Wavefront Curvature in Atomic Beam Clocks

arXiv:2212.00308

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Quantum Research Network
Next Generation Technologies Fund

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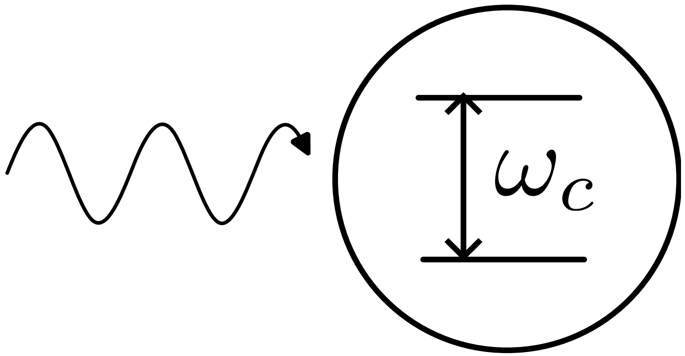
Optical Atomic Clocks

- **Most accurate frequency standards (at lab scale)**
- **Make applicable – compact, portable, cheap**
- **Optimise system subject to constraints**



Our work:

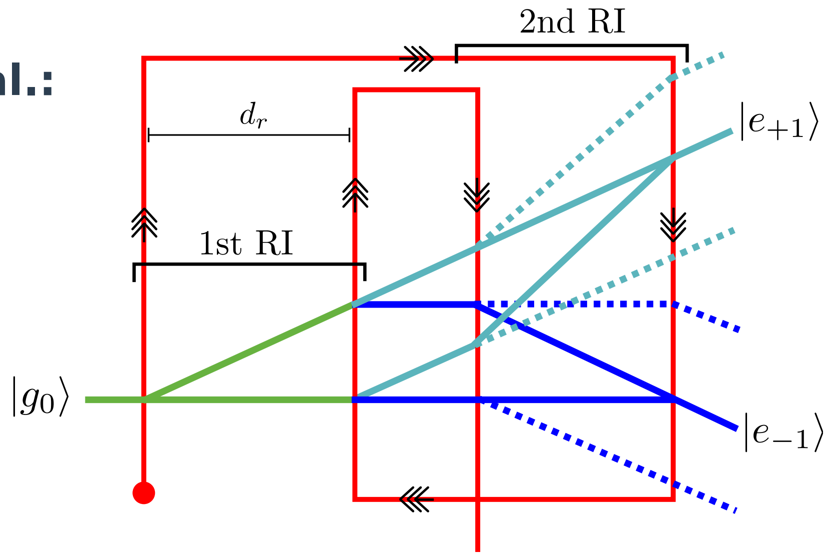
1. **Model for laser wavefront curvature in thermal beam clocks**
2. **Optimisation of laser parameters to maximise Fisher information of clock signal**
3. **Analysis of frequency shifts/instability of the clock**

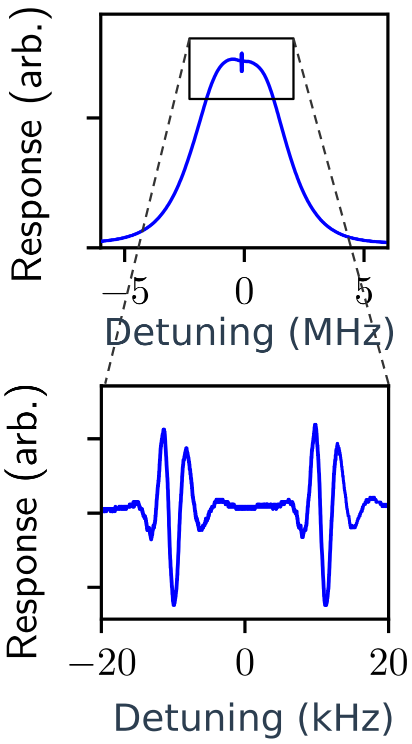
Ramsey-Bordé Interferometry

Promising compact atomic beam clock architecture

Demonstrated by Olson et al.:

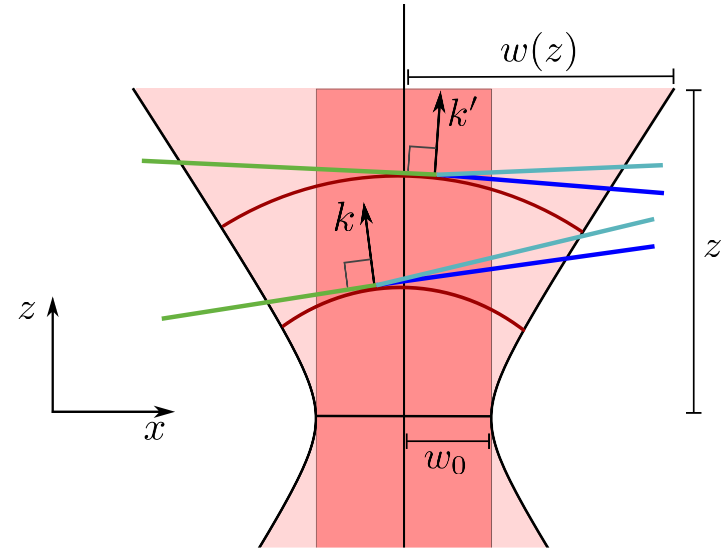
- **Calcium 657nm transition**
- **10^{-16} stability**
- **Sensitivity to laser geometry/collimation**
- **Optical path ~1m**
- **Rayleigh range ~0.5m**

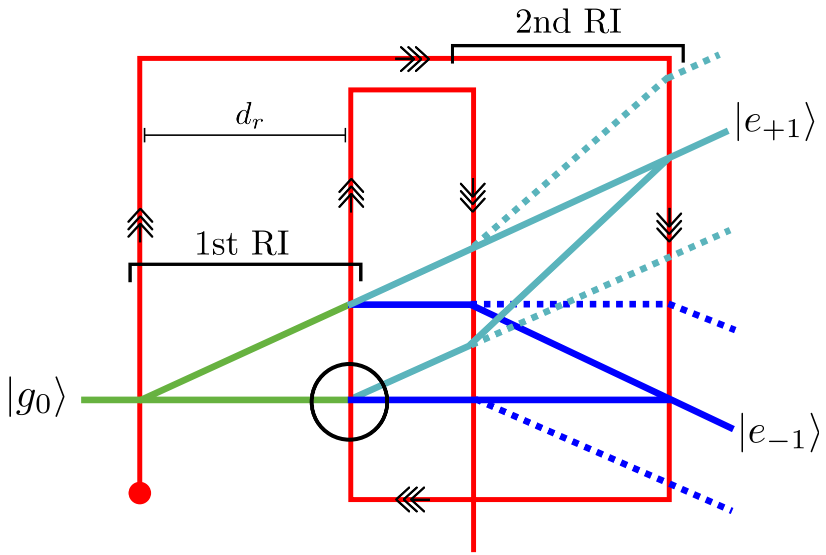


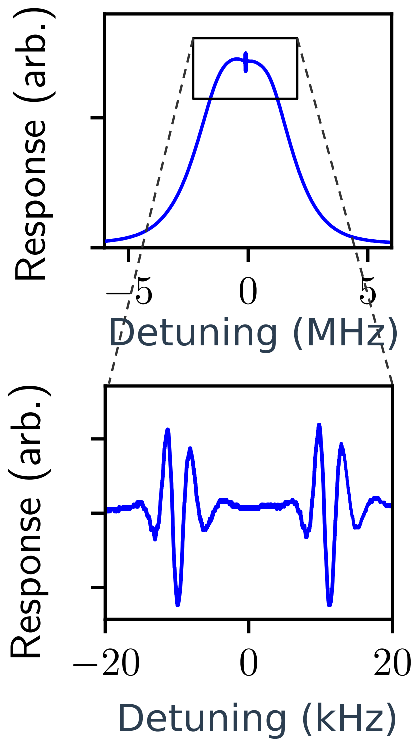


Olson et al., Phys. Rev. Lett. 123, 073202 (2019)

Ramsey-Bordé Interferometry







Olson et al., Phys. Rev. Lett. 123, 073202 (2019)

Gaussian Laser Model

Using Magnus expansion we find:

