1$ for all final states that reconstruct n jets.

- Each step in a clustering process is IR safe, so can "groom" jets by calculating jet properties in terms of only energetic clusters. Such constructions are actually more inclusive in soft radiation. "Mass drop" is one such technique.

- **Could take R small, then jets are "narrow" but to quantify how "good" the jets are a popular and convenient measure is N-jettiness, a sort of generalization of the thrust to multijets:**

$$\tau_N = \frac{1}{Q^2}\sum_k \min\left(q_a \cdot p_k, q_b \cdot p_k, \ldots q_N \cdot p_k\right)$$

generalizable to a class of "N-subjettiness" jet measures

$$\tau_N^{(\alpha)} = \frac{1}{E_J}\sum_i \min\left(\theta_{i,1}^{\alpha}, \theta_{i,2}^{\alpha}, \ldots \theta_{i,N}^{\alpha}\right)$$

Jets in hadron-hadron scattering

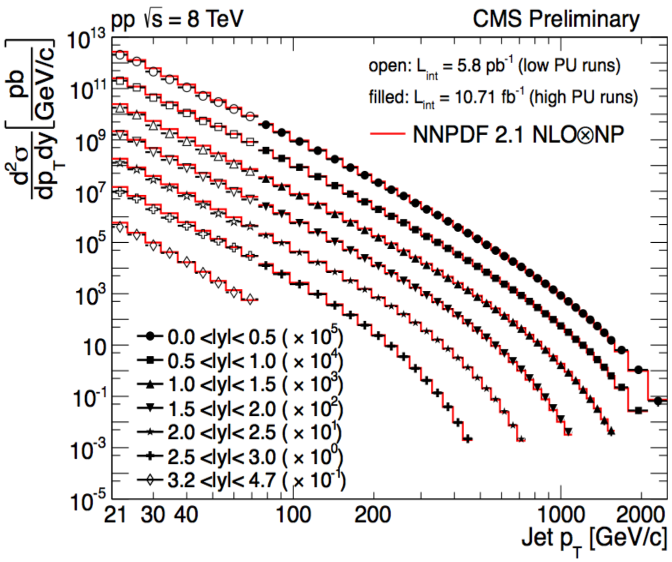
– General relation for hadron-hadron scattering for a hard, inclusive process with momentum transfer M to produce final state $F = J_1 + J_2 + \ldots + X$:

$$d\sigma_{\mathrm{H_1H_2}}(p_1, p_2, J_1, J_2 \ldots) \;=\; \sum_{a,b} \int_0^1 d\xi_a \, d\xi_b \; d\hat{\sigma}_{ab\to F+X}\,(\xi_a p_1, \xi_b p_2, J_1, J_2 \ldots, \mu) \times \phi_{a/H_1}(\xi_a, \mu)\, \phi_{b/H_2}(\xi_b, \mu),$$

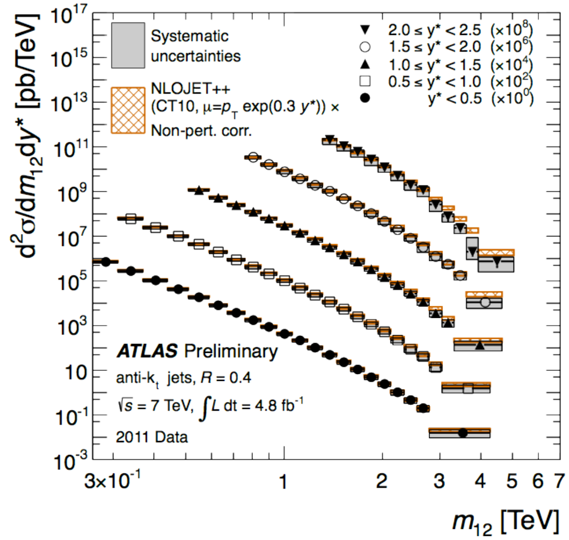
– The jets are calculated by clustering quarks and gluons according to the same algorithm used for hadrons in experiment.

– Parton distributions, short distance “coefficients” and functions of the jet momenta tell a story of autonomous correlated on-shell propagations punctuated by a single short-distance interaction.

