

# $\alpha_s$ determinations overview

**ECFA WG1-PREC miniworkshop**  
**Virtual, 10<sup>th</sup> March 2022**

**David d'Enterria**  
**CERN**

The strong coupling constant: State of the art and the decade ahead

(Snowmass-2021 White Paper)

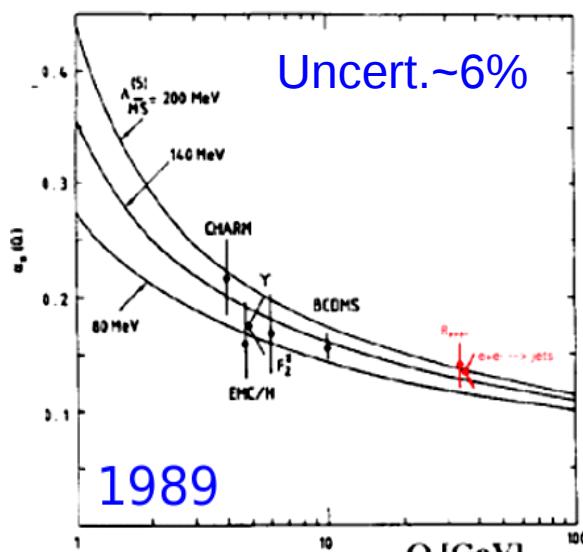
[expected in arXiv next Monday]

*Authors:* C. Ayala,<sup>3</sup> M.A. Benitez-Rathgeb,<sup>4</sup> J. Blümlein,<sup>5</sup> D. Boito,<sup>4,6</sup> N. Brambilla,<sup>7</sup> D. Britzger,<sup>2</sup> S. Camarda,<sup>1</sup> A. M. Cooper-Sarkar,<sup>8</sup> T. Cridge,<sup>9</sup> G. Cvetic,<sup>10</sup> D. d'Enterria,<sup>1</sup> M. Dalla Brida,<sup>11</sup> A. Deur,<sup>12</sup> F. Giuli,<sup>1</sup> M. Golterman,<sup>13,14</sup> A.H. Hoang,<sup>4,15</sup> J. Huston,<sup>16</sup> M. Jamin,<sup>4,17</sup> S. Kluth,<sup>2</sup> A. V. Kotikov,<sup>18</sup> V. G. Krivokhizhin,<sup>18</sup> A.S. Kronfeld,<sup>19</sup> V. Leino,<sup>7</sup> K. Lipka,<sup>20</sup> T. Mäkelä,<sup>20</sup> B. Malaescu,<sup>21</sup> K. Maltman,<sup>22,23</sup> S. Marzani,<sup>24</sup> V. Mateu,<sup>25,26</sup> S. Moch,<sup>27</sup> P. F. Monni,<sup>11</sup> P. Nadolsky,<sup>28</sup> P. Nason,<sup>2,29</sup> A.V. Nesterenko,<sup>18</sup> R. Pérez-Ramos,<sup>30,31</sup> S. Peris,<sup>14</sup> P. Petreczky,<sup>32</sup> A. Pich,<sup>33</sup> K. Rabbertz,<sup>34</sup> A. Ramos,<sup>33</sup> D. Reichelt,<sup>35</sup> A. Rodríguez-Sánchez,<sup>36</sup> J. Rojo,<sup>37,38</sup> M. Saragnese,<sup>5</sup> L. Sawyer,<sup>39</sup> M. Schott,<sup>40</sup> S. Schumann,<sup>41</sup> B. G. Shaikhatalenov,<sup>18</sup> S. Sint,<sup>42</sup> G. Soyez,<sup>43</sup> D. Teca,<sup>10</sup> A. Vairo,<sup>7</sup> M. Vos,<sup>33</sup> C. Waits,<sup>39</sup> J. H. Weber,<sup>44</sup> M. Wobisch,<sup>39</sup> K. Xie,<sup>45</sup> and G. Zanderighi<sup>2</sup>

<https://indico.cern.ch/e/alphas2022>

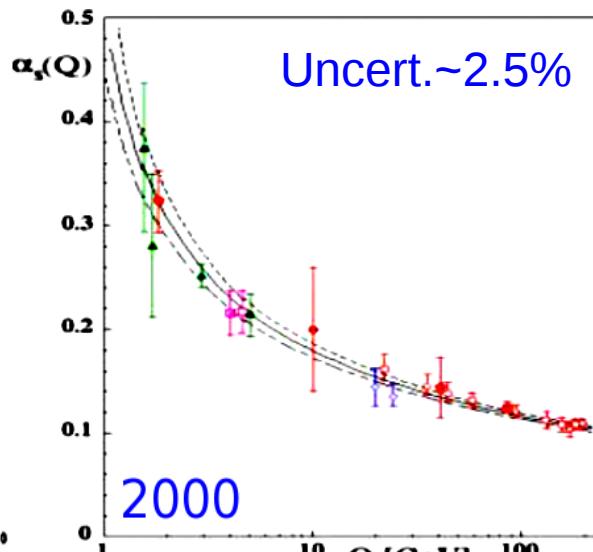
# QCD coupling $\alpha_s$

- Determines **strength of the strong interaction** between quarks & gluons.
- Single free parameter of QCD in the  $m_q \rightarrow 0$  limit.
- Determined at a ref. scale ( $Q = m_Z$ ), decreases as  $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$ ,  $\Lambda \sim 0.2$  GeV



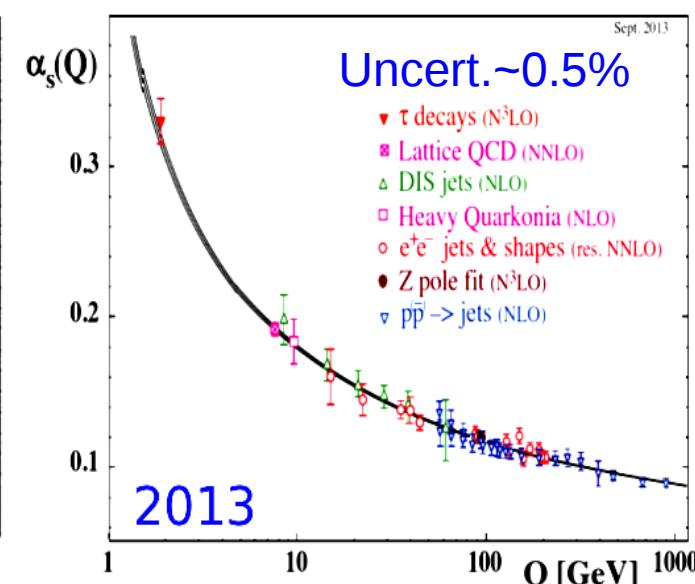
$$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008} \text{ (NLO)}$$

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$$\alpha_s(M_Z) = 0.1184 \pm 0.0031 \text{ (NNLO)}$$

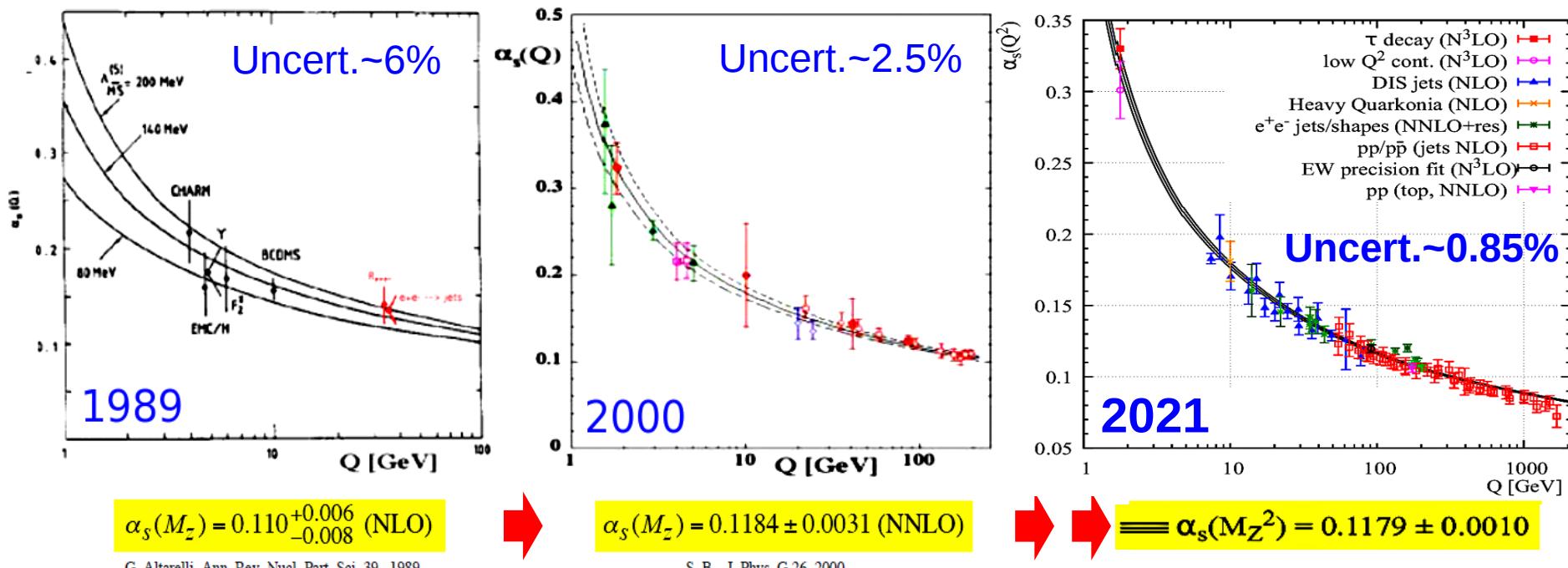
S. B., J. Phys. G 26, 2000



David d'Enterria (CERN)

# QCD coupling $\alpha_s$

- Determines **strength of the strong interaction** between quarks & gluons.
- Single free parameter of QCD in the  $m_q \rightarrow 0$  limit.
- Determined at a ref. scale ( $Q=m_Z$ ), decreases as  $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$ ,  $\Lambda \sim 0.2$  GeV



► Least precisely known of all interaction **couplings** !

$$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$$

# Importance of the QCD coupling $\alpha_s$

→ Impacts all QCD x-sections & decays (H), precision top & parametric EWPO:

Process	$\sigma$ (pb)	$\delta\alpha_s$ (%)	PDF + $\alpha_s$ (%)	Scale(%)
ggH	49.87	$\pm 3.7$	-6.2 +7.4	-2.61 + 0.32
tH	0.611	$\pm 3.0$	$\pm 8.9$	-9.3 + 5.9

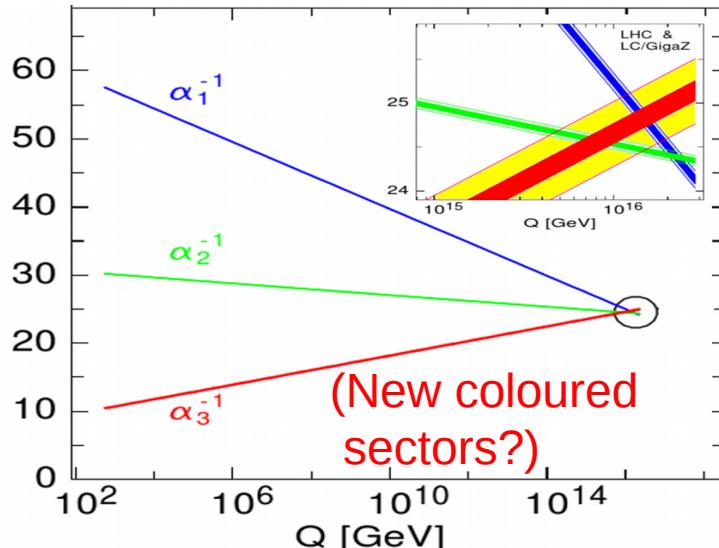
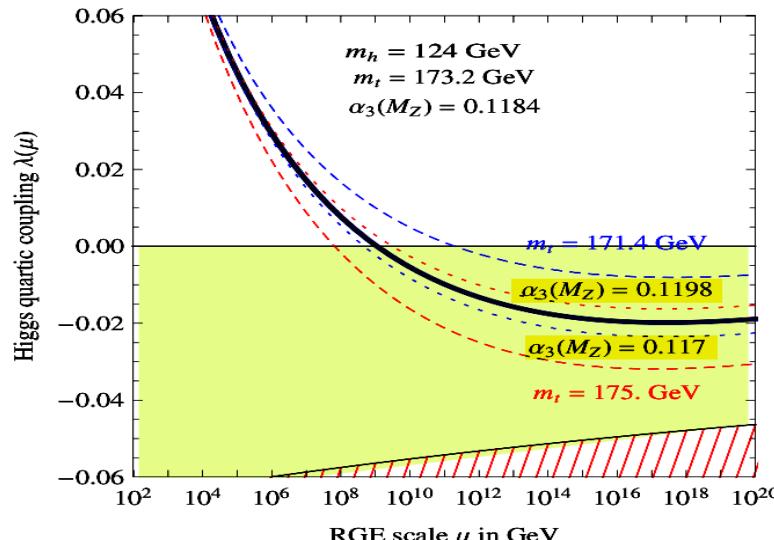
Partial width	intr. QCD	para. $m_q$	para. $\alpha_s$
$H \rightarrow b\bar{b}$	$\sim 0.2\%$	1.4%	0.4%
$H \rightarrow c\bar{c}$	$\sim 0.2\%$	4.0%	0.4%
$H \rightarrow gg$	$\sim 3\%$	< 0.2%	3.7%

Msbar mass error budget (from threshold scan)			
$(\delta M_t^{\text{SD-low}})^{\text{exp}}$	$(\delta M_t^{\text{SD-low}})^{\text{theo}}$	$(\delta \overline{m}_t(\overline{m}_t))^{\text{conversion}}$	$(\delta \overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV
⇒ improvement in $\alpha_s$ crucial			$\delta\alpha_s(M_z) = 0.001$

Quantity	FCC-ee	future param. unc.	Main source
$\Gamma_Z$ [MeV]	0.1	0.1	$\delta\alpha_s$
$R_b$ [ $10^{-5}$ ]	6	< 1	$\delta\alpha_s$
$R_\ell$ [ $10^{-3}$ ]	1	1.3	$\delta\alpha_s$

