α_s from $e^+e^- \to$ jets

Gábor Somogyi

Wigner Research Centre for Physics

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Part I: α_s from jet rates in e^+e^-

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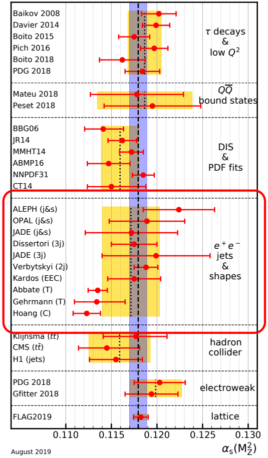


[P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update]

α_s from e^+e^-

- $\alpha_s(M_Z)$ is known with $\sim 0.8\%$ precision (lattice)
- the e^+e^- jets & shapes sub-field alone gives $\sim 2.6\%$ uncertainty: large spread between measurements
- **Can $< 1\%$ precision be achieved?**

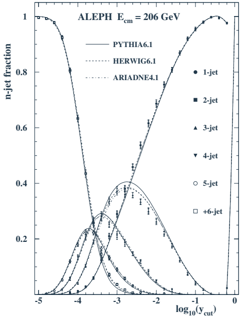
Why jet rates?

- high perturbative accuracy, especially for the two-jet rate R_2
- jet rates are known to be less sensitive to hadronization corrections than event shapes

How best to improve?

Jet rates

Jet rates: R_n is the fraction of n-jet events for given y: $R_n(y) = \frac{\sigma_{n-\mathrm{jet}}(y)}{\sigma_{\mathrm{tot}}}$



[ALEPH Coll., Eur. Phys. J. **C35**, 457 (2004)]

- R_3 was used multiple times in the past to extract $\alpha_s(M_Z)$
- fixed-order perturbative predictions for R_3/R_2 at NNLO/N^{3}LO
  [Gehrmann-De Ridder et al., Phys. Rev. Lett. **100** (2008) 172001, Weinzierl, Phys. Rev. Lett. **101** (2008) 162001]
- resummed predictions for R_2 at NNLL accuracy
  [Banfi et al., Phys. Rev. Lett. **117** (2016) 172001]
- combining R_2 and R_3 in one analysis is possible

Durham jet algorithm: sequential recombination algorithm with distance measure $y_{ij} = 2\frac{\min(E_i^2, E_j^2)}{E_{\mathrm{vis}}^2}(1 - \cos\theta_{ij})$ where E_i is particle energy and θ_{ij} is the angle between tree-momenta of particles i and j; momenta recombined using the E-scheme

Analysis components

Measurement of $\alpha_s(M_Z)$ from the fit of the Durham two-jet rate R2 in e^+e^- annihilation to N^{3}LO+NNLL predictions + hadronization corrections extracted from state-of-the-art MC event generators

[Verbytskyi, Banfi, Kardos, Monni, Kluth, GS, Szőr, Trócsányi, Tulipánt, Zanderighi, JHEP **1908** (2019) 129]

- data from LEP and PETRA + new OPAL measurements used to build correlation model for older measurements
- fixed-order perturbative predictions + some b-mass corrections
- resummation + matching
- non-perturbative corrections from state-of-the-art MC event generators + Lund and cluster hadronization models

Data

Combined analysis using 20+ datasets from 4 collaborations

The data covers a **wide range of cms energies**: $\sqrt{s} = 35 - 207\,\text{GeV}$

Experiment	Data $\sqrt{s}$, (average), GeV	MC $\sqrt{s}$, GeV	Events
OPAL	91.2(91.2)	91.2	1508031
OPAL	189.0(189.0)	189	3300
OPAL	183.0(183.0)	183	1082
OPAL	172.0(172.0)	172	224
OPAL	161.0(161.0)	161	281
OPAL	130.0 — 136.0(133.0)	133	630
L3	201.5 — 209.1(206.2)	206	4146
L3	199.2 — 203.8(200.2)	200	2456
L3	191.4 — 196.0(194.4)	194	2403
L3	188.4 — 189.9(188.6)	189	4479
L3	180.8 — 184.2(182.8)	183	1500
L3	161.2 — 164.7(161.3)	161	424
L3	135.9 — 140.1(136.1)	136	414
L3	129.9 — 130.4(130.1)	130	556
JADE	43.4 — 44.3(43.7)	44	4110
JADE	34.5 — 35.5(34.9)	35	29514
ALEPH	91.2(91.2)	91.2	3600000
ALEPH	206.0(206.0)	206	3578
ALEPH	189.0(189.0)	189	3578
ALEPH	183.0(183.0)	183	1319
ALEPH	172.0(172.0)	172	257
ALEPH	161.0(161.0)	161	319
ALEPH	133.0(133.0)	133	806