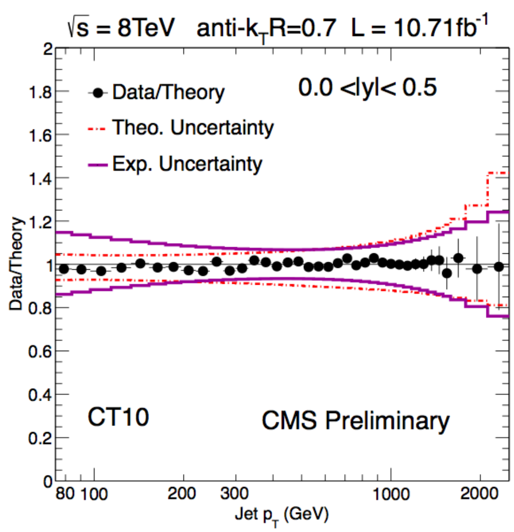
Correlated and "autonomous" dynamics. The data confront calculations . . . (note the "anti-k_T")

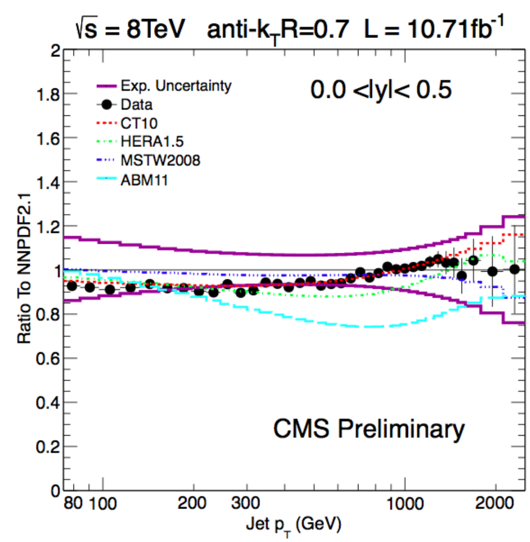


(CMS-PAS-SMP-12-012)



(ATLAS-CONF-2012-021)





Summary, Part D

- **The simplest example is the cone jet in e^+e^- annihilation. Predictions for hadrons from computations with partons.**

- **Thrust illustrates both jet finding and quantification by weight.**

- **Jet algortihms for hadronic collisions and N-jettiness can assemble and quantify hadronic jets.**

- **These methods are phenomenologically successful.**

Conclusion

- There is so much more.
- For more on jet substructure, see Appendices, but also . . .
- Jets in cold and hot nuclear matter.
- Radiation between jets.
- The entropy of jets and their entanglement.
- The LHC is providing unprecedented data on perturbative and nonperturbative dynamics in jets.
- Perhaps it will lead to a theory that ties these together.

Appendix: More on jet shapes, their resummations in and beyond perturbation theory

Summary

- **Jet shapes like angularities and energy correlations are generalizations of thrust that provide varied information on jets substructure.**
- **Modifying jet shapes through soft drop and related grooming procedures can shed light on jet evolution and aid in the identification of jet partonic origins: quark, gluon, standard model vector or Higgs . . .**
- **Thrust resummation illustrates the interplay of perturbative and nonperturbative dynamics.**

Varieties of jet shapes

- Jet shapes describe QCD dynamics and can reveal the origin of jets, individually or statistically: quarks vs. gluons, but also QCD vs. boosted heavy particles. very briefly.

- Angularities as a generalization of thrust. Starting with the thrust axis, define

$$
\begin{aligned}
\tau_a &= \frac{1}{E_J} \sum_{i \in J} k_{i,T} e^{-(1-a)|\eta|} \\
&\sim \frac{1}{E_J} \sum_{i \in J} E_i \, \theta_i^{2-a}
\end{aligned}
$$

Interpolates between the total cross section ($a = \infty$), the thrust ($a = 0$) and "jet broadening" ($a = 1$).

As a changes, we re-weight, to favor wide- or small-angle radiation, depending on a.

- A generalization: energy flow correlations

$$e_2^{(\alpha)} = \frac{1}{E_J^2} \sum_{i<j \in J} E_i E_j \theta_{ij}^{\alpha}$$

As above, but more insensitive to unobserved soft radiation at large a (favors hard, forward).

- These can be single particles, or calorimeter clusters.

- Can generalize further, to three- and more-point when we want to distinguish QCD radiation from boosted particle decays.