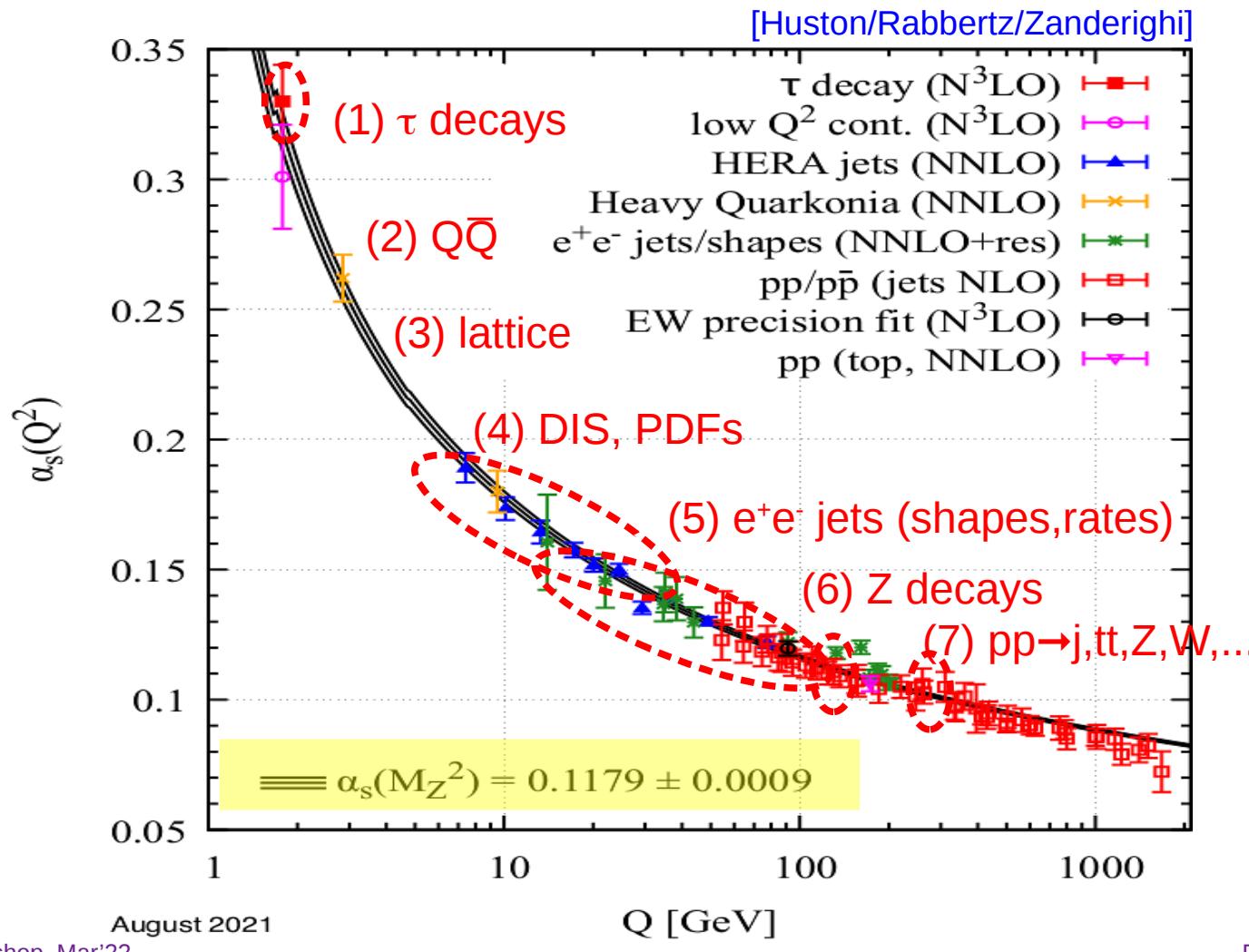
Sven Heinemeyer – 1st FCC physics workshop, CERN, 17.01.2017

→ Impacts physics approaching Planck scale: EW vacuum stability, GUT



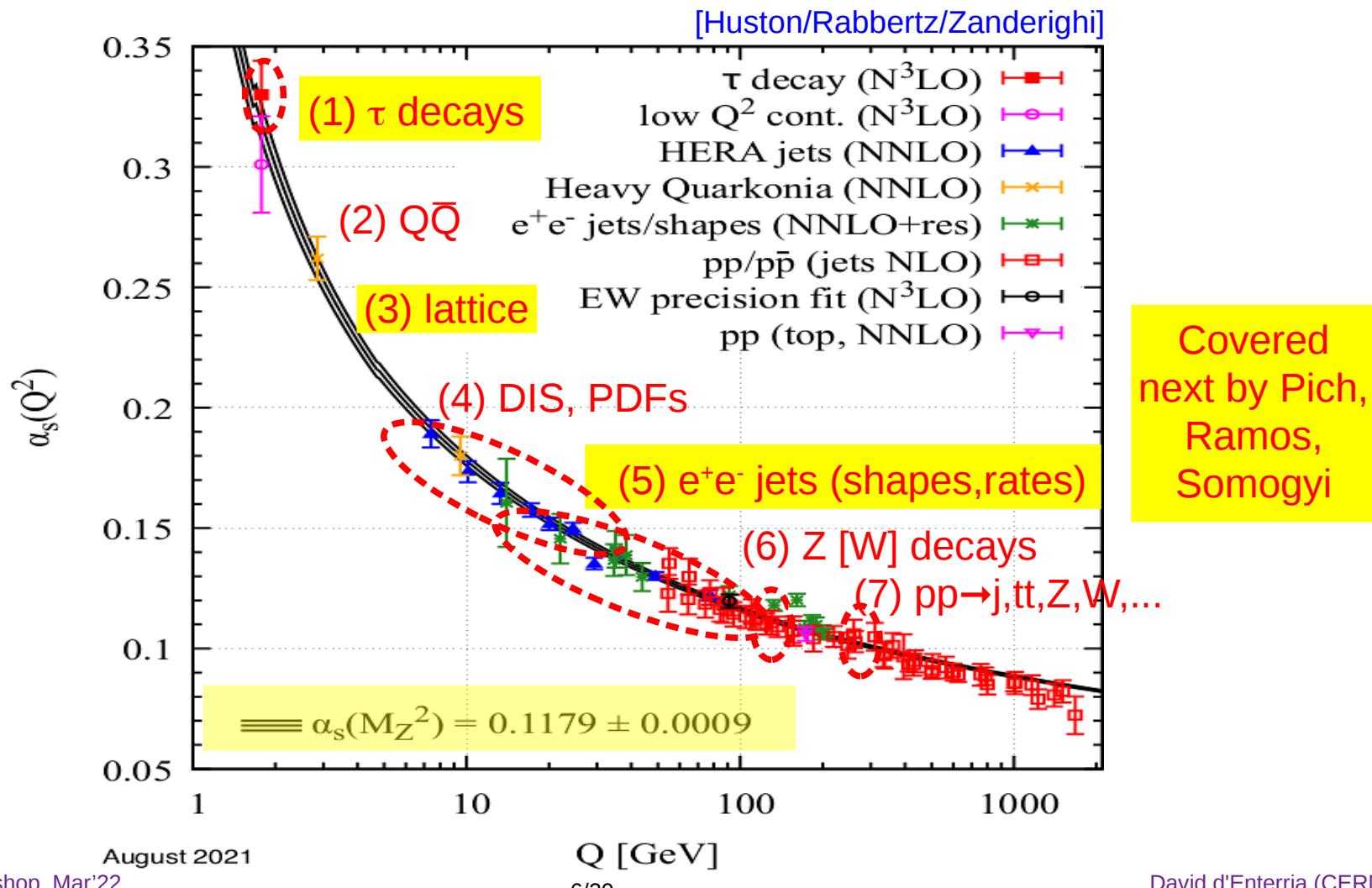
# World $\alpha_s$ determination (PDG 2021)

- Determined today by comparing 7 experimental observables to pQCD NNLO,N<sup>3</sup>LO predictions, plus global average at the Z pole scale:



# World $\alpha_s$ determination (PDG 2021)

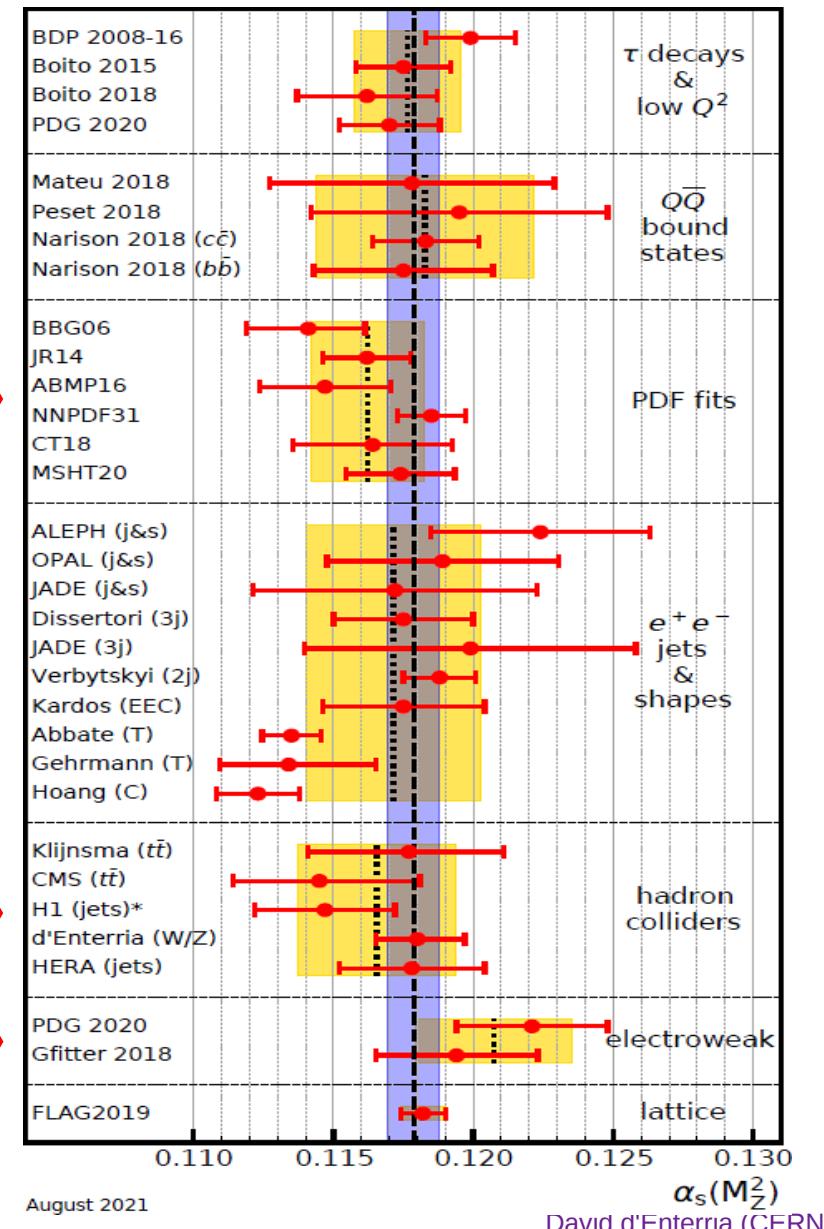
- Determined today by comparing 7 experimental observables to pQCD NNLO,N<sup>3</sup>LO predictions, plus global average at the Z pole scale:



# World-average $\alpha_s$ (PDG 2021)

- Average of 7 pre-averages from categories of observables:  
 $\alpha_s(M_z) = 0.1179 \pm 0.0009 \text{ } (\pm 0.85\%)$

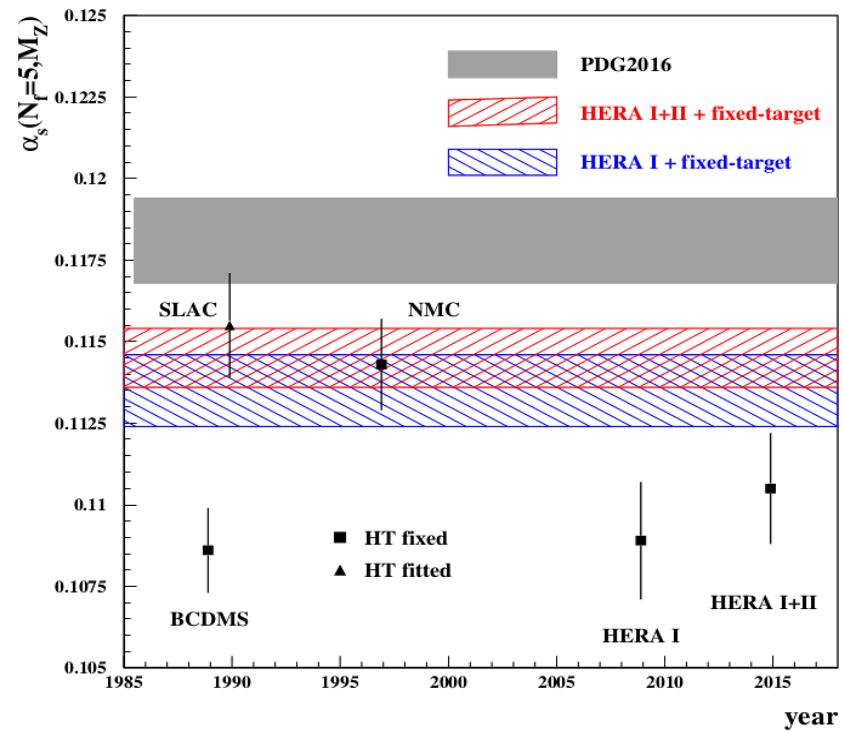
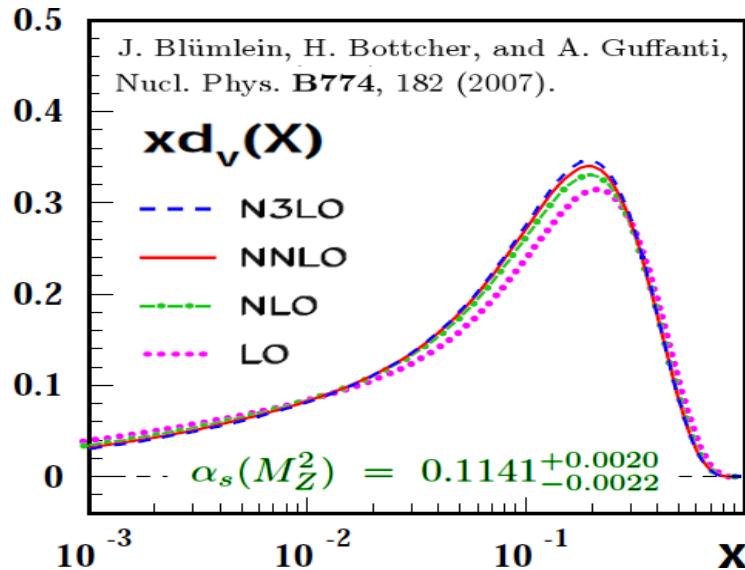
- I will focus on 3 of them:
  - (1) DIS & PDF determinations:  
 Fits of DIS structure functions  
 Global PDF fits.  
 $\alpha_s(M_z) = 0.1162 \pm 0.0020 \text{ } (\pm 1.7\%)$
  - (2) Hadron collider determinations:  
 pp:  $\sigma(t\bar{t})$ ,  $\sigma(W,Z)$ . ep:  $\sigma(jets)$   
 $\alpha_s(M_z) = 0.1165 \pm 0.0028 \text{ } (\pm 2.4\%)$
  - (3) Electroweak precision fits:  
 ee  $\rightarrow Z$  pseudoobserv., Global SM fit  
 $\alpha_s(M_z) = 0.1208 \pm 0.0028 \text{ } (\pm 2.3\%)$