Effective pulse area

Constant transit time

$$\tau_0 = w_0/v_x$$

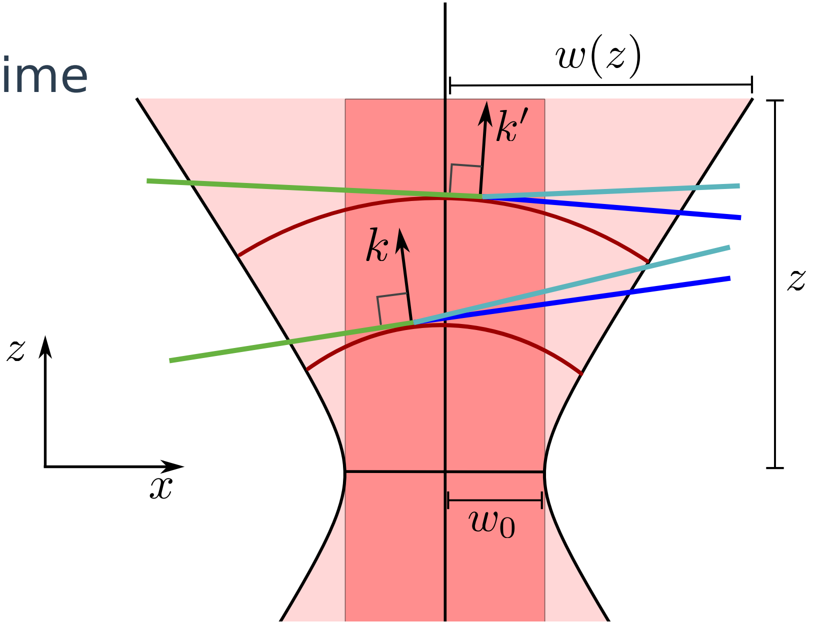
$$\Phi \propto \sqrt{\frac{w_0}{w(z)}} \exp\left(-\frac{1}{4}\Delta^2\tau_0^2\right)$$

Gouy phase

Effective laser phase

$$\psi = kz\left(1 - \frac{1}{2}\frac{\Delta^2}{k^2 v_x^2}\right) - \frac{1}{2}\arctan\left(\frac{z}{z_r}\right)$$

$$\approx zk\cos\left(\frac{v_z}{v_x}\right)$$ on resonance

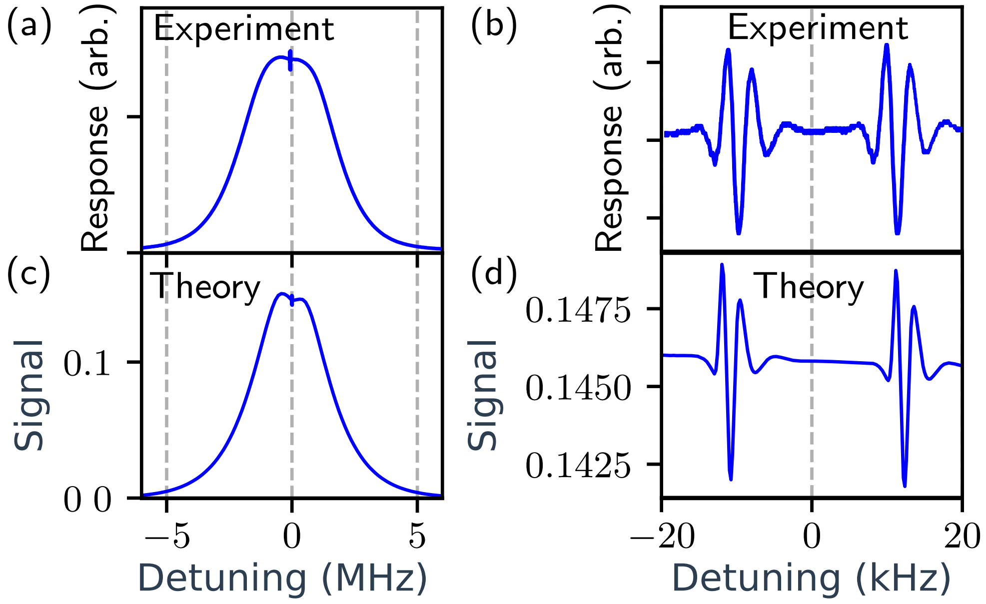


Comparison with Olson et al.

Qualitative agreement with experiment

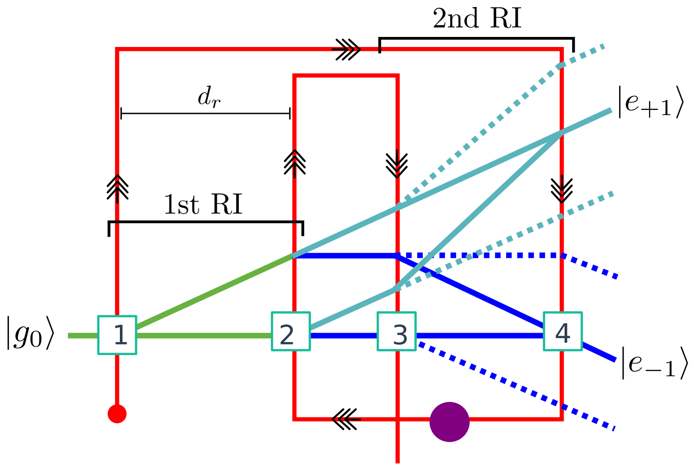
Improve fit with:

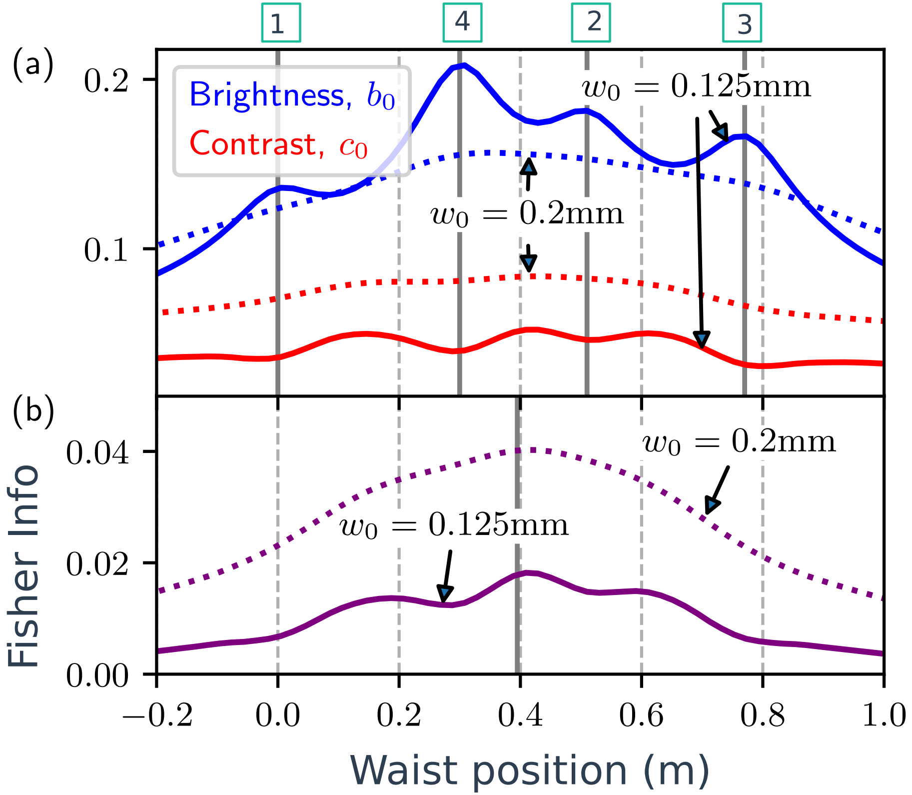
- Better velocity distributions
- Account for measurement protocol
- More realistic laser profile



Optimising Waist Position

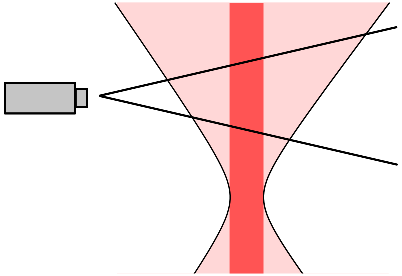
- Locating clock transition requires large background (brightness) and high interference fringe visibility (contrast)
- Quantify using Fisher information – how much info about clock transition in the signal