Every event provide τ_a for all a. That's a lot of information, depending differently on wide- and small-angle radiation – variables like these can play roles in "tagging" jets.

Soft drop: grooming

- **"Grooming"** is a general term for pre-processing a jet, before measuring its mass or other quantity.

- In looking for boosted heavy particles, we'd like to remove "incidental" QCD radiation from pile-up or the underlying event, resolving "subjets" that might represent things like $H \to b\bar{b}$ or $t \to Wb \to u\bar{d}b$. These should "stand out" from a flatter distribution of soft particles. If we want to measure the "true" mass we are better off neglecting these soft particles. To do so, we recluster with a smaller R.

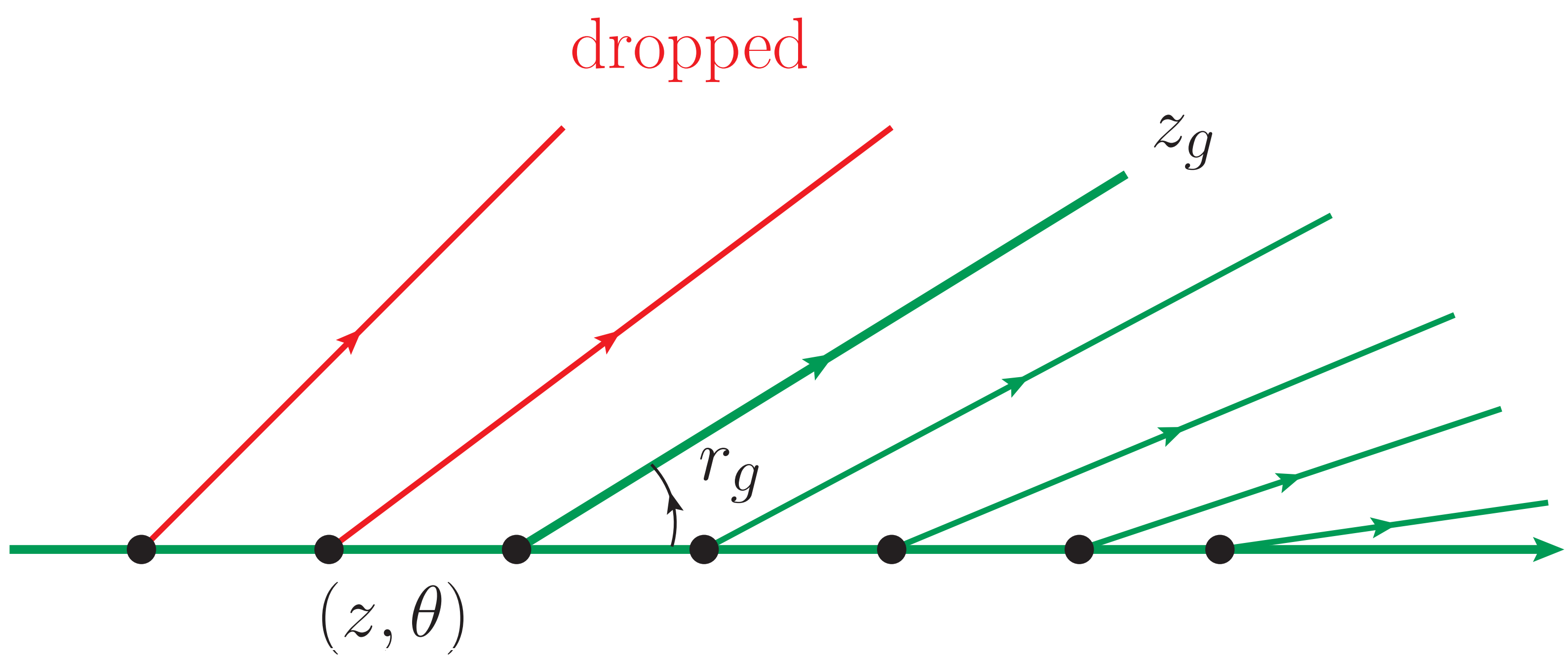
  Examples used by ATLAS and CMS are:

- **Trimming:** drop subjets with less than 5% (or so) of the total jet p_T.