# $\alpha_s$ from DIS struct. functions & PDF fits (I)

- N<sup>3</sup>LO/NNLO analysis of (non)singlet struct. functions (BBG, JR14) and NNLO global PDF fit (AMBP) tend to give “lowish”  $\alpha_s(M_z) \approx 0.1150$

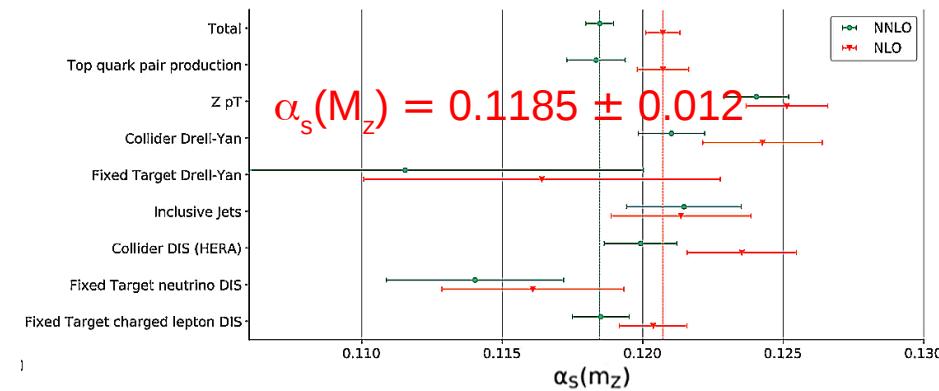
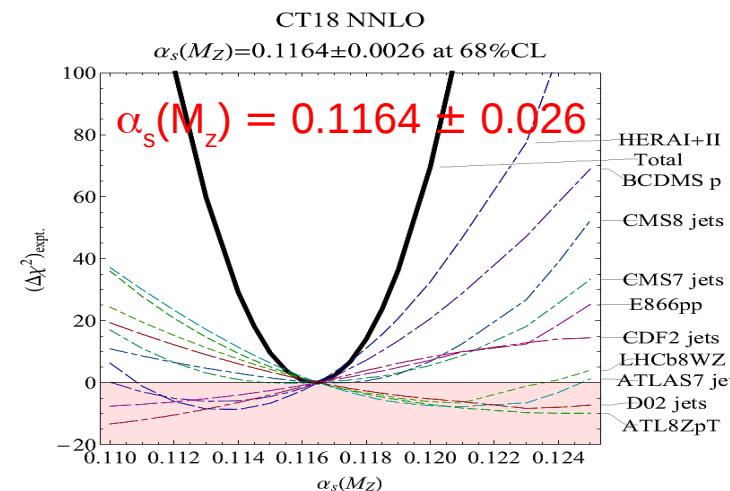
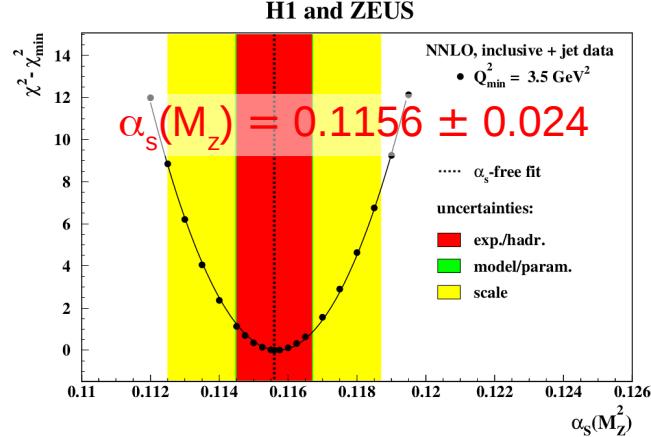
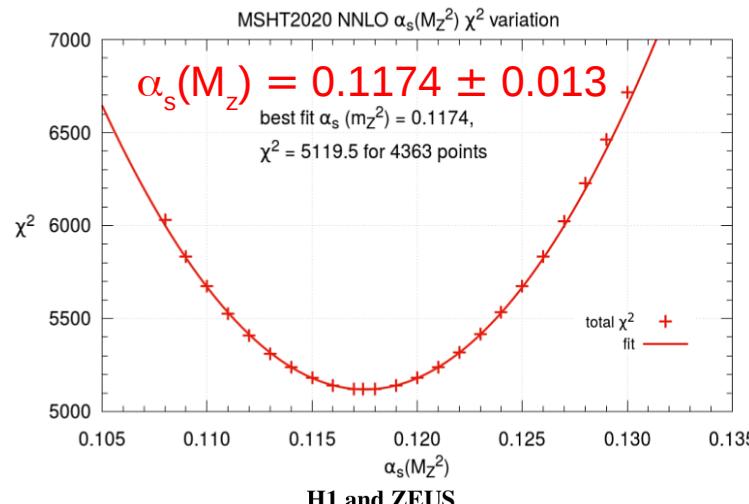
$$F_2(x, Q^2) = x \sum_{n=0}^{\infty} \frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}(z, Q^2, \mu_R^2, \mu_F^2) f_{i/p}\left(\frac{x}{z}, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$



- Neglect of singlet contribs. for  $x>0.3$  in NS fits? Size of higher-order corrs.?
- Future: New high-precision  $F_i(x, Q^2)$  & polarized  $g_i(x, Q^2)$  at EIC

# $\alpha_s$ from DIS struct. functions & PDF fits (II)

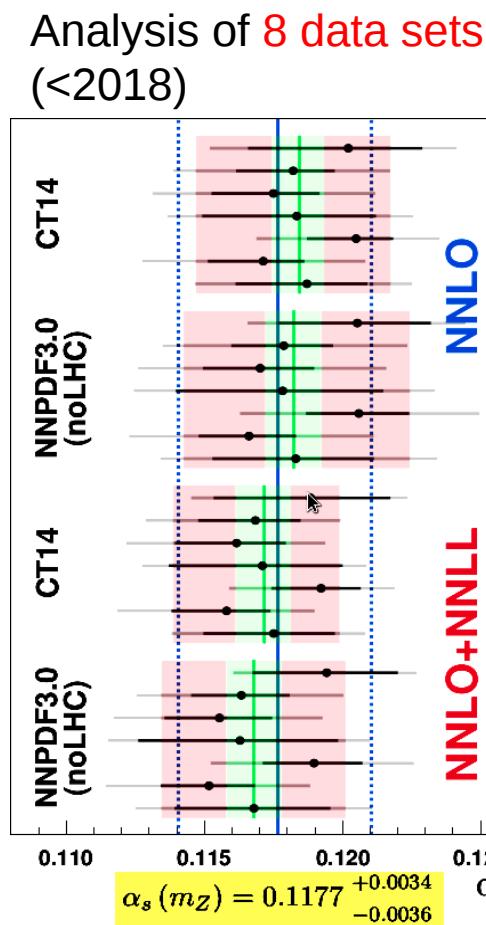
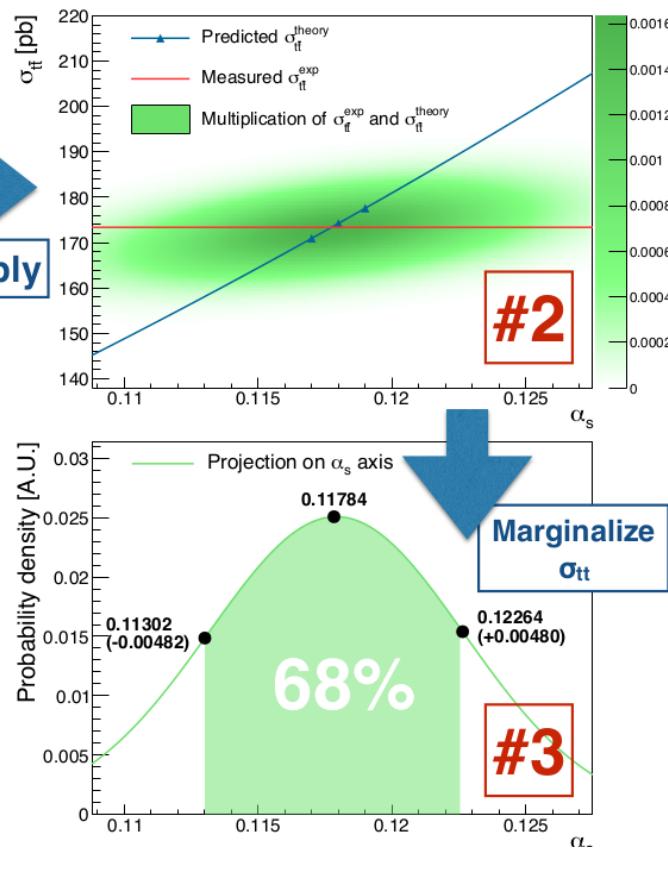
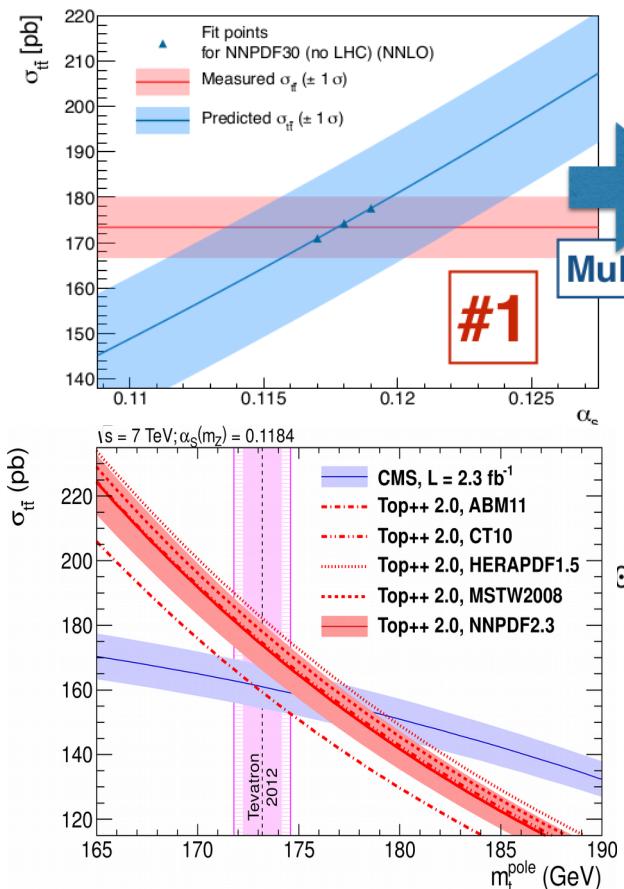
- NNLO global PDF+ $\alpha_s$  fits: CT18, HERAPDF2.0+j, MSTH2020, NNPDF3.1



- DIS/FT (LHC) data tend to prefer lower (higher) values of  $\alpha_s(M_Z)$
- Size of missing higher-order corrections? Global fits at N<sup>3</sup>LO needed

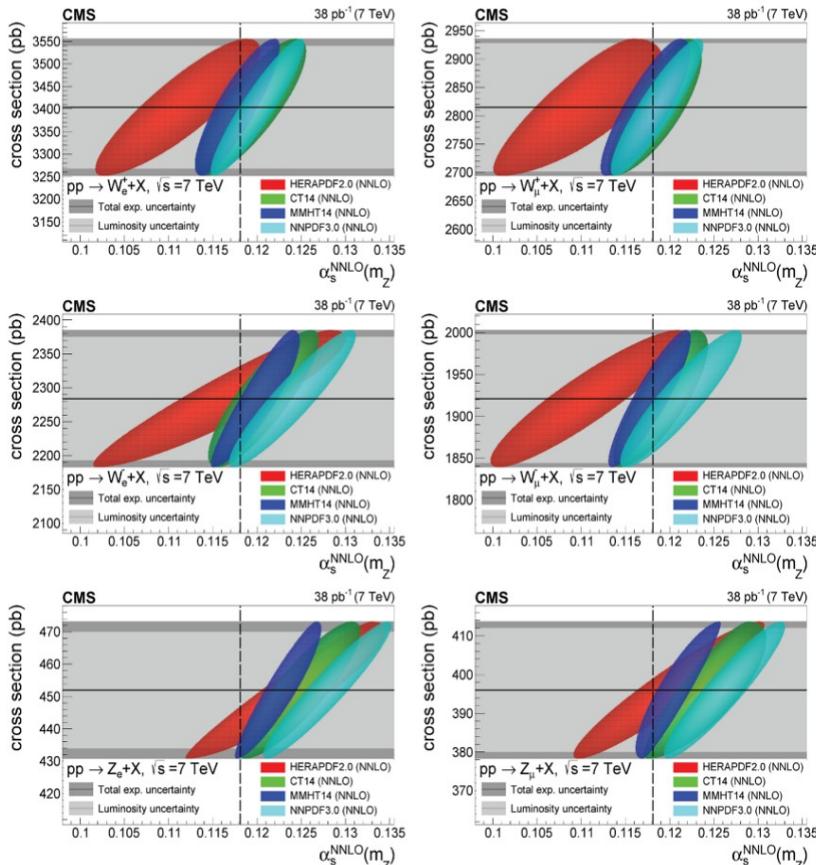
# $\alpha_s$ from hadron collider x-sections (ttbar)