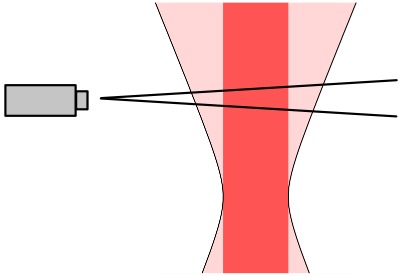


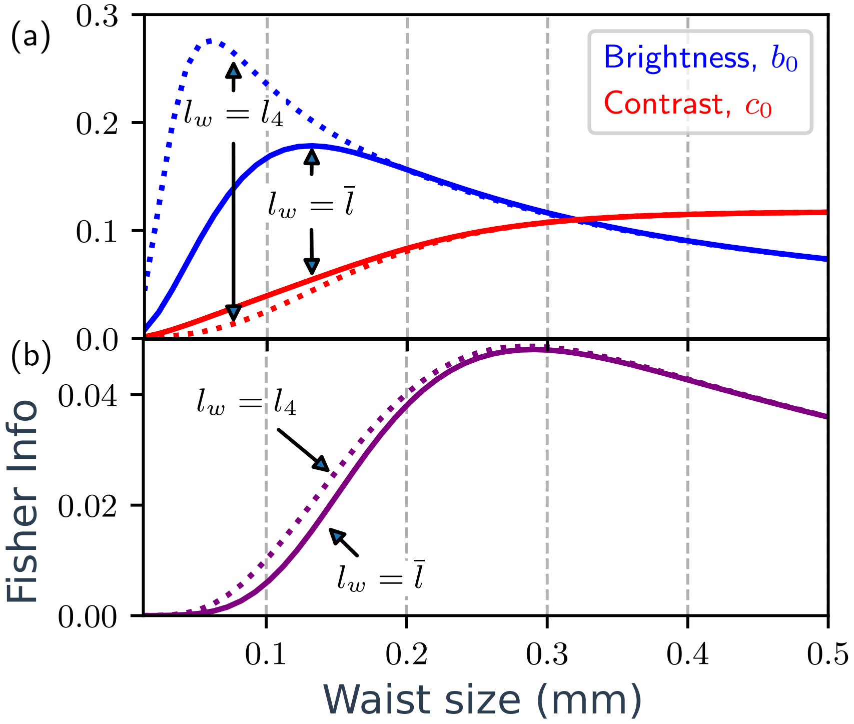
Optimising Waist Size

Small waist:
Low excitation probability
More atoms contribute



Large waist:
High excitation probability
Fewer atoms contribute



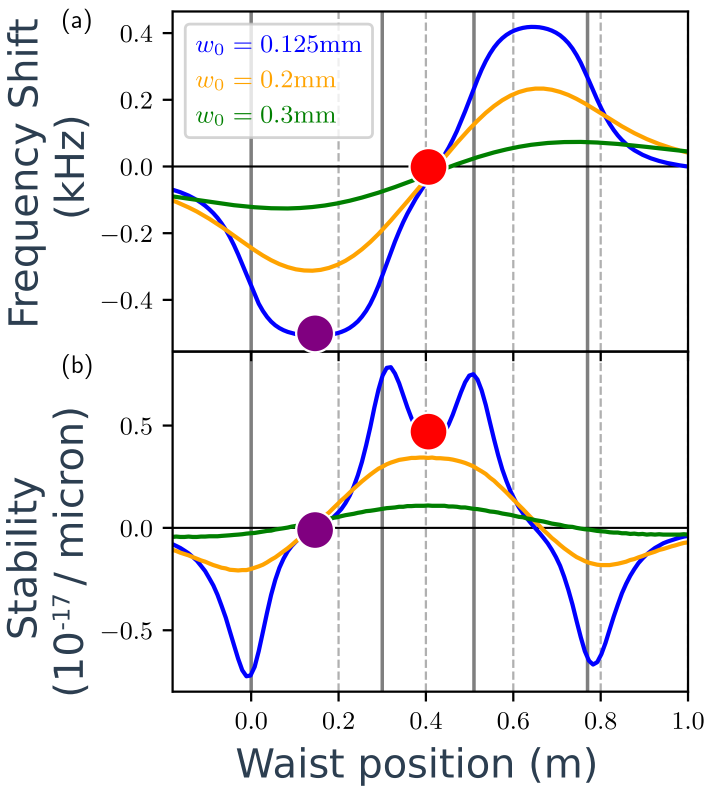


Frequency Shifts and Stability

Gouy phase is dominant source of shift ~ 0.5 kHz

Micrometer fluctuations in waist position give ~10^{-17} fractional stability

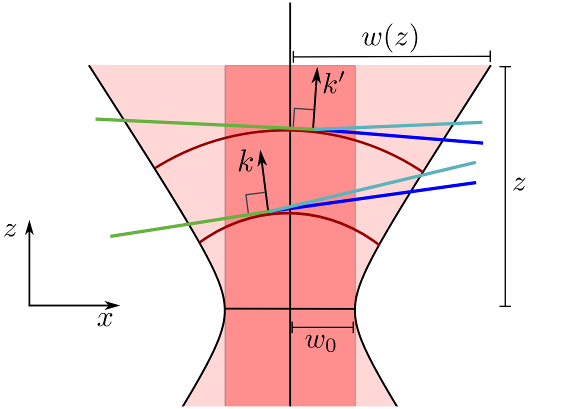
- Accurate and short term stability
- Long term stability

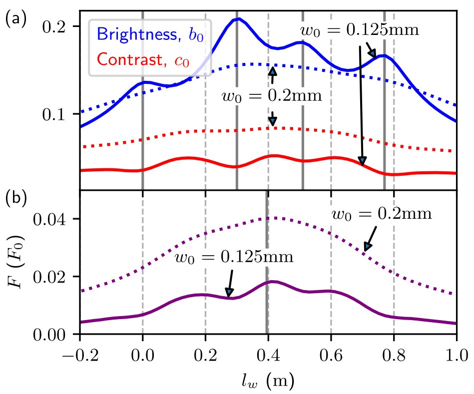


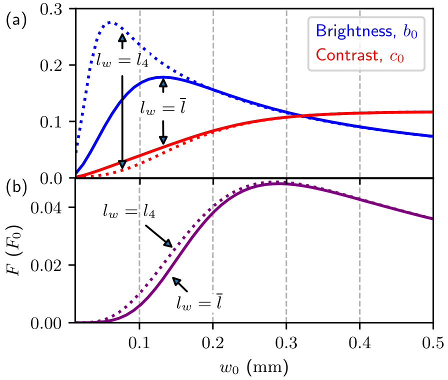
Thanks for Listening! arXiv:2212.00

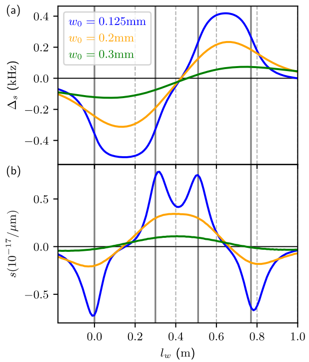
Summary:

1. **Intuitive description of laser wavefront curvature in beam clocks – predictions consistent with experiment**
2. **Optimised position and size of waist to maximise Fisher information**
3. **Shifts/instability of clock frequency dominated by Gouy phase**









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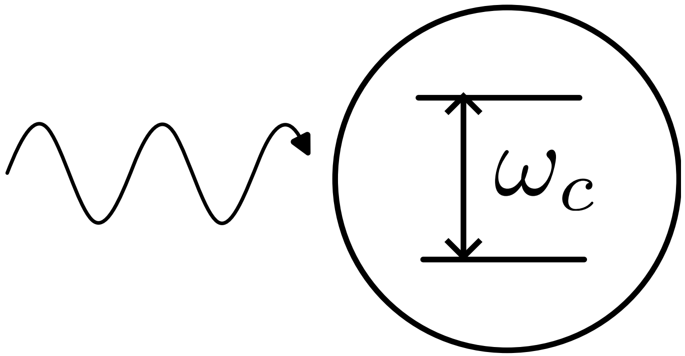