Data selection:

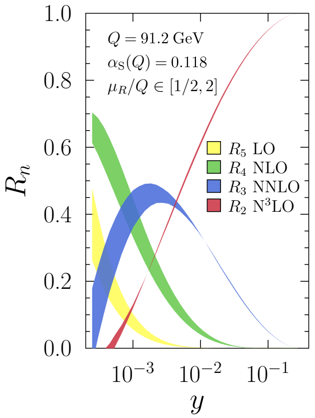
- measurements with both charged and neutral final state particles
- corrected for detector effects
- corrected for QED ISR
- no overlap with other samples
- sufficient precision
- sufficient information on dataset available

Fixed-order predictions

Fixed-order predictions up to and including α_s^3 corrections known for some time

[Gehrmann-De Ridder et al., Phys. Rev. Lett. **100** (2008) 172001, Weinzierl, Phys. Rev. Lett. **101** (2008) 162001]

$$R_n(y) = \delta_{2,n} + \frac{\alpha_s(Q)}{2\pi} A_n(y) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B_n(y) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 C_n(y) + \mathcal{O}(\alpha_s^4)$$



- R_3 computed at **NNLO** accuracy using CoLoRFulNNLO $\Rightarrow$ obtain R_2 at **N^{3}LO**

  [Del Duca et al., Phys. Rev. **D94** (2016) no.7, 074019]
- very good numerical precision and stability
- b-mass corrections from Zbb4: note only **NLO** for R_3 $\Rightarrow$ **NNLO** for R_2

  [Nason, Oleari, Phys. Lett. **B407**, 57 (1997)]
- mass effects included at distribution level, e.g.

$$R_2(y) = (1 - r_b) R_2^{\mathrm{N^3LO}}(y)_{m_b=0} + r_b R_2^{\mathrm{NNLO}}(y)_{m_b\neq 0}$$

where r_b is the fraction of b-quark events

$$r_b = \frac{\sigma_{m_b\neq 0}(e^+e^- \to b\bar{b})}{\sigma_{m_b\neq 0}(e^+e^- \to \text{hardons})}$$

Resummation

Resummed predictions for R_2 at **NNLL** accuracy have been computed more recently

[Banfi et al., Phys. Rev. Lett. **117** (2016) 172001]

$$R_2(y) = e^{-R_{\mathrm{NNLL}}(y)} \left[\left(1 + \frac{\alpha_s(Q)}{2\pi} H^{(1)} + \frac{\alpha_s(Q\sqrt{y})}{2\pi} C_{\mathrm{hc}}^{(1)} \right) \mathcal{F}_{\mathrm{NLL}}(y) + \frac{\alpha_s(Q)}{2\pi} \delta\mathcal{F}_{\mathrm{NNLL}}(y) \right]$$

- resummation performed with the `ARES` program
- matching to fixed-order: $\log R$ scheme
- counting of logs (NNLL) here refers to logs in $\ln R_2$

In contrast, resummed predictions for R_3 have a much lower logarithmic accuracy

- more colored emitters
- state-of-the-art resummation includes only $\mathcal{O}(\alpha_s^n L^{2n})$ and $\mathcal{O}(\alpha_s^n L^{2n-1})$ terms in R_3 (note different logarithmic counting)
- in this analysis, no resummation for R_3 is performed

$$\Downarrow$$

Main focus on N^{3}LO+NNLL for R_2, but also simultaneous analysis with NNLO for R_3

Hadronization corrections

Effects associated with the parton-to-hadron transition cannot be computed in perturbation theory and must be estimated by other means.