- So far, only top-pair and W,Z boson x-sections available at NNLO.  
(Jets also available at NNLO since couple of years: (long) analysis ongoing).
- Method: Compare  $\sigma(\text{exp})$  to  $\sigma(\text{NNLO})$  computed w/ diff. PDFs/ $\alpha_s$ : Extract  $\alpha_s$
- $\sigma(\text{tt})$  (dis)advantages: Direct sensitivity to  $\alpha_s$  (Uncertainties:  $\sim 5\%$  exp./th.,  $m_{\text{top}}$ )

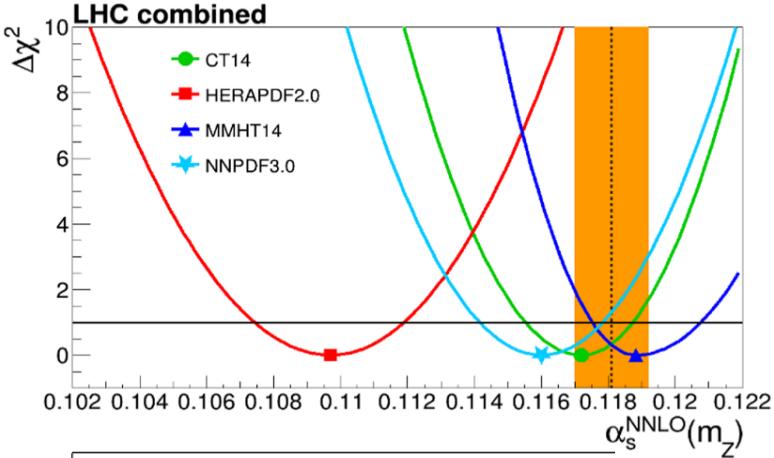


# $\alpha_s$ from hadron collider x-sections (W,Z)

- So far, only top-pair and W,Z boson x-sections available at NNLO.  
(Jets also available at NNLO since couple of years: (long) analysis ongoing).
- Method: Compare  $\sigma(\text{exp})$  to  $\sigma(\text{NNLO})$  computed w/ diff. PDFs/ $\alpha_s$ : Extract  $\alpha_s$
- $\sigma(W,Z)$  (dis)advantages:  $\sim 1\text{--}2\%$  th./exp. uncertainties (not LO sensitivity to  $\alpha_s$ )



Combined fit of 28 LHC data sets (<2019)  
with all correlation matrices:



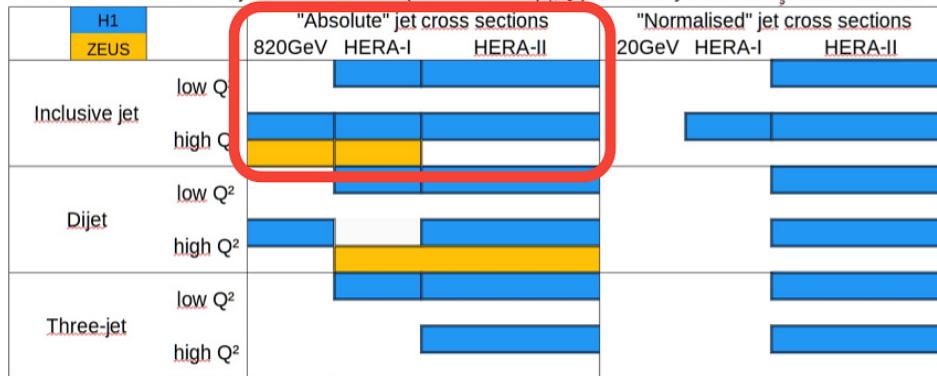
PDF	$\alpha_s(m_Z)$
CT14	$0.1172^{+0.0015}_{-0.0017}$
MMHT14	$0.1188^{+0.0019}_{-0.0013}$
HERAPDF2.0	$0.1097^{+0.0022}_{-0.0023}$
NNPDF3.0	$0.1160 \pm 0.0018$

- Eventually, better to incorporate  $\sigma(t\bar{t})$ ,  $\sigma(W,Z)$  into global PDF+ $\alpha_s$  fits.

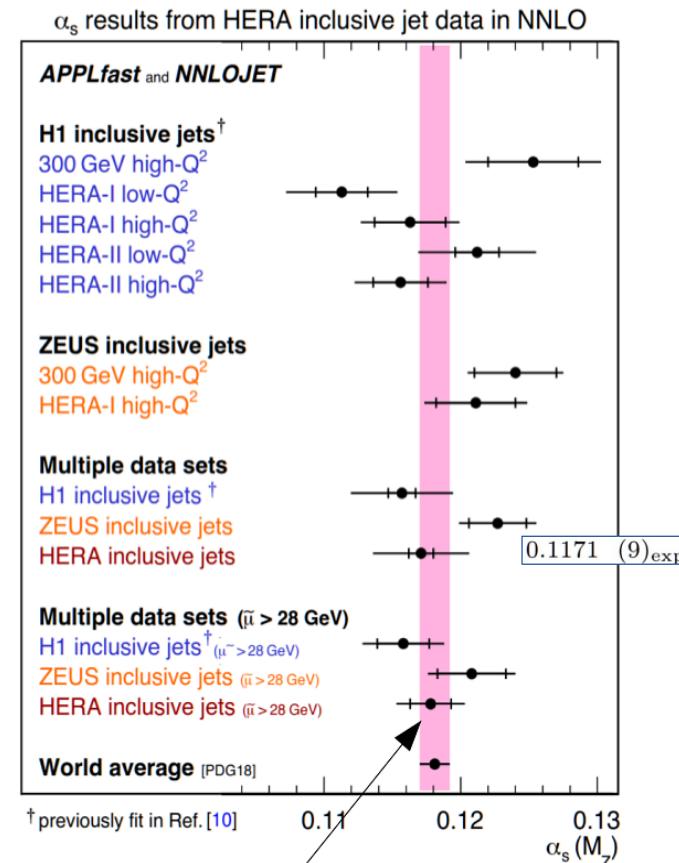
# $\alpha_s$ from hadron collider x-sections (HERA jets)

## ■ DIS HERA jet x-sections employed for $\alpha_s$ extractions via NNLOjet:

Double-differential HERA jet data in NC DIS (as function of  $p_T, Q^2$ ) commonly used for  $\alpha_s$  determinations



- $\alpha_s$  from inclusive jet cross sections in NC DIS
- NNLO pQCD w/ non-pert. hadronisation corrections
- H1 and ZEUS consistent
- Sizeable scale uncertainties (MHOU) since data are at comparably low scales
- Highest precision obtained in fit to data with  $\mu > 28$  GeV



$$\alpha_s(M_Z) = 0.1178(15)_{\text{exp}}(21)_{\text{th}}$$

- ## ■ Largest uncertainty from missing HO corrs. Nice testbed for upcoming LHC jet x-section based extractions.

# $\alpha_s$ from EW precision fits (PDG'21)

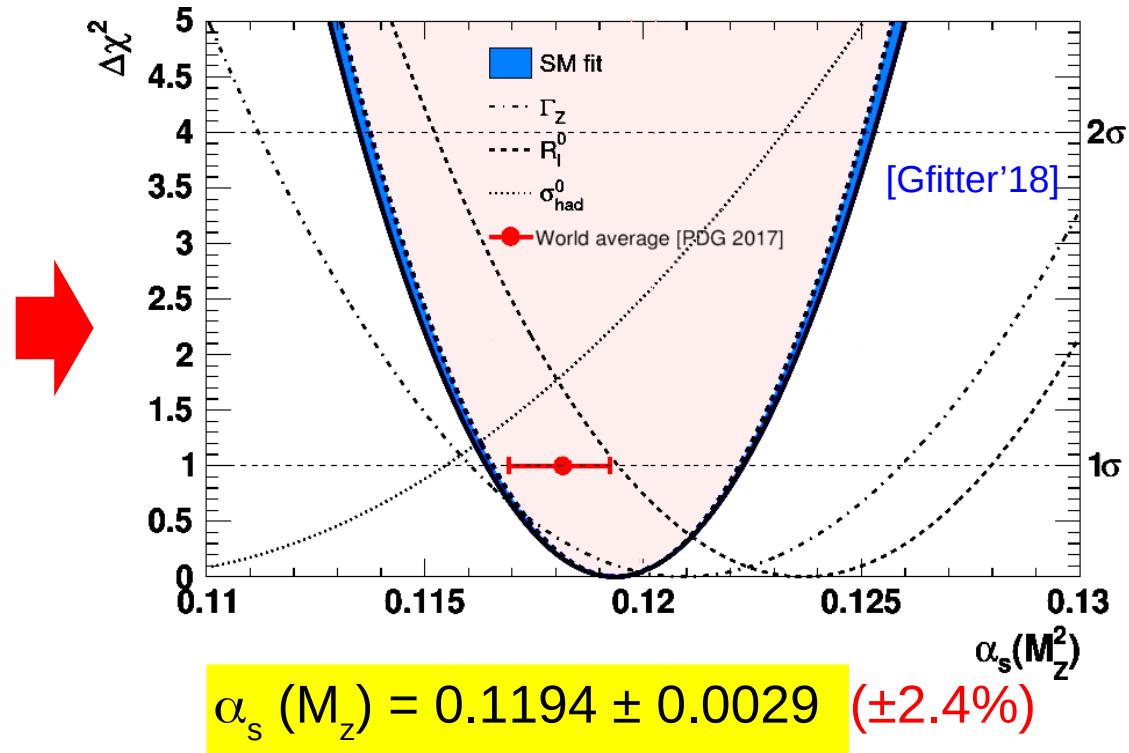
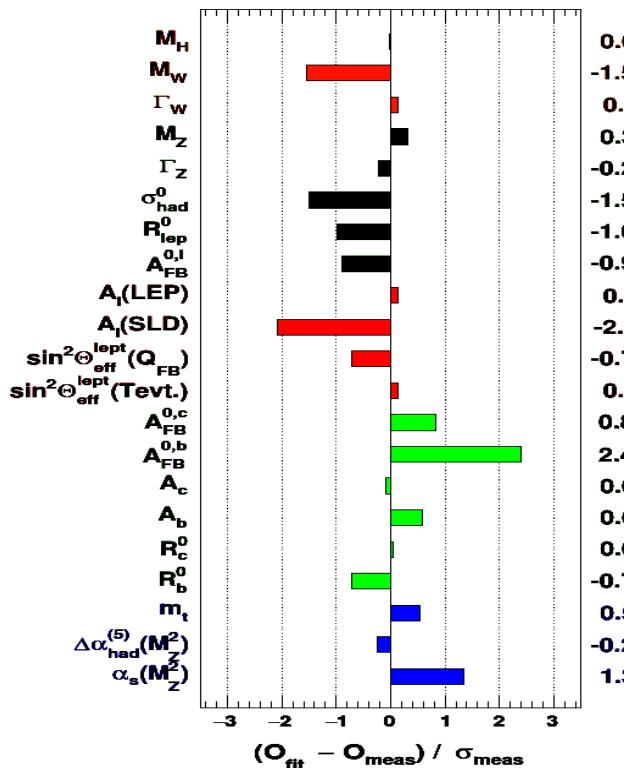
→ Extraction from three Z-peak pseudo-observables (LEP, SLC):

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV } (\pm 0.1\%) \quad R_\ell^0 = \frac{\Gamma_{\text{had}}}{\Gamma_\ell}, \quad \sigma_{\text{had}}^0 = \frac{12\pi}{m_Z} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}, \quad \sigma_\ell^0 = \frac{12\pi}{m_Z} \frac{\Gamma_\ell^2}{\Gamma_Z^2}$$

Known at N<sup>3</sup>LO, no NP uncer.:  $R_l^0 \equiv \frac{\Gamma(Z \rightarrow h)}{\Gamma(Z \rightarrow l)} = R_Z^{\text{EW}} N_C \left( 1 + \sum_{n=1}^4 c_n \left( \frac{\alpha_s}{\pi} \right)^n + \mathcal{O}(\alpha_s^5) + \delta_m + \delta_{\text{np}} \right)$

$$\alpha_s(M_Z) = 0.1221 \pm 0.0027 \text{ } (\pm 2.3\%)$$

→ Also from the global EW fit leaving  $\alpha_s$  as single free parameter:



$$\alpha_s(M_Z) = 0.1194 \pm 0.0029 \text{ } (\pm 2.4\%)$$

# $\alpha_s$ from hadronic EW decays (Update)

→ Incorporated new  $\alpha^2, \alpha^3$  EW corrections(\*) to Z pseudoobserv:

DdE, Jacobsen:  
arXiv:2005.04545 [hep-ph]

- The W and Z hadronic widths : (\*)Dubovsky/Chen/Freitas et al. arXiv:1906.08815, arXiv:2002.05845

$$\Gamma_{W,Z}^{\text{had}}(Q) = \Gamma_{W,Z}^{\text{Born}} \left( 1 + \sum_{i=1}^4 a_i(Q) \left( \frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{EW}} + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

TH uncertainties:  
 $\pm 0.01\%$  (Z)  
 $\pm 0.02\%$  (W)

- The ratio of W, Z hadronic-to-leptonic widths :

$$R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{\text{had}}(Q)}{\Gamma_{W,Z}^{\text{lep}}(Q)} = R_{W,Z}^{\text{EW}} \left( 1 + \sum_{i=1}^4 a_i(Q) \left( \frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

Parametric uncerts.:  
 $(\alpha_s, m_{Z,W}; V_{cs,ud})$ :

