GPS from the ground up - a novel pedagogy for understanding general relativity

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Abstract In this paper we present a novel approach for learning relativity by combining theory with application to the GPS system. The course starts with 2 and 3 dimensional vector theory and moves through to spacetime and special relativity (SR), then to postgraduate level tensors and an introduction to general relativity (GR). A study of the GPS systems includes theoretical physical considerations as well as graded practical construction of electronic GPS receiver modules. The course is applicable as a practical introduction to the applied mathematics of relativistic measurements.

1. Introduction While there is a plethora of literature for teaching the mathematical physics of relativity theory, we believe there is a paucity of manageable, graded and practical training pedagogies in the area. Many texts are excellent in a particular aspect of the theory but generally there is no connected journey from high-school levels through to the general theory. In contrast the outstanding tome "Gravitation" by Misner, Thorne & Wheeler [1], is a complete work this area, but is daunting in its size and detail. We present a workable alternative as an entry point to the area. The project follows from the development of the application of algebraic software developed in a doctoral thesis by the main author [2].

2. The course outline The entry levels introduce 2-, 3- & 4-dimensional vectors and move through to the main features of special relativity. In each level there are standardised problems with solutions, and testing of the students comprehension of the concepts and techniques. (For an example, see Figure 1) SR is worked through next then the theory of tensors and covariant formalism leading to GR.

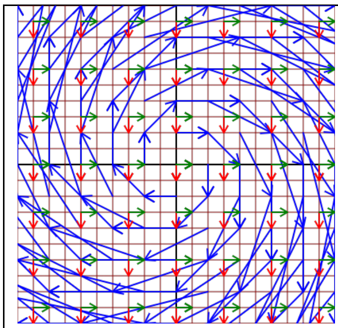

Figure 1. The vector field $\mathbf{w}=w_1\mathbf{i}-w_2\mathbf{j} = y\mathbf{i}-x\mathbf{j}$ is initially shown shown as **blue** arrows. The student is asked to calculate the curl of **w**. Once answered, the differential vector fields for $\partial\mathbf{w}/\partial x$ and $\partial\mathbf{w}/\partial y$ are shown in **red** and **green** respectively. The curl is calculated as $\partial \mathbf{w}_1/\partial y - \partial \mathbf{w}_2/\partial x = 1-(-1)=2$.

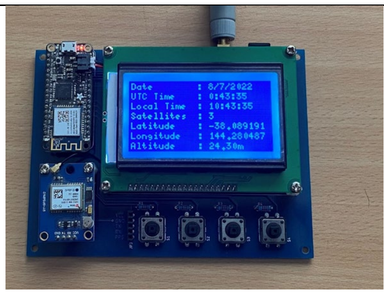

Figure 2. A simple GPS portable module with wifi and LCD display. This is version 4 of the modules constructed in the course.

Most of the syllabus is guided by what is required to effectively understand the mathematics of the GPS system and the relativistic effects on satellite clock rate. There are two main effects: SR effects slow the clock rate while the stronger effect of GR speeds up clock rate, compared to an Earth-bound clock. The importance of these parameters, and the solutions of overcoming the problem of clock-rate mis-match is worked through combining theory and practical outcomes.

The final important aspect of the course is the practical application: GPS receivers are constructed, initially as simple kits but later with increasing complexity and electronic capability (see Figure 2). As a precursor to the practical application there are general modules on the application of electromagnetic radiation (EMR) with respect to the GPS system.

[1] Misner, C, Thorne, KS , Wheeler, JA 1973, *Gravitation*, Freeman, San Francisco

[2] Huf, PA, 2019, Algebraic computing and tensors in general relativity. PhD thesis. Deakin University.