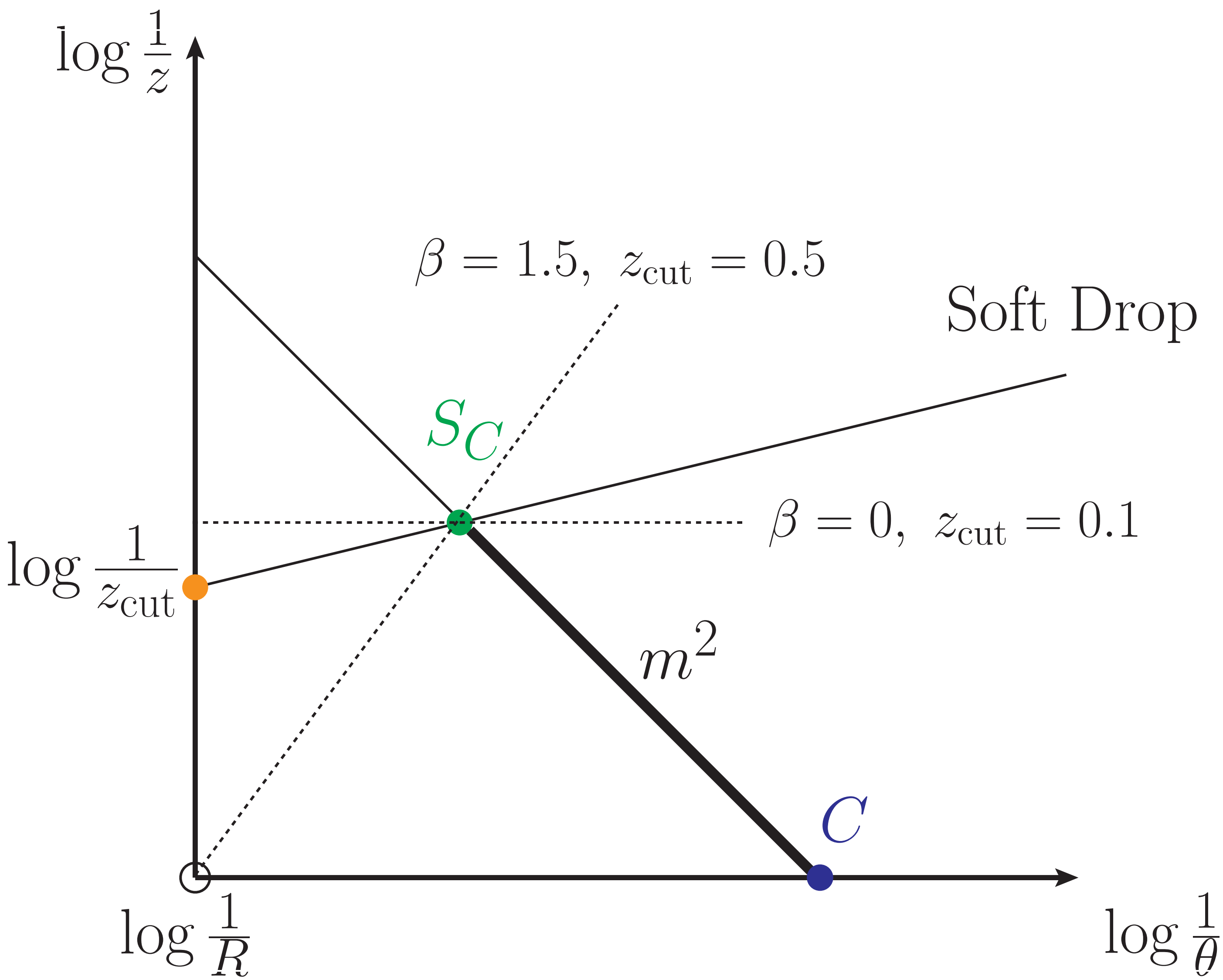
- **Soft drop:** classify the subjets according to their likely ordered branching (roughly formation time). Now they are sequential, and drop those that don't obey

$$\frac{\min\left[p_{T,i}\ ,\ p_{T_j}\right]}{p_{T,i} + p_{T,j}} > z_{\text{cut}} \left(\frac{R_{ij}}{R}\right)^{\beta}$$

- **Soft drop illustrated by ordered branches . . . and on a Lund plane (where β gives the slope separating cut from uncut):**
  (Y.-T. Chien, 2019 SCET)