- In the Z boson case, the hadronic cross section at the resonance peak in  $e^+e^-$ :

$$\sigma_Z^{\text{had}} = \frac{12\pi}{m_Z} \cdot \frac{\Gamma_Z^e \Gamma_Z^{\text{had}}}{(\Gamma_Z^{\text{tot}})^2}$$

$\pm 0.03\%$  (Z)  
 $\pm 1.7\%$  (W)  
 $\pm 0.03\%$  (W, CKM unit)

→ Incorporated modified LEP data due to luminosity bias correction(\*):

	theory			experiment		
	previous	new (this work)	change	previous [6]	new [20, 21]	change
$\Gamma_Z^{\text{tot}}$ (MeV)	$2494.2 \pm 0.8_{\text{th}}$	$2495.2 \pm 0.6_{\text{par}} \pm 0.4_{\text{th}}$	+0.04%	$2495.2 \pm 2.3$	$2495.5 \pm 2.3$	+0.012%
$R_Z$	$20.733 \pm 0.007_{\text{th}}$	$20.750 \pm 0.006_{\text{par}} \pm 0.006_{\text{th}}$	+0.08%	$20.767 \pm 0.025$	$20.7666 \pm 0.0247$	-0.040%
$\sigma_Z^{\text{had}}$ (pb)	$41490 \pm 6_{\text{th}}$	$41494 \pm 5_{\text{par}} \pm 6_{\text{th}}$	+0.01%	$41540 \pm 37$	$41480.2 \pm 32.5$	-0.144%

Recent update of  
LEP luminosity  
bias change the Z  
values by few permil

W boson observables	GFITTER 2.2 (NNLO)			experiment
	this work (N <sup>3</sup> LO)		(exp. CKM)	
$\Gamma_W^{\text{had}}$ (MeV)	–	$1440.3 \pm 23.9_{\text{par}} \pm 0.2_{\text{th}}$		$1410.2 \pm 0.8_{\text{par}} \pm 0.2_{\text{th}}$
$\Gamma_W^{\text{tot}}$ (MeV)	$2091.8 \pm 1.0_{\text{par}}$	$2117.9 \pm 23.9_{\text{par}} \pm 0.7_{\text{th}}$		$2087.9 \pm 1.0_{\text{par}} \pm 0.7_{\text{th}}$
$R_W$	–	$2.1256 \pm 0.0353_{\text{par}} \pm 0.0008_{\text{th}}$		$2.0812 \pm 0.0007_{\text{par}} \pm 0.0008_{\text{th}}$
				$2.069 \pm 0.019$

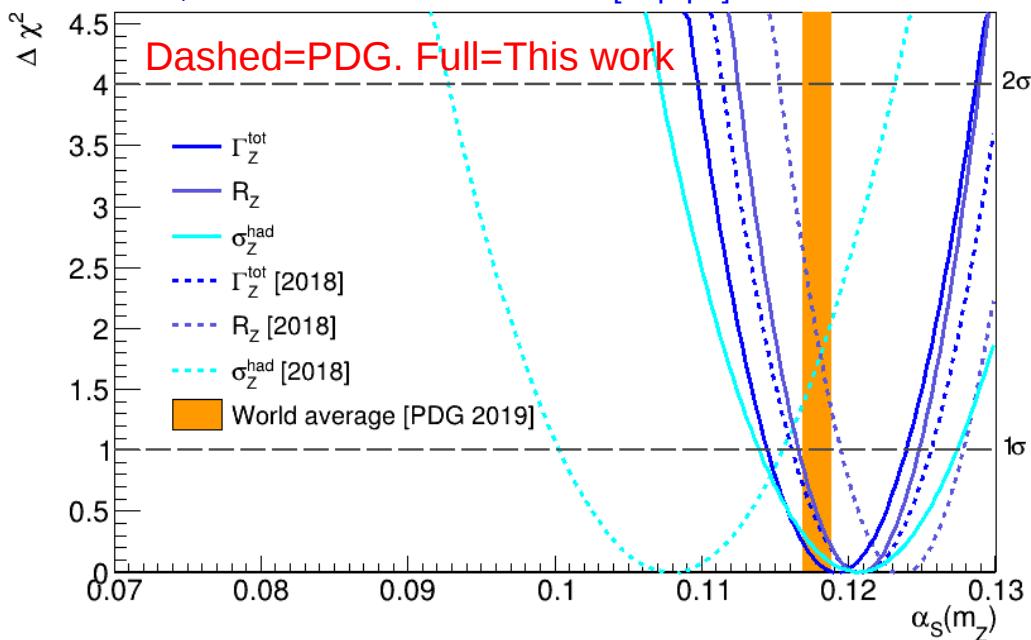
(\*) Voutsinas et al.  
arXiv:1908.01704,  
Janot et al.  
arXiv:1912.02067

# $\alpha_s$ from hadronic Z decays (Update)

► QCD coupling extracted from:

- (i) Combined fit of 3 Z pseudo-observ.
- (ii) Full SM fit (with  $\alpha_s$  free parameter)

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]

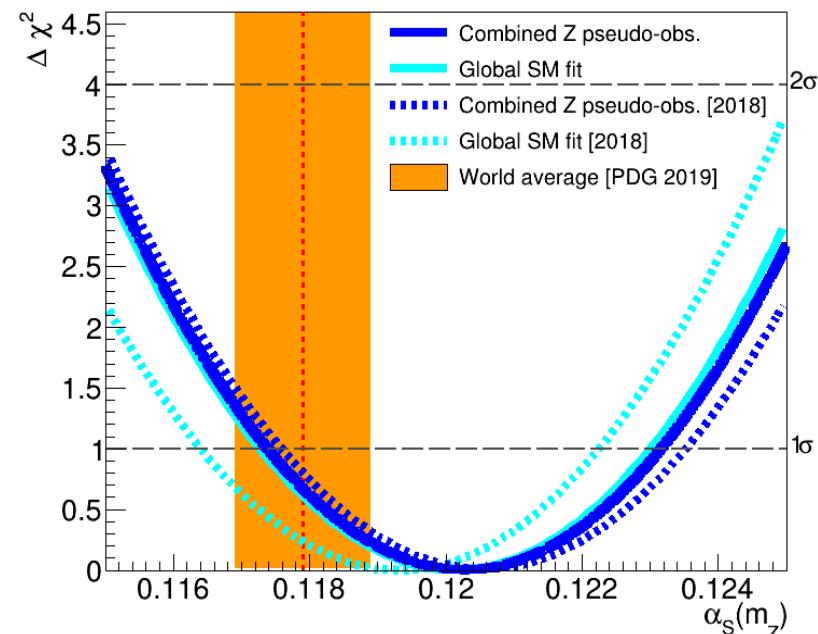


► LEP lumi-bias updates lead to much better agreement among  $\Gamma_z$ ,  $R_z$ ,  $\sigma_0$  extractions:

$$\alpha_s(m_Z) = 0.1203 \pm 0.0028 \quad (\pm 2.3\%)$$

$$PDG'21: \alpha_s(m_Z) = 0.1221 \pm 0.0027 \quad (\pm 2.3\%)$$

Z boson observable	$\alpha_s(m_Z)$ extraction	uncertainties		
		exp.	param.	theor.
$\Gamma_Z^{\text{tot}}$	$0.1192 \pm 0.0047$	$\pm 0.0046$	$\pm 0.0005$	$\pm 0.0008$
$R_Z$	$0.1207 \pm 0.0041$	$\pm 0.0041$	$\pm 0.0001$	$\pm 0.0009$
$\sigma_Z^{\text{had}}$	$0.1206 \pm 0.0068$	$\pm 0.0067$	$\pm 0.0004$	$\pm 0.0012$
All combined	$0.1203 \pm 0.0029$	$\pm 0.0029$	$\pm 0.0002$	$\pm 0.0008$
Global SM fit	$0.1202 \pm 0.0028$	$\pm 0.0028$	$\pm 0.0002$	$\pm 0.0008$



► EXP/TH updates lead to better agreement with full SM fit:

$$\alpha_s(m_Z) = 0.1202 \pm 0.0028$$

$$PDG'21: \alpha_s(m_Z) = 0.1194 \pm 0.0029$$

David d'Enterria (CERN)

# $\alpha_s$ from hadronic W decays (Update)

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]

- Parametrized W boson  $\Gamma_W^{\text{lep}}$ ,  $\Gamma_W^{\text{had}}$ ,  $\Gamma_W^{\text{tot}}$ ,  $R_W$  (with  $\alpha_s^4, \alpha, \alpha_s \alpha$  corrections):

$$\Gamma_W^{\text{lep}} = \Gamma_0 + c_1 \Delta_W + c_4 \Delta_H + c_5 \Delta_t + c_7 \Delta_\tau,$$

$$\Delta_W = \left( \frac{m_W}{80.379} \right)^3 - 1, \quad \Delta_H = \log \left( \frac{m_H}{125.10} \right), \quad \Delta_t = \left( \frac{m_t}{172.9} \right) - 1, \quad \Delta_\tau = \left( \frac{m_\tau}{1.777} \right) - 1 \quad \Delta_{\alpha_S} = \frac{\alpha_S(m_Z)}{0.1179} - 1, \quad \Delta_{\text{CKM}} = \frac{|V_{cd}|^2 + |V_{cs}|^2}{0.218^2 + 0.997^2} - 1$$

W widths (GeV)	$\Gamma_0$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	Max dev.
$\Gamma_W^{\text{lep}}$	679.35	676.78	–	–	0.04674	0.47745	–	-0.347428	< 0.00002
$\Gamma_W^{\text{had}}$ (exp. CKM)	1440.28	1446.61	734.557	53.76	–	–	-1.24411	–	< 0.0002
$\Gamma_W^{\text{had}}$ (CKM unit.)	1410.21	1409.59	–	52.34	–	–	-1.15932	–	< 0.0002
$\Gamma_W^{\text{tot}}$ (exp. CKM)	2119.58	2044.8	732.55	50.67	0.03980	0.46258	-1.0723	-0.36408	< 0.0002
$\Gamma_W^{\text{tot}}$ (CKM unit.)	2089.51	2088.26	–	52.28	0.04790	0.47842	-1.2683	-0.32942	< 0.0002

- Numerical evaluation of W boson ( $N^3\text{LO} + \text{EW corrs.}$ ) pseudo-observables :

W boson observables	GFITTER 2.2 (NNLO)	this work ( $N^3\text{LO}$ )		experiment
		(exp. CKM)	(CKM unit.)	
$\Gamma_W^{\text{lep}}$ (MeV)	–	$679.4 \pm 0.3_{\text{par}} \pm 0.5_{\text{th}}$		$682.2 \pm 10.2$
$\Gamma_W^{\text{had}}$ (MeV)	–	$1440.3 \pm 23.9_{\text{par}} \pm 0.2_{\text{th}}$	$1410.2 \pm 0.8_{\text{par}} \pm 0.2_{\text{th}}$	$1405 \pm 29$
$\Gamma_W^{\text{tot}}$ (MeV)	$2091.8 \pm 1.0_{\text{par}}$	$2119.6 \pm 23.9_{\text{par}} \pm 0.7_{\text{th}}$	$2089.5 \pm 1.1_{\text{par}} \pm 0.7_{\text{th}}$	$2085 \pm 42$
$R_W$	–	$2.1200 \pm 0.0352_{\text{par}} \pm 0.0016_{\text{th}}$	$2.0757 \pm 0.0014_{\text{par}} \pm 0.0015_{\text{th}}$	$2.0684 \pm 0.0254$

# $\alpha_s$ from hadronic W decays (Update)

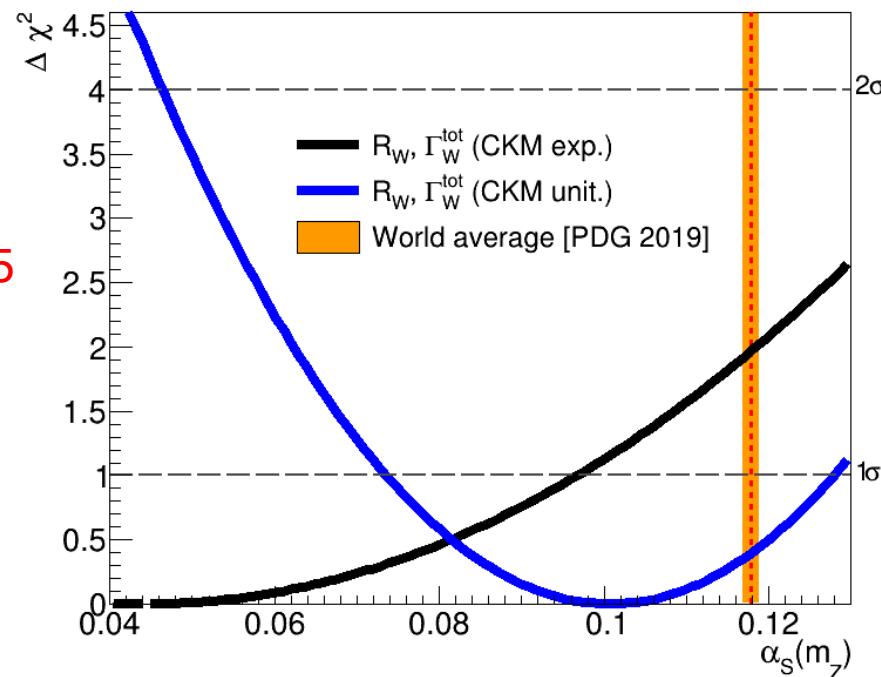
- QCD coupling extracted from new N<sup>3</sup>LO fit of combined  $\Gamma_w$ ,  $R_w$  pseudo-observ.:

W boson observables	$\alpha_S(m_Z)$ extraction	uncertainties		
		exp.	param.	theor.
$\Gamma_W^{\text{tot}}$ , $R_w$ (exp. CKM)	$0.044 \pm 0.052$	$\pm 0.024$	$\pm 0.047$	( $\pm 0.0014$ )
$\Gamma_W^{\text{tot}}$ , $R_w$ (CKM unit.)	$0.101 \pm 0.027$	$\pm 0.027$	( $\pm 0.0002$ )	( $\pm 0.0016$ )
$\Gamma_W^{\text{tot}}$ , $R_w$ (FCC-ee, CKM unit.)	$0.11790 \pm 0.00023$	$\pm 0.00012$	$\pm 0.00004$	$\pm 0.00019$