Obtained using state-of-the-art MC event generators: $e^+e^- \to jjjjj$ merged samples with massive b-quarks

- **default setup "H^L": `Herwig7.1.4` for $e^+e^- \to 2,3,4,5$ jets, 2 and 3 jets at NLO using `MadGraph5` and `OpenLoops` + Lund fragmentation model**
- setup for hadronization systematics "H^C": `Herwig7.1.4` for $e^+e^- \to 2,3,4,5$ jets, 2 and 3 jets at NLO using `MadGraph5` and `OpenLoops` + cluster fragmentation model
- setup for cross-checks "S^C": `Sherpa2.2.6` for $e^+e^- \to 2,3,4,5$ jets, 2 jets at NLO using `AMEGIC`, `COMIX` and `OpenLoops` + cluster fragmentation model

Issues

- the parton level of an MC simulation is not equivalent to a fixed-order calculation
- current hadronization models tuned using MC's with lower accuracy

Fit procedure

To extract the value of α_s, `MINUIT2` is used to minimize

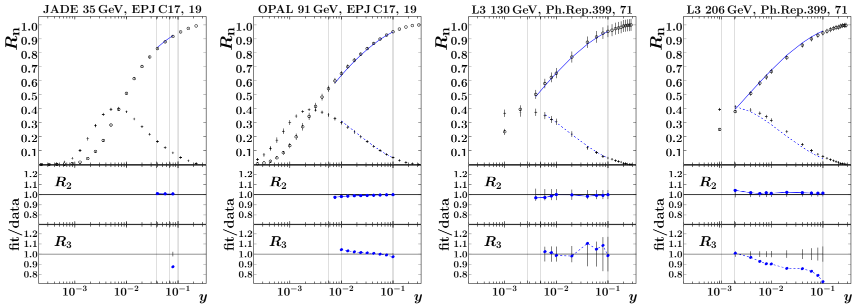
$$\chi^2(\alpha_s) = \sum_{\text{data set}} \chi^2(\alpha_s)_{\text{data set}}$$

where $\chi^2(\alpha_s)$ are computed separately for each data set

$$\chi^2(\alpha_s) = \vec{r}\, V^{-1} \vec{r}^T \,, \qquad \vec{r} = (\vec{D} - \vec{P}(\alpha_s))$$

- $\vec{D}$: vector of data points
- $\vec{P}(\alpha_s)$: vector of theoretical predictions
- V: covariance matrix for $\vec{D}$ (statistical correlations estimated from MC generated samples, systematic correlations modeled to mimic patters observed in OPAL data)

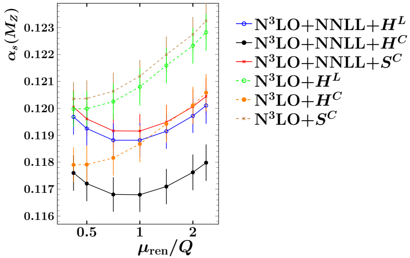
Fit results

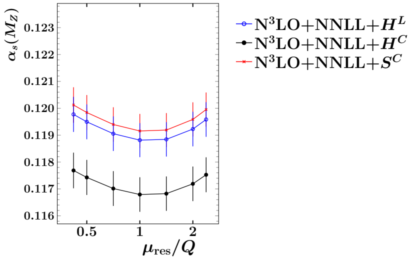


Central result and fit range selection

- avoid regions where theoretical predictions or hadronization model are unreliable
- Q^2-dependent fit range: $[-2.25 + \mathcal{L}, -1]$ for R_2 and $[-2 + \mathcal{L}, -1]$ for R_3 (if used), where $\mathcal{L} = \ln \frac{M_Z^2}{Q^2}$
- note separate fit ranges for R_2 and R_3 (if used)
- smallest $\chi^2/ndof$, low sensitivity to fit range

Uncertainties





Uncertainty is assessed by

- varying the renormalization scale $\mu_{\rm ren} \in [Q/2, 2Q]$: (*ren.*)
- varying the resummation scale $\mu_{\rm res} \in [Q/2, 2Q]$: (*res.*)
- varying the hadronization model H^L vs. H^C: (*hadr.*)
- fit uncertainty is obtained from the $\chi^2 + 1$ criterion as implemented in `MINUIT2`: (*exp.*)

Notice much reduced renormalization scale uncertainty when NNLL resummation for R_2 is included

Results: R_2

Extraction of $\alpha_s(M_Z)$ from the two-jet rate R_2 measured over a wide range of cms energies in e^+e^- collisions has been performed at N^{3}LO+NNLL accuracy yields:

$$\alpha_s(M_Z) = 0.11881 \pm 0.00063(exp.) \pm 0.00101(hadr.) \pm 0.00045(ren.) \pm 0.00034(res.)$$
$$\alpha_s(M_Z) = 0.11881 \pm 0.00131(comb.)$$