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Ramsey-Borde Interferometry

- **Promising compact atomic beam clock**
- **Demonstrated by Olson et al. - ~10^{-16} stability**
- **Sensitivity to laser geometry/collimation**
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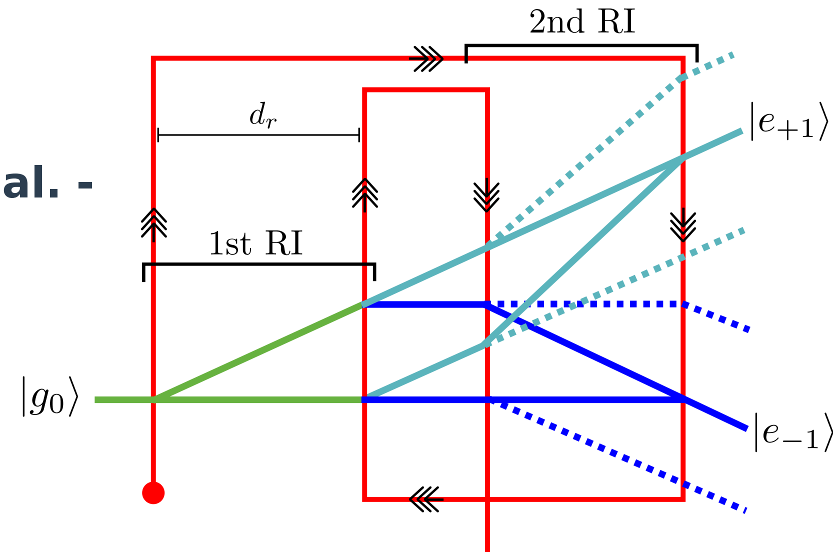
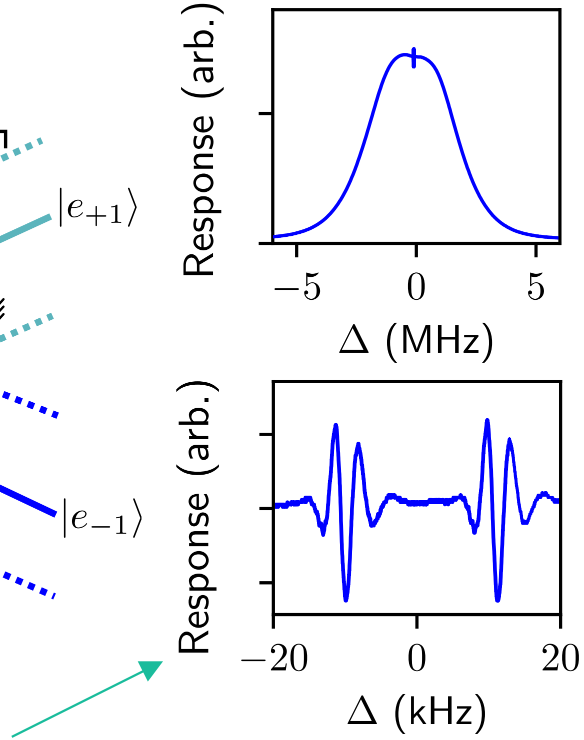
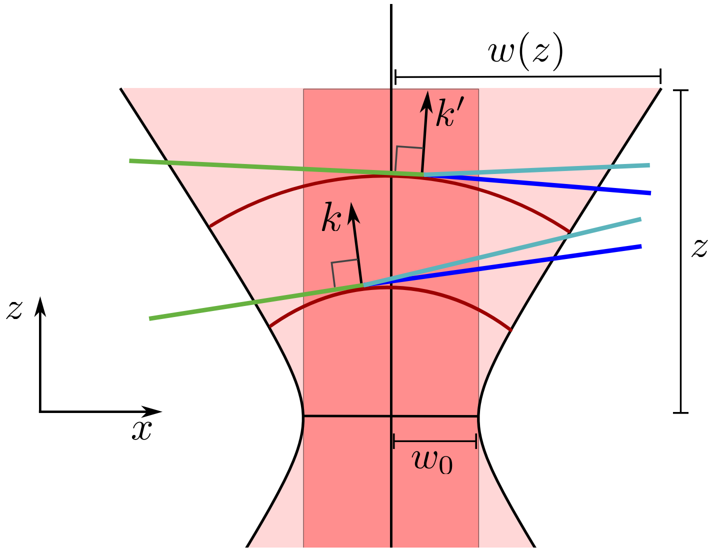
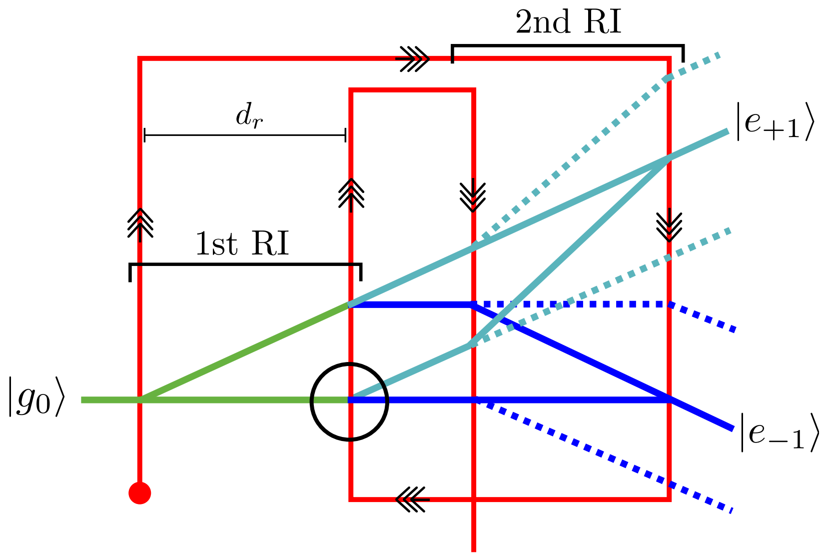


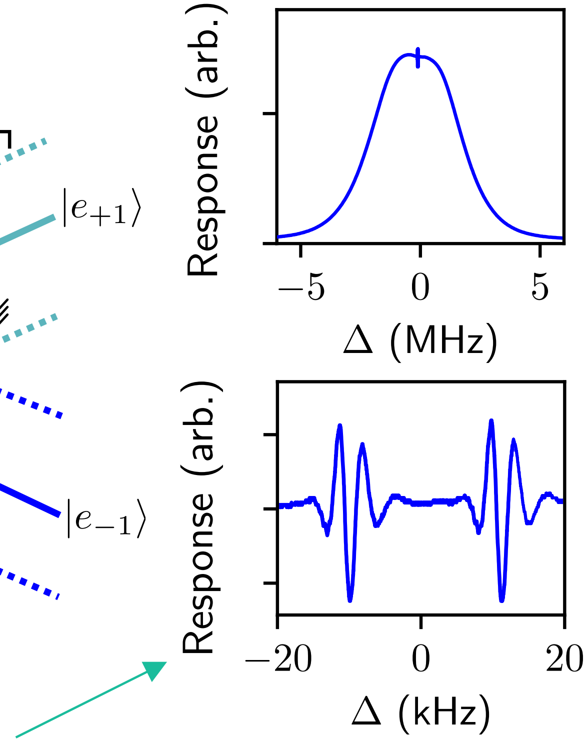


Olson et al., Phys. Rev. Lett. 123, 073202 (2019)

Ramsey-Borde Interferometry







Olson et al., Phys. Rev. Lett. 123, 073202 (2019)