Still very imprecise extraction:

- Large propagated parametric uncert. from poor  $V_{cs}$  exp. precision ( $\pm 2\%$ ): QCD coupling unconstrained:  $0.04 \pm 0.05$
- Imposing CKM unitarity: large exp. uncertainties from  $\Gamma_w$ ,  $R_w$  ( $\pm 0.9\text{--}2\%$ ): QCD coupling with  $\sim 27\%$  precision
- Propagated TH uncertainty much smaller today than exp. ones:  $\sim 1\%$

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



$$\alpha_s(m_Z) = 0.101 \pm 0.027 \quad (\pm 27\%)$$

# $\alpha_s$ from hadronic Z decays (future)

► QCD coupling extracted from:

- (i) Combined fit of 3 Z pseudo-observ:
- (ii) Full SM fit (with  $\alpha_s$  free parameter)

Z boson observable	$\alpha_s(m_Z)$ extraction	uncertainties		
		exp.	param.	theor.
All combined	$0.1203 \pm 0.0029$	$\pm 0.0029$	$\pm 0.0002$	$\pm 0.0008$
Global SM fit	$0.1202 \pm 0.0028$	$\pm 0.0028$	$\pm 0.0002$	$\pm 0.0008$
All combined (FCC-ee)	$0.12030 \pm 0.00026$	$\pm 0.00013$	$\pm 0.00005$	$\pm 0.00022$
Global SM fit (FCC-ee)	$0.12020 \pm 0.00026$	$\pm 0.00013$	$\pm 0.00005$	$\pm 0.00022$

► FCC-ee:

- Huge Z pole stats. ( $\times 10^5$  LEP):
- Exquisite systematic/parametric precision (stat. uncert. negligible):

$$\Delta R_Z = 10^{-3}, \quad R_Z = 20.7500 \pm 0.0010$$

$$\Delta \Gamma_Z^{\text{tot}} = 0.1 \text{ MeV}, \quad \Gamma_Z^{\text{tot}} = 2495.2 \pm 0.1 \text{ MeV}$$

$$\Delta \sigma_Z^{\text{had}} = 4.0 \text{ pb}, \quad \sigma_Z^{\text{had}} = 41494 \pm 4 \text{ pb}$$

$$\Delta m_Z = 0.1 \text{ MeV}, \quad m_Z = 91.18760 \pm 0.00001 \text{ GeV}$$

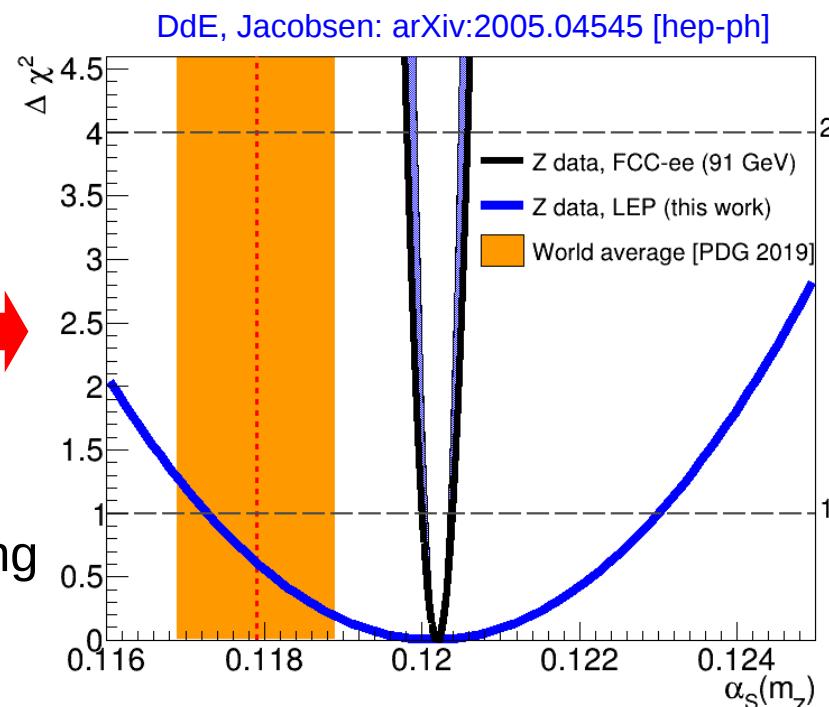
$$\Delta \alpha = 3 \cdot 10^{-5}, \quad \Delta \alpha_{\text{had}}^{(5)}(m_Z) = 0.0275300 \pm 0.0000009$$

- TH uncert. to be reduced by  $\times 4$  computing missing  $\alpha_s^5, \alpha^3, \alpha \alpha_s^2, \alpha \alpha_s^2, \alpha^2 \alpha_s$  terms

► 10 times better precision than today:

$$\delta \alpha_s / \alpha_s \sim \pm 0.2\% \text{ (exp+th)}, \pm 0.1\% \text{ (exp)}$$

Strong (B)SM consistency test.



$$\alpha_s(m_Z) = 0.12030 \pm 0.00028 \quad (\pm 0.2\%)$$

# $\alpha_s$ from hadronic W decays (future)

- QCD coupling extracted from new N<sup>3</sup>LO fit of combined  $\Gamma_W$ ,  $R_W$  pseudo-observ.:

W boson observables	$\alpha_S(m_Z)$ extraction	uncertainties		
		exp.	param.	theor.
$\Gamma_W^{\text{tot}}$ , $R_W$ (exp. CKM)	$0.044 \pm 0.052$	$\pm 0.024$	$\pm 0.047$	( $\pm 0.0014$ )
$\Gamma_W^{\text{tot}}$ , $R_W$ (CKM unit.)	$0.101 \pm 0.027$	$\pm 0.027$	( $\pm 0.0002$ )	( $\pm 0.0016$ )
$\Gamma_W^{\text{tot}}$ , $R_W$ (FCC-ee, CKM unit.)	$0.11790 \pm 0.00023$	$\pm 0.00012$	$\pm 0.00004$	$\pm 0.00019$

## FCC-ee extraction:

- Huge W pole stats. ( $\times 10^4$  LEP-2).
- Exquisite syst./parametric precision:

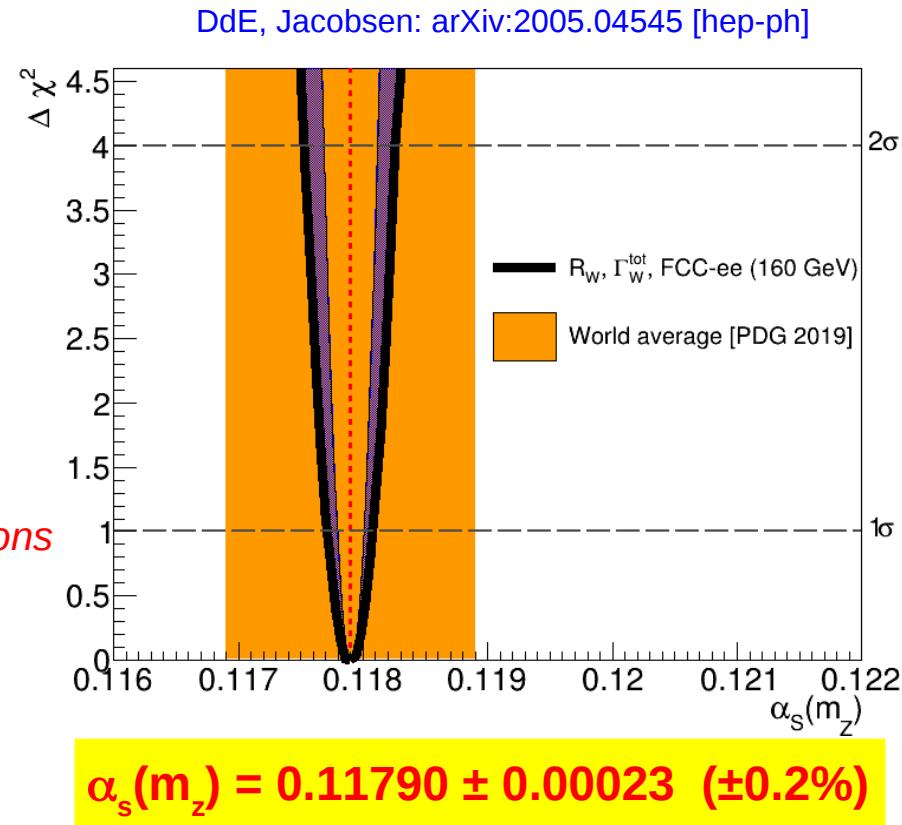
$$\Gamma_W^{\text{tot}} = 2088.0 \pm 1.2 \text{ MeV}$$

$$R_W = 2.08000 \pm 0.00008$$

$$m_W = 80.3800 \pm 0.0005 \text{ GeV}$$

$$|V_{cs}| = 0.97359 \pm 0.00010 \leftarrow O(10^{12}) D \text{ mesons}$$

- TH uncertainty to be reduced by  $\times 10$  after computing missing  $\alpha_s^5$ ,  $\alpha^2$ ,  $\alpha^3$ ,  $\alpha\alpha_s^2$ ,  $\alpha\alpha_s^2$ ,  $\alpha^2\alpha_s$  terms



# Summary: $\alpha_s$ overview

- World-average QCD coupling today:
  - Determined from  $7 N^{2,3}LO$  observables
  - Impacts all LHC QCD x-sections & decays.
  - Role **BSM**: GUT, EWK vacuum stability, New coloured sectors?

## (1) DIS & PDF determinations:

Fits of DIS structure functions, global PDF fits.

$$\alpha_s(M_Z) = 0.1162 \pm 0.0020 \ (\pm 1.7\%)$$

Future:  $N^3LO$  PDFs needed, EIC data

## (2) Hadron collider determinations:

pp:  $\sigma(t\bar{t})$ ,  $\sigma(W,Z)$ . ep:  $\sigma(jets)$

$$\alpha_s(M_Z) = 0.1165 \pm 0.0028 \ (\pm 2.4\%)$$

Future: Incorporate to global PDF+ $\alpha_s$  fits

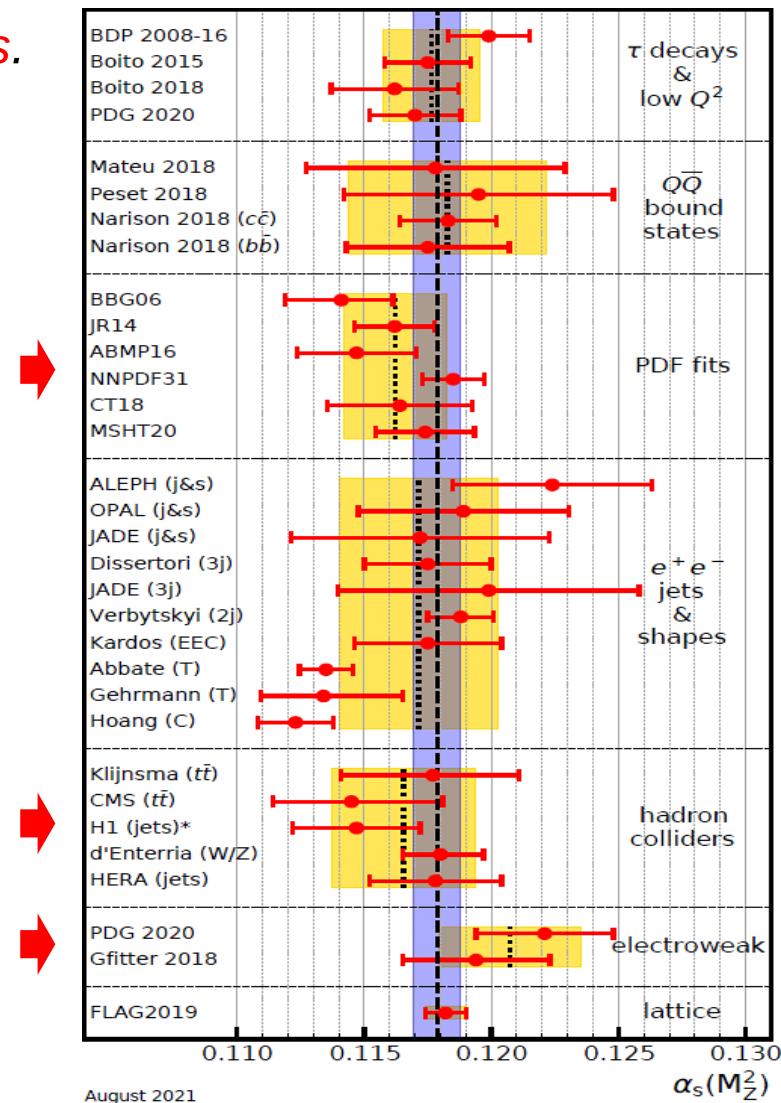
## (3) Electroweak precision fits:

$ee \rightarrow Z$  pseudoobserv., Global SM fit

$$\alpha_s(M_Z) = 0.1208 \pm 0.0028 \ (\pm 2.3\%)$$

Future: Permil uncertainty requires FCC-e<sup>+</sup>e<sup>-</sup>

$$\alpha_s(M_Z) = 0.1179 \pm 0.0009 \ (\pm 0.85\%)$$

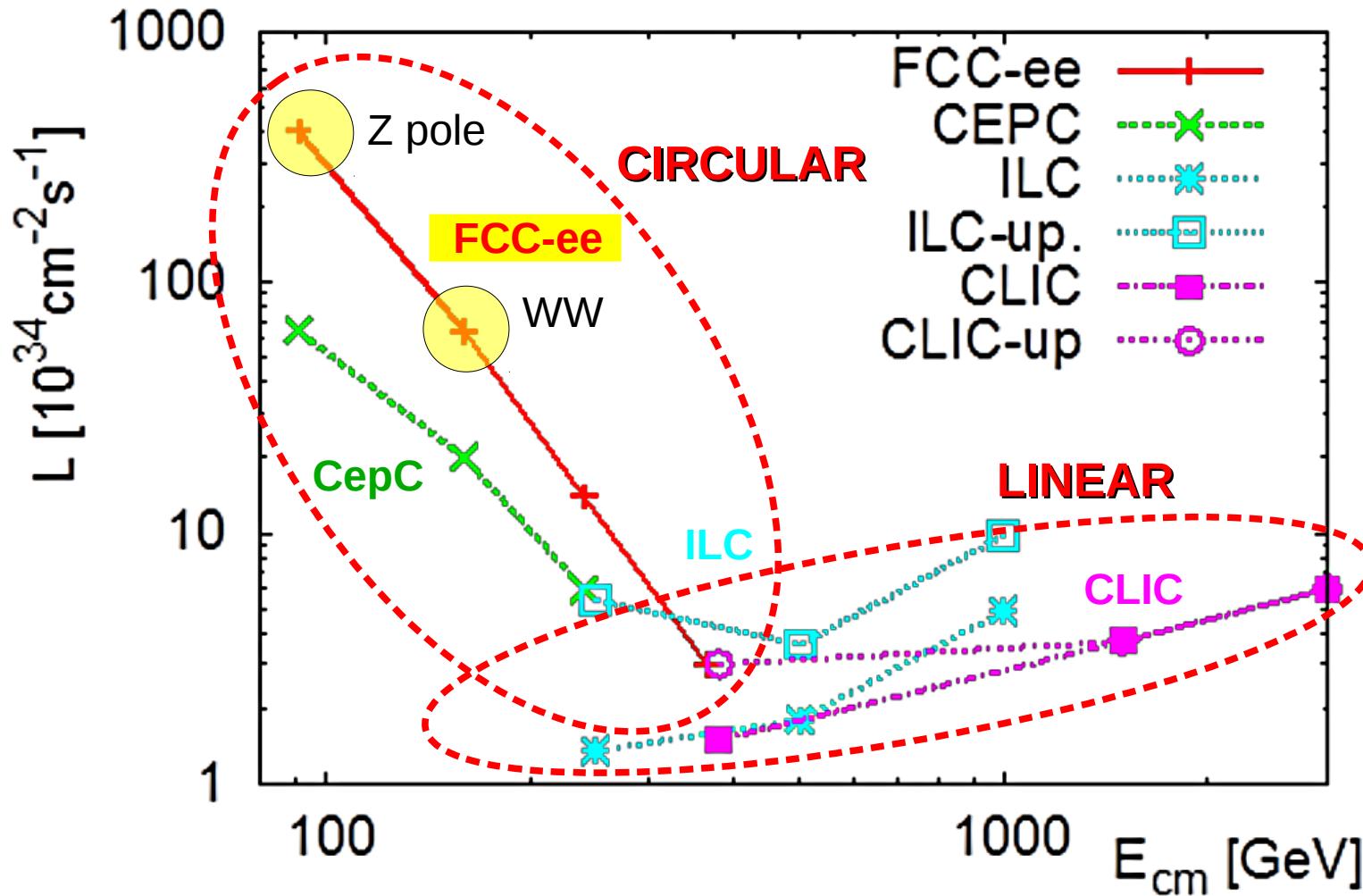


August 2021

David d'Enterria (CERN)

# Backup slides

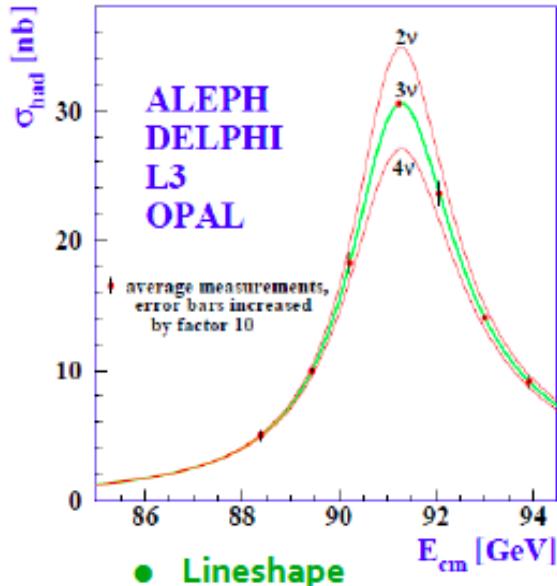
# Future $e^+e^-$ colliders under discussion



- FCC-ee features lumis a few times larger than other machines over 90–240 GeV
- Unparalleled Z, W, jets,  $\tau$ ,... data sets: Negligible  $\alpha_s$  stat. uncertainties

# Ultra-precise W, Z, top physics at FCC-ee

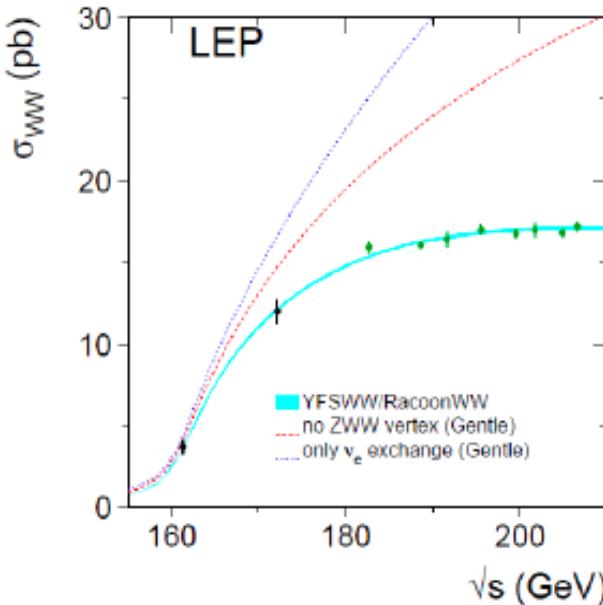
$\sqrt{s}=91 \text{ GeV}, 10^{12} Z's$



- Lineshape
  - Exquisite  $E_{\text{beam}}$  (unique!)
  - $m_Z, \Gamma_Z$  to 10 keV (stat.)
- Asymmetries 100 keV (syst.)
  - $\sin^2\theta_W$  to  $5 \times 10^{-6}$
- Branching ratios,  $R_b, R_{\text{had}}$ 
  - $\alpha_s(m_Z)$  to 0.0002
- Predict  $m_{\text{top}}, m_W$  in SM

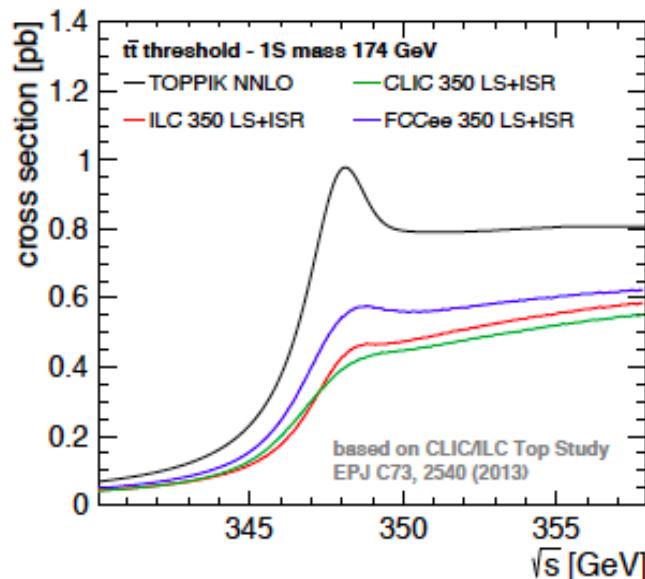
- Unparalleled Z, W, jets,  $\tau, \dots$  data sets: Negligible  $\alpha_s$  stat. uncertainties
- Unparalleled syst. uncert.:  $\delta E_{\text{cm}}(Z, W) \sim 0.1, 0.3 \text{ MeV} \rightarrow$  Very precise  $\Gamma_{W,Z}$

$\sqrt{s}=161 \text{ GeV}, 10^8 W's$



- Threshold scan
  - $m_W$  to 500 keV
- Branching ratios  $R_b, R_{\text{had}}$ 
  - $\alpha_s(m_W)$  to 0.0002
- Radiative returns  $e^+e^- \rightarrow \gamma Z$  ( $Z \rightarrow \nu\nu, \mu^+\mu^-$ )
  - $N_\nu$  to 0.001

$\sqrt{s}=350 \text{ GeV}, 10^6 \text{ tops}$



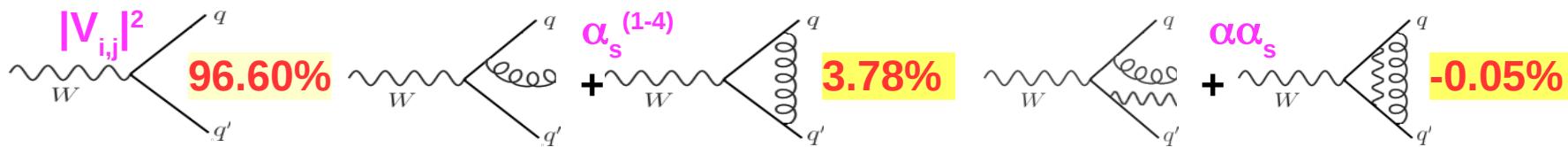
- Threshold scan + 4D fit
  - $m_{\text{top}}$  to 10 MeV (stat.)
  - 40 MeV (th.)
  - $\lambda_{\text{top}}$  to 13%
  - EWK couplings to 1–10%

# $\alpha_s$ from hadronic W decays (today)

→ Width known at N<sup>3</sup>LO. As for Z boson, small sensitivity to  $\alpha_s$  (only beyond Born):

$$\Gamma_{W,\text{had}} = \frac{\sqrt{2}}{4\pi} G_F m_W^3 \sum_{\text{quarks } i,j} |V_{i,j}|^2 \left[ 1 + \sum_{k=1}^4 \left( \frac{\alpha_s}{\pi} \right)^k + \delta_{\text{electroweak}}(\alpha) + \delta_{\text{mixed}}(\alpha \alpha_s) \right]$$

[EWK: -0.35%]



→ Recalculation of partial and total hadronic W widths:

DdE, Srebre:arXiv:1603.06501

Partial widths (MeV)	$\Gamma^{(0)}$	$\Gamma_{\text{QCD}}^{(1)}$	$\Gamma_{\text{QCD}}^{(2)}$	$\Gamma_{\text{QCD}}^{(3)}$	$\Gamma_{\text{QCD}}^{(4)}$	$\Gamma_{\text{ewk}}$	$\Gamma_{\text{mixed}}$	$\Gamma_{\text{had}}^W$
$W \rightarrow qq'$ (exp. $V_{ij}$ )	1379.851	52.931	2.857	-0.992	-0.238	-5.002	-0.755	$1428.65 \pm 22.40_{\text{par}} \pm 0.04_{\text{th}}$
$W \rightarrow qq'$ ( $V_{ij} V_{jk} = \delta_{ik}$ )	1363.197	52.291	2.822	-0.980	-0.235	-4.942	-0.746	$1411.40 \pm 0.96_{\text{par}} \pm 0.04_{\text{th}}$
$W \rightarrow qq'$ (exp. $V_{ij}$ ) [5]	1408.980	54.087	2.927	-1.018	-0.245	-5.132	-0.779	$1458.820 \pm 0.006_{\text{th}}$
$W \rightarrow qq'$ ( $V_{ij} V_{jk} = \delta_{ik}$ ) [5]	1363.640	52.346	2.833	-0.985	-0.237	-4.940	-0.748	$1411.910 \pm 0.006_{\text{th}}$

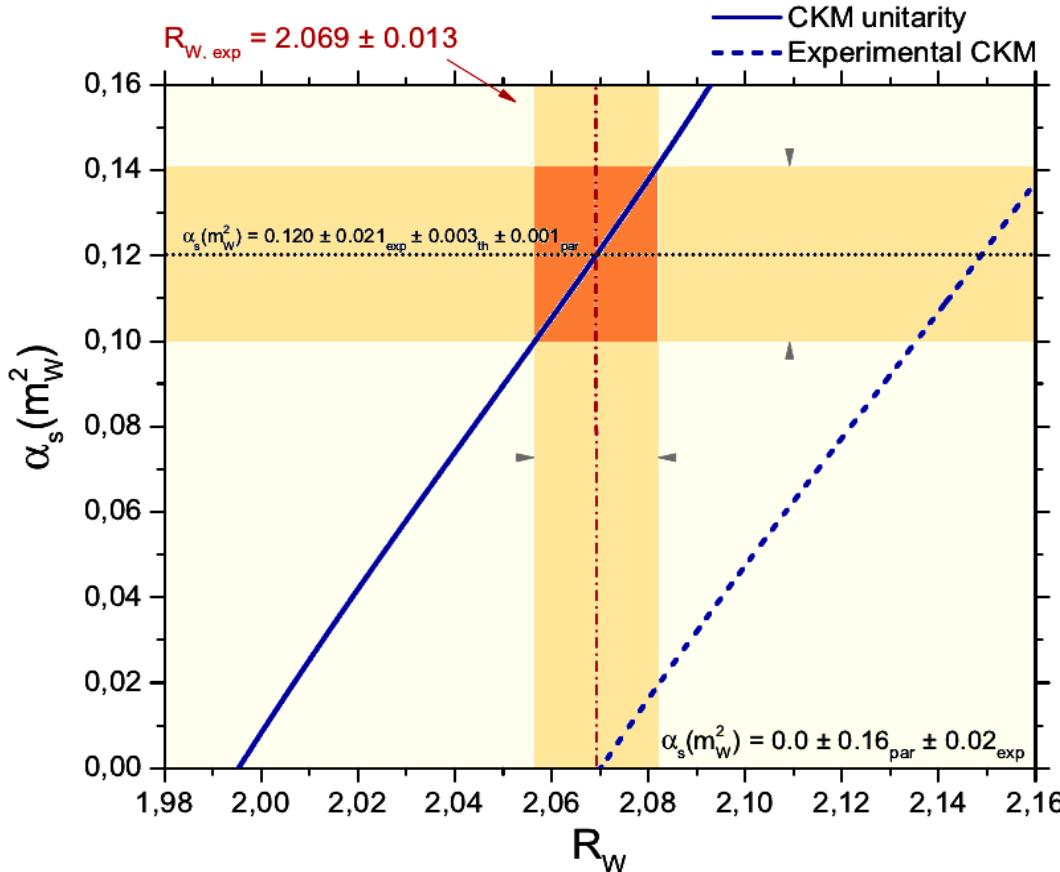
→ Careful evaluation of parametric ( $V_{i,j}$ ,  $m_W$ ) & theoretical uncertainties:

- Parametric uncertainty:  $\pm 22.40$  MeV (dominated by  $V_{cs}$ )
- Higher-order QCD corrections:  $\pm 0.96$  MeV (CKM unitarity, dominated by  $m_W$ )
- Higher-order EWK, mixed corrections:  $\pm 0.035$  MeV (from D.Kara NPB877(2013)683)
- Others (non-pQCD  $(\Lambda_{\text{QCD}}/m_W)^4$ , finite q masses above LO, ren. scheme,...): negligible

# $\alpha_s$ from hadronic W decays (today)

DdE, Srebre:arXiv:1603.06501

- Extract  $\alpha_s$  by comparing the theoretical hadronic branching ratio formula to the experimental world average value



⇒ Setting the CKM matrix to unit matrix instead of using experimental values, we can extract  $\alpha_s$  with 35% uncertainty.