- **and on** **a** **Lund plane (where β gives the slope separating cut from uncut):**
  (Y.-T. Chien, 2019 SCET)



- **Jet autonomy and resummations: the ubiquity of Sudakov logarithms and the concept of "Sudakov safety".**

Factorization structure in the limit of narrow jets:

$$\frac{d\sigma(Q, a+b \to N_{\text{jets}})}{dQ} = H_{IJ} \otimes \prod_{c=a,b} \mathcal{P}_{c'/c} \times S_{JI} \times \prod_{i} J_i$$

- **A story with only these pieces:**

- **Evolved incoming partons $\mathcal{P}_{a'/a}$, $\mathcal{P}_{b'/b}$ collide at H_{IJ}, I, J label color exchange in M and M^*;**

- **Outgoing jets J_i and coherent soft emission S_{JI}.**

- **Holds to any fixed α_s^n, all $\ln^a \mu/Q$ up to $\sim E_{\text{soft}}/E_{\text{jet}}$ corrections.**

- For Angularities: The Cross section is a convolution in contributions of each jet and a soft radiation function

$$\sigma\,(\tau_a, Q, a) \;=\; H_{IJ} \int dt_s \prod_{\text{jets}\ i} \int dt_i\, S_{JI}(t_s) \;\prod_i J_i(t_i, p_{Ji}) \\ \times\, \delta(\sum_i t_i + t_s - \tau_a)$$

- It's convenient to use a Laplace transform

$$\sigma\,(\tau_a, Q, a) \;=\; \int_C d\nu\, \mathrm{e}^{\nu\,\tau_a}\, H_{IJ}\, S_{JI}(\nu) \;\prod_i J_i(\nu, p_{Ji})$$

where we define $f(\nu) = \int_0^\infty dt\, e^{-\nu t}\, f(t)\,.$

- Logs of ν space exponentialte, just like IR divergences in QED!

$$\sigma(\nu, Q, a) \sim e^{E(\nu,a)}$$

where E has double-log (Sudakov) integrals over the running coupling

$$E(\nu, Q, a) \;=\; 2\int_0^1 \frac{du}{u}\Big[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right)\left(\mathrm{e}^{-u^{1-a}\nu(p_T/Q)^a} - 1\right) \\ + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right)\left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}} - 1\right)\Big]$$

Through the coupling nonperturbative scales enter perturbation theory

- **Perturbation theory isn't self-consistent – but it shouldn't be! But its failure may tell us about the form of nonperturbative contributions. Presents the form that its "infrared completion" should take**

- **For $p_T > \kappa$, perturbation theory is fine., $p_T < \kappa$, expand exponentials**

- **for low p_T, replace perturbation theory by $f_{\rm NP}$ "shape function"**

$$
\begin{aligned}
E(\nu, Q, a) \; &= \; E_{\rm PT}(\nu, Q, \kappa, a) \\
&\quad + \frac{2}{1-a} \sum_{n=1}^{\infty} \frac{1}{n\, n!} \left(-\frac{\nu}{Q} \right)^n \int_0^{\kappa^2} \frac{dp_T^2}{p_T^2} \, p_T^n \, A\left(\alpha_s(p_T)\right) + \ldots \\
&\equiv \; E_{\rm PT}(\nu, Q, \kappa, a) + \ln \tilde{f}_{a,{\rm NP}} \left(\frac{\nu}{Q}, \kappa \right)
\end{aligned}
$$

- Shape function factorizes in moments → convolution

$$\sigma(\tau_a, Q) = \int d\xi f_{a,\mathrm{NP}}(\xi)\ \sigma_{\mathrm{PT}}(\tau_a - \xi, Q)$$

- $\mathrm{e}^+\mathrm{e}^-$: fit at $Q = M_{\mathrm{Z}}$ ⇒predictions for all Q, any (quark) jet.

- Portable to jets in hadronic collisions.