Gaussian Laser Model

$$\Delta \propto k v_z$$

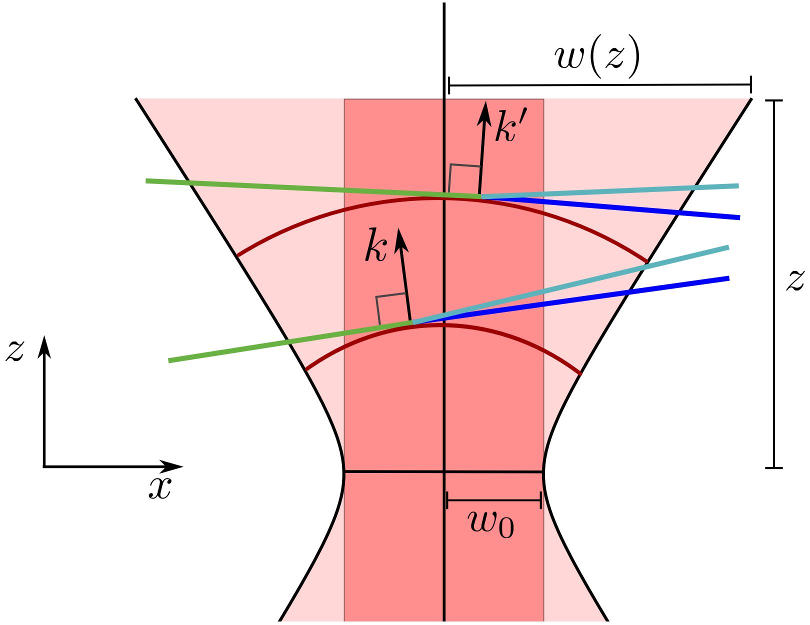
$$H(t) = \Omega(t) e^{-i\Delta t - i\phi(t)} \left|e\right\rangle \left\langle g\right| + h.c.$$

$$\Omega(t) = \frac{v_m A}{2\sqrt{\pi} w(z)} \exp\left(-\frac{v_x^2 t^2}{w(z)^2}\right)$$

Target pulse area

$$\phi(t) = kz + k\frac{v^2 t^2}{2R} - \mathrm{atan}(z/z_R)$$

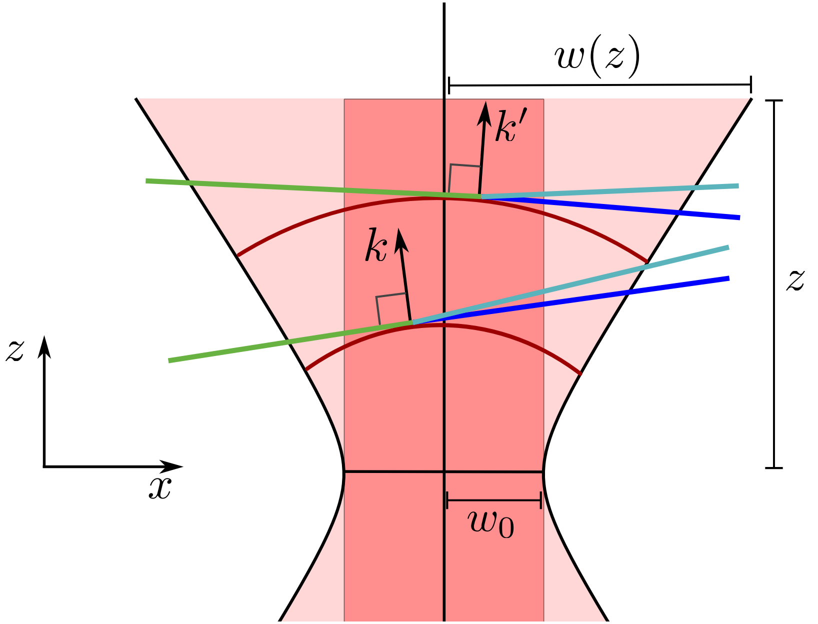
Wavefront radius of curvature



Gaussian Laser Model

Magnus expansion:

$$\log(U) \approx -i \int_{-\infty}^{\infty} H(t)dt$$



Gaussian Laser Model

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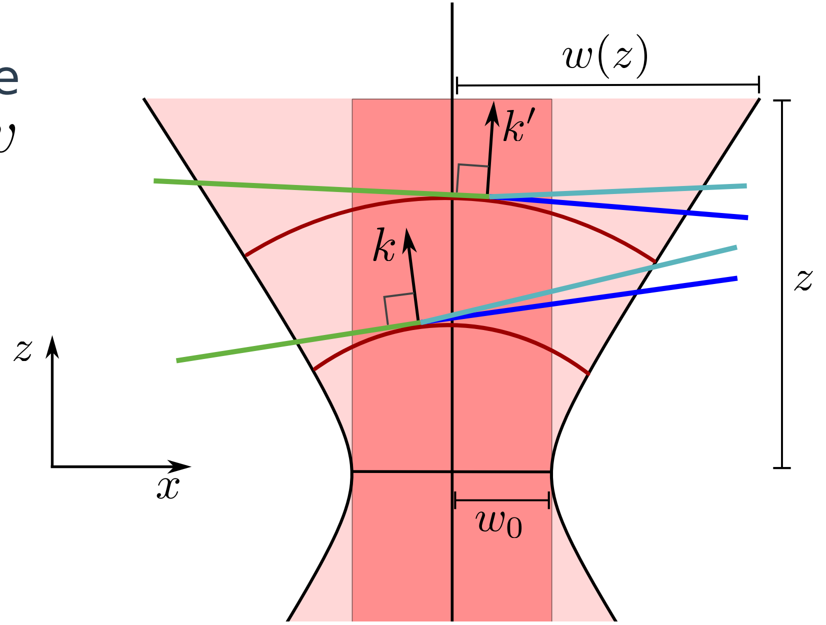
Effective pulse area:

$$\Phi = \frac{A}{2}\frac{v_m}{v_x}\sqrt{\frac{w_0}{w(z)}}\exp\left(-\frac{1}{4}\Delta^2\tau_0^2\right)$$

Transit time

$\tau_0 = w_0/v$



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Transit time
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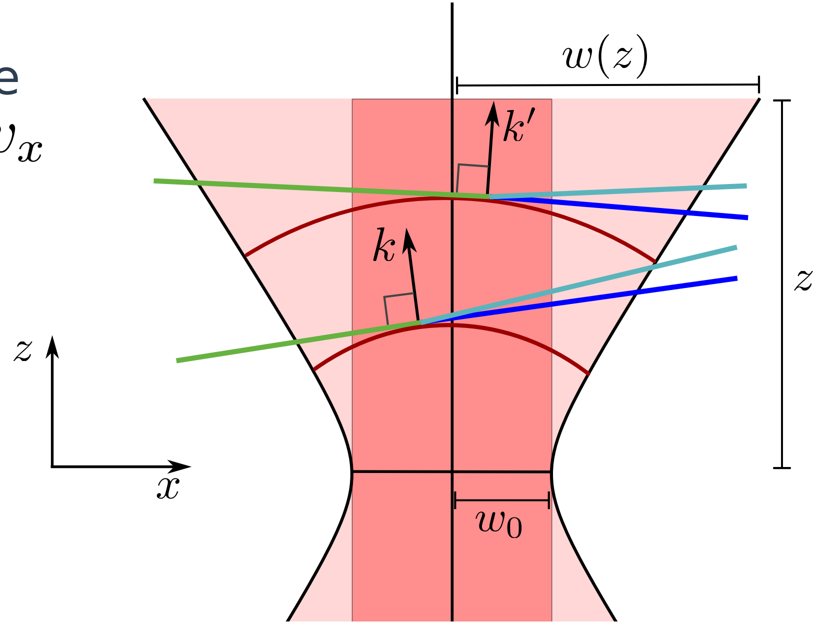
Effective laser phase:

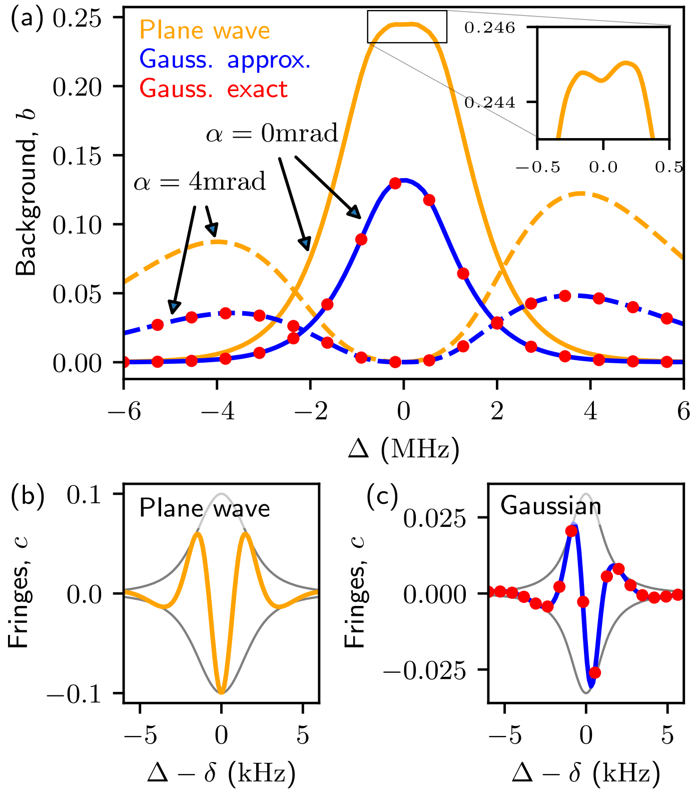
$$\psi = kz\left(1 - \frac{1}{2}\frac{\Delta^2}{k^2 v_x^2}\right) - \frac{1}{2}\arctan\left(\frac{z}{z_r}\right)$$

$$\approx zk\cos\left(\frac{\Delta}{kv_x}\right) \approx zk\cos\left(\frac{v_z}{v_x}\right)$$