- main source of uncertainty: hadronization modeling
- uncertainty from scale variation is considerably smaller than from hadronization
- experimental uncertainty comparable to perturbative one

Inclusion of **NNLL resummation crucial** for reducing perturbative uncertainty

Results: $R_2 + R_3$

Combined fit of R_2 at N^{3}LO+NNLL and R_3 at NNLO, taking into account the correlation between the observables gives:

$$\alpha_s(M_Z) = 0.11989 \pm 0.00045(exp.) \pm 0.00098(hadr.) \pm 0.00046(ren.) \pm 0.00017(res.)$$
$$\alpha_s(M_Z) = 0.11989 \pm 0.00118(comb.)$$

- result is fully compatible with R_2-only fit
- formally more precise than a fit based on R_2 alone,
- but much more sensitive to fit range selection

An accurate resummation of R_3 could potentially reduce the sensitivity to fit range selection and lead to an even more precise determination of $\alpha_s(M_Z)$

Final result

The following value of $\alpha_s(M_Z)$ was obtained

$$\boldsymbol{\alpha_s(M_Z) = 0.11881 \pm 0.00063\ (exp.) \pm 0.00101\ (hadr.) \pm 0.00045\ (ren.) \pm 0.00034\ (res.)}$$
$$\boldsymbol{\alpha_s(M_Z) = 0.11881 \pm 0.00131\ (comb.)}$$

- The result agrees with the world average $\alpha_s(M_Z)_{\mathrm{PDG2020}} = 0.1179 \pm 0.0010$ and has an uncertainty that is of the same size
- The presented result is the most precise in its subclass [Salam, arXiv:1712.05165v2]

Determination	Data and procedure
0.1175 ± 0.0025	ALEPH 3-jet rate (NNLO+MChad)
0.1199 ± 0.0059	JADE 3-jet rate (NNLO+NLL+MChad)
0.1224 ± 0.0039	ALEPH event shapes (NNLO+NLL+MChad)
0.1172 ± 0.0051	JADE event shapes (NNLO+NLL+MChad)
0.1189 ± 0.0041	OPAL event shapes (NNLO+NLL+MChad)
$0.1164^{+0.0028}_{-0.0026}$	Thrust (NNLO+NLL+anlhad)
$0.1134^{+0.0031}_{-0.0025}$	Thrust (NNLO+NNLL+anlhad)
0.1135 ± 0.0011	Thrust (SCET NNLO+N^{3}LL+anlhad)
0.1123 ± 0.0015	C-parameter (SCET NNLO+N^{3}LL+anlhad)

Part II: lessons for the future

Improving perturbative predictions I

More legs, more N's

- beyond NNLO for 3-jet event shapes/rate?
- beyond 3-jet rate/event shapes at NNLO?
- improved logarithmic accuracy for R_2, R_3?

Mass effects, mixed EW$\times$QCD corrections

- R_3 at NNLO with massive b-quarks?
- mixed EW$\times$QCD corrections for R_2, R_3?

Two issues

- full 2- and 3-loop matrix elements that would be needed are presently **not known**, however **great progress**, so expect new results
- computing **physical observables** using those matrix elements is a **separate issue** (definitely beyond NNLO), new ideas may be needed

Improving perturbative predictions I

More legs, more N's

- beyond NNLO for 3-jet event shapes/rate? $\Rightarrow$ not top priority for this fit
- beyond 3-jet rate/event shapes at NNLO? $\Rightarrow$ not top priority for this fit
- **improved logarithmic accuracy for R_2, R_3?** $\Rightarrow$ **already within reach**

Mass effects, mixed EW$\times$QCD corrections

- R_3 at NNLO with massive b-quarks? $\Rightarrow$ **more relevant for this fit**
- mixed EW$\times$QCD corrections for R_2, R_3? $\Rightarrow$ **more relevant for this fit**

Two issues

- full 2- and 3-loop matrix elements that would be needed are presently **not known**, however **great progress**, so expect new results
- computing **physical observables** using those matrix elements is a **separate issue** (definitely beyond NNLO), new ideas may be needed

Aside: role of fixed-order corrections beyond NNLO

Would including **more perturbative orders alone** improve precision?