To extract  $\alpha_s$  with a higher precision:

- reduce the uncertainty of  $|V_{cs}|$  as mentioned earlier,
- reduce the uncertainty of  $m_W$  (measured to 0.02%) which becomes dominant once the  $|V_{cs}|$  uncertainty is reduced below 0.05%,
- measure  $\text{BR}_W(\text{hadronic})$  with a better precision than today (0.4% now).

$\alpha_s$ extraction method	$\alpha_s(m_W^2)$	$\alpha_s(m_Z^2)$
$\mathcal{B}_{\text{had}}^W$ (experimental CKM)	$0.0 \pm 0.04_{\text{exp}} \pm 0.16_{\text{par}}$	$0.0 \pm 0.04_{\text{exp}} \pm 0.16_{\text{par}}$
$\mathcal{B}_{\text{had}}^W$ (CKM unitarity)	$0.119 \pm 0.042_{\text{exp}} \pm 0.004_{\text{th}} \pm 0.001_{\text{par}}$	$0.117 \pm 0.042_{\text{exp}} \pm 0.004_{\text{th}} \pm 0.001_{\text{par}}$

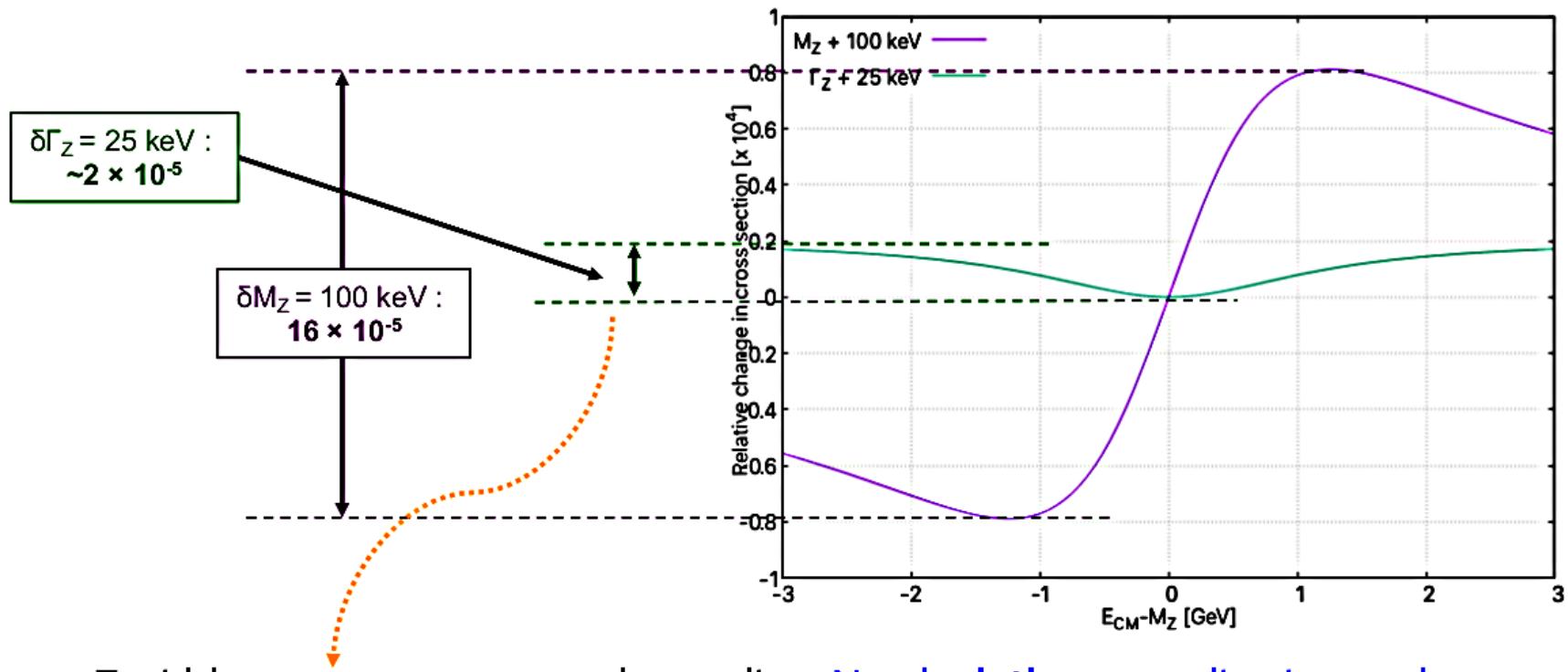
# FCC-ee (91 GeV) syst. uncertainties

- ◆ FCC-ee goal: Via Z line-shape scan, determine Z parameters to precisions:

$$\delta M_Z = 100 \text{ keV}; \quad \delta \Gamma_Z = 25 \text{ keV}$$



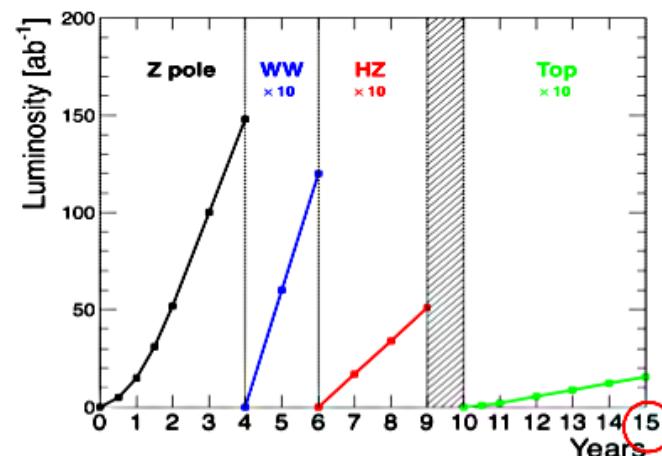
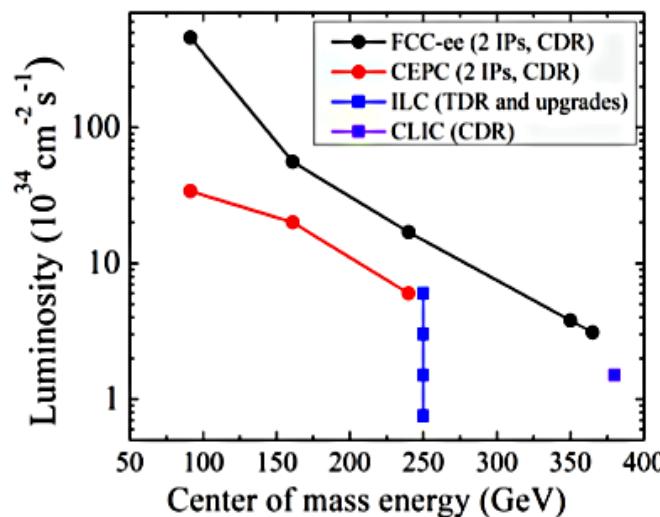
- Plot shows relative change in cross section across Z resonance for parameter variation of this size



- ◆ Z width measurement most demanding: Need **relative** normalisation to about  $10^{-5}$ 
  - Need statistics of order  $10^{10}$
  - Need careful control of energy dependent effects

# FCC-ee Luminosity, Operation, Data samples

Largest luminosities in the 88 – 365 GeV energy range



Event statistics		$\sqrt{s}$ precision
$5 \times 10^{12} e^+e^- \rightarrow Z$	$10^8 e^+e^- \rightarrow W^+W^-$	$100 \text{ keV}$
$10^8 e^+e^- \rightarrow W^+W^-$	$10^6 e^+e^- \rightarrow HZ$	$300 \text{ keV}$
$10^6 e^+e^- \rightarrow HZ$	$10^6 e^+e^- \rightarrow tt$	$1 \text{ MeV}$
$10^6 e^+e^- \rightarrow tt$		$2 \text{ MeV}$

Working point	Z, years 1-2	Z, later	WW	HZ	tt threshold...	... and above
$\sqrt{s} (\text{GeV})$	<b>88, 91, 94</b>		<b>157, 163</b>	<b>240</b>	<b>340 – 350</b>	<b>365</b>
Lumi/IP ( $10^{34} \text{ cm}^{-2} \text{s}^{-1}$ )	<b>100</b>	<b>200</b>	<b>25</b>	<b>7</b>	<b>0.8</b>	<b>1.4</b>
Lumi/year (2 IP)	<b>24 ab<sup>-1</sup></b>	<b>48 ab<sup>-1</sup></b>	<b>6 ab<sup>-1</sup></b>	<b>1.7 ab<sup>-1</sup></b>	<b>0.2 ab<sup>-1</sup></b>	<b>0.34 ab<sup>-1</sup></b>
Physics goal	<b>150 ab<sup>-1</sup></b>		<b>10 ab<sup>-1</sup></b>	<b>5 ab<sup>-1</sup></b>	<b>0.2 ab<sup>-1</sup></b>	<b>1.5 ab<sup>-1</sup></b>
Run time (year)	<b>2</b>	<b>2</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>4</b>

# Summary: $\alpha_s$ from hadronic EW bosons decays

## ■ World-average QCD coupling at $N^{2,3}\text{LO}$ today:

- Determined from *7 observables* with combined  $\pm 0.85\%$  uncertainty
- Impacts *all LHC QCD x-sections & decays*.
- Role *beyond SM*: GUT, EWK vacuum stability, New coloured sectors?

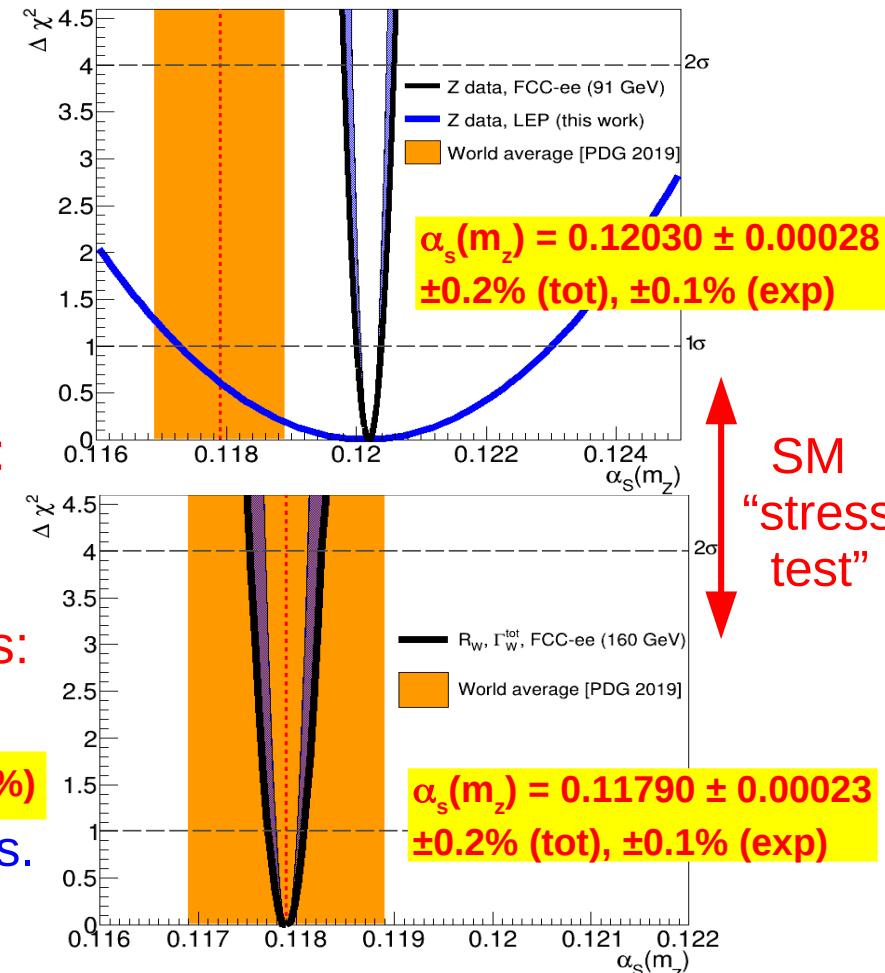
## ■ Current Z,W-based $\alpha_s(m_z)$ extractions:

- Z pseudo-observ.:  $\pm 2.3\%$  (mostly exp.)
- W pseudo-observ.:  $\pm 35\%$  (mostly exp.)

## ■ Updated Z,W-based $\alpha_s(m_z)$ extractions:

- Z boson: New fit with HO EW corrs. + corr. LEP data:  $\alpha_s(m_z) = 0.1203 \pm 0.0028$  ( $\pm 2.3\%$ )  
 $\pm 2.3\%, 0.5\%, 0.1\%$  (exp., th., par.) uncerts.
- W boson: New  $N^3\text{LO}$  fit to  $R_W, \Gamma_W^{\text{tot}}$   
 $\pm 47\%, 27\%, 1\%$  (par., exp., th.) uncerts.  
 $\alpha_s(m_z) = 0.101 \pm 0.027$  ( $\pm 27\%$ )

## ■ Future: Permil uncertainty possible with a machine like FCC-e<sup>+</sup>e<sup>-</sup>



Detector improvements under study to bring propagated syst. uncert. on W,Z pseudo-observ. below 0.1%