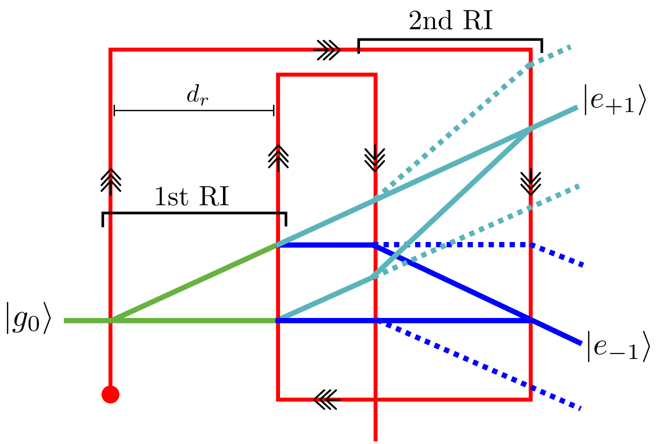
Comparison with Plane Wave

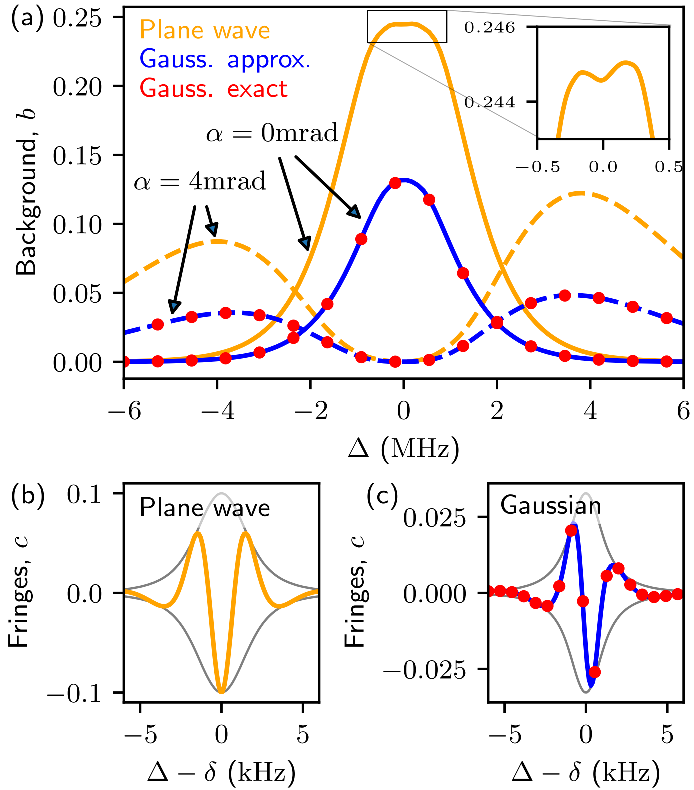
- Numerically average of atomic velocities to get signal
- Inhomogeneity of pulse area reduces background and fringe amplitude compared to plane wave
- Reduced pulse area in Gaussian laser excites lower velocity atoms – narrower envelope
- Frequency shift compared to plane wave
- Approximate analytics are essentially exact



Comparison with Plane Wave

- Intrinsic asymmetry – upper recoil atoms Doppler shifted (inset)
- Misalignment asymmetry – tilting atomic beam by α splits background peak





Comparison with Olson et al.

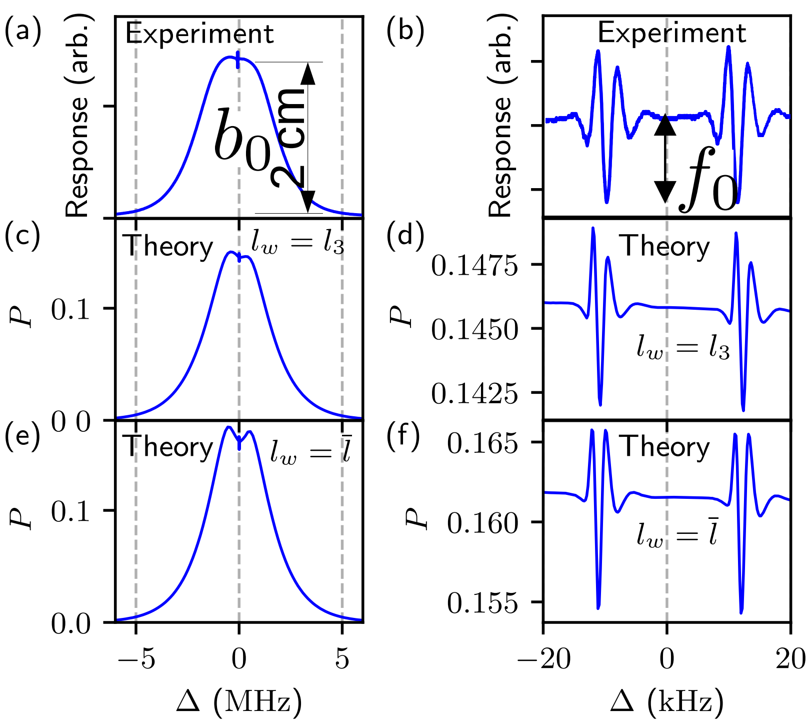
Misalign atomic beam to reproduce asymmetry

Qualitative agreement but lower contrast:
Experiment: $c_0 \sim 0.07$ - 0.09
Theory: $c_0 \sim 0.01$ - 0.03

We find sensitivity to positioning of the waist as reported in Olson et al., Phys. Rev. Lett. 123, 073202 (2019)

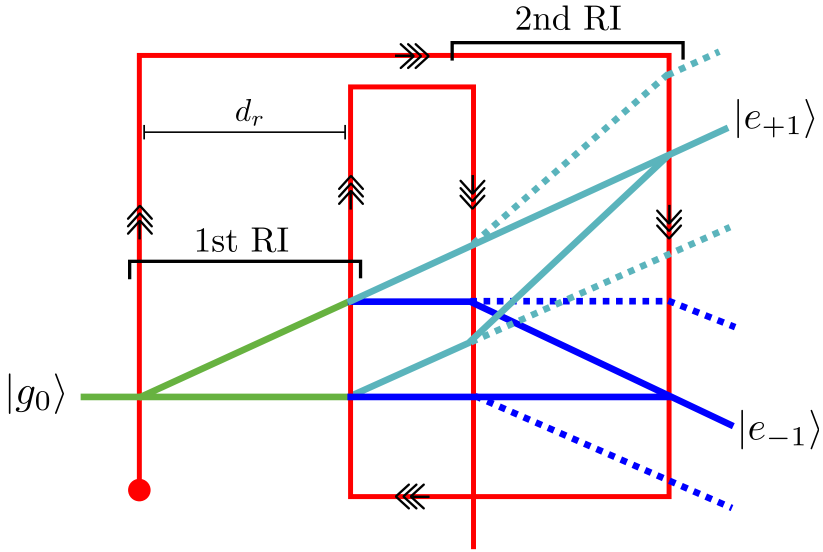
Potentially improve the fit by better modeling the measurement process, velocity distributions, laser field profile