To address these issue, an **analysis of event shape averages** where **unknown perturbative corrections beyond NNLO are estimated from data**. Hadronization corrections are obtained using both Monte Carlo tools as well as analytic models,

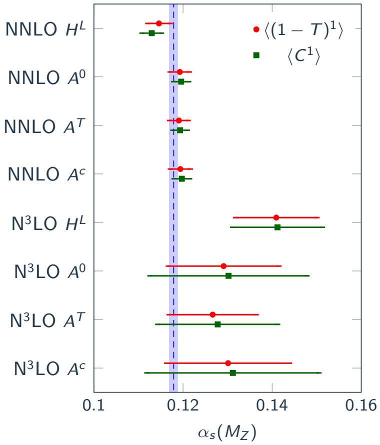
- state-of-the-art MC event generators: $e^+e^- \to 2, 3, 4, 5$ parton processes, 2-parton final state at NLO
- dispersive model of analytic hadronization corrections for event shapes, extended to α_s^4 accuracy

[Kardos, GS, Verbytskyi, Eur. Phys. J. C **81** (2021) 4, 292]

Importantly, the main point of extracting the N^{3}LO coefficients from data is **not** to get an accurate determination of these quantities. **Rather**, it is to model them as best as possible in order to be able to **assess the impact** of including terms beyond NNLO in the extraction of the strong coupling in the absence of an actual calculation of those terms.

Aside: role of fixed-order corrections beyond NNLO

The extractions of $\alpha_s(M_Z)$ from $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data

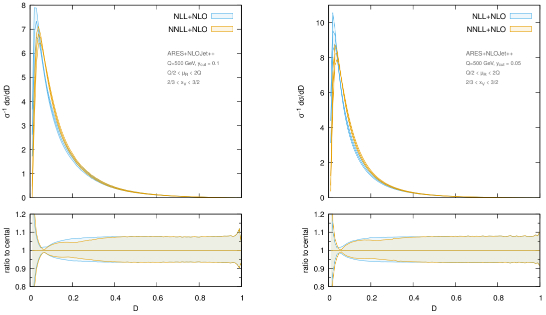


- H^L: **MC hadr.**, `Herwig7.2.0` with Lund fragmentation model
- $A^{0,T,c}$: **analytic hadr.** (dispersive model in various schemes)
- Good agreement between fits to $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data both at NNLO and N^{3}LO $\Rightarrow$ internal consistency of extraction procedure
- Analytic hadr. scheme-dependence is mild.
- **Large discrepancy between results obtained with MC and analytic hadronization models** both at NNLO and N^{3}LO $\Rightarrow$ suggests that the discrepancy has a fundamental origin and would hold even with exact N^{3}LO predictions.
- **Better understanding of hadronization is key.**

Improving perturbative predictions II

Improved logarithmic accuracy for R_2, R_3

- recently the NNLL radiator for three hard emitters has been defined
- allows for NNLL resummation of event shapes in the near-to-planar limit, e.g. D-parameter at NNLL+NLO



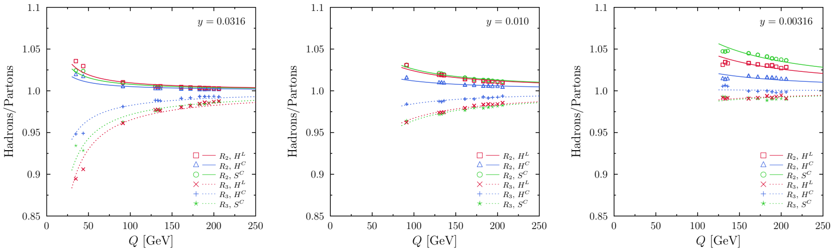
[Arpino et al. arXiv:1912.09341]

Analytic pieces in place for N^{3}LL and NNLL resummation for R_2 and R_3

The role of hadronization corrections

The elephant in the room: the **main source of uncertainty is due to hadronization modeling**

- naively going to higher energies helps: hard. corr. $\sim 1/Q$, however...
- energy is not orders of magnitude larger than LEP
- there is an interplay between smaller hadronization corrections but larger background and much smaller luminosity as we increase energy