- And will be sensitive to gluon/quark origin of the jet

- Scaling property for τ_a event shapes
(C.F. Berger & GS (2003) Berger and Magnea (2004))

- Test of rapidity-independence of NP dynamics (C. Lee, GS (2006); SCET)

$$\ln \tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q}, \kappa\right) = \frac{1}{1-a} \sum_{n=1}^{\infty} \lambda_n(\kappa) \left(-\frac{\nu}{Q}\right)^n$$

$$\tilde{f}_a\left(\frac{\nu}{Q}, \kappa\right) = \left[\ \tilde{f}_0\left(\frac{\nu}{Q}, \kappa\right)\right]^{\frac{1}{1-a}}$$

• **Recent comparison with data seems to work pretty well.** (G. Bell *et al* 1808.07867.)

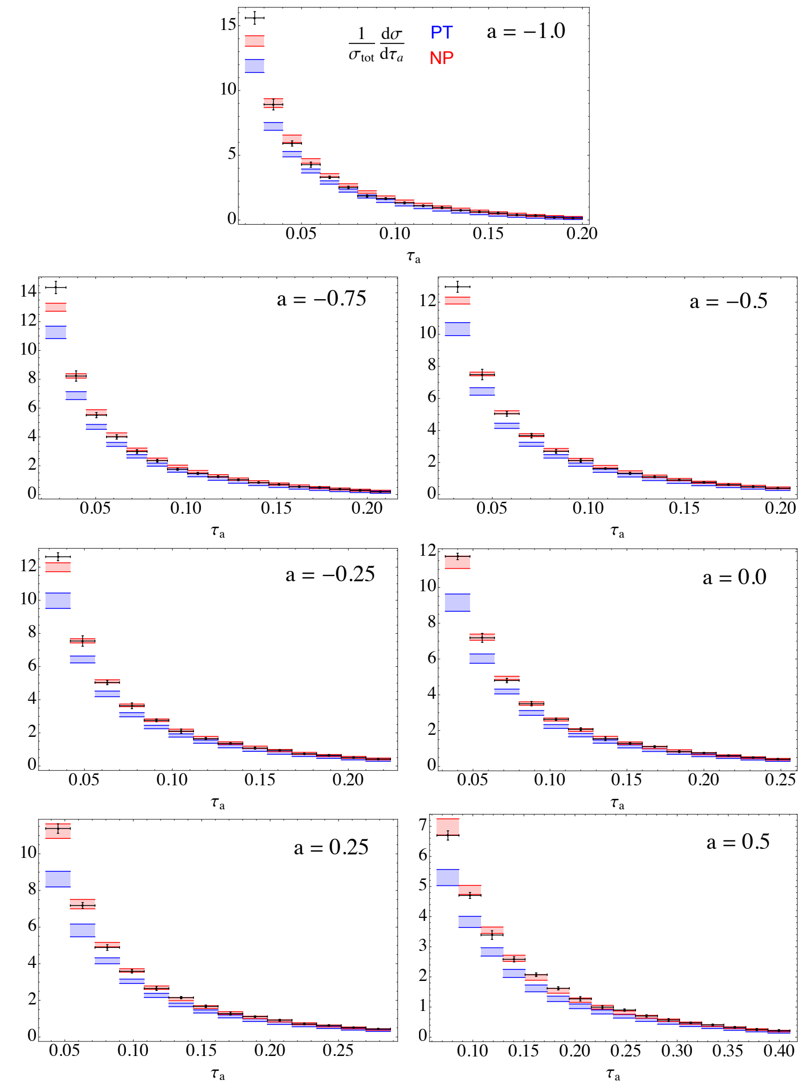


Figure 15. NNLL′ resummed and $\mathcal{O}(\alpha_s^2)$ matched angularity distributions for all values of a considered in this study, $a \in \{-1.0, -0.75, -0.5, -0.25, 0.0, 0.25, 0.5\}$, at $Q = m_Z$, with $\alpha_s(m_Z) = 0.11$. The blue bins represent the purely perturbative prediction and the red bins include a convolution with a gapped and renormalon-subtracted shape function, with a first moment set to $\Omega_1(R_\Delta, R_\Delta) = 0.4$ GeV. Overlaid is the experimental data from [48].

Summary, Appendix

- **Jet shapes like angularities and energy correlations are generalizations of thrust that provide varied information on jets substructure.**

- **Pre-processing jet shapes through soft drop and related grooming procedures can shed light on jet evolution and aid in the identification of jet partonic origins: quark, gluon, standard model vector or Higgs . . .**

- **Thrust resummation illustrates the interplay of perturbative and nonperturbative dynamics.**