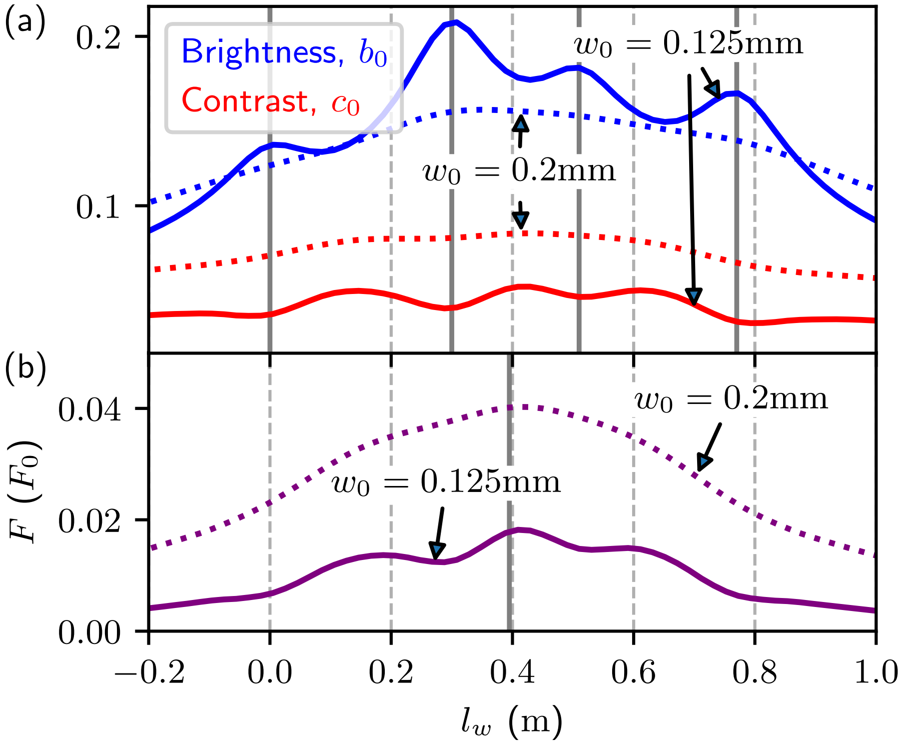
Fringe Contrast: $c_0 = f_0/b_0$



Optimising Waist Position

Peaks in brightness and dips in contrast when laser is focused at interaction zones





Optimising Waist Position

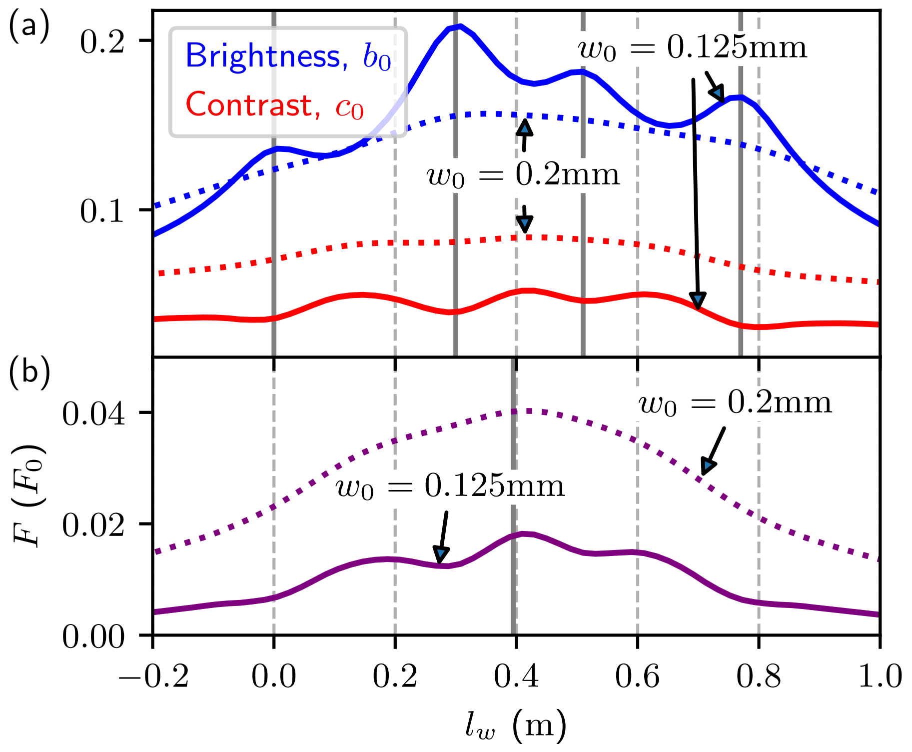
A good signal should have high brightness and high contrast

Quantify using Fisher information – information available about the parameter we are trying to estimate (detuning Δ) given noisy data (shot noise)

Cramer-Rao bound: $\sigma_\Delta \geq 1/F$

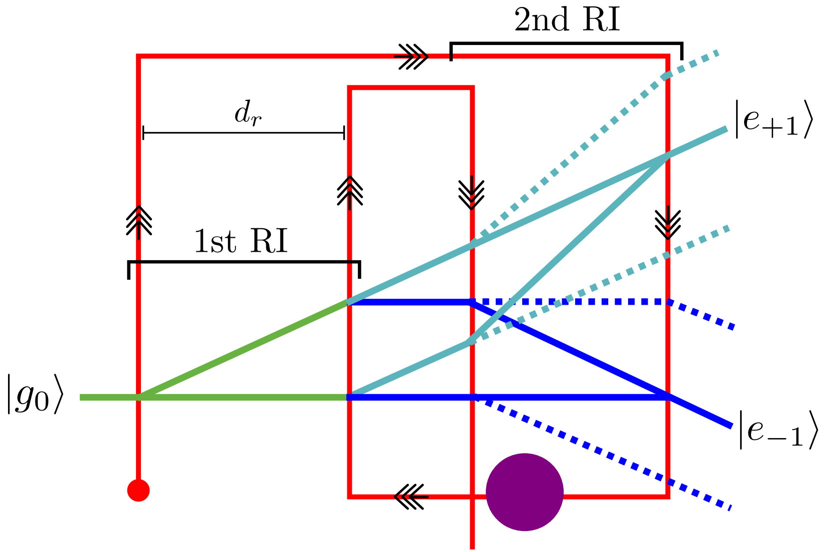
At central fringe:

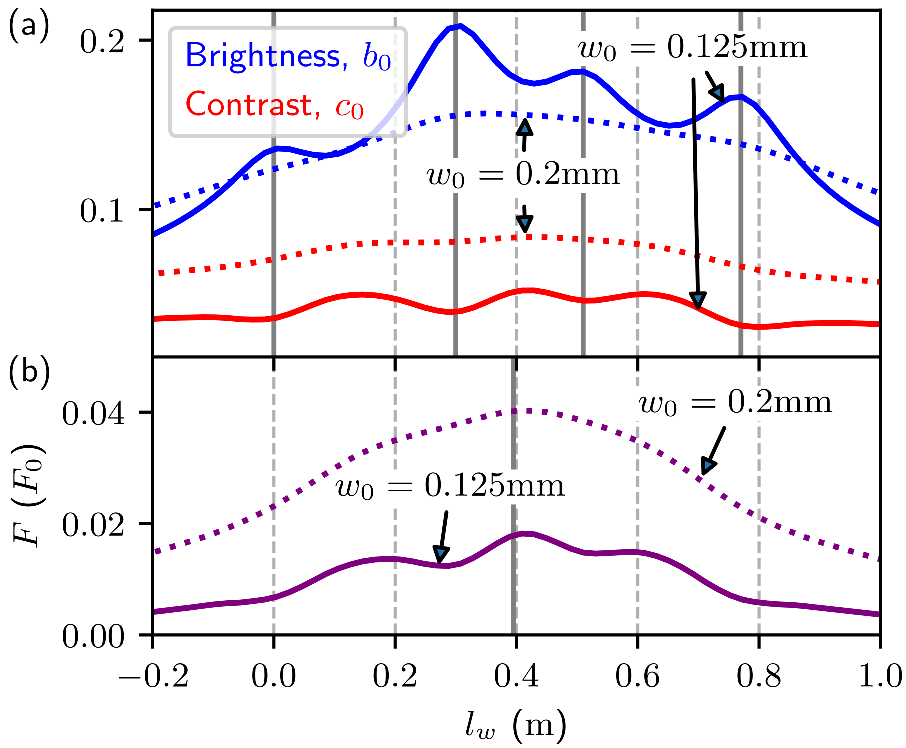
$$F \propto \frac{b_0 c_0^2}{1 + c_0}$$



Optimising Waist Position

Fisher information maximised when waist is positioned near symmetric point between interactions