The role of hadronization corrections

Bottom line: need better MC's + hadronization models/calibration in e^+e^-

In a perfect world

- parton showers with NNLL logarithmic accuracy matched to NNLO
- hadronization models calibrated from scratch with many different observables, since current models were tuned using MC's with lower accuracy

Alternatively

- need a (much) more refined analytical understanding of non-perturbative corrections, for recent advances see e.g.,

[Luisoni, Monni, Salam, Eur. Phys. J. C **81** (2021) 2, 158,
Caola, Ravasio, Limatola, Melnikov, Nason, JHEP 01 (2022) 093]

- look for better observables, with smaller hadronization corrections e.g., groomed event shapes

[Baron, Marzani, Theeuwes, JHEP **08** (2018) 105,
Kardos, Larkoski, Trócsányi, Phys. Lett. B **809** (2020) 135704]

Role of background

At LEP (and before) the signal process was $e^+e^- \to Z/\gamma \to$ hadrons, while $e^+e^- \to VV/ZH \to$ hadrons ($+$ leptons) was background to be subtracted

- introduces a lot of systematic uncertainties
- but this is what could be compared to precisely computed predictions

One way to deal with increased background in the future could be to **redefine the signal process** as $e^+e^- \to$ hadrons

- only background to this is from $e^+e^- \to VV/ZH \to$ hadrons $+$ leptons, e.g. $e^+e^- \to W^+W^- \to q\,\bar{q}\,l\,\bar{\nu}_l$, which can be suppressed almost completely
- however with this redefinition already the Born processes $e^+e^- \to VV/ZH \to q\bar{q}q\bar{q}$ involve four colored particles $\Rightarrow$ the precise theoretical description of all channels is a major challenge
- EW corrections to $e^+e^- \to Z/\gamma \to$ hadrons must also be addressed

Conclusions

State-of-the-art extractions of α_s from $e^+e^- \to$ jets deliver measurements with precision just above 1%.

E.g., the measurement of $\alpha_s(M_Z)$ from the fit of the Durham two-jet rate R_2 in e^+e^- annihilation to N^{3}LO+NNLL predictions + hadronization corrections extracted from state-of-the-art MC event generators:

$$\alpha_s(M_Z) = 0.11881 \pm 0.00063(exp.) \pm 0.00101(hadr.) \pm 0.00045(ren.) \pm 0.00034(res.)$$

- the result is consistent with the world average and the most precise in its subclass
- main source of uncertainty from modeling of hadronization corrections

Conclusions

More perturbative orders alone are not likely to dramatically improve the precision of strong coupling extractions: perturbative uncertainty under control, but improvements possible

- N^4LO/N^3LO for R_2/R_3: not the priority from the point of view of this measurement
- b-quark mass corrections and EW$\times$QCD corrections seem **more relevant**
- N^3LL/NNLL resummation for R_2/R_3 are already **within reach**

Main limiting factor in future studies is likely to be the systematics related to the estimation of **hadronization corrections**

- better understanding of hadronization corrections crucial for improvement: **we must seriously refine our understanding/modeling of non-perturbative effects**
- this would be aided greatly by dedicated low-energy (below the Z-peak) measurements at future e^+e^- facilities
- could consider a redefinition of the signal: $e^+e^- \to$ hadrons

An extraction of α_s from $e^+e^- \to$ jets with sub-percent accuracy will be feasible, given foreseeable theoretical and modeling advances and new data.

Thank you for your attention!

Backup

Hadronization corrections: simultaneous corrections for R_2 and R_3

Challenge: simultaneous corrections for R_2 and R_3

- hadronization corrections derived on a bin-by-bin basis, $R_{n,\mathrm{hadron}} = R_{n,\mathrm{parton}} f_n(y)$, $n = 2, 3, 4, \ldots$ can violate physical constraints: $0 \leq R_n \leq 1$ and $\sum_n R_n = 1$

Solution:

- introduce ξ_1 and ξ_2 such that at parton level $R_{2,\mathrm{parton}} + R_{3,\mathrm{parton}} + R_{\geq 4,\mathrm{parton}} = 1$

$$R_{2,\mathrm{parton}} = \cos^2\xi_1\,, \qquad R_{3,\mathrm{parton}} = \sin^2\xi_1 \cos^2\xi_2\,, \qquad R_{\geq 4,\mathrm{parton}} = \sin^2\xi_1 \sin^2\xi_2\,,$$

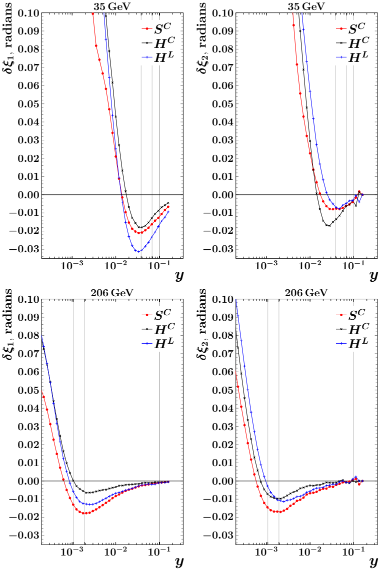
- similarly at hadron level, set

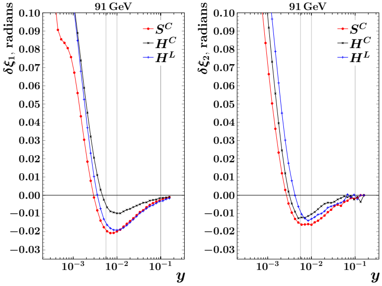
$$R_{2,\mathrm{hadron}} = \cos^2(\xi_1 + \delta\xi_1)\,, \qquad R_{3,\mathrm{hadron}} = \sin^2(\xi_1 + \delta\xi_1)\cos^2(\xi_2 + \delta\xi_2)\,,$$
$$R_{\geq 4,\mathrm{hadron}} = \sin^2(\xi_1 + \delta\xi_1)\sin^2(\xi_2 + \delta\xi_2)$$

- the functions $\delta\xi_1(y)$ and $\delta\xi_2(y)$ account for hadronization corrections and are extracted from the MC samples

This approach clearly preserves physical constraints

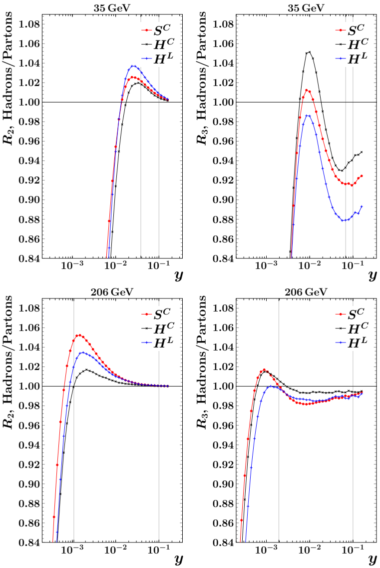
Hadronization corrections: $\delta\xi_1(y)$ and $\delta\xi_2(y)$

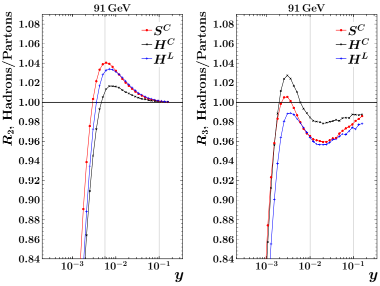




- to avoid binning effects, the hadronization corrections are parametrized with smooth functions
- vertical lines show the fit ranges for the reference fits of R_2 and R_3

Hadronization corrections: hadron to parton ratios





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R_2 fits

Fit of $\alpha_s(M_Z)$ from experimental data for R_2 obtained using N^3LO and $N^3LO+NNLL$ predictions for R_2. The reported uncertainty comes from `MINUIT2`