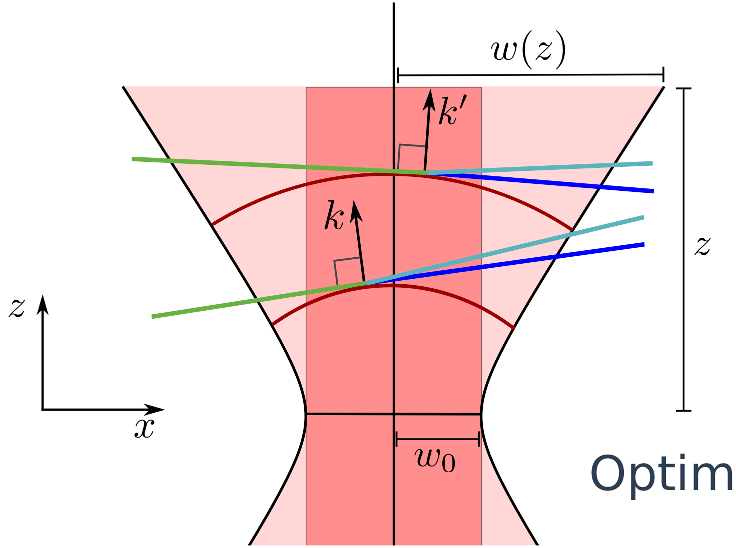


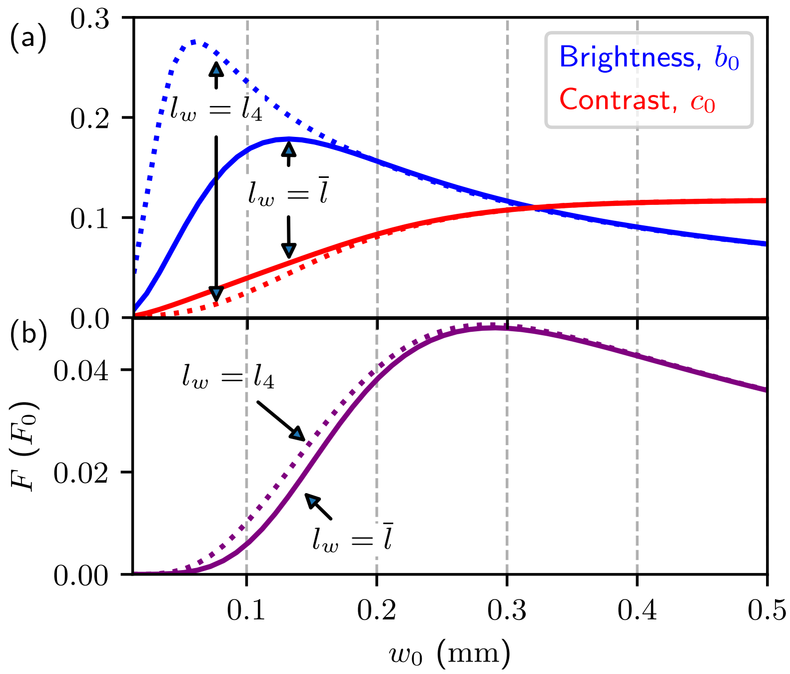


Optimising Waist Size

Small waist – small transit time but small portion of beam with flat wavefronts

Large waist – flat wavefronts but large transit time, only atoms with small transverse velocity are excited





Optimal waist size similar to that used by Olson et al.

Frequency Shifts and Stability

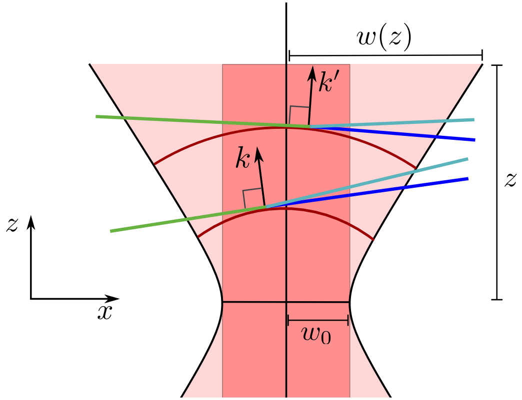
Fringe phase:

$$\theta = 2T\left(1 + \frac{l_s}{d_r}\frac{v_z}{v}\right)(\Delta - \delta) + kl_s\left(1 - \frac{v_z^2}{2v^2}\right) + \frac{g_s}{2}$$

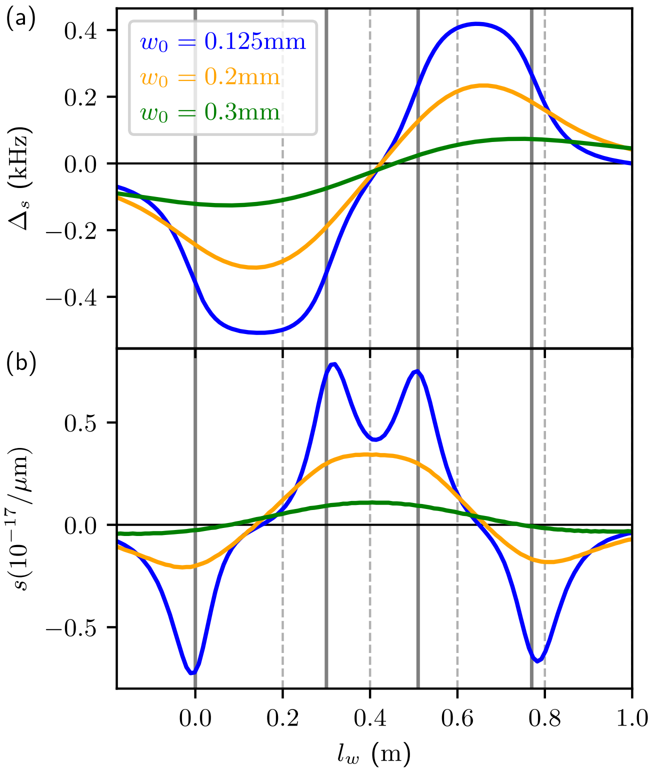
Time of flight correction

Guoy phase

Spatial phase



Guoy phase dominates
~ 0.5kHz shifts



Frequency Shifts and Stability

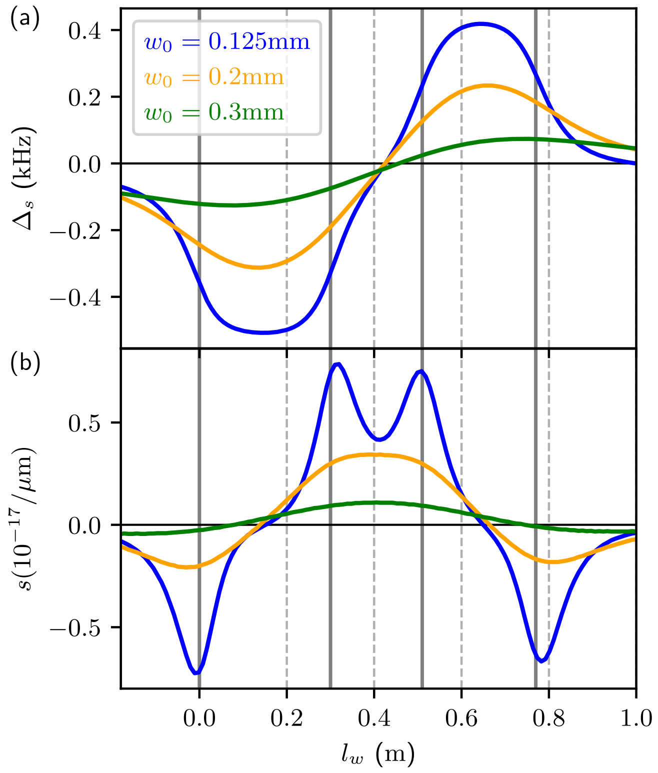
Stability of frequency shifts fluctuation in waist position

$$s = \frac{1}{\omega_c} \frac{d\Delta_s}{dl_w}$$

Micrometer fluctuations in position give ~10^{-17} fractional instability

Frequency shifts minimised when Fisher info maximised but with instability

Instability can be suppressed but Fisher info is suboptimal



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