Fit ranges, $\log y$ Hadronization	N^3LO $\chi^2/ndof$	$N^3LO+NNLL$ $\chi^2/ndof$
$[-1.75+\mathcal{L},-1]$ S^C	0.12121 ± 0.00095 $20/86 = 0.24$	0.11849 ± 0.00092 $20/86 = 0.24$
$[-2+\mathcal{L},-1]$ S^C	0.12114 ± 0.00081 $26/100 = 0.26$	0.11864 ± 0.00075 $26/100 = 0.26$
$[-2.25+\mathcal{L},-1]$ S^C	0.12119 ± 0.00060 $44/150 = 0.29$	0.11916 ± 0.00063 $44/150 = 0.29$
$[-2.5+\mathcal{L},-1]$ S^C	0.12217 ± 0.00052 $89/180 = 0.50$	0.12075 ± 0.00055 $107/180 = 0.59$
$[-1.75+\mathcal{L},-1]$ H^C	0.11957 ± 0.00098 $22/86 = 0.26$	0.11698 ± 0.00093 $22/86 = 0.25$
$[-2+\mathcal{L},-1]$ H^C	0.11923 ± 0.00079 $29/100 = 0.29$	0.11687 ± 0.00076 $28/100 = 0.28$
$[-2.25+\mathcal{L},-1]$ H^C	0.11868 ± 0.00068 $43/150 = 0.28$	0.11679 ± 0.00064 $40/150 = 0.27$
$[-2.5+\mathcal{L},-1]$ H^C	0.11849 ± 0.00050 $58/180 = 0.32$	0.11723 ± 0.00053 $58/180 = 0.32$
$[-1.75+\mathcal{L},-1]$ H^L	0.12171 ± 0.00109 $21/86 = 0.25$	0.11897 ± 0.00092 $21/86 = 0.24$
$[-2+\mathcal{L},-1]$ H^L	0.12144 ± 0.00078 $28/100 = 0.28$	0.11893 ± 0.00075 $26/100 = 0.26$
$[-2.25+\mathcal{L},-1]$ H^L	0.12080 ± 0.00069 $43/150 = 0.28$	0.11881 ± 0.00063 $39/150 = 0.26$
$[-2.5+\mathcal{L},-1]$ H^L	0.12024 ± 0.00051 $57/180 = 0.32$	0.11897 ± 0.00053 $52/180 = 0.29$

$R_2 + R_3$ fits

Simultaneous fit of $\alpha_s(M_Z)$ from experimental data for R_2 and R_3 obtained using N^{3}LO and N^{3}LO+NNLL predictions for R_2 and NNLO predictions for R_3. The reported uncertainty comes from `MINUIT2`

Fit ranges, log y Hadronization	N^{3}LO $\chi^2/ndof$	N^{3}LO+NNLL $\chi^2/ndof$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ S^C	0.12195 ± 0.00072 $120/143 = 0.84$	0.12078 ± 0.00066 $140/143 = 0.98$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ S^C	0.12163 ± 0.00061 $153/187 = 0.82$	0.12065 ± 0.00056 $176/187 = 0.94$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ S^C	0.12075 ± 0.00044 $208/251 = 0.83$	0.11994 ± 0.00041 $222/251 = 0.88$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ S^C	0.12143 ± 0.00043 $321/331 = 0.97$	0.12089 ± 0.00044 $336/331 = 1.01$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ H^C	0.12068 ± 0.00073 $126/143 = 0.88$	0.11956 ± 0.00066 $147/143 = 1.03$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ H^C	0.12006 ± 0.00061 $163/187 = 0.87$	0.11913 ± 0.00054 $188/187 = 1.01$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ H^C	0.11869 ± 0.00043 $221/251 = 0.88$	0.11793 ± 0.00043 $238/251 = 0.95$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ H^C	0.11845 ± 0.00045 $302/331 = 0.91$	0.11799 ± 0.00047 $310/331 = 0.94$
$[-1.75+\mathcal{L},-1][-1.5+\mathcal{L},-1]$ H^L	0.12248 ± 0.00068 $121/143 = 0.85$	0.12129 ± 0.00063 $141/143 = 0.99$
$[-2+\mathcal{L},-1][-1.75+\mathcal{L},-1]$ H^L	0.12211 ± 0.00057 $155/187 = 0.83$	0.12110 ± 0.00053 $180/187 = 0.96$
$[-2.25+\mathcal{L},-1][-2+\mathcal{L},-1]$ H^L	0.12071 ± 0.00044 $209/251 = 0.83$	0.11989 ± 0.00045 $227/251 = 0.90$
$[-2.5+\mathcal{L},-1][-2.25+\mathcal{L},-1]$ H^L	0.12041 ± 0.00044 $266/331 = 0.80$	0.11990 ± 0.00044 $278/331 = 0.84$

Consistency tests

Several consistency tests performed

- simultaneous fit of $R_2 + R_3$ (see above)
- separate R_3 fit
- variation of χ^2 definition
- change of fit ranges
- multiplicative hadronization corrections
- `Sherpa` MC hadronization S^C
- stability across $\sqrt{s}$ (see below)
- exclusion of data with $\sqrt{s} < M_Z$